Physics Letters B 789 (2019) 215-219

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

Blue-tilted primordial gravitational waves from massive gravity

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ARTICLE INFO

Article history: Received 24 October 2018 Received in revised form 6 December 2018 Accepted 11 December 2018 Available online 13 December 2018 Editor: M. Trodden

ABSTRACT

We study a theory of massive tensor gravitons which predicts blue-tilted and largely amplified primordial gravitational waves. After inflation, while their mass is significant until it diminishes to a small value, gravitons are diluted as non-relativistic matter and hence their amplitude can be substantially amplified compared to the massless gravitons which decay as radiation. We show that such gravitational waves can be detected by interferometer experiments, even if their signal is not observed on the CMB scales. © 2018 The Authors. Published by Elsevier B.V. This is an open access article under the CC BY license

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1. Introduction

Cosmic inflation became a standard paradigm in primordial cosmology, while it is still the subject of intensive researches for its unknown nature. A major prediction of the inflation theory is the production of primordial gravitational waves (GWs) which is scaleinvariant and whose amplitude is proportional to the inflationary Hubble scale. Thus, by measuring the amplitude, we can reveal the energy scale of inflation. A number of different experiments such as Planck [1], SKA [2], LISA [3], Advanced-LIGO (A-LIGO) [4] and DECIGO [5,6] put bounds on or aim to detect it. Nevertheless, it should be stressed that even if inflation occurred, the primordial GWs may be different from the conventional prediction based on general relativity. Among various possibilities to generalize the gravity theory, massive gravity attracts conspicuous attention and has been applied to the study on the primordial GWs [7–10].

The study of massive gravity stemmed from one of fundamental questions in classical field theory, "Can a spin-2 field have a non-vanishing mass or not?" This led Fierz and Pauli in 1939 [11] to find a unique Lorentz-invariant mass term for a linearized spin-2 field, for which a nonlinear completion was found in 2010 [12,13]. Another motivation is the accelerated expansion of the universe today: a graviton mass term may lead to acceleration without a need for dark energy. From this point of view, the assumption of

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Once the assumption of Lorentz-invariance is relaxed at the cosmological scale, new possibilities open up [14-19]. In particular, a massive graviton forms a representation of the three-dimensional rotation group instead of four-dimensional Lorentz group, and therefore the number of physical degrees freedom in the gravity sector does not have to be five. The minimal theory of massive gravity (MTMG) introduced in [20,21] is one of such possibilities and propagates only two physical degrees of freedom in the gravity sector, allowing for self-accelerating, homogeneous and isotropic cosmological solutions without pathologies such as strong coupling and ghosts, that are usually unavoidable in Lorentz-invariant massive gravity [22]. The recently developed positivity bounds that significantly shrink the viable parameter space of the Lorentzinvariant massive gravity theory [23-26] also do not apply to those Lorentz-violating theories, including MTMG, since those bounds rely on Lorentz invariance at all scales. Moreover, because of the absence of extra degrees of freedom, MTMG completely evades the so called Higuchi bound, which states that the mass of a Lorentz-invariant massive graviton should be greater than the Hubble expansion rate up to a factor of order unity in order to avoid turning extra degrees of freedom into ghosts in cosmological backgrounds [27]. From the viewpoint of effective field theories, it is plausible to expect that there should be other Lorentz-violating massive gravity theories with similar properties and MTMG is just

https://doi.org/10.1016/j.physletb.2018.12.025





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one concrete example of such theories.¹ As we shall see in the rest of the present paper, those properties stated here open up a new observational window to GWs produced in the early universe.

2. Setup

Our quadratic Lagrangian density for the tensor graviton $h_{ij}(\tau, \vec{x})$ is given by

$$\mathcal{L}_{h}^{(2)} = \frac{a^{2}M_{\rm Pl}^{2}}{8} \left[h_{ij}^{\prime}h_{ij}^{\prime} - \partial_{l}h_{ij}\partial_{l}h_{ij} - a^{2}\mu^{2}h_{ij}h_{ij} \right], \tag{1}$$

where prime denotes a derivative with respect to the conformal time τ , $M_{\rm Pl}$ is the reduced Planck mass, $a(\tau)$ is the scale factor and $\mu(\tau)$ is the mass of the tensor graviton which depends on time. The time dependence of $\mu(\tau)$ may originate in a dynamics of other fields (e.g. a homogeneous scalar field $\mu(\varphi(\tau))$), while we do not discuss any concrete model in this letter.

