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RESEARCH ARTICLE

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Gravitational time advancement and its possible detection

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Abstract The gravitational time advancement (negative time delay) is a natural but a consequence of curve space-time geometry. In the present work the possibility of experimental detection of time advancement effect has been explored.

Keywords Curved space time, Negative time delay, Detection

1 Introduction

The Shapiro time delay (also known as gravitational time delay) [1] constitutes one of the three classic solar system tests of general relativity. The effect arises from both the spatial and temporal coefficients of space-time metric and thus serves as a comprehensive test of general relativity. Nowadays the effect has also been employed as a tool to extract information about the distribution of matter in the Universe, particularly to detect dark matter in our Galaxy.

The general perception about the Shapiro effect is that due to the influence of a gravitating object the average speed of light decreases from its canonical special relativistic value c_0 and hence the signal always suffers an additional (positive) non-Newtonian delay. We wish to point out that this is *not* the case in general; depending on the position of the observer, the delay can as well be negative implying a time advancement. Note that the effect of time advancement does not violate causality, information could not be sent by an observer into his/her own past exploiting the effect, neither can it be used for a warp drive or time machine. The reason is that we are not considering motion in a configuration with matter violating known energy conditions.

So far all conclusive gravitational time delay measurements, including the one using Cassini spacecraft that has verified gravity with a remarkable accuracy of

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about 2.3 parts in 10^5 [2], have tested general relativity in the gravitational field of the Sun and in all such cases observers were far away from the gravitating object. Consequently the delay has been found to be positive in all such measurements, as expected [3; 4]. Instead if we consider the situation that light signal is sent say from the Earth's surface to a certain distance from where the signal is reflected back to the point of transmission, then the observer should notice a time advancement (see the following section). Here we explore the possibility of detecting the gravitational time advancement effect in a future astrometric experiment. We particularly consider the situation in which light signal will be sent from Earth to one of its artificial satellites/space station from where the signal will be reflected back along the same trajectory and the total travel time between signal transmission and reception will be measured with required precision. When a light signal is sent from Earth to one of its artificial satellites, the signal would also come under the influence of Sun's gravity and we need to isolate time advancement effect due solely to Earth's gravity from the resulting motion. However, we will show that the magnitude of the time advancement/delay effect is negligible when the distance between the points of transmission and reflection of the signal is very small in comparison to the impact parameter and hence the gravitational effects of Sun can be ignored for a suitable trajectory of light rays.

The organization of the article is as follows. In the Sect. 2, we describe the gravitational time advancement. In Sect. 3, we obtain the model independent expressions for time advancement/delay when the length of signal propagation is very small. In Sect. 4, we explore the possibility of detecting time advancement effect experimentally. Finally we conclude our results in Sect. 5.

2 Gravitational time advancement

The proposed effect can be understood by considering the following scenario: A radar signal is sent from the surface of the Earth (A) to a point B close to the Sun, (B is the point of closest approach for the trajectory) as in the Fig. 1 from where the signal is reflected back along its original trajectory to the Earth.

Assuming standard Schwarzschild geometry, to first order $\mu \equiv GM/c_0^2$, the well-known coordinate time delay in round trip journey from Earth A to the point B and back is given by [1; 5]

$$c_0 \Delta t_{AB} = 2\sqrt{r_A^2 - r_B^2} + 4\mu_\odot \ln \frac{r_A + \sqrt{r_A^2 - r_B^2}}{r_B} + 2\mu_\odot \left(\frac{r_A - r_B}{r_A + r_B} \right)^{1/2}, \quad (1)$$

where $\mu_\odot \equiv GM_\odot/c_0^2$, $M_\odot$ being the mass of the Sun and r_A and r_B are the values of coordinate r evaluated at the positions of A and B respectively. The difference in proper time between transmission and reception of the signal to be measured

Fig. 1 Schematic view of gravitational time delay/advancement

Fig. 2 Gravitational time delay (along y-axis) as a function of ratio between r_A and r_B (along x-axis) when the observer is at A. The delay has been given in artitrary units

Fig. 3 Gravitational time delay (along y-axis) as a function of ratio between r_A and r_B (along x-axis) when the observer is at B. The delay has been given in arbitrary units

by the observer at A is

$$\begin{aligned} c_0\Delta\tau_{AB} &\simeq (1-\mu_\odot/r_A)\Delta t_{AB} \\ &\simeq 2\sqrt{r_A^2-r_B^2}+4\mu_\odot \ln\frac{r_A+\sqrt{r_A^2-r_B^2}}{r_B} \\ &\quad +2\mu_\odot\left(\frac{r_A-r_B}{r_A+r_B}\right)^{1/2}-2\mu_\odot\frac{\sqrt{r_A^2-r_B^2}}{r_A}. \end{aligned} \tag{2}$$

Hence the signal takes an excess time over the time that it would have taken in the absence of the Sun and the delay (the part proportional to $\mu_\odot$) is positive for any r_A (see Fig. 2).

