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About Fermion Self-Energy Corrections in Perturbative Theory at Finite Temperature

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Abstract

We discuss various aspects of self-energy corrections to the fermion propagator at finite temperature. Several calculational methods not relying on the renormalization of the mass or of the wave function are discussed. General expressions are given. It is shown that, when calculating a physical process, the thermal mass enters essentially the phase space factor and not the dynamical part of the process, at least in some limiting cases. Comparison with the renormalization approach is discussed and it appears that there are no ambiguities in the calculation of the self-energy corrections at finite temperature.

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I. Introduction

Over the last few years there has been an increasing interest in applying Perturbation Theory at finite temperature [1-3]. A point of particular importance is the contribution of the self-energy corrections [4,5] and the question of fermion mass renormalization at finite temperature [6]. A method was proposed some time ago which dealt with this problem in a way very similar to the $T = 0$ case: it involved the introduction of a non scalar mass counter-term in the Lagrangian as well a wave function (Z_2) renormalization. Special care had to be taken because of the non covariance of the expressions at finite temperature (the calculations are usually done in the plasma rest-frame). This method is rather cumbersome and it was suggested recently that it did not lead to the correct results [7]. Since the temperature dependent terms are ultra-violet finite it is not necessary to introduce an explicit renormalization procedure and the self-energy corrections can be evaluated directly. We present here two ways to calculate these contributions which lead to identical results. In sec.2 we state the problem and present our results: they are quite simple in the case of vanishing fermion masses or when $T \ll m_{fermion}$. In sec.3 we present a method of calculating the self-energy contribution based on the resummation of the diagrams: this will help in interpreting the results. In sec.4 we generalize the method of [7] based on an explicit evaluation of derivatives of δ functions. Sec.4 is devoted to a discussion of the results as well as various subtleties related to the expansion of the self-energy contribution around the mass-shell condition.

II. The Results

At $T = 0$ the self-energy diagrams are ultra-violet divergent. The renormalization procedure consists in absorbing the divergences in a redefinition of the physical quantities. We assume here that the standard renormalization procedure has been carried out and we define m as the renormalized fermion mass at zero temperature. In an adequately defined scheme it is then enough to calculate the lowest order diagrams with the renormalized mass and ignore the self energy corrections on the external particles. The renormalized mass appears both in the phase space factor (the external momenta satisfy $p^2 = m^2$), and in the matrix element squared (the virtual lines will involve the spin projection operator $(\not{p} + m)$).

At $T \neq 0$ the situation is different. It is particularly simple if one starts with a massless quark or if one can neglect the temperature dependence on the internal fermion line ($T \ll m$). In both cases the result is that the pole in the fermion propagator is shifted to $m_T^2 = m^2 + cg^2T^2$, a well known result. This thermal mass will appear when taking discontinuities (the propagator pole is then replaced by $\delta(p^2 - m_T^2)$) and therefore will be relevant for the external mass: it consequently enters the phase space factor. On the other hand, when evaluating the self energy diagram contribution to a particular process, it turns out that the matrix element squared is calculated essentially with the $T = 0$ mass: the temperature dependent term disappears when doing the relevant traces. This arises because of the peculiar form of the self energy correction at finite temperature which is of the form [4,5]

$$\Sigma(p)|_{mass-shell} = \gamma_\mu I^\mu \tag{2.1}$$

that is Σ is not a scalar but a matrix such that

$$\begin{aligned}\delta m_T^2 &= 2\ p.I \\ &= c\ g^2 T^2\end{aligned} \tag{2.2}$$

The fact that $\Sigma_{mass-shell}$ transforms as $\not p$ means that the mass shift due to $\not I$ does not break chiral invariance. To put it differently, one could introduce by analogy with the $T = 0$ renormalization procedure, a term $\overline{\Psi}\not I \Psi$ in the Lagrangian and this counter-term would not change the chiral properties of the original Lagrangian. The self energy diagrams contribute other terms which are infra-red or mass singular but it has been shown that they cancel when a physical quantity is considered [8-12].

In the case when the fermion mass cannot be neglected and when the condition $T \ll m$ is not satisfied the situation is more complicated and will be described below.