The tensor gravitons can be decomposed as

$$h_{ij} = \frac{2}{aM_{\rm Pl}} \sum_{\lambda=+,\times} \int \frac{\mathrm{d}^3 k}{(2\pi)^3} e^{i\boldsymbol{k}\cdot\boldsymbol{x}} e_{ij}^{\lambda} \left[v_k^{\lambda}(\tau) \hat{a}_{\boldsymbol{k}}^{\lambda} + \mathrm{h.c.} \right], \tag{2}$$

where $e_{ij}^{\lambda}(\hat{k})$ is the polarization tensor and $\hat{a}_{k}/\hat{a}_{k}^{\dagger}$ are creation/annihilation operators satisfying the commutation relation, $[\hat{a}_{k}^{\lambda}, \hat{a}_{p}^{\dagger\sigma}] = (2\pi)^{3}\delta^{\lambda\sigma}\delta(\mathbf{k} - \mathbf{p})$. From the above action and the decomposition, one finds that the equation of motion (EoM) for the mode function $v_{k}^{\lambda}(\tau)$ is

$$v_k'' + \left[k^2 + a^2 \mu^2 - \frac{a''}{a}\right] v_k = 0,$$
(3)

where we have suppressed the polarization label λ because the EoM does not depend on it.

To solve the above EoM, we need to specify $a(\tau)$, $\mu(\tau)$ and the initial condition for $v_k(\tau)$. For simplicity, we assume the de Sitter expansion $a \propto \tau^{-1}$ during inflation as well as instantaneous reheating followed by the radiation dominated era $a \propto \tau$. Then the scale factor is written as

$$a(\tau) = \begin{cases} -1/(H_{\text{inf}}\tau) & (\tau < -\tau_r) \\ a_r \tau / \tau_r & (\tau > \tau_r) \end{cases},$$
(4)

where H_{inf} is the Hubble expansion rate during inflation and a_r is the scale factor at the reheating time $\tau_r = (a_r H_{\text{inf}})^{-1}$. Note that the conformal time τ jumps from $-1/(aH_{\text{inf}})$ into $1/(aH_{\text{inf}})$ at reheating in this treatment for a and $da/d\tau$ to be continuous. We further assume that the graviton mass vanishes instantaneously,

$$\mu(\tau) = \begin{cases} m & (\tau < \tau_m) \\ 0 & (\tau > \tau_m) \end{cases},$$
(5)

where τ_m is a certain time during radiation dominated era.² Here we consider a simple step-function like behavior of the mass to simplify calculations, while the final graviton amplitude is not sensitive to the detailed behavior of $\mu(\tau)$, if its transition takes place within a sufficiently short time scale, $\Delta t \ll m^{-1}$. Finally, we set the initial condition for the mode function to be that for the Bunch–Davies vacuum during inflation,

$$\lim_{k\tau \to -\infty} v_k(\tau) = \frac{1}{\sqrt{2k}} e^{-ik\tau}.$$
(6)

These conditions suffice to obtain the evolution of the massive tensor gravitons in our setup.

3. Evolution

In this section, we shall study the time evolution of the tensor massive gravitons and calculate their dimensionless power spectrum,

$$\mathcal{P}_{h}(\tau,k) = \frac{4k^{3}|v_{k}(\tau)|^{2}}{\pi^{2}M_{\rm Pl}^{2}a^{2}(\tau)},\tag{7}$$

where the contributions from the two polarization have been summed. The mode function of the tensor gravitons with a long wave length changes its behavior twice; namely at the end of inflation and when their mass vanishes. Therefore we have the following three phases, (i) inflation phase $\tau < \tau_r$, (ii) mass dominant phase $\tau_r < \tau < \tau_m$ and (iii) massless phase $\tau_m < \tau$. We shall discuss these phases in order.

