Ref. TH. 869

RECENT DEVELOPMENTS IN THE STUDY OF THE ANALYTICITY IN J :

FIXED POLES

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(Invited paper at the Conference of the Società Italiana di Fisica - Bologna, October 1967)

Ref. TH. 869

19 January 1968

I. -

It is the aim of this report to sum up some work done recently about fixed poles in the J plane. These singularities are in general forbidden by the unitarity condition, if there are no additional complications in the J plane, like branch points. They have been therefore rediscovered in the study of processes having linear unitarity only, such as by definition the first order weak interactions, or in general the transitions in which external currents appear. These processes are in particular considered in the current algebra framework, and it is there that the fixed poles are shown to be of a special interest.

At the same time there has been a clarification of some aspects of the Gribov-Pomeranchuk phenomenon, in which fixed poles in the left-hand discontinuity of the partial wave are turned by the unitarity condition into essential singularities for the complete function. The Gribov-Pomeranchuk phenomenon is peculiar of the strong processes, i.e., the processes in which the unitarity is valid to all the orders. We can thus distinguish two aspects of the study of the fixed singularities in the J plane. The first one is the role of the fixed singularities in the processes in which higher order terms in a weak coupling constant are neglected and thus quadratic unitarity is not satisfied. The second one is the problem of the fixed singularities of the Gribov-Pomeranchuk type, which may appear also in the strong processes. In Section II, we recall the features of the fixed poles and the connection between fixed poles and asymptotic behaviour in the energy of the crossed channel. In Section III, we discuss the implications of the unitarity. In Section IV, the case in which the external particles have spin is briefly reviewed. In Sections V and VI, we discuss respectively the two problems mentioned before, the role of fixed poles in weak processes and the Gribov-Pomeranchuk phenomenon.

II. -

Let us briefly recall the Gribov-Froissart method of analytic continuation in the angular momentum. We first consider a process without spin and call $A(s,t)$ the scattering amplitude. We assume for simplicity the unsubtracted dispersion relations

$$A(s,t) = \sum_J \frac{2J+1}{2} \left\{ f_J^+(t)\left(P_J(z) + P_J(-z)\right) + f_J^-(t)\left(P_J(z) - P_J(-z)\right)\right\} =$$

$$= \frac{1}{\pi}\int_{s_0}^{+\infty} \frac{ds'}{s'-s} A_s(s',t) + \frac{1}{\pi}\int_{u_0}^{+\infty} \frac{du'}{u'-u} A_u(u',t) =$$

$$= \int \frac{A_\nu(\nu',t)}{\nu'-\nu}\, d\nu' \qquad (1)$$

where

$$s = 2m^2 - \frac{t}{2} + \nu\,, \qquad z = \frac{2\nu}{t-4m^2}\,.$$

A_ν or A_s and A_u may contain some δ function, coming from the Born terms.

The analytic continuation of the partial waves in the J plane is given by the Gribov-Froissart formula

$$f_J^{\pm}(t) = \frac{4}{\pi}\,\frac{1}{t-4m^2}\left[\int_{\nu_0}^{+\infty} Q_J\left(\frac{2x}{t-4m^2}\right) A_s(s(x),t) \pm \right.$$

$$\left. \pm \int_{\nu_0}^{+\infty} Q_J\left(\frac{2x}{t-4m^2}\right) A_u(u(x),t)\, dx\right] \qquad (2)$$

We shall call the subscript $(+)$ right signature for even J, wrong for odd J, and vice versa for $(-)$. We shall use the Mandelstam[1)] version of the Sommerfeld-Watson transform

$$A(\nu,t) = \frac{1}{2i}\oint dJ\,(2J+1)\,\frac{Q_{-J-1}(-z)}{\cos \pi J}\left[f^+(J,t)\,\xi^+(J) + \right.$$

$$\left. + f^-(J,t)\,\xi^-(J)\right] + \Sigma \qquad (3)$$

ξ is the signature factor, Σ' is the sum of the counter terms which cancel the contributions of the poles at the half integers coming from $1/\cos\pi J$. We are not interested here in these terms.

