Studies on Unpolarised and Polarised Non-singlet Structure Functions and Associated Sum Rules Including Higher Order Perturbative and Nonperturbative QCD Corrections

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Dedicated to My beloved

Baba 'Jatin Nath'

and

Maa 'Nanibala Nath'

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Abstract

Particle physics is a quest for the fundamental building blocks of the matter. As more than 99.9 % of the mass of any object in our solar system is carried by its nucleons, exploring the structure of the nucleon is a vital part of our effort to understand the structure of matter. A complete understanding of nucleon structure is expected to achieve through coherent interplay of Deep Inelastic Scattering (DIS) experiments at all energy scale, together with a proper theoretical approach. Our most reliable knowledge concerning the internal structure of the nucleon has been achieved through DIS off high energy beams of electrons, muons and neutrinos. DIS processes are described in terms of the structure functions which are expressed as a functions of the momentum fraction x of the nucleon carried by the parton and the four momentum transfer squared Q^2 and these functions are directly related to the distribution of quarks and gluons inside a nucleon. These DIS structure functions are the objects of intensive investigation both theoretically and experimentally in order to understand the underlying theory of strong interaction. With the recent developments of dedicated experimental facilities significant progresses have been observed in the field of experimental investigation of structure functions. Simultaneously, in this regard, tremendous progress is observed in the field of theoretical investigation with a variety of theoretical approaches.

Quantum Chromodynamics (QCD) and Regge theory are two important approaches in order to account for the strong interaction processes observed at high energy particle colliders. However, the predictive power of both approaches is limited. Specifically, Regge theory has its predictions on the x dependence of the structure functions within the Regge limit ($x \to 0$, Q^2 fixed and of the order of a typical hadronic scale) and QCD is successful in describing Q^2 dependency of the structure functions in accord with DGLAP (Dokshitzer-Gribov-Lipatov-Altarelli-Parisi) evolution equations in the perturbative regime i.e., within the Bjorken limit ($Q^2 \gg 1$, x fixed and not too small). However the most important region in DIS, which has attracted much interest recently is the small-x region, lies between the interface of Bjorken limit and the Regge limit.

Despite limitations in themselves, the combination of QCD and Regge theory is expected to provide proper understanding of the structure functions, particularly in the small-x region. In QCD the structure functions are governed by a set of

integro-differential equations, known as the DGLAP equations. Due to its complicated mathematical structure, an exact analytic determination of the structure functions is currently out of reach and one needs to apply approximated methods to arrive on predictions from the theory. Therefore, in current analysis this set of equations are usually solved numerically by using an initial input distribution of the structure function at a fixed Q^2 , in terms of some free parameters, the parameters are so adjusted that the parameterization best fit the existing data. In order to perform a fit one must start with a particular ansatz for the structure functions at some reference Q_0^2 . In most of the existing fitting analysis, including those in the experimental papers it has been performed by assuming a simple power behavior based on Regge theory. Although many parameterizations are available in literature in order to predict the initial distribution of structure functions to DGLAP equation, but most of them are full of different constraints and suffer of several drawbacks. Therefore explorations of the possibility of obtaining accurate solutions of DGLAP evolution equations with less number of parameters are always interesting. Under this motivation, this thesis is devoted to the exploration of a semi-analytic approach of solving DGLAP equation for non-singlet structure functions using two Regge inspired model with less number of parameters. Here particular emphasis is given to the non-singlet structure functions because they are considered as the starting ground for theoretical description of DIS structure functions. Besides being interesting in themselves, another significant advantage is that QCD analysis by means of non-singlet structure functions is comparatively technically simpler. This thesis concerns with the usefulness of the combination of Regge theory and QCD in order to have reliable understanding of both the spin independent and spin dependent non-singlet structure functions and determination of various sum rules associated with them. Here we explicitly specify how the usefulness of two Q^2 dependent Regge ansatz, utilized as the required initial input to the DGLAP evolution helps in obtaining the small-x behaviour of the non-singlet structure functions, $F_2^{NS}(x,Q^2)$, $xF_3(x,Q^2)$ and $xg_1^{NS}(x,Q^2)$. Obtained small-x behaviour of these non-singlet structure functions are then utilized to calculate the sum rules, Gottfried sum rule(GSR), Gross-Llewellyn Smith(GLS) sum rule and Bjorken Sum Rule (BSR), which are associated to $F_2^{NS}(x, Q^2)$, $xF_3(x, Q^2)$ and $xg_1^{NS}(x, Q^2)$ respectively. In addition to the prediction of structure functions and sum rules we have paid attention to their precision. Precise prediction of structure functions demand

to incorporate the standard higher order approximation of pQCD and several nonperturbative effects. In this regard particular emphasis is given to the determination of structure functions and sum rules with pQCD corrections up to next-next-to-leading order (NNLO) and to the inclusion of the special non-perturbative effects, shadowing effect and higher twist effect.

The outline of the thesis is as follows:

In **Chapter 1**, we have given a brief introduction to our current views of the basic building blocks of matter, deep inelastic scattering, structure functions, parton model, Regge theory and Quantum Chromodynamics and higher order corrections, various sum rules, non-perturbative QCD corrections such as nuclear effect, higher twist effect etc.

Chapter 2 provides a general overview about the recent lepton deep inelastic scattering measurements which have enriched our phenomenological analysis performed in this thesis. Specifically the experimental results for non-singlet structure functions and associated sum rules for both polarized and unpolarized cases measured in electron, muon and neutrino DIS are reviewed. In addition, several parametrization associated with the determination of non-singlet structure functions are discussed.

In Chapter 3, along with a qualitative analysis of the available methods to solve DGLAP equation, I have allude the usefulness of two Q^2 dependent Regge ansatz in solving DGLAP equation in order to have the small-x behaviour of both the spin independent and spin dependent non-singlet structure functions. By means of fitting analysis, we have investigated the compatibility of the two ansatz with the available experimental data and then studied the possible role played by them in evolving the non-singlet structure functions.

Chapter 4 encompasses the evolution of the non-singlet structure function $F_2^{NS}(x,Q^2)$ in charged lepton DIS by means of solving the DGLAP equations in LO, NLO and NNLO using the Regge ansatz as the initial input. Both the Q^2 and x evolutions of $F_2^{NS}(x,Q^2)$ structure functions, thus obtained are analysed phenomenologically in comparison with the experimental measurements taken from NMC and the results of NNPDF parametrization.

Chapter 5 concerns with the determination of small-x behaviour of the nonsinglet structure function, $xF_3(x, Q^2)$ originated in neutrino scattering. The DGLAP equation is solved up to NNLO for $xF_3(x, Q^2)$ structure function and solutions are compared with the experimental data taken from CCFR, NuTeV, CDHSW and CHO-RUS experiments and also with the recent MSTW parametrization results.

Chapter 6 deals with the understanding of the spin dependent non-singlet structure function $xg_1^{NS}(x,Q^2)$ within small-*x* region. The DGLAP equation is solved to have the Q^2 as well as *x* evolution of $xg_1^{NS}(x,Q^2)$ structure function with QCD corrections up to NNLO and perform a phenomenological analysis of our results in comparison with different experimental data taken from COMPASS, HERMES, E143 and JLab experiments, along with other available theoretical as well as phenomenological analysis.

Chapter 7 utilises the small-x behaviour of $F_2^{NS}(x, Q^2)$, $xF_3(x, Q^2)$ and $xg_1^{NS}(x, Q^2)$ structure functions obtained in the previous chapters in prediction of sum rules associated with them, viz., Gottfried sum rule, Gross-Llewellyn Smith sum rule and Bjorken sum rule respectively. These sum rules are calculated incorporating higher order pQCD corrections up to NNLO and analysed phenomenologically by comparing with their respective experimental results and available theoretical predictions.

In Chapter 8, we present an analysis of the non-singlet structure functions and related sum rules taking into account the nuclear effects. In this regard, special attention is given to the nuclear shadowing effect as we are mostly concerning with the small-x region. The corrections due to nuclear shadowing effect, predicted in several earlier analysis are incorporated to our results of structure function and sum rules for free nucleon and calculate the nuclear structure functions as well as sum rules for nuclei. The calculations are analysed phenomenologically in comparison with available experimental data and achieved at a very good phenomenological success in this regard.

In Chapter 9, the higher twist corrections to the non-singlet structure functions and sum rules associated with them are studied. Here, possible improvement in the accuracy of our results for the non-singlet structure functions and sum rules due to the inclusion of relevant higher twist terms is investigated. Based on a simple model we have extracted the higher twist contributions to the non-singlet structure functions and sum rules in NNLO perturbative orders and then incorporated them with our results. Our NNLO results along with higher twist corrections are observed to be compatible with experimental data..

Finally in **Chapter 10**, We have presented the overall conclusion drawn from our work. $\Box\Box$

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Chapter 1

Introduction

In this chapter we have given a brief introduction to our current views of the basic building blocks of matter, deep inelastic scattering, structure functions, parton model, Regge theory and Quantum Chromodynamics and higher order corrections, various sum rules, non-perturbative QCD corrections such as nuclear effect, higher twist effect etc.

1.1 Our Current Views of Nature's Building Blocks

Particle physics is a quest for the fundamental building blocks of the matter, and the fundamental forces that operate to control and shape them. The pursuit for finding the "real" nature of the Universe is not only a means to satisfy instinctive curiosity but also a principal tool for the advancement and progress of civilization[1].

The search for the elementary constituents of nature has occupied generations of human beings since the speculations of the early Greek philosophers and other philosophers from different parts of the world. As far it is known, in the sixth century B.C. Thales proposed that all things reduced to water, and, coming out of the Greek-Roman eras and for centuries to come, the four basic elements were thought to be earth, water, fire, and air. Chinese (in Pinyin, Wu Xing) believed that these were earth, wood, metal, fire and water. Indians (Samkhya-Karika by Isvarakrsna) visualized the world as made of five elements: space, air, fire, water and earth. In about 400 B. C. the Greek philosophers Democritus and Leucippus proposed that matter is composed of indivisible particles called atoms, a word derived from a-(not) and tomos (cut or divided)[2]. This idea lingered in the background for centuries until experimental support and through the work of eighteenth- and nineteenth-century chemists, brought atoms to the fore as the basic building blocks of matter.

The macroscopic quantities of homogeneous material can be divide or cut into parts such that each part retains the basic character of the original. But how far can such division be carried? If we cut a piece of gold into smaller and smaller snips, do we always get pieces of the gold? Is it possible that the divisions can go on forever, generating smaller and smaller snips of gold, or is there a limit such that no further divisions can be made or at least no further pieces leaving the parts as gold? If so what the final divisions consist of? Is there any constituent which is further indivisible?

From the earliest concepts to the resulting periodic table of elements, many small steps had been taken in our pursuit of the fundamental building blocks of nature and up to the end of nineteenth century the answer of this question was "atom". People believed that atoms are immutable and indivisible objects. By the close of the nineteenth century, however, the atoms were also under criticize and evolved the next question, "What are atoms made of?". Before discussing "What are atoms made of?", we would like to discuss something about elementary particles and how they are investigated.

The elementary particles are those particles which have no known structure, i.e., they are structure less or point like. They cannot be resolved into two or more parts. In order to investigate the possible structure of an object firstly we need to probe it by a probing beam which is scattered from the object. Analysing the diffraction pattern of scattered beam, we can remark on their structure. But whether a particle is point like or not it depends on the spatial resolution of the apparatus used. In case of an optical microscope, where the probing beam is light, the resolution is given by

$$\Delta r \simeq \frac{\lambda}{\sin\theta} \tag{1.1}$$

where λ is the de Broglie wavelength of the incident beam of particles, which is given by $\lambda = \frac{h}{p}$. Here p is the beam momentum and θ is the angular aperture of the light beam used to view the structure of an object. For the improvement of resolution we need larger θ and smaller λ . Thus we see that the resolution depends on the initial momenta of the incident particle and to resolve an object we must have a probe whose wavelength is comparable or smaller than the size of the object. Again from the uncertainty principle we have the relation, $\Delta p \Delta x \geq \hbar c \approx 0.2 GeV fm$, which suggests that smaller the distance we want to probe, the beam energy must be higher. This is the underlying idea that is used to study the structure of particles and depending on the energy of the probing beam, the concept of elementary particle has been changing.

To resolve an atom we must have a probe whose wavelength is comparable or smaller than the size of the atom. The requirement of such type of probe was fulfilled by the alpha particle (ionized Helium atoms), which was the result of discovery of radioactivity of the substances in 1886 through the work of French physicist Henri Becquerel. It was observed that the alpha particles can be deflected in magnetic fields and therefore one could expect them to serve as natural weapons to study the atomic structure, in particular its charge distribution. Rutherford utilized this opportunity to investigate the basic structure of atoms. In 1911, he performed an experiment where a beam of alpha particles of a few MeV was fired into a thin sheet of gold foil. He observed that most of the alpha particles passed through the gold completely undisturbed, but a few of them bounced off at wild angles. Based on these observations Rutherford concluded that the positive charge, and virtually all of the mass of the atom was concentrated at the center occupying only a tiny fraction of the volume of the atom[3]. Furthermore, Rutherford was able to show by explicit calculation that the angular distribution of the scattered a particles agreed with that expected if they indeed interacted with a massive scattering center of positive charge Ze and which is in support his intuitive picture of the atom. The nucleus of the lightest atom (hydrogen) was given the name proton by Rutherford and thus the proton was inferred and later it was isolated in the laboratory. However before this, J. J. Thomson demonstrated the existence of a tiny particle which is much smaller in mass than hydrogen, the lightest atom. This was the electron, the first elementary particle which was discovered by Thomson in 1897[4]. In 1932 James Chadwick discovered the other constituent of nucleus, the neutron[5].

The dimension of the atom is typically $\sim 1 \mathring{A} = 10^{-10} m \gg 10^{-15} m$, the dimension of proton. So the low energetic α particles could only resolved the atom and observed that an atom is made of a hard compact nucleus consisting of proton and neutron surrounded by a cloud of electrons. Due to the poor resolution, the proton and the neutron were regarded as point like or elementary particles. Up to 1950, the electron, proton and neutron were considered as the elementary particles. By the early 1960s, accelerators reached higher energies, $\Delta E \gg 1 GeV$, which is required to probe proton. A parallel sequence of events occurred with the proton and neutron. In the 1960s very high energy electron beams were utilized at the Stanford Linear Accelerator Center (SLAC)[6] in an experiment that was analogous to the old alpha particle one in which the atomic structure was revealed. The electron beam was fired at protons and it was observed that the beam of electrons suffered violent collisions when it met the nucleons. The observation of these violent collisions suggested that the proton's charges were concentrated on some discrete scattering centres within, which in turn indicates the evidence of substructure of the proton. Comparison of the data on electron scattering with the analogous probing by neutrino beams has enabled us to learn about the nature, or quantum numbers, of the constituents of the proton. As a result of the above experiments, we have learned that the proton and neutron are therefore not elementary, but are made instead of the pointlike "quarks". The quarks are referred as point like because they have no internal structure or, more probably, that we have not yet resolved any constituent that they may have.

We observe that to Rutherford the nucleus appeared pointlike; more powerful beams of electrons reveal the inner structure of the nuclei and progressively resolved the neutrons and protons and finally using higher energy beams or equivalently shorter wavelength probes the substructure of the proton was uncovered. The use of highenergy particles showed that as the energies of the probing beam of the particles were increased, even smaller particles were obtained, which indicates the possibility of uncovering the substructure of quarks with further higher energetic beams. The quest, "What are the building blocks of nature?" has progressed from everyday objects to molecules, molecules to atoms, atoms to electrons and nuclei, nuclei to protons and neutrons, and protons and neutrons to quarks. Whether this progression to smaller and smaller components go on forever, or there will be the end with a single fundamental particle, that will be reflected in future particle physics research.

The birth of modern experimental particle physics in which particles were used to probe the structure of composite objects began with the famous alpha particle scattering experiment of Rutherford. The experimental effort originated by the end of the 19th century and the beginning of the 20th century with physicists, like Thompson, Rutherford, Chadwick and so on, discovering the presence of subatomic particles like electrons, nucleus etc. The use of high-energy particles as probe showed that as the energies of the colliding particles were increased, even smaller particles were obtained. This led to the subsequent discovery of many particles like mesons, baryons, antiparticles, neutrinos etc. From the world of these particles, which are the outcome of many years of international effort through experiments, theoretical ideas and discussions, physicists have developed a theory called "The Standard Model" that explains the current understanding of elementary particle physics. The standard model is a simple, comprehensive, beautiful as well as the most successful theory in nature. The beauty of the Standard Model is twofold. On one hand it establishes the identity of all the elementary constituents of matter and on the other hand describes the fundamental forces that operate to control and shape matter. In accord with this model all the known matter particles are composites of quarks and leptons, held together by fundamental forces which are represented by the exchange of particles known as gauge bosons. The standard model is summarised in Table 1.1

J	Name	Symbol	Observed
0	Higgs Scalar	Н	Yes
$\frac{1}{2}$	Leptons	$e,\mu,\tau,\nu_e,\nu_\mu,\nu_\tau$	Yes
	Quarks	u, d, c, s, t, b	Yes
	Photon	γ	Yes
1	Vector Mesons	W^+, W^-, Z^0	Yes
	Gluons	$\mid g$	Yes
2	Graviton	G	No

 Table 1.1: Particles in the Standard Model.

1.2 Deep Inelastic Scattering

Deep Inelastic Scattering(DIS)(cf. e.g. [7]) experiments have had an enormous impact towards the understanding of the fundamental constituents of matter. DIS provides one of the cleanest possibilities to probe the space-like short distance structure of nucleon through the interactions

$$l^{\pm} + N \longrightarrow l^{\pm} + X, \tag{1.2}$$

$$\nu_l(\bar{\nu}_l) + N \longrightarrow l^{\pm} + X, \tag{1.3}$$

and

$$l^{\mp} + N \longrightarrow \nu_l(\bar{\nu}_l) + X. \tag{1.4}$$



Figure 1.1: Schematic representation of deep inelastic scattering.

In DIS a charged lepton $(l = e, \mu)$, or a neutrino $(\nu_l = \nu_{e,\mu,\tau})$ is scattered off the nucleon (N) and produces a lepton and a shower of hadrons (X) in the final state. In this regard as the high energetic particles (lepton) probe deep within the target (nucleon), and as the target is disrupted after scattering, it is known as deep inelastic scattering. Various deep inelastic charged and neutral current interactions provide complementary sensitivity to reveal the quark flavor and gluonic structure of the nucleon. Moreover, polarized lepton scattering off polarized targets helps in the investigation of the spin structure of the nucleons.

1.2.1 Kinematics and Variables in DIS

The DIS processes at Born level can be illustrated as shown in Fig. 1.1. Here a lepton with momentum l scatters off a nucleon of mass M and momentum P via the exchange of a virtual vector boson (photon or Z_0 or W^{\pm}) with four momentum q. The four momenta of the outgoing lepton and the hadronic final states are l' and P_F respectively. The virtual boson has space like momentum with a virtuality Q^2 , defined by

$$Q^2 \equiv -q^2, \tag{1.5}$$

where the four momentum transferred q is

$$q = l - l' = P_F - P. (1.6)$$

In addition to Q^2 and q^2 , other two important Lorentz invariant kinematic variables that describe the interaction are

$$s \equiv (P+l)^2 \tag{1.7}$$

and

$$W^2 = (P+q)^2 = P_F^2, (1.8)$$

where s is the total center of mass energy squared and W represents the invariant mass of the hadronic final state. Further, in order to describe the scattering process, the Bjorken scaling variable x, the inelasticity y, and the total total energy transfer ν of the lepton to the nucleon in the nucleon's rest frame are usually referred and they are defined by

$$\nu \equiv \frac{P.q}{M},\tag{1.9}$$

$$x \equiv \frac{-q^2}{2P.q} = \frac{Q^2}{2M\nu},$$
 (1.10)

$$y \equiv \frac{P.q}{P.l} = \frac{2M\nu}{s - M^2}.$$
 (1.11)

1.2.2 Deep Inelastic Scattering Differential Cross Sections Case 1: Charged Lepton DIS

The deep inelastic scattering differential cross section can be written in terms of products of two tensors, the leptonic tensor $L_{\alpha\beta}$ and the hadronic tensor $W_{\alpha\beta}$:

$$\frac{d^2\sigma}{dE'd\Omega'} \propto L_{\alpha\beta}W^{\alpha\beta}.$$
(1.12)

The leptonic tensor describes the lepton-photon interaction. Denoting the spin projections of the initial and final lepton by s and s' and then summing over s' the lepton tensor can be expressed in terms of two pieces which are symmetric and antisymmetric with respect to the Lorentz indices α and β :

$$L_{\alpha\beta} = L^s_{\alpha\beta} + iL^A_{\alpha\beta},\tag{1.13}$$

where $L^s_{\alpha\beta}(k,k') = 2(k_{\alpha}k'_{\beta} + k_{\beta}k'_{\alpha}) + g_{\alpha,\beta}q^2$ and $L^A_{\alpha\beta} = 2m\epsilon_{\alpha\beta\mu\nu}s^{\mu}q^{\beta}$, with the lepton spin vector defined by $2ms^{\mu} = \bar{u}\gamma^{\mu}\gamma_5 u$. For unpolarized lepton scattering the average over the initial lepton polarizations is performed and hence only the symmetric term, $L^s_{\alpha\beta}$, remains.

The hadronic tensor, $W^{\alpha\beta}$ provides complete information about the target response. The hadronic tensor can split into symmetric and antisymmetric parts:

$$W_{\alpha\beta} = W^s_{\alpha\beta} + W^A_{\alpha\beta}. \tag{1.14}$$

Lorentz and gauge invariance and symmetry properties together with parity conservation of the electromagnetic interaction imply the most general forms of these terms:

$$W_{\alpha\beta}^{s} = W_{1}(\nu, Q^{2}) \left(\frac{q_{\alpha}q_{\beta}}{q^{2}} - g_{\alpha\beta}\right) + \frac{W_{2}(\nu, Q^{2})}{M^{2}} \left(P_{\alpha} - \frac{P.q}{q^{2}}q_{\alpha}\right) \left(P_{\beta} - \frac{P.q}{q^{2}}q_{\beta}\right)$$
(1.15)

and

$$W^{A}_{\alpha\beta} = i\epsilon_{\alpha\beta\mu\nu}q^{\alpha} \bigg[G_{1}(\nu,Q^{2})S^{\nu} + \frac{G_{2}(\nu,Q^{2})}{M^{2}} (S^{\nu}P.q - P^{\nu}S.q) \bigg].$$
(1.16)

This defines four response functions $W_1(\nu, Q^2)$, $W_2(\nu, Q^2)$, $G_1(\nu, Q^2)$ and $G_2(\nu, Q^2)$. The first two can be measured in the unpolarized scattering, while the latter two require scattering of polarized leptons on polarized nucleons for their determination.

In the description of deep inelastic scattering process the response functions $W_{1,2}(\nu, Q^2)$ and $G_{1,2}(\nu, Q^2)$ are often replaced by the dimensionless structure functions $F_{1,2}(x, Q^2)$ and $g_{1,2}(x, Q^2)$, expressed in terms of the Bjorken variable x together with Q^2 :

$$F_1(x, Q^2) = MW_1(\nu, Q^2), \qquad (1.17)$$

$$F_2(x, Q^2) = \nu W_2(\nu, Q^2), \qquad (1.18)$$

and

$$G_1(x, Q^2) = M\nu G_1(\nu, Q^2), \qquad (1.19)$$

$$G_2(x,Q^2) = \nu^2 \nu G_2(\nu,Q^2).$$
(1.20)

In terms of these structure functions, the unpolarized and polarized differential cross sections can be written as(with spin denoted by $\uparrow \downarrow$)

$$\frac{d^2\sigma}{dxdy} = \frac{2\pi\alpha^2}{MEx^2y^2} \left[\left(1 - y - \frac{Mxy}{2E} \right) F_2 + xy^2 F_1 \right]$$
(1.21)

and

$$\frac{d^2\sigma\uparrow\Downarrow}{dxdy} - \frac{d^2\sigma\uparrow\Uparrow}{dxdy} = \frac{4\alpha^2}{MExy} \left[\left(2 - y - \frac{Mxy}{E}\right)G_1 - \frac{2Mx}{E}G_2 \right]$$
(1.22)

respectively.

Case 2: Neutrino DIS

Like charged lepton DIS, neutrino-nucleon($\nu - N$) DIS experiments provide a good opportunity to study the structure of nucleon. The advantage of ν -DIS measurements over charged lepton experiments is that $\nu - N$ experiments can measure the structure function xF_3 , in addition to F_1 and F_2 . In the neutrino nucleon scattering, neutrino interacts weakly with the nucleon and due to parity violation in their weak interaction the third structure function xF_3 originates and the resultant differential cross section is given by

$$\frac{d^2 \sigma^{\nu(\bar{\nu})}}{dxdy} = \frac{G_F^2 M E_{\nu}}{\pi (1 + Q^2 / M_W^2)^2} \left[y^2 x F_1 + \left(1 - y - \frac{M_N x y}{2E_{\nu}} \right) F_2 \pm (y - \frac{y^2}{2}) x F_3 \right], \quad (1.23)$$

Where G_F is the Fermi weak coupling constant and M_W is the mass of the W boson mediating the interaction. Here +(-) sign corresponds to the neutrino(antineutrino) scattering cross-section.

1.2.3 Bjorken Scaling

If the nucleon has substructure and that are resolvable for Q, $\nu \gg M$, then these W terms would be functions of the kinematic variables ν and Q^2 :

$$W_1 \longrightarrow W_1(Q^2, \nu), \quad where \quad 2MW_1(Q^2, \nu) = \frac{Q^2}{2M\nu}\delta(1 - \frac{Q^2}{2M\nu}), \quad (1.24)$$

$$W_2 \longrightarrow W_2(Q^2, \nu), \quad where \quad \nu W_2(Q^2, \nu) = \delta(1 - \frac{Q^2}{2M\nu}), \quad (1.25)$$

$$W_3 \longrightarrow W_3(Q^2, \nu), \quad where \quad \nu W_3(Q^2, \nu) = \delta(1 - \frac{Q^2}{2M\nu}).$$
 (1.26)

On the basis of analysis of various sum rules, Bjorken predicted that in the deep inelastic regime, where $Q^2 \to \infty$ and $\nu \to \infty$, the structure functions do not depend individually on (ν, Q^2) but only on their ratio $x = \frac{Q^2}{2M\nu}$. The variable x was first introduced by Bjorken and this feature is known as "Bjorken scaling" [8]. Soon after this prediction, approximate scaling behavior was observed experimentally in electron-proton scattering at SLAC[6]. The fact that the structure functions become independent of Q^2 indicates that the objects inside the nucleon from which one is scattering have no spatially extended structure, that is, one is scattering from point like constituents, known as "partons", about which we have discussed in the section 1.3.1. The scaling behavior of the structure functions are expressed as

$$MW_1 \longrightarrow F_1(x),$$
 (1.27)

$$\nu W_2 \longrightarrow F_2(x), \tag{1.28}$$

and

$$\nu W_3 \longrightarrow F_3(x). \tag{1.29}$$

A similar scaling behavior is expected for the spin-dependent structure functions

$$g_1(x, Q^2) = M^2 \nu G_1(\nu, Q^2), \qquad (1.30)$$

and

$$g_2(x, Q^2) = M\nu^2 G_2(\nu, Q^2), \qquad (1.31)$$

which likewise reduce to functions of x only when the limit $Q^2 \to \infty$ is taken.

However, in the later experiments a small Q^2 dependence of the structure functions was also observed and this phenomena is known as scaling violation. Scaling violation is an important observable of QCD and discussed in the section 1.4.

1.3 Theoretical Models for the DIS Structure Functions

The first electron-proton scattering experiment was carried out at SLAC[6]. Immediately after these experiments several models were proposed to explain the behaviour of the structure functions. The most prominent among them are Light Cone Expansion[9], Quark Parton Model(QPM)[10], Vector Meson Dominance Model(VMD)[11], Regge Pole Model[12, 13] and Dual Resonance Models[14]. Here we have provided a brief introduction to Quark Parton Model and Regge theory.

1.3.1 Quark Parton Model

In the early measurements, nucleon structure function in DIS, a weak dependence of structure functions on Q^2 was revealed, which in turn led to the conclusion that the virtual photon sees point-like constituents in the nucleon. In order to describe the composite nature of nucleons, the quark-parton model[10] was developed. In accord with quark-parton model, the nucleon is composed of free pointlike constituents, the partons, identified later as quarks and gluons. The basis of parton model is the introduction of parton distribution functions, $q_i(x)$ and $\bar{q}_i(x)$ for quarks and anti-quarks respectively, where $q_i(x)dx(\bar{q}_i(x)dx)$ signifies the probability of finding a quark(antiquark) of flavor *i* in a nucleon, which carries a fraction *x* to x + dx of the parent hadron's four-momentum *p*. Here *x* is the fractional four-momentum of the parent nucleon carried by a parton. On the basis of these ideas we can have a simple interpretation of nucleon structure functions F_1 and F_2 measured in charged lepton DIS as

$$F_1(x) = \frac{1}{2} \sum_{i=u,d,\dots} e_i^2 [q_i(x) + \bar{q}_i(x)], \qquad (1.32)$$

and

$$F_2(x) = \frac{1}{2} \sum_{i=u,d,\dots} e_i^2 x[q_i(x) + \bar{q}_i(x)].$$
(1.33)

They are thus related by the Callan-Gross relation[15],

$$F_2(x) = 2xF_1(x). (1.34)$$

The Callan-Gross relation connecting F_1 and F_2 reflects the spin- $\frac{1}{2}$ nature of the quarks.

The interpretation of structure functions measured in neutrino-DIS in accord with parton model is

$$F_2^{\nu(\bar{\nu})}(x) = \sum_{i=u,d,\dots} x[q_i^{\nu(\bar{\nu})}(x) + \bar{q}_i^{\nu(\bar{\nu})}(x)]$$
(1.35)

$$xF_3^{\nu(\bar{\nu})}(x) = \sum_{i=u,d,\dots} x[q_i^{\nu(\bar{\nu})}(x) - \bar{q}_i^{\nu(\bar{\nu})}(x)]$$
(1.36)

In the naive parton model the spin-dependent structure functions g_1 and g_2 are given by

$$g_1(x) = \frac{1}{2} \sum_{i=u,d,\dots} e_i^2 \Delta q_i(x)$$
 (1.37)

and

$$g_2(x) = 0. (1.38)$$

where

$$\Delta q_i(x) = q_i^{\uparrow}(x) - q_i^{\downarrow}(x) + \bar{q}_i^{\uparrow}(x) - \bar{q}_i^{\downarrow}(x).$$
(1.39)

Here the helicity distributions $\Delta q_i(x) = q_i^{\uparrow}(x) - q_i^{\downarrow}(x)$ and $\Delta \bar{q}_i(x) = \bar{q}_i^{\uparrow}(x) - \bar{q}_i^{\downarrow}(x)$ involve the differences of the quark or antiquark distributions with helicities parallel and antiparallel with respect to the helicity of the target nucleon. The interpretation of $g_1(x)$ structure function can be understood from the fact that a virtual photon with spin projection +1 can only be absorbed by a quark with spin projection $-\frac{1}{2}$, and vice versa. In parton model, however the trasverse spin structure function g_2 vanishes identically and has been the subject of much theoretical debate [16].

1.3.2 Regge Theory

The study of scattering of hadronic particles, in the days before QCD was established, was based on Regge theory [12, 13]. The pre-QCD method, Regge theory relied basically on assumptions on the scattering matrix, such as Lorentz invariance, crossing symmetry, unitarity, causality, analyticity, asymptotic states etc., which determines the asymptotic behaviour of cross sections in the high energy limit regardless the strength of the coupling, i.e., independently of perturbation theory.

In accord with Regge theory the scattering amplitude for a two body scattering of hadrons ($2 \rightarrow 2$ process) (Fig. 1.2) is given by the functional form[13]



$$A(s,t) \approx s^{\alpha(t)}.\tag{1.40}$$

for asymptotically large s, such as $s \gg t$. Here s is the center of mass energy, $t = (p_1 p_3)^2$, the momentum transfer and $\alpha(t)$ is a function of the momentum transfer t. Fig. 1.3 represents a typical diagram for the amplitude in Regge theory of the form Eq.(1.40).

The interactions in Regge theory that gives rise to an amplitude of the form Eq.(1.40) is successfully described by the exchange of a quasi-particle called Reggeon. Reggeons, like elementary particles, are characterized by quantum numbers such as charge, spin, etc. The spin of the Reggeons is a function of the momentum transfer t, and more specifically their spin is the function $\alpha(t)$ which appears in the equation for the amplitude Eq.(1.40).



Figure 1.3: An interaction of two particles via the exchange of a Reggeon.

Although Reggeons are not real particles, but there are resonances at (half) integer spins and correspond to real particles of mass m and spin j, where $j = \alpha(m^2)$. By plotting the square of the masses of various particles versus their spin, as shown in Fig. 1.4, it is observed that they lie along straight lines. These lines are the Regge trajectories

$$\alpha(t) = \alpha(0) + \alpha' t, \tag{1.41}$$

which correspond to the various quasi-particles in Regge theory. Here the intercept of the trajectory is $\alpha(0)$ and α' is the slope.



Figure 1.4: Plot of particle mass squared (in GeV^2) versus spin (in units of \hbar). It can be seen that the particles plotted lie along a linear trajectory, data taken from [17].

By utilising the Regge trajectories, asymptotic s dependence of the differential cross section can be obtained as

$$\frac{d\sigma}{dt} \propto s^{2(\alpha(0) + \alpha' t - 1)},\tag{1.42}$$

where the singularity in $\alpha(t)$ with the largest real part, known as the leading singularity, determines the asymptotic behaviour of the scattering amplitude. The scattering amplitude helps in determining the total cross section and in the large *s* regime, where $s \gg t$, the behaviour of the total cross section is given by

$$\sigma_{tot} \propto s^{\sigma(0)-1}.\tag{1.43}$$

It is observed that the requirement for the growth of cross section is $\alpha(0) > 1$, i.e., the intercept has to be greater than one. However, the exchange which leads to this growth in cross section can not be from a charged exchange as this would cause the cross section to vanish asymptotically. Instead the exchange requires to have the quantum numbers of the vacuum: no charge, no isospin, and a parity of +1. The experimental results on proton-proton scattering signifies a significant growth of the total cross section and this behaviour can be well explained by an exchange of a Reggeon with the trajectory [18]

$$\alpha(t) = 1.08 + (0.25GeV^2)t. \tag{1.44}$$

Such an exchange which satisfies this trajectory is known as the Pomeron. More specifically this is referred to as the "soft" Pomeron to differentiate it from a "hard" Pomeron. No particle resonances have been observed on the Pomeron trajectory, however a particle that may lie along this path is proposed to be the glueball [19].

Although these models were seemed to be legitimate as far the early data are concerned, but their predictions show significant deviation from the recent measurements. With the advent of dedicated experimental facilities, now it is possible to determine the structure functions as well as different sum rules over a wide range of x and Q^2 with far greater precision than before. Recent experimental results are well described by Quantum Chromodynamics(QCD) and it is believed that QCD is a correct theory of strong interaction.

1.4 Quantum Chromodynamics

The Quantum Chromodynamics(QCD)(cf. e.g., [7, 20, 21]) is a theory of strong interaction – interactions between hadrons and, in particular, between their inner constituents. The Quark Parton Model(QPM) is based on the idea that DIS scattering cross sections may be determined from free quarks which are bound within the nucleon which is an apparent contradiction. Although QPM was very successful at being able to take parton distribution functions(PDFs) from one scattering process and predicting cross sections for other scattering experiments; it has several difficulties. Firstly QPM fails to describe accurately the violations of scaling and scale dependence of DIS cross sections. The fact that partons are strongly bound into colourless states is an experimental fact, but why they behave as free particles when probed at high momenta is inexplicable in QPM. The QPM is also unable to account for the total momentum of the proton via measurements of the momentum sum rule indicating the existence of gluon. It is only by including the effects of the gluon and gluon radiation in hard scattering processes that an accurate description of experimental data can be given. These developments led to the formulation of Quantum Chromodynamics(QCD).

The successes of QCD in describing the strong interactions are summarized by two terms: asymptotic freedom and confinement. Asymptotic freedom refers to the weakness of the short distance interaction, while the confinement of quarks follows from its strength at long distance. It is an extraordinary feature of QCD that it accommodates both kinds of behaviour. Asymptotic freedom states that, as the distance between two quarks diminishes so does the effective strength of their interaction; and the particles become asymptotically free. On the other hand, as the distance between quarks increases, so does the effective interaction strength. Asymptotic freedom explains the absence of observed free quarks.

In perturbative QCD(pQCD), calculations are performed by expanding terms in a perturbation series in the coupling strength α_s . This is only valid when α_s is small, i.e., at high Q^2 (see Figs.1.5 and 1.6). The calculation of a scattering cross section in pQCD reduces to summing over the amplitudes of all possible intermediate states. Each graph is a symbolic representation for a term in the perturbative calculation. Leading Order(LO) term corresponds to the quark parton model and is considered to be of order α_s^0 in the perturbative expansion. LO Feynman diagrams have no gluon vertices as shown in Fig. 1.5. Next-to-Leading Order (NLO) diagrams add quarkgluon interactions to this pictures. NLO graphs have one gluon vertex and correspond to terms of order α_s^1 in the perturbative expansion. NLO Feynman diagrams for hard scattering are illustrated in Fig. 1.6. Similarly higher order terms such as NNLO, NNNLO etc., would correspond to addition of more gluon vertices two, three etc).