(i) **Inflation phase**: Solving the EoM (3) in the de Sitter universe with the initial condition (6), one finds the solution of the mode function during inflation as

$$v_k^{(i)}(\tau) = \frac{\sqrt{-\pi\tau}}{2} H_v^{(1)}(-k\tau), \quad v \equiv \sqrt{\frac{9}{4} - \frac{m^2}{H_{\inf}^2}}, \tag{8}$$

where $H_{\nu}^{(1)}(z)$ is Hankel function of the first kind. In the superhorizon limit $-k\tau \rightarrow 0$, it asymptotes $v_k^{(i)} \propto \tau^{\frac{1}{2}-\nu}k^{-\nu}$. Thus, massive tensor gravitons produce blue-tilted tensor power spectrum $\mathcal{P}_h \propto (\tau k)^{3-2\nu}$ which decreases on super-horizon scales due to the graviton mass during inflation, $\nu < 3/2$. The usual scale-invariant spectrum is restored in the massless limit $\nu \rightarrow 3/2$. This result can be understood as an analogy to the fluctuation of a massive scalar field.

(ii) Mass dominant phase: Next, let us discuss the evolution of the massive tensor gravitons after inflation. The a''/a term in (3) vanishes during the radiation dominated era, and almost all the modes which exited the horizon during inflation satisfy $k \ll am$ for $m/H_{inf} = \mathcal{O}(1)$. Hence the EoM in this phase reads

$$\partial_{\tau}^2 v_k^{(\text{ii})} + a^2 m^2 v_k^{(\text{ii})} \simeq 0.$$
 (9)

We expect that these modes behave as non-relativistic matter. By using the junction condition,

$$v_k^{(i)}(-\tau_r) = v_k^{(ii)}(\tau_r), \quad \partial_\tau v_k^{(i)}(-\tau_r) = \partial_\tau v_k^{(ii)}(\tau_r), \tag{10}$$

one finds that the mode function shows a damped oscillation at sufficiently late times $m\tau^2 \gg H_{inf}\tau_r^2$,

$$v_k^{(\text{ii})} \simeq \frac{2}{\sqrt{\pi \, am}} \left[C_1 \cos\left(\frac{am\tau}{2} - \frac{\pi}{8}\right) + C_2 \sin\left(\frac{am\tau}{2} + \frac{\pi}{8}\right) \right],\tag{11}$$

with the integration constants, $C_{1,2} \simeq -i\sqrt{\pi}2^{-\frac{7}{2}+\nu}(k\tau_r)^{-\nu}\Gamma(\nu) \times \left[\frac{2m}{H_{inf}}J_{\mp\frac{3}{4}}\left(\frac{m}{2H_{inf}}\right) \pm (1-2\nu)J_{\pm\frac{1}{4}}\left(\frac{m}{2H_{inf}}\right)\right]$. Here $J_{\mu}(z)$ denotes the Bessel function of the first kind.

¹ A generalization of solid inflation [28,29] dubbed supersolid inflation [30] is classified into these theories and the primordial GWs in supersolid inflation are studied in [31–34].

² In principle, gravitons can remain significantly massive for redshift $z \gtrsim 10^{-2}$, because only the recently detected binary neutron star merger (i.e. GW170817 and GRB170817A) occurred at $z \simeq 10^{-2}$ puts a direct bound on the propagation speed of gravitons [35]. However, we conservatively assume that the graviton mass vanishes before the matter-radiation equality in this letter.

(11) implies that $v_k \propto a^{-1/2}$ in this phase and hence the graviton energy density evolves like non-relativistic matter $m^2 h_k^2 \propto a^{-2} v_k^2 \propto a^{-3}$ as expected. Compared to the massless graviton whose energy density decays as a^{-4} , the decay of the massive tensor graviton is slower and thus its final amplitude will be relatively amplified.

(iii) Massless phase: At $\tau = \tau_m$, the graviton mass disappears and gravitons restore their normal behavior as radiation whose energy density decays as a^{-4} . By using the junction conditions same as (10) at $\tau = \tau_m$ between $v_k^{(ii)}$ and $v_k^{(iii)}$, we obtain the mode function as

$$v_k^{(\text{iii)}}(\tau) = \frac{2}{k} \sqrt{\frac{m\tau_m}{\pi H_{\text{inf}} \tau_r^2}} \Big[D_1 \cos(k\tau) + D_2 \sin(k\tau) \Big], \tag{12}$$

with $D_1 \simeq -\sin(k\tau_m)[C_2\cos(\Lambda + \pi/8) - C_1\sin(\Lambda - \pi/8)]$, $D_2 \simeq \cos(k\tau_m)[C_2\cos(\Lambda + \pi/8) - C_1\sin(\Lambda - \pi/8)]$ and $\Lambda \equiv m\tau_m^2/(2H_{inf}\tau_r^2)$. Here we used $\Lambda \gg k\tau_m$. In this phase, the mode function significantly grows and then starts oscillating with a constant amplitude when it re-enters the horizon. In the case with $m/H_{inf} = \mathcal{O}(1)$, the squared amplitude is roughly given by

$$2k|v_k^{(\text{iii})}|^2 \sim \frac{\tau_m}{\tau_r} (k\tau_r)^{-2\nu-1}.$$
(13)