$f(J,t)$ defined in (2) is an analytic function in J as long as the integral converges, except for the poles in J of Q_J at the negative integers. These are the fixed poles in which we are interested. Of course these poles appear in the integrand and they may be no longer present in $f(J,t)$, if the integral over the residuum is zero. Moreover they are in general in conflict with unitarity. However, let us discuss for the moment the features of these poles, assuming they are present. Around a negative integer we have

$$Q_J(z) \approx \frac{\pi\cos\pi J}{\sin\pi J} P_{-J-1}(z) \tag{4}$$

so that, recalling the asymptotic behaviour $Q_J(z) \xrightarrow[z\to\infty]{} z^{-J-1}$ these fixed poles, if present, give a contribution to the asymptotic expansion of $A(\nu,t)$ of the form

$$\frac{1}{\nu^{|J|}}\left\{\int_{\nu_0}^{+\infty} dx\, P_{|J|-1}\left(\frac{2x}{t-4m^2}\right)A_s(s(x),t) + (-1)^{|J|}\int_{\bar{\nu}_0}^{+\infty} dx\, P_{|J|-1}\left(\frac{2x}{t-4m^2}\right)A_u(u(x),t)\right\} \tag{5}$$

We note that, due to the signature factor ξ in (3), only the fixed poles in the right signature give contribution to the asymptotic behaviour. We see that the contribution of the fixed poles has the form of the terms of the asymptotic expansion in inverse powers of ν which can be read from (1)

$$A(\nu,t) \xrightarrow[\nu\to\infty]{} \frac{1}{\nu}A^1(t) + \frac{1}{\nu^2}A^2(t) + \cdots \tag{6}$$

where

$$A^n(t) = -\frac{1}{\pi}\int_{\nu_0}^{+\infty}\nu^{n-1}A_s(s(\nu),t)\,d\nu - \frac{(-1)^n}{\pi}\int_{\bar{\nu}_0}^{+\infty}\nu^{n-1}A_u(u(\nu),t)\,d\nu \tag{7}$$

Fixed poles in the right signature mean fixed (inverse) powers in the asymptotic behaviour. Expression (7) comes directly if we consider the Khuri [2] plane. In that plane, the amplitudes defined by

$$A(\nu,t) = \sum_n \left(a_n^+(t) \frac{\nu^n + (-\nu)^n}{2} + a_n^-(t) \frac{\nu^n - (-\nu)^n}{2} \right) \tag{8}$$

have the analytic continuation

$$a^\mp(n,t) = -\frac{1}{\pi}\left[\int_{\nu_0}^{+\infty} \frac{A_s(s(x),t)}{x^{n+1}}\,dx \pm \int_{\bar{\nu}_0}^{+\infty} \frac{A_u(u(x),t)}{x^{n+1}}\,dx \right] \tag{9}$$

There are no fixed poles coming from the analytic continuation, however, in the Sommerfeld-Watson transform

$$A(\nu,t) = \frac{1}{2i}\oint dn \, \frac{(-\nu)^n}{\sin\pi n}\left[a^+(n,t)\,\xi^+(n) + a^-(n,t)\,\xi^-(n) \right] \tag{10}$$

the integrand shows fixed poles at negative integers because of $1/\sin\pi n$. The asymptotic contribution of these poles is given by (7).

The fixed poles assure the absence of singularities in the amplitude when a moving pole approaches a negative integer. This is rather evident from the opening of the contour in the integration (10) or in (3). For instance, if $a(n)$ is of the form

$$a(n) = \frac{\beta}{n-\alpha}$$

the total contribution of the moving pole plus the fixed pole at $n = -1$ is given by

$$\frac{\beta}{\sin\pi\alpha}\,\nu^\alpha - \frac{1}{\nu}\,\frac{\beta}{\pi(\alpha+1)} \tag{11}$$

The fixed pole realizes a mechanism of cancellation between two singularities, which is different from the usual multiplicative ghost-killer factor.

If there is not a fixed pole, that is

$$a\ (n=-1)=0$$

this means from (9)

$$\int A_\nu (\nu, t)\, d\nu = 0 \qquad (12)$$

The moving pole comes from the divergence of the integral in (9), hence let us assume that for $\nu > L$, $A_\nu \approx \beta \nu^\alpha$

Then (12) means

$$\int^L A_\nu\, d\nu - \frac{\beta L^{\alpha+1}}{\alpha+1} = 0$$

Then $\beta \approx (\alpha + 1)$ and we have the ghost-killer factor.

All these observations can be done equally well for the Regge plane, there is no essential difference between Khuri and Regge plane.

Equation (12) is known as a superconvergence condition [3]. As we have seen it is the statement that there is not a fixed pole at -1. We can write other superconvergence conditions stating the absence of fixed poles at the other negative integers.