However, if the observer is at the point B instead of A, the coordinate time delay for the round trip journey from the position B to A and again back to B would remain the same as given in Eq. (1) but the difference in proper time to be measured by the observer at B now reads

$$\begin{aligned} c_0\Delta\tau_{AB} &\simeq (1-\mu_\odot/r_B)\Delta t_{AB} \\ &\simeq 2\sqrt{r_A^2-r_B^2}+4\mu_\odot \ln\frac{r_A+\sqrt{r_A^2-r_B^2}}{r_B} \\ &\quad +2\mu_\odot\left(\frac{r_A-r_B}{r_A+r_B}\right)^{1/2}-2\mu_\odot\frac{\sqrt{r_A^2-r_B^2}}{r_B}. \end{aligned} \tag{3}$$

Note that the first term in the right hand side of the above equation represents the time taken by light while traveling in a straight line (between the points) at unit velocity (i.e. total elapsed time when $M_\odot = 0$), the next two terms describe the usual gravitational time delay whereas the last term arises because of clock runs differently in gravitational field depending on the curvature. Because of the last term in the right hand side of Eq. (3), the delay works out to be negative as clearly revealed from the Fig. 3. Such a negative time delay is also followed from the expression of time transfer functions for a general post-Minkowskian expansion [6].

Here it is worthwhile to mention that the effect is not a version of red shift effect, though both the effects originate from temporal coefficient of the space-time metric. Unlike the case of time advancement/delay effect, if light signal is transmitted from a point in a gravitational field to another point and again is received at the point of transmission (after reflection), there will be no gravitational red shift at all i.e. the frequencies of the transmitted and received signal would remain the same in that case (Fig. 3).

In the abobe, the mass distribution of the Sun was assumed to be exactly spherically symmetric. When the quadrupole moment of the Sun is also taken into consideration, the gravitational potential at radius r takes the form

$$\phi = -\frac{\mu_{\odot}}{r}\left[1 - J_{2\odot}\frac{R_{\odot}^2}{r^2}P_2\right]. \tag{4}$$

where $J_{2\odot}$ is a measure of the gravitational quadrupole moment of the Sun, $R_{\odot}$ is the average radius of the Sun, $P_2 \equiv \frac{1}{2}(3\cos^2\theta - 1)$ is the second Legendre polynomial, θ being the azimuthal angle from the polar axis. The effect of quadrupole moment of the mass distribution on the time delay/advancement in transmitting the signal from the point A to B and back or vice versa is [5]

$$\delta c_0 \Delta\tau_{AB}(J_2) = -4\frac{J_{2\odot}P_2\mu R_{\odot}^2\sqrt{r_A^2 - r_B^2}}{r_A r_B^2}. \tag{5}$$

For Sun, $J_{2\odot} \sim 10^{-7}$, hence the effect of quadrupole moment is insignificant in comparison to other terms.

3 Gravitational time advancement/delay for small distance travel

As mentioned earlier, in all gravitational time delay measurements conducted so far observers were far away from the gravitating object and the distances between the points of signal transmission and reflection were much larger in comparison to the distance of closest approach. In situations where the signal travel distance is small relative to the distance of closest approach, the standard expression for time advancement as given by Eq. (3) needs to be applied with caution, giving due importance to the operational meaning of the distance of light propagation. In the following we would obtain *model independent* explicit expressions of the time advancement for geodesic motions for the stated circumstances.

