Introduction to Superstring and M-theory-motivated Cosmologies

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Abstract

I review cosmologies which are a consequence of fundamental theories of unified interactions such as superstring and M-theory.

After a short introduction about the basic observational facts in cosmology, I review the basic properties of superstring cosmologies with the main stress put onto the pre-big-bang cosmologies. Next, I discuss M-theory-motivated cosmologies concentrating on the Hořava-Witten, the Randall-Sundrum, and the ekpyrotic/cyclic cosmologies. The latter have a big advantage of a Big-Bang singularity avoidance.

1 Standard cosmology and its observational support

According to a general suggestion for the conference "Hot points in astrophysics and cosmology" I start with the three "Hot Points" in cosmology nowadays. First is that according to observational data the universe accelerates [1, 2, 3, 4, 5], and every "exotic" is admissible to explain this phenomenon. Second is that the theories which move the beginning of time to a "before" Big-Bang are intensively under studies [6, 7]. The third is that within the framework of these new theories, a class of non-singular or cyclic universes is possible [8].

Standard Big-Bang cosmology is based on the Einstein-Hilbert action for gravity [9]

$$S = \frac{1}{16\pi G} \int d^4x R + S_m \ , \tag{1}$$

(G - gravitational constant, R - Ricci scalar, S_m matter action, $c = 1$) which for isotropic and homogeneous Friedmann-Robertson-Walker metric

$$ds^2 = -dt^2 + a^2 \left(\frac{dr^2}{1 - kr^2} + r^2 d\theta^2 + \sin^2 \theta d\phi^2 \right) \ , \tag{2}$$

($a(t)$ - the scale factor, $k = 0, \pm 1$ - the curvature index) together with a matter energy-momentum tensor

$$T^{\mu}_{\mu} = (-\varrho, p, p, p) \ , \tag{3}$$

(ϱ - the energy density, p - the pressure) which is conserved

$$T^{\mu\nu}{}_{;\nu} = 0 \ , \tag{4}$$

and the barotropic equation of state

$$p = (\gamma - 1)\varrho \ , \tag{5}$$

gives the dynamical Friedmann equation

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\varrho - \frac{k}{a^2} \ , \tag{6}$$

which governs the evolution of the universe and tells us how fast the distances grow according to the growth of the scale factor $a(t)$.

Using (5) and (4) we realize that

$$\varrho \propto a^{-3\gamma} \ , \tag{7}$$

and provided $\gamma > 0$ [1] we have a curvature singularity (Big-Bang) $\varrho \to \infty$ for $a \to 0$. A general solution to (6) for $k = 0$ is

$$a(t) = \mid t \mid^{\frac{2}{3\gamma}} \ . \tag{8}$$

Notice that we took modulus of t in (8) - this is for the sake of further discussion despite the fact that in standard cosmology only $t > 0$ is taken into account. The remaining solutions for $k = \pm 1$ are given in standard textbooks, so that we skip them concentrating on the main fact that for $\gamma > 2/3$

$$\ddot{a} < 0 \ , \tag{9}$$

which means that the universe decelerates (slows the speed of expansion). The curvature singularity suggests that together with the achievements of the particle physics we can consider the hot Big-Bang universe scenario. Its main observational support relies on the following facts [5]:

- The universe expands, i.e.,

$$H(t_0) \equiv \frac{\dot{a}(t_0)}{a(t_0)} = H_0 > 0 \ . \tag{10}$$

[1]According to the current observations, also $\gamma < 0$ is admissible (phantom [12, 13]), which leads to a curvature singularity $\varrho \to \infty$ for $a \to \infty$.

- Element abundance in the universe is: hydrogen 75%, helium 24%, and other elements 1%. In particular, the amount of helium is larger than it is possible to be produced in stars, and the only solution to this problem is to assume that its abundance is primordial.

- Cosmic Microwave Background (CMB) - photons once were in thermal equilibrium with charges which further decoupled and formed thermal background with blackbody radiation spectrum with temperature $T = 2.7K$. The information about the density fluctuations $\delta\rho$ at the decoupling epoch was imprinted in the temperature fluctuations according to the formula
$$\frac{\delta T}{T} \propto \frac{\delta\rho}{\rho} \propto 10^{-5} , \tag{11}$$
and the temperature anisotropies can be decomposed onto spherical harmonics
$$\frac{\delta T}{T} = \sigma_{lm} a_{lm} Y_{lm}(\theta, \phi) , \tag{12}$$
where the power spectrum of fluctuations is defined as
$$C_l = <\mid a_{lm} \mid^2> , \tag{13}$$
and it has the peak structure related to an angular scale in the sky
$$\theta_i = \frac{\pi}{l_i} , \tag{14}$$
where l is the multipole moment, and i is the number of the peak. Formally, θ_i is the angle we can see the fluctuation of density in the sky today.

2 Inflationary cosmology and dark energy

However, despite its observational support, the standard Hot Big-Bang cosmology has the puzzles which are as follows:

- **The problem of singularity at $t = 0$.** As I have said for $t = 0$ the energy density and pressure blows up $\varrho, p \to \infty$. Besides, in standard cosmology it was assumed that $t \geq 0$. Was it anything before that?

- **The horizon problem.** The CMB we observe nowadays on the last scattering surface is nearly isotropic (see (11)) in the two opposite directions in the sky, though, according to the standard Friedmann model (8), it has not been in causal contact before.

- **The origin of CMB anisotropies.** How could the observed structures in the universe form in the early universe?

- **Flatness problem.** Nowadays we have strong evidence for the dimensionless energy density parameter
$$\Omega = \frac{\varrho}{\varrho_c} , \tag{15}$$
where
$$\varrho_c = \frac{3H_0^2}{8\pi G} , \tag{16}$$
(ϱ_c - critical density) is to be of order one $O(1)$. Evolving this to the past shows that close to the initial singularity Ω should have been equal to one at the very high precision (one part in 10^{54}).

These problems are solved by the inflationary scenario [10, 11] which assumes a very fast exponential expansion of the universe at the time $t \sim 10^{-35}$ s after Big-Bang. This expansion is accelerated so that we have
$$\ddot{a} > 0 . \tag{17}$$
This very fast expansion is then followed by the standard radiation and matter-dominated evolution. Additionally, inflation gives the origin for the quantum fluctuations of the scalar field which give rise to the observed density fluctuations.

