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Cosmology of f-essence with inhomogeneous viscous fluid

S Myrzakul[1,2], M Arzimbetova[3], M Imankul[3], R Myrzakulov[1]

[1]Eurasian International Center for Theoretical Physics and Department of General and Theoretical Physics, L.N. Gumilyov Eurasian National University, Nur-Sultan, 010008, Kazakhstan

[2] Department of Physics, School of Science and Technology, Nazarbayev University, Nur-Sultan, 010000, Kazakhstan

[3] Department of Theoretical and Nuclear Physics, al-Farabi Kazakh National University, Almaty, 050040, Kazakhstan

E-mail: srmyrzakul@gmail.com

Abstract. F-essence is a generalized form of fermion fields. In this paper, we study the dynamics of f-essence with a viscous fluid in the flat Friedmann-Robertson-Walker universe. Various types of viscous fluids are analyzed and the possibility of reproducing the current accelerating expansion of the Universe is investigated. The cosmological parameters of this model are determined. It is shown that accelerated expansion can also be obtained with a viscous fluid of the fermion field, as in the case of non-interacting perfect fluid considered in most modern models of accelerated expansion of the Universe.

1. Introduction

At the end of the last millennium astronomers, observing type Ia supernovae in distant galaxies from us, found that their brightness is lower than the supposed theory based on the standard candle method. Based on this fact, it was concluded that our Universe is expanding with acceleration. In addition, observations of cosmic microwave background, gravitational lensing, and nucleosynthesis by the Big Bang measurements also confirmed this conclusion. The discovery that our Universe is expanding rapidly [1, 2, 3, 4, 5, 6, 7, 8, 9, 10], led to the revision of the standard gravitational model of the development of the Universe. To describe this phenomenon, two main directions are developing: alternative theories or a modification of the theory of gravity.

The first direction is based on the fact that our Universe is accelerated by some hypothetical substance - dark energy [11, 12, 13, 14, 15, 16], which is evenly distributed throughout the space and is not interwoven under the influence of gravity, and also has strong negative pressure. Based on the above properties, cosmologists build various types of equations of state of dark energy. According to recent cosmological observational data, state parameter of dark energy ω is limited in the range $\omega = -0.972^{+0.061}_{-0.060}$, so there are different forms of dark fluid (phantom, quintessence, inhomogeneous fluids, *etc.*[17, 18]) satisfying a suitable equation of state are alternatives to dark energy [20, 21, 22]. There are various alternatives to the theory of gravity, replacing the ideal fluid with various more real fluids. One of the simplest forms of fluids is to take into account inhomogeneous viscosity, due to which it has become possible to understand some common features of such alternative theories [23, 24, 25].

The second direction is a change in Einstein's gravity and some combination of curvature invariants (Riemann tensor, Weyl tensor, Ricci tensor, etc.), which replace or add to the classical Hilbert-Einstein action of the general theory of relativity. Thus, the current acceleration is created by some (sub)-dominant members of the gravitational effect, which become significant at small curvatures (see [18, 19] for a review).

Earlier in [26, 27], we considered various types of inhomogeneous viscous fluids in flat Friedmann-Robertson-Walker space-time and investigated the possibility of reproducing the current cosmic acceleration, providing various future evolutions relative to the case of the Cosmological constant. In this paper, we want to investigate the equation of state of a viscous fluid for a generalized fermion field - f-essence, which was first proposed in Ref.[28]. Fermion fields, consisting of elementary particles with half spins, especially the Dirac 1/2 spin, play an important role at the micro level. However, in cosmology, the role of fermion fields was usually considered limited. After some noticeable works [29, 30, 31, 32, 33, 34, 35, 36, 37, 38], the importance of fermion fields in the study of the evolution of the Universe became clear. In Ref. [28], the authors investigated the reconstruction of the f-essence and the Chaplygin fermion gas of the dark energy model, then in Ref. [39, 40] we investigated by Noether symmetry approach in f- essence cosmology with scalar-fermion interaction and dynamics of f-essence in the frame of the Starobinsky model, as a result showed a good agreement with the observational data.

The work is organized as follows. First, we form a model of f-essence in flat Friedmann-Robertson-Walker (FRW) space-time and find the equations of motion. Further, using the equation of state of a viscous fluid, we obtain the Lagrangian dependence of the f-essence on the type of viscosity. In this paper, with a constant state parameter, we consider two types of viscosity: constant and proportional to the Hubble parameter. We also find the dependence of cosmological parameters on time. In the end, we give remarks on the results obtained.

We use units of $k_{\rm B} = c = \hbar = 1$ and $8\pi/M_{Pl}^2 = 1$, where M_{Pl} is the Planck Mass.

