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A Simple Program to Calculate the Alvarez-Type Linac Cavity

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I. Introduction

The subject of this report is a rather old one. Essential features of the analytical approach can be found in a paper by Hansen[1] which was published in 1939. An improved version was given by Hahn[2] in 1941 and there are numerous other reports, mostly by W. Walkinshaw and his associates at Harwell, which appeared in the 1950's. With the rapid development of digital computers, a more ambitious project was initiated by R. Christian at MURA in 1961 and a program called MESSYMESH was created.[3] After many revisions and refinements, the program was used extensively for the design of Alvarez-type linacs at Fermilab, Brookhaven and Los Alamos. A similar program called JESSY[4] was also used at Brookhaven, mostly for checking the results of MESSYMESH. Another program LALA,[5] developed at Los Alamos, was primarily for calculating the wave-guide structure although it can be used for the drift-tube structure. All three programs use the relaxation method in solving the partial differential equation when the resonating structure is cylindrically symmetric. The accuracy is limited only by the finite number of mesh points that can be handled by a computer in a reasonable amount of running time. For MESSYMESH, the typical accuracy is 0.2% in the frequency and ~5% in the field values.

Recently, a renewed interest in these programs arose in connection with the proposed construction of deuteron linacs.[6] A few runs of LALA have been tried by D. Johnson with CDC-6600 at Fermilab and with CDC-7600 at Berkeley. The run with CDC-6600 has been too slow to be of a practical use. The use of the Berkeley computer from the terminal at Fermilab is often awkward and expensive for studying many cases. Besides, LALA is intended for arbitrary shapes of boundaries so that its results are generally believed to be less accurate than results from MESSYMESH, which is limited to calculating drift tubes with circular corners. Unfortunately, MESSYMESH was originally written for IBM 704 and often a successful result can be obtained only with an expert interaction with the computer during the course of its running. This is particularly the case when the initial loading of the field to start the relaxation is not a very good one. Without the solution already obtained for a geometry which is very close to the one to be studied, the initial loading is limited to a very simple TM_{010}-type field of a hollow cylindrical cavity. The electric field is everywhere in the axial direction and the resonance frequency is determined by a simple boundary condition on the wall. For the linac cavities at Fermilab with diameters ranging from 0.84 m to 0.94 m, the resulting frequency is from 273 MHz to 244 MHz compared to the correct value of 201 MHz.

The primary purpose of making a simple "home-made" program to calculate the resonant structure is not to replace more elaborate ones like MESSYKESH but rather to alleviate some of the difficulties mentioned above. The program reported here is intended to be run with PDP-10 at Fermilab for approximate but rapid investigations of resonant geometries. It can also supply a much better initial loading of the field for MESSYMESH than the simple TM_{010} field so that the convergence of the relaxation may be improved. Comparison with MESSYMESH results for Fermilab linac indicates that the error in the frequency is $\leq .3\%$. Since the error is almost entirely due to the replacement of the outer corner circle of the drift tube by a rectangular step, it should give a better accuracy when the size of the circular corner (relative to the drift tube size) is less than the one for the linac at Fermilab. The most serious deficiency of the program is its inability to calculate accurately quantities like transit time factor that are directly related to the distribution of the electric field on the axis.[7] In principle, there is no difficulty in adding the bore hole with a straight corner in the program but the resulting complexity and the longer running time do not seem to be justified by the benefits of this extension.

II. Basic Formalism

The simplest shape of the drift tube is a cylinder without curved corners or a bore hole. Fig. 1 shows the cross section of the structure where the solid border line represents the conducting wall. The boundary condition along this line is E(tangential) = 0 which can also be expressed as the normal derivative of $rH_\theta = 0$. Furthermore, all field values must be finite along the axis $r = 0$. Since MESSYMESH solves the differential equation for rH_θ, the magnetic field H_θ and the derivative $\partial(rH_\theta)/\partial_r$ are used in this program as quantities to be matched on the boundary $r = d$ of two regions M and N. Matching of other field components are then automatically assured. Using Bessel functions ($J_{0,1}$, $Y_{0,1}$) and modified Bessel functions ($I_{0,1}$, $K_{0,1}$), one can write down these two quantities in each region as follows:

<u>region M</u> $d \leq r \leq b$, $-L \leq z \leq L$

$$H_\theta = c_o F_1(kr) + \sum_{m=1}^{\infty} c_m G_1(s_m r) \cos(m\pi z/L) \tag{1}$$

$$\partial(rH_\theta)/\partial r = c_o kr F_o(kr) - \sum_{m=1}^{\infty} c_m s_m r G_o(s_m r) \cos(m\pi z/L) \tag{2}$$

<u>region N</u> $0 \leq r \leq d$, $-g \leq z \leq g$

$$H_\theta = a_o J_1(kr) + \sum_{n=1}^{\infty} a_1 I_1(t_n r) \cos(n\pi z/g) \tag{3}$$

$$\partial(rH_\theta)/\partial r = a_o kr J_o(kr) + \sum_{n=1}^{\infty} a_n t_n r I_o(t_n r) \cos(n\pi z/g) \tag{4}$$

where $k = 2\pi/\lambda = \omega/c$, $s_m^2 = (m\pi/L)^2 - k^2$, $t_n^2 = (n\pi/g)^2 - k^2$ and

$$F_o(kr) = Y_o(kr) - [Y_o(kb)/J_o(kb)]\ J_o(kr),$$
$$F_1(kr) = Y_1(kr) - [Y_o(kb)/J_o(kb)]\ J_1(kr),$$
$$G_o(s_m r) = K_o(s_m r) - [K_o(s_m b)/I_o(s_m b)]\ I_o(s_m r),$$
$$G_1(s_m r) = K_1(s_m r) + [K_o(s_m b)/I_o(s_m b)]\ I_1(s_m r).$$

Summations over indices m and n must be terminated at some values, M and N, respectively in the program. It has been found that M = 50 ~ 100 and N = 5 ~ 10 are sufficient for finding the resonant frequency. By matching H_θ and $\partial(rH_\theta)/\partial r$ at the boundary $r = d$, one can establish a set of (N+1) homogeneous linear equations for (a_o, a_1,....a_N) and the resonant frequency is obtained by making the determinant of the coefficient matrix vanish. N inhomogeneous linear equations are then solved for (a_1, a_2,...a_N). The arbitrary value of a_o specifies the voltage across the gap,

$$V = \int_{-g}^{g} E_z dz = Z_o\ g\ a_o$$

where Z_o = 376.7 ohms is the impedance of the vacuum.* Coefficients $(c_o, c_1, \ldots c_M)$ in (1) and (2) are linear functions of a_n's.

It is difficult to introduce a circular corner or even a straight but slanted corner for the analytical treatment. In order to simulate the effect of the circular corner, a simple one-step shape has been tried as shown in Fig. 2. There is no unique way of introducing the step and the choice

$a = d - 2R/3$, $h = g + R/2$ (R = radius of the circular corner)

taken here is more or less arbitrary. In the new region P, there is no obvious boundary condition to fix the radial dependence and the number of unknown expansion coefficients is doubled.

<u>region P</u> $a \leq r \leq d$, $-h \leq z \leq h$

$$H_\theta = q_o J_1(kr) + Q_o Y_1(kr) + \sum_{p=1}^{\infty} [q_p I_1(u_p r) + Q_p K_1(u_p r)] \cos(p\pi z/h) \quad (5)$$

$$\partial(rH_\theta)/\partial r = q_o kr J_o(kr) + Q_o kr Y_o(kr) + \sum_{p=1} u_p k[q_p I_o(u_p r) - Q_p K_o(u_p r)] \cos(p\pi z/h) \quad (6)$$

where $u_p^2 = (p\pi/h)^2 - k^2$.

Again it is not necessary to take many terms in the summation over the index p. In the calculation reported here, ten q_p's (and q_o) and ten Q_p's (and Q_o) are used together with N = 10 for a's and M = 100 for c's. One can construct a set of homogeneous linear equations, twenty-two altogether, for q's and Q's. The procedure for finding the resonant frequency and calculating the

*MKS unit is used. V is in volts, g in meters and a_o (magnetic field) is in ampere/m.

field is identical to the previous case. Analytically, there is no limit to introducing more steps to better represent the circular corner. However, the amount of bookkeeping work and the running time increase very rapidly and the program will go beyond the original scope.