III. The Summation Method

For definiteness we consider the case of a Higgs particle in an QED plasma. The application of the cutting rules at finite temperature [13] allows one to calculate the quantity $\Gamma = \Gamma_d - \Gamma_i$, namely the difference between the rate of decay ($H \rightarrow e^+e^-$) and the rate of formation ($e^+e^- \rightarrow H$) of the Higgs particle. At lowest order, it is in n dimensions

$$\Gamma^0 = -\tfrac{1}{2}(g\mu^\epsilon)^2(e^{Q/T} - 1) \int \frac{d^n p_1}{(2\pi)^n} \frac{d^n p_2}{(2\pi)^n} \left(\theta(-p_1^0) - n_F(E_1)\right) \left(\theta(p_2^0) - n_F(E_2)\right)$$

$$\times\, 2\pi\delta(p_1^2 - m^2) 2\pi\delta(p_2^2 - m^2) \mathrm{Tr}\left[(\not p_1 + m)(\not p_2 + m)\right] (2\pi)^n \delta(p_1 - p_2 - q) \tag{3.1}$$

where Q is the Higgs mass and $E_i = |p_i^0|$. After evaluation of the phase space integrals it comes out

$$\Gamma^0 = (e^{Q/T} - 1)n_F^2(Q/2)\frac{g^2}{8\pi}Q^2\left(1 - \frac{4m^2}{Q^2}\right)^{3/2}\left(\frac{4\pi\mu^2}{Q^2 - 4m^2}\right)^{\epsilon}\frac{\Gamma(1-\epsilon)}{\Gamma(2-2\epsilon)}$$

$$= \tanh(Q/4T)\frac{g^2}{8\pi}Q^2\left(1 - \frac{4m^2}{Q^2}\right)^{3/2} \qquad (3.2)$$

where in the last line we have set $\epsilon = 0$. Note that the phase space factor contributes $v = \sqrt{1 - 4m^2/Q^2}$ while the trace factor gives Q^2v^2. Consider now the self energy corrections to quark line p_1 . After cancellation of ill-defined terms, by adding the contributions of all 'cut diagrams' as required by $T \neq 0$ field theory one finds [9,10,13]

$$\Gamma^{SE} = -(g\mu^{\epsilon})^2(e^{Q/T} - 1)\int \frac{d^np_1}{(2\pi)^n}\frac{d^np_2}{(2\pi)^n}2\pi\left(\theta(-p_1^0) - n_F(E_1)\right)\left(\theta(p_2^0) - n_F(E_2)\right)$$

$$\times\, \delta(p_2^2 - m^2)\mathrm{Re}\left(\mathrm{Tr}\left[(\not p_1 + m)\Delta_1(-i\mathrm{Re}\Sigma(p_1))(\not p_1 + m)\Delta_1(\not p_2 + m)\right]\right)$$

$$\times\, (2\pi)^n\delta^n(p_1 - p_2 - q) \qquad (3.3)$$

with

$$\Delta_1 = \frac{i}{p_1^2 - m^2 + i\eta} \qquad (3.4)$$

The expression for $\mathrm{Re}\Sigma(p_1)$, the temperature dependent part of the self energy loop will be given later. Turning back to eq.(3.1) , we can use the relation

$$\pi\delta(p_1^2 - m^2) = \mathrm{Re}\Delta_1,$$

so that when considering the expression $\Gamma^0 + \Gamma^{SE}$, there appears the following combination

$$(\not p_1 + m)\Delta_1 + (\not p_1 + m)\Delta_1(-i\mathrm{Re}\Sigma(p_1))(\not p_1 + m)\Delta_1. \tag{3.5}$$

To order e^2 ($\mathrm{Re}\Sigma(p_1)$ is proportional to e^2) this can be written as

$$S^{(1)}(p_1) = \frac{i}{\not p_1 - m - \mathrm{Re}\Sigma(p_1) + i\eta} \tag{3.6}$$

Such a result would also have been obtained had we summed the self energy loop to all orders on the fermion p_1 propagator. We turn now to the evaluation of eq.(3.6) for later use in eq.(3.3) . One finds, in the Feynman gauge,

$$\begin{aligned}\mathrm{Re}\Sigma(p) = 2(e\mu^\epsilon)^2 \int \frac{d^n k}{(2\pi)^{n-1}} \Big[& \frac{n_B(k)\delta(k^2)}{(p+k)^2 - m^2 + i\eta}\left((1-\epsilon)(\not p + \not k) - (2-\epsilon)m\right) \\ & - \frac{n_F(k)\delta(k^2 - m^2)}{(p-k)^2 + i\eta}\left((1-\epsilon)\not k - (2-\epsilon)m\right)\Big]\end{aligned} \tag{3.7}$$