However, there are more problems in cosmology - some of them revealed more recently such as:

- **The dark matter problem.** In astrophysics one measures the velocity of stars in galaxies which gives their rotation curves $v = v(r)$. They scale as $v \sim r$ (the mass $M \sim r^3$) for the central core, while for the outer stars they should scale as $v \sim \sqrt{GM/r} \sim r^{-1/2}$. However, the observational curve does not fall according to the latter relation. Instead, it stays flat – $v \approx$ const. for $r > R$, where R is the radius of the central core. The conclusion is that there must be some matter which is not visible which we call dark matter.

- **The dark energy problem.** The second of the Einstein equations which accompanies the Friedmann equation (6) reads as
$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3} (\varrho + 3p) . \tag{18}$$
According to (18) the accelerated expansion requires that the pressure of the fluid which drives the expansion must be negative
$$p < -\frac{1}{3}\varrho . \tag{19}$$

As I mentioned, inflation is just an accelerated expansion, and so it is driven by negative pressure. This did not seem to be a very big problem for cosmologists since it dealt with the early universe rather than with the present.

However, the observations of distant supernovae [1, 2, 4, 3, 5] gave strong evidence that the universe is *accelerating nowadays* - not only close to the Big-Bang. This means that at least 70% of matter in the universe now has negative pressure and it is not the dark matter which can only be some fraction of the remaining 30%. More recent data [4, 5] shows that the dark energy may have really a very strong negative pressure ($p < -\varrho$) and the matter of this type is called *phantom* [12, 13, 14]. The dark energy may be mimicked by a scalar field ϕ with some potential $V(\phi)$ which gives effective energy density and pressure

$$\varrho = \pm\frac{1}{2}\dot{\phi}^2 + V(\phi) , \tag{20}$$

$$p = \pm\frac{1}{2}\dot{\phi}^2 + V(\phi) , \tag{21}$$

where the plus sign refers to an ordinary (though still perhaps of negative pressure) matter and the minus sign refers to a phantom.

I will not be discussing all the details of standard and inflationary cosmology since my main topic is superstring and M-theory cosmology, but I will enumerate a couple of important questions which led people to investigations of these nonstandard cosmologies.

These questions are as follows:

- What is the source of dark matter and dark energy?
- Is there a way to avoid singularity? Putting this differently - was the universe evolving before initial singularity?
- Is there any benefit in unification of gauge interactions with gravity for cosmology? How about running of the gravitational constant G and relating it to the evolution of a scalar field in these theories?
- Is there a quantum theory of gravity?

These questions are addressed in a more fundamental framework than Einstein's general relativity theory, i.e., in superstring and M-theory.

3 Superstring cosmologies

In string theory the unification of gauge interactions with gravity requires "running" of the gravitational constant according to the scheme [15, 16]

$$\Phi(t) = \frac{1}{G(t)} = \frac{e^{-\phi(t)}}{\lambda_s^2} = M_{pl}^2(t), \tag{22}$$

where G is the Newton gravitational "constant", M_{pl} is the Planck mass, λ_s is the fundamental string length, Φ is the Brans-Dicke field, and ϕ is the dilaton. The vacuum expectation value of the dilaton has the interpretation of the string coupling constant

$$g_s \propto < e^{\frac{\phi}{2}} > \ , \tag{23}$$

There are various superstring theories formulated in $D = 10$ spacetime dimensions. Open string theory is called Type I (left- and right-moving modes are equal, and it has $N = 1$ supersymmetry only), while closed string theories are called type IIA, IIB and heterotic. Type II string theories have $N = 2$ supersymmetries. In type IIA left- and right-moving modes are independent and opposite chirality, while in type IIB these modes are also independent, but the same chirality. Heterotic superstring is the most complicated in the way, that the right-moving modes are $N = 1$ supersymmteric, while the left-moving modes have no supersymmetry ($N = 0$) with the gauge group $SO(32)$ or $E_8 \times E_8$ [16].

3.1 Bosonic sector

It is interesting that all superstring theories have a common sector of bosonic particles: the dilaton ϕ, the axion $H_{\mu\nu\rho}$, and the graviton $g_{\mu\nu}$ described by the reduced 4-dimensional action [6]

$$S = \frac{1}{2\lambda_s^2}\int d^4x\sqrt{-{}^{(4)}g}\; e^{-\phi}\left\{{}^{(4)}R + \partial_\mu\phi\partial^\mu\phi - \frac{1}{12}H^2\right\}, \tag{24}$$

where ${}^{(4)}g$ is the determinant of the 4-dimensional metric, ${}^{(4)}R$ the 4-dimensional Ricci scalar, $H^2 \equiv H_{\mu\nu\rho}H^{\mu\nu\rho}$, $H_{\mu\nu\rho} = \partial_{[\mu}B_{\nu\rho]}$ $(dH = 0)$, and $B_{\mu\nu}$ is the antisymmetric tensor potential, $B_{\mu\nu} = -B_{\nu\mu}$.

The action (24) describes the common sector in the string frame which is characterized by the property that in this frame the fundamental string length λ_s = const., while the Planck mass M_{pl} changes with time according to (22). In fact, the reduced action (24) without axion ($H = 0$) is just the Brans-Dicke action in Jordan frame with the Brans-Dicke parameter $\omega = -1$ [17]. The Einstein limit in this frame is recoverable for a constant dilaton.

In the Einstein frame the action (24) takes the form

$$S = \frac{1}{16\pi G}\int d^4x\sqrt{-{}^{(4)}g}\;\left\{{}^{(4)}R + \partial_\mu\phi\partial^\mu\phi - \frac{1}{12}e^{-2\phi}H^2\right\}, \tag{25}$$

The transition from the string frame to the Einstein frame is given by the conformal transformation, as follows:

$$g^S_{\mu\nu} = \exp{(-\phi)} g^E_{\mu\nu}, \tag{26}$$

where $g^S_{\mu\nu}$ is the metric in the string frame, while $g^E_{\mu\nu}$ is the metric in the Einstein frame. In the Einstein frame the fundamental string length $\lambda_s = \lambda_s(t)$ (changes with time) while the Planck mass $M_{pl} =$ const. (compare (22)), i.e.,

$$\frac{1}{G} = \frac{e^{-\phi(t)}}{\lambda_s^2(t)} = M_{pl}^2 \ . \tag{27}$$

3.2 Pre-big-bang cosmology

Let us first notice that for $H = 0$ the solutions of (26) in the Einstein frame are that of stiff-fluid $\gamma = 2$ (taking effectively $V = 0$ in (20)-(21)) in (8), i.e.,

$$a(t) = |\, t\,|^{\frac{1}{3}} \ , \tag{28}$$
$$\phi(t) = \phi_0 \pm \sqrt{3} \ln |\, t\,| \ . \tag{29}$$

It is interesting that the solution of (28) for the negative times $t < 0$ represents an accelerated collapse of the universe, while the one for the positive times $t > 0$ represents a decelerated expansion of the universe. This does not mean anything special as far as the Einstein frame is considered.

