

# The interpretation of quantum cosmological models

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**Abstract.** We consider the problem of extracting physical predictions from the wave function of the universe in quantum cosmological models. We state the features of quantum cosmology an interpretational scheme should confront. We discuss the Everett interpretation, and extensions of it, and their application to quantum cosmology. We review the steps that are normally taken in the process of extracting predictions from solutions to the Wheeler-DeWitt equation for quantum cosmological models. Some difficulties and their possible resolution are discussed. We conclude that the usual wave function-based approach admits at best a rather heuristic interpretation, although it may in the future be justified by appeal to the decoherent histories approach.

## 1. Introduction

Quantum mechanics was originally developed to account for a number of otherwise unexplained phenomena in the microscopic domain. The scope of the theory was not thought to extend beyond the microscopic. Indeed, the existence of an external, macroscopic, classical domain was felt to be necessary for the theory's interpretation. This view of quantum mechanics has persisted for a very long time, with not one shred of experimental evidence against it.

Today, however, more ambitious views of quantum mechanics are entertained. The experimental possibilities afforded by SQUIDS suggest that quantum coherence may act on macroscopic scales (Leggett, 1980). Furthermore, the separation of classical and quantum domains is often felt unnatural from a foundational point of view. The domain of applicability of quantum mechanics may therefore need to be extended. But in extrapolating quantum mechanics to the macroscopic scale, where do we stop? At the scale of large molecules? At the laboratory scale? At the planetary scale? There is only one natural place – at the scale of the entire universe. One rapidly arrives, therefore, at the subject of quantum cosmology, in which quantum mechanics is applied to the entire universe. It might only be through this subject that the form of quantum mechanics as we know it may be fully understood.

Yet quantum cosmology was not originally developed with the foundational problems of quantum mechanics in mind. Rather, it was perhaps viewed as a natural area to investigate as part of a more general drive to understand quantum gravity (DeWitt, 1967; Misner, 1969; Wheeler, 1963, 1968). More recently, the success of models like inflation (Guth, 1981) in early universe cosmology has led to a greater desire to understand the initial conditions with which the universe began. On very general grounds, the universe appears to have emerged from an era in which quantum gravitational effects were important. Quantum cosmology, in which both matter and gravitational fields are taken to be quantized, is therefore the natural framework in which to address

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questions of initial conditions. Indeed, there has been a considerable amount of activity in this subject in recent years.<sup>†</sup>

Much of this recent attention on quantum cosmology has been focused on obtaining a crude idea of the physical predictions made by certain theories of initial conditions. However, little attention has been devoted to understanding or even clarifying the principles involved in extracting these predictions from the mathematical formalism. This contribution represents a small step towards filling this gap. That is, it is concerned with the *interpretation* of quantum cosmology. In particular, I shall be addressing the question, "How are predictions extracted from a given wave function of the universe?". I will state immediately that there is no wholly satisfactory answer to this question. The aim of this contribution, therefore, is to describe the difficulties involved and the attempts to overcome them, the procedures that are actually used in practice and the motivation behind them.

The interpretation of quantum cosmology raises two particular sets of issues which may be discussed separately. The first set concerns the fact that, unlike ordinary quantum mechanics, the system under scrutiny, the universe, is a closed and isolated system. The special features of (non-relativistic) quantum mechanics applied to such systems are discussed in the accompanying paper by Hartle (Hartle, 1992a). For completeness, some of these features are also covered here, but from a somewhat different angle (namely from the wave function point of view, rather than that of the decoherent histories approach). The second set of issues concerns the fact that the wave function for the system is described not by a time-dependent Schrödinger equation, but by the Wheeler-DeWitt equation. Associated with this is the so-called "problem of time", and the absence of the usual machinery of projection operators, unitary evolution, Hilbert space *etc.* It is this second set of issues that will form the primary focus of this paper.

