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$K-$MESON PRODUCTION IN ELECTRON-POSITRON ANNIHILATION

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A model for electromagnetic form factors of the charged and neutral on-shell $K-$mesons is developed. The formalism is based on Lagrangian of Chiral Perturbation Theory which includes vector mesons. Presented terms describe even- as well as odd-intrinsic-parity interactions up to $\mathcal{O}(p^4)$. The kaon form factor, calculated without parameters fine tuning, is compared to experiment for space-like and time-like photon momentum. The status of the muon anomalous magnetic moment (AMM) is reviewed and contribution of the two-kaon channels to AMM is calculated.

1 Introduction

$K-$mesons (or kaons) are the particles with quantum numbers $I\ (J^P) = \frac{1}{2}\ (0^-)$ and nonzero "strangeness", which have lead to discovery of many interesting phenomena due to weak interactions, such as strangeness oscillation, K^0 regeneration, CP violation. These particles have the following basic properties [1].

Quark composition of mass eigenstates reads :

$$\begin{aligned} K^+ &= u\bar{s}, \qquad K^0 = d\bar{s} \qquad (\text{strangeness} = +1), \\ K^- = \overline{K^+} &= \bar{u}s, \qquad \bar{K}^0 = \bar{d}s \qquad (\text{strangeness} = -1). \end{aligned}$$

These particles are created in strong-interaction processes.

Time of life is defined and measured for the states participating in weak decays

$$\begin{aligned} &K^\pm: \ \tau = 1.2 \times 10^{-8}\,\text{s}, \\ &K_S = (1+|\varepsilon|^2)^{-1/2}(K_1 + \varepsilon K_2): \ \tau = 0.9 \times 10^{-10}\,\text{s}, \quad \text{short-lived}, \\ &K_L = (1+|\varepsilon|^2)^{-1/2}(K_2 + \varepsilon K_1): \ \tau = 5.2 \times 10^{-8}\,\text{s}, \quad \text{long-lived}, \end{aligned}$$

where CP-eigenstates are defined as

$$\begin{aligned} K_1 &= \frac{1}{\sqrt{2}}(K^0 + \bar{K}^0): \ \ CP\text{-even}, \\ K_2 &= \frac{1}{\sqrt{2}}(K^0 - \bar{K}^0): \ \ CP\text{-odd} \end{aligned}$$

and parameter $\varepsilon \sim 10^{-3}$ describes small CP violation effects.

Electromagnetic properties of $K-$mesons. Experimental information on $K-$meson electromagnetic (EM) properties in the time-like region ($q^2 \equiv s \geq 4m_K^2$) of photon momentum q comes from measurements of the cross section of electron-positron annihilation $e^+e^- \to K\bar{K}$:

$$\sigma(e^+e^- \to K\bar{K}) = \frac{\pi\alpha^2}{3q^2}\big(1 - \frac{4m_K^2}{q^2}\big)^{3/2}|F_K(q^2)|^2. \qquad (1)$$

High precision measurements are performed by CMD-2 and SND Collaborations in Novosibirsk [2, 3], and KLOE Collaboration in Frascati (Italy) [4].

In the space-like region ($q^2 \equiv s < 0$) the form factor is measured in:

(i) kaon scattering on atomic electrons at relatively small momentum transfer $-s < 0.16$ GeV^2 (CERN, SPS [5]),

(ii) electron-proton scattering with kaon-hyperon production ($ep \to e\Lambda K^+$ and $ep \to e\Sigma^0 K^+$) at large momentum transfer up to $-s \sim 3$ GeV^2 (currently are carried out at Jefferson Laboratory in USA [6]).

Main motivations of the present work are:

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1. Test of effective hadronic models such as
(i) Chiral Perturbation Theory (ChPT) – effective low-energy theory,
(ii) vector-meson dominance of electromagnetic interaction,
(iii) anomalous Lagrangians[1].

2. Study of vector mesons ($J^P = 1^-$): $\rho(770)$, $\omega(782)$, $\phi(1020)$, and their radial excitations $\rho' = \rho(1450)$, $\omega' = \omega(1420)$, $\phi' = \phi(1680)$, etc.

3. Calculation of hadronic contribution to the muon anomalous magnetic moment (AMM). Hadronic contribution is the main source of uncertainty in theoretical prediction for muon AMM. The existing discrepancy between theory and experiment may indicate new physics beyond the Standard Model, thus it is important to precisely calculate every allowed contribution in Standard Model.

Kaon form factors. The quark EM current is

$$j^\mu_{em}(x) = \frac{2}{3}\bar{u}(x)\gamma^\mu u(x) - \frac{1}{3}\bar{d}(x)\gamma^\mu d(x) - \frac{1}{3}\bar{s}(x)\gamma^\mu s(x). \tag{2}$$

The EM form factors (FF's) are defined in terms of this current

$$\langle K(p_1)\bar{K}(p_2)|j^\mu_{em}(x=0)|0\rangle \equiv (p_1 - p_2)^\mu F_K(q^2), \tag{3}$$

where $q^2 = (p_1 + p_2)^2 \equiv s$.

The form factors are analytic functions of q^2 and describe both the time-like and space-like regions of photon momentum.

2 Formalism

Meson interactions in ChPT. At low energies, strong, electromagnetic and weak interactions are described by effective Lagrangian of Chiral Perturbation Theory (ChPT). The underlying theory of strong interactions – Quantum Chromodynamics (QCD) – has global chiral symmetry $SU(3)_L \times SU(3)_R$, if masses of the quarks are zero, and ChPT has this symmetry built in on the hadronic level.