However, it is different, if we consider an appropriate cosmological solutions in the string frame. For $k = 0$ and $H = 0$ (no axion), one has from (25)

$$\begin{aligned} a(t) &= |\, t\,|^{\pm\frac{1}{\sqrt{3}}}, \\ e^{\phi(t)} &= |\, t\,|^{\pm\sqrt{3}-1} \ . \end{aligned} \tag{30}$$

The crucial point is that one makes interpretation of the solutions which are allowed for negative times - this leads to the cosmological scenario which is called *pre-big-bang* cosmology [18, 19, 20, 21, 22]. The name suggests that we consider possible evolution of the universe "before" Big-Bang singularity! However, in view of the fact that we deal with string theory, the singularity at $t = 0$ is not only a curvature singularity – like in general relativity. In fact, it is also a strong coupling singularity in the sense of string theory, since here the string coupling constant $g_s = e^{\frac{\phi}{2}}$ (cf. (23)) diverges at $t \to 0$, too.
From (30) we notice that there are four possibilities for the evolution of the universe.
The ones with '-' sign in (30) are numbered as **1** and **2** while those with '+' sign in (30) are numbered as **3** and **4** (cf. Fig. 1). The four types of evolution are connected by the scale

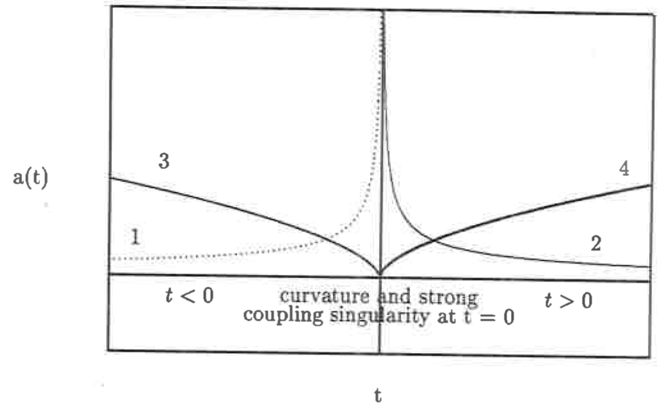

Figure 1: Pre-big-bang cosmology. Branches **1** and **2** correspond to '-' sign in (30) while branches **3** and **4** correspond to '+' sign in (30). Branches **1-3** and **2-4** are duality-related. Branch **1** is superinflationary while branch **4** is deflationary and describes an ordinary radiation-dominated evolution.

factor duality (SFD) [23, 24]. Its mathematical realization is given by the relation which changes the scale factor and the dilaton, leaving field equations unchanged, i.e.,

$$a(t) \Longleftrightarrow \frac{1}{a(t)}, \tag{31}$$

$$\phi(t) \Longleftrightarrow \phi(t) - \ln a^6(t). \tag{32}$$

SFD relates **1** and **3** or **2** and **4** whose domains are either $t < 0$ or $t > 0$. However, an additional time-reflection symmetry

$$t \Longleftrightarrow -t, \tag{33}$$

together with SFD gives relation between **1** and **4**

$$a_1(t) = (-t)^{-\frac{1}{\sqrt{3}}} \Longleftrightarrow t^{\frac{1}{\sqrt{3}}} = \frac{1}{a_4(-t)}. \tag{34}$$

It is easy to show that for branch **1**

$$\frac{\ddot{a}_1}{a_1} > 0, \tag{35}$$

and this is inflation driven only by the *kinetic term* of the dilaton. It is easy to notice that the branch **4** is deflationary, i.e.,

$$\frac{\ddot{a}_4}{a_4} < 0, \tag{36}$$

and it describes standard *radiation-dominated* evolution. Branches **1** and **4** are duality-related, though, they are divided by the singularity of curvature and strong coupling.

In pre-big-bang scenario the universe begins with an asymptotically trivial state ($t \to -\infty$) of weak coupling ($g_s \to 0$) and low curvature ($^{(4)}R \to 0$ which is just Minkowski space - vacuum), undergoes superinflation driven by the kinetic energy of the dilaton, reaches curvature and strong coupling singularity at $t = 0$, then after possible "exit", it reaches an ordinary radiation-dominated evolution [25].

In a way, in the pre-big-bang scenario the problem of singularity is even worse than in standard cosmology since now we have strong coupling and curvature singularity instead of just curvature singularity.

3.3 Duality

The scale-factor duality (31)-(32) has an interesting analogy with the phantom duality. For phantom we have

$$\varrho \propto a^{3|\gamma|} \quad (\gamma < 0) \ , \tag{37}$$

so that he bigger the universe grows the denser it is - this leads to a future curvature singularity called "Big-Rip". It is interesting to notice that standard matter ($\gamma > 0$) and phantom matter ($\gamma < 0$) solutions are dual to each other in the sense that there is an invariance

$$a \to \frac{1}{a} \ , \tag{38}$$

or,

$$\gamma \to -\gamma \ , \tag{39}$$

for these models [13, 26, 27].

3.4 "Graceful-exit" problem

Pre-big-bang cosmology is, in fact, plagued by the big problem which is how to "exit-out" superinflation which undergoes before the curvature and strong coupling singularity appear. This problem is called the "graceful-exit" problem and led to the main criticism of the pre-big-bang scenario [7].

The main approaches which were suggested to cope with this problem referred to:

- Regularization of the strong coupling singularity in the sense of string theory. This consists of the modification of the effective action by the quantum corrections which come from either the quantum nature of the fundamental string length λ_s or from inclusion of the string loop effects.

- Quantum mechanical scattering in minisuperspace on the singularity at $t = 0$.

- "Bounce" due to a negative dilaton potential energy (effective negative cosmological constant).

