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THE DRELL-YAN CONTRIBUTION TO MASSIVE LEPTON-PAIR PRODUCTION*

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ABSTRACT

The Drell-Yan contribution to the massive lepton-pair production in hadron-hadron scattering is calculated using the parton distributions extracted from deep inelastic eN and νN scattering data. The calculated cross sections at BNL energy agree well with the measured data.

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The impulse approximation as applied to elementary particles interactions has been shown to be rather successful in deep inelastic scattering; however, it seems to break down in the low energy annihilation process, especially in the small $\bar{\bar{x}} \equiv \frac{2p\cdot q}{Q^2}$ region. Theoretically, it should be valid only when the partons can be treated as instantaneously free, and when the energy is approximately conserved along with momentum across the interaction vertex of the partons.[1] For these reasons, the fraction $x = Q^2/2p\cdot q$ of the longitudinal momentum must be <u>finite</u> while Q^2 and $2p\cdot q$ must be large in order to apply this approximation. In high energy hadron-hadron scattering we have no long-lived large mass incident particle $Q^2 = M^2$ while $2p\cdot q \rightarrow \infty$ then x is always small or "wee". This approximation will break down. However, when we apply this approximation to the production of lepton-pair created from $q\bar{q}$ annihilation in hadron-hadron interactions (the Drell-Yan mechanism) we can create any arbitrarily large mass Q^2 and large energy $2\,p\cdot q$ (same as deep inelastic eN and e^+e^- annihilation), and the impulse approximation is again valid. In this paper, we performed the elementary calculation of the Drell-Yan mechanism in hadron-hadron interactions, and we compared the predicted cross sections with the BNL data.[2] We did not consider the detailed and subtle points like colors, the experimental value of $R = \frac{\sigma_{hadrons}}{\sigma_{\mu\mu}}$ of e^+e^- annihilation into the calculation. These will change the results by trivial factors which could be introduced by hand by the readers. All the partons are assumed to be pointlike. Due to the limited data in eN and νN scatterings, the uncertainty introduced in the errors in the parton distribution functions are larger than these effects of a non-pointlike parton. Furthermore, we included elementary derivations of the power laws for the sake of completeness. The calculations were done neglecting the transverse momentum of the partons and we stayed away from the kinematical limit where $Q^2_{max} = (\sqrt{s} - M_{target} - M_{projectile})^2$. The parton distributions are lifted from the recent article by R. Blankenbecler et al. We define $f_i^A(x)$ the probability of finding a parton of type i and charge e_i carrying a fraction x of the hadron A; $\xi \equiv \frac{2Q_z}{\sqrt{s}}$ is the fraction of the longitudinal qM momentum of the lepton pair. Following G. Farrar,[3] the differential cross section for a parton q_i having longitudinal momentum $\frac{x\sqrt{s}}{2}$ to annihilate with an antiparton $\bar{q}_i$ of longitudinal momentum $-\frac{\bar{x}\sqrt{s}}{2}$ producing the virtual photon of mass Q^2 weighted by the probability distribution of finding q_i with a fraction x in A, $q_{\bar{i}}$ with fraction $\bar{x}$ in B is

$$\frac{d\sigma}{dQd\xi} = \frac{4\pi\alpha^2}{3Q^2 s} \circ 2Q \int_0^1 dx \int_0^1 d\bar{x}\, \delta(x\bar{x} - \frac{Q^2}{s})\, \delta(x - \bar{x} - \xi)\, G'(x,\bar{x},\xi)$$

where $$G'(x,\bar{x},\xi) \equiv \sum_i e_i^2 \left[f_i^A(x)\, f_{\bar{i}}^B(\bar{x}) + f_{\bar{i}}^A(x)\, f_i^B(\bar{x})\right]$$