So far for the fixed poles in the right signature. The fixed poles in the wrong signature do not give rise to terms in the asymptotic behaviour. They, however, modify the terms coming from the moving singularities. This happens because when a Regge pole goes through an integer of the wrong signature $\xi(J) \to 0$. Then from (3) or (10), we expect, in absence of fixed pole, a zero for the amplitude. If the fixed pole is present, i.e., if $f(J) \approx (1/J-\alpha)(\beta/J+m)$, the residuum of the moving pole contains $\beta\,[\xi(\alpha)/\alpha+m]$, which is finite when $\alpha \to -m$ and the signature is wrong.

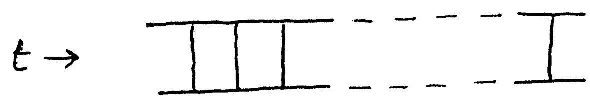
III. -

The quadratic unitarity is in conflict with the fixed poles as long as there are no cuts in the angular momentum plane. The unitarity condition is separately valid for both the signatures and in the region in t in which only the elastic channel is open it reads

$$f^{\pm}(J,t) - \left[f^{\pm}(J^{*},t) \right]^{*} = \rho \left[f^{\pm}(J^{*},t) \right]^{*} f^{\pm}(J,t) \tag{13}$$

The fixed poles occur for real J also in the region in t relevant for (13) and then, if there are no cuts in J the right-hand side of (13) has a double pole if the left-hand side has a single one.

In the models which give rise to a Regge behaviour, the unitarity condition has the role of a dynamical equation. The input of such an equation is given by the "potential", that is the Born term in the crossed channel, whose projection in partial waves can be seen from (2). If we expand the solution in powers of the coupling constant, the various terms have a single pole, a double one, and so on, so that (13) is satisfied. If the input, i.e., the "potential", is not nasty, the sum of the serie is a moving pole. However, it may also result that the negative integer is an essential singularity. These possibilities have been illustrated in a N/D calculation by Jones and Teplitz [4]. Another useful model is the Bethe-Salpeter equation which, in partial waves, is an integral equation of Fredholm type [5]. This equation can be put in a Hilbert-Schmidt form, i.e., having a symmetric kernel. The kernel contains a Q_J function, which has a pole at $J = -n$. However, for most approximations, e.g., the ladder approximation of Fig. 1, the residue of



- Figure 1 -

the kernel at the pole is separable, i.e., of the Pinkerle-Goursat type [see Eq. (16) below]. The resolvent of the integral equation

$$A(x,y) = I(x,y) + \int I(x,z) A(z,y)\, dz \tag{15}$$

where

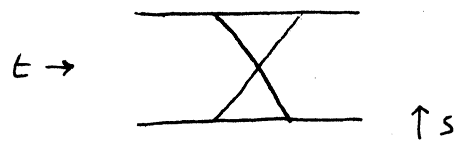
$$I(x,y) = \sum_{k=1}^{m} X_k(x) \frac{X_k(y)}{J+n} \tag{16}$$

has a solution of the form

$$A(x,y) = \sum_{k} X_k(x)\, C_k(y) \tag{17}$$

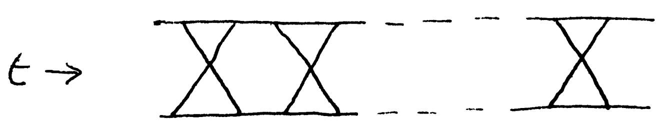
where the C_K's have no pole at $J = -n$ for arbitrary values of the parameters, as can be seen writing explicitly the solution. Then, applying a reasoning like that of Lee and Sawyer in their Section 5 [5], it follows that the solution of the Bethe-Salpeter equation has no fixed singularity at $J = -n$, but only moving poles.

However, for other kernels, for instance those corresponding to an interaction having a non-vanishing third double spectral function such as the one represented by the diagram of Fig. 2



- Figure 2 -

for which the iterative solution for the Bethe-Salpeter equation gives the diagram of Fig. 3



- Figure 3 -

the residue at the pole is not separable in the sense of Eq. (16). If we regard $\lambda \equiv 1/J+n$ as the Fredholm parameter, we know that the solution of Eq. (15), which has a symmetric kernel, has an infinite number of Fredholm poles, clustering to $\lambda = \infty$, i.e., to $J = -n$. The point $J = -n$ is an accumulation of poles and then it is an essential singularity.

We see that in simple models the fixed pole given by the "potential" is turned by unitarity into a moving pole, and, in some case, an essential singularity at the position of the fixed pole of the input.