Introducing the notation

$$\mathrm{Re}\Sigma(p) = A(p)\not p - mB(p) + \not K(p) \tag{3.8}$$

we have

$$\begin{aligned} A(p) &= (1-\epsilon)C_B(p) \\ B(p) &= (2-\epsilon)\left(C_B(p) - C_F(p)\right) \end{aligned} \tag{3.9}$$

with

$$\begin{aligned} C_B(p) &= 2(e\mu^\epsilon)^2 \int \frac{d^n k}{(2\pi)^{n-1}} \frac{n_B(k)\delta(k^2)}{(p+k)^2 - m^2 + i\eta} \\ C_F(p) &= 2(e\mu^\epsilon)^2 \int \frac{d^n k}{(2\pi)^{n-1}} \frac{n_F(k)\delta(k^2 - m^2)}{(p-k)^2 + i\eta} \end{aligned} \tag{3.10}$$

and

$$\not K(p) = 2(1-\epsilon)(e\mu^\epsilon)^2 \int \frac{d^n k}{(2\pi)^{n-1}} \not k \left(\frac{n_B(k)\delta(k^2)}{(p+k)^2 - m^2 + i\eta} - \frac{n_F(k)\delta(k^2 - m^2)}{(p-k)^2 + i\eta} \right) \tag{3.11}$$

The above integrals are for an off mass-shell fermion. Unlike what is done at zero temperature, we defer to a later stage the expansion of these expressions around the mass-shell condition $\not{p} = m$. Introducing now the notation of eq.(3.8) the fermion propagator at $O(e^2)$ can be written

$$-iS^{(1)}(p) = \frac{\not{p}(1 - A(p)) - \not{K}(p) + m(1 - B(p))}{p^2(1 - 2A(p)) - 2p.K(p) - m^2(1 - 2B(p)) + i\eta} \tag{3.12}$$

Considering now the denominator of the above equation and using the definitions eqs.(3.8-3.11), it can be recast in the form

$$[1 - (1 - \epsilon)(C_B(p) - C_F(p))] \left[p^2 - \delta m_T^2 - m^2 \left(1 - 2(C_B(p) - C_F(p))\right)\right]$$

Here we have introduced the notation

$$\delta m_T^2 = 2e^2 \int \frac{d^n k}{(2\pi)^{n-1}} \left(n_B(k)\delta(k^2) + n_F(k)\delta(k^2 - m^2)\right) \tag{3.13}$$

We now expand the integrals $C_B(p)$ and $C_F(p)$ for small $p^2 - m^2$ using the relations

$$\frac{\delta(k^2)}{(p+k)^2 - m^2 + i\eta} = \frac{\delta(k^2)}{(\hat{p}+k)^2 - m^2} - (p^2 - m^2)\frac{\delta(k^2)}{((\hat{p}+k)^2 - m^2)^2}\left(1 + \left.\frac{d(2k.p)}{dp^2}\right|_{p^2=m^2}\right)$$

$$\frac{\delta(k^2 - m^2)}{(p-k)^2 + i\eta} = \frac{\delta(k^2 - m^2)}{(\hat{p}-k)^2} - (p^2 - m^2)\frac{\delta(k^2 - m^2)}{((\hat{p}-k)^2)^2}\left(1 - \left.\frac{d(2k.p)}{dp^2}\right|_{p^2=m^2}\right) \tag{3.14}$$

where the vector $\hat{p} = (E_p, \vec{p})$, with $E_p = \sqrt{\vec{p}^2 + m^2}$. The above equations are not completely defined as we have not specified yet how p^0 and $\vec{p}$ depend on p^2 and therefore $d(k.p)/dp^2$ does not have a precise meaning. The four-vector p is in fact fully determined by the kinematical constraints of the process under study. In our case, they are given by the δ-functions in eq.(3.1) where the quark line , with the self-energy insertion, is taken off-shell (m^2 replaced by p^2 in the corresponding δ-function).