The four-momentum is conserved at each vertex. However, including higher order diagrams, the momentum circulating in the loop is not constrained. The integration over all momentum space for a loop diagram leads to logarithmic divergences when momentum goes to infinity. These type of divergences are treated in a systematic way by the renormalization technique. However the renormalization procedure introduces an arbitrary parameter μ , which has the dimension of mass.

Any physical observable F must be independent of the choice for μ , therefore we impose the following condition:

$$\mu^2 \frac{\partial F}{\partial \mu^2} = \left(\mu^2 \frac{\partial}{\partial \mu^2} + \mu^2 \frac{\partial \alpha_s}{\partial \mu^2} \frac{\partial}{\partial \alpha_s} \right) = 0.$$
(1.45)



Figure 1.5: Leading Order splitting functions diagrams.

Hence, any explicit dependence of the physical observable on the renormalization scale must be cancelled by a proper renormalization scale dependence of α_s . The strong coupling α_s is determined by renormalization group equation given by

$$Q^{2}\frac{\partial\alpha_{s}}{\partial Q^{2}} = \beta(\alpha_{s}) = -\frac{\beta_{0}}{4\pi}\alpha_{s}^{2}(Q^{2}) - \frac{\beta_{1}}{16\pi^{2}}\alpha_{s}^{3}(Q^{2}) - \frac{\beta_{2}}{64\pi^{3}}\alpha_{s}^{2}(Q^{2}) + O(\alpha_{s}^{5}), \qquad (1.46)$$

where the coefficients β_0 , β_1 and β_2 depends on the number of active quark flavors n_f and scale Q^2 as

$$\beta_0 = 11 - \frac{2}{3}n_f, \tag{1.47}$$

$$\beta_1 = 102 - \frac{38}{3}n_f, \tag{1.48}$$

and

$$\beta_2 = \frac{2857}{6} - \frac{6673}{18}n_f + \frac{325}{54}n_f^2. \tag{1.49}$$

Expansion of the β -function is carried out to three loops, which corresponds to a NNLO analysis.



Figure 1.6: Examples of next-to-leading Order splitting functions diagrams.

The solution to renormalization group Eq.(1.46) provides the scale dependence of the strong coupling α_s , i.e. the "running" of α_s . Perturbative QCD predicts the scale dependence of the strong coupling, but α_s at a specific scale is obtained from experiment. Therefore, α_s at a reference scale is a fundamental parameter of the theory of QCD. Measuring the strong coupling from various experiments at different characteristic energy scales is an important test of QCD.

Moreover, one can introduce a dimensional parameter, Λ_{QCD} , [22] to provide a parametrization of the scale dependence of the strong coupling α_s . In accordance with the convention of Ref. [22], $\Lambda_Q CD$ is defined by writing the solution of the renormalization group equation at LO, NLO and NNLO as [23]:

$$\left(\frac{\alpha(t)}{2\pi}\right)_{LO} = \frac{2}{\beta_0 t},\tag{1.50}$$

$$\left(\frac{\alpha(t)}{2\pi}\right)_{NLO} = \frac{2}{\beta_0 t} \left[1 - \frac{\beta_1 \ln t}{\beta_0^2 t}\right],\tag{1.51}$$

$$\left(\frac{\alpha(t)}{2\pi}\right)_{NNLO} = \frac{2}{\beta_0 t} \left[1 - \frac{\beta_1 \ln t}{\beta_0^2 t} + \frac{1}{\beta_0^2 t^2} \left[\left(\frac{\beta_1}{\beta_0}\right)^2 (\ln^2 t - \ln t - 1) + \frac{\beta_2}{\beta_0}\right]\right],$$

where, $t = \frac{Q^2}{\Lambda_{QCD}^2}$ and Λ_{QCD} represents the scale at which perturbative QCD becomes strongly coupled, i.e. the scale for which the coupling α_s is large and perturbative QCD theory breaks down. The scale is comparable with the masses of the


 $\ell n \ Q^2 \longrightarrow$ **Figure 1.7:** Schematic representation of the applicability of various QCD evolution equations across the $x - Q^2$ plane.

light hadrons ($\approx 0.5 GeV$). In other words, Λ_{QCD} determines the boundary between quasi-free state of interacting quarks and gluons (weak coupling) and the state where hadrons are formed (strong coupling).

• QCD Evolution Equations

There exist several QCD evolution equations to obtain the quark and gluon distribution functions such as the Dokshitzer-Gribov-Lipatov-Alterelli-Parisi(DGLAP) equation[24], the Balitsky-Kuraev-Fadin-Lipatov(BKFL) equation[25], the Gribov-Levin-Ryskin(GLR) equation[26] and the Ciafaloni-Catani-Fiorani-Marchesini(CCFM) equation[27]. In spite of them, some other equations are also proposed like the Modified DGLAP equation (by Zhu and Ruan)[28], the Modified BKFL or BK[29] equation (by Balitsky and Kovchegov) and the JIMWLK[30] equation (by Jalilian-Marian, Iancu, McLerran, Weigert, Leonidov and Kovner) etc., in different kinematical regions. Schematic representation of the applicability of various QCD evolution equations across the $x - Q^2$ plane is depicted in Fig.1.7. Among these evolution equations, BFKL or GLR equations are more appealing at small-x, but still the DGLAP evolution equation is used to study various parton distribution functions as well as the structure functions because this equation is a simple perturbative tool which is relevant for the presently accessible $x - Q^2$ range of structure functions.

Nucleon structure functions systematically exhibit a Q^2 -dependence, even at large Q^2 . These scaling violations can be described within the framework of the QCD-improved parton model which incorporates the interaction between quarks and gluons

in the nucleon in a perturbative way. The scale at which this interaction is resolved is determined by the momentum transfer. The Q^2 -dependence of parton distributions, e.g.

$$F_2(x,Q^2) = \sum e_i^2 x [q_i(x,Q^2) + \bar{q}_i(x,Q^2)], \qquad (1.52)$$

$$xF_3(x,Q^2) = \sum x[q_i(x,Q^2) - \bar{q}_i(x,Q^2)]$$
(1.53)

and

$$g_1(x,Q^2) = \frac{1}{2} \sum e_i^2 x [\Delta q_i(x,Q^2) + \Delta \bar{q}_i(x,Q^2)]$$
(1.54)

are described by the DGLAP evolution equations. They are different for flavor nonsinglet and singlet distribution functions. Typical examples of non-singlet combinations are the difference of quark and anti-quark distribution functions, or the difference of up and down quark distributions. The difference of the proton and neutron structure function, $F_2^p - F_2^n$, also behaves as a flavor non-singlet, whereas the deuteron structure function F_2^d is an almost pure flavor singlet combination. For the flavor nonsinglet quark distribution, q^{NS} , and the flavor-singlet quark and gluon distributions, q^S and g, the DGLAP evolution equations read as follows[24]:

$$\frac{dq^{NS}(x,Q^2)}{d\ln Q^2} = \frac{\alpha(Q^2)}{2\pi} \int_x^1 \frac{dy}{y} q^{NS}(y,Q^2) P_{qq}(\frac{x}{y}), \qquad (1.55)$$

$$\frac{d}{d\ln Q^2} \begin{pmatrix} q^s(x,Q^2) \\ g(x,Q^2) \end{pmatrix} = \frac{\alpha(Q^2)}{2\pi} \int_x^1 \frac{dy}{y} \begin{pmatrix} P_{qq}(\frac{x}{y}) & P_{qg}(\frac{x}{y}) \\ P_{gq}(\frac{x}{y}) & P_{gg}(\frac{x}{y}) \end{pmatrix} \begin{pmatrix} q^s(y,Q^2) \\ g(y,Q^2) \end{pmatrix}.$$
 (1.56)

Here $\alpha(Q^2)$ is the running QCD coupling strength. The splitting functions $P_{ij}(x/y)$ are calculable in perturbative QCD as a power series of $\alpha_s(Q^2)$:

$$P_{ij}(z,\alpha_s(Q^2)) = P_{ij}^{(0)}(z) + \frac{\alpha_s}{2\pi} P_{ij}^{(1)}(z) + \left(\frac{\alpha_s}{2\pi}\right)^2 P_{ij}^{(2)}(z).$$
(1.57)

Splitting functions are known up to NNLO[31,32]. The splitting functions $P_{ij}(\frac{x}{y})$ give the probability of parton j with momentum fraction y be resolved as parton i with momentum fraction x < y. They are calculated perturbatively to a given order in α_s . LO and NLO splitting function diagrams are shown in Figs. 1.5-1.6. Evolution equations describe the physical picture in which valence quarks are surrounded by a cloud of virtual particles which are continuously emitted and absorbed. The quarks are also emitting and absorbing virtual particles of their own, corresponding to the branching probability densities. This picture explains why the structure of the hadron appears to change as it is seen at different distance scales. Thus, at low Q^2 there are fewer partons and their PDFs are skewed to high momentum fractions. At high Q^2 , the momentum is shared through the parton branchings, and hence the low x region is filled with gluons and sea quarks which have a high probability to undergo $g \to gg$, $g \to q\bar{q}$ branchings. QCD doesn't predict the PDFs at any scale, rather it predicts how PDFs evolve with the scale through the evolution equations.

1.5 DIS Sum Rules

The structure functions which parameterize the deeply inelastic scattering cross section obey a series of sum rules to which QCD corrections are also available [7,33] and one may perform QCD tests using these relations. Sum rules are integrals over structure functions or parton distributions and they are associated with the conservation law for some quantum number of the nucleon. Parton model sum rules provide information about the distribution of quarks inside nucleon and are very useful to reveal new physics if a sum rule is found to be satisfied or broken. In the following subsections, we have discussed briefly about some important sum rules along with available pQCD corrections.

1.5.1 Gottfried Sum Rule

The Gottfried Sum Rule(GSR)[34,35] involves the difference of F_2 measured in proton and neutron targets using a charged lepton. In accord with parton model it is governed by

$$S_G = \int_0^1 \frac{F_2^{\mu p} - F_2^{\mu n}}{x} dx = \frac{1}{3}.$$
 (1.58)

There are pQCD corrections to GSR up to 3-loop corrections[35] and it is given by

$$S_G = \int_0^1 \frac{F_2^{\mu p} - F_2^{\mu n}}{x} dx = \frac{1}{3} \left[1 + 0.0355 \frac{\alpha_s}{\pi} - 0.811 (\frac{\alpha_s}{\pi})^2 \right].$$
 (1.59)

1.5.2 Adler Sum Rule

The Adler Sum Rule[36,37] predicts the integrated difference between neutrino-neutron and neutrino-proton structure functions. It states

$$S_A = \int_0^1 \frac{F_2^{\nu n} - F_2^{\nu p}}{2x} dx = 1.$$
 (1.60)

The ASR is exact and receives neither QCD nor mass corrections, but its experimental verification is at a very low level of accuracy[37].

1.5.3 Gross-Llewellyn Smith Sum Rule

The Gross-Llewellyn Smith Sum Rule(GLSSR)[38,39] involves an integration over the non-singlet neutrino structure function, $xF_3(x, Q^2)$, which is obtained by subtracting the antineutrino differential cross section on an isoscalar target from the corresponding neutrino cross section. It is the most accurately tested sum rule. The GLSSR predicts that the number of valence quarks in a nucleon, up to finite Q^2 corrections, is three. In the QPM, the GLS sum rule reads[38]:

$$S_{GLS} = \int_0^1 \frac{xF_3}{x} dx = \frac{1}{3},$$
(1.61)

and pQCD correction to GLS sum rule up to NNLO is given by[39]

$$S_{GLS}(Q^2) = \int_0^1 \frac{dx}{x} x F_3(x, Q^2) = 3 \left[1 - \frac{\alpha_s}{\pi} - a(n_f) \left(\frac{\alpha_s}{\pi}\right)^2 - b(n_f) \left(\frac{\alpha_s}{\pi}\right)^3 \right].$$
(1.62)

1.5.4 Unpolarized Bjorken Sum Rule

The Unpolarized Bjorken Sum Rule(UBSR)[40] refers to the integrated difference between neutrino-neutron and neutrino-proton charged current structure functions F_1 .

$$S_{Bj} = \int_0^1 \frac{F_1^{\nu n} - F_1^{\nu p}}{d} x = 1.$$
 (1.63)

It has three loop pQCD correction, which predicts[41]

$$S_{Bj} = \int_0^1 \frac{F_1^{\nu n} - F_1^{\nu p}}{d} x = \left[1 - \frac{\alpha_s}{\pi} - a(n_f) \left(\frac{\alpha_s}{\pi}\right)^2 - b(n_f) \left(\frac{\alpha_s}{\pi}\right)^3\right].$$
 (1.64)

1.5.5 Polarized Bjorken Sum Rule

Polarised Bjorken Sum Rule(BSR)[42] relates the difference of proton and neutron structure functions integrated over all possible values of Bjorken variable, x to the nucleon axial charge g_A as

$$S_{BSR} = \int_0^1 \frac{dx}{x} x g_1^{NS}(x, Q^2) = \frac{g_A}{6}.$$
 (1.65)

However, away from $Q^2 \to \infty$, the polarized BSR is given by a series in powers of the strong coupling constant $\alpha_s(Q^2)[43]$:

$$S_{BSR}(Q^2) = \int_0^1 \frac{dx}{x} x g_1^{NS}(x, Q^2) = \frac{g_A}{6} \left[1 - \frac{\alpha_s}{\pi} - 3.583(\frac{\alpha_s}{\pi})^2 -20.215(\frac{\alpha_s}{\pi})^3 + \dots \right].$$
 (1.66)

Here the BSR consists of pQCD results up to third order of $\alpha_s(Q^2)$. BSR is associated with the conservation of polarised isospin.

1.5.6 Gerasimov-Drell-Hearn Sum Rule

Gerasimov-Drell-Hearn Sum Rule(GDHSR)[44] is given by the first moment of the polarized structure function $g_1^{p,n}(x, Q^2)$ in the form

$$S_{GDHSR}(Q^2) = 2\frac{M^2}{Q^2} \int_0^{x_0} dx g_1^{p,n}(x, Q^2) = \begin{cases} -\frac{1}{4}\mu_{p,n}^2, & Q^2 \to 0\\ \frac{2M^2}{Q^2}\Gamma_1^{p,n}, & Q^2 \to 0 \end{cases}.$$
 (1.67)

at proton and neutron targets, with $x_0 = Q^2/(2Mm_\pi + m_\pi^2 + Q^2)$, $\mu_{p,n}$ the anomalous magnetic moment of the proton or nucleon, and Γ_1 the first moment of the structure function g_1 at infinite space-like momentum transfer. This sum-rule has a very strong Q^2 -evolution for low values of the virtuality. In case of proton targets it changes sign between $Q^2 = 0$ and $Q^2 \approx 1 GeV^2$.

In addition to these there are several important sum rules such as Burkhardt-Cottingham sum rule[45], Efremove-Teryaev-Leader sum rule[46], Ellis-Jaffe sum rule[47], etc., however in this thesis we have concentrated on GSR, GLSSR and BSR, which are associated with the non-singlet structure functions F_2^{NS} , xF_3 and g_1^{NS} respectively. The determination of these sum rules are provided in chapter 7.

1.6 Non-Perturbative QCD Effects

At low Q^2 , the strong coupling constant becomes large and the perturbative calculations fail. In this non-perturbative region the assumption of scattering from massless, point-like, quarks is no longer valid. Also, the resolving power is not large enough to probe a single quark scattering. To obtain high interaction rates, lepton DIS experiments use heavy targets. Therefore, nuclear effects must be considered as well. These non-perturbative effects are discussed in the following subsections.

1.6.1 Target Mass Correction

At low Q^2 and high x, one can not neglect the effects of the target mass. The meaning of x as the fraction of the nucleons momentum carried by the struck quark is not suitable when $Q^2 \approx M^2$. A "target mass" correction must be applied to account for these effects. The target mass correction (TMC) to the structure functions have been first determined in [48]. More recently, new derivations were performed in [49] which lead to the following relations:

$$F_2^{TM}(x,Q^2) = \frac{x^2}{\xi^2} \frac{F_2^{(0)}(x,Q^2)}{k^3} + \frac{6M_p^2 x^3}{Q^2 k^4} \int_{\xi}^1 \frac{F_2^{(0)}(u,Q^2)}{u^2} du + \frac{12M_p^4 x^4}{Q^4 k^5} \int_{\xi}^1 du \int_u^1 \frac{F_2^{(0)}(v,Q^2)}{v^2} dv, \qquad (1.68)$$

$$F_3^{TM}(x,Q^2) = \frac{x}{\xi} \frac{F_3^{(0)}(x,Q^2)}{k^2} + \frac{2M_p^2 x^2}{Q^2 k^3} \int_{\xi}^1 \frac{F_3^{(0)}(u,Q^2)}{u} du$$
(1.69)

with ξ and k are defined as

$$k = \sqrt{1 + \frac{4x^2 M_p^2}{Q^2}},\tag{1.70}$$

and

$$\xi = \frac{2x}{1+k} \tag{1.71}$$

respectively.



Figure 1.8: The contribution from target mass corrections(TMC) to F_2^{NS} structure function. Figure is taken from[50].

The target mass effects are large at high x and low Q^2 . Fig. 1.8 shows the size of the the TMC, as obtained in Ref. [50] for $F_2^{NS}(x, Q^2)$ structure functions along with QCD prediction as a function of x. As our kinematical region of consideration is within low-x and low- Q^2 region in our thesis we have neglected the effects of TMC.

1.6.2 Higher Twist

The Operator Product Expansion(OPE)[51] is a common theoretical framework in analyses of deep inelastic scattering(DIS) in QCD. The operators can be ordered according to their twist yielding the series in $\frac{1}{Q^2}$ for physical observable. For example, for the structure function F_i , this reads

$$F_i(x, Q^2) = F_i^{LT}(x, Q^2) + \frac{h_i(x)}{Q^2}.$$
(1.72)

The first term in this expansion (the leading twist, LT) dominates at sufficiently large



Figure 1.9: Examples of higher twist QCD diagrams.

momentum transfer Q^2 and invariant mass $W^2 = M^2 + \frac{Q^2(1-x)}{x}$. The LT structure functions are constructed in terms of parton distribution functions (PDFs), which are universal for charged lepton and neutrino scattering and have clear probabilistic interpretation. An accurate knowledge of these plays a key role in the extraction of possible contributions of new physics at new collider energies, non-accelerator physics (cosmic neutrinos) and, as observed more recently, in the interpretation of forthcoming high precision experiments on neutrino oscillation.

The higher-twist terms include interactions with other quarks, as shown in Fig. 1.9. Since target mass effects also involve powers of $\frac{1}{Q^2}$, they are referred to as "Kinematic higher-twist" effects. The scattering involving a conglomerate of quarks processes are referred to as "Dynamical higher-twist". These effects are important at low Q^2 and high x. The higher-twist contributions are calculated using phenomenological approaches. However, our analysis uses data in the kinematic range where these effects are negligible. Therefore, only kinematical higher-twist (not the dynamical higher twist) effects are studied.

1.7 Nuclear Effects

Although the primary aim of the DIS experiments is to explore the structure of nucleon, DIS data is collected usually for nuclear targets. The use of nuclei instead of nucleon serve a dual purpose in the studies of in high-energy scattering experiments. Firstly, nuclear DIS provides unique possibilities to study the space-time development of strongly interacting systems and it can provide valuable insights into the origin of nuclear force and properties of hadrons in nuclear medium. Secondly, the nuclear data often serve as the source of information on hadrons otherwise not directly accessible (e.g., extraction of the neutron structure function which is usually obtained from deuterium and proton data). Moreover, in experiments with nucleon targets the products of the scattering processes can only be observed by a detector that is far away from the collision point, whereas a nuclear target can serve as a detector located at the place where the microscopic interaction takes place. Consequently, with nuclei one can study coherence effects in QCD which are not accessible in DIS off nucleons.

When considering DIS on a nuclear target one may expect that the resulting nuclear structure functions were very similar to those measured off a nucleon target. This is so because the nucleons are very loosely tighted inside a nucleus, and the interaction between the external probe, the virtual photon, and the constituent nucleons could be expected to be incoherent.

However, in 1982, the European Muon Collaboration(EMC)[52] reported that the ratio

$$R(x,Q^2) = \frac{F_2^A(x,Q^2)}{F_2^N(x,Q^2)},$$
(1.73)

where F_2^A and F_2^N are the nuclear and nucleon structure functions respectively, is in general, different from one. The observed difference between the nuclear structure function and that corresponding to the simple addition of its constituent nucleons is commonly referred to as the EMC effect[53] and that was the first clear evidence for the nuclear effect in nuclear structure functions. In fact, the EMC effect states that, in the parton point of view, quark distributions in bound nucleon are different from those in free nucleon.

Whether there is enhancement or suppression of the nuclear structure functions with respect to those of the nucleon depends on the kinematical region of interest[53– 55]. The general Bjorken-x dependence of such modification is as follows(see Fig. 1.10):

- 1. The ratio R is smaller than unity within the region x < 0.01. This region is known as **shadowing**.
- 2. Within $0.1 \le x \le 0.25 \sim 0.3$ the ratio R is larger than unity. This region is called **anti-shadowing**.
- 3. Within $0.25 \sim 0.3 \le x \le 0.8$, R is smaller than unity and this region is known as the **EMC region**.



Figure 1.10: x dependence of the ratio $R_{F_2}^A(x, Q^2)$ for a given fixed Q^2 .

4. $x \ge 0.8$, R is greater than unity and this region is known as Fermi motion region.

The nuclear effects are large at low and high x, but are observed to be independent of Q^2 . However, there are new theoretical treatments that consider a Q^2 dependent nuclear target corrections at low x[57, 227]. Recent results from NuTeV hint that neutrino experiments might favor smaller nuclear effects than the charged lepton experiments[58] at high x, but this thesis does not take into account Q^2 dependent nuclear corrections. We can interpret our extracted PDFs from QCD fits as effective nuclear PDFs, which have the nuclear effects absorbed into them. These nuclear effects are discussed in detailed in chapter 8. $\Box\Box$

Chapter 2

Overview of DIS Experiments and Parameterizations

This chapter provides a general overview about the lepton deep inelastic scattering measurements which have enriched our phenomenological analysis performed in this thesis. Specifically the experimental results for non-singlet structure functions and associated sum rules for both polarized and unpolarized cases measured in electron, muon and neutrino DIS are reviewed. In addition, several parametrization associated with the determination of non-singlet structure functions are discussed.

2.1 Introduction

Particle Physics is a subject that can only thrive when there is a coherent interplay between theory and experiment. New theoretical ideas lead to predictions that can be tested experimentally, and new experimental findings challenge theorists to develop better ideas. Phenomenology is research on this boundary between theory and experiment. It is concerned with exploration of interesting physical observables, making theoretical predictions for them and then confronting experimental data gathered at the major international experimental laboratories. The primary goal of phenomenology is to find experimental evidences for new physics and to develop new theories that describe the Universe at a more fundamental level than our current theories can.

The work presented in this thesis is basically a phenomenological analysis of the non-singlet structure functions and the sum rules associated with them, along with higher order perturbative QCD and non-perturbative effects. This phenomenological analysis is enriched by several experimental results, along with a large numbers of

Experiment	Beam	Target	x	Q^2	Energy	Ref.
				(GeV^2)	(GeV)	
NMC	μ	p, d	0.008 - 0.5	0.8 - 65	90, 120	[63]
					200, 280	
CCFR	$ u_{\mu}(ar{ u}_{\mu}) $	Fe	0.015 - 0.65	1.2 - 126	30 - 500	[66]
NuTeV	$ u_{\mu}(ar{ u}_{\mu}) $	Fe	0.015 - 0.75	1.2 - 125	30 - 360	[68]
CHORUS	$ u_{\mu}(ar{ u}_{\mu}) $	Pb	0.02 - 0.65	0.3 - 82	30 - 360	[70]
SLAC(E143)	e^-	NH_3, ND_3	0.03 - 0.5	1 - 10	≤ 28	[75]
SMC	μ^+	H/D-butanol	0.004 - 0.5	1 - 60	100, 190	[74]
		NH_3				
HERMES	e^{-}, e^{+}	H, D, 3 He	0.02 - 0.7	1 - 15	27.5	[73]
COMPASS	μ^+	NH ₃ , ⁶ LiD	0.003 - 0.6	1 - 70	160	[71]
JLAB	e^-	³ He	0.1 - 0.6	1 - 2.5	6	[72]
(Hall A)						
JLAB	e^-	NH_3, ND_3	0.05-0.6	1 - 5	≤ 6	[95]
(Hall B)						

Table 2.1: Table of datasets used in the phenomenological analysis of this thesis. The kinematic range of each measurement in x and Q^2 and the incident beam energy are also given.

parameterizations. A brief description of different experiments and parameterizations, their results for non-singlet structure functions and associated sum rules is given in this chapter.

2.2 DIS Experiments and Results

Many successful experimental programs of both unpolarized and Polarized deep inelastic lepton scattering experiments have been carried out at SLAC, CERN, Fermilab, DESY and Jefferson Laboratory(JLab)(See for more details [59–62]) in order to elucidate the internal structure of the nucleon. With the advent of dedicated experimental facilities, recent experiments were able to determine the structure functions as well as different sum rules over a wide range of x and Q^2 with ever increasing precision.

We have summarized different DIS experiments that have probed the nucleon structure in Table 2.1 along with the x, Q^2 ranges and beam energies of the measurements. However, as our works deal with the non-singlet structure functions and the kinematical region of our consideration is $x \leq 0.05$ and $1.3 \leq Q^2 \leq 20$, we shall restrict our discussion on the following unpolarized DIS experiments: NMC[63– 65], CCFR[66,67], NuTeV[68], CDHSW[69] and CHORUS[70] experiments which are



Figure 2.1: Schematic diagram of the NMC spectrometer.

associated with the measurements of the non-singlet structure functions F_2^{NS} and xF_3 and the sum rules Gottfried Sum rule(GSR)[34,35] and Gross-Llewellyn Smith Sum Rule(GLSSR)[38,39] and polarized experiments: COMPASS[71], HERMES[73], SMC[74], E143[75] and JLab experiments[76–78], which are associated with the measurements of non-singlet spin structure function g_1^{NS} and the Bjorken Sum rule[42,43], associated with g_1^{NS} .

2.2.1 NMC

The New Muon Collaboration(NMC) was constructed at CERN to study DIS using muon beams on proton and deuterium targets. It was an extension and improvement of the European Muon Collaboration(EMC) experiment. Aiming towards better measurements of nuclear effects in DIS, particularly the EMC effect, the NMC was designed. It has also designed to have accurate data on the structure functions F_2^p and F_2^d and to measure structure function ratios with high precision.

A schematic diagram of the NMC apparatus is shown in Fig. 2.1[79]. It is consisted of an upstream beam momentum station(BMS) and hodoscopes, a downstream beam calibration spectrometer(BCS), a target region and a muon spectrometer. The muon beam ran at beam energies of 90, 120, 200, and 280 GeV and illuminated two target cells containing liquid hydrogen and liquid deuterium placed in series along the beam axis. Since the spectrometer acceptance was very different for both targets they were regularly alternated. The muon spectrometer was surrounded by several MWPCs and drift chambers to allow a full reconstruction of the interaction vertex and the scattered muon trajectory. Muons were identified using drift chambers placed behind a thick iron absorber.



Figure 2.2: NMC measurements[63] of F_2^{NS} structure function along with BCDMS data. (Fig. taken from [80].)

The experiment published measurements of the proton and deuteron differential cross sections $d^2/dxdQ^2$ in the region 0.008 < x < 0.5 and 0.8 < Q2 < 65GeV2, from which the structure functions F_2^p and F_2^d were extracted[63]. The NMC data consist of four data sets for the proton and the deuteron structure functions corresponding to beam energies of 90 GeV, 120 GeV, 200 GeV, and 280 GeV. They cover the kinematic range $0.002 \le x \le 0.60$ and $0.5GeV^2 \le Q^2 \le 75GeV^2$.

In 1992 NMC published the first data on the Gottfried sum rule[64]. The initial NMC measurement indicated a violation of this assumption of a flavour symmetric sea. This was verified by the final NMC analysis[65] in which the Gottfried sum was determined to be 0.2350.026 at Q2 = 4GeV2, which implies that $\int dx(\bar{d} - \bar{u}) \sim 0.15$, indicating a significant excess of \bar{d} over \bar{u} .

The experimental results for the non-singlet structure function, F_2^{NS} extracted from NMC measurements are taken from Ref. [63] and the results obtained in [63] are presented in Fig.2.2. Further the GSR results obtained in [65] are shown in Fig. 2.3.



Figure 2.3: NMC measurements of Gottfried sum rule[65].



Figure 2.4: Schematic diagram of the CCFR Dtector.

2.2.2 CCFR

The Chicago-Columbia-Fermilab-Rochester detector (CCFR) was constructed at the Fermi National laboratory, Fermilab to study DIS in neutrino induced lepton beams on an almost isoscalar iron target.

The CCFR detector, shown in Fig. 2.4 uses a wide-band mixed neutrino (ν_{μ}) and antineutrino $(\bar{\nu}_{\mu})$ beam with energies up to 600 GeV. The CCFR detector which was used to observe neutrino interactions consists of an iron based target-calorimeter instrumented with both scintillators and drift chambers and a toroid muon spectrometer. The neutrino DIS data were collected in two high energy high statistics runs, E744 and E770 in Fermilab By CCFR collaboration.



Figure 2.5: CCFR measurements of xF_3 structure function[66].

The non-singlet structure function, xF_3 extracted from the CCFR data is shown in Fig. 2.5. They are with very small systematic errors(which is shown by the corresponding error bars in the Fig. 2.5). In Figs. 2.6 and 2.7, the results for GLS sum rule obtained by CCFR collaboration[67] are shown. In order to perform phenomenological analysis in this thesis we have used these results for both xF_3 structure function and GLSSR.

2.2.3 NUTEV

NeUtrino experiment at the fermlab TeVatron(NuTeV) is a neutrino-iron DIS experiment(E815) that collected separate high statistics neutrino and antineutrino data in 1996-97 at Fermilab. NuTeV's detector(Similar to CCFR, Fig. 2.4)(see [81] for details) consists of an iron target calorimeter (upstream) and a toroid muon spectrometer (downstream). The target calorimeter is an instrumented iron-scintillator sandwich interspersed with drift chambers. The measured hadronic shower energy resolution is $\frac{\sigma_E}{E} = \frac{0.89}{\sqrt{E}} \bigoplus 0.021$. The toroid spectrometer is constructed of steel washers and five drift chambers. Muon energy resolution is limited by multiple Coulomb scattering to $\frac{\sigma_p}{p} = 0.11$.



Figure 2.6: CCFR measurements of GLS sum rule as a function of x[67].



Figure 2.7: CCFR measurements of GLS sum rule as a function of $Q^2[67]$



Figure 2.8: xF_3 structure function measured by NuTeV[68], CCFR[66] and CDHSW[69].



Figure 2.9: Schematic diagram of the CHORUS detector.

The NuTeV experiment took data during the 1996-97 fixed-target run at Fermilab. The main goal of NuTeV was the precise determination of $sin^2\Theta_W$. NuTeV's sign selected quadrupole train (SSQT) beamline was designed to select the sign of the pions and the kaons. The result was high purity separate ν_{μ} and $\bar{\nu}_{\mu}$ beams. Continuous calibration beam, running concurrently with the neutrino beam, illuminated the NuTeV detector with muons, electrons and pions. This allowed understanding of the muon and the hadron energy scales to precisions of 0.43% and 0.7% respectively. The NuTeV experiment has obtained a unique high statistics sample of neutrino and anti-neutrino interactions using its high-energy sign-selected beam. This measurement has significantly improved systematic precision as a consequence of more precise understanding of hadron and muon energy scales.(See [68,81] for more details)

The $xF_3(x, Q^2)$ structure function determined from the linear combination of the neutrino and anti-neutrino differential cross sections measured in NuTeV [68] is shown in Fig. 2.8 along with previous measurements from CCFR and CDHSW.

2.2.4 CHORUS

The CERN Hybrid Oscillation Research ApparatUS (CHORUS)[82] is an experiment for differential measurements of neutrino induced Charged Current DIS and to study the Z/A dependence of the total Charged Current cross section[83].

CHORUSs detector (shown in Fig. 2.9) consisted of two parts (See [82, 84] for more details): a lead-scintillator-fiber calorimeter used as an active target and a magnetized iron muon spectrometer. The experiment utilised proton beam from the



Figure 2.10: CHORUS measurements of xF_3 structure function along with CCFR and CDHSW data.



Figure 2.11: Schematic diagram of the Spin Muon Collaboration spectrometer.

Super Proton Synchrotron (SPS) with 450 GeV/c momentum interact in the target producing pion and kaons, which are focused by two pulsed toroidal magnets called horns. The decay of the mesons results in a wide-band neutrino beam, which was utilised in CHORUS experiment to take data in 1995-1998 using the lead-scintillator calorimeter as an active target[70].

The results for xF_3 structure function extracted from the CHORUS measurement[70] of differential cross sections within the kinematical range $0.02 \le x \le 0.65$ and in the $0.2 \le Q^2 \le 82GeV^2$ are shown in Fig. 2.10 along with the results of CCFR[66] and CDHSW[69] collaborations. We have utilised these results in our phenomenological analysis performed in this thesis.

2.2.5 CDHSW

The CERN-Dortumund-Heidelberg-Saclay-Warsaw (CDHSW) experiment [85,86] measured the total neutrino cross section using 100-, 160-, and 200-GeV narrow-band neutrino beams and performed precision electroweak and structure function measurements with a wideband beam during 1982-1984. CDHSW experimental apparatus [85] consisted of a big iron toroid, interspersed with scintillator to act as a calorimeter, magnetised and equipped with drift chambers to measure the energy of the scattered muon in charged current interactions. This experiment used the CERN neutrino beam in the "wide band" mode which allowed a considerable increase in neutrino intensity and thus in statistics.

The xF_3 structure function data that is used in our analysis are depicted in Fig. 2.8 and 2.10 along with CCFR, NuTeV and CHORUS data.

2.2.6 SMC

The Spin Muon Collaboration(SMC) is a third reincarnation of the EMC detector designed to measure the spin-dependent asymmetries of longitudinally polarized muons scattering from polarized targets. SMC began operation in 1991.

The SMC experimental setup was similar to that used by the EMC collaboration. It used the same beam as the EMC experiment; the beam optics were improved to provide a smaller beam spot at the target location. A beam polarimeter, downstream of the scattered-muon spectrometer, allowed measurement of the beam polarization either by muon scattering on polarized electrons in a magnetized foil[87] or by measuring the Michel spectrum of positrons from C decay [87,88].

The polarized target design[89] was based on the same principles as the EMC target. It consists of two 60*cm*-long cylindric target cells separated by a 30*cm* gap. For most of the data taking solid butanol and deuterated butanol, respectively, were used as target material; however solid ammonia was used for the last data-taking period in 1996.

The SMC spectrometer (see Fig. 2.11) is based on a conventional wide-aperture dipole magnet operated with a bending power of 4.4 Tm at a beam energy of 190 GeV. A large array of multiwire proportional chambers installed before, inside, and behind the magnet is utilised to measure the scattered muons and in a 2m-thick iron absorber the debris of hadrons produced in deep inelastic interactions is stopped. In addition another spectrometer consisting of multiwire proportional chambers, streamer tubes, and drift tubes is used in order to identify the scattered muons by observing tracks. The muon trigger is provided by predefined coincidence patterns between three arrays of plastic scintillation counters, two of which are installed behind the hadron absorber.

SMC measurements for g_1^{NS} and BSR are shown in Figs. 2.13 and 2.14 respectively along with other measurements.

2.2.7 COMPASS

COMPASS[90] is a dedicated polarized deep inelastic muon-scattering experiment installed at CERN SPS and uses a 160 - 200 GeV longitudinally polarized muon beam with a polarization of about 80% and an intensity of $2 \times 10^8 + /spill$. The polarized deuteron (6^LiD) target consists of an upstream and a downstream cell with opposite polarization. The particles produced in the interaction are detected behind the target in a two-stage spectrometer with high momentum resolution and high rate capability.



Figure 2.12: Schematic diagram of the COMPASS spectrometer.

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Figure 2.13: Spin-dependent non-singlet structure function xg_1^{NS} , measured by various experimental collaborations.



Figure 2.14: Experimental results for Bjorken Sum Rule. $(Q^2$'s are taken in the unit of GeV^2).

COMPASS took data from 2002 to 2004, accumulating an integrated luminosity of $\sim 4.6 f b^{-1}$.

The results taken from [71] for spin dependent non-singlet structure function g_1^{NS} and the corresponding Bjorken Sum Rule(BSR) are shown in Figs. 2.13 and 2.14 respectively along with other measurements.

2.2.8 HERMES

The HERMES experiment[91] at DESY was designed to the disentangle the contributions from the different quark flavours to the nucleons spin in semi inclusive deep inelastic scattering reactions. In such reactions, hadrons are detected in coincidence with the scattered lepton. The flavour of the quark probed in the scattering process can be deduced from the charge and the type of the observed hadron in a statistical analysis. The HERMES experiment employed an innovative technique for the polarized target, which is very different from all other polarized DIS experiments. Gas targets of pure nuclear-polarized atoms of hydrogen or deuterium were used, which permit essentially background-free measurements from highly polarized nucleons with



Figure 2.15: Schematic diagram of the HERMES spectrometer.

little or no dilution of the signal from unpolarized nucleons in the target. This choice eliminates one of the main systematic sources in polarized DIS, the uncertainty in the determination of the dilution factor.