The longitudinal momentum and energy conservations are constrained by $\delta(x - \bar{x} - \xi)$ and $\delta(x\bar{x} - Q^2/s)$. Upon integration,

$$\frac{d\sigma}{dQd\xi} = \frac{4\pi\alpha^2}{3Q^4} \cdot 2Q \circ \sum_i e_i^2 \circ \frac{x^2}{x_A + x_B} \cdot \left\{\left[f_i^A(x_A)\, f_{\bar{i}}^B(x_B) + f_{\bar{i}}^A(x_A)\, f_{\bar{i}}^B(x_B)\right]\right\}$$

where

$$x_A = \frac{1}{2}\left[\xi + (\xi^2 + 4\,Q^2/s)^{\frac{1}{2}}\right]$$

$$x_B = \frac{1}{2}\left[-\xi + (\xi^2 + 4\,Q^2/s)^{\frac{1}{2}}\right]$$

The calculations were done for pp, π^+p, and π^-p. The parton distributions for the hadrons are given below.[4] For convenience, we define $u_p(x) = f_u^p(x)$, $d_p(x) = f_d^p(x)$, etc.

Proton:

$$u_p(x) = \frac{.2(1-x)^7}{x} + 1.89\frac{(1-x)^7}{\sqrt{x}} + \begin{cases} 90.2\, x^{3/2} e^{-7.5x} & x < .35 \\ 5(1-x)^3 & x \geq .35 \end{cases}$$

$$d_p(x) = \frac{.2(1-x)^7}{x} + 1.03\frac{(1-x)^7}{\sqrt{x}} + .7(1-x)\begin{cases} 90.2\, x^{3/2} e^{-7.5x} & x < .35 \\ 5(1-x)^3 & x \geq .35 \end{cases}$$

$$\bar{u}_p(x) = \bar{d}_p(x) = \bar{s}_p(x) = s_p(x) = \frac{.2(1-x)^7}{x}$$

Antiproton: same as proton with quarks replaced by antiquark.

Neutron:

$$u_n(x) = d_p(x)$$

$$d_n(x) = u_p(x)$$

$$\text{all others} = \bar{u}_p(x)$$

Positive pions:

$$u_{\pi^+}(x) = \frac{.2(1-x)^5}{x} + .94\frac{(1-x)^5}{\sqrt{x}} + \begin{cases} 45.1\, x^{3/2} e^{-7.5x} & x < .35 \\ 1.03(1-x) & x > .35 \end{cases}$$

$$\bar{d}_{\pi^+}(x) = u_{\pi^+}(x)$$

$$d_{\pi^+}(x) = \bar{u}_{\pi^+}(x) = s_{\pi^+}(x) = \bar{s}_{\pi^+}(x) = \frac{.2(1-x)^5}{x}$$

Negative pions:

$$\bar{u}_{\pi^-}(x) = d_{\pi^-}(x) = u_{\pi^+}(x)$$

$$\text{all others} = \bar{u}_{\pi^+}(x)$$

We will remind the readers of the basic scaling laws at large transverse momentum derived by Brodsky and Farrar[5] by which, via the Drell-Yan-West relation, the power behavior of the u(x) are obtained. In general, the Drell-Yan West relation connects the form factor of the hadron to the distribution of the partons within it. A well-known example is the electron scattering near the limit of elasticity.

$$\frac{d\sigma}{dt\, dM_x^2}(ep \to ex) \underset{M_x \to M_p}{\sim} \frac{d\sigma}{dt}(ep \to ep)$$

Assuming one photon domination,

$$\frac{d\sigma}{dM_x^2}(ep \to ex) \sim \frac{1}{s^3} f\left(\frac{t}{s}\right) g\left(\frac{M_x^2}{s}\right)$$

where $t = Q^2$, and

$$\frac{d\sigma}{dt}(ep \to ep) \sim \frac{1}{t^2} |F(t)|^2$$

then one obtains

$$\frac{1}{s^3} f\left(\frac{t}{s}\right) g\left(\frac{M_x^2}{s}\right) \sim \frac{1}{t^2} |F(t)|^2$$