The 4-vector components are then found to be

$$
\begin{aligned}
p^0 &= \tfrac{Q}{2} + \frac{p^2 - m^2}{2Q} \\
|\vec{p}|^2 &= {p^0}^2 - p^2
\end{aligned}
\tag{3.15}
$$

so that

$$
\begin{aligned}
\left.\frac{dp^0}{dp^2}\right|_{p^2=m^2} &= \frac{1}{2Q} \\
\left.\frac{d|\vec{p}|}{dp^2}\right|_{p^2=m^2} &= -\frac{1}{4|\vec{p}|}
\end{aligned}
\tag{3.16}
$$

which all what is needed to specify the equations above. We dwell in some detail on this problem because it has sometimes been assumed that $|\vec{p}|$ could be kept fixed with the whole off-shellness dependence put into p^0. This hypothesis is inconsistent with the kinematical constraints of the considered process and would lead to the wrong results as far as finite correction terms are concerned.

Defining in an obvious way

$$
C(p) = \widehat{C} + (p^2 - m^2)\widehat{C}'
$$

one finds

$$
\begin{aligned}
\widehat{C}_B &= 0 \\
\widehat{C}_F &= 2(e\mu^\epsilon)^2 \int \frac{d^n k}{(2\pi)^{n-1}} \frac{n_F(k)\delta(k^2 - m^2)}{(\widehat{p} - k)^2} \\
\widehat{C}'_B &= -2(e\mu^\epsilon)^2 \int \frac{d^n k}{(2\pi)^{n-1}} \frac{n_B(k)\delta(k^2)}{((\widehat{p} + k)^2 - m^2)^2} \\
\widehat{C}'_F &= -2(e\mu^\epsilon)^2 \int \frac{d^n k}{(2\pi)^{n-1}} \frac{n_F(k)\delta(k^2 - m^2)}{((\widehat{p} - k)^2)^2} \left(1 - \left.\frac{d(2k.p)}{dp^2}\right|_{p^2=m^2}\right)
\end{aligned}
\tag{3.17}
$$

Note that the derivative term does not contribute in $\widehat{C}'_B$ because of the antisymmetry property of the integrand under the change $k \to -k$. The previous results can then be used to express the fermion propagator at $O(e^2)$ in the following form

$$-iS^{(1)}(p) = (1 - \widehat{C}_F - 2m^2(\widehat{C}'_B - \widehat{C}'_F))\frac{\not{p} - \not{K}(\widehat{p}) + m(1 + 2\widehat{C}_F)}{p^2 - \delta m_T^2 - m^2(1 + 2\widehat{C}_F) + i\eta} \tag{3.18}$$

The pole in the propagator is at

$$\begin{aligned} p^2 &= m_T^2 \\ &= \delta m_T^2 + m^2(1 + 2\widehat{C}_F). \end{aligned} \tag{3.19}$$

In the numerator of eq.(3.18) it is enough to evaluate the integrals at $\widehat{p} = p$ since terms of $O(e^2(p^2 - m^2))$ are in fact of $O(e^4)$ and can therefore safely be neglected.

We are now in a position to evaluate the contribution of the self energy diagrams to the Higgs process. It is a relatively trivial task now. Comparing the form of the $O(e^2)$ propagator, eq.(3.18) , with that of the lowest order propagator it is enough to substitute in the lowest order rate, eq.(3.1) , the expression

$$(1 - \widehat{C}_F - 2m^2(\widehat{C}'_B - \widehat{C}'_F))\left[\not{p} - \not{K}(p) + m(1 + 2\widehat{C}_F)\right]\delta(p^2 - m_T^2)$$

to the usual factor

$$(\not{p} + m)\delta(p^2 - m^2).$$

The net result can then be expressed as

$$\begin{aligned} \Gamma^0 + \Gamma^{SE} = \Gamma^0\, \frac{v_T}{v}\Big[1 - \frac{2q.K(\widehat{p}_1) - 2q.K(\widehat{p}_2)}{Q^2v^2} - 2m^2(\widehat{C}'_B(\widehat{p}_1) + \widehat{C}'_B(\widehat{p}_2) - \widehat{C}'_F(\widehat{p}_1) - \widehat{C}'_F(\widehat{p}_2)) \\ - 2\widehat{C}_F\frac{Q^2 + 4m^2}{Q^2v^2}\Big] \end{aligned} \tag{3.20}$$