The version of ChPT which includes explicit vector meson degrees of freedom (Ecker, Gasser, Pich and de Rafael [7, 8]) has an extended range of applicability. In this approach the vector mesons $\rho, \omega, \phi, ...$ are not considered as gauge bosons of chiral symmetry and are treated on equal footing with other mesons.

The chiral symmetric part of Lagrangian is

$$\begin{aligned}\mathcal{L}_{\mathrm{ch-sym}} &= \frac{F_\pi^2}{4}\mathrm{Tr}(D_\mu U D^\mu U^\dagger) + \frac{eF_V}{2\sqrt{2}}F^{\mu\nu}\mathrm{Tr}(V_{\mu\nu}(uQu^\dagger + u^\dagger Q u)) \\ &+ \frac{iG_V}{\sqrt{2}}\mathrm{Tr}(V_{\mu\nu}u^\mu u^\nu) + \mathcal{L}_{V,\ kin} + [\text{axial-vector + scalar mesons}],\end{aligned} \tag{4}$$

where the nonlinear field representation for pseudoscalar mesons is

$$U \equiv \exp(i\sqrt{2}\Phi/F_\pi), \qquad u = U^{1/2}, \qquad u^\mu = iu^\dagger(D^\mu U)u^\dagger.$$

Here Φ is octet of pseudoscalar mesons ($J^P = 0^-$) – Nambu-Goldstone bosons of spontaneously broken chiral symmetry

$$\Phi = \begin{pmatrix} \pi^0/\sqrt{2} + \eta_8/\sqrt{6} & \pi^+ & K^+ \\ \pi^- & -\pi^0/\sqrt{2} + \eta_8/\sqrt{6} & K^0 \\ K^- & \bar{K}^0 & -2\eta_8/\sqrt{6} \end{pmatrix},$$

and $F_\pi = 92.4$ MeV is constant of the weak pion decay $\pi^+ \to \mu^+\nu_\mu$. The covariant derivative is defined as

$$D_\mu U \equiv \partial_\mu U + \imath e B_\mu [U, Q]$$

with quark charge matrix for flavor $SU(3)_f$

$$Q \equiv \mathrm{diag}\left(\frac{2}{3}, -\frac{1}{3}, -\frac{1}{3}\right),$$

B^μ – electromagnetic field, $F^{\mu\nu} = \partial^\mu B^\nu - \partial^\nu B^\mu$, $V_{\mu\nu}$ is nonet of vector mesons ($J^{PC} = 1^{--}$)

$$V_{\mu\nu} = \begin{pmatrix} \rho^0/\sqrt{2} + \omega_8/\sqrt{6} & \rho^+ & K^{*+} \\ \rho^- & -\rho^0/\sqrt{2} + \omega_8/\sqrt{6} & K^{*0} \\ K^{*-} & \bar{K}^{*0} & -2\omega_8/\sqrt{6} \end{pmatrix}_{\mu\nu} + \frac{(\omega_1)_{\mu\nu}}{\sqrt{3}},$$

[1]By "anomalous" we mean interactions which do not conserve intrinsic parity and thus do not conserve "normality" quantum number $\mathcal{N}$.

Table 1. Electromagnetic coupling constants for vector mesons $V = \rho^0, \omega, \phi$

	ρ^0	ω	ϕ
$f_V = m_\rho/F_V$, $SU(3)$:	f	$3f$	$-3f/\sqrt{2}$
exper. f_V	4.966 ± 0.038	17.06 ± 0.29	-13.38 ± 0.21

Table 2. Vector-meson coupling to two pseudoscalars in $SU(3)_f$

	$\pi^+\pi^-$	K^+K^-	$K^0\bar{K}^0$
ρ^0	G_V	$G_V/2$	$-G_V/2$
ω	–	$G_V/2$	$G_V/2$
ϕ	–	$-G_V/\sqrt{2}$	$-G_V/\sqrt{2}$

in the antisymmetric tensor representation of the vector fields. $\mathcal{L}_{V,\ kin}$ is the kinetic term for vector mesons.

The chiral symmetry breaking part

$$\mathcal{L}_{\mathrm{ch-sym.break}} = \frac{F_\pi^2}{4}\,\mathrm{Tr}\,(\chi U^\dagger + \chi^\dagger U) \tag{5}$$

arises due to non-vanishing quark masses $m_u = 1.5 - 4$ MeV, $m_d = 4 - 8$ MeV, $m_s = 80 - 130$ MeV in QCD and quark condensate

$$\langle 0|\bar{q}q|0\rangle \approx (-240 \pm 10\ \mathrm{MeV})^3 \ \ (\text{at scale } \mu = 1\ \mathrm{GeV})$$

where the vacuum is assumed $SU(3)_f$ symmetric, i.e. $\langle 0|\bar{q}q|0\rangle = \langle 0|\bar{u}u|0\rangle = \langle 0|\bar{d}d|0\rangle = \langle 0|\bar{s}s|0\rangle$.

The condensate value gives typical scale parameter in QCD

$$-\langle 0|\bar{q}q|0\rangle^{1/3} \approx \Lambda_{QCD} = 200 - 300\ \mathrm{MeV}$$

which rules the energy dependence of the running coupling constant

$$\alpha_s(Q) = \frac{2\pi}{(\frac{11}{3}N_c - \frac{2}{3}N_f)\ln(Q/\Lambda_{QCD})}, \tag{6}$$

where N_f (N_c) is the number of "active" quark flavors (quark colors).