As we shall see later, despite its drawbacks, pre-big-bang cosmology, after an appropriate modification related to a choice of the branches, gave an inspiration to nonsingular ekpyrotic/cyclic cosmology.

4 M-theory (Brane) cosmologies

On the one hand, it is the fact that there are five different superstring theories (type I, IIA, IIB, heterotic $SO(32)$ and $E_8 \times E_8$) which are related by different *duality symmetries* and so one is able to transit from one theory to the other [6]. The question is whether this property may suggest that they are not fundamental, but that they are only special cases of a more general theory which encompasses them. This more general theory has been provisionally given the name *M-theory.* Additionally, M-theory is expected to give the description of the physical interactions at strong coupling , i.e., when string coupling parameter g_s given by (23) is large.

On the other hand, there is a hierarchy problem in particle physics. This means that the unification scale of gravity with gauge interactions 10^{19} GeV is many orders of magnitude higher than the electroweak unification scale 100 GeV, for example. Even that, still there is a near miss of the running coupling constants of strong, weak and electromagnetic interactions with gravity at about 10^{16} GeV.

We already know that pre-big-bang cosmology is the result of the admission of the cosmological solutions for the common bosonic sector of superstring effective actions (24) in $D = 10$ spacetime dimensions. It was then proposed that M-theory is 11-dimensional and that its low-energy limit can be described by the supergravity theory - the theory whose bosonic sector contains a three-form antisymmetric tensor potential $A_{\mu\nu\rho}$ and the graviton $g_{\mu\nu}$. The action reads as [28, 29, 30]

$$S_{\rm M} = \frac{1}{16\pi G_{11}} \left(\int d^{11}x\sqrt{-g_{11}} \left[R_{11} - \frac{1}{24}F_4^2\right] + \frac{1}{6}\int A_3 \wedge F_4 \wedge F_4, \right) \tag{40}$$

where G_{11} is 11-dimensional Newton constant, R_{11} is 11-dimensional Ricci scalar, $F_4^2 = F_{\mu\nu\rho\sigma}F^{\mu\nu\rho\sigma}$, $F_4 = dA_3$, $F_{\mu\nu\rho\sigma} = \partial_{[\mu}A_{\nu\rho\sigma]}$, and the last term in (40) is the Chern–Simons term which arises as a consequence of ($N = 1$) supersymmetry.

These arguments led people to consider some exotic cosmological models motivated by supergravity and superstings which are known as *brane universes.* We will discuss three basic types of such models which are: Hořava-Witten cosmologies, Randall-Sundrum cosmologies and ekpyrotic/cyclic cosmologies (for a recent review see [31]).

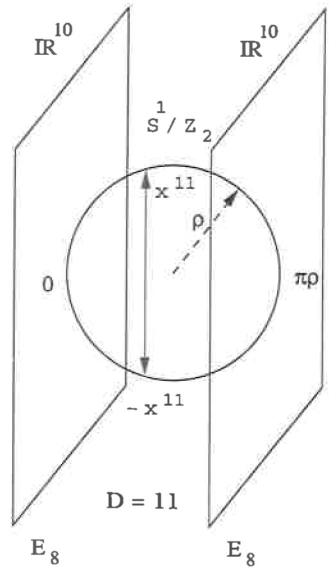

Figure 2: Hořava-Witten model. The 11-dimensional spacetime (the bulk) contains two 10-dimensional branes connected by the orbifold. The gauge interactions are confined to branes only while gravity acts everywhere in the bulk.

4.1 Hořava-Witten cosmology

The compactification of $N = 1$, $D = 11$ supergravity on a circle, S^1, results in the type IIA supergravity theory [32, 33] which can be interpreted as the strongly coupled limit of the type IIA superstring theory (with $N = 2$ supersymmetries) in terms of an 11-dimensional theory [34]. This correspondence gave Hořava and Witten [35] the idea that one can also compactify the $N = 1$, $D = 11$ supergravity on a S^1/Z_2 orbifold (which is a unit interval I) in order to get a heterotic theory with only $N = 1$ supersymmetry. In other words, they proved that the 10-dimensional $E_8 \times E_8$ theory results from an 11-dimensional theory compactified on the orbifold $R^{10} \times S^1/Z_2$ in the same way as the type IIA theory results from an 11-dimensional theory compactified on $R^{10} \times S^1$. This identified strongly coupled limit of heterotic $E_8 \times E_8$ theory as the 11-dimensional supergravity compactified on an orbifold.

A totally new and exotic idea of brane universes is that gravity propagates in *all eleven dimensions* (see e.g. [15]) while E_8 gauge fields are *restricted* only to 10-dimensional orbifold fixed planes. By the way, similar models were investigated already in the 80s by Visser [36, 37].

As pictured in Fig. 2, $y = x^{11}$ is an orbifold coordinate with $y \in [-\pi\lambda, \pi\lambda]$, $\lambda =$ const.

and the orbifold fixed planes are at $y = 0, \pi\lambda$. The Z_2 symmetry acts as $y \to -y$. The 11-dimensional action for such a theory contains a supergravity part S_{SUGRA} and a Yang-Mills part S_{YM} which is composed of the two E_8 Yang-Mills theories on the 10-dimensional orbifold planes as follows:

$$\begin{aligned} S_{SUGRA} &= -\frac{1}{2\kappa_{11}^2}\int_{M_{11}}\sqrt{-g_{11}}\left[R_{11} + \frac{1}{24}G_{IJKL}G^{IJKL}\right. \\ &\left. + \frac{\sqrt{2}}{1728}\epsilon^{I_1\ldots I_{11}}C_{I_1I_2I_3}G_{I_4\ldots I_7}G_{I_8\ldots I_{11}}\right], \end{aligned} \tag{41}$$

and

$$\begin{aligned} S_{YM} = &-\frac{1}{8\pi\kappa_{11}^2}\left(\frac{\kappa_{11}}{4\pi}\right)^{\frac{2}{3}}\int_{M_{10}^{(1)}}\sqrt{-g_{10}}\left[tr\left(F^{(1)}\right)^2 - \frac{1}{2}trR^2\right] \\ &-\frac{1}{8\pi\kappa_{11}^2}\left(\frac{\kappa_{11}}{4\pi}\right)^{\frac{2}{3}}\int_{M_{10}^{(2)}}\sqrt{-g_{10}}\left[tr\left(F^{(2)}\right)^2 - \frac{1}{2}trR^2\right]. \end{aligned} \tag{42}$$

In (41) and (42) $I, J, K, \ldots = 0, \ldots, 11$ while $\bar{I}, \bar{J}, \bar{K}, \ldots = 0, \ldots, 9$ and $M_{10}^{(i)}$ (i = 1,2) are 10-dimensional manifolds orthogonal to the orbifold, $\kappa_{11}^2 = 8\pi G_{11}$. The $F_{\bar{I},\bar{J}}^{(i)}$ are the two gauge field strengths and C_{IJK} is the three-form potential giving the field strength $G_{IJKL} = 24\partial_{[I}C_{JKL]}$.