IV. -

Let us review very briefly the modifications to the previous discussions which are necessary when the external particles have spin. This problem has been studied by Gell-Mann et al. [6], Calogero and Charap [7], and more recently by Drechsler [8].

If λ and μ are the difference of the helicities of the initial and final particles and if $A_{\lambda\mu}(\nu, t)$ is the scattering amplitude,

$$A_{\lambda\mu}(\nu, t) = \sum_J (2J+1)\, f^J_{\lambda\mu}(t)\, d^J_{\lambda\mu}(z) \tag{18}$$

we have the Gribov-Froissart continuation

$$f^J_{\lambda\mu}(t) = K(t) \int \mathrm{Im}\, A_{\lambda\mu}(\nu(z), t)\, (1-z)^{\frac{|\lambda-\mu|}{2}} (1+z)^{\frac{|\lambda+\mu|}{2}} e^J_{\lambda\mu}(z)\, dz \tag{19}$$

We will discuss below the properties of the function of second kind $e^J_{\lambda\mu}$.

It is possible to distinguish two signatured amplitudes $f^{J\pm}_{\lambda\mu}$, suitably modifying (19) in analogy with (2), and write the Sommerfeld-Watson transform

$$A_{\lambda\mu}(\nu, t) = \frac{1}{2i} \oint dJ\, \frac{(2J+1)}{\cos \pi (J-\lambda)}\, e_{-\lambda\mu}(-z, -J-1) \Big[f^{J+}_{\lambda\mu}\, \xi^{+}(J) + f^{J-}_{\lambda\mu}\, \xi^{-}(J) \Big] + \sum \tag{20}$$

The asymptotic behaviour of the function $e^J_{\lambda\mu}(z)$ is

$$e^J_{\lambda\mu}(z) \xrightarrow[z \to \infty]{} z^{-J-1} \tag{21}$$

and the singularities are, if $|\lambda| \geq |\mu|$:

no one	for $J > \lvert\lambda\rvert$		sense-sense region
$\sqrt{\frac{1}{J-m}}$	for $J=m$ and	$\lvert\mu\rvert \le J \le \lvert\lambda\rvert - 1$	sense-nonsense region
$\frac{1}{J-m}$	for $J=m$ and	$-\lvert\mu\rvert \le J \le \lvert\mu\rvert - 1$	nonsense-nonsense region
$\sqrt{\frac{1}{J-m}}$	for $J=m$ and	$-\lvert\lambda\rvert \le J \le -\lvert\mu\rvert - 1$	" " "
$\frac{1}{J-m}$	for $J=m$ and	$J < -\lvert\lambda\rvert$	" " "

(m is an integer).

It is seen from (19) that the integrand of the modified Gribov-Froissart continuation has singularities, poles or inverse square roots, at all the nonsense integers, i.e., at all the integers less than the maximum of $\lvert\lambda\rvert$ and $\lvert\mu\rvert$. We call briefly these singularities "fixed poles". The product $e^{J}_{\lambda\mu}\, e^{-J-1}_{-\lambda\mu}$ has simple poles at all negative and positive integers, except that for $J = m$ integer and $-\mu \le m \le \mu - 1$ where it has double poles. If there are no "fixed poles", i.e., the residue of the singularities of $f(J)$ coming from the singularities of the e^{J} in (19) are zero, the integrand in (20) has, apart from the obvious poles reproducing the development (18), single poles for $-\mu \le J \le \mu - 1$. Drechsler [8] has shown that the contributions of these poles cancel in pairs. The unitarity condition has the matrix form

$$f^{J}_{\lambda\mu}(t+i\varepsilon) - f^{J}_{\lambda\mu}(t-i\varepsilon) = \rho \sum_{\gamma} f^{J}_{\lambda\gamma}(t-i\varepsilon) f^{J}_{\gamma\mu}(t+i\varepsilon) \qquad (22)$$

If we consider a dynamical equation, the "potential" has a singularity of the type $1/\sqrt{J-m}$ if J is in the sense-nonsense region. The unitarity condition, requiring a bounded scattering amplitude, turns the singularity $1/\sqrt{J-m}$ into $\sqrt{J-m}$. The fixed poles of the right signature at $J = m$ give rise to terms in the asymptotic behaviour going as z^{m}.