Pion and kaon masses squared are proportional to quark masses and the condensate value, and the quantity χ is

$$\chi = -\frac{2}{F_\pi^2}\mathrm{diag}(m_u, m_d, m_s)\langle 0|\bar{q}q|0\rangle \overset{SU(2)_f}{=} \mathrm{diag}(m_\pi^2, m_\pi^2, 2m_K^2 - m_\pi^2).$$

Expansion of Lagrangian in powers of meson momenta (or derivatives of meson fields) gives interactions with even number of pseudoscalar mesons

$$\begin{aligned}
\mathcal{L}_{\gamma\Phi\Phi} &= \imath e B_\mu \mathrm{Tr}(Q[\partial_\mu\Phi, \Phi]),\\
\mathcal{L}_{\gamma\gamma\Phi\Phi} &= -\frac{e^2}{2}B^\mu B_\mu \mathrm{Tr}\big([\Phi, Q]^2\big),\\
\mathcal{L}_{\gamma V} &= e\frac{F_V}{\sqrt{2}}F^{\mu\nu}\mathrm{Tr}(V_{\mu\nu}Q), \qquad (\text{vector} - \text{meson dominance})\\
\mathcal{L}_{V\Phi\Phi} &= \imath\frac{\sqrt{2}G_V}{F_\pi^2}\mathrm{Tr}(V_{\mu\nu}\partial^\mu\Phi\partial^\nu\Phi).
\end{aligned}$$

These interactions conserve "normality" quantum number

$$\mathcal{N} = \mathrm{Parity} \times (-1)^{\mathrm{spin}}.$$

The coupling constants F_V and G_V can be found from experimental widths of decays $\Gamma(\rho \to e^+e^-)$ and $\Gamma(\rho \to \pi\pi)$, respectively. It will be further convenient to use other constants, g and f, related to F_V and G_V. Using the data from [1] we obtain

$$\begin{aligned}
F_V &= 156.35\ \mathrm{MeV}, &\qquad \mathrm{G_V} &= 65.65\ \mathrm{MeV},\\
f &\equiv \frac{m_\rho}{F_V} = 4.966, &\qquad g &\equiv \frac{G_V m_\rho}{F_\pi^2} = 5.965.
\end{aligned}$$

Anomalous meson-meson and meson-photon interactions. Interactions of this type are not described by Lagrangians (4) and (5). They are proportional to Levi-Chivita tensor $\epsilon^{\mu\nu\alpha\beta}$, couple odd number of pseudoscalar mesons and do not conserve "normality" $\mathcal{N}$.

Lagrangian of Wess, Zumino and Witten [9, 10] describes interactions of photons with pseudoscalar mesons

$$\mathcal{L}_{WZW} = \mathcal{L}^{(1)}_{WZW} + \mathcal{L}^{(2)}_{WZW}, \tag{7}$$

$$\begin{aligned}
\mathcal{L}^{(1)}_{WZW} &= -\frac{eN_c}{48\pi^2}\epsilon^{\mu\nu\alpha\beta}B_\mu \mathrm{Tr}\Big(Q\big[(\partial_\nu U)(\partial_\alpha U^+)(\partial_\beta U)U^+ \\
&\quad -(\partial_\nu U^+)(\partial_\alpha U)(\partial_\beta U^+)U\big]\Big), \\
\mathcal{L}^{(2)}_{WZW} &= \frac{\imath e^2 N_c}{24\pi^2}\epsilon^{\mu\nu\alpha\beta}(\partial_\mu B_\nu)B_\alpha \\
&\quad \times \mathrm{Tr}\Big(Q^2(\partial_\beta U)U^+ + Q^2U^+(\partial_\beta U) \\
&\quad -\frac{1}{2}QUQ(\partial_\beta U^+) + \frac{1}{2}QU^+Q(\partial_\beta U)\Big).
\end{aligned}$$

The lowest-order WZW interaction is

$$\mathcal{L}_{\gamma\Phi\Phi\Phi} = -\frac{\imath\sqrt{2}eN_c}{12\pi^2 F_\pi^3}\epsilon^{\mu\nu\alpha\beta}B_\mu \mathrm{Tr}\big(Q\partial_\nu\Phi\partial_\alpha\Phi\partial_\beta\Phi\big), \tag{8}$$

$$\mathcal{L}_{\gamma\gamma\Phi} = -\frac{\sqrt{2}e^2N_c}{8\pi^2 F_\pi}\epsilon^{\mu\nu\alpha\beta}\partial_\mu B_\nu\partial_\alpha B_\beta \mathrm{Tr}\big(Q^2\Phi\big). \tag{9}$$

The latter in particular describes well-known $\pi^0\gamma\gamma$ interaction and $\pi^0 \to \gamma\gamma$ decay

$$\mathcal{L}_{\pi^0\gamma\gamma} = -\frac{e^2N_c}{24\pi^2 F_\pi}\epsilon^{\mu\nu\alpha\beta}\partial_\mu B_\nu\partial_\alpha B_\beta\pi^0.$$

