Constructing the Standard Model fermions

Peter Rowlands

Physics Department, University of Liverpool, Oliver Lodge Laboratory, Oxford St, Liverpool. L69 7ZE, UK

p.rowlands@liverpool.ac.uk

Abstract. The Standard Model has three generations of fermions and antifermions, each with two states of isospin, and each of these has both a lepton and a quark in three possible colour states. In total there are 48 states. No known system exists for constructing these from first principles. Here, it is suggested that the number of degrees of freedom required is a consequence of the nilpotent complexified vector-quaternion Dirac algebra, which emerges from the representation of the fundamental parameters mass, time, charge and space as a Klein-4 group, and that these degrees of freedom lead to unique structural representations of each of the individual fermions.

1. The Klein-4 group of mass, time, charge and space

We begin with some fundamental ideas presented in various publications over a long period. The first is the Klein-4 group connecting the fundamental parameters mass, time, charge and space. In principle, we propose that this group is the fundamental basis for physics, that nothing exists in physics outside of it, that it is exact and exclusive [1-10]. It can be seen as a kind of 'Periodic Table' for physics. The parameter group is also a representation of the fundamental principle that the sum total of everything in the universe is zero (even in conceptual terms).

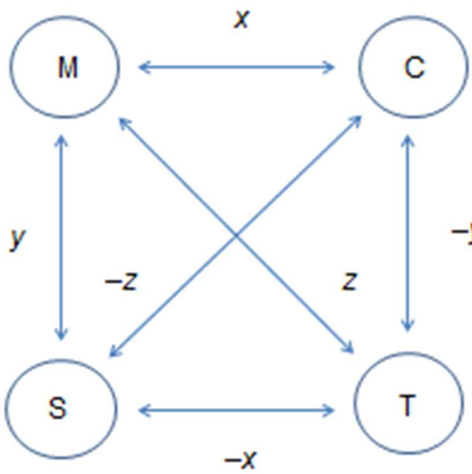
mass	conserved	real	continuouss (1-D) commutative
time	nonconserved	imaginary	continuous (1-D) commutative
charge	conserved	imaginary	discrete (3-D) anticommutative
space	nonconserved	real	discrete (3-D) anticommutative

Mass, here, which includes energy, is a continuous global quantity like the Higgs field. Quantized energy and localized discrete (invariant) mass do not exist at the basic level of the parameter group but are created at the next stage when their algebras are combined and compactified in a larger group structure. Charge, also, at this point is a 3-D quantity subject to global conservation laws and with 3 indistinguishable 'dimensions'. These only become separated into weak, strong and electric charges, each with its own special symmetry group under the compactification process which will be described in the next section. We note that discreteness in physics only ever comes from 3-dimensionality, the creation of a closed system. The discreteness of charge is however, different to that of space, for charge is a conserved quantity, so it has *discrete* and fixed units. Space, on the other hand, is a nonconserved quantity, so its discreteness cannot be fixed but must be endlessly reconstructed. Nevertheless, the uniqueness of space as a directly measurable quantity would be impossible without the fact that it is a discrete in a permanently reconstructible way.

The conceptual zero of the parameter group is particularly evident when we represent the properties and antiproperties in algebraic terms:

mass	x	y	z
time	$-x$	$-y$	z
charge	x	$-y$	$-z$
space	$-x$	y	$-z$

It can also be represented by a diagram which resembles those of category theory:



The next fundamental idea comes with the algebraic representations of these parameters. These algebras are taken to be exactly equivalent to the 'physical meanings' that the parameters generate.

mass	1	scalar
time	*i*	pseudoscalar
charge	***i j k***	quaternion
space	**i j k**	vector

The first three are subalgebras of the last, and combine to produce a version of it, say **I**, **J**, **K**. In other words they are equivalent to a 'vector space', an 'antispace' to counter **i**, **j**, **k**. We see why space appears to have a privileged status. In principle, the properties of the parameters are identical to the algebras representing them. The real / imaginary and dimensional / nondimensional are obviously algebraic. Only the conserved / nonconserved distinction needs further explanation. Nonconserved quantities (space and time, here) seem to involve the complex or pseudoscalar *i* term, connected with the free and continuous flow of time. Space is not a pure 3-D quantity, because it is a complexified quaternion (C ⊗ H), incorporating *i* along with the 3-D units. Nonconservation also includes the rotation symmetry of 3-D quantities, while conservation requires their rotation *a*symmetry.

The algebraic representations of the parameters can be seen as equivalent to the algebraic content of two spaces, one of which is real space (and accessible to observation as such), while the other is a combination of the algebraic quantities associated with the other parameters (and so not representing a single parameter available to observation) [5, 8-9. 11-14]. The totality zero principle suggests that these two 'spaces' effectively cancel each other, and that one is an 'antispace' of the other, containing the same information differently organized. We will find it useful to call them 'real space' and 'vacuum space'. As it contains the charges which create particles, we will also call it 'charge space'.

We note that the algebras do not include the antiassociative octonions (O), even though this is a division algebra, like the reals (R), complex numbers (C) and quaternions (H). It may be tempting to imagine that the fundamental algebra of physics takes the form of a tensor product of the four division algebras R ⊗ C ⊗ H ⊗ O, as, for example, advocated by Dixon [15-16]. However, both the parameter group and the Dirac algebra require it to be R ⊗ C ⊗ H ⊗ H, in line with the tensor product of mass (R), time (C), charge (H) and space (C ⊗ H). It is decided by the requirements of physics and not those of mathematics

We may ask why octonions should be excluded. At the fundamental level anticommutativity is handled by using the spatial positionings of symbols, and their reversal, which is allowed within anticommutativity, but to include antiassociativity would seem to require introducing a temporality into the composition of the products, including an effective reversal in the true time sequence, which physics explicitly excludes. Octonions, however, do have a role to play in the process [5, 8]. Quaternions are based on 2 basic imaginary units with a third as their product. Octonions are based on 3 basic imaginary units with four more as their products (three double and one triple), and we can produce a *mapping* in which the 8 fundamental units of mass, time, charge and space can be structured as a *broken octonion*, using the 7 imaginary units of R ⊗ C ⊗ H ⊗ H.

2. The creation of locality

The combination (tensor product) of the four parameter algebras, taken as commutative to each other, is a 64-part group, which is isomorphic to the gamma algebra of the Dirac equation, the quantum mechanical equation for the fundamental particle or fermionic state [5, 8-9, 17, 18]. This in turn is a version of *Cl*(6, 0) or *G*(6, 0), the Clifford algebra of a double vector space [5]. This suggests that the combined algebra has a truly fundamental role in physics, as we would expect from its origin in the parameter group.

The group can be represented as the four units of complex algebra plus 12 *pentads*, each of which is a generator of the entire algebra (at least when complexified).

1	i				-1	$-i$			
$\boldsymbol{i}\mathbf{i}$	$\boldsymbol{i}\mathbf{j}$	$\boldsymbol{i}\mathbf{k}$	$i\boldsymbol{k}$	$\boldsymbol{j}$	$-\boldsymbol{i}\mathbf{i}$	$-\boldsymbol{i}\mathbf{j}$	$-\boldsymbol{i}\mathbf{k}$	$-i\boldsymbol{k}$	$-\boldsymbol{j}$
$\boldsymbol{j}\mathbf{i}$	$\boldsymbol{j}\mathbf{j}$	$\boldsymbol{j}\mathbf{k}$	$i\boldsymbol{i}$	$\boldsymbol{k}$	$-\boldsymbol{j}\mathbf{i}$	$-\boldsymbol{j}\mathbf{j}$	$-\boldsymbol{j}\mathbf{k}$	$-i\boldsymbol{j}$	$-\boldsymbol{i}$
$\boldsymbol{k}\mathbf{i}$	$\boldsymbol{k}\mathbf{j}$	$\boldsymbol{k}\mathbf{k}$	$i\boldsymbol{j}$	$\boldsymbol{i}$	$-\boldsymbol{k}\mathbf{i}$	$-\boldsymbol{k}\mathbf{j}$	$-\boldsymbol{k}\mathbf{k}$	$-i\boldsymbol{j}$	$-\boldsymbol{i}$
$i\mathbf{i}\boldsymbol{i}$	$i\mathbf{i}\boldsymbol{j}$	$i\mathbf{i}\boldsymbol{k}$	$i\mathbf{k}$	$\mathbf{j}$	$-i\mathbf{i}\boldsymbol{i}$	$-i\mathbf{i}\boldsymbol{j}$	$-i\mathbf{i}\boldsymbol{k}$	$-i\mathbf{k}$	$-\mathbf{j}$
$i\mathbf{j}\boldsymbol{i}$	$i\mathbf{j}\boldsymbol{j}$	$i\mathbf{j}\boldsymbol{k}$	$i\mathbf{i}$	$\mathbf{k}$	$-i\mathbf{j}\boldsymbol{i}$	$-i\mathbf{j}\boldsymbol{j}$	$-i\mathbf{j}\boldsymbol{k}$	$-i\mathbf{i}$	$-\mathbf{k}$
$i\mathbf{k}\boldsymbol{i}$	$i\mathbf{k}\boldsymbol{j}$	$i\mathbf{k}\boldsymbol{k}$	$i\mathbf{j}$	$\mathbf{i}$	$-i\mathbf{k}\boldsymbol{i}$	$-i\mathbf{k}\boldsymbol{j}$	$-i\mathbf{k}\boldsymbol{k}$	$-i\mathbf{j}$	$-\mathbf{i}$

These groupings of five terms, seen on each row of the whole-group representation, can be identified as isomorphs of the five gamma terms which generate the entire Dirac algebra. Typically, they can all be constructed from the eight original units of the algebra (or some equivalent version) according to the same process. This involves the units of one of the two 3-dimensional parameters being attached to the five remaining units, breaking its (rotational) symmetry, while retaining the (rotational) symmetry of the other 3-dimensional quantity.

A nonconserved quantity such as space must preserve its rotation symmetry, but a conserved quantity such as charge need not. In fact, the process helps to establish the physical distinction. Conservation in a 3-D quantity is necessarily a symmetry breaker because the 'dimensions' then become distinguishable and identifiable. This means that the broken symmetry in the combination is usually assigned to charge. So, the first pentad in the table of 12 can be constructed by taking the undifferentiated units of charge (***i***, ***j***, ***k***) and attaching them, respectively, to those of space, mass and time, and creating *entirely new* composite or compound units of momentum, rest mass and energy. The units are now discrete or quantized because of the connection with discrete charge, but retain the respective vector, scalar and pseudoscalar aspects of the other components of the units' composite structures. So we start with

time	space	mass	charge
i	**i j k**	1	***i j k***

Then, taking one of each of ***i***, ***j*** and ***k*** on to one of the units of the other three parameter, we obtain:

ik	**i*i*** **j*i*** **k*i***	1***j***
energy	momentum	rest mass

The scalar values associated with the units (which define the nature of the quantities) are, of course, arbitrary, and so, assuming that they are, say,

E p_x p_y p_z m

we can write versions of the structures which incorporate them, for example:

ik*E* **i*i***p_x **j*i***p_y **k*i***p_z ***j***

No 5-fold structure in either mathematics or physics can be perfectly symmetric. 5 is always the signature of a broken symmetry, as in quintic equations, quasicrystals and Penrose tiling [5, 19-21]. However, the symmetry-breaking works both ways of the 5:3 division [5, 8]. In the first place, the parameters time, space and mass (which is here the undifferentiated continuous energy of something like the Higgs field) acquire some of the characteristics of the discrete charge units they associate with becoming, respectively, quantized energy, quantized momentum, and the discrete rest or invariant mass. At the same time, the charge units become associated with the respective pseudoscalar, vector and scalar units of time, space and continuous mass, leading to the respective *SU*(2), *SU*(3) and *U*(1) symmetries associated with the weak, strong and electric interactions. Here, we can write

ik	**i*i*** **j*i*** **k*i***	1***j***
weak charge	strong charge	electric charge

which, in addition to the characteristics associated with the parameter charge, also respond to algebras that are respectively

pseudoscalar vector scalar

and to symmetry groups that are recognisable as

SU(2) *SU*(3) *U*(1)

Both of these aspects of the compactification of the 64-part algebra into its group generators exist simultaneously and could lead to a concept of 10-'dimensionality' (= 2 × 5), in which 6 of the dimensions are conserved or 'compactified' (five charges plus rest mass) [22].

Of immediate interest is the fact that the physical natures of energy, momentum and rest mass are determined by their algebraic units (***ik***, **i*i***, **j*i***, **k*i***, 1***j***) and not by any scalar coefficient to which they are attached, we can choose coefficients (E, p_x, p_y, p_z, m) such that (***ik****E* + **i*i***p_x + **j*i***p_y + **k*i***p_z + ***j****m*)^2 = 0 or (***ik****E* + **i*i***p_x + **j*i***p_y + **k*i***p_z + ***j****m*) is *nilpotent*. Here we identify

$$(\boldsymbol{ik}E + \mathbf{i}\boldsymbol{i}p_x + \mathbf{j}\boldsymbol{i}p_y + \mathbf{k}\boldsymbol{i}p_y + \boldsymbol{j}m)\ (\boldsymbol{ik}E + \mathbf{i}\boldsymbol{i}p_x + \mathbf{j}\boldsymbol{i}p_y + \mathbf{k}\boldsymbol{i}p_y + \boldsymbol{j}m) = 0$$

as Einstein's relativistic energy equation

$$E^2 - p^2 - m^2 = 0$$

or, in its more usual form,

$$E^2 - p^2c^2 - m^2c^4 = 0.$$

The nilpotent structure is intrinsically relativistic and automatically creates *locality*. The point-like localised particle emerges as a norm-zero singularity at the intersection of two spaces (real and vacuum space) each of which acts to negate the other. The structure is also intrinsically quantized once the ***i***, ***j***, ***k*** operators for discrete charges are applied. The constants *c* and *ħ* are then simply consequences of the historic choices of units for the fundamental quantities of space, time and mass and will be here equated to 1.

The Dirac equation, the quantum mechanical equation for the localized fundamental particle state, now simultaneously applies the nonconservation and conservation principles of the parameter group to the nilpotent structure, using differentials in time and space for *E* and **p** [5, 8-9. 18, 22-29]. We apply a canonical quantization procedure to the first bracket in the squared expressions, to replace the terms *E* and **p** by the operators $E \to i\partial/\partial t$, $\mathbf{p} \to -i\nabla$, and assume that the operators act on the phase factor for a free fermion, $e^{-i(Et - \mathbf{p.r})}$, to obtain the nilpotent Dirac equation for a free fermion:

$$\left(\mp \boldsymbol{k}\frac{\partial}{\partial t} \mp i\boldsymbol{i}\nabla + \boldsymbol{j}m\right)(\pm i\boldsymbol{k}E \pm i\mathbf{p} + \boldsymbol{j}m)e^{-i(Et-\mathbf{p.r})} = 0.$$

Since *E* and **p** represent operators as well as amplitudes, we can also express it as

$$(\pm i\boldsymbol{k}E \pm i\mathbf{p} + \boldsymbol{j}m)(\pm i\boldsymbol{k}E \pm i\mathbf{p} + \boldsymbol{j}m)e^{-i(Et-\mathbf{p.r})} = 0$$

and simply change the phase factor if the particle is not free and the *E* and **p** terms become generic expressions for covariant derivatives or expressions incorporating potentials.