Compared to the usual massless graviton with v = 3/2 and $\tau_m \rightarrow \tau_r$, the power spectrum of our massive graviton is amplified by the two factors: the first factor τ_m/τ_r represents the duration of the epoch where the decay of the graviton energy density is slower by $a \propto \tau$; the second factor $(k\tau_r)^{-2\nu-1}$ for $\nu < 3/2$ represents the damping effect during inflation which leads to a blue-tilted spectrum. Therefore our final power spectrum of the primordial GWs for $\tau > \tau_m$ can be evaluated as

$$\mathcal{P}_{h}^{\text{massive}}(\tau) \sim \frac{\tau_{m}}{\tau_{r}} (k\tau_{r})^{3-2\nu} \mathcal{P}_{h}^{\text{massless}}(\tau),$$
(14)

where $\mathcal{P}_{h}^{\text{massless}}$ denotes the usual power spectrum of the massless tensor modes from inflation.

4. Results

Now we study the parameter region in which the primordial GWs generated in our scenario satisfy the current constraints and can be observed by upcoming experiments. To this end, we consider the energy fraction of the GWs per logarithmic interval of the wave number *k* at the present time, $\Omega_{GW,0}(k) \equiv \rho_{tot}^{-1} d\rho_{GW}/d \ln k$. From (14) and using $\Omega_{GW,0}^{massless}(k) \sim 10^{-15} H_{14}^2$ for the modes which entered the horizon during the radiation dominated era, one finds

$$\Omega_{\rm GW,0}^{\rm massive}(k) \sim \frac{\tau_m}{\tau_r} (k\tau_r)^{3-2\nu} \Omega_{\rm GW,0}^{\rm massless}(k),$$

$$\approx 10^{-15} \frac{\tau_m}{\tau_r} H_{14}^{\nu+\frac{1}{2}} f_8^{3-2\nu}, \qquad (15)$$

where $H_{14} \equiv H_{\rm inf}/(10^{14} \text{ GeV})$ and $f_8 \equiv f/(2 \times 10^8 \text{ Hz})$. Here the GW frequency $f \equiv k/2\pi$ is assumed to be lower than the inflationary UV cutoff, $f_{\rm UV} \simeq a_r H_{\rm inf}/(2\pi) \approx 2 \times 10^8 H_{14}^{1/2}$ Hz and higher than the scale corresponding to the matter-radiation equality, $f_{\rm eq} \simeq 3 \times 10^{-17}$ Hz.

The BBN bound $\Omega_{GW,0}<10^{-5}$ and the CMB bound $\Omega_{GW,0}(f\sim2\times10^{-17}~\text{Hz})<10^{-15}$ are recast as

$$\frac{\tau_m}{\tau_r} \lesssim 10^{10} H_{14}^{-2}, \quad \text{(BBN)}$$
 (16)

$$\nu \lesssim \frac{75 - \log_{10}(H_{14}^{1/2}\tau_m/\tau_r)}{50 + \log_{10}(H_{14})}.$$
 (CMB) (17)



Fig. 1. We plot Ω_{CW} for $H_{\text{inf}} = 10^8$ GeV and $\tau_m/\tau_r = 10^{10}$ (black), 10^{15} (red) and 10^{21} (blue) as thick lines. The graviton mass is $m = 0.5H_{\text{inf}}$ (solid) and $m = 0.8H_{\text{inf}}$ (dashed). The shaded regions are excluded by the BBN (green) and CMB (yellow) constraints. The sensitivity curves of SKA, LISA, A-LIGO and DECIGO (with the original and upgraded sensitivity curves [36]) are also shown as thin dashed lines.