For further cosmological investigations one can compactify Hořava-Witten models on a Calabi-Yau deformed manifold X according to $M^{11} = M^4 \times X \times S^1/Z_2$. It is important that the size of the orbifold is much bigger than the radius of the Calabi-Yau space and we can discuss 5-dimensional effective theory with the action [38]

$$S = \int_{M_5}\sqrt{-g^{(5)}}\left[\frac{1}{2}R^{(5)} - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{6}\alpha_0^2 e^{-2\sqrt{2}\phi}\right] \mp \sqrt{2}\sum_{i=1}^{2}\int_{M_4^{(i)}}\sqrt{-\tilde{g}^{(4)}}\alpha_0 e^{-\sqrt{2}\phi}, \tag{43}$$

where $M_4^{(1)}, M_4^{(1)}$ are orbifold fixed planes, $\phi = 1/\sqrt{2}\ln V$ is a scalar field (dilaton) which parametrizes the radius of Calabi-Yau space and $\tilde{g}_{ij}, i,j = 0,1,2,3$ is the pull-back of 5-dimensional metric onto $M_4^{(1)}$ and $M_4^{(1)}$. In the action (43) we dropped other important fields like p-form fields, gravitini, RR scalar and fermions.

The effective field equations for the action (43) are $(i,j = 0,1,2,3, \mu,\nu = 0,1,2,3,5)$

$$\begin{aligned} R_\mu^\nu &= \nabla_\mu\phi\nabla^\nu\phi + \frac{\alpha_0^2}{9}g_\mu^\nu e^{-2\sqrt{2}\phi} + \sqrt{2}\alpha_0 e^{-\sqrt{2}\phi}\sqrt{\frac{\tilde{g}}{g}}\tilde{g}^{ij} \\ &\times \left[g_{i\mu}g_j^\nu - \frac{1}{3}g_\mu^\nu g_{i\sigma}g_j^\sigma\right]\left[\delta(y) - \delta(y - \pi\lambda)\right] \\ \frac{1}{\sqrt{-g}}\partial_\mu\left(\sqrt{-g}\partial^\mu\phi\right) &= -\frac{\sqrt{2}}{3}\alpha_0^2 e^{-2\sqrt{2}\phi} + 2\alpha_0\sqrt{\frac{\tilde{g}}{g}}e^{-\sqrt{2}\phi}\left[\delta(y) - \delta(y - \pi\lambda)\right]. \end{aligned} \tag{44}$$

In (44) $y \equiv x^5 \in [-\pi\lambda, \pi\lambda]$ is a coordinate in the orbifold direction and the orbifold fixed planes are at $y = 0, \pi\lambda$. Z_2 acts on S^1 by $y \to -y$. The terms involving delta functions arise from the stress energy on the boundary planes.

The 11-dimensional metric for the cosmological solutions is taken to be

$$ds_{11}^2 = e^{-\frac{2\sqrt{2}}{3}\phi} g_{\mu\nu} dx^\mu dx^\nu + e^{\frac{\sqrt{2}}{3}\phi} \Omega_{mn} dy^m dy^n, \tag{45}$$

and since $m, n = 6, \ldots, 11$ so that the last term is just the metric of the Calabi-Yau space. The 5-dimensional metric is given by

$$ds_5^2 = g_{\mu\nu} dx^\mu dx^\nu = -N^2(\tau, y) d\tau^2 + a^2(\tau, y) \left[(dx^1)^2 + (dx^2)^2 + (dx^3)^2 \right] + d^2(\tau, y) dy^2 \ . \tag{46}$$

The separable solutions are of the form [38] (we have taken flat 3-metric - a non-flat solutions are given in [39]; anisotropic models are studied in [40])

$$\begin{aligned} N(\tau, y) &= n(\tau)\tilde{a}(y), \\ a(\tau, y) &= \alpha(\tau)\tilde{a}(y), \\ d(\tau, y) &= \delta(\tau)\tilde{d}(y), \\ V(\tau, y) &= e^{\sqrt{2}\phi(\tau, y)} = \varepsilon(\tau)\tilde{V}(y), . \end{aligned}$$

In fact $\alpha(\tau)$ is the vorldvolume scale factor and $\delta(\tau)$ is an orbifold scale factor. It appears that the appropriate equations of motion are fully separable into the orbifold-dependent part and spacetime-dependent part provided [38]

$$n(\tau) = 1, \qquad \delta(\tau) = \varepsilon(\tau), \tag{47}$$

where the first condition is simply the choice of the lapse function while the second tells us that Calabi-Yau space is tracking the orbifold. One can show that orbifold-dependent part can be solved by

$$\begin{aligned} \tilde{a} &= a_0 H^{1/2}(y), \\ \tilde{d} &= d_0 H^2(y), \\ \tilde{V} &= d_0 H^3(y), \end{aligned} \tag{48}$$

where

$$H(y) = \frac{\sqrt{2}}{3}\alpha_0 \mid y \mid + h_0, \tag{49}$$

$$H''(y) = \frac{2\sqrt{2}}{3}\alpha_0 \left[\delta(y) - \delta(y - \pi\lambda) \right], \tag{50}$$

and we have applied

$$| y |' = \epsilon(y) - \epsilon(y - \pi\lambda) - 1, \tag{51}$$

so that

$$| y |'' = 2\delta(y) - 2\delta(y - \pi\lambda), \tag{52}$$

(factor 2 comes from the fact that y is periodic) and

$$\epsilon(y) = 1 \quad if \quad y \geq 0, \tag{53}$$
$$\epsilon(y) = -1 \quad if \quad y < 0. \tag{54}$$

