ANGULAR INTERMITTENCY IN QCD JETS

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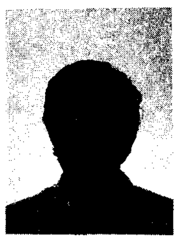

ABSTRACT

The Intermittency patern of perturbative QCD is calculated in the Double Leading log Approximation and related to its fractal dimension. It is shown that QCD is in the same class of universality than a specific Random Fragmentation model involving a two dimensional angular phase space. The observed intermittency breaking in one dimension is directly related to the running of the coupling constant. A specific observable is proposed which does not depends on the observation of the leading quark direction.

Here are presented the main results of a recent paper with Ph. Brax and R. Peschanski[1)], which have worked through the intermittency problem of QCD.

Let us first recall the way the local factorial moments of the multiplicity distribution of particles in a bin of size Δ behave has a power of Δ if the system is intermittent [2)]:

$$\mathcal{F}_q(\Delta) \propto \Delta^{-f_q} \tag{1}$$

where the constants $\{f_q\}$, usually called intermittency indices, play the role of anomalous dimensions of the fluctuation pattern. Indeed, the intermittency indices can in principle be related to the Renyi dimensions $\mathcal{D}_q$ characterizing a set of fluctuations. It reads

$$\mathcal{D}_q = d - \frac{f_q}{q-1} \tag{2}$$

where $\mathcal{D}_q$ measures the defect of phase-space dimensionality due to intermittency and d the dimension of the phase space cell.

In this talk, we are interested with the Intermittent behaviour of the Strong Interaction Theory, QCD.

Our first remark is that the well-known multiplicity generating function [3] of one gluon jet with opening angle Θ_0 and energy E, $\mathcal{Z}(E\Theta_0, v)$, is identical to that obtained from a generating function of the global multiplicity distribution $\mathcal{H}(\Theta, u)$ of a semi-random fragmentation model [4] which obeys the generic evolution equation:

$$\begin{aligned} \dot{\mathcal{H}} &= \mathcal{H}(\widetilde{\mathcal{H}} - 1), \quad \text{with,} \\ \widetilde{\mathcal{H}}(u) &= \int_0^1 r(\mathrm{w})\mathcal{H}(u\mathrm{w})d\mathrm{w} \end{aligned} \tag{3}$$

whose kernel is given by the distribution:

$$r(\mathrm{w}) = \delta(\mathrm{w}) + \frac{\gamma_0}{\mathrm{w}}\Big|_+ \tag{4}$$

More precisely, the equations (2 – 4) are obtained in the framework of Random-Branching Random-Cascading fragmentation models [4], with a "time" variable to be identified, up to a constant, with $\ln \Theta^{-1}$.

In the random fragmentation models described in ref. [4], the natural multi-fractal dimension of the coresponding model is :

$$\mathcal{D}_q = \gamma_0 \frac{q+1}{q} \tag{5}$$

expressing an exact power law (1) in the angular variable Θ, for a fixed coupling constant, i.e. fixed γ_0. As a remark, notice that one recover here,for high q, the fractal dimension founded by Gustafsson and Nielsson in 91 [5] : γ_0.

Let us now introduce the QCD[6] method. For sake of simplicity, we start again considering the fixed coupling regime. The q^{th} factorial moment in this case is sketched in the graphs of Fig.1a-b. Here the gluon jet with initial energy E and production angle Θ_0 evolves producing a soft offspring k which then splits into partons (gluons) with relative angle $\Theta_{12} \leq \Theta \ll \Theta_0$ generating the registered particle flows. Due to angular ordering the resulting q-body correlation can be written as a convolution of the energy spectrum of parton k with the product of the q^{th} power of the multiplicity distribution in an oppening angle Θ (KNO scaling)[6], namely :

$$(4\pi)\frac{\Delta N^{(q)}}{\Delta\Omega} = \frac{4C_f\alpha_s}{2\pi}\int^E \frac{dk}{k}\tilde{D}\left(\frac{E}{k}, \frac{\Theta_0}{\Theta}\right) \cdot N^q(k\Theta) \tag{6}$$

where $\tilde{D}$ denotes the energy spectrum originating from cascades with parton emission angles *larger* than the given Θ. Notice that the colour factor C_F in the emission

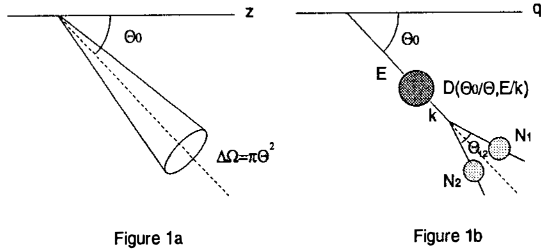

Figure 1 : Kinematics of parton-parton QCD correlations; a) Location of the phase-space cell Δ in polar coordinates; b) schematic representation of the convolution formulae (5).

probability of the gluon E corresponds to the case of the quark as an original parton shown by a horizontal line in Figs.1. This quark line can be thought of as determining the main direction of the hard process under consideration (e.g., the quark jet direction in e^+e^- annihilation).