To focus the discussion, consider the sort of features of the universe one might hope to predict in quantum cosmology. Some of the most important are:

- **Spacetime is Classical.** One of the crudest and most obvious observations about the universe we can make, is that the structure of spacetime is described very accurately by classical laws. The very first requirement of a quantum theory of the universe, therefore, is that it predict this. The question of how classical behaviour emerges in quantum systems is a difficult one, that has not been completely solved even in non-relativistic quantum mechanics. Still, it ought be possible, at least in some crude sense, to see whether a given wave function for the universe is consistent with a prediction of classical spacetime.
- **Initial Inflationary Phase.** Many current observational features of the universe, such as the horizon, flatness and monopole problems, are explained by postulating an inflationary phase at very early times. In models of the universe which admit inflationary solutions, the occurrence and amount of inflation are generally dependent on initial conditions. A correct quantum theory of initial conditions should supply the initial conditions for an inflationary phase to take place.
- **Spectra of Density Fluctuations and Gravitational Waves.** In inflationary models, it is possible to generate gravitational waves and density fluctuations consistent with the observed isotropy of the microwave background, yet sufficiently large

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<sup>†</sup> For general reviews of quantum cosmology, see Fang and Ruffini (1987), Fang and Wu (1986), Halliwell (1990, 1991), Hartle (1985, 1986, 1990), Hawking (1984b), Hu (1989), Laflamme (1991), Page (1986, 1991).

for the formation of large scale structure. These results are obtained by considering the quantum field theory of the fluctuations during the inflationary phase. Again they are initial conditions dependent, and one might hope that quantum cosmology will supply the requisite initial conditions.

- **Low Entropy Initial State.** One of the most striking features of the universe is its time asymmetry. It is very far from equilibrium, indicating that it started out in a very special, low entropy initial state. One would therefore hope to predict this very smooth beginning. This is closely related to the spectra of fluctuations.

In what follows, we will not be concerned with detailed predictions of particular theories of initial conditions. Rather, we will assume that we are given a wave function for the universe (perhaps determined by *some* theory of initial conditions), and discuss the extraction of predictions from it. In particular, we will focus on the emergence of a *semiclassical domain* – an approximately classical spacetime background with quantum fields in it<sup>†</sup>. This is appropriate for two reasons. First, all observational features of the universe are semiclassical in nature. Second, even if the theory could make observable quantum gravitational predictions, as one would certainly hope of a quantum theory of cosmology, observation of them would be through their correlation with the semiclassical domain. Quite generally, therefore, the emergence of the semiclassical domain is the appropriate thing to look for.

## 2. Canonical Quantization

In the interests of conciseness, I shall assume that the formalism of quantum cosmology is known, and give only the briefest account here (see for example Halliwell (1990), and the general references cited earlier). For definiteness, we consider the canonical quantization of Einstein gravity coupled to matter for closed universes. The quantum state of the system, the universe, is represented by a wave functional,  $\Psi[h_{ij}, \phi]$ , a functional on superspace, the space of three-metrics  $h_{ij}$  and matter fields  $\phi$  on a three-surface. The wave function has no explicit dependence on time. There is therefore no Schrödinger equation, only the constraints of the Dirac quantization procedure,

$$\hat{\mathcal{H}}\Psi[h_{ij}, \phi] = 0, \quad \hat{\mathcal{H}}_i\Psi[h_{ij}, \phi] = 0 \quad (2.1)$$

where  $\hat{\mathcal{H}}$  and  $\hat{\mathcal{H}}_i$  are the operator versions of the classical constraints,

$$\mathcal{H} = h^{-\frac{1}{2}} \left( \pi^{ij} \pi_{ij} - \frac{1}{2} \pi_i^i \pi_j^j \right) - h^{\frac{1}{2}} ( {}^3R - 2\Lambda ) + \mathcal{H}^{matter} = 0 \quad (2.2)$$

$$\mathcal{H}_i = -2\pi_i^{[j} + \mathcal{H}_i^{matter} = 0 \quad (2.3)$$

(in the usual notation). Eqs.(2.1) are of course the Wheeler-DeWitt equation and the momentum constraints.

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<sup>†</sup> This is, loosely speaking, the same thing as the *quasi-classical* domain defined by Gell-Mann and Hartle (1990) in the context of the decoherent histories approach, but I give it a slightly different name since the wave function approach considered here does not permit it to be defined in quite the same way.