The vector units **i**, **j**, **k** of **p** are those of a Clifford algebra, and identical to the complexified quaternions, ***ii***, ***ij***, ***ik***. Hestenes [30] saw them as the basis of a *multivariate vector* algebra, in which vectors **a** and **b** would have a *full product*, parallel to that of quaternions:

$$\mathbf{ab} = \mathbf{a.b} + i\ \mathbf{a} \times \mathbf{b}.$$

Hestenes showed, in particular, how the extra cross product term $i\ \mathbf{a} \times \mathbf{b}$ could explain spin. The multivariate properties of **p** similarly allow us to use the 'spin' terms **p** and ∇ instead of the 'helicity' terms **σ.p** and **σ.**∇, in the Dirac equation, with **σ** defined as a unit pseudovector of magnitude –**1**, in a nilpotent structure, since $(\boldsymbol{\sigma}.\mathbf{p})^2 = \mathbf{pp} = p^2$.

From a physical point of view, the significant fact now is that the amplitude is always nilpotent, even when the particle is not free but constrained by various potentials attached to the *E* and **p** terms. In these cases the phase term becomes something more complicated than the simple exponential used for the free particle. The differential operator, which then incorporates the additional phase terms, has to find a phase term such that the amplitude produced after differentiation is nilpotent or squares to zero. This condition or constraint, in effect, substitutes for the Dirac equation and implies that the fundamental expression for a quantum system is an operator rather than an equation. The operator's function is to code for all

IOP Conf. Series: Journal of Physics: Conf. Series **1251** (2019) 012004 doi:10.1088/1742-6596/1251/1/012004

the possible space and time variations associated with the state, with the phase term then decoding the encoding and creating the amplitude.

3. Fermions and bosons

The combination (tensor product) of the four algebras of the parameter group, taken as commutative to each other, is a 64-part group, which is isomorphic to the gamma algebra of the Dirac equation, the quantum mechanical equation for the fundamental particle or fermionic state. This in turn is a version of *Cl*(6, 0) or *G*(6, 0), the double Clifford algebra with 6 real units, representing two complete spaces (and two space-times) [5]. The combined two-space structure creates a hybrid 'energy-momentum space' and physics has largely structured itself on combining real space with this hybrid space. The hybrid space, because of its relation to real space, is not truly independent of it, leading to the anticommutativity of space and momentum, time and energy. Real space and 'charge space' or 'vacuum space' are commutative, but each of these is anticommutative to energy-momentum space.

The nilpotent structure immediately introduces locality and Lorentz invariance (or special relativity). At the same time, the space duality conservation / nonconservation properties of the point-particle, expressed through the complementary pairings of energy / time and momentum / space, leads to the quantum mechanical representations which emerge from canonical quantization. Noether's theorem, which connects conserved and nonconserved quantities, expresses both the global conservation principles of the parameter group and the local conservation principles created by the nilpotent compactification.

The latter process, which creates the fermionic state, also specifically creates discrete invariant or 'rest' mass from the continuous mass distribution of the pure parameter group. It is the incorporation of this structure on the same algebraic basis as energy and momentum that creates the nilpotent version of quantum mechanics (which is the only one which incorporates 'rest mass' as a third quaternionic dimension of the vacuum space, along with E and $\mathbf{p}$). We may note the way that the Higgs mechanism uses the transition from global to local symmetries to create the discrete masses of particles from the continuous vacuum energy.

The quaternionic operators used in the nilpotent structure and the Dirac equation are the equivalent of 4 × 4 matrices, which means that 4 simultaneous solutions are required for the wavefunction: 2 for fermion / antifermion × 2 for spin up / spin down. Here, they are simply $\pm\, \boldsymbol{ik}E \pm \boldsymbol{i}\mathbf{p}$. In the nilpotent formalism we can arrange them as a column vector wavefunction, which may be represented in abbreviated form by $(\pm\, \boldsymbol{ik}E \pm \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)$. So, the four solutions could be represented as:

$(\boldsymbol{ik}E + \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)$	fermion spin up
$(\boldsymbol{ik}E - \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)$	fermion spin down
$(-\boldsymbol{ik}E + \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)$	antifermion spin down
$(-\boldsymbol{ik}E - \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)$	antifermion spin up

The first term in the column represents the actual particle state. So, the column already quoted would represent fermion spin up. Antifermion spin down would place this state first and then go through the same sign charges for the remaining components:

$(-\boldsymbol{ik}E + \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)$	antifermion spin down
$(-\boldsymbol{ik}E - \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)$	antifermion spin up
$(\boldsymbol{ik}E + \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)$	fermion spin up
$(\boldsymbol{ik}E - \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)$	fermion spin down

Negative energy represents continuous (and, inferentially, gravitational) vacuum rather than the local quantized state. Matter seemingly exceeds antimatter in the universe because one set of states exists in observable real space, and the other in the unobservable 'vacuum space', as required by zero totality. Many things are immediate consequences of the algebraic structure in the nilpotent form. The first is the separation of local and nonlocal. The nilpotent structure *creates locality*. In an expression such as ($\pm \boldsymbol{ik}E \pm \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m$), locality is defined as everything inside the bracket, and implies a Lorentzian structure. Everything outside is nonlocal, representing processes not defined by Lorentzian space-time. So any superposition state or combination state is nonlocal in this sense.

Pauli exclusion is also immediate. If a fermion has a nilpotent amplitude, it can never be in the same state as any other fermion as the combination state is zero. A fundamentally nonlocal phenomenon, Pauli exclusion can also be seen as an immediate consequence of defining the total structure of the universe to be exactly zero. Imagine, we are creating a fermion wavefunction of the form $\psi_f = (\boldsymbol{ik}E + \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)$ from *absolutely nothing*; then we must simultaneously create the dual term, $\psi_v = -(\boldsymbol{ik}E + \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)$, which negates it both in superposition and combination:

$$\psi_f + \psi_v = (\boldsymbol{ik}E + \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m) - (\boldsymbol{ik}E + \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m) = 0$$
$$\psi_f \psi_v = -(\boldsymbol{ik}E + \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)(\boldsymbol{ik}E + \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m) = 0$$

Creating a fermion as a singularity simultaneously creates a kind of 'hole in nothing', which we call vacuum, or the rest of the universe, and which is its mirror image.

Wavefunctions or amplitudes are also Pauli exclusive because they are antisymmetric, with nonzero combinations via the Slater determinant:

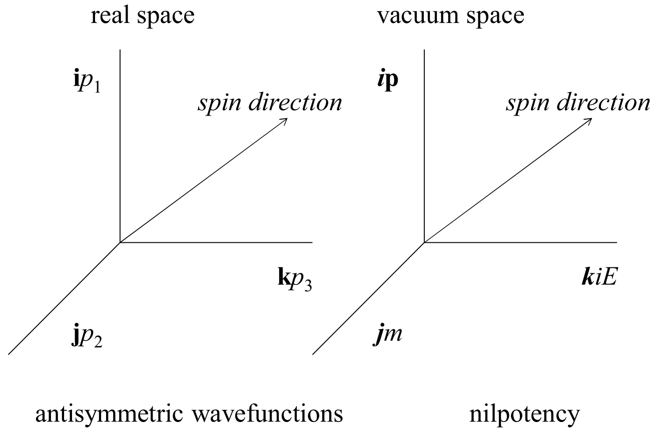
$$\psi_1 \psi_2 - \psi_2 \psi_1 = -(\psi_2 \psi_1 - \psi_1 \psi_2)$$

This is automatic in the nilpotent formalism, where the expression becomes

$$(\pm \boldsymbol{ik}E_1 \pm \boldsymbol{i}\mathbf{p}_1 + \boldsymbol{j}m_1)(\pm \boldsymbol{ik}E_2 \pm \boldsymbol{i}\mathbf{p}_2 + \boldsymbol{j}m_2)$$
$$-(\pm \boldsymbol{ik}E_2 \pm \boldsymbol{i}\mathbf{p}_2 + \boldsymbol{j}m_2)(\pm \boldsymbol{ik}E_1 \pm \boldsymbol{i}\mathbf{p}_1 + \boldsymbol{j}m_1)$$
$$= 4\,\mathbf{p}_1\,\mathbf{p}_2 - 4\,\mathbf{p}_2\,\mathbf{p}_1 = 8\,i\,\mathbf{p}_1 \times \mathbf{p}_2.$$

But, apart from being clearly antisymmetric, the result also tells us something new, for it requires a nilpotent wavefunction to have a **p** vector in spin space at a different orientation to any other. In effect, the only thing that distinguishes two fermionic states is their instantaneous sign of **p** (or **σ.p**). In other words the 3 dimensions of **p** (those of real space) produce the same information as those of the nilpotent structure, which are those of the vacuum space ($\boldsymbol{k}, \boldsymbol{i}, \boldsymbol{j}$). In either case, the combination produces the equivalent of a unique direction in one space or the other.

Pauli exclusion



The norm-zero condition of nilpotency for fermions seemingly creates the 'singularity' (point-like) state out of a combination of two vector spaces, and *no other information*. The information contained in the two spaces of **i**, **j**, **k** and **I**, **J**, **K** is not independent, but each depends on the other. Information about a system in one space can then be translated into information about the system in another. This is important when we are studying structures of particles, where symbols frequently have multiple meanings.

P, *T* and *C* transformations exploit the fact that the three operators, ***i***, ***k*** and ***j*** are on an equal footing in the nilpotent structure:

$$P \qquad \boldsymbol{i}\,(\boldsymbol{ik}E + \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)\,\boldsymbol{i} = (\boldsymbol{ik}E - \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)$$
$$T \qquad \boldsymbol{k}\,(\boldsymbol{ik}E + \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)\,\boldsymbol{k} = (-\boldsymbol{ik}E + \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)$$
$$C \qquad -\boldsymbol{j}\,(\boldsymbol{ik}E + \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)\,\boldsymbol{j} = (-\boldsymbol{ik}E - \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)$$

In the 4-component spinor structure representing the Dirac fermion, the observed particle state is the first in the column, while the others are the accompanying states into which it would transform by *P*, *T* or *C* . Replacing the observed fermion state spin up with any of the others would simultaneously transform all four states by *P*, *T* or *C*.

The *CPT* theorem is an obvious consequence of a nilpotent wavefunction providing a closed algebraic structure, for it is an immediate consequence that $CP \equiv T$, $PT \equiv C$, and $CT \equiv P$ apply, and also that $TCP \equiv CPT \equiv$ identity as

$$\boldsymbol{k}(\boldsymbol{j}(\boldsymbol{i}(\pm\,\boldsymbol{ik}E \pm \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)\boldsymbol{k})\boldsymbol{j})\boldsymbol{j} = \boldsymbol{kji}\,(\pm\,\boldsymbol{ik}E \pm \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)\,\boldsymbol{ijk} = (\pm\,\boldsymbol{ik}E \pm \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m).$$

The *CPT* theorem can be said to combine special relativity and causality. Special relativity says that the square of $(\pm\,\boldsymbol{ik}E \pm \boldsymbol{i}\mathbf{p})$ or its conjugate, $(\pm\,\boldsymbol{ik}t \pm \boldsymbol{i}\mathbf{r})$, is an invariant, but only when we specifically add the invariant term, $\boldsymbol{j}m$ or the equivalent $\boldsymbol{j}t$, do we also get causality. The two principles, relativity and causality, combined require a structure with ***k***, ***i*** and ***j*** on the same footing, and this is also a requirement for *CPT* symmetry.

The three types of boson state can be immediately constructed from combinations of the fermion state with any of the *P*, *T* or *C* transformed ones, the result being a scalar wavefunction.

$(\pm\,\boldsymbol{ik}E \pm \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)\,(\mp\,\boldsymbol{ik}E \pm \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)$	spin 1 boson
$(\pm\,\boldsymbol{ik}E \pm \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)\,(\mp\,\boldsymbol{ik}E \mp \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)$	spin 0 boson
$(\pm\,\boldsymbol{ik}E \pm \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)\,(\pm\,\boldsymbol{ik}E \mp \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)$	fermion-fermion

From the mathematical structures alone we see that a spin 1 boson can be massless, but a spin 0 boson cannot, as then $(\pm\,\boldsymbol{ik}E \pm \boldsymbol{i}\mathbf{p})\,(\mp\,\boldsymbol{ik}E \mp \boldsymbol{i}\mathbf{p})$ would immediately zero itself, which is why Goldstone bosons must become Higgs bosons in the Higgs mechanism.

Though the total vacuum structure can be thought of as continuous, corresponding to the state of mass before localization, fermion singularities create structure in the parts of the vacuum to which they respond. The vacuum structuring directly reflects the structuring of matter. If we take the standard fermion wavefunction $(\pm\,\boldsymbol{ik}E \pm \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)$ and post-multiply it by the idempotent $\boldsymbol{k}(\pm\,\boldsymbol{ik}E \pm \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)$ any number of times, the only effect is to introduce a scalar multiple, which can be normalized away.

$$(\pm\,\boldsymbol{ik}E \pm \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)\,\boldsymbol{k}(\pm\,\boldsymbol{ik}E \pm \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)\,\boldsymbol{k}(\pm\,\boldsymbol{ik}E \pm \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)\,\ldots \;\rightarrow (\pm\,\boldsymbol{ik}E \pm \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)$$

The same thing occurs with with $\boldsymbol{j}(\pm\,\boldsymbol{ik}E \pm \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)$ or $\boldsymbol{i}(\pm\,\boldsymbol{ik}E \pm \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)$, although the latter also produces vector terms in even applications which can also be removed by normalization. Notably, these idempotents (which are similar to the relatively limited minimum left ideal solutions of the conventional Dirac formalism) only give partial pictures of the fermion interactions.

IOP Conf. Series: Journal of Physics: Conf. Series **1251** (2019) 012004 doi:10.1088/1742-6596/1251/1/012004

The partitions can be described as strong, weak and electric 'vacua', and assigned to particular roles within existing physics:

$\boldsymbol{k}\,(\boldsymbol{ik}E + \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)$	weak vacuum	fermion creation
$\boldsymbol{i}\,(\boldsymbol{ik}E + \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)$	strong vacuum	gluon plasma
$\boldsymbol{j}\,(\boldsymbol{ik}E + \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)$	electric vacuum	isospin / hypercharge

The electric vacuum – full or empty in fermionic states – can be seen as responsible for the transition between weak isospin up and down states in what is necessarily a chiral process.

The idea notably connects with the gravity-gauge theory correspondence which has been taken up in some manifestations of string theory but is a natural consequence of particles having a simultaneous existence in two spaces, one a nonlocalized vacuum space with continuous negative energy and the other a localized space with positive energy. In the nilpotent formalism, the whole $-(\pm\,\boldsymbol{ik}E \pm \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)$ can be seen as the gravitational vacuum, with its positive version corresponding to a local Lorentzian manifestation of a fictious inertia.