If we introduce the amplitude free of kinematical singularity

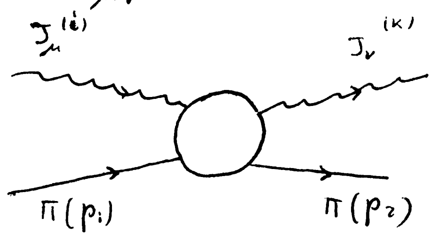
$$\bar{A}_{\lambda\mu} = \frac{1}{(1-z)^{\frac{|\lambda-\mu|}{2}}} \frac{1}{(1+z)^{\frac{|\lambda+\mu|}{2}}} A_{\lambda\mu} \tag{23}$$

the asymptotic behaviour of the term originated from a fixed pole is $z^{m-\lambda}$ (recall $m < \lambda$). The fixed poles of the wrong signature, on the other hand, simply cancel, analogously to the spinless case, the zero of the signature factor $\xi(\alpha)$ in the contribution of the moving poles, leaving a finite result when α goes through an integer. This happens in particular for the integer values of the wrong signature in the sense-nonsense region. In the absence of the fixed pole the corresponding amplitude vanishes.

V. -

We may expect in general fixed poles in a world in which only linear unitarity is valid.

Let us consider first the transitions excited by external fields. These are the type of processes studied by current algebra, Refs. 9),10),11). They can also be seen as first order weak interactions, the currents being coupled to the leptons. As it is well known, current algebra establishes an equation between a "scattering amplitude" of external currents on particles and a form factor of another current of the algebra. To bemore specific, calling π a hadron of spin 0 for simplicity, if $J_\mu^{(i)}, J_\nu^{(k)}$ are two vector or axial vector currents, let us consider the scattering amplitude $T_{\mu\nu}$ for the process indicated in Fig. 4.



- Figure 4 -

$T_{\mu\nu}$ is expressed as a matrix element of a retarded commutator of the currents $J_\mu^{(i)}, J_\nu^{(k)}$. It will have the expansion

$$T_{\mu\nu} = P_\mu P_\nu A + \cdots \tag{24}$$

$P = p_1 + p_2$. Then, from the equal-time commutator $[J^{(i)}, J^{(k)}] = J^{(\ell)}$, it follows the equation, in the hypothesis of non-subtracted dispersion relations 12)

$$\int d\nu \; \mathrm{Im}\, A(\nu, t) = F(t) \tag{25}$$

where $F(t)$ is given by $\langle \pi \, | J_\mu^{(\ell)} | \, \pi \rangle = P_\mu F(t)$.

The left-hand side of (25) is the term in $1/\nu$ of the asymptotic expansion in ν of $A(\nu,t)$ [see Eq. (6) of Section II], and we have seen in Section II that it represents the contribution of a fixed pole in the right signature so that it is now clear that fixed poles in the right signature are necessary in order to reconcile current algebra and analyticity properties of the amplitude describing the process of Fig. 4. Because of the spin of the currents, the fixed pole occurs at $J = 1$, which is the first nonsense point for A (amplitude of double spin flip). The amplitude A which appears in Eq. (24) is actually the amplitude free of kinematical singularities and the discussion following Eq. (23) of Section IV is applied with $m = 1$, $\lambda = 2$. In other processes in which the weak interactions are considered only up to the first order, there may be no necessity of fixed poles, but there is nothing against them too.

The fixed poles in the right signature reassume all the effects of the S channel, strong interaction resonances and so on. If we want to take into account higher order terms in the weak coupling constant, so that quadratic unitarity is satisfied, the fixed poles will move from their position. However, we expect that they remain in the neighbourhood of the fixed position, as long as the coupling constant is small. It will then be possible to approximate their contribution to the asymptotic behaviour in ν by terms having an inverse power behaviour. These ideas can be applied for instance in the pion photoproduction.

VI. -

In 1962, Gribov and Pomeranchuk [13] have found that the left-hand discontinuity in t of the partial wave amplitude $f_J(t)$ has fixed poles at the negative integers (in the spinless case). These fixed poles only occur in the wrong signature. They consider the discontinuity in t of

$$\varphi_J = \frac{f_J}{(t-4m^2)^J}$$

which is given for $t<0$ by

$$\Delta\varphi_J^{\pm} = \frac{1}{(4m^2-t)^{J+1}}\left\{\int_{s_0}^{4m^2-t-u_0} \rho_{su}(x, 4m^2-x-t)\left(Q_J\left(\frac{2x}{4m^2-t}-1\right) \pm \right.\right.$$
$$\left. \pm Q_J\left(1-\frac{2x}{4m^2-t}\right)\right)dx +$$
$$\left. + \int_{s_0}^{4m^2-t} dx\, P_J\left(\frac{2x}{4m^2-t}-1\right)A_s(x,t-i\varepsilon) \pm \int_{u_0}^{4m^2-t} dx\, P_J\left(\frac{2x}{4m^2-t}-1\right)A_u(x,t-i\varepsilon)\right\} \tag{26}$$

ρ_{su} is the third double spectral function. $\Delta\varphi_J^{\pm}$ is given by an integral over a finite region and can be continued analytically for all J. From Eq. (26), we see that $\Delta\varphi_J^{\pm}$ has poles at even negative integers for the (-) signature, at odd negative integers for the (+) signature, so that the poles are at the wrong signature.