ChPT also predicts anomalous interactions of vector mesons with pseudoscalar mesons [11]

$$\mathcal{L}_{VV\Phi} = -\frac{\sqrt{2}\sigma_V}{F_\pi}\epsilon^{\mu\nu\alpha\beta}\mathrm{Tr}(\partial_\mu V_\nu\{\Phi, \partial_\alpha V_\beta\}), \tag{10}$$

$$\mathcal{L}_{V\gamma\Phi} = -\frac{4\sqrt{2}eh_V}{F_\pi}\epsilon^{\mu\nu\alpha\beta}\partial_\mu B_\nu \mathrm{Tr}(V_\alpha\{\partial_\beta\Phi, Q\}), \tag{11}$$

$$\mathcal{L}_{V\Phi\Phi\Phi} = -\frac{2i\sqrt{2}\theta_V}{F_\pi^3}\epsilon^{\mu\nu\alpha\beta}\mathrm{Tr}(V_\mu\,\partial_\nu\Phi\,\partial_\alpha\Phi\,\partial_\beta\Phi) \tag{12}$$

with free parameters σ_V, h_V, θ_V (see Table 3).

An extension of WZW anomaly for vector and axial-vector mesons was suggested by Kaymakcalan, Rajeev, Schechter, Ko and Rudaz [12]

$$\begin{aligned}
\mathcal{L}_{V\Phi\Phi\Phi} &= \frac{\imath g}{4\pi^2 F_\pi^3}\epsilon^{\mu\nu\alpha\beta}\mathrm{Tr}(V_\mu\,\partial_\nu\Phi\,\partial_\alpha\Phi\,\partial_\beta\Phi), \\
\mathcal{L}_{VV\Phi} &= \frac{3g^2}{8\sqrt{2}\pi^2 F_\pi}\epsilon^{\mu\nu\alpha\beta}\mathrm{Tr}(\partial_\mu V_\nu\partial_\alpha V_\beta\Phi)
\end{aligned}$$

with $g = 5.96$ taken from $\rho \to \pi\pi$ decay and EM field is included by the substitution

$$V_\mu \to V_\mu + \frac{\sqrt{2}e}{g}QB_\mu$$

As a result one obtains an effective $V\gamma\Phi$ interaction

$$\mathcal{L}_{V\gamma\Phi} = -\frac{3eg}{4\pi^2 F_\pi}\epsilon^{\mu\nu\alpha\beta}\partial_\mu B_\nu \mathrm{Tr}(QV_\alpha\partial_\beta\Phi).$$

Now we calculate the kaon form factors which in the present model are

$$F_{K^+}(s) = 1 - \sum_{V=\rho,\omega,\phi}\frac{g_{VK^+K^-}}{f_V(s)}A_V(s), \tag{13}$$

$$F_{K^0}(s) = -\sum_{V=\rho,\omega,\phi}\frac{g_{VK^0\bar{K}^0}}{f_V(s)}A_V(s), \tag{14}$$

$$A_V(s) \equiv \frac{s}{s - m_V^2 - \Pi_V(s)},$$

Table 3. Values of parameters σ_V, h_V, θ_V for vector mesons

Coupling constants	h_V	θ_V	σ_V
"ideal" values [12]	$\frac{3g}{32\sqrt{2}\pi^2} = 0.040$	$\frac{g}{8\sqrt{2}\pi^2} = 0.054$	$\frac{3g^2}{32\pi^2} = 0.34$
fixed by experiment	0.039	0.0011	0.33
Nambu-Jona-Lasinio model [11]	0.040	0.053	0.33

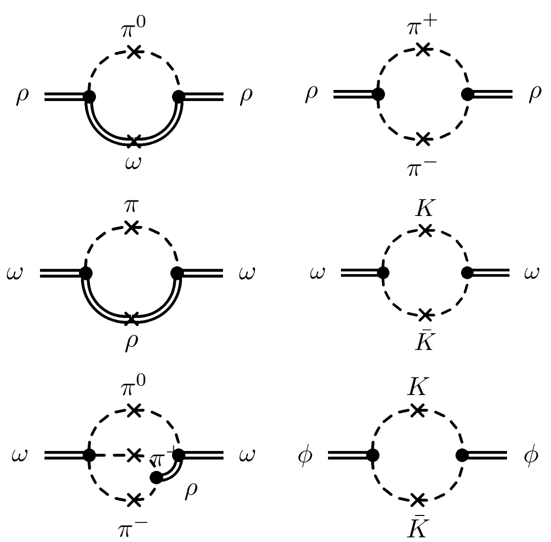

Figure 1. Loops included in self-energy of vector mesons.

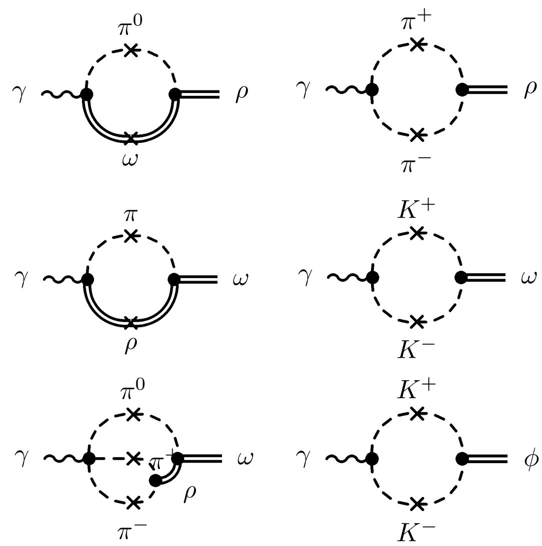

Figure 2. Loop corrections for photon-vector-meson vertex

where $\Pi_V(s)$ is self-energy operator of vector meson $V = \rho, \omega, \phi$.