Wave functions satisfying the constraints (2.1) may also be generated using a (complex) path integral representation (Halliwell and Hartle, 1990, 1991). This is used, for example, in the specification of the no-boundary wave function (Hawking, 1982, 1984a; Hartle and Hawking, 1983). The sum-over-histories approach is more general in that it may be used to construct more complicated amplitudes (*e.g.*, ones depending on the three-metric and matter fields on many three-surfaces) and in fact the latter type of amplitude is likely to be the most useful for interpreting the theory. Here we are concerned with describing what has actually been done, and this means discussing single surface amplitudes.

There are other very different ways of quantizing the classical system described by the constraints, (2.2), (2.3). In particular, one could contemplate solving the constraints classically, prior to quantization, and then quantizing an unconstrained theory. The Dirac quantization scheme outlined here is the one most commonly used in practical quantum cosmology, and it is this that we shall use in what follows.

Of course, all known approaches to quantum gravity are fraught with severe technical difficulties, and the formal framework outlined above is no exception. Moreover, a proper theory of quantum gravity, should it exist, might involve substantial departures from this general framework (*e.g.*, string theory). However, there are a number of reasons why it might nevertheless make sense to persevere with this approach to quantum cosmology. Perhaps the most important is that in quantum cosmology one is frequently concerned with *issues*. Many issues, and in particular the interpretational issues considered here, are not very sensitive to the resolution of technical difficulties. Furthermore, they are likely to be equally present in any approach to quantum gravity. No generality is lost, therefore, in employing the formal scheme envisaged in Eq.(2.1).

### 3. Interpretation

The question of interpretation is that of extracting physical statements about the universe from the wave function,  $\Psi[h_{ij}, \phi]$ . In non-relativistic quantum mechanics there is a well-defined procedure for achieving this. It is generally known as the Copenhagen interpretation. Although it is frequently regarded as problematic from a foundational point of view, it has been very successful in its physical predictions.<sup>†</sup> The Copenhagen interpretation involves a number of features and assumptions that should be highlighted for the purposes of this account:

- C1. It assumes the existence of an *a priori* split of the world into two parts: a (usually microscopic) quantum system, and an external (usually macroscopic) classical agency.
- C2. It concerns systems that are not genuinely closed, because they are occasionally subject to intervention by the external agency.
- C3. The process of prediction places heavy emphasis on the notion of *measurement* by the external agency.
- C4. Predictions are probabilistic in nature, and generally only have meaning when measurements are performed on either a large ensemble of identical systems, or on the same system many times, each time prepared in the same state.

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<sup>†</sup> Many of the key papers on the Copenhagen interpretation may be found in Wheeler and Zurek (1983).

C5. Time plays a very distinguished and central role.

Quantum cosmology, by contrast, involves a number of corresponding features and assumptions that render the Copenhagen interpretation woefully inadequate:

- QC1. It is assumed that quantum mechanics is universal, applying to microscopic and macroscopic systems alike, up to and including the entire universe. There can therefore be no *a priori* split of the universe into quantum and classical parts.
- QC2. The system under scrutiny is the entire universe. It is a genuinely closed and isolated system without exterior.
- QC3. Measurements cannot play a fundamental role, because there can be no external measuring apparatus. Even internal measuring apparatus should not play a role, because the conditions in the early universe were so extreme that they could not exist.
- QC4. The universe is a unique entity. It does not belong to an ensemble of identical systems; nor is it possible to make repeated measurements on it prepared in the same state.
- QC5. The problem of time: general relativity does not obviously supply the time parameter so central to the formulation and interpretation of quantum theory.

Of these features, only (QC5) is specific to quantum gravity. The rest would be true of any closed and isolated system described by non-relativistic quantum mechanics, and may be discussed in this context.

The problem of interpretation is that of finding a scheme which confronts features (QC1)–(QC5), yet which may be shown to be consistent with or to reduce to (C1)–(C5) under suitable approximations and restrictions.

Probably the best currently available approach to these difficulties is the decoherent histories approach, which employs not the wave function, but the decoherence functional as its central tool.<sup>†</sup> It has a number of features which strongly recommend it for quantum cosmology: it specifically applies to closed systems; it assumes no *a priori* separation of classical and quantum domains; it does not rely on the notion of measurement or observation; and its focus on histories rather than events at a single moment of time might sidestep (or at least alleviate) the problem of time in quantum gravity. Unfortunately, a detailed application of this approach to quantum cosmology has not yet been carried out (although efforts in this direction are currently being made (Hartle, 1992b)). Furthermore, my task here is to describe what has actually been done, and for that reason I will describe the somewhat cruder approaches to interpretation based on the wave function. However, I find it useful to remain close to the spirit of the decoherent histories approach, and to think of the wave function approach as an approximation to it (although in a sense yet to be explained). In particular, it is convenient to have as one's aim the assignment of probabilities to histories of the universe.