The largest GWs can be produced when these two conditions are saturated for given H_{inf} . In that case, the graviton mass during inflation is given by

$$\frac{m^2}{H_{\inf}^2}\Big|_{\max GW} \simeq \frac{9}{4} - \left(\frac{65 + \frac{3}{2}\log_{10}H_{14}}{50 + \log_{10}H_{14}}\right)^2.$$
(18)

It should be noted that gravitons are still massive at BBN if

$$\frac{\tau_m}{\tau_r} \gtrsim \frac{\tau_{\rm BBN}}{\tau_r} = \sqrt{\frac{H_{\rm inf}}{H_{\rm BBN}}} \approx 10^{17} H_{14}^{\frac{1}{2}}.$$
(19)

In this case, the BBN bound (16) should be relaxed, because gravitons do not contribute to relativistic degrees of freedom during BBN. In this letter, however, we conservatively respect the original bound (16). Whereas, since we assumed the graviton mass vanishes at τ_m before the radiation–matter equality at τ_{eq} , the amplification factor τ_m/τ_r cannot exceed,

$$\frac{\tau_m}{\tau_r} \le \frac{\tau_{eq}}{\tau_r} = \sqrt{\frac{H_{inf}}{H_{eq}}} \approx 2 \times 10^{25} H_{14}^{\frac{1}{2}}.$$
(20)

In Fig. 1, we show the accurate prediction for the primordial GWs by numerically solving the EoM and compare them with the sensitivity curves of the various experiments. Although the inflationary Hubble scale is chosen as low as $H_{inf} = 10^8$ GeV, the amplification during the mass dominant phase with $\tau_m/\tau_r \gg 1$ makes the GWs large enough to be observed. Furthermore, the blue-tilted spectra made by the graviton mass naturally enable to avoid the CMB constraint and to have detectable amplitudes on smaller scales at the same time. It is interesting to note that all of SKA, LISA and aLIGO can detect the GWs for $m = 0.8H_{inf}$ and $\tau_m/\tau_r = 10^{21}$ (see the blue dashed line). Except for the blue solid line which is excluded by the CMB constraint, all the lines satisfy other constrains including ones from the PTA [37] and the CMB μ distortion [38] which are not shown in Fig. 1. In Fig. 2, the detectable parameter region of $(m, \tau_m/\tau_r)$ is explored. For sufficiently large *m* and τ_m/τ_r , the detectability and the CMB constraint are compatible, while a too large τ_m/τ_r violates (20) and requires to revisit the BBN bound.

5. Discussion

In this section, we discuss the interpretation of the amplification of the gravitational waves and a possibility to further ex-



Fig. 2. The GWs can be detected only by DECIGO (yellow), by LISA and DECIGO (pink), by aLIGO and DECIGO (green) and by all of aLIGO, LISA and DECIGO (blue) in the colored parameter regions. The grey shaded region is excluded by the CMB observation. The Hubble scale is fixed at $H_{\rm inf} = 10^8$ GeV which corresponds to $\Omega_{\rm GW} \sim 10^{-27}$ in the massless graviton case.

tend the model based on the effective field theory (EFT) approach. We also discuss the connection between the graviton mass and the dark energy based on the minimal theory of massive gravity (MTMG).

In our setup, the relative amplification of the graviton energy density obtained in the mass dominant phase remains unchanged while the graviton mass disappears. It can be interpreted as the consequence of particle production. The graviton mode function can be seen as a harmonic oscillator whose energy density ρ_k consists of the time-kinetic energy $(\partial_{\tau} v_k)^2/2$ and the potential energy $\omega^2 v_k^2/2$, with $\omega^2 \equiv k^2 + a^2 m^2$ in our case. When the mass quickly vanishes, since the time-kinetic energy is conserved in such an abrupt process, ρ_k does not change significantly unless it is dominated by the potential energy at the time of the transition. Therefore, after averaging over a range of k and thus over different phases of the oscillation, the energy density remains almost unchanged. As a consequence, the graviton particle number $\mathcal{N}_k = \rho_k / \omega$ increases as $\omega(\tau)$ drops. If the transition process of the mass is adiabatic, however, the particle number \mathcal{N}_k is conserved instead and the energy density (i.e. Ω_{GW}) may be reduced. We leave it for future study to explore the cases with a longer transition time of the graviton mass.