The correct normalization conditions at $q^2 = 0$

$$F_{K^+}(0) = 1, \qquad F_{K^0}(0) = 0 \tag{15}$$

are fulfilled due to gauge invariance of the theory.

Self-energy operators. Dressed ("exact", or full) propagator of vector particles includes self-energy operators $\Pi_V(s)$ which account for intermediate states, such as $\pi^+\pi^-$, $\omega\pi^0$, $K\bar{K}$, $\omega\pi^0 \to \pi^0 K^+ K^-$ for ρ meson, etc. The dominant contributions are (see Fig. 1)

$$\begin{aligned}
\Pi_\rho &= \Pi_{\rho(\pi^0\omega)\rho} + \Pi_{\rho(\pi\pi)\rho}, \\
\Pi_\omega &= \Pi_{\omega(\pi^0\rho)\omega} + \Pi_{\omega(KK)\omega} + 2\Pi_{\omega(3\pi,\pi\rho)\omega}, \\
\Pi_\phi &= \Pi_{\phi(KK)\phi},
\end{aligned}$$

Imaginary part of self-energy gives rise to energy-dependent widths of vector mesons

$$\Gamma_V(s) = -m_V^{-1}\,\mathrm{Im}\,\Pi_V(s)$$

One can find the imaginary parts of self-energy by applying the Cutkosky rules. To restrict fast growth of the partial widths with s we have to introduce a cut-off form factors (for details see [13]).

Electromagnetic vertex modification. Vertex corrections (see Fig. 2) are related to self-energy corrections, for example

$$\mathrm{Im}\,\Pi_{\gamma(\pi^0\omega)\rho}(s) = \frac{\sqrt{2}e\,h_V}{\sigma_V}\mathrm{Im}\,\Pi_{\rho(\pi^0\omega)\rho}(s). \tag{16}$$

Modified (or exact) EM vertex satisfies equation

$$\frac{1}{f_V(s)} = \frac{1}{f_V^{(0)}} - \frac{\imath}{e\,s}\sum_c \mathrm{Im}\Pi_{\gamma(c)V}(s) \tag{17}$$

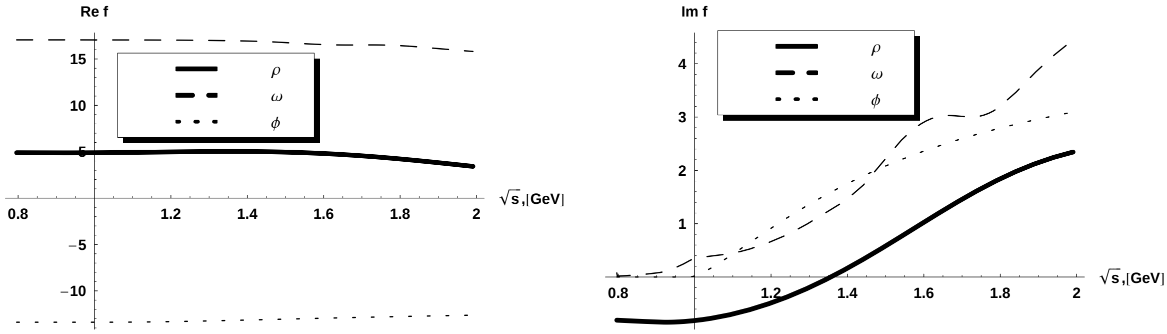

Figure 3. Real and imaginary parts of modified $\gamma - V$ vertex.

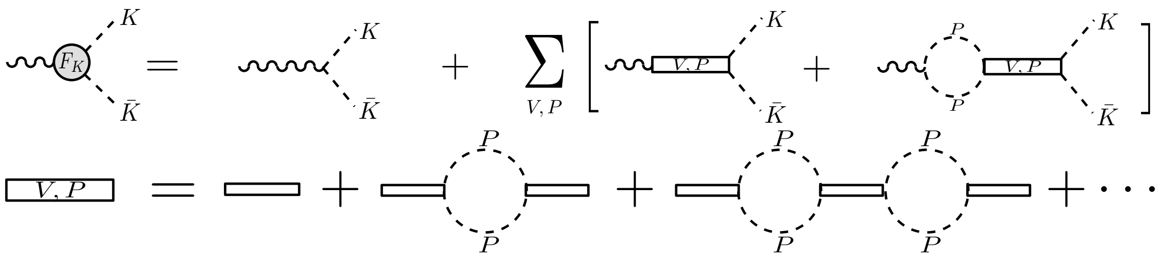

Figure 4. Electromagnetic form factor of K^+ (K^-).

and at $s = m_V^2$ (on the mass shell) describe the leptonic decay widths of vector mesons

$$\left| f_V(s = m_V^2) \right|^2 = \frac{4}{3}\pi\alpha^2 \frac{m_V}{\Gamma(V \to e^+e^-)}.$$

This, together with information from "Particle Data Group" compilation [1] allows one to find the "bare" couplings

$$f_\rho^{(0)} = 5.026, \qquad f_\omega^{(0)} = 17.060, \qquad f_\phi^{(0)} = -13.382$$

and then obtain real and imaginary parts of the momentum-dependent couplings $f_V(s)$ for arbitrary s (see Fig. 3).

Fig. 4 schematically illustrates the model for the form factors including self-energy and EM vertex loop corrections.