We start with the general static and spherically symmetric spacetime in isotropic coordinates given by

$$ds^2 = -B(\rho)c_0^2dt^2 + A(\rho)\left(d\rho^2 + \rho^2 d\theta^2 + \rho^2\sin^2\theta d\phi^2\right). \tag{6}$$

The post-Newtonian (PN) formalism to some orders [5] is generally used to describe the gravitational theories in a weak gravitational field. This description gives additional advantage of comparing predictions of general relativity with those from any alternative metric theory of gravity. In order to discuss light propagation to any given order, knowledge of every component of space-time metric to the same order is required [7]. When considered up to the second-PN correction terms, the metric coefficients read [7]

$$B(\rho) = 1 - 2\frac{\mu}{\rho}\left(1 - J_2\frac{R^2}{\rho^2}P_2\right) + 2\beta\frac{\mu^2}{\rho^2} \tag{7}$$

and

$$A(\rho) = 1 + 2\gamma\frac{\mu}{\rho}\left(1 - J_2\frac{R^2}{\rho^2}P_2\right) + \frac{3}{2}\delta\frac{\mu^2}{\rho^2}, \tag{8}$$

where β, γ are the parametrized Post-Newtonian (PPN) parameters, δ can be considered as the second-PN parameter (these parameters are different for different theories [5; 8]; in general relativity, all of them are equal to 1), J_2 and R are the quadrupole moment parameter and radius of the object respectively. ρ is related to the usual parametrized PN coordinates $\rho = \sqrt{x^2+y^2+z^2}$.

Consider that a light signal is transmitted from point B on the surface of the gravitating object horizontally (with respect to an observer at B) to a nearby point A from where it is reflected back to the point B along the same trajectory. Here B is the point of closest approach for the trajectory. The distance (ΔX) between A and B is very small in comparison to the radius of the gravitating object. The light signal will travel a null curve of the space-time satisfying $ds^2 = 0$. Note that in this section all the quantities, namely J_2, $\Delta R/R$ and μ/R are considered small and since our present discussion is restricted to the second order accuracy, any product of at least three of these symbols (even if some of them are repeated or absent) is being neglected.

To derive time delay to the order of μ^2, one needs to know, to the accuracy of μ, the deviation of photon trajectory from the vertical direction while traveling from B to A. The study of geodesic equations reveal that within such accuracy light trajectory does not involve the azimuthal angle and follows straight Euclidean path between B and A for small Δx [7]. Following the standard practice of choosing the PPN coordinate axes in such a way so that both the transmitter and reflector lie in the $z = 0$ surface, to the second order in μ the proper distance (ΔL_{BA}) between the two points B ($x = 0, y = \rho_o, z = 0$) and A ($x = \Delta x, y = \rho_o, z = 0$) is

$$\begin{aligned}\Delta L_{BA} &= \int_0^{\Delta x} \sqrt{A(x)}dx \\ &= \Delta x \left[1 + \frac{\gamma\mu(1-J_2')}{R} - \frac{1}{4}\left(2\gamma^2 - 3\delta - 4\gamma^2 J_2'\right)\frac{\mu^2}{R^2}\right] \end{aligned} \tag{9}$$

where $J_2' \equiv J_2 P_2$. Hence the lapse of coordinate time in transiting from B to A and back is given by

$$\begin{aligned}\Delta t_t &= 2\int_0^{\Delta x} \sqrt{\frac{A(x)}{B(x)}}dx \\ &= 2\Delta L_{BA}\left[1 + \frac{\mu(1-J_2')}{R} + \left(3/2 - \beta - 3J_2'\right)\frac{\mu^2}{R^2}\right] \end{aligned} \tag{10}$$

In the above expression we have retained coordinate radius R. Since usually R is very large, the difference of coordinate radius and proper radius is very small in comparison to coordinate radius and the relative errors resulting thereby is also very small. Consequently the proper time interval to be measured by the observer at B between transmission and reception of the signal is given by

$$\Delta\tau_t = B^{1/2}(R)\Delta t_t = 2\Delta L_{BA}, \tag{11}$$

which shows that for motion in the transverse direction there is no gravitational time delay (or advancement) effect at least up to the second PN order when the distance between the points of transmission and reflection is small. The effect of rotation of Earth (as gravitating object) on gravitational time delay for geodesic motion in horizontal direction has been studied in [9] where it was found that rotational contribution is much smaller than the second PN effect.

Next consider the case of radial motion. Restricting to orbits in the equatorial plane ($\theta = \pi/2$), the geodesic equation for ϕ leads to

$$\rho^2 \frac{d\phi}{dt} = \frac{B}{A} J^2, \tag{12}$$

where J is a constant of integration which in the far field region describes angular momentum of the photon. If initially the motion of the photon is set along the radial direction (so that $J = 0$), the above equation warrants the motion would remain radial throughout.