The spectrometer (Fig. 2.15) consists of a large magnet followed by a detector package. The magnet bends charged particles produced in the interactions at the target, and the detectors are used for particle identification and momentum determination. The detector package in HERMES relies on a threshold Cerenkov counter and transition radiation detector for particle identification and tracking chambers with a rear lead-glass calorimeter for momentum and energy determination. Recently, a Cerenkov ring imaging detector has been installed at HERMES to tag hadrons and, in particular, kaons produced in the DIS interactions. The beam operates in a continuous mode, so it is straightforward to tag final state particles for studies of semi-inclusive scattering. The strength of the HERMES program lies in its clean identification of the interaction and complete event reconstruction using pure polarized gas targets.

HERMES measurements of g_1^{NS} and BSR, which are used in our analysis are included in Fig.2.13 and 2.14.

2.2.9 JLab Experiments

Thomas Jefferson National Accelerator Facility(TJNAF), commonly called Jafferson Lab or JLab, is a U.S. National laboratory located in Virginia. The experiments at Jefferson Lab utilized the highest polarization electron beams (85%) with energy ranging from 0.8 GeV close to 6 GeV. The technologies of polarizing beam and target

follow those pioneered and further developed at SLAC. The beam was provided by the Continuous Electron Beam Accelerator Facility (CEBAF)[92], which used polarized electron guns based on a "superlattice of a thin gallium arsenide (GaAs) layer on top of GaAs-phosphide bulk matter illuminated by circularly polarized photons from high intensity lasers [93]. Subsequently, the polarized electrons passed up to five times the two linear accelerators based on superconducting radio frequency technology and connected by two recirculation arcs. The spin direction of the electrons was manipulated using the crossed electric and magnetic fields of Wien filters, which allow for rapid spin rotation. Their direction was inverted every about 30 ms. Beam polarimetry was employed at several stages of the acceleration process. CEBAF delivered polarized beams simultaneously to the three experimental halls (Hall A, B and C) with the option to independently dial the energy and intensity. Typical beam intensities ranged from a few nA in Hall B to over 100A in the other two halls[94].

Longitudinal polarized solid state ammonia (NH₃) targets for the proton and ND₃ for the deuteron were employed at Hall B[95]. Hall A used a polarized ³He target. The target polarization was measured by both the NMR technique of adiabatic fast passage and a technique based on electron paramagnetic resonance[96]. Average target polarizations of about 55% were obtained. Hall A and C were both instrumented with small acceptance but high resolution spectrometers that could cope with the highest beam intensities but measured at fixed scattering angles. These spectrometers are equipped for high resolution tracking, precise time-of-flight measurements and lepton/hadron separation[97].

JLab measurements of BSR we have used in the phenomenological analysis of our BSR results are depicted in Fig. 2.14 along with other measurements.

2.3 Parameterizations and Results

Parton distribution functions are determined from fits of perturbative QCD calculations, based on the DGLAP evolution equations, to various sets of experimental data. These fits are regularly updated to account for new experimental input and theoretical developments. Modern pdf's extracted from global analyses of data from DIS and hadronic processes are provided by several groups. For purposes of comparison in this thesis we present a brief discussion on the parameterizations MRST[104–106] and MSTW[107], CTEQ[98], GRV[99, 100]and NNPDF[101–103].

2.3.1 MRST and MSTW2008

Martin-Roberts-Stirling-Thorne(MRST)[104–106] parametrization has the following functional behaviour

$$xf(x, Q_0^2) = A_0 x^{\alpha} (1 - x)^{\beta} (1 + \delta x^{\gamma} + \eta x), \qquad (2.1)$$

parton	A_0	α	β	γ	δ	η
$d_v(x,Q^2)$	0.040	0.27	3.88	52.73	0.5	30.65
$d_v(x,Q^2)$	0.158	0.25	3.33	5.61	0.5	55.49

Table 2.2: Parameters characterizing the MRST2001[104] NLO parton distribution functions at $Q_0^2 = 1 GeV^2$ defined by Eq. (2.1).

and the Parameters characterizing the MRST2001 NLO parton distribution functions at $Q_0^2 = 1 GeV^2$ are listed in Table 2.3. With this parametrization, they have been predicting the parton distribution functions including several data set from H1, Zeus, BCDMS, SLAC, FNAL E665, CCFR, Drell-Yan etc., since 2002. MRST2002[104] and MRST2003[105] were improved in MRST2004[106] by including the new HERA data at moderate values of x and high Q^2 . Also in MRST2004, full NNLO splitting functions were used. Recently published MSTW[107] parametrization represents an update of MRST. This update has a number of new theoretical features aimed at the NNLO parametrization; e.g., NNLO corrections to the Drell-Yan data. This parametrization also includes NuTeV and CHORUS data, the CDFII data, HERA inclusive jet data as well as direct high-x data on the F_L structure function.

2.3.2 CTEQ

The most recent parametrizations from the Coordinated Theoretical-Experimental Project on QCD (CTEQ) is the CTEQ6 series[98]. CTEQ uses the following functional form of the distribution functions,

$$xf(x,Q_0^2) = A_0 x^{A_1} (1-x)^{A_2} e^{A_3 x} (1+e^{A_4} x)^{A_5}, \qquad (2.2)$$

and the Parameters characterizing the CTEQ6M NLO parton distribution functions at $Q_0^2 = 1.69 GeV^2$ are listed in Table 2.3. With this parametrization, they have been predicting different parton distribution functions. The CTEQ collaboration omits

parton	A_0	A_0	β	γ	δ	$\mid \eta$
$d_v(x,Q_0^2)$	1.4473	0.616	4.9670	-0.8408	0.4031	3.00
$u_v(x,Q_0^2)$	1.7199	0.5526	2.9009	-2.3502	1.6123	1.5917
$g(x,Q_0^2)$	30.4571	0.5100	2.3823	4.3945	2.355	-3.000
$\left(\bar{u}+\bar{d}\right)(x,Q_0^2)$	0.0616	-0.2990	7.7170	-0.5283	4.7539	0.6137
$s(x, Q_0^2) = \bar{s}(x, Q_0^2)$	0.0123	-0.2990	7.7170	-0.5283	4.7539	0.6137

Table 2.3: Parameters characterizing the CTEQ6M parton distribution functions at $Q_0^2 = 1.69 GeV^2$ defined by Eq. (2.2).

data for $Q^2 \leq 4GeV^2$. In particular, CTEQ6 omits SLAC data as well as some high- Q^2 H1 data. CTEQ uses 10% systematic errors in quadrature with the statistical errors for the Drell-Yan data in comparison with 5% systematic errors assumed by MRST2002. CTEQ uses a starting scale of $Q_0^2 = 1.69GeV^2$.

2.3.3 GRV

The Gluck-Reya-Vogt(GRV) parton distribution functions were developed in a series of publications throughout the 1990s[99,100]. They are dynamical distributions, which are generated radiatively from valence-like inputs at a low resolution scale. The latest of this series makes use of the 1994-95 HERA data for $Q^2 \ge 2GeV^2$ as well as the SLAC, BCDMS, NMC and E665 data with $Q^2 \ge 4GeV^2$ and the simply extracted ratios F_2^n/F_2^p from the NMC, BCDMS and E665 experiments. This analysis takes into account the Drell-Yan data and the u_v/d_v ratios extracted from the CERN CDHSW and WA21 neutrino data.

The GRV parton distribution functions are parametrized as

$$xf(x, Q_0^2) = A_0 x^{\alpha} (1-x)^{\beta} (1+\delta\sqrt{x}+\eta x), \qquad (2.3)$$

parton	A_0	α	β	δ	η
$d_v(x,Q_0^2)$	0.761	1.48	3.62	-1.8	9.5
$u_v(x,Q_0^2)$	1.239	0.48	2.72	-1.8	9.5
$x\Delta(x,Q_0^2)$	0.23	0.48	11.3	-12.0	50.9
$x(\bar{u}+\bar{d})(x,Q_0^2)$	1.52	0.15	9.1	-3.6	7.8
$xg(x,Q_0^2) = \bar{s}(x,Q_0^2)$	17.47	1.6	3.8	-	-

Table 2.4: Parameters characterizing the GRV98LO parton distribution functions at $Q_0^2 = 1.69 GeV^2$ defined by Eq.(2.3).

parton	A_0	α	β	δ	η
$d_v(x,Q_0^2)$	0.394	1.43	4.09	-	18.2
$u_v(x,Q_0^2)$	0.632	0.43	3.09	-	18.2
$x\Delta(x,Q_0^2)$	0.20	0.43	12.4	-13.3	60.0
$x(\bar{u}+\bar{d})(x,Q_0^2)$	1.24	0.20	8.5	-2.3	5.7
$xg(x,Q_0^2) = \bar{s}(x,Q_0^2)$	20.80	1.6	4.1	-	-

Table 2.5: Parameters characterizing the GRV98NLO parton distribution functions at $Q_0^2 = 1.69 GeV^2$ defined by Eq.(2.3).

and the parameters for GRV98LO and GRV98NLO are listed in tables 2.4 and 2.5 respectively. These distributions are characterized by a relatively low starting scale for evolution: LO, $Q_0 = 0.5 GeV$; and NLO, $Q_0 = 0.63 GeV$.

2.3.4 NNPDF

The consideration of a specific parametrization with large number of parameters is potentially a source of bias, i.e. systematic error which is very difficult to control. Furthermore, when a parametrization is fitted to the data, it is very hard to obtain a determination not only of the best fitting parameters, but also of their errors. Therefore, explorations of the possibility of obtaining accurate solutions of DGLAP evolution equations without an initial input or with initial input, consisting of less number of parameters are always interesting. NNPDF method is one of the most interesting methods which does not require to assume a functional form and it is largely bias free[101–103]. NNPDF uses neural networks to parameterise the densities. The formalism is described in [102] and references therein. Neural networks are just another functional form, that generalises parameterisations like $xf(x) = \sum_{n} \alpha_n P_n(x)$ based on interpolation polynomials $P_n(x)$. They allow non-linear dependencies of the function on the fitted parameters α_n .

The analysis presented in [102] fits the gluon density together with the six densities for light quarks and anti-quarks u, \bar{u} , d, \bar{d} , s, \bar{s} . The neural networks chosen to parameterise these densities have 37 free parameters each. Hence, the resulting parameterisation has a total of $7 \times 37 = 259$ free parameters, which is much larger than the number of free parameters, O(25), which are fitted in QCD analyses based on a standard functional form like Eq. (2.3). The use of such a flexible parameterisation scheme considerably reduces any parameterisation bias. $\Box\Box$.

Chapter 3

On the Solution of DGLAP Evolution Equation

Along with a qualitative analysis of the available methods to solve DGLAP equation, in this chapter we have allude the usefulness of two Q^2 dependent Regge ansatz in solving DGLAP equation in order to have the small-x behaviour of both the spin independent and spin dependent non-singlet structure functions. By means of fitting analysis, we have investigated the compatibility of the two ansatz with the available experimental data and then studied the possible role played by them in evolving the non-singlet structure functions in accord with DGLAP equation.

3.1 Introduction

It is widely believed that QCD is the correct theory of strong interaction. In QCD, the structure functions are governed by a set of integro-differential equations, the Dokshitzer-Gribov-Lipatov-Altarelli-Parisi(DGLAP) evolution equations[24]. The DGLAP equation is a renormalisation group equation for the quarks and gluon inside hadron. It is one of the fundamental equations of perturbative quantum chromodynamics (pQCD), being central to all theoretical predictions for lepton-hadron colliders. The DGLAP evolutions are given in terms of a perturbative expansion of splitting functions (P_{ij}) which describe the probability of a parent parton *i* producing a daughter parton *j* with momentum fraction *z* by the emission of a parton with momentum fraction 1 - z. For the flavor non-singlet ($q_{NS} = q_i - \bar{q}_j$), flavor-singlet($q_s = q_i + \bar{q}_i$) and gluon distributions(*q*), the DGLAP evolution equations read as follows:

$$\frac{dq^{NS}(x,Q^2)}{d\ln Q^2} = \frac{\alpha(Q^2)}{2\pi} \int_x^1 \frac{dy}{y} q^{NS}(y,Q^2) P_{qq}(\frac{x}{y}), \qquad (3.1)$$

$$\frac{d}{d\ln Q^2} \begin{pmatrix} q^s(x,Q^2) \\ g(x,Q^2) \end{pmatrix} = \frac{\alpha(Q^2)}{2\pi} \int_x^1 \frac{dy}{y} \begin{pmatrix} P_{qq}(\frac{x}{y}) & P_{qg}(\frac{x}{y}) \\ P_{gq}(\frac{x}{y}) & P_{gg}(\frac{x}{y}) \end{pmatrix} \begin{pmatrix} q^s(y,Q^2) \\ g(y,Q^2) \end{pmatrix}.$$
 (3.2)

Solutions of DGLAP equations give the Q^2 evolution of both the parton distribution functions as well as various structure functions. Although QCD predicts the Q^2 dependence of structure functions in accord with the DGLAP equations but they have limitations on absolute prediction of structure functions. DGLAP equations cannot predict the initial values from which the evolution starts, they can only predict the evolution of structure functions with Q^2 , once an initial distribution is given. Further, due to its complicated mathematical structure, an exact analytic determination of the structure functions is currently out of reach and one needs to apply approximated methods to arrive on predictions from the DGLAP equation. Accordingly several approximate numerical as well as semi-analytical methods for the solution of DGLAP equation have been discussed considerably over the past years [108–113]. In literature there are essentially two main classes of approaches in order to have solutions of DGLAP equations: those that solve the equation directly in x-space and those that solve it for Mellin transformations of structure functions and invert the transformation back to x-space. The approaches based on Mellin transformation method have been achieved much interest because under Mellin transformation the integro-differential DGLAP equation turns into a continuum of independent matrix differential equations, one for each value of moments(N), which in turn makes the evolution more efficient numerically. However, in this regard as the Mellin transformation of both the splitting functions and the initial input is required, which may not be possible for all functions, especially if higher-order corrections are included in the equations, therefore it is not possible to have exact solution to DGLAP equation in moment space beyond leading order. In contrast to Mellin space, the x-space method is more flexible, since the inputs are only required in x-space; however it is generally considered to less efficient numerically, because of the need to carry out the convolution in DGLAP equations. Taking into account the advantage of being greater flexibility, despite the difficulty in obtaining high accuracy, the x-space methods have been serving as the basis of many widely used programs HOPPET[114], QCDNUM[108], CANDIA[113] etc., and being incorporated by the CTEQ[98], MRST/MSTW(see [107] and references therein) collaborations. In addition, several numerical and semi-analytical methods have been developed[21,100–103,107,109–113,115,138] and achieved significant phenomenological success.

3.2 Methods of Solution of DGLAP Evolution Equation

There exist numerous techniques to solve DGLAP equations. Among them most popular techniques are the Laguerre polynomial method, Mellin transformation method and Brute force method.

The Laguerre Polynomial method for numerical solution of the DGLAP evolution equation is based on the expansion of the structure functions and splitting functions in the basis of orthogonal Laguerre Polynmials[116, 117].

In this method, initially an evolution function $E_{NS}(x,t)$ is defined which describes the evolution of the structure functions from t = 0 to t as

$$F_{NS}(x,t) = \int_{x}^{1} \frac{d\omega}{\omega} E_{NS}(\frac{x}{\omega},t) F_{NS}(\omega,t=0), \qquad (3.3)$$

which satisfies

$$\frac{\partial}{\partial t}E_{NS}(x,t) = \int_{x}^{1} \frac{d\omega}{\omega} P_{NS}(\frac{x}{\omega},t)E_{NS}(\omega,t=0).$$
(3.4)

This integro-differential equation has the similar form as the original DGLAP equation. The advantage of introducing an evolution function is that it should be the delta function at t = 0: $E_{NS}(x, t = 0) = \delta(1 - x)$ because of its definition in Eq.(3.3).

Here the functions are expanded in terms of the polynomials: $P_{NS}(e^{-\prime x}) = \sum_{n} P_{NS}^{n} L_{n}(\prime x)$ and $E_{NS}(e^{-\prime x},t) = \sum_{n} E_{NS}^{n}(t)L_{n}(x')$, where P_{NS}^{n} and $E_{NS}^{n}(t)$ are the expansion coefficients. The coefficient F^{n} for a function F(x) is given by $F^{n} = \int_{0}^{1} L_{n}(\prime x)F(x)$, and it could be calculated analytically for a simple function. If the two functions on the right of Eq.(3.4) are expanded, it becomes an integration of two Laguerre polynomials. Using the formula $\int_{0}^{x'} d\omega' L_{n}(x'-\omega')L_{m}(\omega') = L_{n+m}(x') - L_{n+m+1}(x')$ for this integration, we obtain

$$\frac{d}{dt}E_{NS}(t) = \sum_{m=0}^{n} (P_{NS}^{n-m} - P_{NS}^{n-m-1})E_m(t).$$
(3.5)

At t = 0 all the expansion coefficients are one, as the evolution function is a delta function. Therefore, the solution of this equation gives a summation of the form:

$$E_{NS}^{m}(t) = e^{P_{NS}^{0}t} \sum_{k=0}^{m} \frac{t^{k}}{k!} B_{m}^{k}, B_{m}^{K+1} = \sum_{i=k}^{m-1} (P_{NS}^{m-i} - P_{NS}^{m-i-1}) B_{i}^{k}.$$
 (3.6)

This recursion relation is calculated with the relations $B_i^0 = 1$, $B_i^1 = \sum_{j=1}^i (P_{NS}^j - P_{NS}^{j-1})$ and $B_0^k = B_1^k = \dots = B_{k-1}^k = 0$. After all, the evolution is calculated by simple summation:

$$F_{NS}(x,t) = \sum_{n=0}^{\infty} N_{Lag} \sum_{m=0}^{n} \left[E_{n-m}(t) - E_{n-m-1}(t) \right] L_n(-\ln x) F_{NS}^m(t=0).$$
(3.7)

In this way, the integro-differential equation turns into a simple summation of Laguerre expansion coefficients, so that this method is regarded as a significant and very efficient numerical method for the numerical solution of the equation. However the accuracy of this technique is limited and it is quite accurate up to x-values not smaller than $x \approx 10^3$. on the other hand for small x the convergence of the expansion decreases. Therefore his method results no longer practical for the solution of DGLAP equation within smaller-x region.

The Mellin transformation method is one of the popular evolution methods[118]. The reason behind popularity of the method is its ability to resolve the right hand side of Eq.(3.1) into a simple product of to moments, namely the moments of distribution function and the moments of splitting functions. In order to have solution in this way, the moments of both the splitting function and distribution function are required. Usually, the moments are well known and assuming a simple model for the distribution functions at certain small Q^2 such that its moments can be calculated easily, the analytical solution of the equation can be obtained in moments space. Furthermore, the computation time is fairly short. These are the reasons why this method has been used as a popular method. For example, it is used for the χ^2 analysis of experimental data for obtaining polarized PDFs[119], whereas the brute-force method is employed in Ref. [120].

The Mellin transformation and inversion are defined by

$$\hat{F}_{NS}(s,t) = \int_0^1 dx x^{s-1} F(x,t), F(x,t) = \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} ds x^{-s} \hat{F}(s,t).$$
(3.8)

The Mellin inversion is a complex integral which consists of an arbitrary real constant c, which should be chosen such that absolutely convergency is achieved in the integral $\int_0^1 dx F(x) x^{c-1}$. Under this transformation, the integro-differential equations become very simple. For example, the nonsinglet evolution equation becomes

$$\frac{\partial}{\partial t}\hat{F}_{NS}(s,t) = \hat{P}_{NS}(s)\hat{F}_{NS}(s,t).$$
(3.9)

Its solution is simply given by

$$\hat{F}_{NS}(s,t) = e^{P_{NS}(s)t}\hat{F}_{NS}(s,t=0).$$
(3.10)

The moments of the distribution function $\hat{F}_{NS}(s, t = 0)$, which is initially considered at a certain $Q^2 = Q_0^2$, can be evaluated and using the well known moments of the splitting functions $\hat{P}_{NS}(s)$, the solution of Eq.(7.14) can easily be obtained in the moment space. However, in order to have the distribution in x space, an inverse Mellin transformation is required. In this regard, one important point to be noted is that, the numerical Mellin inversion is relatively CPU time consumig, which happen even if the analytical expressions of the moments of the initial conditions are well known[121]. Moreover, as discussed in [122], since x variable is associated with the invariant energy W^2 of the virtual photon-hadron scattering process by $W^2 = (1x)/x, x \to 0$ is the infinite energy limit and thus can never experimentally be reached. As a consequence of this all moments are plagued by an a priori infinite uncertainty, which can be reduced by means of assumptions implying that any use of the evolution equations for moments is model dependent.

The Brute-force method[122,123] is the simplest method in order to have numerical solution of integro-differential equation. For more complicated equations consisting of higher twist terms[124], which could not be easily handled by other methods, such as Mellin transformation as well as Laguerre-polynomial methods, the Brute-force method is suitable, although it being seemed to be too simple. Furthermore, a computer code is so simple that the possibility of a program mistake is small, which means the code could be used for checking other numerical methods.

In the brute-force method, the two variables t and x are divided into small steps, and then the differentiation and integration are defined by

$$\frac{\partial F_{NS}(x,t)}{\partial t} \Rightarrow \frac{F(x_i, t_{j+1}) - F(x_i, t_j)}{\Delta t_j}, \int dx F(x,t) \Rightarrow \sum_{k=1}^{N_x} \Delta x_k F(x_k, t_j), \quad (3.11)$$

where Δt_j and Δx_k are the steps at the positions j and k, and they are given by $\Delta t_j = t_{j+1} - t_j$ and $\Delta x_k = x_k - x_{k-1}$. The numbers of t and x steps are denoted N_t and N_x , respectively. Applying these equations to Eq.(3.1), we write the non-singlet evolution from t_j to t_{j+1} as

$$F_{NS}(x_i, t_{j+1}) = F_{NS}(x_i, t_j) + \Delta t_j \sum_{k=1}^{N_x} \frac{\Delta x_k}{x_k} P_{qq}(x_i/x_k) F_{NS}(x_k, t_j).$$
(3.12)

If the initial distribution F_{NS} is considered at $t_1 = 0$, the next one $F_{NS}(x, t_2)$ can be determined by the above equation. Proceeding in this way, step by step, upto $N_t 1$ times, the final distribution at t_{N_t} can be obtained. However, accuracy of the results demands a large number of steps N_t and N_x .

In addition to these three, some other numerical as well as semi-analytical methods to solve DGLAP evolution equations are available in literature, such as Matrix approach method, Taylor expansion method, Regge theory method etc. and predict the evolution of various structure functions with considerable phenomenological success.

3.3 A Regge Inspired Approach to Solve the DGLAP Equation

Due to the unavailability of exact analytical way of solving the DGLAP equations, in current analysis this set of equations are solved numerically by using an initial input distribution for the structure function at a fixed Q^2 , in terms of some free parameters, the parameters are so adjusted that the parametrization best fit the existing data. However, the consideration of a specific parametrization with large number of parameters is potentially a source of bias, i.e. systematic error which is very difficult to control. Furthermore, when a parametrization is fitted to the data, it is very hard to obtain a determination not only of the best fitting parameters, but also of their errors. Therefore, explorations of the possibility of obtaining accurate solutions of DGLAP evolution equations without an initial input or with initial input, consisting of less number of parameters are always interesting. Under this motivation, this thesis is devoted to the exploration of a semi-analytic approach of solving DGLAP equation for non-singlet structure functions using two Regge inspired model with less number of parameters. Here particular emphasis is given to the non-singlet structure functions because they are considered as the starting ground for theoretical description
of DIS structure functions. Besides being interesting in themselves, another significant advantage is that QCD analysis by means of non-singlet structure functions is comparatively technically simpler.

In order to perform a fit, one must start with a particular ansatz for the structure functions at some reference Q_0^2 . In most of the existing fitting analysis, including those in the experimental papers it has been performed by assuming a simple power behavior based on Regge theory. Regge theory predicts the x dependence of the structure functions at fixed Q^2 and at small x. In Regge theory the x dependency of the non-singlet structure functions F_i^{NS} , i=2,3 (i.e., $F_{i=2}^{NS}=F_2^{NS}$ and $F_{i=3}^{NS}=xF_3$) are described with a power law, $F_i^{NS}(x) = B_i^{NS} x^{\lambda_i^{NS}}$, for fixed $Q^2[13, 59, 127, 128]$. Besides being x dependency, the structure functions, in accordance with QCD predictions, are dependent on Q^2 also. The Bjorken Scalling violation or the Q^2 dependence of the structure functions is one of the significant predictions of Quantum Chromodynamics and recent experiments also reveal the evidence of Q^2 dependency of the structure functions even at small-x. Therefore in order to have Q^2 behavior of non-singlet structure functions we have to modify the Regge predictions by incorporating Q^2 dependency either to the exponents (λ_i^{NS}) only or to the coefficients (B_i^{NS}) or both. Here, we have preferred to investigate the possibility of first two cases i.e., firstly, the coefficients (B_i^{NS}) are Q^2 dependent with constant exponents and next, the exponents (λ_i^{NS}) are Q^2 dependent with constant coefficients.

3.3.1 Regge Ansatz with Q^2 Dependent Coefficient and Constant Intercept

There are many phenomenological models, developed within the Regge approach for Deep Inelastic Scattering and structure functions. The simple Regge pole exchange model predicts that, towards smaller values of x the non-singlet unpolarized structure functions $F_2^{NS}(x, Q^2)$, $xF_3(x, Q^2)$ and $xg_1(x, Q^2)$ behave as

$$\frac{1}{x}F_2^{NS}(x) = B_2^{NS}x^{-\lambda_2^{NS}},\tag{3.13}$$

$$xF_3(x) = B_3^{NS} x \cdot x^{-\lambda_3^{NS}}$$
(3.14)

and

$$xg_1^{NS}(x) = B_g^{NS} x \cdot x^{-\lambda_3^{NS}}$$
(3.15)

respectively, with the exponents $\lambda_i^{NS} = \alpha_{A_2}(0)$. According to Regge theory, $F_2^{NS}(x)$, $xF_3(x)$ and xg_1^{NS} are governed by the A_2 Regge trajectory with the intercept $\alpha_{A_2}(0)$. For $\alpha_{A_2}(0) \approx 0.5$, the behaviors (3.13) and (3.15) are stable and considerable phenomenological success is observed in this regards. As a consequence, the small x behavior of the unpolarized non-singlet structure functions can be expressed as

$$F_2^{NS}(x) = B_2^{NS} x^{0.5}, (3.16)$$

$$xF_3(x) = B_3^{NS} x^{0.5} (3.17)$$

and

$$xg_1^{NS}(x) = B_g^{NS} x \cdot x^{0.5}.$$
(3.18)

Now, in accordance with QCD, we should expect all the dependence on Q^2 to be in $B_{(i=2,3,g)}^{NS}$, so that the Regge predictions, (7.13), (7.13) and (3.18) for x dependence do not change. Therefore, incorporating the Q^2 behavior of the structure functions in terms of the functions $B_{(i=2,3,g)}^{NS}(Q^2)$, we have the QCD modified Regge like model for both x as well as Q^2 dependent non-singlet structure functions at small x as

$$F_2^{NS}(x,Q^2) = B_2^{NS}(Q^2)x^{0.5},$$
(3.19)

$$xF_3(x,Q^2) = B_3^{NS}(Q^2)x^{0.5} aga{3.20}$$

and

$$xg_1^{NS}(x) = B_g^{NS} x x^{0.5}.$$
(3.21)

The non-singlet structure functions in this form does not contain any fitting parameter. We just need to evaluate the Q^2 dependent function $B_i(Q^2)$ and it can be obtained by means of solving the respective DGLAP evolution equations using these ansatz as the initial input, which is discussed briefly in the section 3.5 and in detailed in the chapters 4, 5, and 6.

3.3.2 Regge Ansatz with Q^2 Dependent Intercept and Constant Coefficient

Instead of being constant, there are several predictions on the Q^2 dependency of the Regge intercept. There were predictions [129, 130] that the exponent would be larger at high values of Q^2 and these types of predictions were born out from two different equations of perturbative QCD: the DGLAP equation and BFKL equation. Although this Regge model seems to legitimate as far the early data are concerned, which were mostly taken at moderate $Q^2 (\approx 10 GeV^2)$ and x values of around $x \ge 0.01$ but the recent measurement of F_i^{NS} for available small-x in the interval 0.0001 < x < 0.01 can be described with a single Regge type exchange $F_i^{NS} = Ax^{\alpha}$, in which the intercept has a smooth Q^2 dependence and varies like x^{α} with $-0.5 \le \alpha \le 0$. In the case of g_1^{NS} similar behaviour was predicted with valon model[131] and a variation from -0.13 to -0.3 was obtained within the interval of Q^2 from $2GeV^2$ to $10GeV^2$. On the other hand, Ref.[132] predicts a behaviour of the type, $g_1^{NS} \simeq \left(\frac{Q^2}{x^2}\right)^{\Delta_{NS/2}}$, with $\Delta_{NS} = 0.42$ in which the asymptotic scaling of g_1^{NS} depends on only one variable $\frac{Q^2}{x^2}$. In addition there are several studies on Q^2 dependency of the intercepts of the non-singlet structure functions[133].

In this section we have investigated the possibility of a simple Regge ansatz of the type $F_i^{NS} = Ax^{-bt}$ with Q^2 dependent intercept in order to describe the small-x behaviour of the structure functions. The underlying idea behind the assumption of this type of model is as follows: HERA measurements [134, 135] suggest that the behavior of F_2 structure function at low-x is consistent with a dependence $F_2(x, Q^2) =$ $Cx^{-\lambda(Q^2)}$, where the coefficient A is independent of Q^2 and the exponent, defined by $\lambda(Q^2) = a \ln\left(\frac{Q^2}{\Lambda^2}\right) = at$, is observed to rise linearly with $\ln Q^2$. Here Λ is the QCD cut off parameter and $t = \ln \left(\frac{Q^2}{\Lambda^2}\right)$. Thus we see that the rise of the un-polarized structure function $(F_2(x, Q^2))$ is much steeper than that predicted by Regge theory and gets steeper and steeper as Q^2 increases. Since this observation it has been the challenging issue to resolve whether the Regge intercepts for $F_2(x, Q^2)$ structure function as well as it's non-singlet, singlet and gluon parts, along with the spin structure functions are Q^2 dependent or not. Further, before the observation at HERA, there are several predictions on the Q^2 dependency of the Regge intercept [129, 130]. These predictions as well as experimental observations at HERA motivated us to consider the possibility that the Regge behaved non-singlet part, $\frac{1}{x}F_2^{NS}$ of $F_2(x,Q^2)$ structure function is also

satisfy a functional behaviour, $F_2^{NS}(x, Q^2) = Ax^{-b\ln(\frac{Q^2}{\Lambda^2})} = Ax^{-bt}$ similar to $F_2(x, Q^2)$. Again as the non-singlet structure functions, $\frac{1}{x}F_2^{NS}$ and $F_3(x,t)$ and $g_1^{NS}(x,t)$ are Regge behaved[131,136], therefore their x dependency will be similar within smallerx region. Further, in QCD the Q^2 behaviour of these structure functions are governed by the same DGLAP equation. Therefore the x and Q^2 dependency for all the nonsinglet structure functions are similar and in accord with F_2 , and hence F_2^{NS} here we assume that the Q^2 dependency of the Regge behaved structure function F_i^{NS} is dominated only by the intercept and it satisfies a relation of the type

$$F_i^{NS}(x,t) = A_i x^{1-b_i t}, (3.22)$$

where A_i and b_i are arbitrary constants, which are to be determined by fitting expressions with respective available experimental results. Here for simplicity, F_i^{NS} is defined to represent all of $F_2^{NS}(x,t)$, $xF_3^{NS}(x,t)$ and $xg_1^{NS}(x,t)$ structure functions.

3.4 Fitting Analysis of Our Models

The Regge like ansatz for F_i^{NS} structure functions in the form of Eqs.(3.19-3.21) does not consists of any parameters to be fitted. It will be seen in the following section as well as next three chapters that the unknown Q^2 dependent coefficient can be obtained by means of solving the DGLAP equation with the ansatz as the initial input. However the Regge like ansatz for F_i^{NS} structure function in the form of Eq. (3.22) consists of two parameters. This parametrization can be expressed in a different form in terms of only one parameter b by eliminating the parameter A, as A has no effect on the structure function in our approach, which is done as follows: The value of the F_i^{NS} structure function at any point (x_0, t_0) in the (x, t) coordinate system is given by

$$F_i^{NS}(x_0, t_0) = A_i x_0^{(1-b_i t_0)}.$$
(3.23)

Dividing (3.22) by (3.23) and rearranging a bit we get

$$F_i^{NS}(x,t) = F_i^{NS}(x_0,t_0) x^{(1-b_it)} x_0^{-(1-b_it_0)}.$$
(3.24)

This reduced form of the structure function consists of only one fitting parameter, the parameter b_i and a known input point $F_i^{NS}(x_0, t_0)$, which can be taken from the available experimental data. If the input point is more accurate and precise, we can

	Input point	Value of b	$\frac{\chi^2}{d.o.f}$	Kinematical region
F_2^{NS}	0.010348 ± 0.006208	0.118 ± 0.028	0.85	$x < 0.05 \text{ and } Q^2 \le 20$
xF_3	0.3298 ± 0.02605	0.0744 ± 0.0136	1.98	$x < 0.05$ and $Q^2 \le 20$
xg_1^{NS}	$0.0133075 \pm 0.0.001938$	0.0759 ± 0.0107	1.41	$x < 0.05 \text{ and } Q^2 \le 20$

Table 3.1: Summary of best fitting results for different structure functions.

expect batter fitting. There are not any specific reason in choosing the input point. Any one of the data points at a certain value of $x = x_0$ and $t = t_0$ can be considered as the input point. Off course, the sensitivity of different inputs will be different. However instead of choosing the input point on the basis of their sensitivity, in our manuscript we have incorporated a suitable condition in determining the input point. We have considered that particular point from the most recent measurements as the input point in which experimental errors are minimum. Under this condition we have selected the points, given in the Table:3.1, for different structure functions as the initial input point and then fitted the expressions with all the available experimental data. We have observed that the above parametrization fit best for the values of b_i which are collected in Table. 1 along with the corresponding $\frac{\chi^2}{d.o.f.}$. In this analysis, we have considered the QCD cut-off parameter λ to be fixed and the considered values are 0.323 GeV², 0.337 GeV² and 0.300 GeV² for F_2^{NS} , xF_3 and xg_1^{NS} respectively. Best fitted results are depicted in Figures 3.1, 3.2 and 3.3 for F_2^{NS} , xF_3 and xg_1^{NS} respectively along with the available experimental data. In addition, we have shown the band due to the uncertainty assolated with input and the fitting parameter b. The figures reflect a very good consistency between the ansatz and the experimental data.

As far the Figures are concerned, we see that the Regge ansatz Eq.(3.24) (with Q^2 dependent intercept) for the non-singlet structure functions are compatible with their respective experimental data within kinematical region of our consideration.

3.5 Solution of DGLAP Equation for F_i^{NS} with the Regge Ansatz

We now investigate how the two analytic ansatz help in solving the DGLAP evolution equations in order to have the Q^2 behavior of non-singlet structure functions. For simplicity, as an example, here we would like to discuss only the solution of LO DGLAP equation for F_2^{NS} structure function. When the two ansatz are introduced to the LO DGLAP evolution equation (3.1)



Figure 3.1: Our best fit results of Eq.(3.24) for $F_2^{NS}(x, Q^2)$ structure functions to NMC[63] results. For clarity, the points are offset by the amount given in parenthesis.(Q^2 's are taken in the unit of GeV^2).



Figure 3.2: Our best fit results of Eq.(3.24) for $xF_3(x, Q^2)$ structure functions to CCFR[66] results. For clarity, the points are offset by the amount given in parenthesis. (Q^2 's are taken in the unit of GeV^2).



Figure 3.3: Our best fit results of Eq.(3.24) for $xg_1^{NS}(x,Q^2)$ structure functions to the experimental data taken from SMC[74], HERMES[73], COMPASS[71] and E143[75]. Here the results are plotted against x.

$$\frac{\partial F_2^{NS}(x,t)}{\partial t} = \left(\frac{\alpha(t)}{2\pi}\right)_{LO} \left[\frac{2}{3}\{3+4\ln(1-x)\}F_2^{NS}(x,t) + I_1(x,t)\right],\tag{3.25}$$

and rearrange a bit, we obtain

$$\frac{\partial F_2^{NS}(x,t)}{\partial t} = \left(\frac{\alpha(t)}{2\pi}\right)_{LO} \left[\frac{2}{3}\{3+4\ln(1-x)\} + \frac{4}{3}\int_x^1 \frac{d\omega}{1-\omega} \left\{\frac{1+\omega^2}{\omega}\omega^{(-0.5)} - 2\right\}\right] F_2^{NS}(x,t), \quad (3.26)$$

$$\frac{\partial F_2^{NS}(x,t)}{\partial t} = \left(\frac{\alpha(t)}{2\pi}\right)_{LO} \left[\frac{2}{3}\left\{3 + 4ln(1-x)\right\} + \frac{4}{3}\int_x^1 \frac{d\omega}{1-\omega} \left\{\frac{1+\omega^2}{\omega}\omega^{(bt-1)} - 2\right\}\right] F_2^{NS}(x,t), \quad (3.27)$$

which have the form ordinary differential equations

$$\frac{\partial F_2^{NS}(x,t)}{\partial t} = \frac{\alpha(t)}{2\pi} U(x) F_i^{NS}(x,t), \qquad (3.28)$$

$$\frac{\partial F_2^{NS}(x,t)}{\partial t} = \frac{\alpha(t)}{2\pi} U(x,t) F_i^{NS}(x,t)$$
(3.29)

respectively. These two equations can be easily solved to have

$$F_2^{NS}(x,t)\bigg|_{LO} = C_1 \exp\left[U(x) \int \left(\frac{\alpha(t)}{2\pi}\right)_{LO} dt\right]$$
(3.30)

and

$$F_2^{NS}(x,t)\bigg|_{LO} = C_1 \exp\left[\int \left(\frac{\alpha(t)}{2\pi}\right)_{LO} U(x,t)dt\right],$$
(3.31)

respectively.