If $F(t) \sim t^{1-n}$, one gets

$$\frac{1}{s^3} f\left(\frac{t}{s}\right) g\left(\frac{M_x^2}{s}\right) \sim t^{2(1-n)-2} = t^{-2n}$$

or

$$f\left(\frac{t}{s}\right) g\left(\frac{M^2}{s}\right) \sim \frac{s^3}{t^{2n}} = \left(\frac{s}{t}\right)^{2n} s^{3-2n}$$

Then

$$g\left(\frac{M_x^2}{s}\right) \sim \left(\frac{M_x^2}{s}\right)^{2n-3} \xrightarrow[\text{large } s]{} (1-x)^{2n-3}$$

This illustrates the more general Drell-Yan-West relation which states that if the proton form factor contribution of a n-quark state behaves as t^{1-n}, the structure function $xf(x)$ would be proportional to $(1-x)^{2n-3}$. The power laws of the parton distribution $f(x)$ listed above were derived from the scaling laws at large t. Brodsky and Farrar have shown that under quite general conditions an irreducible system of n non-wee constituents has a spin averaged form factor at large t of t^{1-n}.

For protons, in the qqq state where $n = 3$, $F_p(t) \sim t^{-2}$ and $x\, f_u(x) = x\, u_p(x) \sim (1-x)^3$. In order to have a non-wee antiquark in the proton, the minimum number of fields is five ($qqq\bar{q}q$), so, near the threshold, $x\bar{u}_p(x) \sim (1-x)^{5 \cdot 2-3} = (1-x)^7$. For pions, in the $\bar{q}q$ state, $n = 2$, $F_\pi(t) = t^{-2}$ and $x\bar{u}_{\pi^-}(x) \sim (1-x)^{3 \cdot 2-2} = (1-x)^3$. In order to have a non-wee up-quark in the π^- (or down quark in the π^+) the minimum number of fields is four ($q\bar{q}q\bar{q}$), so, near the threshold, $xu_{\pi^-}(x) \sim (1-x)^{4 \cdot 2-3} = (1-x)^5$. The coefficients in the quark distribution functions were obtained experimentally. Of course, near $x = 0$ or $x = 1$, they were not easily determined. As stated above, x must be finite in order to use the impulse approximation. Otherwise, as x approaches very close to 0 or 1, we will be forced to deal with the very slow partons in the infinite momentum system, or, as seen in the proton rest frame, with the high momentum extremities of the bound state structure and for these cases the impulse approximation will break down.

In Fig. 1, the results of the Drell-Yan contribution to the massive lepton pair productions were compared with the extracted data with the A dependence taken into consideration (we have followed Farrar's argument on this dependence).[6] Since the error resulted from the quark distribution assignment might be large, the good agreement may be coincidental. However, we have not noticed orders of magnitude discrepancy from the calculated and measured cross sections as claimed previously. The departure of the two curves at $Q > 2.8$ GeV is due to the newly discovered resonances. The agreement at Q = 1.1 GeV should be considered accidental, due to the fact that the contribution to the tail of the ρ to the cross section $d\sigma/dQ$ at this value of Q is very large. One should only limit oneself to the agreement at $Q \simeq 2.5$ GeV where the continuum lepton pair is far away from known resonances.

We conclude that the calculation of the Drell-Yan process, which is very much dependent on the quark distribution extracted from existing eN and νN scattering data, is consistent with the measured data. We should not trust these distributions more than a factor of 3, especially the assignment of the non-valence quarks. Any firm conclusion concerning colors, $R = \frac{\sigma h}{\sigma \mu\mu}$, etc., should await more accurate quark assignments and better resolution lepton pair experiments.