The three bosonic states are also clearly related to the vacua produced by the three quaternionic operators:

weak spin 1

$$(\boldsymbol{ik}E + \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)\,\boldsymbol{k}\,(\boldsymbol{ik}E + \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)\,\boldsymbol{k}\,(\boldsymbol{ik}E + \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)\,\boldsymbol{k}\,(\boldsymbol{ik}E + \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)\,\ldots$$
$$(\boldsymbol{ik}E + \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)\,(-\boldsymbol{ik}E + \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)\,(\boldsymbol{ik}E + \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)\,(-\boldsymbol{ik}E + \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)\,\ldots$$

electric spin 0

$$(\boldsymbol{ik}E + \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)\,\boldsymbol{j}\,(\boldsymbol{ik}E + \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)\,\boldsymbol{j}\,(\boldsymbol{ik}E + \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)\,\boldsymbol{j}\,(\boldsymbol{ik}E + \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)\,\ldots$$
$$(\boldsymbol{ik}E + \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)\,(-\,\boldsymbol{ik}E - \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)\,(\boldsymbol{ik}E + \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)\,(-\,\boldsymbol{ik}E - \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)\,\ldots$$

strong paired fermion state

$$(\boldsymbol{ik}E + \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)\,\boldsymbol{i}\,(\boldsymbol{ik}E + \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)\,\boldsymbol{i}\,(\boldsymbol{ik}E + \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)\,\boldsymbol{i}\,(\boldsymbol{ik}E + \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)\,\ldots$$
$$(\boldsymbol{ik}E + \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)\,(\boldsymbol{ik}E - \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)\,(\boldsymbol{ik}E + \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)\,(\boldsymbol{ik}E - \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)\,\ldots$$

The processes described further indicate that repeated post-multiplication of a fermion operator by any of the discrete idempotent vacuum operators creates an alternate series of antifermion and fermion vacuum states, or, equivalently, an alternate series of boson and fermion states without changing the character of the real particle state. The equivalence of these vacuum bosons to the original real fermion state suggests a real answer to some of the problems of renormalization. A fermion produces a boson state by combining with its own vacuum image in any of the three vacuum interaction 'mirrors' (***i***, ***k*** and ***j***), and the two states then form a *supersymmetric* partnership. The fermion / boson nilpotent creation and annihilation operators are thus intrinsically supersymmetric, with supersymmetry operators typically of the form:

Boson to fermion: $Q = (\pm\,\boldsymbol{ik}E \pm \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)$
Fermion to boson: $Q^{\dagger} = (\mp\,\boldsymbol{ik}E \pm \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)$

Here, a fermion converts to a boson by multiplication by an antifermionic operator $Q^{\dagger}$; a boson converts to a fermion by multiplication by a fermionic operator Q, and the sequence $(\boldsymbol{ik}E + \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)\,\boldsymbol{k}\,(\boldsymbol{ik}E + \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)$ ... can be represented by the supersymmetric

$$Q\,Q^{\dagger}\,Q\,Q^{\dagger}\,Q\,Q^{\dagger}\,Q\,Q^{\dagger}\,Q\,\ldots$$

IOP Conf. Series: Journal of Physics: Conf. Series **1251** (2019) 012004 doi:10.1088/1742-6596/1251/1/012004

If this is interpreted as the series of boson and fermion loops, of the same energy and momentum, required by the exact supersymmetry, then the self-energy renormalization can be eliminated and the hierarchy problem removed altogether [5, 8, 31]

4. Baryons and gluons

The two fermionic spaces become important when we look at particle structures. Baryons can be seen in real space through the rotation-symmetric momentum operator [5, 8]. But this exists at the same time as a rotation asymmetric vacuum or charge space. The dual aspects of angular momentum are seen in the distinction between space and charge: it is both nonconserved and symmetric in real space, and conserved and asymmetric in vacuum or charge space. The necessary duality, which occurs for all 3-dimensional quantities, is encapsulated in Noether's theorem.

The strong interaction can be completely explained by the spatial 3-dimensionality of the momentum operator in the nilpotent wavefunction, but it also requires the other 3-dimensionality associated with charge. Effectively, the vector aspect of the strong charge requires a source term and corresponding vacuum with three components. Though we clearly cannot combine three components in the form:

$$(\boldsymbol{ik}E \pm \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)\,(\boldsymbol{ik}E \pm \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)\,(\boldsymbol{ik}E \pm \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)$$

as this will automatically reduce to zero, we can imagine a three-component structure in which the vector nature of $\mathbf{p}$ plays an explicit role, specifically

$$(\boldsymbol{ik}E \pm \boldsymbol{i}\,\mathbf{i}p_x + \boldsymbol{j}m)\,(\boldsymbol{ik}E \pm \boldsymbol{i}\,\mathbf{j}p_y + \boldsymbol{j}m)\,(\boldsymbol{ik}E \pm \boldsymbol{i}\,\mathbf{k}p_z + \boldsymbol{j}m) \tag{1}$$

For convenience, only the first term of the 4-component spinors is specified, but the two spin states are retained, as these are needed explicitly.

Nilpotent solutions are possible when $\mathbf{p} = \pm\,\boldsymbol{i}\,\mathbf{i}p_x$, $\mathbf{p} = \pm\,\boldsymbol{i}\,\mathbf{j}p_y$, or $\mathbf{p} = \pm\,\boldsymbol{i}\,\mathbf{k}p_z$, or when the momentum is directed entirely along the x, y, or z axes, in either direction, these, of course, being arbitrarily defined. All other phases will be superposition of these. If we multiply out (1) for each possible phase, using the appropriate normalization to remove arbitrary scalars, the amplitudes reduce to

$(\boldsymbol{ik}E + \boldsymbol{i}\mathbf{i}p_x + \boldsymbol{j}m)$	$+RGB$
$(\boldsymbol{ik}E - \boldsymbol{i}\mathbf{i}p_x + \boldsymbol{j}m)$	$-RBG$
$(\boldsymbol{ik}E - \boldsymbol{i}\mathbf{j}p_y + \boldsymbol{j}m)$	$+BRG$
$(\boldsymbol{ik}E + \boldsymbol{i}\mathbf{j}p_y + \boldsymbol{j}m)$	$-GRB$
$(\boldsymbol{ik}E + \boldsymbol{i}\mathbf{k}p_z + \boldsymbol{j}m)$	$+GBR$
$(\boldsymbol{ik}E - \boldsymbol{i}\mathbf{k}p_z + \boldsymbol{j}m)$	$-BGR$

The third and fourth of these, very significantly, change the sign of the $\mathbf{p}$ component. Because of this, there has to be a maximal superposition of left- and right-handed components, thus explaining the zero observed chirality in the interaction and the mass of the baryon (because equal left- and right-handedness creates mass by the Higgs mechanism).

This cycle of six phases requires an $SU(3)$ group structure, with eight generators and a wavefunction superposition, exactly like that of the conventional model using coloured quarks,

$$\psi \sim (BGR - BRG + GRB - GBR + RBG - RGB).$$

'Colour' transitions in the 3-component structures require either an exchange of the components of $\mathbf{p}$ between the individual quarks or baryon components, or a relative switching of the component positions, independently of any actual distance between the components. No direction is privileged, meaning that the transition is gauge invariant, with massless mediators, exactly as happens with the eight massless gluons of the conventional formalism.

The complete wavefunction contains information from the equivalent of six allowed independent nonlocally gauge invariant phases, which all exist simultaneously and are subject to continual transitions at a constant rate:

$$
\begin{array}{ll}
(\boldsymbol{ik}E + \boldsymbol{i}\mathbf{i}p_x + \boldsymbol{j}m)\ (\boldsymbol{ik}E + \ ...\ + \boldsymbol{j}m)\ (\boldsymbol{ik}E + \ ...\ + \boldsymbol{j}m) & +RGB \\
(\boldsymbol{ik}E - \boldsymbol{i}\mathbf{i}p_x + \boldsymbol{j}m)\ (\boldsymbol{ik}E - \ ...\ + \boldsymbol{j}m)\ (\boldsymbol{ik}E - \ ...\ + \boldsymbol{j}m) & -RBG \\
(\boldsymbol{ik}E + \ ...\ + \boldsymbol{j}m)\ (\boldsymbol{ik}E + \boldsymbol{i}\mathbf{j}p_y + \boldsymbol{j}m)\ (\boldsymbol{ik}E + \ ...\ + \boldsymbol{j}m) & +BRG \\
(\boldsymbol{ik}E - \ ...\ + \boldsymbol{j}m)\ (\boldsymbol{ik}E - \boldsymbol{i}\mathbf{j}p_y + \boldsymbol{j}m)\ (\boldsymbol{ik}E - \ ...\ + \boldsymbol{j}m) & -GRB \\
(\boldsymbol{ik}E + \ ...\ + \boldsymbol{j}m)\ (\boldsymbol{ik}E + \ ...\ + \boldsymbol{j}m)\ (\boldsymbol{ik}E + \boldsymbol{i}\mathbf{k}p_z + \boldsymbol{j}m) & +GBR \\
(\boldsymbol{ik}E - \ ...\ + \boldsymbol{j}m)\ (\boldsymbol{ik}E - \ ...\ + \boldsymbol{j}m)\ (\boldsymbol{ik}E - \boldsymbol{i}\mathbf{k}p_z + \boldsymbol{j}m) & -BGR
\end{array}
$$

The six ’coloured’ gluons required are then:

$$
\begin{array}{ll}
(\boldsymbol{ik}E + \boldsymbol{i}\mathbf{i}p_x)\ (-\boldsymbol{ik}E + \boldsymbol{i}\mathbf{j}p_y) & (\boldsymbol{ik}E + \boldsymbol{i}\mathbf{j}p_y)\ (-\boldsymbol{ik}E + \boldsymbol{i}\mathbf{i}p_x) \\
(\boldsymbol{ik}E + \boldsymbol{i}\mathbf{j}p_y)\ (-\boldsymbol{ik}E + \boldsymbol{i}\mathbf{k}p_z) & (\boldsymbol{ik}E + \boldsymbol{i}\mathbf{k}p_z)\ (-\boldsymbol{ik}E + \boldsymbol{i}\mathbf{j}p_y) \\
(\boldsymbol{ik}E + \boldsymbol{i}\mathbf{k}p_z)\ (-\boldsymbol{ik}E + \boldsymbol{i}\mathbf{i}p_x) & (\boldsymbol{ik}E + \boldsymbol{i}\mathbf{i}p_x)\ (-\boldsymbol{ik}E + \boldsymbol{i}\mathbf{k}p_z)
\end{array}
$$

Two ‘colourless’ ones emerge from combinations of the three possible structures:

$$
\begin{array}{ll}
(\boldsymbol{ik}E + \boldsymbol{i}\mathbf{i}p_x)\ (-\boldsymbol{ik}E + \boldsymbol{i}\mathbf{i}p_x) & (\boldsymbol{ik}E + \boldsymbol{i}\mathbf{j}p_y)\ (-\boldsymbol{ik}E + \boldsymbol{i}\mathbf{j}p_y) \\
(\boldsymbol{ik}E + \boldsymbol{i}\mathbf{k}p_z)\ (-\boldsymbol{ik}E + \boldsymbol{i}\mathbf{k}p_z) &
\end{array}
$$

As with the baryons, only the lead term is shown for each 4-component spinor.

The ‘coloured’ gluons introduce pseudovector terms because includes a product of orthogonal vector units. The ‘directional’ aspects of the gluons show why they are different from the other massless spin 1 gauge bosons or photons, which take the form $(\pm\, \boldsymbol{ik}E \pm \boldsymbol{i}\mathbf{p})\ (\mp\, \boldsymbol{ik}E \pm \boldsymbol{i}\mathbf{p})$, and these in turn are distinguishable from the massive spin 1 gauge bosons, W and Z, which are of the form $(\pm\, \boldsymbol{ik}E \pm \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)$ $(\mp\, \boldsymbol{ik}E \pm \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)$, and the massive spin 0 Higgs boson, $(\pm\, \boldsymbol{ik}E \pm \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)\ (\mp\, \boldsymbol{ik}E \mp \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)$.

5. Fermion structures from the group of order 64

The two fermionic spaces become important when we look at particle structures. Baryons can be seen in real space through the rotation-symmetric momentum operator. But this exists at the same time as a rotation asymmetric vacuum. In general, there are two structures for all fermions. There is the nilpotent or phase space structure, expressed in terms of E, $\mathbf{p}$ and m, which determines the quantum mechanics, and the charge structure which states the composition in terms of s, e and w charges, independently of space and time. The relationship between these is often subtle, sometimes using the same algebraic symbols in quite different ways. The nilpotent structure is substantially the same for all fermions, though differs mainly in the mass values and some aspects of E and $\mathbf{p}$. The charge structure, on the other hand, which is different for all fermions and leads to the unique rest masses which are incorporated into the nilpotent structure, cannot easily be obtained from the nilpotent structure. However, as we will see, the charge structures can ultimately be used to determine the nilpotent structures, on an effectively 1 to 1 basis.

The charge structures have a simple origin in fundamental terms [1, 5, 8-9]. They are a description of unit point charges, electric, strong and weak. The charges may be present or absent, which we can symbolize by 1 or 0; they may be + or –; they may be e, s or w. There are also considerations related to *CPT* symmetry and the chirality of fermions under the weak interaction. The nilpotents, or *possible* nilpotents, must in some way determine charge structures. How? Let us start with the group of order 64 from the nilpotent structures are derived.

The nilpotent structure actually needs only one out of the 12 pentads which compose the 64-component group of the Dirac algebra. What is the significance of the others? It seems that they can be used to determine the degrees of freedom available to fermions. The 36 Standard Model quarks and antiquarks and the 12 Standard Model leptons and antileptons can each be represented in full by using the 12 pentad structures, and defining respective coefficients of momentum (p_x, p_y, p_z), energy (E) and rest mass (m) [8, 19-21]. The only difference between the leptons and quarks is the fact that the vector structures in the latter represent a choice between three possibilities (represented here by the OR symbol, V), because the quarks take the three vector directions one at a time.

generation		*isospin*					
1	electron neutrino	up	ii	ij	ik	ik	j
	electron	down	iii	iij	iik	ik	j
2	muon neutrino	up	ji	jj	jk	ii	k
	muon	down	iji	ijj	ijk	ii	k
3	tau neutrino	up	ki	kj	kk	ij	i
	tau	down	iki	ikj	ikk	ij	i
1	antielectron-neutrino	up	–ii	–ij	–ik	–ik	–j
	antielectron	down	–iii	–iij	–iik	–ik	–j
2	antimuon-neutrino	up	–ji	–jj	–jk	–ii	–k
	antimuon	down	–iji	–ijj	–ijk	–ii	–k
3	antitau-neutrino	up	–ki	–kj	–kk	–ij	–i
	antitau	down	–iki	–ikj	–ikk	–ij	–i

generation		*isospin*							
1	up quark	up	ii	V	ij	V	ik	ik	j
	down quark	down	iii	V	iij	V	iik	ik	j
2	charm quark	up	ji	V	jj	V	jk	ii	k
	strange quark	down	iji	V	ijj	V	ijk	ii	k
3	top quark	up	ki	V	kj	V	kk	ij	i
	bottom quark	down	iki	V	ikj	V	ikk	ij	i
1	anti-up quark	up	–ii	V	–ij	V	–ik	–ik	–j
	anti-down quark	down	–iii	V	–iij	V	–iik	–ik	–j
2	anti-charm quark	up	–ji	V	–jj	V	–jk	–ii	–k
	anti-strange quark	down	–iji	V	–ijj	V	–ijk	–ii	–k
3	antitop-quark	up	–ki	V	–kj	V	–kk	–ij	–i
	antibottom-quark	down	–iki	V	–ikj	V	–ikk	–ij	–i

In fact, the pentad structures have two meanings, either as components of fermions or as *fermions themselves*. For, as the related geometrical structures based on Platonic solids suggest, we can invert the derivation of 12 structures from a 5-unit pentad, and map the fermions onto a new pentad structure, of which the pseudoscalar component (the *iE* term) is 12 leptons / antileptons, and the vector component (the **p** term) 36 quarks / antiquarks (the fifth term being like a boson). Such representations are not unique. There are alternative versions, as we will see, though the *degrees of freedom* for the fermions and antifermions of all three generations remain the same. The table now becomes:

1	up quark	$\boldsymbol{i}\mathbf{i}$	V	$\boldsymbol{i}\mathbf{j}$	V	$\boldsymbol{i}\mathbf{k}$	electron neutrino	$i\boldsymbol{k}$
	down quark	$i\mathbf{i}\boldsymbol{i}$	V	$i\mathbf{i}\boldsymbol{j}$	V	$i\mathbf{i}\boldsymbol{k}$	electron	$i\mathbf{k}$
2	charm quark	$\boldsymbol{j}\mathbf{i}$	V	$\boldsymbol{j}\mathbf{j}$	V	$\boldsymbol{j}\mathbf{k}$	muon	$i\boldsymbol{i}$
	strange quark	$i\mathbf{j}\boldsymbol{i}$	V	$i\mathbf{j}\boldsymbol{j}$	V	$i\mathbf{j}\boldsymbol{k}$	muon neutrino	$i\mathbf{i}$
3	top quark	$\boldsymbol{k}\mathbf{i}$	V	$\boldsymbol{k}\mathbf{j}$	V	$\boldsymbol{k}\mathbf{k}$	tau neutrino	$i\boldsymbol{j}$
	bottom quark	$i\mathbf{k}\boldsymbol{i}$	V	$i\mathbf{k}\boldsymbol{j}$	V	$i\mathbf{k}\boldsymbol{k}$	tau	$i\mathbf{j}$
1	anti-up quark	$-\boldsymbol{i}\mathbf{i}$	V	$-\boldsymbol{i}\mathbf{j}$	V	$-\boldsymbol{i}\mathbf{k}$	antielectron neutrino	$-i\boldsymbol{k}$
	anti-down quark	$-i\mathbf{i}\boldsymbol{i}$	V	$-i\mathbf{i}\boldsymbol{j}$	V	$-i\mathbf{i}\boldsymbol{k}$	antielectron	$-i\mathbf{k}$
2	anti-charm quark	$-\boldsymbol{j}\mathbf{i}$	V	$-\boldsymbol{j}\mathbf{j}$	V	$-\boldsymbol{j}\mathbf{k}$	antimuon neutrino	$-i\boldsymbol{i}$
	anti-strange quark	$-i\mathbf{j}\boldsymbol{i}$	V	$-i\mathbf{j}\boldsymbol{j}$	V	$-i\mathbf{j}\boldsymbol{k}$	antimuon	$-i\mathbf{i}$
3	antitop-quark	$-\boldsymbol{k}\mathbf{i}$	V	$-\boldsymbol{k}\mathbf{j}$	V	$-\boldsymbol{k}\mathbf{k}$	antitau neutrino	$-i\boldsymbol{j}$
	antibottom-quark	$-i\mathbf{k}\boldsymbol{i}$	V	$-i\mathbf{k}\boldsymbol{j}$	V	$-i\mathbf{k}\boldsymbol{k}$	antitau	$-i\mathbf{j}$

The remaining units $\boldsymbol{j}$, $\mathbf{j}$, $\boldsymbol{k}$, $\mathbf{j}$, $\boldsymbol{i}$, $\mathbf{i}$, $-\boldsymbol{j}$, $-\mathbf{j}$, $-\boldsymbol{k}$, $-\mathbf{j}$, $-\boldsymbol{i}$, $-\mathbf{i}$, can possibly be taken, in some way, to represent the 12 gauge bosons.

The minimal degrees of freedom required to set up the 48 Standard Model particles can be represented as follows:

+ and – represent particles and antiparticles
separate and combined vector terms represent quarks and leptons
$\mathbf{i}$, $\mathbf{j}$ or $\mathbf{k}$ represent the 3 colours of quark
$\boldsymbol{i}$, $\boldsymbol{j}$ or $\boldsymbol{k}$ can be used to represent the 3 generations
absence or presence of i represents up / down isospin states

These degrees of freedom are derived only from the algebra itself. While the first three are obvious, the fourth can probably be accommodated by C, P, T (in the order $\boldsymbol{j}$, $\boldsymbol{i}$, $\boldsymbol{k}$), and the fifth is a possible version of the electroweak broken symmetry. Here, the pseudoscalar i is used to switch between quaternions and vectors, simulating an inversion between real and vacuum space and between the filled and empty electric vacuum, but it could equally be used as a simple multiplier, as we will show below.

These options can then be arranged in the form:

$(1, \boldsymbol{i}, \boldsymbol{j}, \boldsymbol{k})$	$(1, \mathbf{i}, \mathbf{j}, \mathbf{k})$	$(1, i)$	$(1, -1)$
$\boldsymbol{i}$, $\boldsymbol{j}$, $\boldsymbol{k}$ are 3 generations 1 is the 'zeroth generation' – no charges	1 represents leptons $\mathbf{i}$, $\mathbf{j}$, $\mathbf{k}$ are 3 colours	1 isospin up i isospin down	1 particle –1 antiparticle

It will be shown subsequently that the first three options represent the possibilities available to respective weak, strong and electric charges. All transfer information about these charges to the angular momentum operator through its three axes in dual spaces. In a significant earlier result by the author [2, 5, 8], the symmetries of the Klein-4 parameter group can be shown to lead to an extension of Noether's theorem in which the conservation of the type of charge (electric, strong or weak), necessarily taken to be rotation asymmetric, is identified with the conservation of angular momentum through the respective conservation laws of magnitude, direction and handedness, and from these to the respective distinctive symmetry groups $U(1)$, $SU(3)$ and $SU(2)$.

As previously stated, the algebraic options allow a variation in the representation of isospin in the structures of fermions and antifermions as derived from the group of order 64, though the isospin still involves a multiplication by the pseudoscalar unit i:

Quarks			*Leptons*	*Antiquarks*			*Antileptons*
$\boldsymbol{i}\mathbf{i}$	$\boldsymbol{i}\mathbf{j}$	$\boldsymbol{i}\mathbf{k}$	$\boldsymbol{i}$	$-\boldsymbol{i}\mathbf{i}$	$-\boldsymbol{i}\mathbf{j}$	$-\boldsymbol{i}\mathbf{k}$	$-\boldsymbol{i}$
$i\boldsymbol{i}\mathbf{i}$	$i\boldsymbol{i}\mathbf{j}$	$i\boldsymbol{i}\mathbf{k}$	$i\boldsymbol{i}$	$-i\boldsymbol{i}\mathbf{i}$	$-i\boldsymbol{i}\mathbf{j}$	$-i\boldsymbol{i}\mathbf{k}$	$-i\boldsymbol{i}$
$\boldsymbol{j}\mathbf{i}$	$\boldsymbol{j}\mathbf{j}$	$\boldsymbol{j}\mathbf{k}$	$\boldsymbol{j}$	$-\boldsymbol{j}\mathbf{i}$	$-\boldsymbol{j}\mathbf{j}$	$-\boldsymbol{j}\mathbf{k}$	$-\boldsymbol{j}$
$i\boldsymbol{j}\mathbf{i}$	$i\boldsymbol{j}\mathbf{j}$	$i\boldsymbol{j}\mathbf{k}$	$i\boldsymbol{j}$	$-i\boldsymbol{j}\mathbf{i}$	$-i\boldsymbol{j}\mathbf{j}$	$-i\boldsymbol{j}\mathbf{k}$	$-i\boldsymbol{j}$
$\boldsymbol{k}\mathbf{i}$	$\boldsymbol{k}\mathbf{j}$	$\boldsymbol{k}\mathbf{k}$	$\boldsymbol{k}$	$-\boldsymbol{k}\mathbf{i}$	$-\boldsymbol{k}\mathbf{j}$	$-\boldsymbol{k}\mathbf{k}$	$-\boldsymbol{k}$
$i\boldsymbol{k}\mathbf{i}$	$\boldsymbol{k}\mathbf{j}$	$i\boldsymbol{k}\mathbf{k}$	$i\boldsymbol{k}$	$-i\boldsymbol{k}\mathbf{i}$	$-i\boldsymbol{k}\mathbf{j}$	$-i\boldsymbol{k}\mathbf{k}$	$-i\boldsymbol{k}$

The options are, of course, possible because vectors and quaternions transform into each other via multiplication by a pseudoscalar unit. In addition, the simultaneous 3-fold distribution of quaternion units between generations and *within* them can be accomplished either by changing only one or both at any one time. The advantage of the present structure is that it allows all *charged* (quaternionic) states to be differentiated from uncharged ones as fermions, with the 12 bosons possibly taking up the uncharged units, excluding 1 –1, *i* and –*i* (although the W^- and the W^+ bosons are, of course, in reality, charged).

Transitions between quarks and leptons in the present structure involve multiplication by unit vectors. Transitions between quarks involve pseudovectors (gluons). Isospin transitions involve ± *i*. Transitions between generations involve quaternion units (acting through *C*, *P* and *T*). Creation of fermions from the 'zeroth generation' also involves quaternion units. A fermion / antifermion transition requires a factor of –1. The fact that quaternions are not included in 16 components of the 64-component group means that there are elements of space, or space and time, or space and time and mass. which don't include charges – leading to the fact charges exist in separated positions in space and time.

The components with charge (symbolized by quaternions) are also separated into two groups, those with no vector components, and so no spatial element (leptons) and those with vector components, indicating spatial elements (quarks), leading to the three quarks in a baryon requiring spatial separation. Mass, by contrast with charge, is a scalar, and so is included in all 64 of the elements of the algebra and is a universal component of everything. The simplicity of this classification has an obvious appeal if we are only concerned with quarks and leptons. However, the alternative structure previously discussed allows an easier overall classification involving quarks, leptons and bosons, which (as we will see) can be extended to higher group classifications such as E_8 with the inclusion of spin states and vacuum.

The symbols cannot directly represent fermion structures in either classification, and certainly not structures requiring different charge types in all three generations, but they can code for fermion charges if we remember the complementary description of the particles in terms of the momentum operator. They are the components of the double space corresponding to the fermions, which need to be translated into single space equivalents (often through **p**) for observed particles. Here, notably, the $\boldsymbol{i}$, $\boldsymbol{j}$, $\boldsymbol{k}$ units become the generators of *P*, *C*, *T*, rather than being direct representations of the charges.

6. An equation for the charge structures of fermions

Given the known characteristics of each interaction, it is possible to set down a single expression related to the momentum operator which generates the charge structures of all fermionic and antifermionic states [5, 8]. Given also the general principles behind the Higgs mechanism, the same formula additionally serves to explain the mass generated when an element of partial right-handedness is introduced into an intrinsically left-handed system.

$$\boldsymbol{\sigma}_{\mathbf{z}}.\left(\boldsymbol{j}(\hat{p}_a - \mathbf{1}\delta_{0m}) + \boldsymbol{i}\hat{p}_b(\delta_{bc} - 1) + \boldsymbol{k}\hat{p}_c\left((-1)^{\delta_{1g}}g\right)\right). \quad (2)$$

The spin pseudovector component $\boldsymbol{\sigma}_{\mathbf{z}}$ (with *z* defined as the reference direction) is taken as intrinsically left-handed from the structure of the Dirac equation; $\boldsymbol{\sigma}_{\mathbf{z}} = -1$ defines left-handed states, = 1 defines right-handed. Assuming that chirality requires a filled weak vacuum, left-handed states are predominantly fermionic, with right-handed states appearing as antifermionic 'holes' in the vacuum (0 in this representation). The filled weak vacuum implies that the 'ground state of the universe' can be specified

IOP Conf. Series: Journal of Physics: Conf. Series **1251** (2019) 012004 doi:10.1088/1742-6596/1251/1/012004

in terms of positive, but not negative, energy (*E*). In physical terms, this is because the universal mass-energy of the universe is a continuum state.

Any reduction of the size of the bracket, for example, by reducing any of its terms to 0, will have the effect of reducing the degree of left-handedness by introducing the opposite sign of σ_z or a partially right-handed state. The degree of right-handedness determines the *zitterbewegung* frequency and so the amount of mass. The three terms in the bracket effectively represent weak isospin, quark confinement and weak charge conjugation violation, responding respectively to the electric, strong and weak charges. The last is particularly significant as successive violations of *P* and *T* symmetry involve 'step functions' in the introduction of a right-handed component and hence large mass creations.

The coding of the degrees of freedom in the previous section enables us to locate which parts of the expression are active: ($\boldsymbol{i}$, $\boldsymbol{j}$, $\boldsymbol{k}$) switch on the third term (weak); (**i**, **j**, **k**) the second term (strong); and *i* the isospin down part of the first (electric). The quaternion operators in the expression, $\boldsymbol{j}$, $\boldsymbol{i}$ and $\boldsymbol{k}$ (which are to be distinguished from the same units as used in the degrees of freedom), are respectively electric, strong and weak charge units. $\hat{p}_a$, $\hat{p}_b$ and $\hat{p}_c$ are quantized angular momentum units, which randomly and independently take on values from the three orthogonal components $\hat{p}_x$, $\hat{p}_y$ and $\hat{p}_z$. We note that the information is entirely conveyed in terms of the angular momentum operator (**p**).

In the first term, *m* is an electromagnetic mass unit, which selects the state of weak isospin by becoming 1 when present and 0 when absent. The respective values of *m* define the weak isospin up and down states; $m = 1$ corresponds to an empty electromagnetic vacuum, $m = 0$ to a filled one.

In the second term, $b = c$ leads to the generation of leptons; $b \neq c$ to that of quarks. When $b = c$ we can define a single direction, but $b \neq c$ forces us to take into account the three directions of **p** at once, and define baryons as composed of three quarks (with mesons as quark and antiquark), with each of *a*, *b*, *c* cycling through the directions *x*, *y*, *z*.

In the third term, *g* corresponds to a conjugation of weak charge units; $g = -1$ represents maximal conjugation. If the conjugation fails maximally, then *g* becomes 1; *g* is a composite term, containing a parity element (*P*) and a time-reversal element (*T*). There are then two ways in which the conjugated *PT* may remain at the unconjugated value ($g = 1$). The first generation of *u*, *d*, ν_e, *e* requires $g = -1$; the second generation of *c*, *s*, ν_μ, μ comes from $g = 1$, with *P* responsible; the third generation of *t*, *b*, ν_τ, τ. Is the result when $g = 1$ with *T* responsible.

The weak interaction can only identify the handedness of $\boldsymbol{\sigma}_z$. In the in the anticommuting Dirac pentad ($\boldsymbol{i}\boldsymbol{k}E + \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m$), this occupies the $\boldsymbol{ik}E$ site, and the *i* term is responsible for the distinction between fermions and antifermions. This complex operator in $\boldsymbol{ik}E$ means that the sign of $\boldsymbol{k}$ has two possible values even when those of $\boldsymbol{i}$ and $\boldsymbol{j}$ are fixed. The sign of the weak charge associated with $\boldsymbol{k}$ is, consequently, determined in physical terms only by the sign of $\boldsymbol{\sigma}_z$.