The factor

$$v_T = \sqrt{1 - 4m_T^2/Q^2} \tag{3.21}$$

is the threshold factor appropriate for the $O(e^2)$ calculation. Its origin is clear: it comes from the kinematics of two external particles of mass m_T. The dynamical part, related to the trace evaluation, is contained in the expression in square brackets in eq.(3.20) . One may remark that the mass shift δm_T^2 does not appear in this factor since it cancels in terms such as $p^2 - 2p.K(p)$ leaving in the final result only pieces like $m^2\widehat{C}_F$. At zero temperature the term $2p.K(p)$ of course does not exist leaving in the trace factor the full mass shift contribution. Further discussion of this result will be given in Sec.5.

IV. The Direct Calculation

We now turn back to eq.(3.3) and carry out the integration directly. The basic step is to recognize that [14]

$$\mathrm{Re}(-i\Delta_1^2) = \pi\frac{\partial}{\partial m^2}\delta(p_1^2 - m^2) \tag{4.1}$$

Since after performing the p_2 integration there still remains in the integrand of eq.(3.3) another δ function, whose argument also depends on m^2, the usual method of integration by part is not very useful. We use here the trick of ref.[7] who define in general

$$\int d^n p F(p,m)\frac{\partial}{\partial m^2}\delta(p^2 - m^2) = \frac{\partial}{\partial \widehat{m}^2}\int d^n p F(p,m)\delta(p^2 - \widehat{m}^2)\Big|_{\widehat{m}=m} \tag{4.2}$$

Upon using this relation and carrying out the phase space integration on p_1 in eq.(3.3) we arrive at the relatively simple formula

$$\Gamma_{SE} = (\pi\mu^2)^\epsilon\frac{\Gamma(1-\epsilon)}{\Gamma(2-2\epsilon)}\frac{g^2}{8\pi Q}\frac{\partial}{\partial\widehat{m}^2}\left(|\vec{p}_1|^{1-2\epsilon}n_F(|p_1^0|)n_F(|Q-p_1^0|)G(p_1,\widehat{m})\right)\Big|_{\widehat{m}=m} \tag{4.3}$$

The function $G(p_1, \widehat{m})$ contains all the dynamical part of the process and it is defined as

$$G(p_1, \widehat{m}) = 2\mathrm{Tr}((\not{p}_1 + m)\mathrm{Re}\Sigma(p_1)(\not{p}_1 + m)(\not{p}_2 + m)) \tag{4.4}$$

It explicitly depends on $\widehat{m}$ because of the relations

$$\begin{aligned} p_1^0 &= \tfrac{Q}{2} + \frac{\widehat{m}^2 - m^2}{2Q} \\ |\vec{p}_1|^2 &= p_1^{0\,2} - \widehat{m}^2 \end{aligned} \tag{4.5}$$

This set of equations is identical to the set eq.(3.15) . The derivation on the statistical factors do not give any contribution so that the final result is simply expressed as

$$\Gamma^{SE} = \frac{\Gamma^0}{4Q^4v^4}\left[(1-2\epsilon)G(p_1, m) - Q^2v^2\frac{\partial}{\partial\widehat{m}^2}G(p_1, \widehat{m})\Big|_{\widehat{m}=m}\right] \tag{4.6}$$

where the first term on the right hand side arises from the derivation of the phase space factor. The tedious part is now the evaluation of the eq.(4.4) . We shall not give here the full off-shell expression but only the relevant on-shell results. We find

$$\begin{aligned} G(p_1, \widehat{m})|_{\widehat{m}=m} &= -4Q^2v^2(\delta m_T^2 + 2m^2\widehat{C}_F) \\ \frac{\partial}{\partial\widehat{m}^2}G(p_1, \widehat{m})|_{\widehat{m}=m} &= 4(2q.K(\widehat{p}_1) + 2m^2Q^2v^2(\widehat{C}_B' - \widehat{C}_F') + (Q^2 + 4m^2)\widehat{C}_F) \end{aligned} \tag{4.7}$$

where the various functions are defined in eq.(3.17) . Combining all these results and similar ones for the fermion p_2 we obtain again eq.(3.20) where the phase space factor v_T has now been expanded to $O(e^2)$.