III. Results

Fifteen different geometries in Table 1 have been tested to compare resonant frequencies with MESSYMESH results. The correct frequency is very close to the design value of 201.25 MHz for all cases. Results are tabulated in Table 2 together with MESSYMESH values. Without a step, the frequency is always underestimated and the relative error is between two to eight percent. The underestimate is reasonable since a drift tube without the circular corner should give a heavier loading (more capacitance across the gap) compared to the one of the same dimension but with the corner. The effect of the corner on the resonant frequency is by no means negligibly small. By taking a crude representation of the corner, one improves the accuracy significantly. The relative error is reduced to less than 0.3 per cent in all cases. It is hoped that the resulting field values provide a good initial loading for MESSYMESH runs when they are needed for the design of new linacs.

Tutorial conversations with Tom Collins have been very helpful during the course of this work and they are greatly appreciated.

References

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3. T.W. Edwards, MURA-622, June 15, 1961; D.E. Young, Minutes of the Conference on Linear Accelerators for High Energies, Brookhaven National Laboratory, August 1962, p. 76.

4. P.F. Dahl et. al., Proceedings of the 1966 Linear Accelerator Conference, Los Alamos Scientific Laboratory, October 1966, p. 115.

5. H.C. Hoyt, ibid., p. 119.

6. Proposal for an Accelerator-Based Neutron Generator, BNL 20159, Brookhaven National Laboratory, July 1975 (Edited by P. Grand). A similar proposal is being prepared by Holifield (Oak Ridge) National Laboratory.

7. R. Taylor, A.E.R.E Report R3012, 1959; A. Carne and P.M. Lapostolle, Proceedings of the 1966 Linear Accelerator Conference, p. 201.

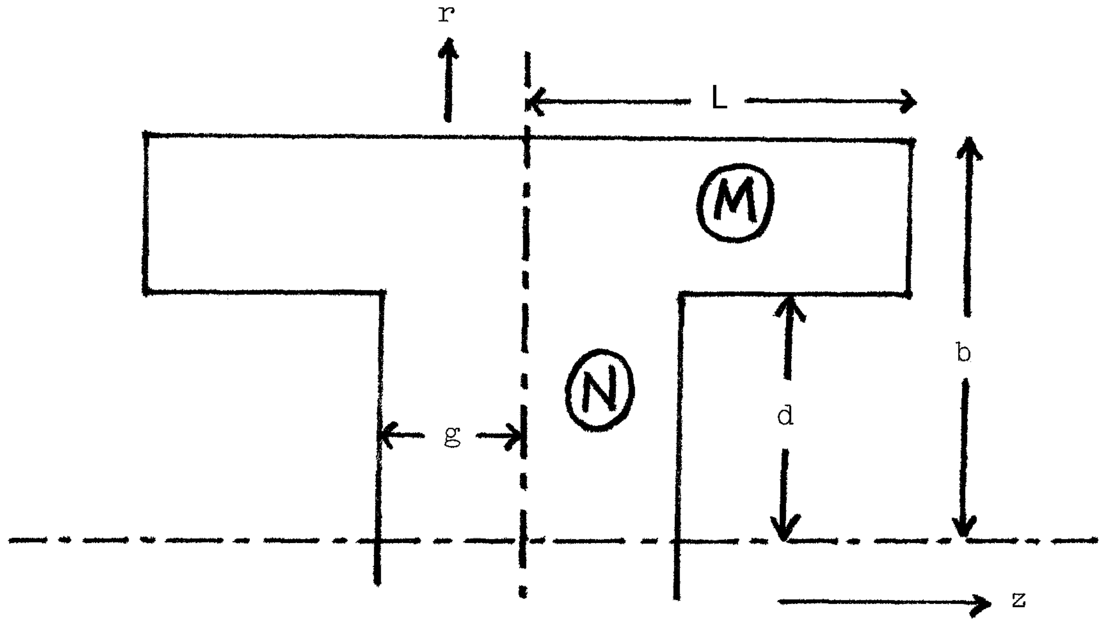

Fig. 1. Cell geometry with the simplest drift tube.