After all these substitutions one can write down 5-dimensional metric in the form

$$\begin{aligned} ds_5^2 &= -a_0^2 H(y) d\tau^2 + a_0^2 H(y) \alpha^2(\tau, y) \left((dx^1)^2 + (dx^2)^2 + (dx^3)^2 \right) \\ &+ d_0^2 H^4(y) \delta^2(\tau) dy^2. \end{aligned} \tag{55}$$

Elementary Friedmann-Hořava-Witten ($k = 0$) solutions are a bit analogous to (30) in pre-big-bang cosmology and read

$$\bar{a}(\tau) = | \tau |^{p_\mp}, \quad p_\mp = \frac{3}{11} \mp \frac{4}{11\sqrt{3}}, \tag{56}$$
$$\delta(\tau) = | \tau |^{q_\mp}, \quad q_\pm = \frac{2}{11} \pm \frac{4\sqrt{3}}{11}, \tag{57}$$

for the worldvolume and orbifold respectively. From (56) and (57) we conclude that there are four types of evolution of the worldvolume M^4 and the orbifold. Namely: both the worldvolume and the orbifold contract, both the worldvolume and the orbifold expand, the worldvolume contracts while the orbifold expands (superinflationary), and the worldvolume expands while the orbifold contracts. The former case corresponds to a superinflation while the latter to a standard radiation-dominated evolution in pre-big-bang scenario.

4.2 Randall-Sundrum cosmology

Hořava-Witten were followed by Randall and Sundrum [41, 42] who mainly took care of the hierarchy problem in particle physics [43, 44, 45, 46, 47, 48, 49, 50]. As a result, they obtained a 5-dimensional Z_2 symmetric bulk (Randall-Sundrum I model) with two 3-brane(s) connected by the orbifold which are embedded in it. All the gauge interactions are confined to the branes while gravity is propagating in the whole bulk. In a one-brane scenario [41] (Randall-Sundrum II model), the brane appears at the $y = 0$ position, where y is an orbifold dimension, and the 5-dimensional spacetime is that of an anti-deSitter space with negative 5-dimensional cosmological constant. The extra dimension can be infinite due to an exponential "warp" factor in the metric. In general, not only extra spatial dimensions, but also extra time dimensions are possible [51].

The main point of the Randall-Sundrum approach is the fact that the scale of unification of gravity with other interactions can be as low as the *electroweak scale*, i.e., 1 TeV. The weakness of gravity compared to other interactions is explained by the fact that some of its strength is "leaking out" into extra dimensions while gauge interactions are *confined* to an ordinary number of dimensions.

As a result of the existence of extra dimensions for propagation of gravity, there is a modification of the Newton's potential in a 4-dimensional world

$$V(r) = \frac{GM}{r}\left(1 + \frac{2l^2}{3r^2}\right) , \tag{58}$$

where l is the size of an extra dimension, and in order to not contradict observations [45, 46] this modification can only be detectable in a submillimeter scale

$$r \leq 1\text{mm} . \tag{59}$$

As we have mentioned, there are two Randall-Sundrum theories: the first one consists of two branes (RSI model), and the second considers only one brane (RSII model).

The action for the Randall-Sundrum I model is given by

$$S = \int d^4x \int_{-\pi}^{\pi} dy \sqrt{-g^{(5)}} \left[2R^{(5)(5)}M_p^3 - \Lambda_{(5)}\right] \tag{60}$$

$$+ \sum_{i=1}^{2} \int d^4x \sqrt{-g_i^{(4)}} \left[\mathcal{L}_i - V_i\right] , \tag{61}$$

where $i = 1, 2$ refer to a "visible" and a "hidden" brane, while the indices "(5)" and "(4)" refer to 5-dimensional and 4-dimensional quantities, respectively. This action gives 5-dimensional Einstein equations in the form

$$\sqrt{-g^{(5)}}\left(R_{\mu\nu}^{(5)} - \frac{1}{2}g^{(5)}R^{(5)}\right) = -\frac{1}{4^{(5)}M_p^3}\left[\Lambda_{(5)}\sqrt{-g^{(5)}}g_{\mu\nu}^{(5)}\right.$$
$$\left. + V_1\sqrt{-g_1^{(4)}}g_{ij}^{(4)}\delta_\mu^i\delta_\nu j\delta(y-\pi) + V_2\sqrt{-g_2^{(4)}}g_{ij}^{(4)}\delta_\mu^i\delta_\nu j\delta(y)\right] , \tag{62}$$

where $\mu\nu = 0, 1, 2, 3, 5$ and $i, j = 0, 1, 2, 3$.

The 5-dimensional metric is assumed to be

$$ds^2 = e^{-2\sigma(y)}\eta_{ij}dx^i dx^j + r_c^2 dy^2 . \tag{63}$$

The Einstein equations give

$$\frac{6\sigma'^2}{r_c^2} = -\frac{\Lambda_{(5)}}{4^{(5)}M_p^3} , \tag{64}$$

$$\frac{3\sigma''}{r_c^2} = \frac{1}{4^{(5)}M_p^3 r_c}\left[V_2\delta y + V_1\delta(y-\pi)\right] , \tag{65}$$

so that

$$\sigma = r_c \mid y \mid \sqrt{-\frac{\Lambda_{(5)}}{4^{(5)}M_p^3}} , \tag{66}$$

$$\sigma'' = 2r_c\sqrt{-\frac{\Lambda_{(5)}}{4^{(5)}M_p^3}}\left[\delta(y) - \delta(y-\pi)\right] , \tag{67}$$

and this requires

$$V_2 = -V_1 = 24^{(5)}M_p^3 k, \quad \Lambda_{(5)} = -24^{(5)}M_p^3 k^2 < 0 , \tag{68}$$

which means that the tension of the visible brane is negative, while that of the hidden brane is positive and that the 5-dimensional cosmological constant is negative which means that we deal with the theory of flat branes in a 5-dimensional anti-deSitter spacetime. There is also an interesting fine-tuning of the 4-dimensional cosmological constant: $\Lambda_{(4)} = 0$.