2. R gravity model with f-essence

The action of f-essence reads as

$$S = \int d^4x\sqrt{-g}[R + L_f], \tag{1}$$

where g is the determinant of the metric tensor $g_{\mu\nu}$ and R denotes the scalar curvature (the Ricci scalar). Here

$$L_f = 2K(Y, \psi, \bar{\psi}) \tag{2}$$

is the Lagrangian density of the f-essence field, Y is the canonical kinetic term of the fermionic field, $\psi = (\psi_1, \psi_2, \psi_3, \psi_4)^T$ is a fermionic function and $\bar{\psi} = \psi^\dagger\gamma^0$ is its adjoint function, the dagger represents complex conjugation. The canonical kinetic term for the fermionic field is

$$Y = 0.5i[\bar{\psi}\Gamma^\mu D_\mu\psi - (D_\mu\bar{\psi})\Gamma^\mu\psi], \tag{3}$$

where D_μ is the covariant derivative and

$$\gamma^0 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}, \quad \gamma^k = \begin{pmatrix} 0 & \sigma^k \\ -\sigma^k & 0 \end{pmatrix}, \quad \gamma^5 = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}, \tag{4}$$

are the Dirac gamma matrices. Note that the fermionic fields are treated here as classically commuting fields.

In general, the equations corresponding to the action (1) have a very complicated form. For clarity we consider here the simple cosmological metric, namely, the homogeneous, isotropic and flat FRW universe filled with f-essence. This metric is given by

$$ds^2 = -dt^2 + a(t)^2(dx^2 + dy^2 + dz^2), \tag{5}$$

where $a(t)$ is the scale factor of the Universe. Also we have

$$R = 6\left(\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2}\right), \ Y = \frac{1}{2}i\left(\bar{\psi}\gamma^0\dot{\psi} - \dot{\bar{\psi}}\gamma^0\psi\right), \tag{6}$$

where a dot denotes a time derivative and since we consider a homogeneous and isotropic metric, from this we assume that the field of the f-essence depends only on time, i.e. $\psi = \psi(t)$.

The preliminary set-up for writing the equations of motion is now complete. So for the FRW metric (3), taking into account $H = \dot{a}/a$ denotes the Hubble parameter, the equations of motion corresponding to the action (1) look like [37]

$$3H^2 - \rho_f = 0, \tag{7}$$

$$2\dot{H} + 3H^2 + p_f = 0, \tag{8}$$

$$K_Y\dot{\psi} + 0.5(3HK_Y + \dot{K}_Y)\psi - K_{\bar{\psi}}i\gamma^0 = 0, \tag{9}$$

$$K_Y\dot{\bar{\psi}} + 0.5(3HK_Y + \dot{K}_Y)\bar{\psi} + K_{\psi}i\gamma^0 = 0, \tag{10}$$

where $K_Y = \partial K/\partial Y, K_\psi = \partial K/\partial\psi, K_{\bar{\psi}} = \partial K/\partial\bar{\psi}$. In the Friedmann equations, p and ρ are the pressure and the energy density of the fluid contents of the universe which must satisfy the conservation law,

$$\dot{\rho_f} + 3H(\rho_f + p_f) = 0. \tag{11}$$

Here the energy density and the pressure of the f-essence take the following form

$$\rho_f = K_Y Y - K, \quad p_f = K, \tag{12}$$

respectively. Also we remark that as $K = Y - V(\bar{\psi}, \psi)$, the model (1) gives the usual Einstein-Dirac theory.

From the equations (9),(10) we obtain

$$\bar{\psi}\psi = \frac{\alpha}{a^3 K_Y}, \tag{13}$$

where α is a integrable constant. We have four independent equations (7)-(10), and five unknown variables, namely $H, \rho_f, p_f, \psi, \bar{\psi}$ and K to be solved as functions of time. In the following section we choose the equation of state of inhomogeneous viscous fluids and try to solve for all cosmological parameters.

3. Inhomogeneous viscous fluids in the flat FRW space-time

The equation of state of inhomogeneous viscous fluids in the flat FRW space-time is given by [20, 21, 26]

$$p = \omega(\rho)\rho + B(\rho, a(t), H, \dot{H}...), \tag{14}$$

where the EoS parameter, $\omega(\rho)$, may depend on the energy density, and the bulk viscosity $B(\rho, a(t), H, \dot{H}...)$ is a general function of the fluid energy density, the scale factor, the Hubble parameter and its derivatives. In this paper we will consider a simple formulation of such equation with $\omega = const$, namely

$$p = \omega\rho_f - 3H\zeta(H), \tag{15}$$

where $\zeta(H)$ is the bulk viscosity and it depends on the Hubble parameter H only. On thermodynamical grounds, in order to have the positive sign of the entropy change in an

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irreversible process, $\zeta(H)$ has to be a positive quantity, so that we assume $\zeta(H) > 0$ [24, 25]. Moreover, for the stress-energy tensor of fluid $T_{\mu\nu}$ one has

$$T_{\mu\nu} = \rho_f u_\mu u_\nu + [\omega(\rho_f)\rho_f - 3H\zeta(H)]\,(g_{\mu\nu} + u_\mu u_\nu)\,, \tag{16}$$

where $u_\mu = (1, 0, 0, 0)$ is the four velocity vector. The fluid energy conservation law reads

$$\dot{\rho_f} + 3H\rho_f(1 + \omega(\rho_f)) = 9H^2\zeta(H)\,. \tag{17}$$

In what follows, we will analyze the behavior of such a kind of fluids in the FRW universe with the f-essence. We are interested in fluids which provide a viable cosmology today but a different future evolution with respect to the Cosmological Constant case.