Higher vector resonances. Contribution from higher resonances $\rho' = \rho(1450)$, $\omega' = \omega(1420)$ and $\phi' = \phi(1680)$ is

$$\Delta F_{K^+}(s) = -\sum_{V'=\rho',\omega',\phi'} \frac{g_{V'K^+K^-}}{f_{V'}(s)} A_{V'}(s), \tag{18}$$

$$\Delta F_{K^0}(s) = -\sum_{V'=\rho',\omega',\phi'} \frac{g_{V'K^0\bar{K}^0}}{f_{V'}(s)} A_{V'}(s). \tag{19}$$

We assume $SU(3)_f$ for ratios of the strong and EM couplings for "primed" resonances

$$\begin{aligned} \frac{g_{\rho' K^+K^-}}{f_{\rho'}} : \frac{g_{\omega' K^+K^-}}{f_{\omega'}} : \frac{g_{\phi' K^+K^-}}{f_{\phi'}} &= \frac{1}{2} : \frac{1}{6} : \frac{1}{3}, \\ \frac{g_{\rho' K^0\bar{K}^0}}{f_{\rho'}} : \frac{g_{\omega' K^0\bar{K}^0}}{f_{\omega'}} : \frac{g_{\phi' K^0\bar{K}^0}}{f_{\phi'}} &= -\frac{1}{2} : \frac{1}{6} : \frac{1}{3} \end{aligned}$$

and use the known branching ratios from [1], then obtain

$$\begin{aligned} g_{\rho' K^+K^-}/f_{\rho'} &= -0.063, \qquad g_{\omega' K^+K^-}/f_{\omega'} = -0.021, \\ g_{\phi' K^+K^-}/f_{\phi'} &= -0.036. \end{aligned}$$

High q^2 behavior of form factors. On the basis of quark counting rule in perturbative QCD (Lepage, Brodsky, Farrar and Jackson [14])

$$F_{K^+}(s) \to \frac{A}{s} \quad \text{at} \quad s \to -\infty, \qquad A = -16\pi F_\pi^2 \alpha_s(s). \tag{20}$$

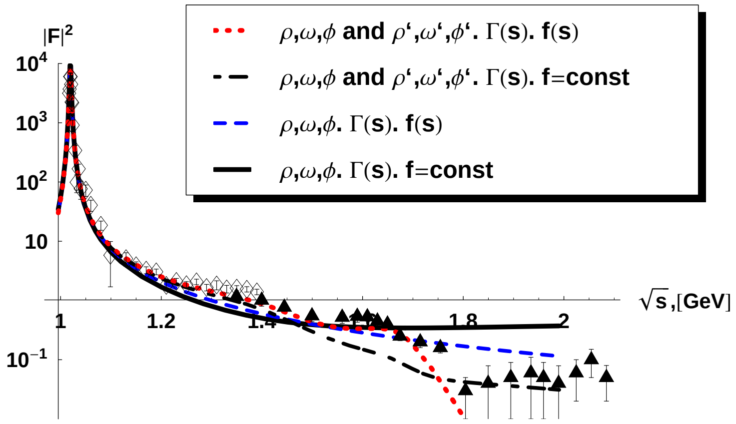

Figure 5. Charged kaon form factor in the time-like region $s \geq 4m^2_{K\pm}$. Data: diamonds (open) are from [15], triangles – from [16].

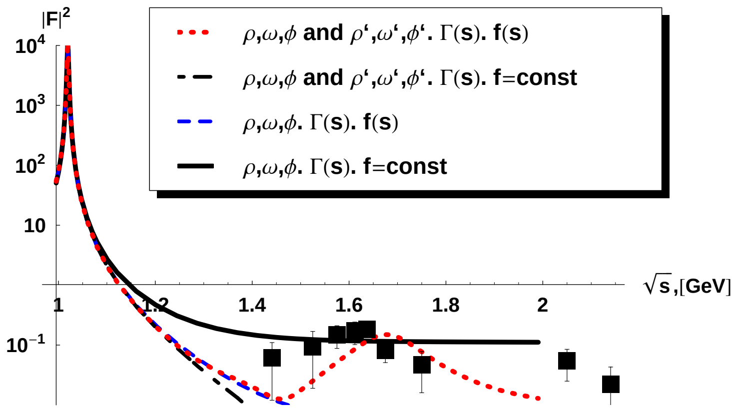

Figure 6. Neutral kaon form factor in the time-like region $s \geq 4m^2_{K0}$. Data (boxes) are from [17].

In the present model we obtain at $s \to -\infty$

$$
\begin{aligned}
&F_{K^+}(s) \to B + \frac{A'}{s}, \\
&B = 1 - \sum_{V=\rho,\omega,\phi} \frac{g_{VK^+K^-}}{f_V} - \sum_{V'=\rho',\omega',\phi'} \frac{g_{V'K^+K^-}}{f_{V'}}, \\
&A' = - \sum_{V=\rho,\omega,\phi} \frac{g_{VK^+K^-} m_V^2}{f_V} - \sum_{V'=\rho',\omega',\phi'} \frac{g_{V'K^+K^-} m_{V'}^2}{f_{V'}}.
\end{aligned}
\tag{21}
$$

For the correct asymptotic behavior the constant B should be zero. Contribution from ρ, ω, ϕ with $g_{VK^+K^-}/f_V$ taken from experiment does not lead to $B = 0$. If we add the higher resonances ρ', ω', ϕ' and choose negative relative sign of couplings $g_{V'K^+K^-}/f_{V'}$ with respect to $g_{VK^+K^-}/f_V$, then $B \approx 0$ and asymptotic behavior of the form factors is improved.