Now at a fixed value of $x = x_0$, the *t* dependence of the structure function $F_2^{NS}(x,t)$ in accord with (4.20) is given by

$$F_2^{NS}(x_0,t)\bigg|_{LO} = C_1 \exp\left[U(x_0) \int \left(\frac{\alpha(t)}{2\pi}\right)_{LO} dt\right].$$
(3.32)

Again the value of the structure function at $x = x_0$ and $t = t_0$ in accord with (4.26) is

$$F_2^{NS}(x_0, t_0) \bigg|_{LO} = C_1 \exp\left[U(x_0) \int \left(\frac{\alpha(t)}{2\pi} \right)_{LO} dt \right] \bigg|_{t=t_0}.$$
 (3.33)

Dividing (4.26) by (4.27) and rearranging a bit we obtain the t evolution of $F_2^{NS}(x,t)$ in accord with the LO DGLAP equation with respect to the point $F_2^{NS}(x_0,t_0)$ as

$$F_2^{NS}(x_0, t) \bigg|_{LO} = F_2^{NS}(x_0, t_0) \exp\left[U(x_0) \int_{t_0}^t \left(\frac{\alpha(t)}{2\pi}\right)_{LO} dt\right].$$
 (3.34)

Again in accord with our preassumptions (3.19), (3.20) and (3.21), the t dependence of $F_2^{NS}(x,t)$ at a particular value of $x = x_0$ is given by

$$F_2^{NS}(x_0, t) = B(Q^2) x_0^{0.5}.$$
(3.35)

Dividing any of (3.19), (3.20) and (3.21) by (4.29), we have the following relation

$$F_2^{NS}(x,t) = F_2^{NS}(x_0,t) \left(\frac{x}{x_0}\right)^{0.5},$$
(3.36)

which describes both t and x dependence of $F_2^{NS}(x,t)$ structure function in terms of the t dependent function $F_2^{NS}(x_0,t)$.

Now combining (4.28) and (4.30) we obtain the relation,

$$F_2^{NS}(x,t)\Big|_{LO} = F_2^{NS}(x_0,t_0) \exp\left[U(x_0)\int_{t_0}^t \left(\frac{\alpha(t)}{2\pi}\right)_{LO}dt\right]\left(\frac{x}{x_0}\right)^{0.5},$$
 (3.37)

which describes both t and x dependence of $F_2^{NS}(x,t)$ structure function in LO in terms of the input point $F_2^{NS}(x_0,t_0)$.

Proceeding in a similar way, from Eq. (4.37) we can have both t and x dependence of $F_2^{NS}(x,t)$ structure function as

$$F_2^{NS}(x,t)\bigg|_{LO} = F_2^{NS}(x_0,t_0) \exp\left[\int_{t_0}^t \left(\frac{\alpha(t)}{2\pi}\right)_{LO} U(x_0,t)dt\right] \left(\frac{x}{x_0}\right)^{(1-bt)}.$$
 (3.38)

As far the equations (4.31) and (4.49) are concerned, they are the analytic expressions representing both x and Q^2 dependence of $F_2^{NS}(x, Q^2)$ structure function jointly, which are obtained by means of solving the DGLAP equations in LO incorporating the Regge ansatz, $F_2^{NS}(x, Q^2) = A(Q^2)x^{0.5}$ and $F_2^{NS}(x, Q^2) = Bx^{1-bt}$ as the initial inputs respectively. These expressions are consisting of an input point $F_2^{NS}(x_0, t_0)$, which can be taken from the available experimental data. Moreover, the Eq. (4.31) does not contain any fitting parameter, however the Eq. (4.49) consists of only one fitting parameter b. Using a suitable input point, $F_2^{NS}(x_0, t_0)$ from experimental data, we can obtain both x and Q^2 -evolution of $F_2^{NS}(x, Q^2)$ structure function with the best fitted value of b. The calculation as well as phenomenological studies of un-polarised and polarised structure functions, F_2^{NS} , xF_3 and xg_1^{NS} with pQCD corrections upto NNLO is discussed in detailed in the chapter 4, chapter 5 and chapter 6 respectively.

Chapter 4

Small-x Behaviour of $F_2^{NS}(x,Q^2)$ Structure Function

In this chapter I present the full calculation of non-singlet structure function $F_2^{NS}(x, Q^2)$ by means of solving DGLAP equation with QCD corrections up to next-next-to-leading order. Using the two ansatz, discussed in the previous chapter, developed by combining the features of perturbative Quantum Chromodynamics and Regge theory, as the initial input we have solved the DGLAP equations. The solutions, along with the ansatz allow us to obtain some analytic expressions which represent the joint Bjorken x and Q^2 dependence of $F_2^{NS}(x,Q^2)$ structure function. The expressions are studied phenomenologically in comparison with experimental results taken from New Muon Collaboration (NMC) and the results of NNPDF parameterizations. A great phenomenological success is achieved in this regards, which signifies the capability of the expressions in describing the small-x behaviour of the non-singlet structure function and their usefulness in determining the structure functions with a reasonable precision.

4.1 Introduction

The structure function $F_2^{NS}(x, Q^2)$ is the non-singlet part of $F_2(x, Q^2)$ structure function originated in the unpolarized charged lepton DIS and it is given by the difference of proton and neutron structure functions as $F_2^{NS} = F_2^p - F_2^n$ [59]. The non-singlet structure function $F_2^{NS}(x, Q^2)$ provides a very good mean to investigate QCD as a theory of strong interaction. Besides being interesting in themselves, the non-singlet structure functions are not marred by the presence of the sea quark and gluon densities about which we have very poor information in particular in the small-*x* region and hence theoretical analysis by means of them are comparatively technically simpler. Therefore they are regarded as a starting ground for a theoretical description of DIS structure functions.

The Gottfried sum rule[34,35], associated with $F_2^{NS}(x,Q^2)$ is also an important observable of QCD. The determination of the Gottfried sum rule requires knowledge of $F_2^{NS}(x,Q^2)$ structure functions over the entire region of $x \in (0;1)$. However, the experimentally accessible x range for DIS is limited for the available data and therefore one should extrapolate results to x = 0 and x = 1. The extrapolation to $x \to 0$, where F_2^{NS} structure functions grow strongly, is much more important than the extrapolation to $x \to 1$, where structure functions vanish. Again, it is known that maximum contribution (about 90%) to the Gottfried sum rule come from the small $x(\leq 0.1)$ region. Because of the large contribution to the Gottfried sum rule from small x, the small x region is particularly important. Therefore this chapter is an attempt to have the small-x behaviour of $F_2^{NS}(x,Q^2)$ structure function by means of solving the DGLAP equation using the two Regge ansatz discussed in chapter 3 as the initial input.

The Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) evolution equation[24] which describe the Q^2 behavior of unpolarised non-singlet structure function $F_2^{NS}(x, Q^2)$ in perturbative Quantum Chromodynamics (QCD) formalism is given by

$$\frac{\partial F_2^{NS}(x,Q^2)}{\partial \ln Q^2} = \int_x^1 \frac{d\omega}{\omega} F_2^{NS}(\frac{x}{\omega},Q^2) P(\omega).$$
(4.1)

Where, $P(\omega)$ is the splitting function associated with $F_2^{NS}(x, Q^2)$ structure function, which is defined up to NNLO by[31]

$$P(\omega) = \frac{\alpha(Q^2)}{2\pi} P^{(0)}(\omega) + \left(\frac{\alpha(Q^2)}{2\pi}\right)^2 P^{(1)}(\omega) + \left(\frac{\alpha(Q^2)}{2\pi}\right)^3 P^{(2)}(\omega).$$
(4.2)

Here, $P^{(0)}(\omega)$, $P^{(1)}(\omega)$ and $P^{(2)}(\omega)$ are the corresponding leading order(LO), next-toleading order (NLO) and next-next-to-leading order(NNLO) corrections to the splitting functions. Splitting functions are given in Appendices.

Again, in LO, NLO and NNLO, the running coupling constant $\frac{\alpha(Q^2)}{2\pi}$ has the forms[23],

$$\left(\frac{\alpha(t)}{2\pi}\right)_{LO} = \frac{2}{\beta_0 t},\tag{4.3}$$

$$\left(\frac{\alpha(t)}{2\pi}\right)_{NLO} = \frac{2}{\beta_0 t} \left[1 - \frac{\beta_1 \ln t}{\beta_0^2 t}\right],\tag{4.4}$$

$$\left(\frac{\alpha(t)}{2\pi}\right)_{NNLO} = \frac{2}{\beta_0 t} \left[1 - \frac{\beta_1 \ln t}{\beta_0^2 t} + \frac{1}{\beta_0^2 t^2} \left[\left(\frac{\beta_1}{\beta_0}\right)^2 (\ln^2 t - \ln t + 1) + \frac{\beta_2}{\beta_0} \right] \right], \quad (4.5)$$

where $\beta_0 = 11 - \frac{2}{3}N_F$, $\beta_1 = 102 - \frac{38}{3}N_F$ and $\beta_2 = \frac{2857}{6} - \frac{6673}{18}N_F + \frac{325}{54}N_F^2$ are the oneloop, two-loop and three-loop corrections to the QCD β -function. Here the running coupling constant is expressed in terms of the variable t, which is defined by $t = \ln(\frac{Q^2}{\Lambda^2})$.

Substituting the respective splitting functions along with the corresponding running coupling constant in (4.1), the DGLAP evolution equations in LO, NLO and NNLO become

$$\frac{\partial F_2^{NS}(x,t)}{\partial t} = \left(\frac{\alpha(t)}{2\pi}\right)_{LO} \left[\frac{2}{3}\{3+4\ln(1-x)\}F_2^{NS}(x,t) + I_1(x,t)\right], \quad (4.6)$$

$$\frac{\partial F_2^{NS}(x,t)}{\partial t} = \left(\frac{\alpha(t)}{2\pi}\right)_{NLO} \left[\frac{2}{3}\{3+4ln(1-x)\}F_2^{NS}(x,t) + I_1(x,t)\right] + \left(\frac{\alpha(t)}{2\pi}\right)_{NLO}^2 I_2(x,t), \quad (4.7)$$

and

$$\frac{\partial F_2^{NS}(x,t)}{\partial t} = \left(\frac{\alpha(t)}{2\pi}\right)_{NNLO} \left[\frac{2}{3}\{3+4ln(1-x)\}F_2^{NS}(x,t)+I_1(x,t)\right] \\ + \left(\frac{\alpha(t)}{2\pi}\right)_{NNLO}^2 I_2(x,t) + \left(\frac{\alpha(t)}{2\pi}\right)_{NNLO}^3 I_3(x,t)$$
(4.8)

respectively. Here Λ is the QCD cut-off parameter and the integral functions are given by

$$I_{1}(x,t) = \int_{x}^{1} \frac{d\omega}{1-\omega} \left\{ \frac{1+\omega^{2}}{\omega} F_{2}^{NS}\left(\frac{x}{\omega},t\right) - 2F_{2}^{NS}(x,t) \right\},$$
(4.9)

$$I_2(x,t) = \int_x^1 \frac{d\omega}{\omega} P^{(1)}(\omega) F_2^{NS}\left(\frac{x}{\omega},t\right)$$
(4.10)

$$I_3(x,t) = \int_x^1 \frac{d\omega}{\omega} P^{(2)}(\omega) F_2^{NS}\left(\frac{x}{\omega},t\right).$$
(4.11)

The DGLAP equations up to NNLO ((4.6)-(4.8)) can be solved analytically using the ansatz $F_2^{NS}(x,t) = A(t)x^{0.5}$ and $F_2^{NS}(x,t) = Bx^{(1-at)}$ as the initial inputs and I have discussed below in detailed.

4.2 Solution of DGLAP Evolution Equations with the Initial Input $F_2^{NS}(x,t) = A(t)x^{0.5}$

On substitution of

$$F_2^{NS}(x,t) = F_2^{NS}(x,t) = A(t)x^{0.5}$$
(4.12)

and hence

$$F_2^{NS}(\frac{x}{\omega},t) = F_2^{NS}(\frac{x}{\omega},t) = A(t)x^{0.5}\omega^{-0.5} = F_2^{NS}(x,t)\omega^{-0.5}$$
(4.13)

in the equations (4.6), (4.7) and (4.8), we obtain

$$\frac{\partial F_2^{NS}(x,t)}{\partial t} = \left(\frac{\alpha(t)}{2\pi}\right)_{LO} \left[\frac{2}{3}\{3+4\ln(1-x)\} + \frac{4}{3}\int_x^1 \frac{d\omega}{1-\omega} \left\{\frac{1+\omega^2}{\omega}\omega^{-0.5} - 2\right\}\right] F_2^{NS}(x,t), \quad (4.14)$$

$$\frac{\partial F_2^{NS}(x,t)}{\partial t} = \left(\frac{\alpha(t)}{2\pi}\right)_{NLO} \left[\frac{2}{3}\left\{3 + 4\ln(1-x)\right\} + \frac{4}{3}\int_x^1 \frac{d\omega}{1-\omega} \left\{\frac{1+\omega^2}{\omega}\omega^{-0.5} - 2\right\}\right] F_2^{NS}(x,t) + \left(\frac{\alpha(t)}{2\pi}\right)_{NLO}^2 \int_x^1 \frac{d\omega}{\omega} P^{(1)}(\omega)\omega^{-0.5}F_2^{NS}(x,t) \quad (4.15)$$

$$\begin{aligned} \frac{\partial F_2^{NS}(x,t)}{\partial t} &= \left(\frac{\alpha(t)}{2\pi}\right)_{NNLO} \left[\frac{2}{3}\{3+4ln(1-x)\} + \frac{4}{3}\int_x^1 \frac{d\omega}{1-\omega} \left\{\frac{1+\omega^2}{\omega}\omega^{-0.5}\right\} \\ &-2\left\{\right\} F_2^{NS}(x,t) + \left(\frac{\alpha(t)}{2\pi}\right)^2_{NNLO}\int_x^1 \frac{d\omega}{\omega} P^{(1)}(\omega)\omega^{-0.5}F_2^{NS}(x,t) \\ &+ \left(\frac{\alpha(t)}{2\pi}\right)^3_{NNLO}\int_x^1 \frac{d\omega}{\omega} P^{(2)}(\omega)\omega^{-0.5}F_2^{NS}(x,t) (4.16) \end{aligned}$$

respectively. These equations can be rearranged to have three ordinary differential equations in terms of $F_2^{NS}(x,t)$,

$$\frac{\partial F_2^{NS}(x,t)}{\partial t} = \frac{\alpha(t)}{2\pi} U(x) F_2^{NS}(x,t), \qquad (4.17)$$

$$\frac{\partial F_2^{NS}(x,t)}{\partial t} = \left[\left(\frac{\alpha(t)}{2\pi} \right)_{NLO} U(x) + \left(\frac{\alpha(t)}{2\pi} \right)_{NLO}^2 V(x) \right] F_2^{NS}(x,t), \tag{4.18}$$

and

$$\frac{\partial F_2^{NS}(x,t)}{\partial t} = \left[\left(\frac{\alpha(t)}{2\pi} \right)_{NNLO} U(x) + \left(\frac{\alpha(t)}{2\pi} \right)_{NNLO}^2 V(x) + \left(\frac{\alpha(t)}{2\pi} \right)_{NNLO}^3 W(x) \right] F_2^{NS}(x,t)$$
(4.19)

which can be easily solved to have

$$F_2^{NS}(x,t)\bigg|_{LO} = C_1 \exp\left[U(x) \int \left(\frac{\alpha(t)}{2\pi}\right)_{LO} dt\right],\tag{4.20}$$

$$F_2^{NS}(x,t)\bigg|_{NLO} = C_2 \exp\left[U(x) \int \left(\frac{\alpha(t)}{2\pi}\right)_{NLO} dt + V(x) \int \left(\frac{\alpha(t)}{2\pi}\right)_{NLO}^2 dt\right], (4.21)$$

and

$$F_2^{NS}(x,t)\bigg|_{NNLO} = C_3 \exp\left[U(x) \int \left(\frac{\alpha(t)}{2\pi}\right)_{NNLO} dt + V(x) \int \left(\frac{\alpha(t)}{2\pi}\right)_{NNLO}^2 dt + W(x) \int \left(\frac{\alpha(t)}{2\pi}\right)_{NNLO}^3 dt\right].$$
(4.22)

respectively. Here,

$$U(x) = \frac{2}{3} \{3 + 4\ln(1-x)\} + \frac{4}{3} \int_{x}^{1} \frac{d\omega}{1-\omega} \left\{ \frac{1+\omega^{2}}{\omega} \omega^{-0.5} - 2 \right\},$$
(4.23)

$$V(x) = \int_{x}^{1} \frac{d\omega}{\omega} P^{(1)}(\omega) \omega^{-0.5}, \qquad (4.24)$$

$$W(x) = \int_x^1 \frac{d\omega}{\omega} P^{(2)}(\omega) \omega^{-0.5}, \qquad (4.25)$$

and C_1, C_2, C_3 are the constants originated due to integration .

Now at a fixed value of $x = x_0$, the *t* dependence of the structure function $F_2^{NS}(x,t)$ in LO is given by

$$F_2^{NS}(x_0,t)\bigg|_{LO} = C_1 \exp\left[U(x_0) \int \left(\frac{\alpha(t)}{2\pi}\right)_{LO} dt\right].$$
(4.26)

Again the value of the structure function at $x = x_0$ and $t = t_0$ in accord with (4.26) is

$$F_2^{NS}(x_0, t_0) \bigg|_{LO} = C_1 \exp\left[U(x_0) \int \left(\frac{\alpha(t)}{2\pi} \right)_{LO} dt \right] \bigg|_{t=t_0}.$$
 (4.27)

Dividing (4.26) by (4.27) and rearranging a bit we obtain the t evolution of $F_2^{NS}(x,t)$ in accord with the LO DGLAP equation with respect to the point $F_2^{NS}(x_0,t_0)$ as

$$F_2^{NS}(x_0, t) \bigg|_{LO} = F_2^{NS}(x_0, t_0) \exp\left[U(x_0) \int_{t_0}^t \left(\frac{\alpha(t)}{2\pi}\right)_{LO} dt\right].$$
 (4.28)

Again in accord with our preassumption (4.12), the t dependence of $F_2^{NS}(x,t)$ at a particular value of $x = x_0$ is given by

$$F_2^{NS}(x_0, t) = A(t)x_0^{0.5}.$$
(4.29)

Dividing (4.12) by (4.29), we have the following relation

$$F_2^{NS}(x,t) = F_2^{NS}(x_0,t) \left(\frac{x}{x_0}\right)^{0.5},$$
(4.30)

which describes both t and x dependence of $F_2^{NS}(x,t)$ structure function in terms of the t dependent function $F_2^{NS}(x_0,t)$.

Now combining (4.28) and (4.30) we obtain the relation,

$$F_2^{NS}(x,t)\bigg|_{LO} = F_2^{NS}(x_0,t_0) \exp\left[U(x_0)\int_{t_0}^t \left(\frac{\alpha(t)}{2\pi}\right)_{LO}dt\right] \left(\frac{x}{x_0}\right)^{0.5},$$
 (4.31)

which describes both t and x dependence of $F_2^{NS}(x,t)$ structure function in LO in terms of the input point $F_2^{NS}(x_0,t_0)$.

Proceeding in the similar way we can obtain the expressions representing both xand t dependence of $F_2^{NS}(x,t)$ structure function in terms of an input point $F_2^{NS}(x_0,t_0)$ in NLO and NNLO as

$$F_2^{NS}(x,t)\bigg|_{NLO} = F_2^{NS}(x_0,t_0) \exp\left[U(x_0)\int_{t_0}^t \left(\frac{\alpha(t)}{2\pi}\right)_{NLO}^{-1}dt + V(x_0)\int_{t_0}^t \left(\frac{\alpha(t)}{2\pi}\right)_{NLO}^{-2}dt\right]\left(\frac{x}{x_0}\right)^{0.5}$$
(4.32)

and

$$F_2^{NS}(x,t)\Big|_{NNLO} = F_2^{NS}(x_0,t_0) \exp\left[U(x_0)\int_{t_0}^t \left(\frac{\alpha(t)}{2\pi}\right)_{NNLO} dt +V(x_0)\int_{t_0}^t \left(\frac{\alpha(t)}{2\pi}\right)_{NNLO}^2 dt +W(x_0)\int_{t_0}^t \left(\frac{\alpha(t)}{2\pi}\right)_{NNLO}^3 dt\right] \left(\frac{x}{x_0}\right)^{0.5}$$
(4.33)

respectively.

4.3 Solution of DGLAP Evolution Equations with the Initial Input $F_2^{NS}(x,t) = Bx^{(1-bt)}$

Now considering the ansatz, $F_2^{NS}(x,t) = Bx^{(1-bt)}$ as the initial input we obtain the DGLAP equations in LO, NLO and NNLO as

$$\frac{\partial F_2^{NS}(x,t)}{\partial t} = \left(\frac{\alpha(t)}{2\pi}\right)_{LO} \left[\frac{2}{3}\{3+4\ln(1-x)\} + \frac{4}{3}\int_x^1 \frac{d\omega}{1-\omega} \left\{\frac{1+\omega^2}{\omega}\omega^{(bt-1)} - 2\right\} \right] F_2^{NS}(x,t), \quad (4.34)$$

$$\frac{\partial F_2^{NS}(x,t)}{\partial t} = \left(\frac{\alpha(t)}{2\pi}\right)_{NLO} \left[\frac{2}{3}\{3+4ln(1-x)\} + \frac{4}{3}\int_x^1 \frac{d\omega}{1-\omega} \left\{\frac{1+\omega^2}{\omega}\omega^{(bt-1)} - 2\right\}\right] F_2^{NS}(x,t) + \left(\frac{\alpha(t)}{2\pi}\right)_{NLO}^2 \int_x^1 \frac{d\omega}{\omega} P^{(1)}(\omega)\omega^{(bt-1)} F_2^{NS}(x,t)$$
(4.35)

$$\frac{\partial F_2^{NS}(x,t)}{\partial t} = \left(\frac{\alpha(t)}{2\pi}\right)_{NNLO} \left[\frac{2}{3}\left\{3+4ln(1-x)\right\} + \frac{4}{3}\int_x^1 \frac{d\omega}{1-\omega} \left\{\frac{1+\omega^2}{\omega}\omega^{(bt-1)}\right\} - 2\left\{\frac{1+\omega^2}{\omega}\right\}\right] F_2^{NS}(x,t) + \left(\frac{\alpha(t)}{2\pi}\right)_{NNLO}^2 \int_x^1 \frac{d\omega}{\omega} P^{(1)}(\omega)\omega^{(bt-1)}F_2^{NS}(x,t) + \left(\frac{\alpha(t)}{2\pi}\right)_{NNLO}^3 \int_x^1 \frac{d\omega}{\omega} P^{(2)}(\omega)\omega^{(bt-1)}F_2^{NS}(x,t) + \left(\frac{\alpha(t)}{2\pi}\right)_{NNLO}^3 \int_x^1 \frac{d\omega}{\omega} P^{(2)}(\omega)\omega^{(bt-1$$

respectively, which can be easily solved to have

$$F_2^{NS}(x,t)\bigg|_{LO} = C_1 \exp\left[\int \left(\frac{\alpha(t)}{2\pi}\right)_{LO} U(x,t)dt\right],\tag{4.37}$$

$$F_2^{NS}(x,t)\bigg|_{NLO} = C_2 \exp\left[\int \left(\frac{\alpha(t)}{2\pi}\right)_{NLO} U(x,t)dt + \int \left(\frac{\alpha(t)}{2\pi}\right)_{NLO}^2 V(x,t)dt\right] (4.38)$$

and

$$F_2^{NS}(x,t) \bigg|_{NNLO} = C_3 \exp\left[\int \left(\frac{\alpha(t)}{2\pi}\right)_{NNLO} U(x,t)dt + \int \left(\frac{\alpha(t)}{2\pi}\right)_{NNLO}^2 V(x,t)dt + \int \left(\frac{\alpha(t)}{2\pi}\right)_{NNLO}^3 W(x,t)dt \right] 4.39$$

respectively. Here

$$U(x,t) = \frac{2}{3} \{3 + 4\ln(1-x)\} + \frac{4}{3} \int_{x}^{1} \frac{d\omega}{1-\omega} \left\{ \frac{1+\omega^{2}}{\omega} \omega^{(bt-1)} - 2 \right\},$$
(4.40)

$$V(x,t) = \int_{x}^{1} \frac{d\omega}{\omega} P^{(1)}(\omega) \omega^{(bt-1)}, \qquad (4.41)$$

$$W(x,t) = \int_{x}^{1} \frac{d\omega}{\omega} P^{(2)}(\omega) \omega^{(bt-1)}$$
(4.42)

and C_1 , C_2 and C_3 are the constants originated due to integration.

At a fixed value of $x = x_0$, the t dependence of the structure function in LO is given by

$$F_2^{NS}(x_0,t) = C_1 \exp\left[\int \left(\frac{\alpha(t)}{2\pi}\right)_{LO} U(x_0,t)dt\right].$$
(4.43)

Again the value of the structure function at $x = x_0$ and $t = t_0$ in accord with (4.43) is given by

$$F_2^{NS}(x_0, t_0) = C_1 \exp\left[\int \frac{\alpha(t)}{2\pi} U(x_0, t) dt\right]\Big|_{t=t_0}.$$
(4.44)

Dividing (4.43) by (4.44) and rearranging a bit we obtain the t dependence of $F_2^{NS}(x,t)$ in accord with LO DGLAP evolution equation with respect to the point $F_2^{NS}(x_0,t_0)$ as

$$F_2^{NS}(x_0, t) = F_2^{NS}(x_0, t_0) \exp\left[\int_{t_0}^t \left(\frac{\alpha(t)}{2\pi}\right)_{LO} U(x_0, t) dt\right].$$
 (4.45)

Again, as both t and x dependence of $F_2^{NS}(x,t)$ is assumed to satisfy

$$F_2^{NS}(x,t) = B.x^{(1-bt)}$$
(4.46)

relation, and at any fixed $x = x_0$ we have

$$F_2^{NS}(x_0, t) = B.x_0^{(1-bt)}, (4.47)$$

which represents the t dependence of the structure function at any fixed value of $x = x_0$. Dividing (4.46) by (4.47) we have the following relation

$$F_2^{NS}(x,t) = F_2^{NS}(x_0,t) \left(\frac{x}{x_0}\right)^{(1-bt)},$$
(4.48)

which gives both t and x dependence of $F_2^{NS}(x,t)$ structure function in terms of the t dependent function $F_2^{NS}(x_0,t)$ at fixed $x = x_0$.

Now combining (4.45) and (4.48) we obtain the expression representing both x and t dependence of $F_2^{NS}(x,t)$ structure function in terms of an input point $F_2^{NS}(x_0,t_0)$ in LO as

$$F_2^{NS}(x,t)\bigg|_{LO} = F_2^{NS}(x_0,t_0) \exp\left[\int_{t_0}^t \left(\frac{\alpha(t)}{2\pi}\right)_{LO} U(x_0,t)dt\right] \left(\frac{x}{x_0}\right)^{(1-bt)}.$$
 (4.49)

Similarly we may have the joint x and t dependence of $F_2^{NS}(x,t)$ structure function in NLO and NNLO as

$$F_{2}^{NS}(x,t)\Big|_{NLO} = F_{2}^{NS}(x_{0},t_{0}) \exp\left[\int_{t_{0}}^{t} \left(\frac{\alpha(t)}{2\pi}\right)_{NLO} U(x_{0},t)dt + \int_{t_{0}}^{t} \left(\frac{\alpha(t)}{2\pi}\right)_{NLO}^{2} V(x_{0},t)dt\right] \left(\frac{x}{x_{0}}\right)^{(1-bt)}$$
(4.50)

and

$$F_2^{NS}(x,t)\Big|_{NNLO} = F_2^{NS}(x_0,t_0) \exp\left[\int_{t_0}^t \left(\frac{\alpha(t)}{2\pi}\right)_{NNLO} U(x_0,t)dt + \int_{t_0}^t \left(\frac{\alpha(t)}{2\pi}\right)^2_{NNLO} V(x_0,t)dt + \int_{t_0}^t \left(\frac{\alpha(t)}{2\pi}\right)^3_{NNLO} W(x_0,t)dt\right] \left(\frac{x}{x_0}\right)^{(1-bt)}.$$
(4.51)

4.4 **Results and Discussion**

The equations (4.31)-(4.33) and (4.49)-(4.51) are the analytic expressions representing both x and Q^2 dependence of $F_2^{NS}(x,Q^2)$ structure function jointly, obtained by means of solving the DGLAP equations in LO, NLO and NNLO incorporating the Regge ansatz, $F_2^{NS}(x,Q^2) = A(Q^2)x^{0.5}$ and $F_2^{NS}(x,Q^2) = Bx^{1-bt}$ as the initial inputs respectively. These expressions are consisting of an input point $F_2^{NS}(x_0,t_0)$, which can be taken from the available experimental data. If the input point is more accurate and precise, we can expect batter results. There are not any specific reason in choosing the input point. Any one of the data points at a certain value of $x = x_0$ and $t = t_0$ can be considered as the input point. Off course, the sensitivity of different inputs will be



Figure 4.1: Q^2 evolution of $F_2^{NS}(x, Q^2)$ structure functions in accord with (4.31)-(4.33) in comparison with NMC[63] and NNPDF[101] results. For clarity, the points are offset by the amount given in parenthesis. (Q^2 's are taken in the unit of GeV^2).

different. However instead of choosing the input point on the basis of their sensitivity, in our manuscript we have incorporated a suitable condition in determining the input point. We have considered that particular point from the most recent measurements as the input point in which experimental errors are minimum. Under this condition we have selected the point $F_2^{NS}(x_0, t_0) = 0.010348 \pm 0.006208$ at $x_0 = 0.025$ and $Q^2 =$ $2.34686 GeV^2$ from the experimental results of NMC[63]. Here we have considered the central value of the input point. Further the expressions (4.49)-(4.51) consists of the additional parameter *b* which has the value $b = 0.118 \pm 0.028$ for $F_2^{NS}(x, Q^2)$ as obtained in Chapter 3.

With the input point $F_2^{NS}(x_0, t_0) = 0.010348$, substituting the respective expressions in LO, NLO and NNLO for running coupling constant, $\frac{\alpha_s(t)}{2\pi}$ and performing the corresponding integrations, we have obtained both x as well as Q^2 evolution of $F_2^{NS}(x, Q^2)$ structure function in accord with the equations (4.31), (4.32) and (4.33) respectively. The Q^2 evolution results at fixed value of x are depicted in Fig. 4.1 in comparison with the experimental data taken from NMC[63] and with the results of NNPDF collaboration[101]. In Fig. 4.2, the x evolution of $F_2^{NS}(x, Q^2)$ for fixed values of Q^2 are depicted along with NMC and NNPDF results. In all figures, as indicated,



Figure 4.2: x evolution of $F_2^{NS}(x, Q^2)$ structure functions in accord with (4.31)-(4.33) in comparison with NMC[63] results. For clarity, the points are offset by the amount given in parenthesis.



Figure 4.3: Q^2 (in the unit of GeV^2) evolution of $F_2^{NS}(x, Q^2)$ structure functions in accord with (4.49)-(4.51) in comparison with NMC[63] results. For clarity, the points are offset by the amount given in parenthesis. (Q^2 's are taken in the unit of GeV^2).



Figure 4.4: x evolution of $F_2^{NS}(x, Q^2)$ structure functions in accord with (4.49)-(4.51) in comparison with NMC[63] and NNPDF[101] results. For clarity, the points are offset by the amount given in parenthesis.



Figure 4.5: Q^2 evolution of $F_2^{NS}(x, Q^2)$ structure functions in accord with (4.33) and (4.51) in comparison with NMC[63] and NNPDF[101] results. For clarity, the points are offset by the amount given in parenthesis. (Q^2 's are taken in the unit of GeV^2).



Figure 4.6: x evolution of $F_2^{NS}(x, Q^2)$ structure functions in accord with (4.33) and (4.51) in comparison with NMC[63] results. For clarity, the points are offset by the amount given in parenthesis.



Figure 4.7: Q^2 evolution of $F_2^{NS}(x, Q^2)$ structure functions in accord with NNLO corrections, (4.33) and (4.51) in comparison with NMC[63] results. For clarity, the points are offset by the amount given in parenthesis. (Q^2 's are taken in the unit of GeV^2).

the dotted curves represent the LO results, the dashed curves represent NLO results and the solid lines are representing NNLO results. Experimental data are given with vertical upper and lower error bars for total uncertainties of statistical and systematic errors.

Again the results from equations (4.49), (4.50) and (4.51) for Q^2 and x evolution of $F_2^{NS}(x,Q^2)$ structure function with $F_2^{NS}(x_0,t_0) = 0.010348$ and b = 0.118 are depicted in Fig. 4.3 and Fig. 4.4 respectively. The experimental results from NMC and the results of NNPDF collaboration are also plotted along with our results. Here, our LO, NLO and NNLO results are represented by the dotted, dashed and solid curves respectively. The solid circles are used to represent the NMC data point and they are along with vertical upper and lower error bars for total uncertainties of statistical and systematic errors.

As far the figures, 4.1-4.4 are concerned, we observe a very good consistency between theoretical and experimental as well as parametrization results within the kinematical region x < 0.05 and $Q^2 \le 20 GeV^2$ of our consideration, especially, if the NNLO results are concerned. The most consistent results, the NNLO results for both the inputs along with NMC and NNPDF results are plotted in Figs. 4.5 and 4.6. It reflects the comparative picture of the results obtained by means of the two ansatz. However within our kinematical region of consideration we do not observe any significant differences among them. This implies that the analytic expressions, we have obtained by means of solving the DGLAP equations with both the ansatz as the initial input, are applicable in describing the small x behaviour of $F_2^{NS}(x, Q^2)$ structure function with a considerable precision.

In addition, we have shown in the Fig. 4.7, the band due to the uncertainty associated with input and the fitting parameter b. Here the uncertainty due to the fitting parameter is considerably less than that of due to input point.

4.5 Summary

We have employed the usefulness of two ansatz as the initial input in order to solve DGLAP equation up to NNLO and obtain Q^2 evolution of the unpolarized non-singlet structure function $F_2^{NS}(x, Q^2)$. The structure function, evolved as the solutions of the DGLAP equations are studied phenomenologically in comparison with the results taken from NMC and NNPDF collaborations. We observe a very good agreement between our theoretical results and other experimental results as well as parametrization, within the kinematical range x < 0.05 and $Q^2 = 20 GeV^2$ of our consideration. The phenomenological success achieved in this study suggests that the two simple QCD featured Regge behaved ansatz $F_2^{NS}(x,Q^2) = A(Q^2)x^{0.5}$ and $F_2^{NS}(x,Q^2) = Bx^{1-bt}$ are capable of evolving $F_2^{NS}(x,Q^2)$ structure functions with Q^2 in accord with DGLAP equations at small-x. However we could not distinguish the efficiencies among the two models in comparison with experimental data within the kinematical range of our consideration. We hope future experimental measurements at very very small values of Bjorken x will clarify their differences and help us in batter understanding of the structure of nucleon. \Box

Chapter 5

Small-x Behaviour of $xF_3(x, Q^2)$ Structure Function

This chapter is devoted to the determination of Q^2 and x evolutions of $xF_3(x, Q^2)$ structure function in accord with the leading order(LO), next-to-leading order(NLO) and next-next-to-leading order(NNLO) DGLAP evolution equations within the small-xregion. The DGLAP equation is solved up to NNLO for $xF_3(x, Q^2)$ structure function using two Regge ansatz as initial input and solutions for both the inputs are compared with the experimental data from CCFR, NuTeV, CDHSW and CHORUS experiments as well as with the recent MSTW parametrization results. A great phenomenological success is achieved in this regards, which signifies the capability of the expressions in describing the small-x behaviour of this non-singlet structure function and their usefulness in determining the structure functions with a reasonable precision.

5.1 Introduction

One of the significant contributions that neutrino-nucleon interaction has towards the understanding of hadron structure is its ability to produce the parity violating term, $xF_3(x, Q^2)$ which receives contributions from the non-singlet part of the co-efficient function and reflects only the valence quark distribution[61]. It is not marred by the presence of the sea quark and gluon densities about which we have very poor information in particular in the small-x region. Therefore, the neutrino-nucleon scattering as well as $xF_3(x, Q^2)$ structure function are becoming more important theoretically as well as experimentally for the study of different nuclear effects such as shadowing, anti-shadowing, EMC in parton distribution in nuclei etc. Also the study of neutrino interaction provides the understanding of neutrino propagation in matter, whose importance is seen in astrophysics, cosmology and even geology application.

