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Large-scale structure

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Abstract.
Recent observational surveys have made substantial progress in quantifying the structure of the Universe on large scales. Galaxy density and galaxy velocity fields show deviations from the predictions of a homogeneous and isotropic world model on scales approaching one percent of the current horizon scale. A comparison of the amplitudes in density and in velocity provides the first direct dynamical evidence in favour of a high mean density similar to that required for closure. The fluctuations observed on these scales have the amplitude predicted by the standard Cold Dark Matter (CDM) model when this model is normalised to agree with the microwave background fluctuations measured on much larger scales by the COBE satellite. However, a CDM model with this amplitude appears *inconsistent* with observational data on smaller scales. In addition it predicts a scale dependence of fluctuation amplitude which disagrees with that observed for galaxies in the APM survey of two million faint galaxies. The COBE measurement also strongly excludes the standard neutrino-dominated Hot Dark Matter model. Finally, the baryon fraction in rich clusters of galaxies appears much larger than the baryon fraction allowed in an Einstein-de Sitter universe by the theory of Big Bang nucleosynthesis, and so conflicts with both models. Several modifications of these standard models have been proposed in order to avoid some of these difficulties, but none avoids all of them.

1. Introduction

Current theories for the formation of structure in the Universe embrace aspects of quantum gravity, of high energy particle physics, and of nonlinear gravitational collapse, together with a lot of rather messy astrophysics. The global geometry of space-time presumably finds its origin at the Planck time, either as an initial condition, or as a result of some consistency requirement in quantum gravity. The very large (possibly infinite) length-scale characterising the curvature of the universe may reflect an initial phase of chaotic inflation, or may arise from a later inflationary phase occurring, for example, at the symmetry breaking epoch of a Grand Unified Theory. The latter phase-transition could also produce the baryon asymmetry of the Universe, as could processes at later times. Quantum fluctuations present in the gravitationally dominant field during either inflationary phase could create small-scale structure on the world model which might ultimately develop into present-day galaxies and larger structures. As an alternative, breaking of the symmetry of some non-dominant field to a state with non-trivial local orientation properties might lead to topological defects, regions of space where topological constraints force the field to remain in its unbroken state. The energy density associated with these defects could then induce gravitational perturbations in the dominant component of the Universe and so lead to galaxy formation. Thus well before the end of the first second, the structure and the particle and radiation content of the Universe may all be determined [1].

A minute or so later the temperature drops to the point where atomic nuclei can bind. The abundances of the light elements (^{2}H, ^{3}He, ^{4}He and ^{7}Li) produced at this time can compared with observation. Good agreement is found for a simple model where the early universe is effectively homogeneous and its baryon content is a few percent of that required for closure. This has long been taken as one of the main pieces of evidence in favour of the Hot Big Bang [1, 5]. The techniques needed for calculating the linear evolution of fluctuations on an FRW background are now well developed and can be used to predict the statistical properties of angular fluctuations in the microwave background [2]. The detection of such fluctuations by the COBE satellite has opened up what should prove to be a very rich source of information about the contents and structure of the early universe [3]. However, comparison with observations of structure in the present universe requires some treatment of the nonlinear evolution of structure, and in particular of galaxy formation (since it is galaxies that we are able to observe). This has been done most effectively through large-scale computer simulations, although such work has so far treated only a subset of the relevant physical processes [4, 2].

2. The Standard Model

The complex of ideas briefly sketched in the last section has led to a "standard" model for the content and structure of the Universe. This model, in variety of forms, is currently the subject of intensive exploration and testing. Its major elements may be enumerated as follows:

1. The inflationary model indicates that the present Universe should be flat to a very good approximation. Thus in the absence of a cosmological constant, the present density parameter, $\Omega_0 \approx 1$ [1].

2. Comparison of light element abundances with cosmic nucleosynthesis calculations leads to the conclusion that Ω_b, the mean baryon density in units of the critical density, and h, Hubble's constant in units of 100 km/s/Mpc, must satisfy, $\Omega_b h^2 = 0.0125 \pm 0.0025$ [5].