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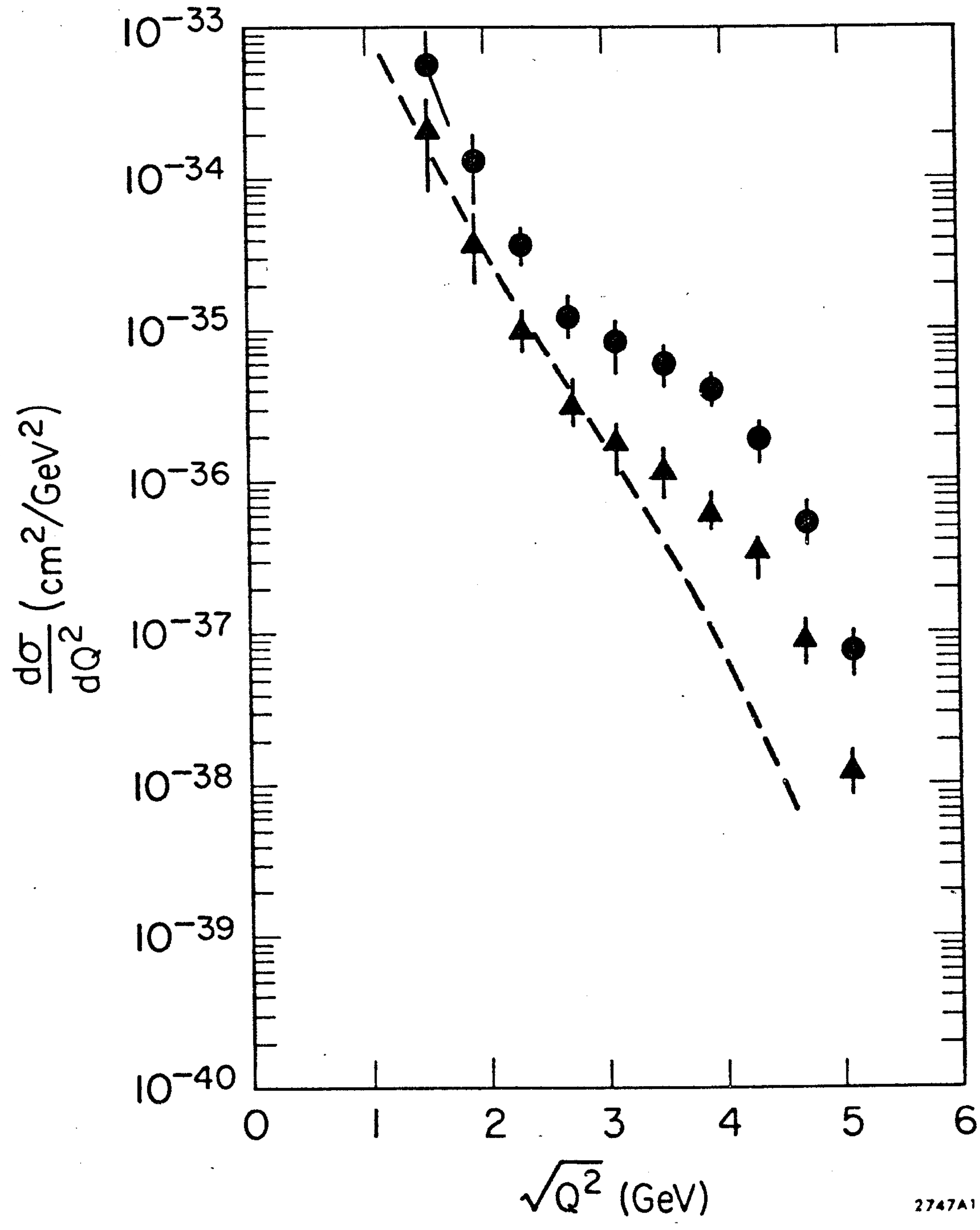

FIG. 1. The lepton pair cross section. The dots are the published data without modification of A dependence. The triangles are the data with this A dependence. The dotted line is the calculated Drell-Yan process on proton target.