Further, the Gross-Llewellyn Smith(GLS) sum rule[33, 38, 39] associated with the non-singlet $xF_3(x, Q^2)$ structure function measured in neutrino-nucleon $(\nu - N)$ scattering is one of the best observables to investigate Quantum Chromodynamics(QCD) as a theory of strong interaction. As $xF_3(x,Q^2)$ structure function is not marred by the presence of the sea quark and gluon densities about which we have very poor information in particular in the small-x region and higher order QCD calculations are observed to be largely independent of renormalization scheme, the prediction of GLS sum rule is considered as the robust prediction in pQCD. The determination of the GLS sum rule requires knowledge of $xF_3(x,Q^2)$ structure functions over the entire region of $x \in (0, 1)$. The experimentally accessible x range for the neutrino DIS is however limited for the available data and therefore one should extrapolate results to x = 0 and x = 1. The extrapolation to $x \to 0$, where F_3 structure functions grow strongly, is much more important than the extrapolation to $x \to 1$, where structure functions vanish. Again, it is known that maximum contribution (about 90%) to the GLS sum rule come from the small $x (\leq 0.1)$ region. Because of the large contribution to the GLS sum rule from small x, the small x region is particularly important. Therefore this chapter is an attempt to have the small-x behaviour of $xF_3(x,Q^2)$ structure function by means of solving the DGLAP equation using the two Regge ansatz discussed in chapter 3 as the initial input.

The Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) evolution equation [24, 137] which describe the Q^2 behavior of unpolarised non-singlet structure function $xF_3(x,Q^2)$ in perturbative Quantum Chromodynamics (QCD) formalism is given by

$$\frac{\partial x F_3(x, Q^2)}{\partial \ln Q^2} = \int_x^1 \frac{d\omega}{\omega} \frac{x}{\omega} F_3(\frac{x}{\omega}, Q^2) P(\omega), \qquad (5.1)$$

where, $P(\omega)$ is the splitting function associated with $xF_3(x, Q^2)$ structure function, which is defined up to NNLO by[31]

$$P(\omega) = \frac{\alpha(Q^2)}{2\pi} P^{(0)}(\omega) + \left(\frac{\alpha(Q^2)}{2\pi}\right)^2 P^{(1)}(\omega) + \left(\frac{\alpha(Q^2)}{2\pi}\right)^3 P^{(2)}(\omega).$$
(5.2)

Here, $P^{(0)}(\omega)$, $P^{(1)}(\omega)$ and $P^{(2)}(\omega)$ are the corresponding leading order(LO), next-toleading order (NLO) and next-next-to-leading order(NNLO) corrections to the splitting functions. These splitting functions are given in Appendices.

Again, in LO, NLO and NNLO, the running coupling constant $\frac{\alpha(Q^2)}{2\pi}$ has the forms[23],

$$\left(\frac{\alpha(t)}{2\pi}\right)_{LO} = \frac{2}{\beta_0 t},\tag{5.3}$$

$$\left(\frac{\alpha(t)}{2\pi}\right)_{NLO} = \frac{2}{\beta_0 t} \left[1 - \frac{\beta_1 \ln t}{\beta_0^2 t}\right]$$
(5.4)

and

$$\left(\frac{\alpha(t)}{2\pi}\right)_{NNLO} = \frac{2}{\beta_0 t} \left[1 - \frac{\beta_1 \ln t}{\beta_0^2 t} + \frac{1}{\beta_0^2 t^2} \left[\left(\frac{\beta_1}{\beta_0}\right)^2 (\ln^2 t - \ln t + 1) + \frac{\beta_2}{\beta_0}\right]\right], \quad (5.5)$$

where $\beta_0 = 11 - \frac{2}{3}N_F$, $\beta_1 = 102 - \frac{38}{3}N_F$ and $\beta_2 = \frac{2857}{6} - \frac{6673}{18}N_F + \frac{325}{54}N_F^2$ are the oneloop, two-loop and three-loop corrections to the QCD β -function. Here the running coupling constant is expressed in terms of the variable t, which is defined by $t = \ln(\frac{Q^2}{\Lambda^2})$.

For simplicity, defining $xF_3(x,Q^2) = F_3^{NS}(x,Q^2)$ and then substituting the respective splitting functions along with the corresponding running coupling constant in (5.1), the DGLAP evolution equations in LO, NLO and NNLO become

$$\frac{\partial F_3^{NS}(x,t)}{\partial t} = \left(\frac{\alpha(t)}{2\pi}\right)_{LO} \left[\frac{2}{3}\{3+4\ln(1-x)\}F_3^{NS}(x,t) + I_1(x,t)\right], \quad (5.6)$$

$$\frac{\partial F_3^{NS}(x,t)}{\partial t} = \left(\frac{\alpha(t)}{2\pi}\right)_{NLO} \left[\frac{2}{3}\{3+4\ln(1-x)\}F_3^{NS}(x,t) + I_1(x,t)\right] + \left(\frac{\alpha(t)}{2\pi}\right)_{NLO}^2 I_2(x,t), \quad (5.7)$$

and

$$\frac{\partial F_3^{NS}(x,t)}{\partial t} = \left(\frac{\alpha(t)}{2\pi}\right)_{NNLO} \left[\frac{2}{3}\left\{3 + 4ln(1-x)\right\}F_3^{NS}(x,t) + I_1(x,t)\right] + \left(\frac{\alpha(t)}{2\pi}\right)_{NNLO}^2 I_2(x,t) + \left(\frac{\alpha(t)}{2\pi}\right)_{NNLO}^3 I_3(x,t)$$
(5.8)

respectively. Here Λ is the QCD cut-off parameter and the integral functions are given by

$$I_{1}(x,t) = \int_{x}^{1} \frac{d\omega}{1-\omega} \left\{ \frac{1+\omega^{2}}{\omega} F_{3}^{NS}\left(\frac{x}{\omega},t\right) - 2F_{3}^{NS}(x,t) \right\},$$
(5.9)

$$I_2(x,t) = \int_x^1 \frac{d\omega}{\omega} P^{(1)}(\omega) F_3^{NS}\left(\frac{x}{\omega},t\right)$$
(5.10)

and

$$I_3(x,t) = \int_x^1 \frac{d\omega}{\omega} P^{(2)}(\omega) F_3^{NS}\left(\frac{x}{\omega}, t\right).$$
(5.11)

The DGLAP equations up to NNLO ((5.6)-(5.8)) can be solved analytically using the ansatz $xF_3(x,t) = A(t)x^{0.5}$ and $xF_3(x,t) = Bx^{(1-at)}$ as the initial inputs and I have discussed below in detailed.

5.2 Solution of DGLAP Evolution Equations with the Initial Input $xF_3(x,t) = A(t)x^{0.5}$

On substitution of

$$xF_3(x,t) = F_3^{NS}(x,t) = A(t)x^{0.5}$$
(5.12)

and hence

$$xF_3(\frac{x}{\omega},t) = F_3^{NS}(\frac{x}{\omega},t) = A(t)x^{0.5}\omega^{-0.5} = F_3^{NS}(x,t)\omega^{-0.5}$$
(5.13)

in the equations (5.6), (5.7) and (5.8) we obtain

$$\frac{\partial F_3^{NS}(x,t)}{\partial t} = \left(\frac{\alpha(t)}{2\pi}\right)_{LO} \left[\frac{2}{3}\{3+4\ln(1-x)\} + \frac{4}{3}\int_x^1 \frac{d\omega}{1-\omega} \left\{\frac{1+\omega^2}{\omega}\omega^{-0.5} - 2\right\}\right] F_3^{NS}(x,t),$$
(5.14)

$$\frac{\partial F_3^{NS}(x,t)}{\partial t} = \left(\frac{\alpha(t)}{2\pi}\right)_{NLO} \left[\frac{2}{3}\{3+4\ln(1-x)\} + \frac{4}{3}\int_x^1 \frac{d\omega}{1-\omega} \left\{\frac{1+\omega^2}{\omega}\omega^{-0.5} -2\right\}\right] F_3^{NS}(x,t) + \left(\frac{\alpha(t)}{2\pi}\right)_{NLO}^2 \int_x^1 \frac{d\omega}{\omega} P^{(1)}(\omega)\omega^{-0.5} F_3^{NS}(x,t)$$
(5.15)

$$\begin{aligned} \frac{\partial F_3^{NS}(x,t)}{\partial t} &= \left(\frac{\alpha(t)}{2\pi}\right)_{NNLO} \left[\frac{2}{3}\{3+4ln(1-x)\} + \frac{4}{3}\int_x^1 \frac{d\omega}{1-\omega} \left\{\frac{1+\omega^2}{\omega}\omega^{-0.5}\right\} \\ &-2\left\{\right\} F_3^{NS}(x,t) + \left(\frac{\alpha(t)}{2\pi}\right)^2_{NNLO}\int_x^1 \frac{d\omega}{\omega} P^{(1)}(\omega)\omega^{-0.5}F_3^{NS}(x,t) \\ &+ \left(\frac{\alpha(t)}{2\pi}\right)^3_{NNLO}\int_x^1 \frac{d\omega}{\omega} P^{(2)}(\omega)\omega^{-0.5}F_3^{NS}(x,t) (5.16) \end{aligned}$$

respectively. These equations can be rearranged to have three ordinary differential equations in terms of $F_3^{NS}(x,t)$,

$$\frac{\partial F_3^{NS}(x,t)}{\partial t} = \frac{\alpha(t)}{2\pi} U(x) F_3^{NS}(x,t), \qquad (5.17)$$

$$\frac{\partial F_3^{NS}(x,t)}{\partial t} = \left[\left(\frac{\alpha(t)}{2\pi} \right)_{NLO} U(x) + \left(\frac{\alpha(t)}{2\pi} \right)_{NLO}^2 V(x) \right] F_3^{NS}(x,t), \tag{5.18}$$

and

$$\frac{\partial F_3^{NS}(x,t)}{\partial t} = \left[\left(\frac{\alpha(t)}{2\pi} \right)_{NNLO} U(x) + \left(\frac{\alpha(t)}{2\pi} \right)_{NNLO}^2 V(x) + \left(\frac{\alpha(t)}{2\pi} \right)_{NNLO}^3 W(x) \right] F_3^{NS}(x,t),$$
(5.19)

which can be easily solved to have

$$F_3^{NS}(x,t)\bigg|_{LO} = C_1 \exp\left[U(x) \int \left(\frac{\alpha(t)}{2\pi}\right)_{LO} dt\right],$$
(5.20)

$$F_3^{NS}(x,t)\bigg|_{NLO} = C_2 \exp\left[U(x) \int \left(\frac{\alpha(t)}{2\pi}\right)_{NLO} dt + V(x) \int \left(\frac{\alpha(t)}{2\pi}\right)_{NLO}^2 dt\right] (5.21)$$

and

$$F_3^{NS}(x,t)\bigg|_{NNLO} = C_3 \exp\left[U(x) \int \left(\frac{\alpha(t)}{2\pi}\right)_{NNLO} dt + V(x) \int \left(\frac{\alpha(t)}{2\pi}\right)_{NNLO}^2 dt + W(x) \int \left(\frac{\alpha(t)}{2\pi}\right)_{NNLO}^3 dt\right] (5.22)$$

respectively. Here,

$$U(x) = \frac{2}{3} \{3 + 4\ln(1-x)\} + \frac{4}{3} \int_{x}^{1} \frac{d\omega}{1-\omega} \left\{ \frac{1+\omega^{2}}{\omega} \omega^{-0.5} - 2 \right\},$$
 (5.23)

$$V(x) = \int_{x}^{1} \frac{d\omega}{\omega} P^{(1)}(\omega) \omega^{-0.5}, \qquad (5.24)$$

$$W(x) = \int_x^1 \frac{d\omega}{\omega} P^{(2)}(\omega) \omega^{-0.5},$$
(5.25)

and C_1, C_2, C_3 are the constants originated due to integration.

Now at a fixed value of $x = x_0$, the *t* dependence of the structure function $F_3^{NS}(x,t)$ in LO is given by

$$F_3^{NS}(x_0,t)\bigg|_{LO} = C_1 \exp\left[U(x_0) \int \left(\frac{\alpha(t)}{2\pi}\right)_{LO} dt\right].$$
(5.26)

Again the value of the structure function at $x = x_0$ and $t = t_0$ in accord with (5.26) is

$$F_3^{NS}(x_0, t_0) \bigg|_{LO} = C_1 \exp\left[U(x_0) \int \left(\frac{\alpha(t)}{2\pi} \right)_{LO} dt \right] \bigg|_{t=t_0}.$$
 (5.27)

Dividing (5.26) by (5.27) and rearranging a bit we obtain the t evolution of $F_3^{NS}(x,t)$ in accord with the LO DGLAP equation with respect to the point $F_3^{NS}(x_0,t_0)$ as

$$F_3^{NS}(x_0, t) \bigg|_{LO} = F_3^{NS}(x_0, t_0) \exp\left[U(x_0) \int_{t_0}^t \left(\frac{\alpha(t)}{2\pi}\right)_{LO} dt\right].$$
 (5.28)

Again in accord with our preassumption (5.12), the t dependence of $F_3^{NS}(x,t)$ at a particular value of $x = x_0$ is given by

$$F_3^{NS}(x_0, t) = A(t)x_0^{0.5}.$$
(5.29)

Dividing (5.12) by (5.29), we have the following relation

$$F_3^{NS}(x,t) = F_3^{NS}(x_0,t) \left(\frac{x}{x_0}\right)^{0.5},$$
(5.30)

which describes both t and x dependence of $F_3^{NS}(x,t)$ structure function in terms of the t dependent function $F_3^{NS}(x_0,t)$.

Now combining (5.28) and (5.30) we obtain the relation,

$$F_3^{NS}(x,t)\bigg|_{LO} = F_3^{NS}(x_0,t_0) \exp\left[U(x_0)\int_{t_0}^t \left(\frac{\alpha(t)}{2\pi}\right)_{LO}dt\right] \left(\frac{x}{x_0}\right)^{0.5},$$
 (5.31)

which describes both t and x dependence of $F_3^{NS}(x,t)$ structure function in LO in terms of the input point $F_3^{NS}(x_0,t_0)$.

Proceeding in the similar way we can obtain the expressions representing both xand t dependence of $F_3^{NS}(x,t)$ structure function in terms of an input point $F_3^{NS}(x_0,t_0)$ in NLO and NNLO as

$$F_{3}^{NS}(x,t)\Big|_{NLO} = F_{3}^{NS}(x_{0},t_{0}) \exp\left[U(x_{0})\int_{t_{0}}^{t}\left(\frac{\alpha(t)}{2\pi}\right)_{NLO}^{}dt +V(x_{0})\int_{t_{0}}^{t}\left(\frac{\alpha(t)}{2\pi}\right)_{NLO}^{2}dt\right]\left(\frac{x}{x_{0}}\right)^{0.5}$$
(5.32)

and

$$F_{3}^{NS}(x,t)\Big|_{NNLO} = F_{3}^{NS}(x_{0},t_{0}) \exp\left[U(x_{0})\int_{t_{0}}^{t}\left(\frac{\alpha(t)}{2\pi}\right)_{NNLO}^{NNLO}dt + V(x_{0})\int_{t_{0}}^{t}\left(\frac{\alpha(t)}{2\pi}\right)_{NNLO}^{2}dt + W(x_{0})\int_{t_{0}}^{t}\left(\frac{\alpha(t)}{2\pi}\right)_{NNLO}^{3}dt\right]\left(\frac{x}{x_{0}}\right)^{0.5}$$
(5.33)

respectively.

5.3 Solution of DGLAP Evolution Equations with the Initial Input $xF_3(x,t) = Bx^{(1-bt)}$

Now considering the ansatz, $xF_3(x,t) = Bx^{(1-bt)}$ as the initial input we obtain the DGLAP equations in LO, NLO and NNLO as

$$\frac{\partial F_3^{NS}(x,t)}{\partial t} = \left(\frac{\alpha(t)}{2\pi}\right)_{LO} \left[\frac{2}{3}\{3+4\ln(1-x)\} + \frac{4}{3}\int_x^1 \frac{d\omega}{1-\omega} \left\{\frac{1+\omega^2}{\omega}\omega^{(bt-1)} - 2\right\} F_3^{NS}(x,t), \quad (5.34)$$

$$\frac{\partial F_3^{NS}(x,t)}{\partial t} = \left(\frac{\alpha(t)}{2\pi}\right)_{NLO} \left[\frac{2}{3}\{3+4ln(1-x)\} + \frac{4}{3}\int_x^1 \frac{d\omega}{1-\omega} \left\{\frac{1+\omega^2}{\omega}\omega^{(bt-1)} - 2\right\} \right] F_3^{NS}(x,t) + \left(\frac{\alpha(t)}{2\pi}\right)_{NLO}^2 \int_x^1 \frac{d\omega}{\omega} P^{(1)}(\omega)\omega^{(bt-1)} F_3^{NS}(x,t), (5.35)$$

$$\begin{aligned} \frac{\partial F_3^{NS}(x,t)}{\partial t} &= \left(\frac{\alpha(t)}{2\pi}\right)_{NNLO} \left[\frac{2}{3}\{3+4ln(1-x)\} + \frac{4}{3}\int_x^1 \frac{d\omega}{1-\omega} \left\{\frac{1+\omega^2}{\omega}\omega^{(bt-1)}\right\} \\ &-2\left\{\right\} F_3^{NS}(x,t) + \left(\frac{\alpha(t)}{2\pi}\right)_{NNLO}^2 \int_x^1 \frac{d\omega}{\omega} P^{(1)}(\omega)\omega^{(bt-1)}F_3^{NS}(x,t) \\ &+ \left(\frac{\alpha(t)}{2\pi}\right)_{NNLO}^3 \int_x^1 \frac{d\omega}{\omega} P^{(2)}(\omega)\omega^{(bt-1)}F_3^{NS}(x,t) (5.36) \end{aligned}$$

respectively, which can be easily solved to have

$$F_3^{NS}(x,t)\bigg|_{LO} = C_1 \exp\left[\int \left(\frac{\alpha(t)}{2\pi}\right)_{LO} U(x,t)dt\right],\tag{5.37}$$

$$F_3^{NS}(x,t)\bigg|_{NLO} = C_2 \exp\left[\int \left(\frac{\alpha(t)}{2\pi}\right)_{NLO} U(x,t)dt + \int \left(\frac{\alpha(t)}{2\pi}\right)_{NLO}^2 V(x,t)dt\right] (5.38)$$

and

$$F_3^{NS}(x,t) \bigg|_{NNLO} = C_3 \exp\left[\int \left(\frac{\alpha(t)}{2\pi}\right)_{NNLO} U(x,t)dt + \int \left(\frac{\alpha(t)}{2\pi}\right)_{NNLO}^2 V(x,t)dt + \int \left(\frac{\alpha(t)}{2\pi}\right)_{NNLO}^3 W(x,t)dt \right] 5.39$$

respectively. Here

$$U(x,t) = \frac{2}{3} \{3 + 4\ln(1-x)\} + \frac{4}{3} \int_{x}^{1} \frac{d\omega}{1-\omega} \left\{ \frac{1+\omega^{2}}{\omega} \omega^{(bt-1)} - 2 \right\},$$
 (5.40)

$$V(x,t) = \int_{x}^{1} \frac{d\omega}{\omega} P^{(1)}(\omega) \omega^{(bt-1)},$$
(5.41)

$$W(x,t) = \int_{x}^{1} \frac{d\omega}{\omega} P^{(2)}(\omega) \omega^{(bt-1)}$$
(5.42)

and C_1 , C_2 and C_3 are the constants originated due to integration.

At a fixed value of $x = x_0$, the t dependence of the structure function in LO is given by

$$F_3^{NS}(x_0,t) = C_1 \exp\left[\int \left(\frac{\alpha(t)}{2\pi}\right)_{LO} U(x_0,t)dt\right].$$
(5.43)

Again the value of the structure function at $x = x_0$ and $t = t_0$ in accord with (5.43) is given by

$$F_3^{NS}(x_0, t_0) = C_1 \exp\left[\int \frac{\alpha(t)}{2\pi} U(x_0, t) dt\right]\Big|_{t=t_0}.$$
(5.44)

Dividing (5.43) by (5.44) and rearranging a bit we obtain the t dependence of $F_3^{NS}(x,t)$ in accord with LO DGLAP evolution equation with respect to the point $F_3^{NS}(x_0,t_0)$ as

$$F_3^{NS}(x_0, t) = F_3^{NS}(x_0, t_0) \exp\left[\int_{t_0}^t \left(\frac{\alpha(t)}{2\pi}\right)_{LO} U(x_0, t) dt\right].$$
 (5.45)

Again, as both t and x dependence of $F_3^{NS}(x,t)$ is assumed to satisfy

$$F_3^{NS}(x,t) = B.x^{(1-bt)}$$
(5.46)

relation, and at any fixed $x = x_0$ we have

$$F_3^{NS}(x_0, t) = B.x_0^{(1-bt)}, (5.47)$$

which represents the t dependence of the structure function at any fixed value of $x = x_0$. Dividing (5.46) by (5.47) we have the following relation

$$F_3^{NS}(x,t) = F_3^{NS}(x_0,t) \left(\frac{x}{x_0}\right)^{(1-bt)},$$
(5.48)

which gives both t and x dependence of $F_3^{NS}(x,t)$ structure function in terms of the t dependent function $F_3^{NS}(x_0,t)$ at fixed $x = x_0$.

Now combining (5.45) and (5.48) we obtain the expression representing both x and t dependence of $F_3^{NS}(x,t)$ structure function in terms of an input point $F_3^{NS}(x_0,t_0)$ in LO as

$$F_3^{NS}(x,t)\bigg|_{LO} = F_3^{NS}(x_0,t_0) \exp\left[\int_{t_0}^t \left(\frac{\alpha(t)}{2\pi}\right)_{LO} U(x_0,t)dt\right] \left(\frac{x}{x_0}\right)^{(1-bt)}.$$
 (5.49)

Similarly we may have the joint x and t dependence of $F_3^{NS}(x,t)$ structure function in NLO and NNLO as

$$F_{3}^{NS}(x,t)\Big|_{NLO} = F_{3}^{NS}(x_{0},t_{0}) \exp\left[\int_{t_{0}}^{t} \left(\frac{\alpha(t)}{2\pi}\right)_{NLO} U(x_{0},t)dt + \int_{t_{0}}^{t} \left(\frac{\alpha(t)}{2\pi}\right)_{NLO}^{2} V(x_{0},t)dt\right] \left(\frac{x}{x_{0}}\right)^{(1-bt)}$$
(5.50)

and

$$F_{3}^{NS}(x,t)\bigg|_{NNLO} = F_{3}^{NS}(x_{0},t_{0}) \exp\left[\int_{t_{0}}^{t} \left(\frac{\alpha(t)}{2\pi}\right)_{NNLO} U(x_{0},t)dt + \int_{t_{0}}^{t} \left(\frac{\alpha(t)}{2\pi}\right)^{2}_{NNLO} V(x_{0},t)dt + \int_{t_{0}}^{t} \left(\frac{\alpha(t)}{2\pi}\right)^{3}_{NNLO} W(x_{0},t)dt\right] \left(\frac{x}{x_{0}}\right)^{(1-bt)}.$$
(5.51)

5.4 Results and Discussion

The equations (5.31)-(5.33) and (5.49)-(5.51) are the analytic expressions representing both x and Q^2 dependence of $xF_3(x, Q^2)$ structure function jointly, obtained by means of solving the DGLAP equations in LO, NLO and NNLO incorporating the Regge ansatz, $xF_3(x, Q^2) = A(Q^2)x^{0.5}$ and $xF_3(x, Q^2) = Bx^{1-bt}$ as the initial inputs respectively. These expressions are consisting of an input point $xF_3(x_0, t_0)$, which can be taken from the available experimental data. If the input point is more accurate and precise, we can expect batter results. There are not any specific reason in choosing the input point. Any one of the data points at a certain value of $x = x_0$ and $t = t_0$ can be considered as the input point. Off course, the sensitivity of different inputs


Figure 5.1: Q^2 evolution of $xF_3(x, Q^2)$ structure functions in accord with (5.31)-(5.33). For clarity, the points are offset by the amount given in parenthesis. (Q^2 's are taken in the unit of GeV^2).



Figure 5.2: Q^2 evolution of $xF_3(x, Q^2)$ structure functions in accord with (5.31)-(5.33). For clarity, the points are offset by the amount given in parenthesis. (Q^2 's are taken in the unit of GeV^2).



Figure 5.3: x evolution of $xF_3(x, Q^2)$ structure functions in accord with (5.31)-(5.33) in comparison with CCFR[66] data. For clarity, the points are offset by the amount given in parenthesis.



Figure 5.4: x evolution of $xF_3(x, Q^2)$ structure functions in accord with (5.31)-(5.33) in comparison with NuTeV[68] results. For clarity, the points are offset by the amount given in parenthesis.



Figure 5.5: Q^2 evolution of $xF_3(x, Q^2)$ structure functions in accord with (5.49)-(5.51) in comparison with CCFR[66] and MSTW[107] results. For clarity, the points are offset by the amount given in parenthesis. (Q^2 's are taken in the unit of GeV^2)

will be different. However instead of choosing the input point on the basis of their sensitivity, in our manuscript we have incorporated a suitable condition in determining the input point. We have considered that particular point from the most recent measurements as the input point in which experimental errors are minimum. Under this condition we have selected the point $xF_3(x_0, t_0) = 0.3298$ at $x_0 = 0.025$ and $Q^2 = 3.2 GeV^2$ from the experimental results of CCFR[66]. Here we have considered the central value of the input point. Further the expressions (5.49)-(5.51) consists of the additional parameter *a* which has the value $b = 0.0744 \pm 0.0136$ for $xF_3(x, Q^2)$ as obtained in Chapter 3.

With the input point $xF_3(x_0, t_0)$, substituting the respective expressions in LO, NLO and NNLO for running coupling constant, $\frac{\alpha_s(t)}{2\pi}$ and performing the corresponding integrations, we have obtained both x as well as Q^2 evolution of $xF_3(x, Q^2)$ structure function in accord with the equations (5.31), (5.32) and (5.33) respectively. The Q^2 evolution results at fixed value of x are depicted in Fig. 5.1 and Fig. 5.2 in comparison with the experimental data taken from CCFR[66], NuTeV[68], CDHSW[69], CHORUS[70] collaborations and with the parametrization results of MRST98[138],



Figure 5.6: Q^2 evolution of $xF_3(x, Q^2)$ structure functions in accord with (5.49)-(5.51) in comparison with NuTeV[68], CHORUS[70], CDHSW[69] and MSTW[107] results. For clarity, the points are offset by the amount given in parenthesis. (Q^2 's are taken in the unit of GeV^2)



Figure 5.7: x evolution of $xF_3(x, Q^2)$ structure functions in accord with (5.49)-(5.51) in comparison with CCFR[66] and MSTW[107] results. For clarity, the points are offset by the amount given in parenthesis.



Figure 5.8: x evolution of $xF_3(x, Q^2)$ structure functions in accord with (5.49)-(5.51) in comparison with NuTeV[68], CHORUS[70] and CDHSW[69] results. For clarity, the points are offset by the amount given in parenthesis.

CTEQ4[139], MSTW[107] and KPS[140] results. In Fig. 5.3 and Fig. 5.4, the x evolution of $xF_3(x, Q^2)$ for fixed values of Q^2 are depicted along with CCFR[66], NuTeV[68] results. In all figures, as indicated, the dotted curves represent the LO results, the dashed curves represent NLO results and the solid lines are representing NNLO results. Experimental data are given with vertical upper and lower error bars for total uncertainties of statistical and systematic errors.

Again the results from equations (5.49),(5.50) and (5.51) for Q^2 and x evolution of $xF_3(x,Q^2)$ structure function with $xF_3(x_0,t_0) = 0.3298$ and b = 0.0744 are depicted in Fig. 5.5, Fig. 5.6, Fig. 5.7 and Fig. 5.8 respectively. The experimental results from CCFR, NuTeV, CDHSW, CHORUS collaborations and those of MRST98, CTEQ4, MSTW and KPS results are also plotted along with our results. Here, our LO, NLO and NNLO results are represented by the dotted, dashed and solid curves respectively.

As far the figures (Fig. 5.1 - Fig. 5.4) and (Fig. 5.5 - Fig. 5.8) are concerned, we observe a very good consistency between our theoretical and experimental as well as parametrization results within the kinematical region x < 0.05 and $Q^2 = 20 GeV^2$ of our consideration, especially, if the NNLO results are concerned. The most consistent results, the NNLO results for both the inputs along with other experimental and



Figure 5.9: Q^2 evolution of $xF_3(x, Q^2)$ structure functions in accord with (5.33) and (5.51) in comparison with CCFR[66] data. For clarity, the points are offset by the amount given in parenthesis. (Q^2 's are taken in the unit of GeV^2)



Figure 5.10: Q^2 evolution of $xF_3(x, Q^2)$ structure functions in accord with (5.33) and (5.51) in comparison with NuTeV[68], CHORUS[70], CDHSW[69] and MSTW[107] results. For clarity, the points are offset by the amount given in parenthesis. (Q^2 's are taken in the unit of GeV^2)



Figure 5.11: NNLO results for $xF_3(x, Q^2)$ structure functions predicted by(5.51) along with the uncertainty band associated with the fitting parameter *b* and the chosen input point. Our results are compared with CCFR[66] data. For clarity, the points are offset by the amount given in parenthesis. (Q^2) 's are taken in the unit of GeV^2).

parametrization results are plotted in the figures Fig. 5.9 and Fig. 5.10. They reflect the comparative picture of the results obtained by means of the two ansatz. However within our kinematical region of consideration we do not observe any significant differences among them. Further in Fig. 5.11 our NNLO results predicted by Eq.(5.51) are plotted along with the uncertainty band associated with the fitting parameter and the chosen input point. The uncertainties are observed to be small in both the cases and the uncertainty due to the fitting parameter is considerably less than that of due to input point. Along with the estimated uncertainties, we observe that the Eq.(5.51) has the capability of describing the experimental results with considerable precession.

5.5 Summary

In this chapter the non-singlet structure function $xF_3(x, Q^2)$ has been calculated at small-x. We have employed a unified approach incorporating QCD and Regge theory in this regard. Our results for $xF_3(x, Q^2)$ structure function have been found for two different input to the DGLAP equation. Both the inputs are Regge behaved. One of them consists of constant intercept (= 0.5) with Q^2 dependent residue and the other has Q^2 dependent intercept with constant residue. The structure function, evolved as the solutions of the DGLAP equations are studied phenomenologically in comparison with the results taken from CCFR, NuTeV, CHORUS and CDHSW experimental measurements. In addition, our results are compared with those obtained by MRST98, CTEQ4, MSTW and KPS collaborations. We observe a very good agreement between our theoretical results and other experimental results as well as parametrization, within the kinematical range x < 0.05 and $Q^2 = 20 GeV^2$ of our consideration. The phenomenological success achieved in this study suggests that the two simple QCD featured Regge behaved ansatz $xF_3(x, Q^2) = A(Q^2)x^{0.5}$ and $xF_3(x, Q^2) = Bx^{1-bt}$ are capable of evolving $xF_3(x, Q^2)$ structure functions with Q^2 in accord with DGLAP equations at small-x. However we could not distinguish the efficiencies among the two models in comparison with experimental data within the kinematical range of our consideration. We hope future experimental measurements at very very small values of Bjorken x will clarify their differences and help us in batter understanding of the structure of nucleon. $\Box\Box$

Chapter 6

Small-x Behaviour of $xg_1^{NS}(x,Q^2)$ Structure Function

This chapter encompasses the calculation of spin-dependent non-singlet structure function $xg_1^{NS}(x,Q^2)$ by means of solving DGLAP equation with QCD corrections up to next-next-to-leading order. Using the two ansatz, discussed in the chapter 3, developed by combining the features of perturbative Quantum Chromodynamics and Regge theory, as the initial input we have solved the DGLAP equations. The solutions, along with the ansatz allow us to obtain some analytic expressions which represent the joint Bjorken x and Q^2 dependence of $xg_1^{NS}(x,Q^2)$ structure function. The expressions are studied phenomenologically in comparison with experimental data taken from SMC, E143, HERMES, COMPASS and JLab experiments. In addition, our results are compared with some other strong analysis. We have achieved at a great phenomenological success, which signifies the capability of the expressions in describing the small-x behaviour of this non-singlet structure function and their usefulness in determining the structure functions with a reasonable precision.

6.1 Introduction

Proper understanding of the spin structure of nucleon and associated sum rules is expected to offer an important opportunity to investigate Quantum Chromodynamics(QCD) as a theory of strong interaction and hence these observables have been the active frontiers in recent years [62, 141–146]. Many successful experimental programs of polarized deep-inelastic lepton-nucleon scattering in combination with remarkable theoretical efforts have been devoted in order to elucidate the internal spin structure of the nucleon. Polarized deep inelastic lepton scattering experiment have been carried out at SLAC, CERN, DESY and Jefferson Laboratory(JLab)[62]. With the advent of dedicated experimental facilities, recent experiments were able to determine the spin structure functions as well as different sum rules over a wide range of x and Q^2 with ever increasing precision. Simultaneously, tremendous progress is observed in the field of theoretical investigation in determining and understanding the shape of quarks and gluon spin distribution functions with perturbative QCD, non-perturbative QCD, chiral perturbation theory[147], lattice QCD[148], anti-de Sitter/conformal field theory (AdS/CFT)[149], etc., along with different reliable theoretical models. In addition, recently available several dedicated phenomenological works[131,150–158] in extracting polarized parton distribution function(PPDF) as well as spin structure functions from different experiments within NLO QCD analysis have also significant contributions towards the understanding of spin structure of the nucleon.

In Quantum Chromodynamics, the spin structure function $g_1(x, Q^2)$ is described as Mellin convolutions between parton distribution functions $(\Delta q_i, \Delta g)$ and the Wilson coefficients C_i [159]

$$g_1(x,Q^2) = \frac{1}{2n_f} \sum_{i=1}^n e_i^2 [C_{NS} \otimes \Delta q_{NS} + C_S \otimes \Delta q_S + 2n_f C_g \otimes \Delta g], \tag{6.1}$$

which consists of three parts, non-singlet $g_1^{NS}(x,Q^2) = \frac{1}{2n_f} \sum_{i=1}^n e_i^2 [C_{NS} \otimes \Delta q_{NS}]$, singlet $g_1^S(x,Q^2) = \frac{1}{2n_f} \sum_{i=1}^n e_i^2 [C_S \otimes \Delta q_S]$ and gluon $\Delta G(x,Q^2) = \frac{1}{2n_f} \sum_{i=1}^n e_i^2 [2n_f C_g \otimes \Delta q_S]$. The Q^2 distribution of these spin dependent non-singlet, singlet and gluon distribution functions are governed by a set of integro-differential equations, the Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) equations which are given by [24]

$$Q^{2} \frac{\partial x g_{1}^{NS}(x,Q^{2})}{\partial \ln Q^{2}} = \frac{\alpha(Q^{2})}{2\pi} P_{qq}^{NS}(x,Q^{2}) \otimes x g_{1}^{NS}(x,Q^{2}), \qquad (6.2)$$

$$Q^{2} \frac{\partial \left(\begin{array}{c}g_{1}^{S}(x,Q^{2})\\\Delta G(x,Q^{2})\end{array}\right)}{\partial \ln Q^{2}} = \left(\begin{array}{c}P_{qq}^{S}(x,Q^{2}) & 2n_{f}P_{qg}^{S}(x,Q^{2})\\P_{gq}^{S}(x,Q^{2}) & P_{gg}^{S}(x,Q^{2})\end{array}\right) \otimes \left(\begin{array}{c}g_{1}^{S}(x,Q^{2})\\\Delta G(x,Q^{2})\end{array}\right).$$
(6.3)

Here P_i are the polarized splitting functions [24, 32]. These equations are valid to all orders in the strong coupling constant $\frac{\alpha(Q^2)}{2\pi}$.

In this chapter we have concentrated on the non-singlet part of the polarized nucleon structure function. Here we have investigated the small-x behaviour of xg_1^{NS} structure function. The investigation is based on the solution of the DGLAP evolution equation in LO, NLO and NNLO using the two ansatz $xg_1^{NS}(x,Q^2) = A(Q^2)x^{0.5}$ and $xg_1^{NS}(x,Q^2) = Ax^{(1-at)}$ as the initial inputs. We have performed a phenomenological analysis of these solutions in comparison with different experimental measurements[71,73–75] as well as the predictions due to different models [131,160–162] and achieved at a very good phenomenological success. The phenomenological success achieved in this regard reflects, on one hand the acceptability of the Regge ansatz in describing the small x behavior of the non-singlet part of spin structure function and on the other hand, the usefulness of the Regge ansatz in evolving the spin structure function, $g_1^{NS}(x,Q^2)$ in accord with DGLAP equation with a considerable precision within smaller x region.

For simplicity, defining $xg_1^{NS}(x,Q^2) = g^{NS}(x,Q^2)$, the Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) evolution equation which describe the Q^2 behavior of polarised non-singlet structure function $xg_1^{NS}(x,Q^2)$ in perturbative Quantum Chromodynamics (QCD) formalism is given by

$$\frac{\partial g^{NS}(x,t)}{\partial t} = \frac{\alpha(t)}{2\pi} \int_{x}^{1} \frac{d\omega}{\omega} g^{NS}\left(\frac{x}{\omega},t\right) P_{qq}^{NS}(\omega), \qquad (6.4)$$

in terms of the variable $t = \ln(\frac{Q^2}{\Lambda^2})$. Here the splitting function, $P_{qq}^{NS}(\omega)$ is defined up to next-next-to-leading order by

$$P_{qq}^{NS}(\omega) = \frac{\alpha(t)}{2\pi} P^{(0)}(\omega) + \left(\frac{\alpha(t)}{2\pi}\right)^2 P^1(\omega) + \left(\frac{\alpha(Q^2)}{2\pi}\right)^3 P_i^2(\omega), \tag{6.5}$$

where, $P^{(0)}(\omega)$, $P^{(1)}(\omega)$ and $P^{(2)}(\omega)$ are the corresponding LO, NLO and NNLO corrections to the splitting functions [24,32]. Splitting functions are given in Appendices.