3. The bulk of the present matter content of the Universe must therefore be in some nonbaryonic form, for example neutrinos with a mass of ~ 30 eV (Hot Dark Matter

or HDM), or more exotic particles with smaller thermal motions, such as axions or the lightest supersymmetric partner of known particles (generically such particles are termed Cold Dark Matter, CDM).

4. Structure is imposed either: (a) by quantum fluctuations in the field which dominates during the inflationary period, giving rise to a gaussian fluctuation field obeying the Harrison-Zel'dovich scaling of amplitude with spatial scale, or (b) by gravitational effects due to topological defects (*e.g.* cosmic strings) or to the re-alignment of orientable random fields (*e.g.* cosmic texture) [1, 6].

5. Observed structure grows primarily through gravitational instability, and the galaxy distribution reflects the underlying mass distribution in the simple way suggested by schematic models for galaxy formation [7]

In the absence of a cosmological constant, a flat universe can only approach consistency with the inferred ages of globular star clusters if Hubble's constant is small, $h \sim 0.5$. The evolution of structure has so far been studied most thoroughly in the gaussian case, 4a, for which a HDM model seems unable to produce the observed structure [4].

Recent progress in assessing the viability of this picture has come from a variety of directions. Surveys of galaxy motions are now providing the first direct dynamical evidence in favour of $\Omega_0 \sim 1$, but other observational developments sit less well with these ideas. Although the COBE fluctuation measurement appears to confirm that structure grew through gravitational instability, the actual fluctuation amplitude measured is not in agreement with prior expectations for the simplest models. For standard CDM the result is a factor of two larger than predicted based on the strength of galaxy clustering, while for standard HDM it gives such a low amplitude that the model develops almost no structure at all and can therefore be ruled out. (Neutrino dark matter may still be viable if structure originates through cosmic strings or textures.) Another discrepancy comes from measurements of galaxy clustering which imply a scaling of fluctuation amplitude with spatial scale which is inconsistent with a standard CDM model. Finally, new estimates of the baryon fraction in rich clusters of galaxies appear much too large to be compatible with the baryon fraction allowed by nucleosynthesis constraints in an $\Omega_0 = 1$ universe. The rest of this contribution amplifies these points.

3. Ω_0 estimates from streaming motions

For the growing mode of linear density fluctuations in a dust-filled FRW universe, the present *peculiar velocity* of matter relative to the local fundamental (unperturbed) frame is related to a quasi-Newtonian gravitational potential through:

$$\mathbf{v}(\mathbf{x}) = \frac{2}{3} H_o^{-1} g(\Omega_0) \nabla \Phi$$

where $\mathbf{x}$ is a comoving spatial coordinate, H_0 and Ω_0 are the present values of Hubble's constant and the density parameter, g is a dimensionless function well approximated by $\Omega_0^{-0.4}$, and Φ is related to the overdensity, $\delta(\mathbf{x}) = \rho(\mathbf{x})/\overline{\rho} - 1$, by

$$\nabla^2 \Phi = \frac{3}{2} H_0^2 \Omega_0 \delta.$$

Recent improvements in observational techniques have made it possible to measure distances to large numbers of galaxies. Subtracting H_0 times the distance from the observed recession velocity of an object gives the projection along the line-of-sight of the difference beween the values of $\mathbf{v}$ at its position and at our own. Since $\mathbf{v}(\mathbf{x})$ is curl-free, estimates of this quantity for a dense enough sample of galaxies suffice to reconstruct the whole field up to a constant value. This constant is the peculiar motion of our own Galaxy which can be measured directly through the dipole asymmetry it induces in the apparent temperature of the microwave background. Hence the observational data permit the reconstruction of the full peculiar velocity field. Its divergence is then $\Omega_0 g(\Omega_0)\delta(\mathbf{x}) \approx \Omega_0^{0.6}\delta$.