For the PN metric as given by Eq. (6) through Eqs. (7) and (8), the proper distance between the point B (R, θ, ϕ) and the point $(R + \Delta R, \theta, \phi)$ (denoted as point C) is given by

$$\begin{aligned} \Delta L_{BC} &= \int\limits_{R}^{R+\Delta R} \sqrt{A(r)} dr \\ &= \Delta R \left[1 + \frac{\gamma\mu(1 - J_2')}{R} - \frac{1}{4}\left(2\gamma^2 - 3\delta - 4\gamma^2 J_2'\right)\frac{\mu^2}{R^2} - \frac{1}{2}\gamma\mu\frac{\Delta R}{R^2} \right]. \end{aligned} \tag{13}$$

The coordinate time interval in transiting a light signal from B to C and back, up to the order in μ^2, is given by

$$\begin{aligned} \Delta t_t &= 2 \int\limits_{R}^{R+\Delta R} \sqrt{\frac{A(r)}{B(r)}} dr \\ &= 2\Delta L_{BC} \left[1 + \frac{\mu(1 - J_2')}{R} - \frac{\mu \Delta R}{2R^2} + \left(3/2 - \beta - 3J_2'\right)\frac{\mu^2}{R^2} \right]. \end{aligned} \tag{14}$$

Translating from difference in coordinate time to that in proper time to be measured by the observer at B between transmission and reception of the signal is given by

$$\Delta\tau_t = B^{1/2}(R)\Delta t_t = 2\Delta L_{BC}\left(1 - \frac{\mu\Delta R}{2R^2}\right). \tag{15}$$

Clearly in this case the signal suffers a negative delay due to the negative sign i.e., gravitational time advancement occurs. It is worthwhile to mention that the corrective term follows also from dimensional arguments [10]. Please note that the calculated effect is independent of PPN γ parameter unlike the standard expression for time delay [1; 3; 4; 5] because we expressed the time delay/advancement results in terms of proper length that includes the γ factor (in a gravitational field physical parameters should be expressed in terms of proper quantities particularly in situations like the present one where proper quantities differ substantially from those of coordinate expressions).

A curious aspect of the above expression is that the time advancement factor is in second order in $1/\rho$ though it is first order in μ. Till now gravitational theories have been tested only to the first order both in μ and $1/\rho$ in the solar system.

4 Possibility of experimental detection of time advancement

The gravitational time advancement due to Earth's gravity can be measured by sending light signal to one of its artificial satellites/space station from where the signal will be reflected back to Earth along the same trajectory and then measuring the total travel time. Note that when a light signal is sent from Earth to one of its artificial satellites/space station, the signal would also come under the influence of Sun's gravity. To overcome this light signal may be sent (to a satellite) in the perpendicular direction to the axis passing through the Sun and the Earth. In that case the motion of light signal would be in the transverse direction with respect to the Sun and since the distance involved is small, according to the Eq. (8) up to the second PPN order there will be no gravitational time advancement or delay effect due to the Sun. On the other hand with respect to the Earth the propagation distance is considerable and hence expression given in Eq. (3) will be applicable with the identification r_B and r_A as $R_\oplus$ and R_{sat} (R_{sat} and $R_\oplus$ are the coordinate positions of the satellite and the observer at Earth's surface) respectively provided the motion is transverse as in Fig. 1. In that case to the leading order the gravitational time advancement due to Earth's gravity would be

$$\begin{aligned} c_0\Delta\tau_{adv} &= \sqrt{R_{sat}^2 - R_\oplus^2} - c_0\Delta\tau \\ &= 2\mu_\oplus \frac{\sqrt{R_{sat}^2 - R_\oplus^2}}{R_\oplus} - 4\mu_\oplus \ln\frac{R_{sat} - \sqrt{R_{sat}^2 - R_\oplus^2}}{R_\oplus} \\ &\quad - 2\mu_\oplus \left(\frac{R_{sat} - R_\oplus}{R_\oplus + R_{sat}}\right)^{1/2} + 4\frac{J_{2\oplus}P_2\mu_\oplus R_\oplus^2\sqrt{R_{sat}^2 - R_\oplus^2}}{R_{sat}R_\oplus^2}, \end{aligned} \tag{16}$$