In order to deal with the issues noted above, conventional wave function-based approaches invoke the Everett (or "Many Worlds") interpretation.<sup>††</sup> Above all, the

<sup>†</sup> The consistent histories approach to quantum mechanics was introduced by Griffiths (1984), and later developed by Omnès (reviewed in Omnès, 1990, 1992). Much of it was developed independently under the name decoherent histories, by Gell-Mann and Hartle (1990). See also Hartle (1990, 1992a). For applications and generalizations, see Blencowe (1991), Albrecht (1990) and Dowker and Halliwell (1992).

<sup>††</sup> Everett (1957). A useful collection of papers on the Everett interpretation may be found in DeWitt

Everett interpretation is a scheme specifically designed for quantum mechanical systems that are closed and isolated. Everett asserted that quantum mechanics should be applicable to the entire universe, and there should be no separation into quantum and classical domains. These features of the Everett interpretation are therefore consistent with features (QC1) and (QC2) of quantum cosmology. Furthermore, the state of the entire system should evolve solely according to a wave equation, such as the Schrödinger equation, or in quantum cosmology the Wheeler-DeWitt equation, and there should be no discontinuous changes (collapse of the wave function).

Everett went on to model the measurement process by considering a world divided into a large number of subsystems, and showed how the conventional Copenhagen view of quantum mechanics can emerge. It is this part of the Everett interpretation that leads to its "Many Worlds" feature – the idea that the universe splits into many copies of itself whenever a measurement is performed. It is at this stage, however, that the original version of the Everett interpretation departs in its usefulness from practical quantum cosmology. For the sort of models one is often interested in interpreting in quantum cosmology are minisuperspace models, which are typically very simple, and do not contain a large number of subsystems. Furthermore, the many worlds aspect of the interpretation has, I believe, been rather over-emphasized, perhaps at the expense of undermining the credibility of the overall set of ideas. The Everett interpretation, especially as it applies to practical quantum cosmology, is not so much about many worlds, but rather, about how one might make sense of quantum mechanics applied to genuinely closed systems.

It is, therefore, convenient to pass to a restatement of the Everett interpretation due to Geroch (1984). Geroch translated Everett's modeling of the measurement process using a large number of subsystems into statements about the wave function of the entire system. He argued that predictions for closed quantum systems essentially boil down to statements about "precluded" regions – regions of configuration space in which the wave function for the entire system is very small. From here, it is a small step to a slightly more comprehensive statement given by Hartle, specifically for quantum cosmology (Hartle, 1986). It is the following:

*If the wave function for the closed system is strongly peaked about a particular region of configuration space, then we predict the correlations associated with that region; if it is very small, we predict the lack of the corresponding correlations; if it is neither strongly peaked, nor very small, we make no prediction.*

This basic idea appears to have been adopted, perhaps provisionally, in most, if not all, attempts to interpret the wave function in quantum cosmology (see also Wada (1988)).

The statement is admittedly rather vague and a number of qualifying remarks are in order. Firstly, to say that a wave function is "strongly peaked" necessarily involves some notion of a measure, and this has to be specified. For example, do we use  $|\Psi|^2$ ? The Klein-Gordon current constructed from  $\Psi$ ? We will return to this below. Second, the interesting correlations in the wave function are often not evident in the configuration space form. It is therefore appropriate to interpret the above statement rather broadly, as referring to not just the wave function, but any distribution constructed from the wave function. For example, phase space distributions such as the Wigner function, and related functions, have been the focus of attention in a number of pa-

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and Graham (1973). See also Bell (1981), Deutsch (1985), Kent (1990), Smolin (1984) and Tipler (1986).

pers, and these have sometimes proved quite useful in identifying classical correlations (Anderson, 1990; Calzetta and Hu, 1989; Habib, 1990; Habib and Laflamme, 1990; Halliwell, 1987, 1992; Kodama, 1988; Singh and Padmanabhan, 1989).