Using the evolution equations for $\tilde{D}$ and N, and the stationnary phase argument in a Mellin transform (to diaganolize the multiplicative convolution), the scaled moments can be calculated, and one retrieves formula (3) under the guise:

$$\mathcal{F}_q(\Delta\Omega) \propto \left[\frac{\Theta_0}{\Theta}\right]^{(q-1)(d-\mathcal{D}_q)} \text{, with} \qquad \mathcal{D}_q = \frac{\gamma_0}{2}\frac{q+1}{q} \tag{7}$$

We can now introduce the running coupling constant $\alpha_s(P_\perp = k\Theta)$. We use again the Mellin transform, but in the context of a running α_s, one gets a violation of the intermittent behaviour:

$$\mathcal{D}_q = \frac{q+1}{q}\frac{\int_\Theta^{\Theta_0}\gamma_0(\alpha_s(k\theta))d\theta/\theta}{\ln\Theta_0/\Theta} \simeq \simeq \frac{q+1}{q}\gamma_0(\alpha_s(E\Theta_0))\frac{2}{x}(1-\sqrt{1-x}), \tag{8}$$

where $x = \frac{\ln\Theta_0/\Theta}{\ln E\Theta_0/\Lambda}$.

The experimental data on factorial moments and intermittency in e^+-e^-annihilation into hadrons and especially on hadronic decays of $\mathcal{Z}_0$'s are already remarkable[7)]. A few features, like the saturation effects, the increase of intermittency with dimensionality, the dependence of the diffusion direction angle can be seen in the data. However, the tests of angular intermittency need a reappraisal of the most convenient observables. The best indication for confronting angular intermittency with data - if experimentally feasible - would be to give one- and two-dimensional factorial moments directly for small cells in angular variables. Indeed, let us consider a given direction Θ_0, and two angular cells of aperture Θ_1 and Θ_2 around that direction.One gets:

$$\mathcal{F}_q(\Theta_1) = \mathcal{F}_q(\Theta_2)\exp\left[(q-1)\left(\log\Theta_2/\Theta_1 - \frac{q+1}{q}\int_{\Theta_1}^{\Theta_2}\gamma_0(\alpha_s(E\theta))d\theta/\theta\right)\right] \tag{9}$$

(10) is formally independent of the chosen direction Θ_0, except that one has to take into account the application range of perturbative calculations, namely: $0 < \log \frac{\mu}{\Lambda} < \log \frac{E\Theta_1}{\Lambda} < \log \frac{E\Theta_2}{\Lambda} < \log \frac{E\Theta_0}{\Lambda} = \epsilon$. In Fig. 2, we display the predictions for $\mathcal{Z}_0$ jet decays corresponding to 1-dimensional (φ-averaged) and 2-dimensional moments of rank 3 as a function of $\log \Theta_2/\Theta_1$ for two typical values of $\epsilon_2 = \log E\Theta_2/\Lambda$. The range of the variable $\log \Theta_2/\Theta_1$ between 0 and $\log E\Theta_0/\mu \simeq 3$.

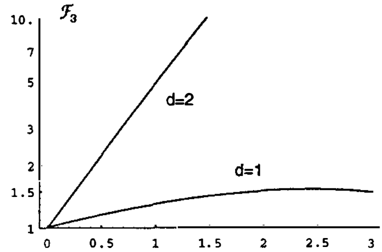

Figure 2a

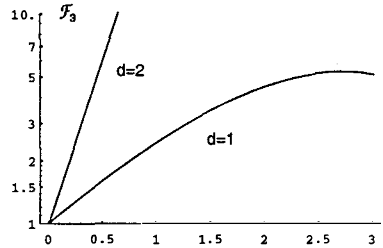

Figure 2b

Figure 2 : $\mathcal{F}_3$ *ratios* as a function of Θ_2/Θ_1 (in log/log units); a) $\epsilon_2 = 6$; b) $\epsilon_2 = 4$.

Note that the saturation effect, clearly present in the one dimensional case, almost desappears for 2-d solid angles cells. Data on factorial moments are in general (except those from ALEPH collaboration[8)]) "horizontally" averaged over different rapidities - i.e. different Θ_0 angles -. We suggest to perform *ratios* of moments for different apertures before an eventual horizontal averaging in order to test the prediction (10). It is interesting to notice that, if this ratio can be measured event by event, the result is predicted to be independent from the angular direction Θ_0, and thus from the definition of the jet axis! However, one has to require to avoid small angles Θ_0. In any case, the check of (10) represents a challenge for QCD phenomenology, taking into account that higher-order and hadronisation corrections are probably diminished in the ratio of moments .

References

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