where $J_{2\oplus}$ is the quadrupole moment of Earth. In the above equation the first term of the right hand side will dominate over the other terms and thus clearly there will be time advancement. Note that for Earth $J_{2\oplus} \sim 10^{-3}$ and hence the effect due to quadrupole moment is about 500 times smaller than the first term. For a high altitude satellite of typical distance 36,000 km, the time of advancement would be about $0.2\,nsec$ when $\gamma = 1$ i.e. for general relativity. In order to measure the time advancement with such a high precision, one has to know the distances with accuracy better than 10 cm. Instead, in measuring the usual Shapiro effect [3; 4] the distances are treated as unknown parameters and they are determined by fitting the observed times for various positions of reflector. In the proposed case, however, the requirement of high accuracy in distance measurements can be avoided in a novel way by repeating the measurement from the satellite i.e. by sending light signal to the Earth from the satellite from where the signal will be reflected back to the satellite and then measuring the total travel time between transmission and reception of the signal (for satellite bound measurement the Eq. (2) will be applicable). To the leading order the difference in total travel times as measured from Earth and the satellite would be

$$\delta\left(c_0\Delta\tau_{adv}\right) \simeq 2\mu_\oplus\sqrt{R_{sat}^2 - R_\oplus^2}\left(\frac{1}{R_\oplus} - \frac{1}{R_{sat}}\right). \tag{17}$$

The magnitude of this difference in travel times would be nearly 0.3 ns (for $R_{sat} \sim 36{,}000$ km). Here it is worthwhile to mention that the ionospheric refraction remains a major error source in measuring time interval for a photon to travel between a satellite and the receiver at Earth for real-time applications such as in a Global Positioning System. The magnitude of the ionospheric delay is of the order of 10 nsec which is much higher than the estimated gravitational advancement in the proposed scenario. However, the ionospheric delay is inversely proportional to the frequency squared. Hence in principle the effect can be disentangled from the gravitational effect by using signals of different frequencies. Further note that the ionospheric delay should be the same for the Earth bound and the satelite bound (simultaneous) measurements and hence the difference of these measurements as predicted via Eq. (17) should be free from ionospheric delay effect.

On the other hand if the light propagation is radial, straight forward calculations gives that to the leading order the difference in total travel times as measured from Earth and the satellite (ignoring the quadrupole effect)

$$\begin{aligned} \delta\left(c_0 \Delta \tau_{adv}\right) &= 2\int_{R_\oplus}^{R_{sat}} \sqrt{\frac{A(r)}{B(r)}} dr \\ &= 2\mu_\oplus \left(\frac{R_{sat}}{R_\oplus} - \frac{R_\oplus}{R_{sat}}\right), \end{aligned} \tag{18}$$

and the magnitude of the difference in travel times would remain nearly the same of that for transverse motion. If a space station is used as reflector, the Eq. (15) have to be applied, as the altitude of a space stations is normally small ($\sim$300 km) compare to the radius of Earth.

5 Conclusion

The gravitational time advancement effect arises predominantly because clock runs differently in gravitational field depending on the curvature i.e. due to gravitational time dilation. The time dilation in a gravitational field has already been confirmed through measurements of gravitational redshift. Thus indirect experimental support to the gravitational time advancement effect with magnitude as dictated by the Eq. (3) is already in existence. But no direct observation of the effect exits so far.

Despite of their common origin, the gravitational time advancement effect differs from redshift effect in various aspects. As mentioned before, if a light signal is transmitted from a point to another point in a gravitational field from where it is reflected back to the point of transmission then there will be no gravitational red shift at all unlike the case of gravitational time advancement/delay. Moreover the gravitational time advancement has a cumulative nature i.e. magnitude of gravitational time advancement increases (proportionally) with the number of turns of light ray trajectory (i.e. with the number of times that a light ray propagates from the point of transmission to the point of reflection and back to the transmission point). Hence direct experimental observation of gravitational time advancement appears imperative.

Gravitational time advancement effect also has some interesting consequences. For instance, the effect suggests that an observer at a stronger gravitational field has a better chance to communicate with a distant observer during his/her life period than the one at weaker gravitational field.

We thus conclude the following: Contrary to the common belief gravitational time delay could be negative as well leading to time advancement. A possible way of detecting this effect in future is through radar echo delay like experiment with Earth's satellites as reflector.

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