For practical quantum cosmology, the implication of the above interpretational scheme is that rather than find the probability distributions for quantities felt to be of interest, as in ordinary quantum mechanics, it is necessary to determine those quantities for which the theory gives probabilities close to zero or one, and hence, for which it makes predictions. On the face of it, this may seem to suggest that the predictive power of quantum cosmology is very limited in comparison to ordinary quantum mechanics. However, by studying isolated systems consisting of a large number of identical subsystems, it may be shown that this interpretation implies the usual statistical interpretation of ordinary quantum mechanics, in which subsystem probabilities are given by  $|\psi|^2$  where  $\psi$  is the subsystem wave function (Farhi, Goldstone and Gutmann, 1989; Finkelstein, 1963; Graham, 1973; Hartle, 1968). It is in this sense that feature (QC4) of quantum cosmology is reconciled with (C4) of the Copenhagen interpretation.

Given a measure on a set of possible histories for the universe, it is often not peaked about a particular history or family of histories. To obtain probabilities close to one or zero, it is often necessary to restrict attention to a certain subset of the possible histories of the universe, and make predictions within that subset. That is, one looks at *conditional probabilities*. The motivation behind this is anthropic reasoning – we as observers do not look out into a generic universe, but to one in which the conditions for our own existence have necessarily been realized (see, for example, Barrow and Tipler, 1986). Obviously the detailed conditions for our existence could be very complicated and difficult to work out. However, it is possible to get away with exploiting only the weakest of anthropic assumptions in order to make useful and interesting predictions about the universe. It is, for example, extremely plausible that the existence of life requires the existence of stars like our sun. It therefore makes sense to restrict attention only to those histories of the universe which exist long enough for stars to form before recollapsing.

This completes our brief survey of the special features of closed quantum systems. We have discussed features (QC1), (QC2) and (QC4) of quantum cosmology. The status of measurements, (QC3), in the above discussion is admittedly somewhat vague, although it is clear that they do not play a significant role. Their status is perhaps clarified more fully in the decoherent histories approach. Finally, although it is an important issue for the interpretation of quantum cosmology, I have not discussed the problem of time, (QC5). This will be briefly mentioned below, but it would take a lot of space to do it justice. The interested reader is referred to the reviews by Kuchař (1989, 1992), which cover the problem of time and its connections with the interpretation of quantum cosmology. See also Unruh and Wald (1989).

#### 4. Interpretation of Solutions to the Wheeler-DeWitt Equation

We now come to the central part of this contribution, which is to describe how in practice predictions are actually extracted from solutions to the Wheeler-DeWitt equation. As mentioned earlier, the primary aim is to determine the location and features of the semiclassical domain. Different papers in the literature treat the process of prediction in different ways, but it seems to boil down to four distinct steps, which I now describe in turn.

### A. Restriction to Perturbative Minisuperspace.

The full theory described by Eq.(2.1) is not only difficult to handle technically, it is not even properly defined. This problem is normally avoided by artificially restricting the fields to lie in the region of superspace in the neighbourhood of homogeneity (and often isotropy). That is, one restricts attention to the finite dimensional subspace of superspace called "minisuperspace", and considers small but completely general inhomogeneous perturbations about it.<sup>†</sup>

In more detail, the sort of restrictions entailed are as follows. One restricts the three-metric and matter fields to be of the form,

$$h_{ij}(\mathbf{x}, t) = h_{ij}^{(0)}(t) + \delta h_{ij}(\mathbf{x}, t), \quad \Phi(\mathbf{x}, t) = \phi(t) + \delta\phi(\mathbf{x}, t) \quad (4.1)$$

Here, the minisuperspace background is described by the homogeneous fields  $h_{ij}^{(0)}$  and  $\phi(t)$ . For example, the three-metric could be restricted to be homogeneous and isotropic, described by a single scale factor  $a$ . We will denote the minisuperspace background coordinates by the finite set of functions  $q^\alpha(t)$ , where  $\alpha = 1, \dots, n$ .  $\delta h_{ij}$  and  $\delta\phi$  are small inhomogeneous perturbations about the minisuperspace background, describing gravitational waves and scalar field density perturbations. They are retained only up to second order in the action and Hamiltonian (and therefore to first order in the field equations). For convenience, we will denote the perturbation modes simply by  $\delta\phi$ .