As mentioned before, we did not expand the self-energy contribution until late in the calculation when we had to deal with scalar expressions. This was done in order to avoid expanding matrix expressions which are more complicated to deal with than scalar ones. Had we followed the usual approach and expanded Σ we would have, of course, gotten the same result both in the summation method and in the direct calculation. In order to make the connection with the usual renormalization more obvious we will come back to this point later.

V. Discussion of the Results

To ease the comparison with previous works we rewrite eq.(3.20) so as to separate off the explicit dependence on the quark mass m and we obtain

$$\Gamma^0 + \Gamma^{SE} = \Gamma^0 \, \frac{v_T}{v}\Big[1 - \frac{2q.K(\hat{p}_1) - 2q.K(\hat{p}_2)}{Q^2} - 2\hat{C}_F$$
$$- 2m^2(\hat{C}'_B(\hat{p}_1) + \hat{C}'_B(\hat{p}_2) - \hat{C}'_F(\hat{p}_1) - \hat{C}'_F(\hat{p}_2)) \qquad (5.1)$$
$$- \frac{4m^2}{Q^2 v^2}(\frac{2q.K(\hat{p}_1) - 2q.K(\hat{p}_2)}{Q^2} + 4\hat{C}_F)\Big]$$

We turn now to special cases. The simplest one is the case of a massless fermion at $T = 0$. As is obvious from the above equation, many terms drop out and the evaluation of the remaining integrals is very simple. In that limit we find

$$2q.K(\hat{p}) = -F(\epsilon)\frac{1-\epsilon}{\epsilon}\text{sign}(p^0)\frac{\alpha}{\pi}Q^2\int_0^\infty z^{1-2\epsilon}(n_B(\tfrac{1}{2}Qz) + n_F(\tfrac{1}{2}Qz))\,dz \qquad (5.2)$$

where we have defined

$$F(\epsilon) = \left(\frac{4\pi\mu^2}{Q^2}\right)^\epsilon \frac{\Gamma(1-\epsilon)}{\Gamma(1-2\epsilon)} \qquad (5.3)$$

Introducing a notation reminiscent of the wave function renormalization approach we write

$$2q.K(\hat{p}) = Q^2(Z_2^{-1} - 1) \qquad (5.4)$$

with this definition of Z_2 identical to that of ref.[9] (see eq.(3.24) in that paper) where the color factor has been ignored here since we work in QED. We also note that the function $\widehat{C}_F$ vanishes in the massless fermion limit so that the self-energy correction takes the very simple form [15]

$$\begin{aligned}\Gamma^0 + \Gamma^{SE} &= \Gamma^0 v_T(1 - 2(Z_2^{-1} - 1)) \\ &= \tanh(Q/4T)\frac{g^2}{8\pi}Q^2\left(1 - \frac{4m_T^2}{Q^2}\right)^{1/2}(1 - 2(Z_2^{-1} - 1))\end{aligned} \tag{5.5}$$

The factor v is equal to 1 in the limit we consider. The divergent factor $Z_2^{-1} - 1$ compensates with the contribution coming from the emission of a gluon [8-12]. The behavior in m_T^2 of eq.(5.5) is to be contrasted with the behavior in m^2 of eq. (3.2): the dynamically generated mass appears only in the phase space factor.

The next simplest example is with $m \neq 0$ but assuming $T \ll m$: all thermal factors on the internal fermion legs can then be neglected because they are of $O(e^{-m/T})$ and therefore the integrals $\widehat{C}_F$ and $\widehat{C}'_F$ drop out. We also set $\epsilon = 0$ so that the results can be compared with those of refs.[7] and [10]. Following the notation and the normalization conventions of ref.[10] we find in that limit

$$\begin{aligned}2q.K(\widehat{p}) &= \mathrm{sign}(p^0)\frac{\alpha}{\pi}\frac{4}{v}\ln\left(\frac{1+v}{1-v}\right)\int \omega n_B(\omega)\, d\omega \\ &= \mathrm{sign}(p^0)\frac{2\alpha}{\pi^2}QI^0\end{aligned} \tag{5.6}$$

and

$$\begin{aligned}\widehat{C}'_B &= -\frac{\alpha}{\pi}\frac{1}{m^2}\int n_B(\omega)\frac{d\omega}{\omega} \\ &= -\frac{\alpha}{\pi^2}\frac{1}{2m^2}I_A\end{aligned} \tag{5.7}$$

We can now define the wave function renormalization factor from the first line of eq.(5.1)

$$Z_2^{-1} = 1 - \frac{\alpha}{\pi^2}(I_A - 2\frac{I^0}{Q}) \tag{5.8}$$

in agreement with ref.[10]. The constant terms cannot be compared since they were not calculated in the previous work. Needless to say that our result is also in complete agreement with ref.[7] for Z_2 as well as for the constant terms.