We remind that the effective EoS paramter for fluid (15) reads

$$\omega_{\text{eff}} := \frac{p}{\rho} = \omega(\rho) - \frac{3H\zeta(H)}{\rho}\,. \tag{18}$$

Cosmological data imply $\omega_{\text{eff}} \simeq -1$. In the case of perfect fluids, it follows that ω must be very close to the value of minus one, but for different kinds of non perfect fluid other possibilities are allowed.

3.1. Constant viscosity

Let us remind the case of constant bulk viscosity $\zeta(H) = \zeta_0$, $\zeta_0 > 0$. From the equations of motion (7), (8) with (15) we find the dynamics of the scale factor, the Hubble parameter, density of the f-essence as

$$a(t) = a_0\left(e^{\frac{3}{2}\zeta_0(t-t_0)}(1+\omega) - 1\right)^{\frac{2}{3(1+\omega)}}\,, \tag{19}$$

$$H(t) = \frac{\zeta_0}{1+\omega - e^{-\frac{3}{2}\zeta_0(t-t_0)}}\,, \tag{20}$$

$$\rho_f(t) = \frac{3\zeta_0^2}{\left(1+\omega - e^{-\frac{3}{2}\zeta_0(t-t_0)}\right)^2}, \tag{21}$$

where a_0 is an integration constant. Then we obtain with the gravitational equation of motion (9), (10) the Lagrangian of the f-essence field

$$K(Y, \psi, \bar{\psi}) = C_1 Y^{\frac{1+\omega}{\omega}} - \frac{3\zeta_0^2}{(1+\omega)\left(1+\omega - e^{-\frac{3}{2}\zeta_0(t-t_0)}\right)}. \tag{22}$$

We immediately see that, if $\omega \to -1$, then the kinetic term $C_1 Y^{\frac{1+\omega}{\omega}} \to 0$ and the potential term $\frac{3\zeta_0^2}{(1+\omega)\left(1+\omega - e^{-\frac{3}{2}\zeta_0(t-t_0)}\right)} \to \infty$.

The deceleration parameter q is

$$q = -\frac{a\ddot{a}}{\dot{a}^2} = -1 + \frac{3}{2}e^{-\frac{3}{2}\zeta_0 t}, \tag{23}$$

if $q(t \to \infty) = -1 < 0$ negative, then $\ddot{a} > 0$ so the expansion of the universe is "accelerating".

3.2. Viscosity proportional to H

This is the case $\zeta(H) = H$. We find the dynamics of the scale factor, the Hubble parameter and density in the following form

$$a(t) = a_0\,(t-t_0)^{\frac{2}{3\omega}}\ , \tag{24}$$

$$H(t) = \frac{2}{3\omega}\frac{1}{t-t_0}\,, \tag{25}$$

$$\rho_f(t) = \frac{4}{3\omega^2}\frac{1}{(t-t_0)^2}. \tag{26}$$

Thus, the f-essence field Lagrangian corresponding to a inhomogeneous viscous fluid for this case reads

$$K(Y,\psi,\bar{\psi}) = C_2 Y^{\frac{1+\omega}{\omega}} - \frac{4}{3\omega^2(1+\omega)(t-t_0)^2}, \tag{27}$$

the deceleration parameter is

$$q = -\frac{a\ddot{a}}{\dot{a}^2} = -1 + \frac{3}{2}\omega = const, \tag{28}$$

which is constant throughout the evolution of the universe. For q to be positive, we must take ω larger than $\frac{2}{3}$, which determines the expansion of the universe at a slower rate. If we accept the negative value of q, i.e. ω will be less than $\frac{2}{3}$, then the universe expands rapidly. Thus, for suitable values of ω we can get a slow and accelerated expansion of the Universe. In this case, the model does not show a phase transition due to a constant q value.

4. Conclusion

Within the framework of cosmological gravitational field the equivalence between the inhomogeneous viscous fluid (and dark energy) and nonlinear f-essence has been established. It is shown that different types of the inhomogeneous viscous fluid can be simulated by means of the nonlinear f-essence. Using the new description of the inhomogeneous viscous fluid or dark energy evolution of the Universe has been studied within the scope of the isotropic FRW model. The corresponding Einstein equations have been solved. Two types of viscosity are considered: constant and proportional to the Hubble parameter. It is shown that at a constant viscosity the Universe expands with acceleration, and for the second case it depends on the choice of the parameter ω. Furthermore, in both cases, when the cosmological constant with $\omega = -1$ it is shown that the potential energy of the f-essence field tends to infinity.

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