A galaxy overdensity field, $\delta_g(\mathbf{x})$, can be estimated quite independently by measuring the spatial density of objects directly in a suitable (different) sample of galaxies. Clearly, if the structures seen in the field, δ_g, correspond well to those seen in the field, $\Omega_0^{0.6}\delta$, this suggests that the observed peculiar motions are indeed induced gravitationally, that the measurements are not dominated by observational error, and that the observable galaxy density field is closely related to the invisible mass density field. This programme was first suggested and applied by [8]. There really does seem to be quite a good correspondance between the density field measured by counting galaxies, and that inferred from peculiar velocities. Furthermore, if complete samples of galaxies from catalogues constructed using the Infrared Astronomical Satellite (IRAS) are used to define the galaxy density field, the amplitudes also agree approximately *i.e.* $\delta_g(\mathbf{x}) \approx \Omega_0^{0.6}\delta(\mathbf{x})$. Thus if IRAS galaxies trace the mass distribution (so that $\delta_g \approx \delta$) we infer that $\Omega_0 \approx 1$.

In this subject it is often assumed that the galaxies are *biased* relative to the mass in the sense that the contrast of their density field is enhanced; in the simplest model $\delta_g = b\delta$, where the bias constant is taken to be $b > 1$ in order to account for the small mass to luminosity ratios measured for galaxy clusters. In this model the comparison of peculiar velocity and galaxy density fields leads to the estimate, $b \approx \Omega_0^{0.6}$, for the IRAS galaxy samples. Hence mass would need to be substantially *more* clustered than the observed IRAS galaxies in order to produce the observed peculiar velocities in a low density universe (*e.g.* $\Omega_0 \sim 0.2$). This is difficult to reconcile with our present, admittedly poor understanding of how galaxies form, and so the current situation is usually taken as a positive indication in favour of $\Omega_0 = 1$. At present a number of long-term projects are acquiring larger and more accurate datasets to carry out this test, so the conclusions should become much more solid over the next few years.

4. The COBE amplitude measurement

The Differential Microwave Radiometer on board COBE has detected fluctuations on all angular scales larger than the instrument's resolution of 7°. Smoothed to 10° the *rms* temperature fluctuation on the sky is $30 \pm 5\mu K$ in the first year's data, or one part in 10^5 [9]. The spatial scale corresponding to 10° is considerably larger than any scale for which we have an estimate of fluctuation amplitude from studies of clustering in the present universe. Comparison of the COBE result with such data therefore requires not only an assumption about the evolution of fluctuation amplitudes (so that the present amplitude on the COBE scale can be inferred from the observed amplitude on the surface where the radiation was last scattered, at a redshift between 30 and 10^3) but also an assumption about the scaling of fluctuation amplitude with spatial scale. The time evolution depends only on the global cosmological parameters, Ω_0 and the cosmological constant, Λ, but the spatial scaling depends in addition on the details of the fluctuation generation mechanism, on the nature of the dark matter, and on the baryon density, Ω_b. A further complication is that the COBE measurements could contain a significant contribution from gravitational wave modes which would have no effect on structure formation and hence would not be reflected in measurements of galaxy clustering.

In one of the announcement papers [9] the COBE team discussed the consequences of their fluctuation measurement for the models of §2. For standard CDM models in which inflation-generated gaussian fluctuations have the Harrison-Zel'dovich scaling of amplitude with spatial scale, the COBE measurement implies that the *rms* amplitude of mass fluctuations on small scales is about equal to the observed amplitude of galaxy fluctuations, *i.e.* that the bias parameter, $b \sim 1$. This amplitude is almost exactly that required in a CDM universe to produce the observed peculiar motions discussed in the last section. However, it has generally been argued that substantially larger values of b, (and hence smaller fluctuation amplitudes) are required for a flat universe to be consistent with the observed dynamics of galaxy clustering on smaller scales and with the observed, relatively low abundance of massive objects such as galaxy clusters [7, 10]. A dissenting opinion that $b \sim 1$ is actually required to explain the observed properties of galaxy clustering was expressed in [11] and the situation is still somewhat controversial. If it is accepted that CDM with $b \sim 1$ is unacceptable, then a variety of possibilities have been suggested which might reconcile the COBE observations with observed clustering within a CDM-like model (see, for example [12]).