Just twelve fermionic structures arise from the various permutations of the general equation (2), and these are created by discrete operations with differing degrees of right-handedness.

up quark	$-\boldsymbol{\sigma}.(-\boldsymbol{j}(\hat{p}_a - \mathbf{1}) + \boldsymbol{j}\hat{p}_b + \boldsymbol{k}\hat{p}_c)$
down quark	$-\boldsymbol{\sigma}.(-\boldsymbol{j}\hat{p}_a + \boldsymbol{j}\hat{p}_b + \boldsymbol{k}\hat{p}_c)$
charm quark	$-\boldsymbol{\sigma}.(-\boldsymbol{j}(\hat{p}_a - \mathbf{1}) + \boldsymbol{j}\hat{p}_b - z_P\boldsymbol{k}\hat{p}_c)$
strange quark	$-\boldsymbol{\sigma}.(-\boldsymbol{j}\hat{p}_a + \boldsymbol{j}\hat{p}_b - z_P\boldsymbol{k}\hat{p}_c)$
top quark	$-\boldsymbol{\sigma}.(-\boldsymbol{j}(\hat{p}_a - \mathbf{1}) + \boldsymbol{j}\hat{p}_b - z_T\boldsymbol{k}\hat{p}_c)$
bottom quark	$-\boldsymbol{\sigma}.(-\boldsymbol{j}\hat{p}_a + \boldsymbol{j}\hat{p}_b - z_T\boldsymbol{k}\hat{p}_c)$
electron neutrino	$-\boldsymbol{\sigma}.(-\boldsymbol{j}(\hat{p}_a - \mathbf{1}) + \boldsymbol{k}\hat{p}_c)$
electron	$-\boldsymbol{\sigma}.(-\boldsymbol{j}\hat{p}_a + \boldsymbol{k}\hat{p}_c)$
muon neutrino	$-\boldsymbol{\sigma}.(-\boldsymbol{j}(\hat{p}_a - \mathbf{1}) - z_P\boldsymbol{k}\hat{p}_c)$
muon	$-\boldsymbol{\sigma}.(-\boldsymbol{j}\hat{p}_a - z_P\boldsymbol{k}\hat{p}_c)$
tau neutrino	$-\boldsymbol{\sigma}.(-\boldsymbol{j}(\hat{p}_a - \mathbf{1}) - z_T\boldsymbol{k}\hat{p}_c)$
tau	$-\boldsymbol{\sigma}.(-\boldsymbol{j}\hat{p}_a - z_T\boldsymbol{k}\hat{p}_c)$

It is notable that the fermions of the second and third generations only match those of the first generation in their weak charges if parity and time reversal symmetry are violated in their weak interactions. The weak charge signs would be opposite but for these symmetry violations, which are here symbolised by z_P and z_T.

7. The charge structures of fermions in tabular form

The charge structures arising from the general equation (2) can be set out in four tables. These are the oldest part of the current work and were the first method used of explaining particle structures in terms of the interaction between two spaces or 3-dimensionalities (here, that of the three colours, derived from ordinary space, and that of the three charges) [1-2, 5, 8-9, 32-36]. They required three different methods of incorporating a sign variation in three different charges. The multipliers z_P and z_T are significant here as well as in the structures derived from equation (2).

A

		B	**G**	**R**
u	$+e$	$1\mathbf{j}$	$1\mathbf{j}$	$0\mathbf{i}$
	$+s$	$1\mathbf{i}$	$0\mathbf{k}$	$0\mathbf{j}$
	$+w$	$1\mathbf{k}$	$0\mathbf{i}$	$0\mathbf{k}$
d	$-e$	$0\mathbf{j}$	$0\mathbf{k}$	$1\mathbf{j}$
	$+s$	$1\mathbf{i}$	$0\mathbf{i}$	$0\mathbf{k}$
	$+w$	$1\mathbf{k}$	$0\mathbf{j}$	$0\mathbf{i}$
c	$+e$	$1\mathbf{j}$	$1\mathbf{j}$	$0\mathbf{i}$
	$+s$	$1\mathbf{i}$	$0\mathbf{k}$	$0\mathbf{j}$
	$-w$	$z_P\mathbf{k}$	$0\mathbf{i}$	$0\mathbf{k}$
s	$-e$	$0\mathbf{j}$	$0\mathbf{k}$	$1\mathbf{j}$
	$+s$	$1\mathbf{i}$	$0\mathbf{i}$	$0\mathbf{k}$
	$-w$	$z_P\mathbf{k}$	$0\mathbf{j}$	$0\mathbf{i}$
t	$+e$	$1\mathbf{j}$	$1\mathbf{j}$	$0\mathbf{i}$
	$+s$	$1\mathbf{i}$	$0\mathbf{k}$	$0\mathbf{j}$
	$-w$	$z_T\mathbf{k}$	$0\mathbf{i}$	$0\mathbf{k}$
b	$-e$	$0\mathbf{j}$	$0\mathbf{k}$	$1\mathbf{j}$
	$+s$	$1\mathbf{i}$	$0\mathbf{i}$	$0\mathbf{k}$
	$-w$	$z_T\mathbf{k}$	$0\mathbf{j}$	$0\mathbf{i}$

B

		B	**G**	**R**
u	$+e$	$1\mathbf{j}$	$1\mathbf{j}$	$0\mathbf{k}$
	$+s$	$0\mathbf{i}$	$0\mathbf{k}$	$1\mathbf{i}$
	$+w$	$1\mathbf{k}$	$0\mathbf{i}$	$0\mathbf{j}$
d	$-e$	$0\mathbf{i}$	$0\mathbf{k}$	$1\mathbf{j}$
	$+s$	$0\mathbf{j}$	$0\mathbf{i}$	$1\mathbf{i}$
	$+w$	$1\mathbf{k}$	$0\mathbf{j}$	$0\mathbf{k}$
c	$+e$	$1\mathbf{j}$	$1\mathbf{j}$	$0\mathbf{k}$
	$+s$	$0\mathbf{i}$	$0\mathbf{k}$	$1\mathbf{i}$
	$-w$	$z_P\mathbf{k}$	$0\mathbf{i}$	$0\mathbf{j}$
s	$-e$	$0\mathbf{i}$	$0\mathbf{k}$	$1\mathbf{j}$
	$+s$	$0\mathbf{j}$	$0\mathbf{i}$	$1\mathbf{i}$
	$-w$	$z_P\mathbf{k}$	$0\mathbf{j}$	$0\mathbf{k}$
t	$+e$	$1\mathbf{j}$	$1\mathbf{j}$	$0\mathbf{k}$
	$+s$	$0\mathbf{i}$	$0\mathbf{k}$	$1\mathbf{i}$
	$-w$	$z_T\mathbf{k}$	$0\mathbf{i}$	$0\mathbf{j}$
b	$-e$	$0\mathbf{i}$	$0\mathbf{k}$	$1\mathbf{j}$
	$+s$	$0\mathbf{j}$	$0\mathbf{i}$	$1\mathbf{i}$
	$-w$	$z_T\mathbf{k}$	$0\mathbf{j}$	$0\mathbf{k}$

C

		B	**G**	**R**
u	$+e$	$1\boldsymbol{j}$	$1\boldsymbol{j}$	$0\boldsymbol{k}$
	$+s$	$0\boldsymbol{i}$	$1\boldsymbol{i}$	$0\boldsymbol{j}$
	$+w$	$1\boldsymbol{k}$	$0\boldsymbol{k}$	$0\boldsymbol{i}$
d	$-e$	$0\boldsymbol{j}$	$0\boldsymbol{k}$	$1\boldsymbol{j}$
	$+s$	$0\boldsymbol{i}$	$1\boldsymbol{i}$	$0\boldsymbol{k}$
	$+w$	$1\boldsymbol{k}$	$0\boldsymbol{j}$	$0\boldsymbol{i}$
c	$+e$	$1\boldsymbol{j}$	$1\boldsymbol{j}$	$0\boldsymbol{k}$
	$+s$	$0\boldsymbol{i}$	$1\boldsymbol{i}$	$0\boldsymbol{j}$
	$-w$	$z_P\boldsymbol{k}$	$0\boldsymbol{k}$	$0\boldsymbol{i}$
s	$-e$	$0\boldsymbol{j}$	$0\boldsymbol{k}$	$1\boldsymbol{j}$
	$+s$	$0\boldsymbol{i}$	$1\boldsymbol{i}$	$0\boldsymbol{k}$
	$-w$	$z_P\boldsymbol{k}$	$0\boldsymbol{j}$	$0\boldsymbol{i}$
t	$+e$	$1\boldsymbol{j}$	$1\boldsymbol{j}$	$0\boldsymbol{k}$
	$+s$	$0\boldsymbol{i}$	$1\boldsymbol{i}$	$0\boldsymbol{j}$
	$-w$	$z_T\boldsymbol{k}$	$0\boldsymbol{k}$	$0\boldsymbol{i}$
b	$-e$	$0\boldsymbol{j}$	$0\boldsymbol{k}$	$1\boldsymbol{j}$
	$+s$	$0\boldsymbol{i}$	$1\boldsymbol{i}$	$0\boldsymbol{k}$
	$-w$	$z_T\boldsymbol{k}$	$0\boldsymbol{j}$	$0\boldsymbol{i}$

L

				ν_e
	$+e$	$1\boldsymbol{j}$	$1\boldsymbol{j}$	$0\boldsymbol{j}$
	$+s$	$0\boldsymbol{k}$	$0\boldsymbol{i}$	$0\boldsymbol{i}$
	$+w$	$0\boldsymbol{i}$	$0\boldsymbol{k}$	$1\boldsymbol{k}$
				e
	$-e$	$0\boldsymbol{i}$	$0\boldsymbol{k}$	$1\boldsymbol{j}$
	$+s$	$0\boldsymbol{j}$	$0\boldsymbol{i}$	$0\boldsymbol{i}$
	$+w$	$0\boldsymbol{k}$	$0\boldsymbol{j}$	$1\boldsymbol{k}$
				ν_μ
	$+e$	$1\boldsymbol{j}$	$1\boldsymbol{j}$	$0\boldsymbol{j}$
	$+s$	$0\boldsymbol{k}$	$0\boldsymbol{i}$	$0\boldsymbol{i}$
	$-w$	$0\boldsymbol{i}$	$0\boldsymbol{k}$	$z_P\boldsymbol{k}$
				μ
	$-e$	$0\boldsymbol{i}$	$0\boldsymbol{k}$	$1\boldsymbol{j}$
	$+s$	$0\boldsymbol{j}$	$0\boldsymbol{i}$	$0\boldsymbol{i}$
	$-w$	$0\boldsymbol{k}$	$0\boldsymbol{j}$	$z_P\boldsymbol{k}$
				ν_τ
	$+e$	$1\boldsymbol{j}$	$1\boldsymbol{j}$	$0\boldsymbol{j}$
	$+s$	$0\boldsymbol{k}$	$0\boldsymbol{i}$	$0\boldsymbol{i}$
	$-w$	$0\boldsymbol{i}$	$0\boldsymbol{k}$	$z_T\boldsymbol{k}$
				τ
	$-e$	$0\boldsymbol{i}$	$0\boldsymbol{k}$	$1\boldsymbol{j}$
	$+s$	$0\boldsymbol{j}$	$0\boldsymbol{i}$	$0\boldsymbol{i}$
	$-w$	$0\boldsymbol{k}$	$0\boldsymbol{j}$	$z_T\boldsymbol{k}$

The tables A-C give us quarks, table L gives us leptons. The rotation of *s* from A to C is identical to the rotation of **p** in the baryonic nilpotent structure, and actually led to that discovery. The EMC experiment [37] provided an indication that the **p** component (or angular momentum) did not arise from that of three physically quark-like objects. In the second and third generations, as in other representations, the weak charge components retain the + sign only after respective violations of *P* and *T*. We assume that the fermions are left-handed states, with *w* becoming effectively 0 in right-handed ones, with a gain in mass. It is even possible that the first two columns of table L (with their filled electric vacuum) are giving us the left-handed antistates of the leptons and neutrinos.

The tables produce significant results in themselves, for example, they can be used to describe both strong and weak interactions by transitions between A, B and C and between A-C and L. They provide charge structures for composite particles which provide an insight into *CP* violation, which occurs only with those mesons whose weak charge structure is $\pm 2w$ in some configurations and 0 in others. Here, we assume $\pm 2w$ becomes 0 by *CP* violation. Fermions invariably have $\pm w$ weak charge structure, bosons have 0; baryons uniquely have nonzero strong charge structure. Neutrinos have total charge structure w, $-z_P w$ and $-z_T w$, in the three generations, antineutrinos have $-w$, $z_P w$ and $z_T w$. The difficulties of distinguishing between $-z_P w$ and $-z_T w$ suggests a maximal mixing between muon and tau neutrinos, and other results in neutrino mixing. Majorana behaviour (if any) would depend on difficulties in distinguishing between w and $-w$. Other results include possible mass predictions for composite particles based on zero charge units in their components parts [5,8].

Of course, electric charges in quarks have always been observed to be $2e / 3$ or $-e / 3$, and many attempts at constructing fermions start from that basis. It is important to note that this is *exactly the result predicted by the current representation*, and we would expect QED to reflect this with precision. The charges as proposed in the tables would always result in such conventional fractional values because of the perfect gauge invariance of the strong interaction, and there is no energy regime at which we would expect unit values to appear. We now know that the same applies also in the fractional quantum Hall effect where charges of $e / 3$ are observed because of the perfect gauge invariance of the *weak* interaction when an electron creates a quasi-bosonic state with three magnetic flux lines. In this sense, the correct table for the electric and strong charges (conventionally replaced by baryon number) in the first generation will necessarily be:

		B	**G**	**R**
u	$+e$	2 / 3	2 / 3	2 / 3
	$+s$	1 / 3	1 / 3	1 / 3
d	$-e$	1 / 3	1 / 3	1 / 3
	$+s$	1 / 3	1 / 3	1 / 3

However, although there is no question that quark charges are fractional in QED, the use of integral charges for the underlying groups (which appeared originally in the first model for coloured quarks [38]) has distinct advantages. The present author has found that it leads to Grand Unification at the Planck mass (at which energy all the interactions would be purely Columbic) and resolves difficulties which arise in the application of the Higgs mechanism to fermion masses. In addition, it resolves the anomaly of the electron having a composite charge while seemingly being an elementary particle, solves the fact that leptons can be free of electric charges but quarks seemingly cannot, and makes a 'unification' between quarks and leptons more achievable with a unified $\sin^2\theta_W$ at the experimentally-favoured value of 0.25 [5. 8. 33, 34, 39-40].

One further result of the tables is that the generators of the Dirac group, which become the coefficients of the nilpotent summation of energy, momentum and rest mass, $\boldsymbol{ik}$, $\mathbf{i}\boldsymbol{i}$, $\mathbf{j}\boldsymbol{i}$, $\mathbf{k}\boldsymbol{i}$, $\boldsymbol{j}$, can be produced by a matrix multiplication of the separate E-$\mathbf{p}$-m coefficients, and *those of charge in the tables A-C*, so providing a connection between charge structures and nilpotent wavefunctions. Here, we take the trace of the product

$$\begin{pmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & i & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \boldsymbol{i} & 0 & 0 \\ \boldsymbol{i} & \boldsymbol{k} & 0 \\ \boldsymbol{i} & 0 & \boldsymbol{j} \end{pmatrix}$$

8. The nilpotents derived from charge structures

Ultimately, the charge structures, like the fermions themselves, involve some kind of combination of two 3-dimensionalities, one with symmetry broken and the other with symmetry preserved. This is, as we have previously shown, how the point-like particle structure is created. One way of doing this is to go straight to the nilpotent representation, in which the four components of the spinor represent the full potentiality of what any fermionic state could be transformed into, with the weak, strong and electric interactions as the means of making this transfer [5, 8]. The ultimate representation would be a nilpotent one derived directly from the charge structure. This is, in fact, possible as we will show in this section.