In Randall-Sundrum II theory one removes the second brane to infinity by taking the limit

$$0 \leq y \leq \pi r_c \quad (r_c \to \infty) \tag{69}$$

in the metric

$$ds^2 = e^{-2k|y|}\eta_{ij}dx^i dx^j + dy^2 , \tag{70}$$

which is the solution to the field equations provided

$$V_2 = 24^{(5)}M_p^3 k, \quad \Lambda_{(5)} = -24^{(5)}M_p^3 k^2 < 0 , \tag{71}$$

and the appropriate relation between the 5-dimensional and 4-dimensional Planck masses makes sense in the $r_c \to \infty$ limit, i.e.,

$$^{(4)}M_p^2 = \frac{^{(5)}M_p^3}{k}\left[1 - e^{-2kr_c\pi}\right] . \tag{72}$$

The admission of a non-zero energy-momentum tensor on the brane to the Randall-Sundrum II model with brane at $y = 0$ location gives the following 5-dimensional field equations [52, 53, 54] (compare (62))

$$\tilde{G}_{\mu\nu}^{(5)} = \kappa_{(5)}^2[-\Lambda_{(5)}g_{\mu\nu}^{(5)} + \delta(y)(-\lambda h_{\mu\nu}^{(4)} + T_{\mu\nu}^{(4)})] \tag{73}$$

where $\tilde{G}_{\mu\nu}^{(5)}$ is a 5-dimensional Einstein tensor, $g_{\mu\nu}^{(5)}$ is a 5-dimensional metric, $\Lambda_{(5)}$ is a 5-dimensional cosmological constant, and $h_{\mu\nu}^{(4)} = g_{\mu\nu}^{(5)} - n_\mu n_\nu$ is a 4-dimensional induced metric, n^α a unit normal vector to the brane, $T_{\mu\nu}^{(4)}$ – a 4-dimensional energy-momentum tensor, $\lambda = V_2$ is a brane tension, and

$$\kappa_{(5)}^2 = 8\pi^{(5)}G_N = \frac{8\pi}{^{(5)}M_p^3}. \tag{74}$$

In order to cope with the hierarchy problem the 5-dimensional Planck mass ${}^{(5)}M_p \sim TeV$ ("real" unification energy scale) is always much less than the 4-dimensional Planck mass ${}^{(4)}M_p$ ("fake" unification energy scale) as measured on the brane, i.e., ${}^{(5)}M_p \sim TeV \ll {}^{(4)}M_p = 1.2 \cdot 10^{19}$ GeV.

Integrating out the y coordinate in (60) we obtain the 4-dimensional effective action in the form

$$S_4 = {}^{(4)} M_p^2 \int d^4x \sqrt{-g^{(4)}} \left(R^{(4)} - \Lambda_{(4)} \right) , \tag{75}$$

so that the induced 4-dimensional Einstein equations on the brane are [55]

$$G^{(4)}_{\mu\nu} = -\Lambda_{(4)} h^{(4)}_{\mu\nu} + \kappa^2_{(4)} T_{\mu\nu} + \kappa^2_{(5)} \Pi_{\mu\nu} - E_{\mu\nu} \tag{76}$$

where

$$\kappa^2_{(4)} = 8\pi {}^{(4)}G_N = \frac{8\pi}{{}^{(4)}M_p^2} = \frac{\lambda}{6} \kappa^4_{(5)} \tag{77}$$

$$\Lambda_{(4)} = \frac{1}{2} \kappa^2_{(5)} \left[\Lambda_{(5)} + \frac{1}{6} \kappa^2_{(5)} \lambda^2 \right] \tag{78}$$

$$\Pi_{\mu\nu} = \frac{1}{12} T T_{\mu\nu} - \frac{1}{4} T_{\mu\alpha} T^{\alpha}{}_{\nu} + \frac{1}{24} g_{\mu\nu} [3 T_{\alpha\beta} T^{\alpha\beta} - T^2], \tag{79}$$

$$T_{\mu\nu} = \rho V_\mu V_\nu + p h^{(4)}_{\mu\nu}, \tag{80}$$

where V_μ is the 4-velocity of an observer on the brane, $\Lambda_{(4)}$ is the 4-dimensional cosmological constant on the brane [56, 57] and $E_{\mu\nu}$ is the correction which appears from the *Weyl tensor* in the bulk which reads as [55]

$$E_{\mu\nu} = -\frac{6\mathcal{U}}{\lambda \kappa^2_{(4)}} \left[V_\mu V_\nu + \frac{1}{3} h^{(4)}_{\mu\nu} + \mathcal{P}_{\mu\nu} + \mathcal{Q}_\mu V_\nu + \mathcal{Q}_\nu V_\mu \right] . \tag{81}$$

Here $\mathcal{U}$ is an effective nonlocal energy density on the brane which arises from the gravitational field in the bulk which is not necessarily positive and reads

$$\mathcal{U} = -\frac{1}{6} \kappa^2_{(4)} \lambda E_{\mu\nu} V^\mu V^\nu . \tag{82}$$

Since $E_{\mu\nu}$ is traceless, then its effective local pressure is $p = (1/3)\mathcal{U}$ and this is why it is called *dark radiation.* It is interesting that the contribution to the pressure from $\mathcal{U}$ can be both positive (gravitons flowing into the brane) and negative (gravitons leaking out of the brane). On the other hand, an effective nonlocal anisotropic stress is

$$\mathcal{P}_{\mu\nu} = -\frac{1}{6} \kappa^2_{(4)} \lambda E_{[\mu\nu]}, \tag{83}$$

while an effective energy flux on the brane is

$$\mathcal{Q}_\mu = -\frac{1}{6} \kappa_{(4)} \lambda \left(E_{\mu\nu} V^\nu + E_{\nu\mu} V^\mu \right) . \tag{84}$$

After admission the perfect fluid with barotropic equation of state (5) into (79) one has

$$\Pi_{\mu\nu} = \frac{1}{12}\varrho^2 V_\mu V_\nu + \frac{1}{12}\varrho^2(2\gamma - 1)h^{(4)}_{\mu\nu} , \tag{85}$$

and the dynamics of homogeneous models in the RSII brane-world scenario can be described by the generalized Friedmann equation (compare (6))

$$H^2 = \frac{\kappa^2_{(4)}}{3}\rho + \frac{\kappa^2_{(4)}}{6\lambda}\rho^2 - \frac{k}{a^2} + \frac{\Lambda_{(4)}}{3} + \frac{2\mathcal{U}}{\lambda\kappa^2_{(4)}}. \tag{86}$$

One can easily see from (86) that in the limit $\lambda \to \infty$ one recovers general relativity. The main difference from the standard general relativistic Friedmann equation is the appearance of the ϱ^2 correction. Apart from *dark radiation* $\mathcal{U}$, this term comes as a *unique contribution* from the brane. Qualitative evolution of the cosmological models in Friedmann-Randall-Sundrum cosmology is the same as in general relativity, although there is a quantitative difference which comes from the fact that it is more difficult to gain negative pressure contribution for inflation due to the existence of the ϱ^2 term in the equation (86) [58, 59].