Again, in LO, NLO and NNLO, the running coupling constant $\frac{\alpha(Q^2)}{2\pi}$ has the forms[23],

$$\left(\frac{\alpha(t)}{2\pi}\right)_{LO} = \frac{2}{\beta_0 t},\tag{6.6}$$

$$\left(\frac{\alpha(t)}{2\pi}\right)_{NLO} = \frac{2}{\beta_0 t} \left[1 - \frac{\beta_1 \ln t}{\beta_0^2 t}\right],\tag{6.7}$$

and

$$\left(\frac{\alpha(t)}{2\pi}\right)_{NNLO} = \frac{2}{\beta_0 t} \left[1 - \frac{\beta_1 \ln t}{\beta_0^2 t} + \frac{1}{\beta_0^2 t^2} \left[\left(\frac{\beta_1}{\beta_0}\right)^2 (\ln^2 t - \ln t + 1) + \frac{\beta_2}{\beta_0} \right] \right]$$
(6.8)

respectively, where $\beta_0 = 11 - \frac{2}{3}N_F$, $\beta_1 = 102 - \frac{38}{3}N_F$ and $\beta_2 = \frac{2857}{6} - \frac{6673}{18}N_F + \frac{325}{54}N_F^2$ are the one-loop, two-loop and three-loop corrections to the QCD β -function. Here the running coupling constant is expressed in terms of the variable t, which is defined by $t = \ln(\frac{Q^2}{\Lambda^2})$. Substituting the respective splitting functions along with the corresponding running coupling constant in (6.4) the DGLAP evolution equations in LO, NLO and NNLO become

$$\frac{\partial g^{NS}(x,t)}{\partial t} = \left(\frac{\alpha(t)}{2\pi}\right)_{LO} \left[\frac{2}{3}\{3+4\ln(1-x)\}g^{NS}(x,t) + I_1(x,t)\right], \quad (6.9)$$

$$\frac{\partial g^{NS}(x,t)}{\partial t} = \left(\frac{\alpha(t)}{2\pi}\right)_{NLO} \left[\frac{2}{3}\left\{3 + 4ln(1-x)\right\}g^{NS}(x,t) + I_1(x,t)\right] + \left(\frac{\alpha(t)}{2\pi}\right)_{NLO}^2 I_2(x,t), \quad (6.10)$$

and

$$\frac{\partial g^{NS}(x,t)}{\partial t} = \left(\frac{\alpha(t)}{2\pi}\right)_{NNLO} \left[\frac{2}{3}\left\{3 + 4ln(1-x)\right\}g^{NS}(x,t) + I_1(x,t)\right] \\ + \left(\frac{\alpha(t)}{2\pi}\right)_{NNLO}^2 I_2(x,t) + \left(\frac{\alpha(t)}{2\pi}\right)_{NNLO}^3 I_3(x,t) \tag{6.11}$$

respectively. Here the integral functions are given by

$$I_1(x,t) = \int_x^1 \frac{d\omega}{1-\omega} \left\{ \frac{(1+\omega^2)}{\omega} g^{NS}\left(\frac{x}{\omega},t\right) - 2g^{NS}(x,t) \right\},\tag{6.12}$$

$$I_2(x,t) = \int_x^1 \frac{d\omega}{\omega} P^{(2)}(\omega) g^{NS}\left(\frac{x}{\omega},t\right)$$
(6.13)

and

$$I_3(x,t) = \int_x^1 \frac{d\omega}{\omega} P^{(3)}(\omega) g^{NS}\left(\frac{x}{\omega},t\right).$$
(6.14)

We now solve the DGLAP equations up to NNLO ((6.9)-(6.11)) analytically using the ansatz $xg_1^{NS}(x,t) = A(t)x^{0.5}$ and $xg_1^{NS}(x,t) = Bx^{(1-at)}$ as the initial inputs. Here in both the case we have discussed in detailed the LO solution and then the same formalism is extended to have corresponding NLO and NNLO solutions.

6.2 Solution of DGLAP Evolution Equations with the Initial Input $xg_1^{NS}(x,t) = A(t)x^{0.5}$

If we consider that the non-singlet part of the spin structure function satisfies the following Regge ansatz:

$$g_1^{NS}(x,t) = g^{NS}(x,t) = A(t)x^{0.5},$$
 (6.15)

then the t dependence of $xg_1^{NS}(x,t)$ structure function at a particular value of $x = x_0$ is given by

$$g^{NS}(x_0, t) = A(t)x_0^{0.5}.$$
(6.16)

Dividing (6.15) by (6.16) we have the following relation

$$g^{NS}(x,t) = g^{NS}(x_0,t) \left(\frac{x}{x_0}\right)^{0.5},$$
(6.17)

which gives both t and x dependence of $g^{NS}(x,t)$ structure function in terms of the t dependent function $g^{NS}(x_0,t)$ at fixed $x = x_0$. The t dependent function, $g^{NS}(x_0,t)$ can be obtained from the DGLAP equation.

Substituting $g^{NS}(x,t) = A(t)x^{0.5}$ and $g^{NS}(\frac{x}{\omega},t) = \omega^{-0.5}g^{NS}(x,t)$ in equation, (6.9), we obtain

$$\frac{\partial g^{NS}(x,t)}{\partial t} = \left(\frac{\alpha(t)}{2\pi}\right)_{LO} \left[\frac{2}{3}\left\{3 + 4\ln(1-x)\right\} + \frac{4}{3}\int_{x}^{1}\frac{d\omega}{1-\omega} \left\{\frac{1+\omega^{2}}{\omega}\omega^{-0.5} - 2\right\}\right]g^{NS}(x,t), \quad (6.18)$$

which can be rearranged to have an ordinary differential equation and can be solved easily to have

$$g^{NS}(x,t)\bigg|_{LO} = C_1 \exp\left[U(x) \int \left(\frac{\alpha(t)}{2\pi}\right)_{LO} dt\right].$$
(6.19)

Here

$$U(x) = \frac{2}{3} \{3 + 4\ln(1-x)\} + \frac{4}{3} \int_{x}^{1} \frac{d\omega}{1-\omega} \left\{ \frac{1+\omega^{2}}{\omega} \omega^{-0.5} - 2 \right\}$$
(6.20)

and C is the constant of integration.

Now at a fixed value of $x = x_0$, the t dependence of the structure function $g_1^{NS}(x,t)$ in LO is given by

$$g^{NS}(x_0,t)\bigg|_{LO} = C_1 \exp\left[U(x_0) \int \left(\frac{\alpha(t)}{2\pi}\right)_{LO} dt\right].$$
(6.21)

Again the value of the structure function at $x = x_0$ and $t = t_0$ in accord with (6.21) is

$$g^{NS}(x_0, t_0) \bigg|_{LO} = C_1 \exp\left[U(x_0) \int \left(\frac{\alpha(t)}{2\pi} \right)_{LO} dt \right] \bigg|_{t=t_0}.$$
 (6.22)

Dividing (6.21) by (6.22) and rearranging a bit we obtain the t evolution of $g^{NS}(x,t)$ in accord with the LO DGLAP equation with respect to the point $g^{NS}(x_0,t_0)$ as

$$g^{NS}(x_0, t) \bigg|_{LO} = g^{NS}(x_0, t_0) \exp\left[U(x_0) \int_{t_0}^t \left(\frac{\alpha(t)}{2\pi}\right)_{LO} dt\right].$$
 (6.23)

Now substituting $g^{NS}(x_0, t) \Big|_{LO}$ from (6.23) in (6.17), we have a relation representing both x and t dependence of structure function in LO, in terms of the input point $g^{NS}(x_0, t_0)$ given by

$$g^{NS}(x,t)\Big|_{LO} = g^{NS}(x_0,t_0) \exp\left[\int_{t_0}^t \left(\frac{\alpha(t)}{2\pi}\right)_{LO} U(x_0,t)dt\right] \left(\frac{x}{x_0}\right)^{0.5}.$$
 (6.24)

Proceeding in the similar way we can obtain the relation for $g^{NS}(x,t)$ structure function in NLO and NNLO as

$$g^{NS}(x,t)\Big|_{NLO} = g^{NS}(x_0,t_0) \exp\left[U(x_0)\int_{t_0}^t \left(\frac{\alpha(t)}{2\pi}\right)_{NLO} dt + V(x_0)\int_{t_0}^t \left(\frac{\alpha(t)}{2\pi}\right)_{NLO}^2 dt\right] \left(\frac{x}{x_0}\right)^{0.5}$$
(6.25)

and

$$g^{NS}(x,t)\Big|_{NNLO} = g^{NS}(x_0,t_0) \exp\left[U(x_0)\int_{t_0}^t \left(\frac{\alpha(t)}{2\pi}\right)_{NNLO} dt + V(x_0)\int_{t_0}^t \left(\frac{\alpha(t)}{2\pi}\right)_{NNLO}^2 dt + W(x_0)\int_{t_0}^t \left(\frac{\alpha(t)}{2\pi}\right)_{NNLO}^3 dt\right] \left(\frac{x}{x_0}\right)^{0.5} (6.26)$$

respectively, where

$$V(x) = \int_{x}^{1} \frac{d\omega}{\omega} P^{(1)}(\omega) \omega^{-0.5}$$
(6.27)

and

$$W(x) = \int_x^1 \frac{d\omega}{\omega} P^{(2)}(\omega) \omega^{-0.5}.$$
(6.28)

6.3 Solution of DGLAP Evolution Equations with the Initial Input $xg_1^{NS}(x,t) = Bx^{(1-bt)}$

Now we considered that the non-singlet part of the spin structure function satisfies the following Regge ansatz:

$$g_1^{NS}(x,t) = Ax^{(-bt)}$$
(6.29)

and hence we have

$$xg_1^{NS}(x,t) = g^{NS}(x,t) = A.x^{(1-bt)}.$$
 (6.30)

The t dependence of $xg_1^{NS}(x,t)$ structure function at a particular value of $x = x_0$ is givent by

$$g^{NS}(x_0, t) = A \cdot x_0^{(1-bt)}.$$
(6.31)

Dividing (6.30) by (6.31) we have the following relation

$$g^{NS}(x,t) = g^{NS}(x_0,t) \left(\frac{x}{x_0}\right)^{(1-bt)},$$
(6.32)

which gives both t and x dependence of $g^{NS}(x,t)$ structure function in terms of the t dependent function $g^{NS}(x_0,t)$ at fixed $x = x_0$. The t dependent function, $g^{NS}(x_0,t)$ can be obtained from the DGLAP equation.

Substituting $g^{NS}(x,t) = Ax^{1-bt}$ and $g^{NS}(\frac{x}{\omega},t) = \omega^{-(1-bt)}g^{NS}(x,t)$ in equation (6.9) and rearranging a bit we can convert the LO DGLAP equation into an ordinary differential equation which can be easily solved to have

$$g^{NS}(x,t)\Big|_{LO} = C \exp\left[\int \left(\frac{\alpha(t)}{2\pi}\right)_{LO} U(x,t)dt\right].$$
(6.33)

Here

$$U(x,t) = \frac{2}{3} \{3 + 4ln(1-x)\} + \frac{4}{3} \int_{x}^{1} \frac{d\omega}{1-\omega} \left\{ \frac{1+\omega^{2}}{\omega} \omega^{-(1-bt)} - 2 \right\},$$
(6.34)

and C is the constant of integration.

At a fixed value of $x = x_0$, the t dependence of the structure function in LO is given by

$$g^{NS}(x_0, t) = C \exp\left[\int \left(\frac{\alpha(t)}{2\pi}\right)_{LO} U(x_0, t) dt\right].$$
(6.35)

Again the value of the structure function at $x = x_0$ and $t = t_0$ in accord with (6.35) is given by

$$g^{NS}(x_0, t_0) = C \exp\left[\left.\int \frac{\alpha(t)}{2\pi} U(x_0, t) dt\right]\right|_{t=t_0}.$$
 (6.36)

Dividing (6.35) by (6.36) and rearranging a bit we obtain the t dependence of $g^{NS}(x_0, t)$ in accord with LO DGLAP evolution equation with respect to the point $g^{NS}(x_0, t_0)$ as

$$g^{NS}(x_0, t) = g^{NS}(x_0, t_0) \exp\left[\int_{t_0}^t \left(\frac{\alpha(t)}{2\pi}\right)_{LO} U(x_0, t) dt\right].$$
 (6.37)

Now substituting $g^{NS}(x_0, t) \Big|_{LO}$ from (6.37) in (6.32), we have a relation representing both x and t dependence of structure function in LO, in terms of the input point $g^{NS}(x_0, t_0)$ given by

$$g^{NS}(x,t)\Big|_{LO} = g^{NS}(x_0,t_0) \exp\left[\int_{t_0}^t \left(\frac{\alpha(t)}{2\pi}\right)_{LO} U(x_0,t)dt\right] \left(\frac{x}{x_0}\right)^{(1-bt)}.$$
 (6.38)

Proceeding in the similar way we can obtain the relation for $g^{NS}(x,t)$ structure function in NLO and NNLO as

$$g^{NS}(x,t)\Big|_{NLO} = g^{NS}(x_0,t_0) \exp\left[\int_{t_0}^t \left(\frac{\alpha(t)}{2\pi}\right)_{NLO} U(x_0,t)dt + \int_{t_0}^t \left(\frac{\alpha(t)}{2\pi}\right)_{NLO}^2 V(x_0,t)dt\right] \left(\frac{x}{x_0}\right)^{(1-bt)},$$
(6.39)

and

$$g^{NS}(x,t)\Big|_{NNLO} = g^{NS}(x_0,t_0) \exp\left[\int_{t_0}^t \left(\frac{\alpha(t)}{2\pi}\right)_{NNLO} U(x_0,t)dt + \int_{t_0}^t \left(\frac{\alpha(t)}{2\pi}\right)^2_{NNLO} V(x_0,t)dt + \int_{t_0}^t \left(\frac{\alpha(t)}{2\pi}\right)^3_{NNLO} W(x_0,t)dt\right] \left(\frac{x}{x_0}\right)^{(1-bt)}, \quad (6.40)$$

respectively, where

$$V(x,t) = \int_{x}^{1} \frac{d\omega}{\omega} P^{(1)}(\omega) \omega^{-(1-bt)}$$
(6.41)

and

$$W(x,t) = \int_x^1 \frac{d\omega}{\omega} P^{(2)}(\omega) \omega^{-(1-bt)}.$$
(6.42)

6.4 Results and Discussion

The equations (6.24)-(6.26) and (6.38)-(6.40) are the analytic expressions representing both x and Q^2 dependence of $xg_1^{NS}(x,Q^2)$ structure function jointly, obtained by means of solving the DGLAP equations in LO, NLO and NNLO incorporating the Regge ansatz, $xg_1^{NS}(x,t) = A(t)x^{0.5}$ and $xg_1^{NS}(x,t) = Bx^{1-bt}$ as the initial inputs respectively. These expressions are consisting of an input point $xg_1^{NS}(x_0,t_0)$, which can be taken from the available experimental data. If the input point is more accurate and precise, we can expect batter results. There are not any specific reason in choosing the input point. Any one of the data points at a certain value of $x = x_0$ and $t = t_0$ can be considered as the input point. Off course, the sensitivity of different inputs will be ---



Figure 6.1: xg_1^{NS} structure function in accord with (6.24)-(6.26), compared with the data taken from SMC[74], HERMES[73], COMPASS[71] and E143[75] experiments and the results of TSA[131], AAC[160], BB[161] and GRSV[162] collaborations.

different. However instead of choosing the input point on the basis of their sensitivity, in our manuscript we have incorporated a suitable condition in determining the input point. We have considered that particular point from the most recent measurements as the input point in which experimental errors are minimum. Under this condition we have selected the point $g^{NS}(x_0 = 0.0143955, Q_0^2 = 5GeV^2) = 0.0133075$ at $x_0 =$ 0.0143955 and $Q^2 = 5GeV^2$ from COMPASS[71] experimental data. Here we have considered the central value of the input point. Further the expressions (6.38)-(6.40) consists of the additional parameter b which has the value $b = 0.0759 \pm 0.0107$ for xg_1^{NS} as obtained in Chapter 3.

With the input point $g^{NS}(x_0 = 0.0143955, Q_0^2 = 5GeV^2) = 0.0133075$, substituting the respective expressions in LO, NLO and NNLO for running coupling constant, $\frac{\alpha_s(t)}{2\pi}$ and performing the corresponding integrations, we have obtained the x evolution of $xg_1^{NS}(x, Q^2)$ structure function in accord with the equations (6.24), (6.25) and (6.26) respectively. The x evolution results for two fixed value of $Q^2 = 5.0GeV^2$ are depicted in Fig. 6.1. However, as there are not any available experimental results for --



Figure 6.2: xg_1^{NS} structure function in accord with (6.38)-(6.40), compared with the data taken from SMC[74], HERMES[73], COMPASS[71] and E143[75] experiments and the results of TSA[131], AAC[160], BB[161] and GRSV[162] collaborations.

different Q^2 , we could not have comparative analysis of our Q^2 evolution results. Our x evolution results are plotted along with the experimental results taken from SMC[74], HERMES[73], COMPASS[71] and E143[75] experiments. In addition to these, we have also included the predictions made by Taghavi-Shahri and Arash(TSA)[131], Asymmetry Analysis Collaboration(AAC)[160], Blumlein and Bottcher(BB)[161] and Gluck, Reya, Startmann and Vogelsang(GRSV)[162] based on various models, in our comparative analysis. We see that $g^{NS}(x, Q^2)$ structure functions evolved with respect to the input point are consistent with those of experimental measurements as well as other models. This implies that the expressions, we have obtained by means of solving the DGLAP equations analytically, are applicable in describing small x behaviour of $xg_1^{NS}(x, Q^2)$ structure function with a considerable precision.

Again the results from equations (6.38),(6.39) and (6.40) for x evolution of $xg_1^{NS}(x,Q^2)$ structure function with $xg_1^{NS}(x_0,Q_0^2) = 0.0133075$ and b = 0.0759 are depicted in Fig. 6.2. In this case also as we do not have experimental data point for various Q^2 , we could not perform the comparative analysis of our Q^2 evolution results.



Figure 6.3: xg_1^{NS} structure function in accord with (6.26) and (6.40) and in comparison with the data taken from SMC[74], HERMES[73], COMPASS[71] and E143[75] experiments.

However, the x evolution results are compared with SMC, E143, HERMES and COM-PASS experimental results and with several predictions made in Ref [131, 160–162] based on various model.

Also we have estimated the uncertainty associated with the fitting parameter b and the chosen input point and the respective uncertainty bands are shown in Fig. 6.4 separately. Here the uncertainty due to the fitting parameter is considerably less than that of due to input point. However both the uncertainties are observed to be decreasing as x decreases.

As far the figures, 6.2 - 6.4 are concerned, we observe a very good consistency among our theoretical results and other experimental as well as parametrization results within the kinematical region x < 0.05 of our consideration. Our x evolution results for both the inputs along with other experimental results are plotted in Fig. 6.3. It reflects the comparative picture of the results obtained by means of the two ansatz. However within our kinematical region of consideration we do not observe any significant differences among them. This implies that the analytic expressions, we have obtained by means of solving the DGLAP equations with both the ansatz as the initial input, are applicable in describing the small x behaviour of $xg_1^{NS}(x, Q^2)$ structure



Figure 6.4: xg_1^{NS} structure function in accord with (6.40) and in comparison with different experimental data and theoretical as well as phenomenological analysis, along with the uncertainty band associated with the fitting parameter(b) and the chosen input point.

function with a considerable precision.

6.5 Summary

In this paper we have obtained some expressions for the non-singlet part of spin structure function, $xg_1^{NS}(x, Q^2)$ at small-x by means of analytical solution of DGLAP equation in LO, NLO and NNLO using a Regge like ansatz with Q^2 dependent intercept as the initial input. Both the Regge inspired ansatz in accord with DGLAP equations provides a very good description of the small-x behaviour of $g_1^{NS}(x, Q^2)$, which are consistent with other experimental results. The consistency of the results for $xg_1^{NS}(x, Q^2)$ due to the Regge like models $g^{NS}(x, t) = Ax^{0.5}$ and $g^{NS}(x, t) = Ax^{1-bt}$ with different experimental results taken from SMC[74], HERMES[73], COMPASS[71] and E143[75] and other strong analysis [131,160–162] signifies that the model is applicable in describing the small-x behaviour of $xg_1^{NS}(x, Q^2)$ structure function although it being simple. Moreover, in this method we do not require the knowledge of initial distributions of structure functions at all values of x from 0 to 1. Here, we just require one input point at any fixed x and Q^2 and with respect to that point both the x and

 Q^2 evolution of structure functions can be obtained. $\Box \Box$

Chapter 7

Sum Rules Associated with Non-singlet Structure Functions

In this chapter, we have determined the three sum rules viz., the Gottfired Sum rule(GSR), the Gross-Llewellyn Smith sum rule(GLSSR) and the Bjorken sum rule(BSR), which are associated with the non-singlet structure functions F_2^{NS} , xF_3 and xg_1^{NS} respectively with QCD corrections up to NNLO. The determination of sum rules requires the knowledge of structure functions only at small-x and these requirements are obtained from the previous chapters, where we have successfully evolved the non-singlet structure functions in accord with DGLAP equation through an approach unifying Regge theory and pQCD. We have also perform a phenomenological analysis of our results for various sum rules in comparison with their respective experimental and parametrization results.

7.1 Introduction

Deep inelastic structure functions obey a series of Sum rules, which are integrals over structure functions or parton distributions, expressing usually the conservation law for some quantum number of the nucleon. These sum rules provide information about the distribution of quarks inside nucleon and are very useful to reveal new physics if a sum rule is observed to be satisfied or broken. Perturbative Quantum Chromodynamics has predictions of a wide variety sum rules and they are expected to provide us with a stringent test of QCD. Because, the sum rules are expressed as the integrals of the form $\int_0^1 dx F(x, Q^2) = A$, and in this representation, one gets rid of the unknown x-dependence which is due to non-perturbative effects. Further, sum rules can be computed up to much higher orders in perturbative QCD than other quantities. Therefore the sum rules have been the subject of great experimental, theoretical as well as phenomenological investigation.

In order to investigate the validity of QCD as a theory of strong interaction by means of sum rules, many successful experimental programs of both polarized and unpolarized deep-inelastic lepton nucleon scattering have been performed. With the advent of dedicated experimental facilities the recent measurements of the structure functions of both polarized and un-polarized DIS[7] in the wide interval of the $x = \frac{Q^2}{2pq}$ variable open the possibility of a more precise determination of the number of the DIS sum rules. In view of this experimental progress the detailed studies of the theoretical predictions for the DIS sum rules started to attract special attention.

Brief overview of the basic parton model sum rules have already been given in section 1.5 and commented on the status of their available QCD corrections. QCD corrections to sum rules mainly fall into two classes; those that are strongly suppressed at high energy (higher twist corrections) and those that vanish only logarithmically with the momentum transfer. The latter are fully calculable in terms of the coupling constant α_s of QCD.

The determination of these sum rules requires knowledge of the corresponding structure functions over the entire region of $x \in (0; 1)$. The experimentally accessible x range for the lepton DIS is however limited for the available data and therefore one should extrapolate results to x = 0 and x = 1. The extrapolation to $x \to 0$, where structure functions $\frac{1}{x}F_2^{NS}$, F_3 and g_1^{NS} grow strongly, is much more important than the extrapolation to $x \to 1$, where structure functions vanish. Again, it is known that maximum contribution (about 90%) to the GSR, GLSSR and BSR come from the small $x (\leq 0.1)$ region. Because of the large contribution to these sum rules from small x, the small x region is particularly important. In the following sections we will observe that the determination of sum rules requires the knowledge of structure functions only at small-x and the requirements are obtained from the previous chapters, where we have successfully evolved the non-singlet structure functions in accord with DGLAP equation through a unified approach unifying Regge theory and pQCD.

This chapter is divided into six sections. In the next section 7.2, we have presented a generalized formalism which is adopted in the determination of various sum rules. Then the same formalism is extended to incorporate the sum rules, GSR, GLSSR and BSR in the section 7.3, 7.4 and 7.5 respectively. In the respective sections we have



Figure 7.1: General interpretation of sum rule. The curve represents the variation of the structure function F_i^{NS} with x and the area under the curve represents the sum rule.

also provided a detailed analysis of our results for the sum rules in comparison with other available experimental and parametrization results. In the last section 7.6, the works performed and results obtained in this chapter are summarised.

7.2 The General Strategy Adopted in Determining Sum Rules

Away from $Q^2 \to \infty$, the Sum Rules, GSR, GLSSR and BSR are expressed in terms of a sum of two series in powers of the strong coupling constant $\alpha_s(Q^2)$ (leading twist pQCD correction) and in powers of $\frac{1}{Q^2}$ (nonperturbative higher twist corrections):

$$S_i(Q^2) = \int_0^1 \frac{dx}{x} F_i^{NS}(x, Q^2) = S_i^{pQCD} + \sum_{i=2}^\infty \frac{\mu_{2i}^{p-n}(Q^2)}{Q^{2i-2}},$$
(7.1)

where S_i denotes the sum rules associated with $F_i^{NS} = F_2^{NS}, xF_3, xg_1^{NS}$. Here the leading twist term (bracket term) consists of pQCD results and the second term on the r.h.s. is known as higher twist term. The higher order pQCD corrections and higher twist power corrections are significant at low- Q^2 region(see Ref. [163,164] and references therein). In this chapter we have paid attention to only the first part i.e., the pQCD corrected term, S_i^{pQCD} .

In general the integral associated with the sum rules represents the area under the curve $F_i^{NS}(x, Q^2)$ from x = 0 to x = 1 (Shown in Fig. 7.1), which can be resolved as

$$S_i(Q^2) = \int_0^{x_{min}} \frac{F_i^{NS}(x,Q^2)}{x} dx + \int_{x_{min}}^1 \frac{F_i^{NS}(x,Q^2)}{x} dx$$
(7.2)

and it gives

$$S_i(x_{min}, Q^2) = \int_{x_{min}}^1 \frac{F_i^{NS}(x, Q^2)}{x} dx = S_i(Q^2) - \int_0^{x_{min}} \frac{F_i^{NS}(x, Q^2)}{x} dx.$$
(7.3)

The integral on the left hand side of (7.3) represents the area under the curve $\frac{F_i^{NS}(x,Q^2)}{x}$ from $x = x_{min}$ to x = 1. For $x = x_{min} \to 0$, this integral will tend to cover the whole area under the curve from $x = x_{min} = 0$ to x = 1, that is, it will represent the whole integral associated with the sum rule. Again the second part on the right side of (7.3) represents the part of total area $\int_0^1 \frac{F_i^{NS}(x,Q^2)}{x} dx$, laying under the curve $\frac{F_i^{NS}(x,Q^2)}{x}$ within smaller x region i.e., from x = 0 to any smaller value $x = x_{min}$. Thus we see that in order to investigate the sum rules, we just require the knowledge of corresponding structure function $F_i^{NS}(x,Q^2)$ within smaller x region. This requirement can be fulfilled by using the solutions of DGLAP equations obtained in our previous chapters.

Based on this general formalism, in the following sections we have investigated the GSR, GLSSR and BSR with pQCD corrections up to NNLO utilising the well behaved solutions of the DGLAP evolution equations for F_2^{NS} , xF_3 and xg_1^{NS} obtained in the previous chapters 4, 5 and 6 respectively.

7.3 Determination of Gottfried Sum Rule

The Gottfried Sum Rule(GSR)[34] is associated with the non-singlet structure function $F_2^{ep} - F_2^{en}$, the difference of F_2 measured on proton and on neutron in charged lepton scattering. In accord with parton model this sum rule expresses the fact that there is one more u valence quark than d valence quark in the proton and is only valid under the assumption that the seas of u and d quarks in the proton are equal($\bar{u} = \bar{d}$). It is written as

$$S_{GSR}(Q^2) = \int_0^1 \frac{dx}{x} \left[F_2^{ep}(x,Q^2) - F_2^{en}(x,Q^2) \right]$$
$$= \int_0^1 dx \left[\frac{1}{3} (u_v(x,Q^2) - d_v(x,Q^2)) + \frac{2}{3} (\bar{u}(x,Q^2) - \bar{d}(x,Q^2)) \right]$$
$$= \frac{1}{3} + \frac{2}{3} \int_0^1 \left[\bar{u}(x,Q^2) - \bar{d}(x,Q^2) \right] dx.$$
(7.4)

In fact if the sea were flavour symmetric, namely $\bar{u} = \bar{d}$, we expect

$$S_{GSR} = \frac{1}{3}.\tag{7.5}$$

However, the most detailed analysis of muon-nucleon DIS data of NMC Collaboration gives the following result[65]

$$S_{GSR}(Q^2 = 4GeV^2) = 0.235 \pm 0.026, \tag{7.6}$$

which in turn indicates the violation of theoretical expression of Eq. 7.5 and necessitates more detailed investigations of different effects, related to the Gottfried sum rule.

In QCD, the leading twist pQCD correction up to NNLO for GSR is expressed as a series in powers of the strong coupling constant $\alpha_s(Q^2)[35]$:

$$S_{GSR}(Q^2) = \int_0^1 \frac{dx}{x} F_2^{NS}(x, Q^2) = \frac{1}{3} \left[1 + 0.0355 \frac{\alpha_s}{\pi} - 0.811 \left(\frac{\alpha_s}{\pi}\right)^2 \right].$$
 (7.7)

Here the GSR consists of pQCD results up to second order of $\alpha_s(Q^2)$.

In accord with Eq. 7.3, the GSR integral can be represented as

$$S_{GSR}(x_{min},Q^2) = \int_{x_{min}}^1 \frac{F_2^{NS}(x,Q^2)}{x} dx = S_{GSR}(Q^2) - \int_0^{x_{min}} \frac{xg_1^{NS}(x,Q^2)}{x} dx.$$
 (7.8)

From Eq. 7.8, it is clear that in order to calculate the integral on the l.h.s., which represents the GSR for $x_{min} \rightarrow 0$ limite, we need to know $F_2^{NS}(x, Q^2)$ structure function within smaller x region. This requirement can be fulfilled by using the solutions of DGLAP equations for F_2^{NS} , obtained in the chapter 4, which provide well description of the small-x behaviour of $F_2^{NS}(x, Q^2)$ structure function. Therefore, substituting (4.49), (4.50) and (4.51) in (7.8) (although the expressions (4.31)-(4.33)) and using the corresponding expressions for S_{GSR} in LO, NLO and NNLO, we obtain the GSR integral with LO, NLO and NNLO pQCD corrections as

$$S_{GSR}(x_{min}, Q^2) \Big|_{LO} = S_{GSR}(Q^2) \Big|_{LO} - \int_0^{x_{min}} \frac{dx}{x} \bigg[F_2^{NS}(x_0, t_0) \\ \left(\frac{x}{x_0}\right)^{(1-bt)} \exp\bigg\{ \int_{t_0}^t \bigg(\frac{\alpha(t)}{2\pi}\bigg)_{LO} P(x_0, t) dt \bigg\} \bigg],$$
(7.9)

$$S_{GSR}(x_{min}, Q^2)\Big|_{NLO} = S_{GSR}(Q^2)\Big|_{NLO} - \int_0^{x_{min}} \frac{dx}{x} \left[F_2^{NS}(x_0, t_0) \left(\frac{x}{x_0}\right)^{(1-bt)} \exp\left\{\int_{t_0}^t \left(\frac{\alpha(t)}{2\pi}\right)_{NLO} P(x_0, t)dt + \int_{t_0}^t \left(\frac{\alpha(t)}{2\pi}\right)^2_{NLO} Q(x_0, t)dt\right\}\right]$$
(7.10)

and

$$S_{GSR}(x_{min}, Q^{2})\Big|_{NNLO} = S_{GSR}(Q^{2})\Big|_{NNLO} - \int_{0}^{x_{min}} \frac{dx}{x} \left[F_{2}^{NS}(x_{0}, t_{0}) \left(\frac{x}{x_{0}}\right)^{(1-bt)} \exp\left\{ \int_{t_{0}}^{t} \left(\frac{\alpha(t)}{2\pi}\right)_{NNLO} P(x_{0}, t) dt + \int_{t_{0}}^{t} \left(\frac{\alpha(t)}{2\pi}\right)^{2}_{NNLO} Q(x_{0}, t) dt + \int_{t_{0}}^{t} \left(\frac{\alpha(t)}{2\pi}\right)^{3}_{NNLO} R(x_{0}, t) dt \right\} \right] (7.11)$$

respectively. Considering a known input point $F_2^{NS}(x_0, t_0)$ from experimental data, we will be able to calculate the GSR integral up to NNLO corrections using the expressions, (7.9), (7.10) and (7.11) respectively.

In our calculation of GSR, we have used the NMC[63] experimental data point, $F_2^{NS}(x_0, t_0) = 0.010348 \pm 0.006208$ at $x_0 = 0.025$ and $Q^2 = 2.34686 GeV^2$ as the input point. With this input point we have calculated the Gottfired Sum rule and the results in accord with equations (7.9), (7.10) and (7.11) are depicted in Fig. 7.2 and Fig. 7.3 as a function of Q^2 in comparison with the results predicted by QCD in accord with Eq.(7.7).



Figure 7.2: LO, NLO and NNLO results for GSR along with parton model and pQCD predictions. (Q^2 's are taken in the unit of GeV^2).



Figure 7.3: Our NNLO results for GSR in comparison with parton model and NNLO pQCD predictions as well as the results of NNPDF collaboration. (Q^2 's are taken in the unit of GeV^2).



Figure 7.4: GSR as a function of x_{min} in comparison with NMC, NNPDF and KMRS results. The uncertainty band shown here is the uncertainty associated with the iput point.

In Fig. 7.3, the NNLO results are compared with the NNPDF[101] results. In Fig. 7.4 our results for GSR are depicted as a function of low x limit of integration x_{min} in comparison with those obtained by NMC, NNPDF and KMRS. Here we have also shown the estimated uncertainty band associated with the chosen input point. Although our result for GSR, as far the figures 7.2, 7.3 and 7.4 are concerned, do not agree well with those of NMC, NNPDF as well as KMRS, however a very good agreement with pQCD predictions is observed.

7.4 Determination of Gross-Llewellyn Smith Sum Rule

The Gross-Llewellyn Smith(GLS) sum rule[38, 39] associated with the non-singlet $xF_3(x, Q^2)$ structure function measured in neutrino-nucleon ($\nu - N$) scattering is one of the best observables to investigate Quantum Chromodynamics(QCD) as a theory of strong interaction. Perturbative Quantum Chromodynamics (pQCD) predicts the value of the GLS integral up to next-next-to-leading order(NNLO) as a function of

strong coupling constant(α_s), the four momentum transfer(Q^2) and the number of accessible quark flavour (n_f). Up to NNLO pQCD corrections, the GLS integral can be written as[39]

$$S_{GLS}(Q^2) = \int_0^1 \frac{dx}{x} x F_3(x, Q^2) = 3 \left[1 - \frac{\alpha_s}{\pi} - a(n_f) \left(\frac{\alpha_s}{\pi}\right)^2 - b(n_f) \left(\frac{\alpha_s}{\pi}\right)^3 \right], \quad (7.12)$$

where the flavour dependent functions are given by $a(n_f) = \frac{55}{12} - \frac{n_f}{3}$ and $b(n_f) = 41.441 - 8.02n_f + 0.177n_f^2$.

As $xF_3(x, Q^2)$ structure function is not marred by the presence of the sea quark and gluon densities about which we have very poor information in particular in the small-x region and higher order QCD calculations are observed to be largely independent of renormalization scheme [165], this prediction is considered as the robust prediction in pQCD. In order to verify the GLS sum rule, experiments have been performed by CCFR collaboration[166] and obtained a precision of roughly 3% in accordance with the analysis in Ref. [167], using a leading-order(LO) QCD-based fit to extrapolate all data to $Q^2 = 3.2 GeV^2$. However, some small but important corrections due to quark mass thresholds, target mass or higher twist effects, which were not included in previous analysis were reported in Ref. [165, 168–172]. In addition to these, some small but significant corrections arising from strange quark distributions and from charge symmetry violating parton distributions were also investigated recently in Ref. [173]. However, in this chapter we have focused only on the pQCD corrections up to NNLO.

In order to determine GLS sum rule we have adopted the similar formalism used in determining GSR. Here firstly we have resolved the GLS integral as

$$S_{GLS}(Q^2) = \int_0^{x_{min}} \frac{xF_3(x,Q^2)}{x} dx + \int_{x_{min}}^1 \frac{xF_3(x,Q^2)}{x} dx, \qquad (7.13)$$

which gives

$$S_{GLS}(x_{min}, Q^2) = \int_{x_{min}}^{1} \frac{xF_3(x, Q^2)}{x} dx = S_{GLS}(Q^2) - \int_0^{x_{min}} \frac{xF_3(x, Q^2)}{x} dx.$$
 (7.14)

The integral on the left hand side of (7.14) similarly represents the area under the curve $\frac{xF_3(x,Q^2)}{x}$ from $x = x_{min}$ to x = 1. For $x = x_{min} \to 0$, this integral will tend to cover the whole area under the curve from $x = x_{min} = 0$ to x = 1, that is, it will represent the GLS integral. Again the second part on the right side of (7.14) represents

the part of total area lying under the curve, $\frac{xF_3(x,Q^2)}{x}$ within smaller x region i.e., from x = 0 to any smaller value $x = x_{min}$. Thus we see that in order to investigate the GLS integral, we just require the knowledge of $xF_3(x,Q^2)$ structure function within smaller x region, not the entire region.