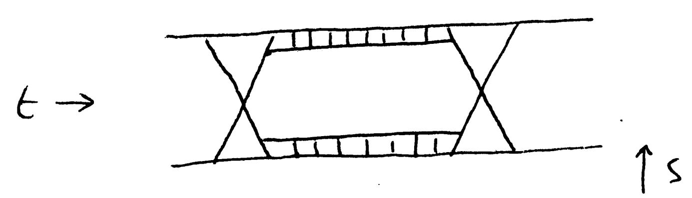
We may verify that the terms of the asymptotic expansion (6) of Section II, which come from the fixed poles at the right signature have no left-hand discontinuity in t

$$\Delta A^n(t) = -\int_{s_0}^{4m^2-t-u_0} x^n \rho_{su}(x, 4m^2-x-t)\,dx +$$
$$+\int_{u_0}^{4m^2-t-s_0} (4m^2-y-t)^n \rho_{su}(4m^2-y-t, y)\,dy = 0 \tag{27}$$

The residuum of the fixed pole of $\Delta\varphi_J$ is given by an integral over a fixed t line in the region of definition of the ρ_{su} function. The parameter t can be varied in order to pick up a line in a region of definition of ρ_{su} to which only some reduced diagrams can contribute (for instance for scalar particles only the diagram of Fig. 2). Sometimes the ρ_{su} function can be calculated in that region and it is found that the residuum of $\Delta\varphi_J$ is different from zero.

If $\Delta\varphi_J$ has a pole, f_J cannot be regular, and Gribov and Pomeranchuk have shown that in absence of cuts in the J plane, it follows from unitarity that there is an essential singularity. The same point is now a pole on the left-hand side of the unitarity condition (13) and a double pole on the right-hand side and so on, to all orders of a perturbative expansion. We have seen in Section III how the essential singularity occurs in the Bethe-Salpeter equation.

The essential singularity contributes to the asymptotic behaviour (even if it is at the wrong signature). The external spin moves the singularity to the first nonsense value of J, which may be positive. Paradoxically, the Froissart bound could be violated. The other possibility, mentioned by Gribov and Pomeranchuk, is the presence of cuts in the angular momentum plane. In order to reconcile the fixed pole of f_J, which cannot be cancelled because $\Delta\varphi_J$ has the pole, with the elastic unitarity the cut has to cover the point $J = -1$ (for the spinless case) when t is over the elastic threshold. Moreover, as a consequence of the cut, the kinematical factor ρ in the elastic unitarity (13) must be replaced by a function of J which vanishes at the negative integers [4]. Mandelstam [14] has found in a model cuts in the J plane, which at least satisfy the first condition. They come from diagrams of the form



- Figure 5 -

where the internal ladders are supposed to give rise to a moving Regge pole whose trajectory contains as a bound state the external particle.

If $\alpha\ (t)$ is the trajectory we have said, the position of the cut is $\alpha_c(t) = 2\alpha(t/4m^2)-1$, showing the correct behaviour. The moving cut in J can be seen by mapping as a cut in the t plane. The branch point comes out from the four particles cut.

Even if the cut can eliminate the essential singularity, the fixed pole is still there. However, it does not give directly a contribution to the asymptotic behaviour, but only modifies the form of the contribution of the moving pole [15], as seen at the end of Sections II and IV.

In particular the presence of a fixed pole in the wrong signature destroys the simplicity of the explanation of the dip in π -p charge exchange which was interpreted as due to a nonsense value in the wrong signature of the Regge trajectory. Of course, one can say in this case that there is an experimental evidence of the smallness of the residuum of the fixed pole and then of the strength of the third double spectral function.

As a last point, we mention the possibility of allowing fixed poles of all kinds, whenever the unitarity condition is shielded by moving cuts, which are the mapping in t of the cuts in the J plane. In partidular, Oehme [16] has proposed the interpretation of the Pomeranchon as a fixed pole.

I want to thank Professor Amati for discussions and comments.

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