3 Comparison with experiment

In this section we present results for the charged and neutral kaons. Figs. 5 and 6 show FF in the time-like region, while Fig. 7 shows FF in the space-like region.

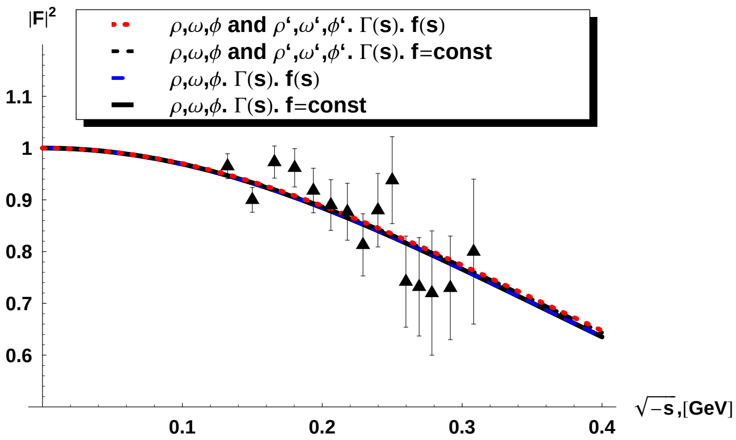

Figure 7. Charged kaon form factor in the space-like region $s < 0$. Data from NA7 Collaboration (CERN, SPS) [5].

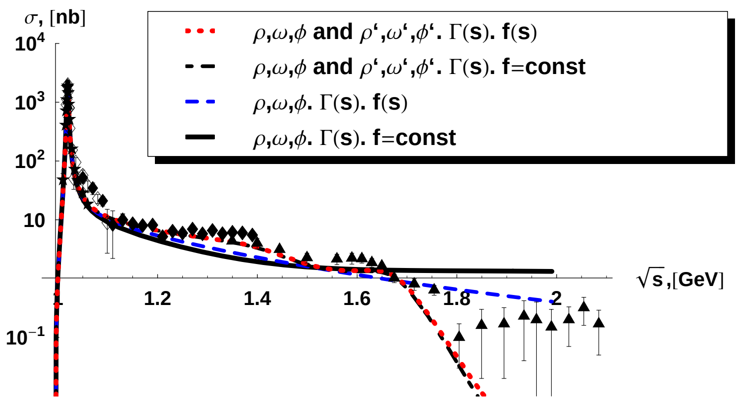

Figure 8. Total cross section of e^+e^- annihilation into charged kaons. Data: stars are from [18], diamonds (filled) – from [19], triangles – from [16], diamonds (open) – from [15].

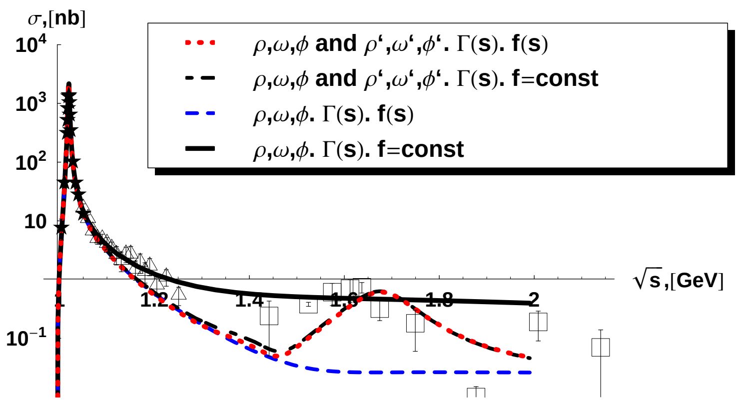

Figure 9. Total cross section of e^+e^- annihilation into neutral kaons. Data: stars are from [18], triangles (open) – from [2], boxes (open) – from [17].

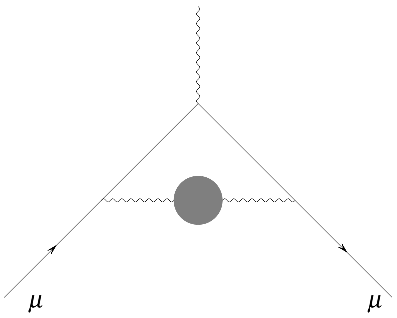

Figure 10. Hadronic contributions to muon AMM.

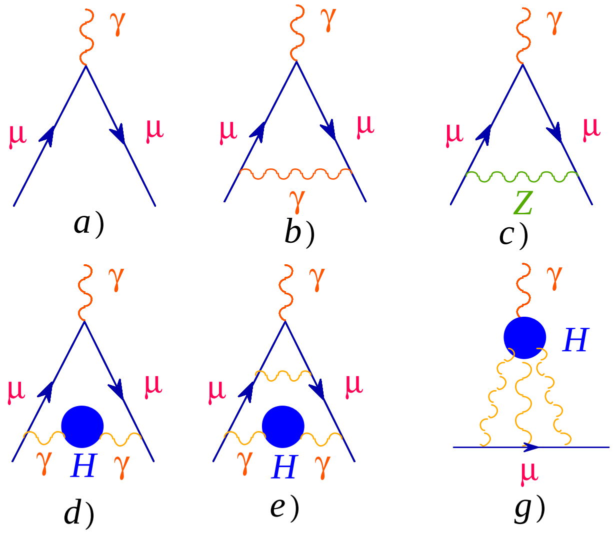

Figure 11. Typical diagrams contributing to muon AMM.