We have already obtained the small-x behaviour of $xF_3(x, Q^2)$ structure function by means of solving DGLAP evolution equation in chapter 5 and they are observed to be consistent with other experimental as well as parametrization results. Which implies that the analytical expressions, we have obtained in chapter 5 for $xF_3(x, Q^2)$ are applicable in describing small x behaviour of $xF_3(x, Q^2)$ structure function with a considerable precision and therefore those expressions can be successfully incorporated in $\int_0^{x_{min}} \frac{xF_3(x,Q^2)}{x} dx$ for $xF_3(x,Q^2)$ term and hence we can obtain the GLS integral (7.14) with LO, NLO and NNLO corrections as

$$S_{GLS}(x_{min}, Q^2)\Big|_{LO} = S_{GLS}(Q^2)\Big|_{LO} - \int_0^{x_{min}} \frac{dx}{x} \bigg[F_3^{NS}(x_0, t_0) \bigg(\frac{x}{x_0}\bigg)^{(1-bt)} \exp\bigg\{\int_{t_0}^t \bigg(\frac{\alpha(t)}{2\pi}\bigg)_{LO} P(x_0, t) dt\bigg\}\bigg], \quad (7.15)$$

$$S_{GLS}(x_{min}, Q^{2})\Big|_{NLO} = S_{GLS}(Q^{2})\Big|_{NLO} - \int_{0}^{x_{min}} \frac{dx}{x} \left[F_{3}^{NS}(x_{0}, t_{0}) \left(\frac{x}{x_{0}}\right)^{(1-\delta t)} \exp\left\{ \int_{t_{0}}^{t} \left(\frac{\alpha(t)}{2\pi}\right)_{NLO} P(x_{0}, t) dt + \int_{t_{0}}^{t} \left(\frac{\alpha(t)}{2\pi}\right)^{2}_{NLO} Q(x_{0}, t) dt \right\} \right]$$
(7.16)

and

$$S_{GLS}(x_{min}, Q^2)\Big|_{NNLO} = S_{GLS}(Q^2)\Big|_{NNLO} - \int_0^{x_{min}} \frac{dx}{x} \left[F_3^{NS}(x_0, t_0) \left(\frac{x}{x_0}\right)^{(1-bt)} \exp\left\{ \int_{t_0}^t \left(\frac{\alpha(t)}{2\pi}\right)_{NNLO}^{0} \right\} \right] + \int_{t_0}^t \left(\frac{\alpha(t)}{2\pi}\right)^2_{NNLO} Q(x_0, t) dt + \int_{t_0}^t \left(\frac{\alpha(t)}{2\pi}\right)^3_{NNLO} R(x_0, t) dt \right\} \right] (7.17)$$



Figure 7.5: Results for the Gross - Llewellyn Smith sum rule at LO, NLO and NNLO, as a function of Q^2 . The data are from the CCFR experiment [67]. The LO, NLO and NNLO curves are offset by the amount given in parenthesis. (Q^2 's are taken in the unit of GeV^2).

respectively. Considering the input point, $xF_3(x_0 = 0.025, t_0 = 3.2GeV^2) = 0.3298$ from CCFR data we have calculated the GLS sum rule with QCD corrections up to NNLO using the expressions (7.15), (7.16) and (7.17) respectively and the results are depicted in Fig. 7.5, Fig. 7.6 and Fig. 7.7.

The Q^2 dependence of GLS integral as obtained from Eqs.(7.15), (7.16) and (7.17) are depicted in Fig. 7.6 in comparison with the experimental data taken from CCFR collaborations and the corresponding perturbative QCD predictions Eq.(7.12) in LO, NLO and NNLO. Here the inner error bar shows statistical errors and the outer one, a combination of statistical and systematic errors associated with CCFR data. Our results for LO, NLO and NNLO are represented by solid curves with the corresponding dashed curves representing theoretical QCD predictions Eq.(7.12) using higher-order QCD corrections (LO, NLO and NNLO) from [39]. Fig. 7.6 reflects the comparative picture of our NNLO results with those of CCFR data and theoretical predictions of QCD at NNLO and results obtained by Kataev and Sidorov(KS) in Ref. [167]. In Fig. 7.7 our results of GLSSR are plotted as a function of x_{min} in comparison with CCFR measurements. Here the uncertainties due to the chosen input is also estimated and



Figure 7.6: Our NNLO results for the Gross-Llewellyn Smith sum rule, for various values of Q^2 , along with QCD predictions Eq.(7.12) in NNLO, in comparison with CCFR experiment [67]. The results with up triangle symbols along with uncertainty bars are the KS[167] results. (Q^2 's are taken in the unit of GeV^2).



Figure 7.7: Results for the Gross-Llewellyn Smith sum rule, for various values of x. The data are taken from the CCFR experiment [67].

they are shown by the green band. From these figures one can see that our LO, NLO and NNLO results are within the statistical uncertainties of measurements by CCFR collaboration and also consistent with QCD predictions as well as KS results.

7.5 Determination of Bjorken Sum Rule

The Bjorken sum rule is associated with the spin dependent non-singlet structure function $xg_1^{NS}(x,Q^2)$. BSR relates the difference of proton and neutron structure functions integrated over all possible values of Bjorken variable, x to the nucleon axial charge g_A . At infinite four-momentum transfer squared, Q^2 , the sum rule reads

$$S_{BSR} = \int_0^1 \frac{dx}{x} x g_1^{NS}(x, Q^2) = \frac{g_A}{6}.$$
 (7.18)

In accord with QCD prediction, the leading twist pQCD correction up to NNLO for BSR is expressed as follows :

$$S_{BSR}(Q^2) = \int_0^1 \frac{dx}{x} x g_1^{NS}(x, Q^2) = \frac{g_A}{6} \left[1 - \frac{\alpha_s}{\pi} - 3.583 \left(\frac{\alpha_s}{\pi}\right)^2 - 20.215 \left(\frac{\alpha_s}{\pi}\right)^3 \right] (7.19)$$

which can be resolved to have

$$S_{BSR}(x_{min},Q^2) = \int_{x_{min}}^1 \frac{xg_1^{NS}(x,Q^2)}{x} dx = S_1^{p-n}(Q^2 - \int_0^{x_{min}} \frac{xg_1^{NS}(x,Q^2)}{x} dx.$$
 (7.20)

Using the solutions of DGLAP equations, obtained in chapter 6, which provide well description of the small-x behaviour of $xg_1^{NS}(x, Q^2)$ structure function we can determine the integral on l.h.s. of Eq. 7.20, which will tend to represent the BSR for the limit $x = x_{min} \rightarrow 0$. Therefore, substituting (6.38),(6.39) and (6.40) in (7.20) and using the corresponding expressions for S_{BSR} in LO, NLO and NNLO, we obtain the Bjorken integral with LO, NLO and NNLO QCD corrections as

$$S_{BSR}(x_{min}, Q^2) \Big|_{LO} = S_{BSR}(Q^2) \Big|_{LO} - \int_0^{x_{min}} \frac{dx}{x} \bigg[g^{NS}(x_0, t_0) \\ \left(\frac{x}{x_0}\right)^{(1-bt)} \exp\bigg\{ \int_{t_0}^t \bigg(\frac{\alpha(t)}{2\pi}\bigg)_{LO} P(x_0, t) dt \bigg\} \bigg],$$
(7.21)

$$S_{BSR}(x_{min}, Q^2) \Big|_{NLO} = S_{BSR}(Q^2) \Big|_{NLO} - \int_0^{x_{min}} \frac{dx}{x} \left[g^{NS}(x_0, t_0) \left(\frac{x}{x_0} \right)^{(1-bt)} \exp\left\{ \int_{t_0}^t \left(\frac{\alpha(t)}{2\pi} \right)_{NLO} P(x_0, t) dt + \int_{t_0}^t \left(\frac{\alpha(t)}{2\pi} \right)^2_{NLO} Q(x_0, t) dt \right\} \right]$$
(7.22)

and

$$S_{BSR}(x_{min}, Q^2)\Big|_{NNLO} = S_{BSR}(Q^2)\Big|_{NNLO} - \int_0^{x_{min}} \frac{dx}{x} \left[g^{NS}(x_0, t_0) \left(\frac{x}{x_0}\right)^{(1-bt)} \exp\left\{\int_{t_0}^t \left(\frac{\alpha(t)}{2\pi}\right)_{NLO} P(x_0, t)dt\right\} + \int_{t_0}^t \left(\frac{\alpha(t)}{2\pi}\right)^3_{NNLO} P(x_0, t)dt\right\} \right] + \int_{t_0}^t \left(\frac{\alpha(t)}{2\pi}\right)^3_{NNLO} R(x_0, t)dt\right\} \right]$$
(7.23)

respectively. Considering a known input point $g^{NS}(x_0, t_0)$ from experimental data, we will be able to calculate the BSR integral up to NNLO corrections using the expressions, (7.21), (7.22) and (7.23) respectively. In our calculations we have used $g^{NS}(x_0 = 0.0143955, Q_0^2 = 5GeV^2) = 0.0133075$ as the input point, which is taken from the COMPASS[71] experimental data. With this input point we have calculated the Bjorken integral and the results in accord with equations (7.21), (7.22) and (7.23) are depicted in Fig. 7.8 and Fig. 7.9.

In Fig. 7.8, we have plotted our results for BSR integral in LO, NLO and NNLO as a function of low x limit of integration x_{min} , in comparison with COMPASS and HERMES measurements along with the results due to valon model(TSA)[131]. The uncertainties due to the parameter, b and the input point are estimated only for the NNLO results and as seen from the Fig. 7.8, they decrease with decrease in x_{min} . From Fig. 7.8 we observe an overall batter description of both COMPASS and HERMES data by our results with respect to the predictions due to valon model. Again our approach expects batter results for $x_{min} \rightarrow 0$, but there are no COMPASS measurement beyond $x \approx 0.004$ and HERMES measurement beyond $x \approx 0.02$ for our comparative analysis. Saturation of the COMPASS data for BSR is observed within x > 0.004, however available HERMES results have not saturated at $x \approx 0.01 - 0.02$.


Figure 7.8: The results of Bjorken integral as a function of the low x limit of integration, x_{min} , in LO, NLO and NNLO in comparison with COMPASS [71] and HERMES[73] experimental data along with the predictions based on Valon model[131].



Figure 7.9: The results of Bjorken integral as a function of momentum transfer squared Q^2 in LO, NLO and NNLO against COMPASS [71] and HERMES[73] E143[75] and JLab [76–78] experimental data along with the theoretical as well as phenomenological analysis, Ref. [176–179]. (Q^2 's are taken in the unit of GeV^2).



Figure 7.10: Our results of Bjorken integral in comparison with the QCD predictions up to NNLO[39]. (Q^2 's are taken in the unit of GeV^2).

Thus we may expect to occur saturation within the smaller x region and within this region both HERMES and COMPASS results might agree with each other and reach an overall compatibility with our measurements.

The Q^2 dependency of Bjorken Sum Rule, as predicted by our expressions 7.21, 7.22 and 7.23 is depicted in Fig. 7.9. Here our results are compared with different experimental data taken from COMPASS [71], HERMES[73], E143[75] and JLab experiments [76–78] and with the theoretical as well as phenomenological analysis, Ref. [176–179]. The results depicted in this figure are calculated using the value of $\Lambda = 0.300 GeV$. Here we have also estimated the uncertainty associated with the NNLO results due to the fitting parameter, b and the input point, and they are observed to be very small in this regard. It is also observed that the uncertainty decreases with decrease in Q^2 .

In Fig. 7.10, we have compared our results with theoretical pQCD predictions (7.19) for Bjorken integral up to NNLO. Here our results are calculated with $\Lambda = 0.300 GeV$. Within the estimated uncertainty our results show a very good consistency with those of pQCD predictions.

7.6 Summary

In the above sections we have presented some analytical expressions for the determination of the Gottfired sum rule, the Gross-Llewellyn Smith sum rule and the Bjorken sum rule. The expressions for sum rules are consisting of the solutions of the DGLAP evolution equations for F_2^{NS} , $xF_3(x, Q^2)$ and $xg_1^{NS}(x, Q^2)$ respectively, which are obtained in previous chapters, along with the input points $F_i^{NS}(x_0, t_0)$. Considering suitable input point as mentioned above, we have calculated the sum rule with pQCD corrections up to NNLO. We would like to emphasize that our results for various sum rules are in a overall good agreement with the corresponding experimental results as well as several strong theoretical, phenomenological predictions and also with the QCD predictions up to NNLO. These agreement, suggests that the Regge ansatz along with available data and QCD formalism allows to have a clean test of QCD predictions for various sum rule. \Box .

Chapter 8

Nuclear Effects in Non-Singlet Structure Functions and Sum Rules

In this chapter we present an analysis of the non-singlet structure functions and related sum rules taking into account the nuclear effects. In this regard, special attention is given to the nuclear shadowing effect as we are mostly concerning with the small-x region. The corrections due to nuclear shadowing effect, predicted in several earlier analysis are incorporated to our results of structure function and sum rules for free nucleon and calculate the nuclear structure functions as well as sum rules for nuclei. The calculations are analysed phenomenologically in comparison with available experimental data and achieved at a very good phenomenological success in this regard.

8.1 Introduction

Higher order pQCD corrections have a significant contribution towards the precise predictions of the structure functions as well as the sum rules. In the Chapters 4, 5, 6 and 7 we have discussed in detailed about the evolution of non-singlet structure functions and associated sum rules in accord with pQCD, along with their QCD corrections up to NNLO. In addition to pQCD corrections there are several non-perturbative effects such as higher twist effects, nuclear corrections, target mass corrections etc., to be included into the joint QCD analysis of structure functions and sum rules. In this chapter we present an analysis of the non-singlet structure functions and related sum rules taking into account the nuclear effects. Particular emphasise is given to the shadowing effect as we are mostly dealing with the small-x region.

We have already given a brief introduction about the nuclear effects in section 1.7. The fact that the structure functions of bound and free nucleons are not equal was discovered in a deep inelastic muon experiment carried out by the European Muon Collaboration at CERN in 1982[52]. Since then the nuclear effect has been actively investigated with ever more sophisticated and ingenious deep inelastic scattering experiment with charged lepton and neutrinos.

Available experimental information on nuclear structure functions are mainly from charged-lepton scattering DIS experiments performed at CERN [180–186], SLAC [187, 188], DESY [189], FNAL[190, 191] and recently at JLab [192, 193]. In addition, data from the DrellYan reaction of protons off nuclear targets are also available [194]. The experiments usually measures the ratio R_2 of the structure function F_2 of a complex nucleus to deuterium. The studies on the behaviour of the ratio R_2 as a function of x for a given fixed Q^2 reflects four distinct region of characteristic nuclear effects: shadowing region(x < 0.1), anti-shadowing region(0.1 < x < 0.3), EMC region (0.3 < x < 0.8) and fermi motion region(x > 0.8). In addition there are several theoretical treatments that predicts a Q^2 dependent nuclear effect only in the shadowing region, while for $0.1 < x < 0.8 R_2$ is almost Q^2 independent. However, the data available on the Q^2 dependence of nuclear effects are still scarce. In this thesis we have not take into account the Q^2 dependent nuclear corrections and considered only the x-dependency of nuclear effects for structure functions.

A quantitative understanding of the nuclear effects in deep inelastic nuclear scattering is important for a number of reasons. A proper interpretation of experimental data can provide valuable insights into the origin of nuclear force and helps us in understanding the possible modification of the properties of hadrons in a nuclear medium. Further, nuclear data provides the opportunity to have reliable information on the hadrons, otherwise not accessible directly. As for example, the extraction of the neutron structure function usually requires the deuterium and proton data, which in turn requires a proper understanding of nuclear effects[195]. Similarly, the use of charged-lepton and neutrino nuclear DIS data in global analysis of QCD observables aiming towards better determination of the proton and neutron pdfs and the higher twist terms [196–198] are the other examples in this regard.

The understanding of nuclear effects is particularly relevant for neutrino physics. For precision measurements in neutrino physics the use of heavy nuclear targets is required in order to collect a significant number of interactions. The presence of an axial-vector component in the weak current and the quark flavour selection distinguishes neutrinos from charged leptons and imply a more complex description of nuclear effects in neutrino scattering. The role of nuclear corrections to neutrino structure functions has been recently emphasized[199] after the NuTeV collaboration reported a deviation from the Standard Model prediction for the value of the weak mixing angle $(sin^2\Theta_W)$ measured in neutrino DIS [200]. It must be mention that nuclear effects are important not only in the determination of electroweak parameters, but also for the understanding of neutrino masses and mixing. The recent high-intensity NuMI[201] and JPARC [202] neutrino facilities offer the possibility to perform a detailed study of nuclear effects in neutrino interactions on a relatively short time scale. The construction of a future neutrino factories[203] are expecting to reach the ultimate precision of the neutrino probe.

8.2 Shadowing Effect in Nuclear Deep Inelastic Scattering

As this thesis is concerned with the small-x behaviour of the non-singlet structure functions and sum rules, we would like to concentrate only on the shadowing effect. Shadowing effect is the most pronounced nuclear effect in lepton nuclear DIS. Several theoretical models to this shadowing have been proposed. In literature there are essentially two main classes of approaches in order to have information about shadowing effect: one concerns with the origin of the shadowing effect and the other one addresses the evolution of shadowing effect by means of parameterizations.

Some models associated with the origin of shadowing effect provides a qualitative understanding using the fact that that in the rest frame of the nucleus the incoming photon splits into a $q\bar{q}$ pair long before reaching the nucleus, and this $q\bar{q}$ pair interacts with it with typical hadronic cross sections, which results in absorption [204–210]; in this way nuclear shadowing is a consequence of multiple scattering which in turn is related to diffraction [207, 211, 212]. On the other hand, in a frame in which the nucleus is moving fast, gluon recombination due to the overlap of the gluon clouds from different nucleons reduces gluon density in nucleus with mass number A by A times that in a free nucleon[213, 214]. These studies have received a great theoretical impulse with the development of semiclassical ideas in QCD and the appearance of non-linear equations for evolution in x in this framework(see [215–217] and references therein.).

Other models do not address the origin of the nuclear shadowing but contains a parametrization at Q_0^2 , which is obtained from a fit to experimental data. Distribution of partons inside nucleus are parameterized at some scale Q_0^2 and then evolved using the DGLAP[24] evolution equations. Nuclear effects are usually studied through a global χ^2 analysis method by using all the available charged-lepton DIS data, and then by adding Drell-Yan data to the data set [218]. In order to determine nuclear effects, various global analysis have been reported [219–224]. The analysis performed by Eskola et al. [225] and Hirai et al. [218, 224] are based on the leadingorder(LO) Dokshitzer-Gribov-Lipatov-Altarelli-Parisi(DGLAP) evolution, while the next-to-leading-order (NLO) evolution was performed by de Florian and Sassot[219]. In 1999, Eskola, Kolhinen, Ruuskanen and Salgado(EKRS)proposed a set of nuclear parton distributions by using the F_2^A/F_2^D data in deep inelastic lA collisions and the nuclear Drell-Yan dilepton cross sections measured in pA collisions and their results were observed to agree very well with the relevant EMC data and the E772 data at Fermilab [226] within the kinematical ranges $10^6 \leq x \leq 1$ and $2.25 GeV^2 \leq Q^2 \leq 10^4 GeV^2$. A reasonable explanation of the measured data of F_2 was provided by Hirai, Komano and Miyama(HKM)[224] based on two (quadratic and cubic) types of nuclear parton distributions whose parameters were determined by a χ^2 global fit to the available experimental data, except those from the protonnucleus Drell-Yan process. The covered kinematical ranges were $10^9 \le x \le 1$ and $1GeV^2 < Q^2 < 105GeV^2$ for deuteron and heavy nuclear targets. Further, in 2004, Hirai, Komano and Nagai(HKN)[218] re-analyzed the measured ratios of nuclear structure functions $F_2^A/F_2^{A'}$ and the ratios of Drell-Yan cross sections between different nuclei for obtaining another parton distribution function in nuclei.

In Ref. [219–222] the nuclear parton distribution have been determined, whereas Ref. [223] concentrated on the determination of nuclear structure functions using conventional nuclear models. The results from different models usually depend on additional semi-phenomenological assumptions and often contradict each other. Some recent parametrization are provided bellow as examples

• FS04[219]($Q_0^2 = 0.4 GeV^2$): $f_i^{N/A}(x) = \int \frac{dy}{y} W_i(y, A, Z) f_i^N(x/y)$

$$W_{i}(y, A, Z) = \begin{cases} A[a_{v}\delta(1 - \varepsilon_{v} - y) + (1 - a_{v})\delta(1 - \varepsilon_{v}' - y)] \\ +n_{v}(y/A)^{\alpha_{v}}(1 - y/A)^{\beta_{v}} + n_{s}(y/A)^{\alpha_{s}}(1 - y/A)^{\beta_{s}} \\ A\delta(1 - y) + \frac{a_{i}}{N_{i}}(\frac{y}{A})^{\alpha}_{i}(1 - \frac{y}{A})^{\beta_{i}} \quad (i = s, g) \end{cases}$$

- HKN07[220]($Q_0^2 = 1 GeV^2$): $f_i^A(x) = W_i(y, A, Z) \frac{1}{A} [Z f_a^p(x) + (A Z) f_a^n(x)]$ $W_i(y, A, Z) = 1 + (1 - \frac{1}{A^{\alpha}}) \frac{a_i + b_i x + c_i x^2 + d_i x^3}{(1 - x)^{\beta}}$
- SYKMOO08[221]($Q_0^2 = 1.69GeV^2$): $f_i^A(x) = W_i(y, A, Z) \frac{1}{A} [Z f_a^{p/A}(x) + (A Z) f_a^{n/A}(x)]$ $x f_i^{N/A}(x) = \begin{cases} A_0 x^{A_1} (1-x)^{A_2} e^{A_3 x} (1+e^{A_4} x)^{A_5} & (i=u_v, d_v, g, \bar{u}+\bar{d}, \bar{s}) \\ A_0 x^{A_1} (1-x)^{A_2} + (1+A_3 x) (1-x)^{A_4} & (i=\bar{d}/\bar{u}) \end{cases}$
- EPS09[222] $(Q_0^2 = 1.69 GeV^2)$: $f_i^A(x) = R_i^A(x) \frac{1}{A} [Zf_a^p(x) + (A Z) * f_a^n(x)],$ $a_0 + (a_1 + a_2 x) [e^{-x} - e^{-x_a}] \qquad (x \le x_a : shadowing)$ $R_i^A(x) = \begin{cases} b_0 + b_1 x + b_2 x^2 + b_3 x^3 \qquad (x_a \le x \le x_e : antishadowing) \\ c_0 + (c_1 - c_2 x)(1 - x)^{-\beta} \qquad (x_e \le x \le 1 : EMC \text{ and Fermi Motion}) \end{cases}$

Here f_i^A is the nuclear parton distribution function for the parton type *i* and f_i^p and f_i^n are the corresponding proton and neutron contribution. The parameters in these equations are determined by global χ^2 analyses of world experimental data on nuclear structure functions. Experimental data are generally obtained in different Q^2 points from Q_0^2 . The standard DGLAP evolution equations are used for evolving the distributions to the experimental points. There are three conditions to be satisfied for the NPDFs, so that three parameters should be fixed by the following relations [218,224]:

- Baryon number: $A \int dx [\frac{1}{3}u_v^A(x) + \frac{1}{3}d_v^A(x)] = A$
- Charge: $A \int dx [\frac{2}{3}u_v^A(x) \frac{1}{3}d_v^A(x)] = Z$
- Momentum: $A \sum_{i=q,\bar{q},q} \int dx x f_i^A(x) = A$

Like the charged-lepton DIS, the deep inelastic neutrino scattering is also a significant process for investigating the structures of hadrons and nuclei. In neutrino-DIS process, the structure functions $F_2(x, Q^2)$ and the parity-violating structure function $xF_3(x, Q^2)$ can simultaneously be measured. Big European Bubble Chamber Collaboration (BEBC) published the antineutrino-neon/deuterium DIS data in 1984, within the kinematic region of 0 < x < 0.7 and $0.25 < Q^2 < 26 GeV^2$ [229]. BEBC results for differential cross section ratio in the high Q^2 and 0.3 < x < 0.6 region[229] is compatible with the muon and electron scattering data from EMC and SLAC. In the same year, CERN-Dortmund-Heidelberg-Saclay Collaboration (CDHS) measured events originating in a tank of liquid hydrogen and in the iron of detector in the 400 GeV neutrino wide-band beam of the CERN Super Proton Synchrotron(SPS)[230]. In their measurements on total cross sections, differential cross sections and structure functions for hydrogen and iron, no significant difference between the structure functions for proton and iron was observed. E545 Collaboration at Fermilab [231], once more measured the cross sections in the deep inelastic neutrino scattering on neon or deuterium. However they were not able to give a definite conclusion due to substantial statistical uncertainties. In fact, many neutrino DIS experiments were carried out with their own primary physical goals, for instance the structure of proton, the mixing angles of electro-weak interaction etc., but none of them can individually confirm the EMC effect.

Although there is no individual neutrino experiment on EMC effect, the differential cross sections and structure functions have been measured in neutrino-nucleus experiments in CCFR[66, 232] and NuTeV[68] at Fermilab, and in CDHSW[69] and CHORUS[70] at CERN. These experimental data would help us to understand the nuclear effects in the neutrino-nucleus interaction further.

Along with the experimental efforts, several groups have been performed theoretical as well as phenomenological analysis of the nuclear effects in neutrino-nucleus DIS. Among them most prominent are the Kulagin and Petti(KP)[223,227], Qiu and Vitev(QV)[233] and Hirai, Komano and Nagai(HKN) groups, which have predicted the nuclear corrections in the low x region. Kulagin and Petti's approach is quite different from the above ones in the sense that they try to calculate the nuclear corrections in conventional nuclear models as far as they can, and then they try to attribute remaining factors to off-shell effects of bound nucleons for explaining the data.

8.3 Nuclear Shadowing Effect in the Non-singlet Structure Functions

In our previous chapters 4,5, and 6, we have calculated the non-singlet structure functions by means of solving DGLAP equations using two Regge ansatz as the initial input. Our calculations predicts the structure functions for a nucleon(single or free) as

$$F_{i}(x,t) = F_{i}(x_{0},t_{0}) \exp\left[\int_{t_{0}}^{t} \left(\frac{\alpha(t)}{2\pi}\right)_{NNLO} U(x_{0},t)dt + \int_{t_{0}}^{t} \left(\frac{\alpha(t)}{2\pi}\right)^{2}_{NNLO} V(x_{0},t)dt + \int_{t_{0}}^{t} \left(\frac{\alpha(t)}{2\pi}\right)^{3}_{NNLO} W(x_{0},t)dt \right] \left(\frac{x}{x_{0}}\right)^{(1-bt)}.$$
(8.1)

However in predicting the free nucleon structure functions, we need to consider the input point $F_i(x_0, t_0)$, a free nucleon structure function at $x = x_0$ and $t = t_0$. In our previous analysis, the points were taken from the available experimental data. It is known that the experimental data for nucleon structure functions are extracted from nuclear targets and hence they are with several nuclear effects. Thus the experimental input points, we considered in our previous analysis are nothing but nuclear structure function $F_i^A(x_0, t_0)$, which in turn leads to inaccuracy in predicting free nucleon structure function. Therefore accurate prediction of free nucleon structure function requires a nuclear effect free input point.

The experimental results are the structure functions for bound nucleon F_i^A which is related to the free nucleon structure function as

$$R(x,t) = \frac{F^{A}(x,t)}{F^{N}(x,t)}.$$
(8.2)

Here $F^A(x,t)$ represents the nucleon structure function per nucleon and $F^N(x,t)$, the free nucleon structure function. At $x = x_0$ and $t = t_0$, if we consider the value of the nuclear correction factor to be $R(x,t) = R_0$, the input point in (8.1) can be replaced with $F_i^N(x_0,t_0) = \frac{F_i^A(x_0,t_0)}{R_0}$ and provides

$$F_i^N(x,t) = \frac{F_i^A(x_0,t_0)}{R_0} \exp\left[\int_{t_0}^t \left(\frac{\alpha(t)}{2\pi}\right)_{NNLO} U(x_0,t)dt + \int_{t_0}^t \left(\frac{\alpha(t)}{2\pi}\right)_{NNLO}^2 V(x_0,t)dt + \int_{t_0}^t \left(\frac{\alpha(t)}{2\pi}\right)_{NNLO}^3 W(x_0,t)dt\right] \left(\frac{x}{x_0}\right)^{(1-bt)}.$$
(8.3)

Structure functions/	Referred analysis	References
F_2^{NS}	S. A. Kulagin and R. Petti	[227]
xF_3	S. A Kulagin and R. Petti	[227]
xg_1^{NS}	V. Guzey and M. Strikman	[228]

Table 8.1: Summary of referred analysis in our study of nuclear effects in variousstructure functions and sum rules.

Above expression is capable of predicting the free nucleon structure function through the experimental data $F_i^A(x_0, t_0)$ along with the correction factor R_0 .

Moreover, due to the unavailability of free nucleon structure function data, direct phenomenological analysis of (8.3) is not possible. In order to perform phenomenological analysis of our results with the experimental data either we need to remove nuclear effects from the data points or include the corresponding effects to our results of free nucleon. Here we have considered the later one, i.e., we have incorporated the nuclear correction factor R(x) with our calculations as

$$F_{i}^{A}(x,t) = R(x)F_{i}^{N}(x,t) = R(x)\frac{F_{i}^{A}(x_{0},t_{0})}{R_{0}}\exp\left[\int_{t_{0}}^{t}\left(\frac{\alpha(t)}{2\pi}\right)_{NNLO}U(x_{0},t)dt + \int_{t_{0}}^{t}\left(\frac{\alpha(t)}{2\pi}\right)^{2}_{NNLO}V(x_{0},t)dt + \int_{t_{0}}^{t}\left(\frac{\alpha(t)}{2\pi}\right)^{3}_{NNLO}W(x_{0},t)dt\right]\left(\frac{x}{x_{0}}\right)^{(1-bt)}, \quad (8.4)$$

in order to describe properly the experimental results.

The results for the nuclear correction factor R(x) predicted in different analysis for different structure functions, which are utilized in our analysis are summarised in Table8.1. Incorporating these nuclear effects we have calculated the nuclear structure functions. Here we have also considered the fact that the nuclear effects in the nonsinglet parts F_2^{NS} and g_1^{NS} are equivalent to the corresponding structure functions F_2 and $g_1[55]$.

8.3.1 Shadowing Effect in $F_2^{NS}(x, Q^2)$

In accord with (8.4), our expressions obtained in chapter 4 for F_2^{NS} nucleon structure functions predicts the corresponding nuclear structure functions as



Figure 8.1: Our NNLO results for F_2^{NS} structure function with and without nuclear effect, in comparison with the NMC measurement.

$$F_{2}^{NS(A)}(x,t) = R_{2}(x)F_{i}^{NS(N)}(x,t) = R_{2}(x)\frac{F_{2}^{NS(A)}(x_{0},t_{0})}{R_{0}} \times \exp\left[\int_{t_{0}}^{t} \left(\frac{\alpha(t)}{2\pi}\right)_{NNLO}^{U(x_{0},t)dt} + \int_{t_{0}}^{t} \left(\frac{\alpha(t)}{2\pi}\right)_{NNLO}^{2} V(x_{0},t)dt + \int_{t_{0}}^{t} \left(\frac{\alpha(t)}{2\pi}\right)_{NNLO}^{3} W(x_{0},t)dt\right] \left(\frac{x}{x_{0}}\right)^{(1-bt)}.$$
(8.5)

In this regard we have used the results for nuclear correction factor, R_2 from Ref.[218, 227, 233]. Incorporating the corrections to our calculations of F_2^{NS} structure function, we have obtained the nuclear structure function $F_2^{NS(A)}$ and depicted it in Fig. 8.1. Here we have shown only the modification of our NNLO results in comparison with NMC experimental data. We observe that our results for free nucleon structure functions, along with nuclear effect predicted by KP provides a well description of available experimental data for nuclear structure functions.



Figure 8.2: Our NNLO results for xF_3 structure function with and without nuclear effect, in comparison with the CCFR data.

8.3.2 Shadowing Effect in $xF_3^{NS}(x,Q^2)$

Our result (5.51) for xF_3 nucleon structure functions along with necessary corrections due to nuclear effect predicts the nuclear structure functions as

$$xF_{3}^{(A)}(x,t) = R_{3}(x)xF_{3}^{(N)}(x,t) = R_{3}(x)\frac{xF_{3}^{(A)}(x_{0},t_{0})}{R_{0}} \times \exp\left[\int_{t_{0}}^{t} \left(\frac{\alpha(t)}{2\pi}\right)_{NNLO}^{U(x_{0},t)dt} + \int_{t_{0}}^{t} \left(\frac{\alpha(t)}{2\pi}\right)^{2}_{NNLO}^{V(x_{0},t)dt} + \int_{t_{0}}^{t} \left(\frac{\alpha(t)}{2\pi}\right)^{3}_{NNLO}^{W(x_{0},t)dt} \left[\left(\frac{x}{x_{0}}\right)^{(1-bt)}\right].$$
(8.6)

In this case we have used the results for nuclear correction factor, R_3 from KP[227]. Incorporating the corresponding corrections to our calculations of xF_3 structure function, we have obtained the nuclear structure function $xF_3^{(A)}$ and depicted them in Fig. 8.2. Here we have shown only the modification of our NNLO results in comparison ---



Figure 8.3: Our NNLO results for xg_1^{NS} structure function with and without nuclear effect, in comparison with SMC, HERMES, COMPASS and E143 data.

with CCFR, NuTeV, CHORUS and CDHSW experimental data. We observe that our results for free nucleon structure functions, along with nuclear effect predicted by KP provides a well description of available experimental data for nuclear structure functions.

8.3.3 Shadowing Effect in $xg_1^{NS}(x, Q^2)$

Similarly using the results for R_g obtained in [228] we can obtain the spin dependent nonsinglet nuclear structure functions as

$$xg_{1}^{NS(A)}(x,t) = R_{g}(x)xg_{1}^{NS(N)}(x,t) = R_{g}(x)\frac{xg_{1}^{NS(A)}(x_{0},t_{0})}{R_{g}} \times \exp\left[\int_{t_{0}}^{t}\left(\frac{\alpha(t)}{2\pi}\right)_{NNLO}U(x_{0},t)dt + \int_{t_{0}}^{t}\left(\frac{\alpha(t)}{2\pi}\right)_{NNLO}^{2}V(x_{0},t)dt + \int_{t_{0}}^{t}\left(\frac{\alpha(t)}{2\pi}\right)_{NNLO}^{3}W(x_{0},t)dt\right]\left(\frac{x}{x_{0}}\right)^{(1-bt)}.$$

$$(8.7)$$

The results for (8.7) are depicted in Fig. 8.3. In this regard our results including nuclear effects are observed to be compatible with the experimetal data.

8.4 Shadowing Effect in the Sum Rules

Analogous to the structure functions, experimental determination of the DIS sum rules consists of considerable nuclear effects. As DIS sum rules are associated with the underlying symmetry as well as conservation laws of interactions, they provide strong normalization constraints on the structure functions. Therefore the sum rules are expected to provide an important bridge between different nuclear effects.

In this section we briefly discuss the nuclear effects in DIS sum rules, specifically in the GSR, GLSSR and BSR based on several earlier analysis. We then incorporate possible nuclear corrections to our results of sum rules, obtained in the previous chapter and perform phenomenological analysis in comparison with the experimental measurements.

8.4.1 Shadowing Correction to Gottfried Sum Rule

In the NMC experiment, due to the unavailability of fixed target for neutron, deuteron is usually used for measuring neutron structure function and combining the relations

$$F_2^p - F_2^n = 2F_2^d \frac{1 - F_2^n / F_2^p}{1 + F_2^n / F_2^p},$$
(8.8)

$$\frac{F_2^n}{F_2^p} = 2\frac{F_2^d}{F_2^p} - 1 \tag{8.9}$$

and

$$F_2^d = F_2^p + F_2^n \tag{8.10}$$

together with world averaged deuteron structure functions, the difference $F_2^p - F_2^n$ is calculated and these calculations are used in determining the GSR. The results can be compared with the GSR only if there is no nuclear modification in the deuteron. However it is well known that nuclear structure functions are modified and the major contribution to the modification comes from the small-x region i.e., shadowing region. Nuclear corrections in the deuteron to the GSR, in particular the shadowing effect, were

calculated in various models [234–240]. So far, VMD, Pomeron and meson-exchange mechanisms have been studied. In Ref. [240], using VMD model, including ρ , ω and ϕ as the vector mesons, estimated the shadowing correction $\delta S_{GSR} = -0.039to - 0.017$ to the GSR $S_{GSR} = S_{GSR}^{NMC} + \delta S_{GSR}$. There are other studies in the Pomeron and meson exchange models. Historically, the first estimate of shadowing contribution to S_{GSR} is discussed by the Pomeron exchange model [234, 235]. A possible way of describing the high-energy scattering in the diffractive region is in terms of Pomeron exchange. The virtual photon transforms into a $q\bar{q}$ pair which then interacts with the deuteron. In the diffractive case, the target is remain intact and only vacuum quantum number, namely the Pomeron, could be exchanged between the $q\bar{q}$ pair and the nucleons. In the earlier works, the shadowing correction in this model was rather large $\delta S_{GSR} \approx -0.08$ [235, 238]. However, the Pomeron contribution is reduced if more realistic deuteron wave functions are used according to Ref. [239]. Next, meson-exchange corrections were investigated in Refs. [236,239]. The studied mesons are π , ω and σ in Ref. [236], and ρ is also included in Ref.[239]. If the corrections due to the π , ω and σ mesons were taken into account, the NMC result became $S_{GSR} = 0.29 \pm 0.03$ [236]. Therefore, meson-exchange contributions reduce the discrepancy between the NMC data and the Gottfried sum rule.