For the standard HDM model with $H_0 = 50$ km/s/Mpc, with $\Omega_0 = 1$ contributed predominantly by a single species of massive neutrino, and with the Harrison-Zel'dovich spectrum of inflationary perturbations, the COBE amplitude implies an *rms* neutrino density fluctuation from point to point in the present universe of only $\langle\delta^2\rangle^{1/2} \approx 0.7$ [13]. For such a small amplitude only a few percent of all the matter in the universe is predicted to be in nonlinear collapsed objects by the present day, and virtually no nonlinear objects should exist at redshifts of one or greater. This is clearly inconsistent with the substantial structures seen in the present universe and with the existence of quasars at redshifts approaching five.

In universes where fluctuations result from the presence of topological defects or the realignment of orientable random fields (*e.g.* cosmic strings or textures) the present nonlinear structure of the mass distribution is considerably more difficult to calculate than in the gaussian fluctuation models just discussed. As a result it has not yet been possible to compare their predictions for galaxy clustering and for the COBE data to observation at the same level of precision as for the other models. At present it still appears that these models may be viable [6].

5. The shape of the galaxy correlation function

The amplitude of fluctuations as a function of spatial scale can be estimated for the galaxy distribution by measuring its spatial autocorrelation function. So far the most sensitive estimate of this two-point statistic has come from the Automated Plate-measuring Machine (APM) survey which catalogued the positions of more than 2×10^6 galaxies over a large area of the southern sky [14]. Individual distances to these galaxies are not known. However, the very large number of objects means that estimates of galaxy clustering from their projected positions are still more precise than those obtained from the much smaller samples for which complete three-dimensional information is available. The scaling of fluctuation amplitude with spatial scale found from this survey is not consistent with that predicted for the *mass* in a standard CDM model [14]. If the model is matched to the data on scales where the amplitude is of order unity ($\sim 5h^{-1}$Mpc), it predicts fluctuations which are too weak on somewhat larger scales ($\sim 20h^{-1}$Mpc). This is often stated as showing that standard CDM has too little power on large scales. In fact, however, if the overall level of mass fluctuations is taken to be fixed by COBE, then the problem seems rather to be that standard CDM predicts too much small-scale power.

The APM result has survived a number of challenges and seems to be supported by similar results for a number of three-dimensional catalogues. At present the latter are less statistically significant than the original, and in addition their interpretation is complicated by the distortion of the distribution which results from the peculiar velocities of galaxies. Extensions of the CDM model which reconcile the COBE amplitude with the dynamics of galaxy clustering generally change the scaling of fluctuation amplitude in a way which makes it more compatible with the APM data [12]. An alternative resolution of the difficulty may lie in the physics of galaxy formation. The assumption that $\delta_g = b\delta$, and thus that the shape of the galaxy correlation function should parallel that of the mass, is based on a rather simple and schematic model for galaxy formation [7]. This assumption does not hold in models where the formation of galaxies is modulated by some long-range nongravitational effect (for example, is inhibited or is stimulated by radiation from nearby quasars). Such models can reproduce the shape of the APM *galaxy* correlations within a standard CDM universe [15].