It is interesting to remark that agreement with ref.[10] is obtained even though it is chosen there, by convention, to expand around the mass shell condition keeping $|\vec{p}|$ fixed, a choice which, we believe, is not appropriate in the case we consider. A difference would appear, however, in the general case (not restricting $T \ll m$) when the thermal factor on the fermion propagator also contributes. It is localized in the term $m^2\ \widehat{C}'_F$ in eq.(3.20) , since $\widehat{C}'_F$ depends explicitly , unlike all the other terms, on the explicit expression of $d(k.p)/dp^2$.

In the methods for calculating the contribution of the self-energy presented in Sec.3 and Sec.4 the separation between Z_2 and the rest of the terms appears rather artificial. We discuss here how to make the connection between the various approaches clearer and show that all methods give, indeed, the same results. This relies on the expansion of $\Sigma(p)$ around the mass shell condition. For simplicity we again neglect the terms proportional to n_F. Following [6] we decompose $\mathrm{Re}\Sigma(p)$ as

$$\mathrm{Re}\Sigma(p) = \frac{\alpha}{\pi^2}(I_A(\not p - m) + \not J + (p^2 - m^2)\not L) \tag{5.9}$$

where the various pieces are defined by

$$\begin{aligned} \frac{\alpha}{\pi^2} I_A = &\ 2m^2\ 2(e\mu^\epsilon)^2 \int \frac{d^n k}{(2\pi)^{n-1}} \frac{n_B(k)\delta(k^2)}{((\widehat{p}+k)^2 - m^2)^2} \\ = &-2m^2\ \widehat{C}'_B \end{aligned} \tag{5.10}$$

$$
\begin{aligned}
\frac{\alpha}{\pi^2} \not{I} = & \ 2(1-\epsilon)(e\mu^\epsilon)^2 \int \frac{d^n k}{(2\pi)^{n-1}} \not{k} \frac{n_B(k)\delta(k^2)}{(\widehat{p}+k)^2 - m^2} \\
= & \ \not{K}(\widehat{p})
\end{aligned}
\tag{5.11}
$$

$$
\frac{\alpha}{\pi^2} \not{L} = -2(1-\epsilon)\,(e\mu^\epsilon)^2 \int \frac{d^n k}{(2\pi)^{n-1}} \frac{n_B(k)\delta(k^2)}{((\widehat{p}+k)^2 - m^2)^2} \not{k} \left.\frac{d(2k.p)}{dp^2}\right|_{p^2=m^2}
\tag{5.12}
$$

The last expression is a new integral not introduced in the previous sections. The propagator at $O(e^2)$

$$
\begin{aligned}
S^{(1)}(p) = & \ \frac{i}{\not{p} - m - \mathrm{Re}\Sigma(p) + i\eta} \\
= & \ i\,(1 + \frac{\alpha}{\pi^2} I_A) \frac{\not{p} - \frac{\alpha}{\pi^2}\not{I} + m}{p^2 - \frac{\alpha}{\pi^2} 2p.I - \frac{\alpha}{\pi^2}(p^2 - m^2) 2p.L - m^2}
\end{aligned}
\tag{5.13}
$$

The integral $\not{I}$ depends on the on-shell vector $\widehat{p}$ and one can expand

$$
2p.I = 2\widehat{p}.I + (p^2 - m^2) \left.\frac{dp.I}{dp^2}\right|_{p^2=m^2}
\tag{5.14}
$$

The first term is related to the mass shift δm_T^2 while the second one compensates the term in $2p.L$ in eq.(5.13) . We then find

$$
S^{(1)}(p) = \ i\,(1 + \frac{\alpha}{\pi^2} I_A) \frac{\not{p} - \frac{\alpha}{\pi^2}\not{I} + m}{p^2 - m^2 - \delta m_T^2}
\tag{5.15}
$$

which is identical to eq.(3.18) up to terms in $\widehat{C}_F$ and $\widehat{C}'_F$ which are neglected here. It also allows, following [6], to calculate Z_2 which is defined in coordinate space as

$$
S^R(x-y) = Z_2^{-1}\ S^{(1)}(x-y)
\tag{5.16}
$$

and with the procedure of ref.[6] eq.(5.8) is immediately recovered.