As far Figs.7.2, 7.3 and 7.4 are concerned, it is observed that our results do not agree well with the available experimental data of NMC. Again as the nuclear effects predicted by the available analysis(discussed above), are observed to be large and inclusion of these effects to our results will deviate from the experimental data farther, hence we have not included the nuclear corrections to our results of GSR.

8.4.2 Shadowing Correction to GLS Sum Rule

Experimental measurements of GLS sum rule was performed by CCFR and the results were extracted from Fe target. In order to compare our results for GLS sum rule obtained in chapter7, we refer the nuclear corrections estimated in [223, 241]. The detailed investigation on the nuclear corrections to GLS sum rule was performed in Ref. [223]. They explicitly separated the nuclear corrections to the GLS integral as $S_{GLS}^A = S_{GLS}^N + \delta S_{GLS}$, where S_{GLS}^N refers to the GLS integral for nucleon. In accord with their predictions, the nuclear corrections to the GLS sum rule cancel out as $x \to 0$ in the leading order, which is due to the baryon charge conservation. They have also calculated the GLS integral S_{GLS} for different nuclear targets. In Ref. [223, 241], they obtained the corrections for iron and deuteron nuclei as $\frac{\delta S_{GLS}^{Fe}}{3} = -\frac{4.0 \times 10^{-3}}{Q^2}$ and $\frac{\delta S_{GLS}^D}{3} = -\frac{6.3 \times 10^{-4}}{Q^2}$ respectively. In Ref. [223] they have nicely presented their result in Fig. 10. From Fig. 10 we observe that the nuclear correction δ_{GLS} decreases progressively by increasing Q^2 .

The GLS sum rule for nuclei can be expressed as

$$S_{GLS}^{A}(x_{min}, Q^{2})\Big|_{NNLO} = S_{GLS}^{N}(Q^{2}) + \delta S_{GLS}, \qquad (8.11)$$

where the first term on the right hand side of above equation represents the GLS sum rule for free nucleon and the second term for the nuclear correction. Using the NNLO pQCD corrected expression 7.17, obtained in chapter 7 as $S_{GLS}^N(Q^2)$ we get

$$S_{GLS}^{A}(x_{min}, Q^{2})\Big|_{NNLO} = S_{GLS}(Q^{2})\Big|_{NNLO} - \int_{0}^{x_{min}} \frac{dx}{x} \left[F_{3}^{NS}(x_{0}, t_{0}) \left(\frac{x}{x_{0}}\right)^{(1-bt)} \exp\left\{ \int_{t_{0}}^{t} \left(\frac{\alpha(t)}{2\pi}\right)_{NNLO}^{t} \right. \\ \left. \left. \left. P(x_{0}, t)dt + \int_{t_{0}}^{t} \left(\frac{\alpha(t)}{2\pi}\right)^{2}_{NNLO} Q(x_{0}, t)dt \right. \\ \left. + \int_{t_{0}}^{t} \left(\frac{\alpha(t)}{2\pi}\right)^{3}_{NNLO} R(x_{0}, t)dt \right\} \right] + \delta S_{GLS}. (8.12)$$

Now incorporating the KP[223,241] prediction $\frac{\delta S_{GLS}^{Fe}}{3} = -\frac{4.0 \times 10^{-3}}{Q^2}$, for the nuclear correction term, we have calculated S_{GLS}^A and depicted the results in Fig. 8.4, in comparison with CCFR measurements of xF_3 structure function with Fe as the target. In addition, we have plotted our NNLO results and the results of KS[179] prediction. From the figure we see that the our NNLO expression for GLSSR along with necessary nuclear correction has the capability of describing the experimental data of GLSSR for nuclei.

8.4.3 Shadowing Correction to Bjorken Sum Rule

Aiming at measuring the polarized structure functions of protons and neutrons and in order to test the Bjorken sum rule several experiment have been performed. The measurement of $g_1^n(x)$ involves necessarily nuclear targets and several experiments have been performed using ²H and ³He targets. However ³He target has advantage



Chapter 8

Figure 8.4: Our NNLO results for Gross-Llewelln Smith sum rule with and without nuclear effect, in comparison with those of CCFR measurements. (Q^2) 's are taken in the unit of GeV^2).



Figure 8.5: Our NNLO results for Bjorken sum rule with and without nuclear effect, in comparison with several experimental data as indicated in the figure. (Q^2) 's are taken in the unit of GeV^2).

over ²*H* target. On the other hand the use of heavy nucleus as target yields nuclear effects. Nuclear effects for the Bjorken sum rule were first discussed in Ref. [242] and Ref. [243]. In particular it was pointed out in Ref. [243] that convolution models and three nucleon description of A = 3 system lead to results for $g_{1A=3}$ inconsistent with the Bjorken sum rule. This observation was left unnoticed in Ref. [244, 245] and in all analyses of the experimental data. In Ref.[55], the ratio of the Bjorken sum rule for A = 3 to A = 1 within impulse approximation was found to be

$$R = \frac{\int_0^1 dx \left[g_1^{He}(x, Q^2) - g_1^H(x, Q^2) \right]}{\int_0^1 dx \left[g_1^n(x, Q^2) - g_1^p(x, Q^2) \right]} = \frac{G_A^{3H}}{G_A(n)},$$
(8.13)

where G_A is the axial coupling constant for β decay of the nucleus A. Combining the most recent experimental data on $\frac{G_A^{3H}}{G_A(n)}$ for tritium β -decay [246] it was found to be $\frac{G_A^{3H}}{G_A(n)} = 0.9634 \pm 0.003$. However, for the case of ⁷Li the ratio was obtained to be 0.73. Further in Ref. [55], this value was estimated to be $\frac{G_A^{3H}}{G_A(n)} = 0.922 \pm 0.006$.

Using the values of $\frac{G_A^{3H}}{G_A(n)}$ obtained in Ref. [55, 246], we can calculate the BSR integral $S_{BSR}^A = \int_0^1 dx \left[g_1^{He}(x, Q^2) - g_1^H(x, Q^2) \right]$ using our NNLO results for $S_{BSR} = \int_0^1 dx \left[g_1^n(x, Q^2) - g_1^p(x, Q^2) \right]$, obtained in chapter 7 as

$$S_{BSR}^{A} = \frac{G_{A}^{^{3}H}}{G_{A}(n)} \left[S_{BSR}(Q^{2}) \Big|_{NNLO} - \int_{0}^{x_{min}} \frac{dx}{x} \left[g^{NS}(x_{0},t_{0}) \left(\frac{x}{x_{0}} \right)^{(1-bt)} \times \right] \right] \\ \exp \left\{ \int_{t_{0}}^{t} \left(\frac{\alpha(t)}{2\pi} \right)_{NLO} P(x_{0},t) dt + \int_{t_{0}}^{t} \left(\frac{\alpha(t)}{2\pi} \right)_{NLO}^{2} Q(x_{0},t) dt \right\} \\ + \int_{t_{0}}^{t} \left(\frac{\alpha(t)}{2\pi} \right)_{NNLO}^{3} R(x_{0},t) dt \right\} \right] \left]. \quad (8.14)$$

In Fig. 8.5, we have shown the results for BSR in accord with Eq.(8.14) along with other experimental. Nuclear correction incorporated results are observed to be consistent with other measurements.

8.5 Summary

In this chapter we present an analysis of the non-singlet structure functions and related sum rules taking into account the nuclear effects. In this regard, special attention is given to the nuclear shadowing effect as we are mostly concerning with the small-x region. Incorporating the results of corrections due to shadowing nuclear effect obtained in several earlier analysis for different structure functions as well as sum rules to our results of the structure functions and sum rules for free nucleon, we obtain structure functions and sum rules for nuclei. Nuclear correction incorporated results are studied phenomenologically and it is observed that along with the nuclear correction, our NNLO results of the non-singlet structure functions and sum rules have the capability of providing well description of their respective experimental data collected using nuclear target. capable of describing well the obtained in previous chapters analysis in comparison with the available data and parametrization. $\Box\Box$.

Chapter 9

Higher Twist Effects in Non-Singlet Structure Functions and Sum Rules

In this chapter the higher twist corrections to the non-singlet structure functions and sum rules associated with them are studied. Here, possible improvement in the accuracy of our results for the non-singlet structure functions and sum rules due to the inclusion of relevant higher twist terms is investigated. Based on a simple model we have extracted the higher twist contributions to the non-singlet structure functions and sum rules in NNLO perturbative orders and then incorporated them with our results. Our NNLO results along with higher twist corrections are observed to be compatible with experimental data.

9.1 Introduction

The behaviour of the deep inelastic structure functions can be analyzed with the perturbative QCD. A method used for this analysis is the operator product expansion method(OPE)[51]. The OPE is successful in describing the contributions from different quark-gluon operators to hadronic tensor and helps in ordering them according to their twist. In accord with OPE, the DIS structure functions and sum rules consist of two parts, the leading twist(LT) and the higher twist(HT) contributions:

$$F_i(x,Q^2) = F_i^{LT}(x,Q^2) + \frac{H_i(x,Q^2)}{Q^2},$$
(9.1)

where i labels the type of the structure function $(F_i = F_2, F_3, g_1)$. The leading twist

term is associated with the single particle properties of quarks and gluons inside the nucleon and is responsible for the scaling of DIS structure function via perturbative QCD $\alpha_s(Q^2)$ corrections. The higher twist terms reflect instead the strength of multiparton interactions (qq and qg). Since such interactions spoil factorization one has to consider their impact on the parton distribution functions extracted in the analysis of low- Q^2 data. Because of the non-perturbative origin it is difficult to quantify the magnitude and shape of the higher twist terms from first principles and current models can only provide a qualitative description for such contributions, which must then be determined phenomenologically from data.

The higher twist terms are governed by the terms contributing at different orders of $1/Q^2$:

$$\frac{H_i(x,Q^2)}{Q^2} = \frac{h_1(x)}{Q^2} + \frac{h_2(x)}{Q^4} + \dots,$$
(9.2)

the leading term in this expansion is known as twist-two, the sub-leading ones twistthree, etcetera. The higher twist terms are suppressed by terms of order $1/Q^2$, $1/Q^4$..., respectively.

The currently available experimental data on deep inelastic structure functions covers a large kinematical regime with high precision measurements. This provides an interesting challenge for theoretical physics when it comes to describing this data in the low- Q^2 domain. pQCD predictions, even with higher order corrections up to NNLO and NNNLO observed to be not sufficient for a precise description of deep inelastic structure function data, which in turn reveals that the discrepancy among data and pQCD predictions are not primarily the sub-leading terms in powers of α_s , but corrections which are proportional to the reciprocal value of the photon virtuality Q^2 , viz. higher-twist terms.

The extraction of higher twist terms from the data is a longstanding problem, as recognized from the very first developments of a pQCD phenomenology [247,248]. Existing information about higher twist terms in lepton-nucleon structure functions is scarce and somewhat controversial. Early analysis [249,250] suggested a significant HT contribution to the longitudinal structure function F_L . The subsequent studies with both charged leptons [251–253] and neutrinos [140] raised the question of a possible dependence on order of QCD calculation used for the leading twist. The common wisdom is generally that HTs only affect the region of $Q^2 \sim 1 - 3GeV^2$ and can be neglected in the extraction of the leading twist.

The higher twist terms are presently poorly known and currently is a subject of both theoretical and phenomenological studies. A better understanding of HT terms, in particular their role in describing low Q^2 and high x DIS data is important and provides valuable information on quark gluon correlations inside the nucleon. The importance of highertwist (HT) contribution to structure functions was pointed from the very beginning of QCD comparison with experimental data^[247] on structure functions. Despite a fast progress in theoretical QCD calculations of power corrections to non-singlet structure functions and sum rules [254, 255] (for reviews and references see [256]), the shape of HT (order $1/Q^2$) contributions is measured only for F_2 SF [257] and is still only estimated for xF_3 [258]. Several reports are available on the determination of the higher twist contributions in the deeply-inelastic structure functions $F_2^{ep,ed}(x, Q^2)$, (see [259] for details). Higher twist contributions were also studied in deep-inelastic neutrino scattering in Ref. [260–263]. Also in the case of polarized deeply inelastic scattering higher twist corrections are present in general. Since the polarized structure functions are measured through an asymmetry, the effect of higher twist contributions in the denominator function has to be known in detail. In [264] no significant higher twist contributions were found. Other authors claim contributions in the low x region [265], which is also the region of very low values of Q^2 .

The non-singlet structure functions as well as associated sum rules obtained in the previous chapters in this thesis by means of incorporating the ansatz $F_i = Ax^{1-bt}$ as the initial input to DGLAP equation are the results of pQCD effect with higher order corrections up to NNLO. Although our results are capable of describing the available experimental data with considerable phenomenological success, in the following sections we report on better description of the data by our results along with higher twist corrections. We have incorporated relevant higher twist terms, proposed in different theoretical as well as phenomenological analysis to our results and analysed their effect on possible improvement in accuracy of our results in describing available experimental data.

The usual approach in analyses whose main aim is the extraction of leading twist PDFs is either to parametrize the higher twist contributions by a phenomenological form and fit the parameters to the experimental data[197,266], or to extract the Q^2 dependence by fitting it in individual bins in x [267–271]. Such an approach effectively includes contributions from multiparton correlations (the true higher twist contributions) along with other power corrections that are not yet part of the theoretical

treatment of DIS at low Q^2 . These include $O(1/Q^2)$ contributions such as jet mass corrections [272] and soft gluon resummation [273], as well as contributions which are of higher order in α_s but whose logarithmic Q^2 behavior mimics terms $\propto \frac{1}{Q^2}$ at low virtuality[271, 274].

9.2 Higher Twist in Non-Singlet Structure Functions

In order to estimate the higher twist contribution to the non-singlet structure functions, we have performed an analysis based on a simple model. Here the first higher twist term is extracted and to do so we have parameterised the non-singlet structure functions as

$$F_i^{data}(x_i, Q^2) = F_i^{LT}(x_i, Q^2) + \frac{h_1(x_i)}{Q^2}.$$
(9.3)

Here leading twist(LT) term corresponds to the pQCD contribution to structure functions and the constants $h_1(x_i)$ (one per x - bin) parameterize the x dependence of higher twist contributions. For the leading twist term, we have utilised the results for the non-singlet structure functions obtained in our previous chapters. Incorporating our results for non-singlet structure functions in NNLO as the LT terms we have extracted the difference, $F_i^{data}(x_i, Q^2) - F_i^{LT}(x_i, Q^2)$ from their corresponding experimental data and then fitted with $h_1(x_i)/Q^2$. From the best fitting values, we have determined the higher twist contribution terms h_i per x-bin. In this analysis we have performed our fitting analysis within the kinematical region $0.0125 \le x \le 0.5$ and $1 \le Q^2 \le 20 GeV^2$. In this analysis we have extracted the higher twist contribution to the F_2^{NS} and xF_3 structure functions only. Due to unavailability of g_1^{NS} data at different Q^2 we could not include the g_1^{NS} structure function. The higher twist effects in F_2^{NS} and xF_3 are presented in the subsection 9.2.1 and 9.2.2 respectively bellow.

9.2.1 Higher Twist Effect in F_2^{NS} Structure Function

As discussed above, the simple parametrization

$$F_2^{data}(x_i, Q^2) = (F_2^{NS})^{LT}(x_i, Q^2) + \frac{h_1(x_i)}{Q^2}, \qquad (9.4)$$



Figure 9.1: Higher twist corrections to F_2^{NS} structure function at NNLO. (Q^2 's are taken in the unit of GeV^2).

for the F_2^{NS} structure function incorporating higher twist contributions in terms of the parameter $h_1(x_i)$ is fitted to the NMC data for the *x*-bins $x_i = 0.0125, 0.0175, 0.025, 0.035$. Here we have used the NNLO results (4.51) for the term $(F_2^{NS})^{LT}(x_i, Q^2)$. Best fitted values of h_1 at different values of x for the F_2^{NS} structure functions are presented in Table 9.1 and Fig. 9.1 along with the $\frac{\chi^2}{d.o.f.}$ value.

x_i	h_1^{NNLO}
0.0125	-0.00397 ± 0.0025
0.0175	-0.00283 ± 0.0029
0.025	-0.0045 ± 0.0026
0.035	-0.0022 ± 0.0052
$\frac{\chi^2}{d.o.f.}$	0.85

Table 9.1: Higher Twist corrections to F_2^{NS} structure functions at NNLO.

In Fig. 9.1 we have presented the best fitting results of (9.4) for F_2^{NS} in comparison with NMC experimental data. Here both the NNLO results, with HT and without HT are shown. Significant higher twist contribution to F_2^{NS} structure function is observed in the low-*x*, low- Q^2 region. We observe that our expressions along with the HT corrections provide better description of NMC data than without HT within our kinematical region of consideration.

9.2.2 Higher Twist Effect in xF_3^{NS} Structure Function

In a similar way, the parametrization

$$xF_3^{data}(x_i, Q^2) = xF_2^{LT}(x_i, Q^2) + \frac{h_1(x_i)}{Q^2},$$
(9.5)

is used for the xF_3^{NS} structure function with higher twist contributions in terms of the parameter $h_1(x_i)$. Incorporating the NNLO result (5.51) as the LT term, we have fitted the parametrization with 9.5 with the CCFR, NuTeV, CHORUS and CDHSW data for the x-bins $x_i = 0.0125, 0.015, 0.0175, 0.025, 0.035, 0.045$. Best fitted values of h_1 at different values of x for the xF_3^{NS} structure functions are presented in Table 9.2 and Fig. 9.2 along with the $\frac{\chi^2}{d.o.f.}$ value.

x_i	h_1^{NNLO}
0.0125	0.064 ± 0.0258
0.015	0.00504 ± 0.00804
0.0175	0.0189 ± 0.034
0.025	0.00797 ± 0.0368
0.035	-0.0118 ± 0.0295
0.045	-0.0429 ± 0.0306
$\frac{\chi^2}{d.o.f.}$	1.03

Table 9.2: Higher Twist corrections to xF_3 structure functions at NNLO.

In Fig. 9.2 we have presented the best fitting results of (9.5) for xF_3^{NS} in comparison with CCFR experimental data. Here both the NNLO results, with HT and without HT are shown. Significant higher twist contribution to xF_3^{NS} structure function is observed in the low-x, low- Q^2 region. We observe that our expressions along with the HT corrections provide better description of CCFR data than without HT within our kinematical region of consideration.

9.3 Higher Twist Effect in Sum Rules

In the previous section, the higher twist effects in non-singlet structure functions are estimated by means of a simple model. We now extend the similar formalism in



Figure 9.2: Higher twist corrections to xF_3^{NS} structure function at NNLO. (Q^2 's are taken in the unit of GeV^2).

order to extract the higher twist contribution to the sum rules associated with the non-singlet structure functions. Here we have parameterized the sum rules as

$$S_i(Q^2) = S_i(Q^2) \Big|_{LT} + \frac{\mu_4}{Q^2},$$
(9.6)

where leading twist(LT) term corresponds to the pQCD contribution to the respective sum rules and μ_4 signifies the contribution from first higher twist term. Our results for the sum rules, obtained in chapter 7 can be utilised as the LT term and then by means of fitting the model (9.6) with the low Q^2 ($0.5 \leq Q^2 \leq 5 GeV^2$) experimental data taken from their respective experiments we can estimate the respective higher twist terms. In the following subsections we have presented the results of higher twist effects for Gross-Llewellyn Smith sum rule(GLSSR) and Bjorken sum rule(BSR). Due to unavailability sufficient experimental data, we could not include the Gottfried sum rule in this chapter.



Figure 9.3: Hgher Twist corrections to GLS sum rule at NNLO. (Q^2 's are taken in the unit of GeV^2).

9.3.1 Higher Twist Effect in Gross-Llewellyn Smith Sum Rule

The Gross-Llewellyn Smith sum rule (GLSSR), with the higher twist term, $\frac{\mu_4}{Q^2}$ is given by

$$S_{GLS}^{data}(Q^2) = S_{GLS}^{pQCD}(Q^2) \bigg|_{LT} + \frac{\mu_4}{Q^2}.$$
(9.7)

Incorporating the results in accord with our NNLO prediction, (7.17) in (9.7), we have fitted the the expression with the available CCFR experimental data for GLSSR. The corresponding value of μ_4 for which best fitting is obtained in NNLO are summarised in Table 9.3 and depicted in Fig.9.3, along with the respective $\frac{\chi^2}{d.o.f}$ values.

	NNLO
μ_4	0.1840 ± 0.0842
$\frac{\chi^2}{d.o.f.}$	0.56

 Table 9.3: Higher Twist corrections to GLS sum rule at NNLO.

In Fig. 9.3 we have presented the best fitting results of (9.7) for GLSSR in NNLO in comparison with CCFR experimental data. Our pQCD corrected results up to



Figure 9.4: Hgher Twist corrections to BSR at NNLO. (Q^2 's are taken in the unit of GeV^2).

NNLO are also included in this figure along with HT corrected results. We observe that our expressions along with the HT corrections provide better description of CCFR measurement of GLS sum rule.

9.3.2 Higher Twist Effect in Bjorken Sum Rule

The Bjorkan sum rule(BSR), with the higher twist term, $\frac{\mu_4}{Q^2}$ is given by

$$S_{BSR}^{data}(Q^2) = S_{BSR}^{pQCD}(Q^2) \bigg|_{LT} + \frac{\mu_4}{Q^2}.$$
(9.8)

Incorporating our Q^2 dependent expressions (7.23) for BSR in NNLO as the LT term, we have fitted above parametrisation to the low Q^2 ($0.5 \le Q^2 \le 5 GeV^2$) experimental data taken from COMPASS [71], HERMES[73], E143[75] and JLab experiments [76– 78]. The corresponding value of μ_4 for which best fitting is obtained in NNLO is summarised in Table 9.4, along with the $\frac{\chi^2}{d.o.f.}$ value. In Fig. 9.4, we have presented the best fitting results in comparison with other experimental data... Here both the results, with HT and without HT are shown. We observe that our expressions along with the HT corrections provide well description of BSR data.

	NNLO
μ_4	-0.007 ± 0.0024
$\frac{\chi^2}{d.o.f.}$	1.3

Table 9.4: Higher Twist corrections to BSR at NNLO.

9.4 Summary

In this chapter we have extracted the higher twist contribution to F_2^{NS} and xF_3 structure functions and to the GLSSR and BSR using a simple model. We then incorporated the higher twist contributions to our NNLO results for all of F_2^{NS} , xF_3 , GLSSR and BSR. We observe that our NNLO expressions for these structure functions and sum rules along with the higher twist corrections provide well description of their respective experimental data. $\Box\Box$

Chapter 10

Conclusion

This thesis concerns with the determination of both the spin independent and spin dependent non-singlet structure functions and sum rules associated with them. We have employed a unified approach incorporating Regge theory and the theoretical framework of perturbative Quantum Chromodynamics. Incorporating two Regge ansatz, one with constant intercept and Q^2 dependent coefficient and the other one with Q^2 dependent intercept and constant coefficient as the initial input, we have solved the Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) evolution equation for the non-singlet structure functions $F_2^{NS}(x,Q^2)$, $xF_3(x,Q^2)$ and $xg_1^{NS}(x,Q^2)$. Here we explicitly specify how the usefulness of two Q^2 dependent Regge ansatz, utilized as the required initial input to the DGLAP evolution helps in obtaining the small-x behaviour of the non-singlet structure functions. Obtained small-x behaviour of these non-singlet structure functions are then utilized to calculate the sum rules – Gottfried sum rule(GSR), Gross-Llewellyn Smith sum rule(GLSSR) and Bjorken sum rule (BSR), which are associated with $F_2^{NS}(x,Q^2)$, $xF_3(x,Q^2)$ and $xg_1^{NS}(x,Q^2)$ respectively. In addition to the prediction of structure functions and sum rules we have paid attention to their precision. Precise prediction of structure functions demand to incorporate the standard higher order approximation of pQCD and several nonperturbative effects. In this regard particular emphasis is given to the determination of structure functions and sum rules with pQCD corrections up to next-next-to-leading order (NNLO) and to the inclusion of the special non-perturbative effects, the nuclear effect and higher twist effect.

The non-singlet structure functions evolved in accord with the DGLAP evolution equations are studied phenomenologically and the analysis is presented in the chapters 4, 5 and 6 for $F_2^{NS}(x, Q^2)$, $xF_3(x, Q^2)$ and $xg_1^{NS}(x, Q^2)$ respectively in comparison with several experimental as well as parametrization results. We observe a very good consistency between our calculation and other experimental as well as parametrization results within the kinematical region x < 0.05 and $Q^2 = 20 GeV^2$ of our consideration, especially, if the NNLO results are concerned. The phenomenological success achieved in this study suggests that the two simple QCD featured Regge behaved ansatz are capable of evolving non-singlet structure functions in accord with DGLAP equations at small-x. However from the comparative picture between the most consistent results, the NNLO results for both the inputs along with other experimental and parametrization results we do not observe any significant differences among them within our region of consideration. We hope future experimental measurements at extremely small values of Bjorken x will clarify their differences and help us in better understanding of the structure of nucleon.

As the small-x behaviour of the non-singlet structure functions $F_2^{NS}(x, Q^2)$, $xF_3(x, Q^2)$ and $xg_1^{NS}(x, Q^2)$ are well explicable through our analytic expressions obtained by means of solving DGLAP evolution equations, we have employed them in the determination of the corresponding sum rules GSR, GLSSR and BSR through a simple but efficient technique, discussed in chapter 7. As we do not observe any significant differences among the results of the two ansatz, therefore in the determination of sum rules we have utilised the results of the ansatz with Q^2 dependent intercept only. The phenomenological analysis of our results in comparison with other experimental, theoretical as well as phenomenological results suggest that our calculations are compatible with other strong measurements.

We also consider the contribution of nuclear shadowing effect to the non-singlet structure functions and sum rules. Incorporating the corrections due to shadowing nuclear effect, proposed in different theoretical as well as phenomenological analysis to our results of the structure functions and sum rules for free nucleon, we obtain nuclear structure functions and sum rules and perform phenomenological analysis in comparison with available data and parametrization. The nuclear structure functions thus obtained are observed to be consistent with other experimental measurements.

Further we have extracted the higher twist effects in the non-singlet structure functions and sum rules based on a simple model, which is discussed in chapter 9. The phenomenological analysis of our results for structure functions and sum rules, along with considerable higher twist correction provide a very good description of their respective experimental data and parameterizations. From the phenomenological analysis discussed above we have the following observations:

i. The Regge inspired ansatz in accord with DGLAP equations provides a very good description of the small-x behaviour of non-singlet structure functions, which are consistent with other results taken from different experiments and parameterizations.

ii. Inclusion of nuclear effect and higher twist corrections lead to a better description of the experimental results for various non-singlet structure functions.

iii. Our results for the sum rules associated with the non-singlet structure functions are observed to be compatible with their respective available data as well as parameterizations.

iv. Our expressions for GLS sum rule and Bjorken sum rule, along with considerable higher twist correction provide a very good description of the experimental measurements which indicates that the experimental data strongly confirm the QCD predictions for different sum rules.

v. Our results for Q^2 behaviour of different sum rules are also consistent with the QCD predictions up to NNLO. This consistency between our results and theoretical QCD predictions suggests that available data, the Regge ansatz and the theoretical framework of pQCD, through this simple method allow us to have a clean test of pQCD predictions on the respective sum rules.

vi. The consistency of the results for the non-singlet structure functions and sum rules due to the Regge like model, $xg_1^{NS}(x,t) = Ax^{1-bt}$ with different experimental results and other strong analysis signifies that the model is applicable in describing the small-x behaviour of structure function although it being simple. Moreover, in this method we do not require the knowledge of initial distributions of structure functions at all values of x from 0 to 1. Here, we just require one input point at any fixed x and Q^2 and with respect to that point both the x and Q^2 evolution of structure functions can be obtained.

Our concluding impression based on all these observations is that the simple but efficient Q^2 dependent Regge ansatz for non-singlet structure functions is capable of evolving successfully the structure functions in accord with DGLAP equation at small-x and the Regge ansatz and the theoretical framework of pQCD, along with available experimental data lead towards a clean test of pQCD predictions of Sum Rules. $\Box\Box$

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Appendices

A. Unpolarised Non-singlet Splitting Function in Leading Order(LO)

The explicit form of the unpolarised non-singlet splitting function in LO is

$$P^{(0)}(x) = C_F \left[\frac{1+x^2}{(1-x)_+} + \frac{3}{2}\delta(1-x) \right],$$
(10.1)

where the + sign is defined by

$$\int_0^1 dx \frac{f(x)}{(1-x)_+} = \int_0^1 \frac{dx}{1-x} [f(x) - f(1)] + f(1)\ln(1-x) \quad (10.2)$$

B. Unpolarised Non-singlet Splitting Function in Next-to-Leading Order(NLO)

The unpolarised non-singlet splitting function in NLO is given by

$$P^{(1)}(x) = C_F^2 \left[P_F(x) - P_A(x) + \delta(1-x) \left\{ \frac{3}{8} - \frac{1}{2}\pi^2 + \zeta(3) - 8\tilde{S}(\infty) \right\} \right] + \frac{1}{2} C_F C_A \left[P_G(x) + P_A(x) + \delta(1-x) \left\{ \frac{17}{12} + \frac{11}{9}\pi^2 - \zeta(3) + 8\tilde{S}(\infty) \right\} \right] + C_F T_R N_F \left[P_{N_F}(x) - \delta(1-x) \left\{ \frac{1}{6} + \frac{2}{9}\pi^2 \right\} \right] 0.3)$$

where,

$$P_F(x) = -2\frac{1+x^2}{1-x}\ln x\ln(1-x) - \left(\frac{3}{1-x} + 2x\right)\ln x - \frac{1}{2}(1+x)\ln^2 x - 5(1-x)0.4$$

$$P_G(x) = \frac{1+x^2}{(1-x)_+} \left[\ln^2 x + \frac{11}{3} \ln x + \frac{67}{9} - \frac{1}{3}\pi^2 \right] + 2(1+x) \ln x + \frac{40}{3}(1-x), (10.5)$$

$$P_{N_F}(x) = \frac{2}{3} \left[\frac{1+x^2}{(1-x)_+} \left(-\ln x - \frac{5}{3} \right) - 2(1-x) \right], \tag{10.6}$$

$$P_A(x) = 2\frac{1+x^2}{1+x} \int_{x/(1+x)}^{1/(1+x)} \frac{dz}{z} \ln \frac{1-z}{z} + 2(1+x)\ln x + 4(1-x)(10.7)$$

C. Unpolarised Non-singlet Splitting Function in Next-to-Next-to-Leading Order(NNLO)

Unpolarised non-singlet splitting function in NNLO has the form,

$$P^{(2)}(x) = N_F \bigg[-183.187D_0 - 173.927\delta(1-x) - \frac{5120}{81}L_1 - 197.0 +381.1x + 72.94x^2 + 44.79x^3 - 1.497xL_0^3 - 56.66L_0L_1 -152.6L_0 - \frac{2608}{81}L_0^2 - \frac{64}{27}L_0^3 \bigg] +N_F^2 \frac{64}{81} \bigg[-D_0 - \bigg(\frac{51}{16} + 3\zeta_3 - 5\zeta_2\bigg)\delta(1-x) + \frac{x}{1-x}L_0\bigg(\frac{3}{2} + 5\bigg) +1 + (1-x)\bigg(6 + \frac{11}{2}L_0 + \frac{3}{4}L_0^2\bigg) \bigg] (10.8)$$

D. Polarised Non-singlet Splitting Function in Leading Order(LO)

In leading order the polarised non-singlet splitting function is given by

$$\Delta P_{ns}^{(0)}(x) = 2C_F \left(\Delta p_{qq}(x) + \frac{3}{2} \delta(1-x) \right)$$
(10.9)

Here

$$\Delta p_{qq}(x) = \frac{2}{1-x} - 1 - x. \tag{10.10}$$

E. Polarised Non-singlet Splitting Function in Next-to-Leading Order(NLO)

$$\begin{split} \Delta P_{NS}^{+(1)}(x) &= 4C_F^2 \left[2\Delta p_{qq}(-x)(\zeta_3 + 2H_{-1,0} - H_{0,0}) + 2\Delta p_{qq}(H_{1,0} \\ &+ H_2 - 3/4H_0) - 9(1-x) - (1+x)H_{0,0} - 1/2(7+11x)H_0 \\ &+ \delta(1-x)(3/8 + 6\zeta_3 - 3\zeta_2) \right] \\ &+ 4C_A C_F \left[-\Delta p_{qq}(-x)(\zeta_2 + 2H_{-1,0} - H_{0,0}) + \Delta p_{qq}(x)(H_{0,0} \\ &+ 11/3H_0 - \zeta_3 + 67/18) + 26/3(1-x) + 2(1+x)H_0 \\ &+ \delta(1-x)(17/24 - 3\zeta_3 + 11/3\zeta_2) \right] + 4/3C_F N_F \\ &\left[-\Delta p_{qq}(x)(5/3 + H_0) - 2(1-x) - \delta(1-x)(1/4 + 2\zeta_2) \right] (10.11) \end{split}$$

with

$$\Delta p_{qq}(x) = \frac{2}{1-x} - 1 - x. \tag{10.12}$$

F. Polarised Non-singlet Splitting Function in Next-to-Next-to-Leading Order(NNLO)

$$P_{NS}^{+(2)}(x) \cong 1174898D_0 + 1295.470\delta(1-x) + 714.1L_1 + 1860.2$$

$$-3505x + 297.0x^2 - 433.2x^3 + L_0L_1(684 + 251.2L_0)$$

$$+1465.2L_0 + 399.2L_0^2 + 320/9L_0^3 + 116/81L_0^4$$

$$+N_F \left[-183.187D_0 - 173.927\delta(1-x) - \frac{5120}{81}L_1 - 197.0 + 381.1x + 72.94x^2 + 44.79x^3 - 1.497xL_0^3 - 56.66L_0L_1 - 152.6L_0 - \frac{2608}{81}L_0^2 - \frac{64}{27}L_0^3 \right] + N_F^2 \frac{64}{81} \left[-D_0 - \left(\frac{51}{16} + 3\zeta_3 - 5\zeta_2\right)\delta(1-x) + \frac{x}{1-x}L_0\left(\frac{3}{2} + 5\right) + 1 + (1-x)\left(6 + \frac{11}{2}L_0 + \frac{3}{4}L_0^2\right) \right] (10.13)$$

Here the following abbreviations are used,

$$D_0 = \frac{1}{(1-x)_+}, \quad L_1 = \ln(1-x), \quad L_0 = \ln x.$$
 (10.14)

List of Publication

I. Papers published/accepted for publication in journals

- 1. Nath, N. M., Das, M. K. and Sarma, J. K. Q^2 evolution of $xF_3(x, Q^2)$ structure function and Gross Llewellyn-Smith sum rule up to nextnext-to-leading order at low-x and low- Q^2 using a Q^2 dependent Regge ansatz, *Indian J. Phys.* (Accepted for publication)
- Nath, N. M., Das, M. K. and Sarma, J. K. Regge-like initial input and evolution of non-singlet structure functions from DGLAP equation up to next-next-to-leading order at low-x and low-Q², *Pramana J. Phys.*, 2014.(DOI: 10.1007/s12043-014-0902-7)
- Nath, N. M., Das, M. K. and Sarma, J. K. Regge Inspired QCD Based Investigation on the Q² Dependence of Gross-Llewellyn Smith Sum Rule Up to NNLO QCD Corrections, J. Adv. Phys. 4(2), 1-7, 2015.
- 4. Nath, N. M. & Sarma, J. K. Usefulness of Regge theory in the evolution of non-singlet structure functions from DGLAP equations up to NNLO at small-x, Int. J. App. Math. Eng. Sc. 2014 (Accepted for publication)
- 5. Nath, N. M., Baruah, N. & Sarma, J. K. Analytical solution of DokshitzerGribovLipatovAltarelliParisi evolution equation for $xF_3(x, Q^2)$ structure functions at low-x and low- Q^2 , using a Q2 dependent Regge ansatz *Indian J. Phys.*, **88**(7), 745-749, 2014.
- 6. Nath, N. M., Das, M. K. and Sarma, J. K. Solution of DGLAP Evolution Equation for xF_3 Structure Function in Leading and Next-to-Leading Order at Small-x, Univ. J. Phys. App. **2**(2) 80-84, 2014.
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II. Papers under review in referred journals/in preparation

- 1. Nath, N. M. & Sarma, J. K. Small-x behavior of non-singlet spin structure function and Bjorken sum rule, Euro. Phys. J. C (Under review)
- 2. Nath, N. M. & Sarma, J. K. Extraction of higher twist effect in nonsinglet structure functions and associated sum rules(Under review)
- 3. Nath, N. M., Mukharjee, A. & Sarma, J. K. Nuclear effects in non-singlet structure functions and associated sum rules (in preparation)
- 4. Nath, N. M. & Sarma, J. K. Perturbative and non-perturbative effects in non-singlet structure functions: a phenomenological analysis(in preparation)

III. Books and Book Chapters

- Nath, N. M. & Sarma, J. K. Semi analytical solution of DGLAP evolution equation up to next-to-leading order for non-singlet spin structure function using a Q² dependent Regge ansatz, *Recent Trends of Mathematics and its applications*, EBH publishers, Guwahati, 2014.
- 2. Nath, N. M. Our current views on nature's building blocks, Ticol printers and Publishers, Tezpur, 2015.(In press)