6. The baryon fraction in rich clusters of galaxies

If the first two assumptions of the standard model of §2 are correct, then the fraction of the matter in the universe which is baryonic is,

$$F_b = \frac{\Omega_b}{\Omega_0} = 0.0125 \pm 0.0025 \quad h^{-2}.$$

The largest objects for which it is possible to get reliable estimates both of total mass and of baryon content are rich clusters of galaxies. These are the most massive quasi-equilibrium systems known, and baryons are observed within them both in the form of stars within the individual galaxies, and in the form of a pervasive intergalactic medium which is sufficiently hot that it emits X-rays. The total mass of a cluster can be obtained using the virial theorem, or by applying the equations of hydrostatic equilibrium either to the gas or to the galaxy population. The baryonic mass in stars can be estimated from the optical luminosity of the cluster, and the baryonic mass in hot gas from X-ray imaging and spectroscopy. For the best observed rich cluster, the nearby Coma cluster, the results found by applying standard techniques are,

$$M_{tot} = 7.4 \times 10^{14} h^{-1} \mathrm{M}_\odot, \qquad M_* = 3.2 \times 10^{13} h^{-1} \mathrm{M}_\odot, \qquad M_{gas} = 5.6 \times 10^{13} h^{-2.5} \mathrm{M}_\odot,$$

where all three masses are estimated within a radius of $1.5h^{-1}$ Mpc and have errors of 20 to 30% [16]. Since some of the unseen dark matter might be made of baryons, we then get a lower limit to the baryon fraction in the Coma cluster:

$$F_{b,Coma} \geq \frac{M_* + M_{gas}}{M_{tot}} = 0.043 + 0.076h^{-1.5}.$$

Although the observational uncertainties in this limit are substantial, it is clearly much too high to be compatible with the baryon fraction in the standard model. Retaining the standard model requires the total mass of the Coma cluster to be much larger than is usually believed and the gas and star masses to be much smaller. Since Coma does not appear to be in any way atypical, similar systematic errors would have to apply to mass estimates in other clusters. One might hope to resolve this paradox by arguing that baryons are preferentially concentrated into the centres of rich clusters during cluster formation. Numerical simulations of cluster formation can put upper limits on the enhancement attainable, and, for a region as large as that used above, the maximum possible enhancement is a few tens of percent; indeed, most simulations of cluster formation conclude that the gas should end up slightly *less* concentrated than the galaxies, thus making the discrepancy worse [17].

7. Conclusions

While the observational data reported in §3 tend to support the standard picture outlined in §2, those discussed in §§4 – 6 disagree with various parts of it. The fluctuation amplitude measured by COBE rules out a standard HDM model with a Harrison-Zel'dovich spectrum of gaussian initial fluctuations. *A priori* this is, of course, the most attractive of all the models dominated by nonbaryonic dark matter. The corresponding standard CDM model is also in trouble because COBE requires a higher normalisation of the fluctuation amplitude than seems consistent with the abundance of rich galaxy clusters and with the dynamics of galaxy clustering on scales of a few Mpc. The shape of the APM galaxy correlation function also suggests the need for initial density fluctuations with less small-scale power than the standard CDM model, and several modifications of the model have been suggested which accomplish this (for example, replacing some of the CDM by HDM, or adding an additional relativistic component such as nonthermalised neutrinos [12]).

Unfortunately the difficulty highlighted in §6 applies in any Einstein-de Sitter universe and so to these modifications of the standard CDM model. Unless some flaw can be found in the interpretation of the observational data, it forces the abandonment of:

1. the Einstein-de Sitter model, or
2. the standard theory of cosmic nucleosynthesis, or
3. the growth of structure through hierarchical clustering driven by gravity.

An example of the first way out is the introduction of a cosmological constant. In an otherwise standard flat CDM model this allows a large value of $F_b = \Omega_b/\Omega_0$, a correlation function shape consistent with the APM survey, and an age for the Universe consistent with those of globular star clusters despite a high value of Hubble's constant.

However, such a model does not produce large enough peculiar velocities to be consistent with the data described in §3. An example of the third way out is explosive galaxy formation. A first generation of objects (early quasars?) might produce hydrodynamic flows which pile the baryonic gas into large-scale structures. These could then act as accretion nuclei for the dark matter. Such a model is unattractive because it decouples the properties of present large-scale structure from primordial density fluctuations. In addition, the energetic shocks it requires are almost certainly inconsistent with the very precise black-body spectrum and the relative weak angular fluctuations measured by COBE. No current model appears consistent with a straightforward interpretation of all the observational data.

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