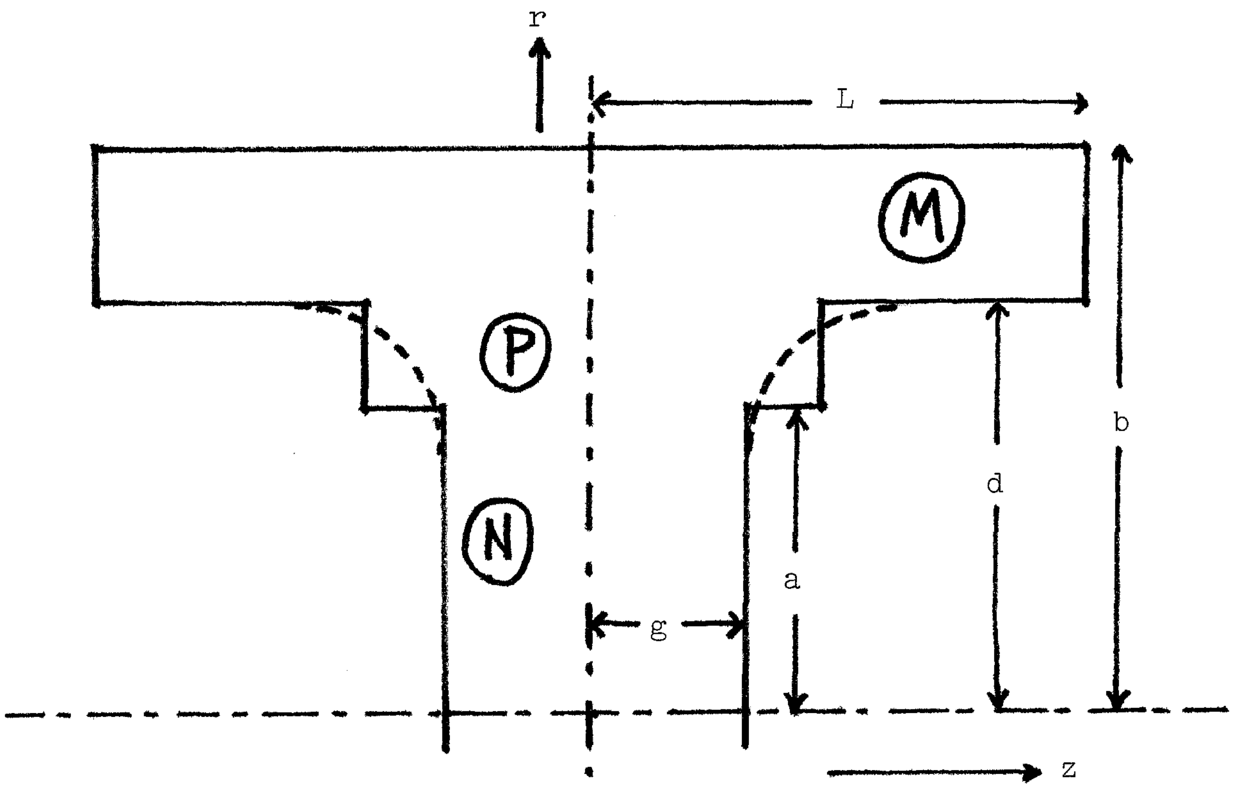

Fig. 2. Cell geometry with the one-step corner. When the corner radius of the real drift tube is R, $a = d - 2R/3$ and $h = g + R/2$.

Table 1. Linac Cell Geometry

Cavity diameter = 2b

Drift tube diameter = 2d

Cell length = 2L

Gap length = 2g

Outer corner radius = R

Bore hole radius = r

MESSYMESH #	2b	2d	2L	2g	R	r
32012	94 cm	18 cm	6.037 cm	1.292 cm	2 cm	1 cm
32020			7.824	1.766		
32028			9.852	2.354		
32029	94	18	10.117	2.430	2	1.25
32049			15.802	4.324		
32069			21.824	6.731		
32141	90	16	22.176	4.386	4	1.5
32203			31.342	8.002		
32234			40.763	12.749		
32163	88	16	41.096	12.229	4	1.5
32264			51.380	18.263		
32166			61.535	25.087		
32167	84	16	61.815	22.620	5	2
32336			67.420	26.583		
32170			73.056	30.787		

Table 2. Comparison of Resonant Frequency

MESSYMESH #	MESSYMESH	no-step	one-step
32012	201.34 MHz	192.17 MHz	200.74 MHz
32020	201.29	193.24	201.01
32028	201.34	194.27	201.26
32029	201.33	194.30	201.21
32049	201.30	195.88	201.29
32069	201.26	196.92	201.34
32141	201.22	185.55	201.76
32203	201.24	189.89	201.74
32234	201.30	192.33	201.65
32163	201.24	191.78	201.63
32264	201.25	193.37	201.50
32166	201.25	194.26	201.47
32167	201.27	191.05	201.51
32336	201.28	191.59	201.47
32170	201.24	192.00	201.43