The gravitational or inertial interaction here is 'passive', the vacuum reflection (which can be expressed as 1ψ or scalar $\times$ ψ) leading to the state itself; ψ is taken to be the local, 'inertial', manifestation of the fermion, with $-\psi$ the nonlocal, gravitational, dual. The other three interactions can be shown to transition from nonlocal structures, based on $SU(3)$, $SU(2)$ and $U(1)$ symmetries, to local ones reflecting the interactions as they are observed. Only quarks incorporate the explicit vector behaviour (that is, one showing a structure made of their components) of the momentum operator in their spinor state vectors. In our postulated nilpotent formalism for a baryon, each of the three nilpotents used to construct the state, conventionally called valence quarks, contain, at any instant, only one component of the total momentum vector $\mathbf{p} = (\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3)$ of the baryon. In the allowed *phases* of the interaction, the total momentum is in *just one* of these components. So a baryon state vector might be expressed a form such as

$$\begin{matrix} inertial \\ strong \\ weak \\ electric \end{matrix} \begin{pmatrix} i\boldsymbol{k}E \pm \boldsymbol{i\sigma}.\mathbf{p_1} + \boldsymbol{j}m \\ i\boldsymbol{k}E \mp \boldsymbol{i\sigma}.\mathbf{p_1} + \boldsymbol{j}m \\ -\boldsymbol{k}E \pm \boldsymbol{i\sigma}.\mathbf{p_1} + \boldsymbol{j}m \\ -i\boldsymbol{k}E \mp \boldsymbol{i\sigma}.\mathbf{p_3} + \boldsymbol{j}m \end{pmatrix} \begin{pmatrix} i\boldsymbol{k}E \pm \boldsymbol{i\sigma}.\mathbf{p_2} + \boldsymbol{j}m \\ i\boldsymbol{k}E \mp \boldsymbol{i\sigma}.\mathbf{p_2} + \boldsymbol{j}m \\ -\boldsymbol{k}E \pm \boldsymbol{i\sigma}.\mathbf{p_3} + \boldsymbol{j}m \\ -i\boldsymbol{k}E \mp \boldsymbol{i\sigma}.\mathbf{p_2} + \boldsymbol{j}m \end{pmatrix} \begin{pmatrix} i\boldsymbol{k}E \pm \boldsymbol{i\sigma}.\mathbf{p_3} + \boldsymbol{j}m \\ i\boldsymbol{k}E \mp \boldsymbol{i\sigma}.\mathbf{p_3} + \boldsymbol{j}m \\ -\boldsymbol{k}E \pm \boldsymbol{i\sigma}.\mathbf{p_2} + \boldsymbol{j}m \\ -i\boldsymbol{k}E \mp \boldsymbol{i\sigma}.\mathbf{p_1} + \boldsymbol{j}m \end{pmatrix} \quad (3)$$

or

$$\begin{matrix} inertial \\ strong \\ weak \\ electric \end{matrix} \begin{pmatrix} i\boldsymbol{k}E \pm \boldsymbol{i\sigma}.\mathbf{p_1} + \boldsymbol{j}m \\ i\boldsymbol{k}E \mp \boldsymbol{i\sigma}.\mathbf{p_1} + \boldsymbol{j}m \\ -\boldsymbol{k}E \pm \boldsymbol{i\sigma}.\mathbf{p_1} + \boldsymbol{j}m \\ -i\boldsymbol{k}E \mp \boldsymbol{i\sigma}.\mathbf{p_3} + \boldsymbol{j}m \end{pmatrix} \begin{pmatrix} i\boldsymbol{k}E \mp \boldsymbol{i\sigma}.\mathbf{p_2} + \boldsymbol{j}m \\ i\boldsymbol{k}E \pm \boldsymbol{i\sigma}.\mathbf{p_2} + \boldsymbol{j}m \\ -\boldsymbol{k}E \mp \boldsymbol{i\sigma}.\mathbf{p_3} + \boldsymbol{j}m \\ -i\boldsymbol{k}E \pm \boldsymbol{i\sigma}.\mathbf{p_2} + \boldsymbol{j}m \end{pmatrix} \begin{pmatrix} i\boldsymbol{k}E \pm \boldsymbol{i\sigma}.\mathbf{p_3} + \boldsymbol{j}m \\ i\boldsymbol{k}E \mp \boldsymbol{i\sigma}.\mathbf{p_3} + \boldsymbol{j}m \\ -\boldsymbol{k}E \pm \boldsymbol{i\sigma}.\mathbf{p_2} + \boldsymbol{j}m \\ -i\boldsymbol{k}E \mp \boldsymbol{i\sigma}.\mathbf{p_1} + \boldsymbol{j}m \end{pmatrix}$$

Here, we use the principle that the **p** term contains the entire information about the nilpotent state. This term determines which of the three components of a baryon carries an 'active' component of the particular kind described in the first column. Suppose $\mathbf{p}_3$ determines 'activity' in (3) at any given time. Then the electric component is the active one in the first quark, the weak component in the second, and the inertial and strong components in the third. This corresponds to the presence or absence of charges (1 or 0) in these cases. The 'activity' at any moment is determined by the inertial phase, which is governed by the unique direction of **σ** for each fermion.

The labels are arbitrary and can be changed to preserve gauge invariance. All six phases of the strong interaction, taking into account the two possible signs of **p**, are simultaneously possible, and the strong charge goes through all possible phases; the weak and electric charges remain relatively (although not absolutely) fixed on single phases, just as the charges do in the discrete charge reputations. If one 'active' direction of the **p** vector instantaneously contains all the information about the system, then we can align this along one of the three axes specifying the three quark momentum components.

The strong interaction goes through all three possible directions, with the 'active' one defined at any moment by coincidence with the one defined as 'inertial', an element which goes through all possible phases in fixing the direction of spin **σ**. But the weak and electric **p** components are only be aligned along that of the 'active' inertial' one in one of three cases. Weak and electric phases must be on different quarks in a baryon. In principle, the nilpotent structure is a vacuum rather than a charge representation, so the reverse should apply when we consider weak isospin. The down state is straightforward because the electric vacuum is empty. The up state, however, has a full electric vacuum and this can be represented by adding the Coulomb potential from a positive electronic charge to the *E* term.

Baryons are not only composed of one kind of quark, but, for convenience, we can describe baryonic quarks in this way, using a superposition of states to represent baryons with different kinds of quarks. This allows us to write nilpotents for all types of baryonic quark. Antiquarks would reverse the signs of the energy term. The filled vacuum for the up isospin state can be represented using a Coulomb term from a positive electric charge incorporated into *E*. We write it as E_C. The second and third generations can be accommodated by using a reverse sign for the weak $\boldsymbol{i\sigma}.\mathbf{p_2}$ component, together with the respective Z_P and Z_T operators, exactly as in the charge structures in the previous section.

The other composite particles, mesons, have the same structure as baryons, except that they are constructed from single fermions combined with antifermions. The three phases or 'colours' in mesons should be considered in a purely temporal (i.e. non-spatial) sequence.

up

$$\begin{matrix} inertial \\ strong \\ weak \\ electric \end{matrix} \begin{pmatrix} i\boldsymbol{k}E_C \pm \boldsymbol{i\sigma}.\mathbf{p_1} + \boldsymbol{j}m \\ i\boldsymbol{k}E_C \mp \boldsymbol{i\sigma}.\mathbf{p_1} + \boldsymbol{j}m \\ -\boldsymbol{k}E_C \pm \boldsymbol{i\sigma}.\mathbf{p_1} + \boldsymbol{j}m \\ -i\boldsymbol{k}E_C \mp \boldsymbol{i\sigma}.\mathbf{p_3} + \boldsymbol{j}m \end{pmatrix} \begin{pmatrix} i\boldsymbol{k}E_C \pm \boldsymbol{i\sigma}.\mathbf{p_2} + \boldsymbol{j}m \\ i\boldsymbol{k}E_C \mp \boldsymbol{i\sigma}.\mathbf{p_2} + \boldsymbol{j}m \\ -\boldsymbol{k}E_C \pm \boldsymbol{i\sigma}.\mathbf{p_3} + \boldsymbol{j}m \\ -i\boldsymbol{k}E_C \mp \boldsymbol{i\sigma}.\mathbf{p_2} + \boldsymbol{j}m \end{pmatrix} \begin{pmatrix} i\boldsymbol{k}E_C \pm \boldsymbol{i\sigma}.\mathbf{p_3} + \boldsymbol{j}m \\ i\boldsymbol{k}E_C \mp \boldsymbol{i\sigma}.\mathbf{p_3} + \boldsymbol{j}m \\ -\boldsymbol{k}E_C \pm \boldsymbol{i\sigma}.\mathbf{p_2} + \boldsymbol{j}m \\ -i\boldsymbol{k}E_C \mp \boldsymbol{i\sigma}.\mathbf{p_1} + \boldsymbol{j}m \end{pmatrix}$$

down

$$\begin{matrix} inertial \\ strong \\ weak \\ electric \end{matrix} \begin{pmatrix} i\boldsymbol{k}E \pm \boldsymbol{i\sigma}.\mathbf{p_1} + \boldsymbol{j}m \\ i\boldsymbol{k}E \mp \boldsymbol{i\sigma}.\mathbf{p_1} + \boldsymbol{j}m \\ -\boldsymbol{k}E \pm \boldsymbol{i\sigma}.\mathbf{p_1} + \boldsymbol{j}m \\ -i\boldsymbol{k}E \mp \boldsymbol{i\sigma}.\mathbf{p_3} + \boldsymbol{j}m \end{pmatrix} \begin{pmatrix} i\boldsymbol{k}E \pm \boldsymbol{i\sigma}.\mathbf{p_2} + \boldsymbol{j}m \\ i\boldsymbol{k}E \mp \boldsymbol{i\sigma}.\mathbf{p_2} + \boldsymbol{j}m \\ -\boldsymbol{k}E \pm \boldsymbol{i\sigma}.\mathbf{p_3} + \boldsymbol{j}m \\ -i\boldsymbol{k}E \mp \boldsymbol{i\sigma}.\mathbf{p_2} + \boldsymbol{j}m \end{pmatrix} \begin{pmatrix} i\boldsymbol{k}E \pm \boldsymbol{i\sigma}.\mathbf{p_3} + \boldsymbol{j}m \\ i\boldsymbol{k}E \mp \boldsymbol{i\sigma}.\mathbf{p_3} + \boldsymbol{j}m \\ -\boldsymbol{k}E \pm \boldsymbol{i\sigma}.\mathbf{p_2} + \boldsymbol{j}m \\ -i\boldsymbol{k}E \mp \boldsymbol{i\sigma}.\mathbf{p_1} + \boldsymbol{j}m \end{pmatrix}$$

strange

$$\begin{matrix} inertial \\ strong \\ weak \\ electric \end{matrix} \begin{pmatrix} i\boldsymbol{k}E_C \pm \boldsymbol{i\sigma}.\mathbf{p_1} + \boldsymbol{j}m \\ i\boldsymbol{k}E_C \mp \boldsymbol{i\sigma}.\mathbf{p_1} + \boldsymbol{j}m \\ -\boldsymbol{k}E_C \mp z_P\boldsymbol{i\sigma}.\mathbf{p_1} + \boldsymbol{j}m \\ -i\boldsymbol{k}E_C \mp \boldsymbol{i\sigma}.\mathbf{p_3} + \boldsymbol{j}m \end{pmatrix} \begin{pmatrix} i\boldsymbol{k}E_C \pm \boldsymbol{i\sigma}.\mathbf{p_2} + \boldsymbol{j}m \\ i\boldsymbol{k}E_C \mp \boldsymbol{i\sigma}.\mathbf{p_2} + \boldsymbol{j}m \\ -\boldsymbol{k}E_C \mp z_P\boldsymbol{i\sigma}.\mathbf{p_3} + \boldsymbol{j}m \\ -i\boldsymbol{k}E_C \mp \boldsymbol{i\sigma}.\mathbf{p_2} + \boldsymbol{j}m \end{pmatrix} \begin{pmatrix} i\boldsymbol{k}E_C \pm \boldsymbol{i\sigma}.\mathbf{p_3} + \boldsymbol{j}m \\ i\boldsymbol{k}E_C \mp \boldsymbol{i\sigma}.\mathbf{p_3} + \boldsymbol{j}m \\ -\boldsymbol{k}E_C \mp z_P\boldsymbol{i\sigma}.\mathbf{p_2} + \boldsymbol{j}m \\ -i\boldsymbol{k}E_C \mp \boldsymbol{i\sigma}.\mathbf{p_1} + \boldsymbol{j}m \end{pmatrix}$$

charm

$$\begin{matrix} inertial \\ strong \\ weak \\ electric \end{matrix} \begin{pmatrix} i\boldsymbol{k}E \pm \boldsymbol{i\sigma}.\mathbf{p_1} + \boldsymbol{j}m \\ i\boldsymbol{k}E \mp \boldsymbol{i\sigma}.\mathbf{p_1} + \boldsymbol{j}m \\ -\boldsymbol{k}E \mp z_P\boldsymbol{i\sigma}.\mathbf{p_1} + \boldsymbol{j}m \\ -i\boldsymbol{k}E \mp \boldsymbol{i\sigma}.\mathbf{p_3} + \boldsymbol{j}m \end{pmatrix} \begin{pmatrix} i\boldsymbol{k}E \pm \boldsymbol{i\sigma}.\mathbf{p_2} + \boldsymbol{j}m \\ i\boldsymbol{k}E \mp \boldsymbol{i\sigma}.\mathbf{p_2} + \boldsymbol{j}m \\ -\boldsymbol{k}E \mp z_P\boldsymbol{i\sigma}.\mathbf{p_3} + \boldsymbol{j}m \\ -i\boldsymbol{k}E \mp \boldsymbol{i\sigma}.\mathbf{p_2} + \boldsymbol{j}m \end{pmatrix} \begin{pmatrix} i\boldsymbol{k}E \pm \boldsymbol{i\sigma}.\mathbf{p_3} + \boldsymbol{j}m \\ i\boldsymbol{k}E \mp \boldsymbol{i\sigma}.\mathbf{p_3} + \boldsymbol{j}m \\ -\boldsymbol{k}E \mp z_P\boldsymbol{i\sigma}.\mathbf{p_2} + \boldsymbol{j}m \\ -i\boldsymbol{k}E \mp \boldsymbol{i\sigma}.\mathbf{p_1} + \boldsymbol{j}m \end{pmatrix}$$

top

$$\begin{matrix} inertial \\ strong \\ weak \\ electric \end{matrix} \begin{pmatrix} i\boldsymbol{k}E_C \pm \boldsymbol{i\sigma}.\mathbf{p_1} + \boldsymbol{j}m \\ i\boldsymbol{k}E_C \mp \boldsymbol{i\sigma}.\mathbf{p_1} + \boldsymbol{j}m \\ -\boldsymbol{k}E_C \mp z_T\boldsymbol{i\sigma}.\mathbf{p_1} + \boldsymbol{j}m \\ -i\boldsymbol{k}E_C \mp \boldsymbol{i\sigma}.\mathbf{p_3} + \boldsymbol{j}m \end{pmatrix} \begin{pmatrix} i\boldsymbol{k}E_C \pm \boldsymbol{i\sigma}.\mathbf{p_2} + \boldsymbol{j}m \\ i\boldsymbol{k}E_C \mp \boldsymbol{i\sigma}.\mathbf{p_2} + \boldsymbol{j}m \\ -\boldsymbol{k}E_C \mp z_T\boldsymbol{i\sigma}.\mathbf{p_3} + \boldsymbol{j}m \\ -i\boldsymbol{k}E_C \mp \boldsymbol{i\sigma}.\mathbf{p_2} + \boldsymbol{j}m \end{pmatrix} \begin{pmatrix} i\boldsymbol{k}E_C \pm \boldsymbol{i\sigma}.\mathbf{p_3} + \boldsymbol{j}m \\ i\boldsymbol{k}E_C \mp \boldsymbol{i\sigma}.\mathbf{p_3} + \boldsymbol{j}m \\ -\boldsymbol{k}E_C \mp z_T\boldsymbol{i\sigma}.\mathbf{p_2} + \boldsymbol{j}m \\ -i\boldsymbol{k}E_C \mp \boldsymbol{i\sigma}.\mathbf{p_1} + \boldsymbol{j}m \end{pmatrix}$$