Instead of using eq.(5.15) directly in the calculation, Donoghue et al. perform a finite mass renormalization and introduce a thermal wave function which satisfies

$$
(\not{p} - \frac{\alpha}{\pi^2}\not{I} - m)\ \psi_T = 0
\tag{5.17}
$$

and a counter-term $\frac{\alpha}{\pi^2} I\!\!/$ in the Lagrangian. The propagator to $O(e^2)$ in the renormalized theory is then

$$\frac{i}{\not p - m - \frac{\alpha}{\pi^2} I\!\!/} + \frac{i}{\not p - m - \frac{\alpha}{\pi^2} I\!\!/}(-i\frac{\alpha}{\pi^2}\mathrm{Re}\Sigma(p))\frac{i}{\not p - m - \frac{\alpha}{\pi^2} I\!\!/} + \frac{i}{\not p - m - \frac{\alpha}{\pi^2} I\!\!/}(i\frac{\alpha}{\pi^2} I\!\!/)\frac{i}{\not p - m - \frac{\alpha}{\pi^2} I\!\!/} \tag{5.18}$$

The first term is the renormalized lowest order contribution while the last two pieces are the $O(e^2)$ correction and the counter-term respectively. Expanding the lowest order propagator in terms of $\frac{\alpha}{\pi^2} I\!\!/ /(\not p - m)$ one immediately cancels, to $O(e^2)$, the counter-term. The higher order correction can be evaluated as before and we recover eq.(5.15) . It has been said that the renormalization approach with a non scalar counter-term gave the wrong results. The reason for this claim was that the lowest order contribution was then incorrectly assumed to be $i/(\not p - m - \delta m_T)$ with $\delta m_T = 2\frac{\alpha}{\pi^2} p.I/2m$ and the non scalar nature of the thermal contribution to the mass was consequently not properly taken into account.

On the other hand, following ref.[7], we can adopt δm_T as defined above as a counter-term and $(\not p - m - \frac{\alpha}{\pi^2} I\!\!/)\psi_T = 0$ as the Dirac equation at finite temperature. Then the perturbative series takes the form

$$\frac{i}{\not p - m - \delta m_T} + \frac{i}{\not p - m - \delta m_T}(-i\frac{\alpha}{\pi^2}\; Re\Sigma(p))\frac{i}{\not p - m - \delta m_T} + \frac{i}{\not p - m - \delta m_T}(i\delta m_T)\frac{i}{\not p - m - \delta m_T} \tag{5.19}$$

It is easy to see, expanding the first term in $\delta m_T/(\not p - m)$, that the counter-term is exactly cancelled leaving us once again with the the result eq.(5.15) . Unlike what is said in ref.[7] we therefore do not see any ambiguity related to the choice of the counter-term.

It is clear, in view of the above discussion, that, for practical calculations the renormalization approach, besides being unnecessary, is not particularly elegant since the counter-terms do not have the usual simplicity of the $T = 0$ calculation. In the first case it amounts to introduce a chirality preserving but momentum dependent piece in the Lagrangian. In the second approach, the counter-term breaks chiral symmetry which is against standard knowledge concerning the high temperature corrections. Futhermore, in the general case when the fermion thermal contribution is not neglected, it would be $\delta m_T = 2\frac{\alpha}{\pi^2}p.I/2m + m\widehat{C}_F/2$ which is again momentum dependent as can be seen from eq.(3.17) .

VI. Conclusions

We have presented several ways to calculate the contribution of the self-energy diagram to a physical process such as Higgs production in a plasma in equilibrium. The importance of correctly defining the off-shell behavior of the self-energy correction was discussed. We have shown that all methods agree and that no ambiguity remains in the evaluation of this diagram. The dynamically generated fermion mass at finite temperature cannot be treated simply as a scalar mass term. This has consequences on the structure of the finite correction terms.

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