4.3 Ekpyrotic/Cyclic cosmology

The name ekpyrotic comes from Ancient Greece philosopher Anaximander and means "out of fire". The main framework for ekpyrosis is Hořava-Witten theory in which additional branes called the "bulk" branes, which move between boundary branes are possible. In this scenario a bulk brane is supposed to hit a boundary brane in order to "produce" the Big-Bang.

It is interesting to notice that the effective 4-dimensional description of ekpyrotic cosmology is the same as in pre-big-bang cosmology given by (30) where the four different branches are possible. However, unlike in pre-big-bang, one makes use of a different branch as describing the evolution in pre-big-bang phase. Referring to what we see in Fig. 1 one chooses branch 3 *instead of* branch 1 in pre-big-bang phase. This means that at Big-Bang there is no strong coupling singularity since now

$$g_s = e^\phi = \mid t \mid^{\sqrt{3}-1} \to 0 \tag{87}$$

at $t = 0$. This is very beneficial compared to what we have in pre-big-bang, but still a Big-Bang curvature singularity is present in the theory.

Apparently, in the simple model of a scalar field Friedmann cosmology there is no way to go from a contraction

$$H = \frac{\dot{a}}{a} < 0 , \tag{88}$$

to an expansion

$$H = \frac{\dot{a}}{a} > 0 \ , \tag{89}$$

through singularity, since H should have a minimum, whereas from the standard Einstein equations it follows that it always decreases, i.e.,

$$\dot{H} = \left(\frac{\dot{a}}{a}\right)^{\cdot} = -4\pi G(\varrho + p) = -4\pi G\dot{\phi}^2 \leq 0 \ , \tag{90}$$

and the only way to make such a transition is to violate the null energy condition

$$\varrho + p \geq 0 \ . \tag{91}$$

This is fulfilled by phantom, for example, but without appealing to such an exotic type of matter, one is also able to avoid curvature singularity by a special choice of the potential $V(\phi)$ and the coupling $\beta(\phi)$ to an energy momentum tensor in ekpyrotic models which reads as (in the Einstein frame)

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{16\pi G}R - \frac{1}{2}\partial_\mu\phi\partial^\nu\phi - V(\phi) + \beta^4(\phi)(\varrho_R + \varrho_m)\right] \ , \tag{92}$$

where

$$V(\phi) = V_0\left(1 - e^{-c\phi}\right)F(\phi), \quad F(\phi) \propto e^{-\frac{1}{g_s}} \ , \tag{93}$$

and ϱ_R is the energy density of radiation, while ϱ_m is the energy density of matter.

This scenario is admissible only for the collision of the boundary branes and composes of infinitely many such collisions - each of them produces Big-Bang, after which the evolution repeats, so that it was given the name *cyclic universe* in contrast to a single brane collision in the ekpyrotic scenario.

The main point is that in cyclic universe the 5th dimension (orbifold) *collapses*, while the 4-dimensional theory has no singularity. This can be seen by having a look onto the generalized Friedmann equations which read as

$$H^2 = \frac{8\pi G}{3}\left(\frac{1}{2}\dot{\phi}^2 + V + \beta^4\varrho_R + \beta^4\varrho_m\right) \ , \tag{94}$$

$$\frac{\ddot{a}}{a} = -\frac{8\pi G}{3}\left(\dot{\phi}^2 - V + \beta^4\varrho_R + \frac{1}{2}\beta^4\varrho_m\right) \ . \tag{95}$$

By an appropriate choice of the coupling $\beta(\phi) \sim 1/a$

$$\varrho_R \sim \frac{1}{[a\beta(\phi)]^4} \sim \frac{1}{\left(a\frac{1}{a}\right)^4} \sim \text{const.} \ , \tag{96}$$

$$\varrho_m \sim \frac{1}{[a\beta(\phi)]^3} \sim \frac{1}{\left(a\frac{1}{a}\right)^3} \sim \text{const.} \ , \tag{97}$$

in the limit $a \to 0$, and the standard Big-Bang singularity does not appear at all.

Now let us describe the consecutive stages of the evolution in these cyclic universes [60, 61, 8, 62, 63].

1. An accelerated expansion with the domination of the small positive potential energy $V(\phi)$ (positive pressure) and negligible kinetic energy - lasts about trillion years. This is in agreement with current observational data [4, 5].
2. A decelerated expansion which begins since the kinetic energy starts to matter - lasts about billion years.
3. Expansion changes into contraction ($\dot{a} = 0$).
4. During contraction the density fluctuations are generated - this happens about 1 ms before Big-Crunch. The Crunch is a consequence of the negative potential (negative cosmological term), though the model can be of zero curvature ($k = 0$).
5. Kinetic energy starts dominating, which drives further contraction - this happens about 10^{-30} seconds before Big-Crunch.
6. The *smooth* transition from Big-Crunch to Big-Bang appears - the orbifold collapses in a 5-dimensional theory while the 4-dimensional theory (brane) is not singular at all.
7. Kinetic energy stops dominating though it still drives the expansion - this happens about 10^{-30} seconds after Big-Bang.
8. Standard radiation-dominated era begins - about 10^{-25} seconds after Big-Bang.
9. Matter-dominated era begins - about 10^{10} seconds after Big-Bang.

The cycle of these stages repeats again and again.

5 Conclusions

Referring to what I have said in the Introduction I should emphasize again that the observational data put very severe constraints on the theory in cosmology. In particular, most of the pillars of the established scenario of cosmic evolution were put in doubts in recent years due to the observations of supernovae. These observations appended with the observations of large-scale structure of galaxies distribution and the cosmic microwave background temperature anisotropies suggest a very exotic type of matter (negative pressure, quintessence, dark energy, phantom or something with more exotic names) being the main ingredient of the universe. This makes the standard stream of theories of gravity to be gradually more and more extended towards the more fundamental theories which include the unification of all the known interactions in nature, i.e., the superstring and M-theories.

These more general theories of gravity allow for more freedom in constructing the cosmological scenarios. In particular, they provide us with some extra contribution to the basic cosmological equations, as considered in analogy to standard cosmology, which can give interesting consequences for explanation of the observational data.

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