bottom

$$\begin{matrix} inertial \\ strong \\ weak \\ electric \end{matrix} \begin{pmatrix} i\boldsymbol{k}E \pm \boldsymbol{i\sigma}.\mathbf{p_1} + \boldsymbol{j}m \\ i\boldsymbol{k}E \mp \boldsymbol{i\sigma}.\mathbf{p_1} + \boldsymbol{j}m \\ -\boldsymbol{k}E \mp z_T\boldsymbol{i\sigma}.\mathbf{p_1} + \boldsymbol{j}m \\ -i\boldsymbol{k}E \mp \boldsymbol{i\sigma}.\mathbf{p_3} + \boldsymbol{j}m \end{pmatrix} \begin{pmatrix} i\boldsymbol{k}E \pm \boldsymbol{i\sigma}.\mathbf{p_2} + \boldsymbol{j}m \\ i\boldsymbol{k}E \mp \boldsymbol{i\sigma}.\mathbf{p_2} + \boldsymbol{j}m \\ -\boldsymbol{k}E \mp z_T\boldsymbol{i\sigma}.\mathbf{p_3} + \boldsymbol{j}m \\ -i\boldsymbol{k}E \mp \boldsymbol{i\sigma}.\mathbf{p_2} + \boldsymbol{j}m \end{pmatrix} \begin{pmatrix} i\boldsymbol{k}E \pm \boldsymbol{i\sigma}.\mathbf{p_3} + \boldsymbol{j}m \\ i\boldsymbol{k}E \mp \boldsymbol{i\sigma}.\mathbf{p_3} + \boldsymbol{j}m \\ -\boldsymbol{k}E \mp z_T\boldsymbol{i\sigma}.\mathbf{p_2} + \boldsymbol{j}m \\ -i\boldsymbol{k}E \mp \boldsymbol{i\sigma}.\mathbf{p_1} + \boldsymbol{j}m \end{pmatrix}$$

We can, of course, create a nilpotent in which **p** is not specified as a vector divisible into components, and no initial separation into component parts. The final structure looks similar to that of the baryon but has fewer simultaneous options. The distinction can be accommodated using the vector phase, which is an instantaneous choice of direction containing all the information about the fermion. In such a *lepton* or free fermion, the phases are purely the inertial phases, and the weak and electric charges switch on or off as the phase changes through the components 1, 2, 3. Leptons have weak and electric occupancy on the same phase, with a temporal cycle, 1-2-3, as the structure rotates through the three directions involved in **p**. Only the direction of the vector properties of **p**, of course, define a strong phase – the magnitude is determined by the combination of *E* and *m*. For free fermions, there is no strong charge because no information is carried about direction, and there is no *SU*(3) symmetry.

We can write leptonic nilpotent structures in forms such as:

electron neutrino

$$\begin{matrix} inertial \\ strong \\ weak \\ electric \end{matrix} \begin{pmatrix} \boldsymbol{ik}E_C \pm \boldsymbol{i\sigma}.\mathbf{p_1} + \boldsymbol{j}m \\ \boldsymbol{ik}E_C \mp \boldsymbol{i\sigma}.\mathbf{p_1} + \boldsymbol{j}m \\ -\boldsymbol{k}E_C \pm \boldsymbol{i\sigma}.\mathbf{p_1} + \boldsymbol{j}m \\ -\boldsymbol{ik}E_C \mp \boldsymbol{i\sigma}.\mathbf{p_1} + \boldsymbol{j}m \end{pmatrix}$$

electron

$$\begin{matrix} inertial \\ strong \\ weak \\ electric \end{matrix} \begin{pmatrix} \boldsymbol{ik}E \pm \boldsymbol{i\sigma}.\mathbf{p_1} + \boldsymbol{j}m \\ \boldsymbol{ik}E \mp \boldsymbol{i\sigma}.\mathbf{p_1} + \boldsymbol{j}m \\ -\boldsymbol{k}E \pm \boldsymbol{i\sigma}.\mathbf{p_1} + \boldsymbol{j}m \\ -\boldsymbol{ik}E \mp \boldsymbol{i\sigma}.\mathbf{p_1} + \boldsymbol{j}m \end{pmatrix}$$

muon neutrino

$$\begin{matrix} inertial \\ strong \\ weak \\ electric \end{matrix} \begin{pmatrix} \boldsymbol{ik}E_C \pm \boldsymbol{i\sigma}.\mathbf{p_1} + \boldsymbol{j}m \\ \boldsymbol{ik}E_C \mp \boldsymbol{i\sigma}.\mathbf{p_1} + \boldsymbol{j}m \\ -\boldsymbol{k}E_C \mp z_P\boldsymbol{i\sigma}.\mathbf{p_1} + \boldsymbol{j}m \\ -\boldsymbol{ik}E_C \mp \boldsymbol{i\sigma}.\mathbf{p_1} + \boldsymbol{j}m \end{pmatrix}$$

muon

$$\begin{matrix} inertial \\ strong \\ weak \\ electric \end{matrix} \begin{pmatrix} \boldsymbol{ik}E \pm \boldsymbol{i\sigma}.\mathbf{p_1} + \boldsymbol{j}m \\ \boldsymbol{ik}E \mp \boldsymbol{i\sigma}.\mathbf{p_1} + \boldsymbol{j}m \\ -\boldsymbol{k}E \mp z_P\boldsymbol{i\sigma}.\mathbf{p_1} + \boldsymbol{j}m \\ -\boldsymbol{ik}E \mp \boldsymbol{i\sigma}.\mathbf{p_1} + \boldsymbol{j}m \end{pmatrix}$$

tau neutrino

$$\begin{matrix} inertial \\ strong \\ weak \\ electric \end{matrix} \begin{pmatrix} \boldsymbol{ik}E_C \pm \boldsymbol{i\sigma}.\mathbf{p_1} + \boldsymbol{j}m \\ \boldsymbol{ik}E_C \mp \boldsymbol{i\sigma}.\mathbf{p_1} + \boldsymbol{j}m \\ -\boldsymbol{k}E_C \mp \boldsymbol{i}z_T\boldsymbol{\sigma}.\mathbf{p_1} + \boldsymbol{j}m \\ -\boldsymbol{ik}E_C \mp \boldsymbol{i\sigma}.\mathbf{p_1} + \boldsymbol{j}m \end{pmatrix}$$

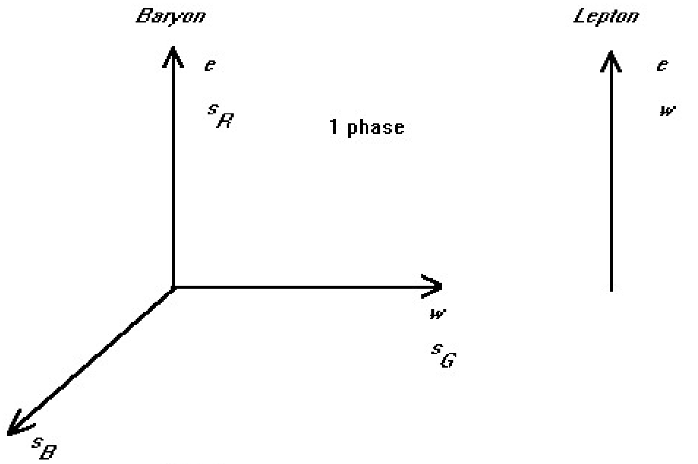
tau

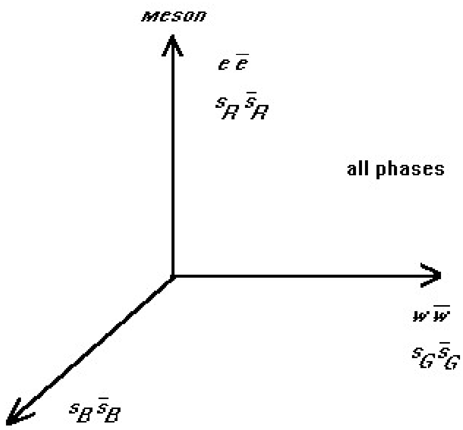
$$\begin{matrix} inertial \\ strong \\ weak \\ electric \end{matrix} \begin{pmatrix} \boldsymbol{ik}E \pm \boldsymbol{i\sigma}.\mathbf{p_1} + \boldsymbol{j}m \\ \boldsymbol{ik}E \mp \boldsymbol{i\sigma}.\mathbf{p_1} + \boldsymbol{j}m \\ -\boldsymbol{k}E \mp z_T\boldsymbol{i\sigma}.\mathbf{p_1} + \boldsymbol{j}m \\ -\boldsymbol{ik}E \mp \boldsymbol{i\sigma}.\mathbf{p_1} + \boldsymbol{j}m \end{pmatrix}$$

In all these representations, *iE*, **σ.p** and *m* can be seen as the respective coefficients for the weak, strong and electric vacuum terms. In total, there are two pseudoscalar terms ± *iE*; six vector terms ± $\boldsymbol{\sigma}.\mathbf{p}_1$, ± $\boldsymbol{\sigma}.\mathbf{p}_2$, ± $\boldsymbol{\sigma}.\mathbf{p}_3$; and one scalar term *m*. The weak component involves a switching between *iE* and –*iE*, and involves dipolarity. The strong switching converts $\boldsymbol{\sigma}.\mathbf{p}_1$ into – $\boldsymbol{\sigma}.\mathbf{p}_1$, $\boldsymbol{\sigma}.\mathbf{p}_2$, – **σ.p**, $\boldsymbol{\sigma}.\mathbf{p}_3$ and – $\boldsymbol{\sigma}.\mathbf{p}_3$. The switching occurs at a constant rate of change of **p**, which can be attributed to a linear potential. The electric component preserves the invariant mass *m*. The respective group structures for the interactions are *SU*(2), *SU*(3) and *U*(1).

The filled electric and empty electric background, which constitute the *SU*(2) of isospin, and global, and so automatically set with respect to *E*. The background becomes incorporated automatically into *E* as the potential producing a scalar or *U*(1) phase, which becomes a Coulomb potential in the spherically symmetric. All the information can be incorporated into a single expression when we use Lorentz invariance for a purely point source with spherical symmetry to transfer the information contained in **σ.p** to the *E* term [5]. This requires the addition of a potential function of *r* which reproduces that aspect of spherical symmetry (*SU*(3), *SU*(2) or *U*(1)) incorporated in the covariant part of **σ.p**, that is, the part responsible for the interaction. The strong term becomes a linear function ($\propto r$) through the rotation of vector **p**. The options are ultimately a consequence of applying the condition of spherical symmetry to the fermionic state. The electric term becomes a scalar phase or Coulombic term ($\propto 1 / r$) through the scalar nature of *m*. The weak term becomes a dipolar equivalent of the scalar phase ($\propto 1 / r^3$) through the dipolarity of $\pm iE$. All specific phase information is lost when we choose the frame such that all the information is transferred to the *E* term.

Baryon, lepton and meson arrangements can be displayed diagrammatically using a vector rotating over a complete spherical surface to represent the 'privileged' direction states for the charges [5, 8]. Each charge is then only 'active' in one phase out of three at any given time to fix the angular momentum direction. The symbols *e*, *s*, and *w* in the diagrams refer to the active phases, not the charges, which are different in the electric case. For baryons and mesons, which are composed of quarks, the information about the angular momentum state is split between three axes, whereas, in leptons, it is carried on a single axis.





As these structures and the paired graph for Pauli exclusion in section 3 show, the information to build up an entire particle state can come from either **p** (in charge terms *s*) or through the *combination* of *E* and *m* (in charge terms *w* and *e*), explaining why the electric and weak interactions are intrinsically linked. (Details on the electroweak link are given in [5].)

9. Larger structures for fermions and bosons

We have demonstrated in the previous sections that the Standard Model fermions can be completely explained on the basis of a Dirac-type algebra derived from the Klein-4 parameter group of mass, time, charge and space, and that both charge-structure and nilpotent representations can be derived which parallel each other directly. Many physicists believe that there may be higher group representations involving such things as left- and right-handed spin states, with the idea that some 'mechanism' exists to break the higher symmetry into what we now observe.

Many broken symmetries can certainly be found in particle physics, but it seems most likely that these are a result of complexity or synthesis, rather than from unknown 'symmetry-breaking principles' acting on large-scale structures. A study of fundamental physics suggests that the basic symmetry principles are very simple and stem from just two major symmetry patterns, based on the numbers 2 and 3, namely duality and anticommutativity. All the higher symmetries that are relevant to physics, particularly those based on the octonion symmetries, such as the exceptional groups E_6, E_7 and E_8, are based on combinations of these two patterns in ways that are already broken, a characteristic example being the appearance of the fundamentally asymmetric number 5 in the creation of fermion point particles.

In many ways, the two fundamental patterns also tend to oppose each other, anticommutativity being a symmetry-breaker within dualities (as is manifested, for example, in quarks and the 3 particle generations). The 3-dimensionality inherent within the process means that the symmetries involved in particle groupings tend to map naturally onto geometries in 3-dimensional space, especially where they are inherently dual (as with Platonic solids). However, the higher groupings which collect unlike particles such as quarks and leptons, or fermions and bosons, have relationships with structures in higher-dimensional spaces and the groups connected with them. The structural relationships within such groups remain unbroken, the 'brokenness' arising from the bringing together of disparate parts, not from the structure in which they are embedded. The culmination of such relationships may be the unbroken root vector structure in E_8, the highest group symmetry to emerge from this type of mathematics. The octonions (although necessarily broken in physical applications) have a distinct role here as they are the main source of these higher group structures. In all cases, the numbers involved seem to be based only on powers of 2 and 3, with 5 where there is a specific symmetry-breaking. In the case of higher-dimensions, we find that symmetrical figures, such as Platonic solids, carry with them the numbers associated with similar figures from lower dimensions.

The group E_8 has long been thought of as a possible unifying group for the fundamental particles. This was discussed as such, among other places, in my own *Zero to Infinity*, published in 2007 [5]. In the same year, Garrett Lisi made a well-publicised claim that all known fermions and gauge bosons could be fitted into the 240 root vectors of the E_8 group [41]. Even with some dubious additions, he was unable to make the number of particles equal 240 or reproduce the 3 generations, and he had no intrinsic explanation of gravity, but incorporated a very speculative version of quantum loop gravity as a bolt-on extra. Because he was unable to find his assumed number of states, Lisi made completely *ad hoc* speculation about particles needed to make up the numbers, and some of the assignments seem very difficult to understand.