Although we have assumed the quadratic Lagrangian (1) so far, according to the general philosophy of the EFT approach, one should consider a non-trivial sound speed of graviton c_T introduced as $-c_T^2 \partial_t h_{ij} \partial_t h_{ij}$ in (1). Provided that $c_T < 1$ is constant for $\tau < \tau_m$ and becomes unity at $\tau = \tau_m$ in the same way as the graviton mass, the varying c_T leads to the following three modifications. (i) The UV cutoff frequency f_{UV} increases by c_T^{-1} . (ii) The tensor power spectrum is amplified by $c_T^{-2\nu}$ for $k \ll a(\tau_m)m$. (iii) For the modes with $a(\tau_m)m \lesssim k$, which are produced only if $c_T \lesssim \tau_r/\tau_m$, the tilt of the tensor power spectrum becomes bluer. A detailed study on the cases with a non-trivial c_T is left for future work. Supersolid inflation [30] based on EFT can also generate a highly blue-tilted GWs [34]. However, the post-inflationary dynamics was not considered and the amplification mechanism was missed in the work. It would be interesting to combine such models with our analysis.

In the present paper we have studied impacts of a class of massive gravity theories on GWs observations without specifying a concrete theory. This is a totally rational attitude from the viewpoint of EFTs. It is nonetheless interesting to discuss concrete examples. Here we thus consider one such example based on MTMG [20,21]. The FLRW cosmology in this theory has two branches of solutions, the self-accelerating branch and the normal branch. In the former branch the effective cosmological constant is

$$\Lambda_{\rm eff} = \frac{m_{\rm g}^2}{2} X(c_1 X^2 + 3c_2 X + 3c_3), \qquad (21)$$

where c_i (i = 1, 2, 3) are dimensionless constants in the gravity action, m_g is a mass scale and X is a constant satisfying $c_1X^2 + 2c_2X + c_3 = 0$. The graviton acquires a squared mass,

$$\mu^{2} = \frac{m_{g}^{2}}{2} X \left[c_{2} X + c_{3} + \frac{H}{H_{f}} (c_{1} X + c_{2}) \right], \qquad (22)$$

where H_f is the Hubble expansion rate of the fiducial metric that can be freely specified as a part of the definition of the model. One can promote the constants c_i to functions of a scalar field, $c_i = c_i(\phi)$. When ϕ is constant, Λ_{eff} and μ^2 in the self-accelerating branch are given by the above formulas. If ϕ starts with a constant and changes to another constant then Λ_{eff} and μ^2 also exhibit a transition. By tuning $c_i(\phi)$ and H_f , one can in principle realize such a model that $\mu^2 \gg |\Lambda_{\text{eff}}|$ before the transition.

6. Conclusion

In this letter, we have investigated the primordial GWs in the theory of massive tensor gravitons (1). Contrary to the massless graviton case, the massive gravitons with a mass comparable to the inflationary Hubble scale $m = \mathcal{O}(H_{inf})$ generate a blue-tilted tensor spectrum during inflation. Moreover, while their mass is significant after inflation, the dilution of the energy density of the massive gravitons becomes slower $\rho_h^{\text{massive}} \propto a^{-3}$ than the massless ones $\rho_{h}^{\text{massless}} \propto a^{-4}$. Thus, $\Omega_{\text{GW},0}$ in the massive case can be substantially amplified compared to the massless case. Consequently, we have obtained the blue-tilted and largely amplified primordial GWs which are suitable for the detection by the interferometers and to avoid the CMB constraint at the same time. We have derived the analytic expression for $\Omega_{GW,0}$ (15) and illustrated its detectability in Fig. 1 and 2. We have found that it is even possible to generate primordial GWs detectable for all of SKA, LISA and advanced-LIGO. Our findings further motivate the theoretical works on massive gravitons and the experimental efforts to detect stochastic GWs.

Acknowledgements

We would like to thank Claudia de Rham, Tsutomu Kobayashi, Misao Sasaki, Takahiro Tanaka and Gianmassimo Tasinato for useful comments. In this work, TF is supported by Japan Society for the Promotion of Science (JSPS) Grants-in-Aid for Scientific Research (KAKENHI) No. 17J09103. SK is supported by JSPS KAKENHI No. 17K14282 and Career Development Project for Researchers of Allied Universities. The work of SMi was supported by JSPS KAKENHI No. 16K17709. The work of SMu was supported by JSPS KAKENHI No. 17H02890, No. 17H06359, and by WPI, Ministry of Education, Culture, Sports, Science and Technology (MEXT), Japan.

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