4 $K\bar{K}$ production and anomalous magnetic moment of muon

If g_μ is gyromagnetic ratio defined through the relation between magnetic moment and spin of the muon

$$\vec{M} = g_\mu \frac{e}{2m_\mu}\vec{s}, \tag{22}$$

and $a_\mu \equiv g_\mu/2 - 1$ is a measure of AMM, then $K\bar{K}$ contribution to a_μ can be determined via the dispersion integral (Brodsky and de Rafael [20]) which follows from analyticity of the photon polarization operator:

$$a_\mu^{had,K\bar{K}} = \frac{\alpha^2}{3\pi^2}\int_{4m_K^2}^{\infty} W(s)R(s)\frac{\mathrm{d}s}{s}, \tag{23}$$

$$W(s) = \int_0^1 \frac{x^2(1-x)}{x^2+(1-x)s/m_\mu^2}\mathrm{d}x,$$

where m_μ is muon mass and $R(s)$ is ratio of cross sections

$$R(s) = \frac{\sigma(e^+e^- \to K\bar{K})}{\sigma(e^+e^- \to \mu^+\mu^-)} = \frac{(1-\frac{4m_K^2}{s})^{3/2}}{4(1+2\frac{m_\mu^2}{s})(1-\frac{4m_\mu^2}{s})^{1/2}}|F_K(s)|^2.$$

The calculated values are presented in Table 4 together with the inaccuracy caused by uncertainty in the model parameters. The value $(34.70 \pm 1.01)\times 10^{-10}$ is close (within 1.5%) to results from e^+e^- annihilation by CMD-2 and SND Collaborations in Novosibirsk [21].

Note that $K\bar{K}$ channels contribute about 5% of the hadronic contributions to AMM (Fig. 10).

The total AMM in the Standard Model includes various contributions (see Fig. 11) and is equal to [22]

$$a_\mu^{theor} = a_\mu^{QED} + a_\mu^{weak} + a_\mu^{\gamma\ by\ \gamma} + a_\mu^{had,\ LO} + a_\mu^{had,\ HO}, \tag{24}$$

where

$$a_\mu^{QED} = (11658471.81 \pm 1.45_{\ loops} \pm 0.08_\alpha)\times 10^{-10},$$
$$a_\mu^{weak} = (15.4 \pm 0.1_{had} \pm 0.2_{Higgs,\ 3\ loops})\times 10^{-10},$$
$$a_\mu^{\gamma\ by\ \gamma} = (8 \pm 4)\times 10^{-10},$$
$$a_\mu^{had,\ LO} = (690.9 \pm 3.9_{exp} + 1.9_{rad} + 0.7_{QCD})\times 10^{-10},$$
$$a_\mu^{had,\ HO} = (-9.79 \pm 0.09_{exp} + 0.03_{rad})\times 10^{-10}.$$

The difference between the most precise experimental value ($g-2$ Collaboration, experiment E821, BNL, Brookhaven) and the theoretical value is (in units 10^{-10})

$$a_\mu^{exp} - a_\mu^{theor} = (11659208.0 \pm 6.3) - (11659176.3 \pm 6) = 31.7 \pm 8.7$$

Table 4. Contribution of $K\bar{K}-$channels to muon AMM $a_\mu^{had,K\bar{K}}$

channel	K^+K^-	$K^0\bar{K}^0$	total $K\bar{K}$
$a_\mu^{had,K\bar{K}}$, 10^{-10}	19.06 ± 0.57	15.64 ± 0.44	34.70 ± 1.01

The discrepancy is only about 3×10^{-6} of the experimental value. Nevertheless it is more than 3 "standard deviations" σ and is therefore important. Whether this indicates new physics beyond the Standard Model remains to be studied further. There are other possible contributions which may add to the theoretical value, and from the experimental side there is a puzzling discrepancy between results from $e^+e^- \to \pi^+\pi^-$ and $\tau-$decay $\tau^- \to \pi^-\pi^0\nu_\tau$.

New experiment E969 is scheduled at BNL [23] aiming to reduce experimental error in muon AMM from 6.3×10^{-10} to 2.5×10^{-10}.

5 Conclusions

1. A model for electromagnetic form factors of the $K-$mesons in the time-like ($s \geq 4m_K^2$) and space-like ($s < 0$) regions of the photon momentum is developed up to $\mathcal{O}(p^4)$ [13].

2. Agreement with experiments on $e^+e^- \to K\bar{K}$ annihilation at $\sqrt{s} = 1 - 1.75$ GeV is obtained without fitting parameters. Deviations from the data at $\sqrt{s} > 2$ GeV are probably related to higher resonances $\rho(1700)$ and $\omega(1650)$.

3. Form factor agrees with the data in the space-like region at $-q^2 < 0.16$ GeV^2. Results from Jefferson Laboratory at large momentum transfer $-q^2 \sim 3$ GeV^2 [6] which are coming soon may help to test further the model.

4. Contribution of $K\bar{K}$ channels to anomalous magnetic moment of the muon is found to be:

$$a_\mu^{had,K^+K^-} + a_\mu^{had,K^0\bar{K}^0} = (34.70 \pm 1.01) \times 10^{-10}$$

which agrees with e^+e^- annihilation results from Novosibirsk.

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