However, the basic idea may be correct, and it would fit in with previous ideas on the significance of E_8, which is the highest of the octonion-based symmetries. Lisi was criticized for the inclusion of spin ½ fermions and spin 1 gauge bosons in the same representation, which he argued was possible, but only through the exceptional groups E_6, E_7, E_8. If this idea is correct (and it has gained some support), it may be his most significant contribution, along with the emphasis on root vectors, for it may be possible to reach 240 particle states from the known fermions and bosons without adding any extra particles (except the 12 X and Y bosons assumed to exist in basic theories of Grand Unification).

As we have seen, the Standard Model requires 6 quarks arranged in 3 generations, each of which has 2 weak isospin states (up / down; charm / strange; top / bottom), and each of which comes in 3 varieties of 'colour'. Alongside these are 6 leptons, again in 3 generations, each with 2 weak isospin states (electron neutrino / electron; muon neutrino / muon; tau neutrino / tau). As there are no colours associated with the leptons, each set of 3 coloured quarks and 1 lepton, in each isospin state in each generation, represents a kind of 4-dimensional structure, parallel to that of space and time. The total of real fermions in the Standard Model is therefore 24 (18 coloured quarks + 6 leptons). There are an equal number of antistates, so the total number of fermions and antifermions is 48.

The spin 1 gauge bosons are represented by (±`***k****E* ± ***i*p** + ***j****m*) (∓ ***k****E* ± ***i*p** + ***j****m*) in the nilpotent formalism, which means that 4 fermionic states are required to produce 1 boson, and 12 such bosons are known: 8 gluons plus W^+, W^-, Z^0 and γ. The total of fermions / antifermions plus bosons is therefore 60, and some of the representations in the previous sections have suggested that this total may in some way be represented by the 12 pentads of the Dirac algebra.

Now, the 2 spin states and fermion / antifermion options are an intrinsic aspect of the fermion's spinor structure, (± ***ik****E* ± ***i*p** + ***j****m*). Though left- and right-handed fermions are not separate particles, we could take this as a doubling of the total number of fermionic states to 96, and, as previously indicated, the majority of Grand Unification theories (and certainly those involving *SU*(5) or containing it) predict the existence of another 12 gauge bosons (6 *X* and 6 *Y*) to unify strong and electroweak interactions, leading to a total of 96 fermionic plus 24 bosonic states, or 120 real particle states.

Finally, the nilpotent theory suggests that 1 vacuum boson state (never seen, but still mathematically necessary) exists for every 4 real fermion states, and that 4 vacuum fermionic states (again never seen, but still necessary) exist for every real bosonic state, which would require the addition of 24 vacuum boson states and 96 vacuum fermionic states. So a total of 120 real states would be accompanied by another 120 vacuum states. The total of states (real and vacuum) then becomes 240, the kissing number in 8 dimensions and the number of root vectors in E_8 [8, 20-21]. We set out the total in tables such as:

	q	*l*	*b*		*f*	*b*							
1	3	1	1	=	4	1	=	5					
2	6	2	2	=	8	2	=	10	I				
3	9	3	3	=	12	3	=	15					G
4	12	4	4	=	16	4	=	20	I		A		
5	18	6	6	=	24	6	=	30	I				G
6	24	8	8	=	32	8	=	40	I	S	A		
7	**36**	**12**	**12**	**=**	**48**	**12**	**=**	**60**	**I**		**A**		**G**
8	48	16	16	=	64	16	=	80	I	S	A	V	
9	72	24	24	=	96	24	=	120	I	S	A		G
10	144	48	48	=	192	48	=	240	I	S	A	V	G

or (if we privilege spin, rather than isospin):

	q	*l*	*b*		*f*	*b*							
1	3	1	1	=	4	1	=	5					
2	6	2	2	=	8	2	=	10	S				
3	9	3	3	=	12	3	=	15					G
4	12	4	4	=	16	4	=	20	S	I			
5	18	6	6	=	24	6	=	30	S				G
6	24	8	8	=	32	8	=	40	S	I	A		
7	36	12	12	=	48	12	=	60	S	I			G
8	48	16	16	=	64	16	=	80	S	I	A	V	
9	72	24	24	=	96	24	=	120	S	I	A		G
10	144	48	48	=	192	48	=	240	S	I	A	V	G

The tables incorporate 4 factor 2 dualities: isospin up / down (I), spin up / down (**S**), fermion / antifermion (A), particle / vacuum (V), which could be said to relate to charge, space, time and mass. The generations are represented by 1 factor 3 triplet, symbolized by G. The factors I, S, A, V and G then act on a row of 5, representing 3 quarks + 1 lepton + 1 boson. The order of application is not intrinsically significant in the dualities, but the tables here show the most immediate connection to the physics of particles. (We may note here that, though R ⊗ C ⊗ H ⊗ O and R ⊗ C ⊗ O ⊗ O respectively double and quadruple the parameter space of the R ⊗ C ⊗ H ⊗ H we have used, they have the wrong composition to give the 120 and 240 of these tables, even if we use O_L and O_R.)

Of course, the 240 represents a broken symmetry because the parts are already disparate. In particular the rows of 5 are artificial constructs, linking fermions with bosons, and allowing this in the exceptional groups E_6 to E_8, through the parallel fact that the last term in the 5 components of a Dirac pentad is a scalar. Bosons are scalar particles, and scalars are the squared products of pseudoscalars and vectors, just as bosons are the squared products of fermions / antifermions. In this sense, it is the nilpotent structure that gives us a physical as well as mathematical reason for combining fermions and bosons in the same representation. In another sense, there is a connection because a fermion, in the nilpotent structure, is necessarily its own vacuum boson, and vice versa.

Significantly, in these tables, we can follow through many of the fundamental algebras derived from real and complex numbers, quaternions and octonions, also geometries in spaces from 3 to 8 dimensions, and many associated groups, and find all the key numbers there. We see complexity building up from the simplest symmetries in a way that suggest why the higher symmetries have physical meaning and why they are always broken.

10. A route to particle masses

The previous sections have given a comprehensive treatment of the structure and origin of the Standard Model fermions, although related subjects such as the $SU(3) \times \mathrm{SU}(2) \times U(1)$ symmetry, the electroweak synthesis, chirality, the Higgs mechanism and Grand Unification are treated more extensively elsewhere [5, 8]. One thing that remains to be resolved, however, is the question of fermion masses. They must, in some way, be derivable from the charge or nilpotent structures, but the route to finding them is far from obvious. There are 12 parameters to be found, and it is likely that factors that might be involved include the vacuum expectation value of the Higgs field (f = 246 GeV), which seems to be a genuine constant, and the electric, strong and weak coupling constants (a, a_2 and a_3, or even the electroweak a_1), all of which vary with the energy of interaction, as do the masses of the quarks. The following diagrams give an approximate idea of how this might work, with the corresponding masses in GeV (though some of these are uncertain or very approximate):

ν_e	e		d	u
			↑ α_2	↑ α
ν_μ	μ	←	s	c
		α_3	↑ α_2	↑ α
ν_τ	τ	←	b	t
		α_3		

unknown	**0.5 × 10^{-3}**		**4.6 × 10^{-3}**	**2.2 × 10^{-3}**
			↑ 1/21	**↑**
unknown	**0.1057**	**←**	**0.096**	**1.28**
		1.1	**↑ 1/43.5**	**↑ 1/133**
unknown	**1.777**	**←**	**4.18**	**173.1**
		1/2.35		

It seems probable that the top mass is determined, almost exactly, as the maximal coupling to the Higgs field ($f/\sqrt{2} \approx 174$ GeV). It is possible also that the heaviest lepton, the tau, may derive its mass by coupling its electric charge unit to the Higgs field as $f\alpha$. There is a good case for saying that the relative masses of b and τ, and s and μ, are determined by the strong couplings at the appropriate energies, which are certainly close to these ratios. While the strong coupling might be expected to distinguish between quarks and leptons, the weak coupling is probably more significant between generations. The relative masses of b and s, and s and d, show a rough correlation to the weak coupling constant, which could actually be more impressive than it looks, as the masses of s and especially d are not well established. The electric coupling constant might be involved in the 'up' weak isospin states, with their filled electric vacuum, and the t / c ratio shows a relatively good correlation to this coupling. The quoted u quark mass is too uncertain and energy-dependent to be easily compared with that c.

No fermion mass is determined by a single coupling, but the relations might make sense where one particular coupling or relative coupling between two states is dominant. The results seem to be promising enough to suggest that the coupling constants are a highly significant factor in fixing the fermion masses. A few other suggestions have been considered elsewhere [8, 42], including ones which fix the electron mass and suggest values of 0.13 eV for the lowest mass neutrino and between 3 and 12 MeV for the masses of the lightest quarks, and the peculiar numerical relation of the muon mass to those of the composite baryons and mesons may suggest some deeper significance, but these require more speculative additions not completely confined to the Standard Model.

References

[1] Rowlands P 1983 The fundamental parameters of physics *Speculat Sci Tech* **6** 69-80
[2] Rowlands P 1991 *The Fundamental Parameters of Physics: An Approach towards a Unified Theory* (Liverpool: PD Publications)
[3] Rowlands P 2001 A foundational approach to physics arXiv:physics/0106054 *Prespacetime Journal* 2012 **31** 14 1278-1310
[4] Rowlands P Cullerne J P and Koberlein B D 2001 The group structure bases of a foundational approach to physics arXiv:physics/0110092
[5] Rowlands P *Zero to Infinity The Foundations of Physics* 2007 (Singapore: World Scientific)
[6] Rowlands P 2009 Are There Alternatives to Our Present Theories of Physical Reality? arXiv:0912.3433 *Prespacetime Journal* 2012 **31** 14 1387-1403 December 2012
[7] Rowlands P 2013 Symmetry in physics from the foundations, *Symmetry* **24** 41-56
[8] Rowlands P 2014 *The Foundations of Physical Law* (Singapore: World Scientific)
[9] Rowlands P 2915 *How Schrödinger's Cat Escaped the Box* (Singapore: World Scientific)
[10] Rowlands P 2017 How symmetries become broken, *Symmetry* **28** 3 244-254
[11] Rowlands P 2010 Dual Vector Spaces and Physical Singularities, AIP Conference Proceedings **1316** 102-111
[12] Rowlands P 2013 Space and antispace in Amoroso R L Kauffman L H and Rowlands P (eds) *The Physics of Reality Space, Time, Matter, Cosmos* (Singapore: World Scientific) pp 29-37.
[13] Rowlands P 2014 A dual space as the basis of quantum mechanics and other aspects of physics, in Amson J C and Kauffman LH (eds) *Scientific Essays in Honor of H Pierre Noyes on the Occasion of His 90th Birthday* (Singapore: World Scientific) pp 318-338
[14] Rowlands P 2015 Dual vector spaces and physical singularities in Amoroso R L Kauffman L H and Rowlands P (eds) *Unified Field Mechanics Natural Science Beyond the Veil of Spacetime* (Singapore: World Scientific) pp 92-101
[15] Dixon, G M, 1994 *Division Algebras: Octonions Quaternions Complex Numbers and the Algebraic Design of Physics* (Springer US)
[16] Dixon, G M 2011 *Division Algebras, Lattices, Physics, Windmill Tilting* (CreateSpace Independent Publishing Platform)
[17] Rowlands P 1994 An algebra combining vectors and quaternions: A comment on James D. Edmonds' paper *Speculat. Sci Tech* **17** 279-282

[18] Rowlands P and Rowlands S 2018 Representations of the nilpotent Dirac matrices in Amoroso R L Kauffman L H Rowlands P and Albertini G (eds) *Unified Field Mechanics II: Formulations and Empirical Tests* (Singapore: World Scientific) pp 26-33
[19] Hill, V J and Rowlands P 2008 Nature's code *AIP Conference Proceedings* **1051** 117-126
[20] Hill, V J and Rowlands P 2010 Nature's fundamental symmetry breaking *International Journal of Computing Anticipatory Systems* **25** 144-159
[21] Hill, V J and Rowlands P 2010 The numbers of Nature's code *International Journal of Computing Anticipatory Systems* **25** 160-175
[22] Rowlands P 1998 The physical consequences of a new version of the Dirac equation in Hunter G Jeffers S and Vigier J-P (eds), *Causality and Locality in Modern Physics and Astronomy: Open Questions and Possible Solutions* (*Fundamental Theories of Physics* **97**) (Dordrecht: Kluwer Academic Publishers) 397-402
[23] Rowlands P 1996 Some interpretations of the Dirac algebra *Speculat Sci Tech* **19** 243-51
[24] Rowlands P Symmetry breaking and the nilpotent Dirac equation, *AIP Conference Proceedings*, **718**, 102-115, 2004
[25] Rowlands P The nilpotent Dirac equation and its applications in particle physics, arXiv:quant-ph/0301071.
[26] Rowlands P 2005 Removing redundancy in relativistic quantum mechanics arXiv.org:physics/0507188 2012 *Prespacetime Journal* **31** 14 1311-1354
[27] Rowlands P 2008 What is vacuum? arXiv:0810.0224 2012 *Prespacetime Journal* **31** 14 1356-1386
[28] Rowlands P 2010 Physical interpretations of nilpotent quantum mechanics arXiv: 1004.1523 2012 *Prespacetime Journal* **31** 14 1404-1441
[29] Marcer P and Rowlands P 2017 Nilpotent quantum mechanics: analogs and applications *Frontiers in Physics* **5** article 28 pp 1-8
[30] Hestenes, D 1966 *Space-Time Algebras* (New York: Gordon and Breach)
[31] Hauser J and Dröscher W 2017 Gravity beyond Einstein? Part I: physics and the trouble with experiments *Z Naturforsch* **72**(6)a 493-525, p 496, note 3
[32] Rowlands P and Cullerne, J P 2001 The connection between the Han-Nambu quark theory, the Dirac equation and fundamental symmetries *Nuclear Physics* A **684** 713-5
[33] Rowlands P and Cullerne, J P 2002 Can gravity be included in grand unification? in Amoroso R L Hunter G Kafatos M and Vigier J-P (eds) *Gravitation and Cosmology: From the Hubble Radius to the Planck Scale* (Dordrecht: Kluwer) 279-86.
[34] Rowlands P and Cullerne, J P 2001 The Dirac algebra and grand unification arXiv:quant-ph/0106111
[35] Rowlands P and Cullerne, J P 2001 QED using the nilpotent formalism arXiv:quant-ph/0109069
[36] Rowlands P Fermion interactions and mass generation in the nilpotent formalism 2006 *AIP Conference Proceedings* **839** 225-35, 2006
[37] Ashman, J et al 1988 A measurement of the spin asymmetry and determination of the structure function *g*(1) in deep inelastic muon-proton scattering, CERN-EP-87-230, December 1987 *Phys Lett B* **206** 364
[38] Han M Y and Nambu Y 1965 Three-triplet model with double *SU*(3) symmetry *Phys Rev* **139** B 1006-10
[39] Rowlands P What Is the meaning of fractional electric charges? 2012 *Prespacetime Journal* **3** 12 1159-1167
[40] Rowlands P 2018 Why do quarks have electric charges?, in Amoroso R L Kauffman L H Rowlands P and Albertini G (eds) *Unified Field Mechanics II: Formulations and Empirical Tests* (Singapore: World Scientific) pp 399-411
[41] Lisi, A G 2007, An exceptionally simple theory of everything arXiv:0711.0770
[42] Rowlands P 2015 Reducing the number of free parameters in Croll G J and Gregory N G (eds) *Explorations, Proceedings of ANPA* ***35*** 223-259