With these restrictions on the class of fields considered, the Wheeler-DeWitt equation, after integration over the three-surface, takes the form

$$\left[ -\frac{1}{2m_p^2} \nabla^2 + m_p^2 U(q) + H_2(q, \delta\phi) \right] \Psi(q, \delta\phi) = 0 \quad (4.2)$$

Here,  $\nabla^2$  is the Laplacian operator in the minisuperspace modes  $q$  and we have explicitly included the Planck mass  $m_p$ , since it is to be used as a large parameter in a perturbative expansion.  $H_2$  is the Hamiltonian of the perturbation modes,  $\delta\phi$ , and is quadratic in them. There are more constraint equations associated with the remaining parts of the Wheeler-DeWitt equation, and with the momentum constraints. These are all linear in the perturbations and can be solved, after gauge-fixing. When this is done, only Eq.(4.2) remains, in which  $\delta\phi$  may be thought of as a gauge-invariant perturbation variable.

The restriction to perturbative minisuperspace is of course very difficult to justify from the point of view of the full theory. A number of attempts have been made to understand the sense in which minisuperspace models might be part of a systematic approximation to the full theory, but the answer seems to be that their connection is at best tenuous (Kuchař and Ryan, 1986, 1989). What one can say, however, is that solutions to the minisuperspace field equations, including perturbations about them, will (with a little care) be solutions to the full field equations, and thus the lowest order semiclassical approximation to perturbative minisuperspace quantum cosmological models will agree with the lowest order semiclassical approximation to the full theory.

<sup>†</sup> This general approach has been the topic of many papers, including Banks (1985), Banks, Fischler and Susskind (1985), Fischler, Ratra and Susskind (1986), Halliwell and Hawking (1985), Lapchinsky and Rubakov (1979), Shirai and Wada (1988), Vachaspati and Vilenkin (1988), Wada (1986).



These models may therefore be thought of as useful models in which a number of issues can be profitably investigated, but which may also give some crude predictions about the physical universe.

The essential ideas of the practical interpretational scheme described here will very probably be applicable to situations more general than perturbative minisuperspace, but very little work on such situations has been carried out. We will not go into that here.

### B. Identification of the Semiclassical Regime.

The next step involves inspecting the wave function  $\Psi(q^\alpha, \delta\phi)$ , asking how it behaves as a function of the minisuperspace variables  $q$ , and in particular, identifying the regions in which the wave function is exponentially growing or decaying in  $q$ , or oscillatory in  $q$ .

The regions in which the wave function is rapidly oscillating in  $q$  are regarded as the semiclassical domain, in which the modes  $q$ , are approximately classical, whilst the perturbation modes  $\delta\phi$  need not be. This interpretation comes partly from analogy with ordinary quantum mechanics. But also one can often argue that certain distributions constructed from a rapidly oscillating wave function are peaked about classical configurations. For example, the Wigner function, mentioned earlier, is often used to support this interpretation.

The other regions, in which the wave function tends to be predominantly exponential in behaviour are regarded as non-classical, like the under-the-barrier wave function in tunneling situations. Were this ordinary quantum mechanics, then the wave function would typically be exponentially small in the tunneling regions. One could then invoke Geroch's version of the Everett interpretation, and just that the system will not be found in this region, because it is "precluded". However, a peculiar feature of gravity (readily traced back to the indefiniteness of the action) is that the wave function may be either exponentially small or exponentially large in the regions where it is of exponential form. Nevertheless, one still says that the system is not approximately classical in this regime, even when the wave function is exponentially large. To support this claim, one can argue that, in contrast to the oscillatory regions, a predominantly exponential wave function, either growing or decaying, is *not peaked* about classical configurations.

### C. WKB Solution in the Oscillatory Regime.

The next stage of the scheme involves solving in more detail in the region in which the wave function is rapidly oscillating in the minisuperspace variables. This involves focusing on a particular type of state, namely the WKB state,

$$\Psi(q, \delta\phi) = C(q) \exp(im_p^2 S_0(q)) \psi(q, \delta\phi) + O(m_p^{-2}) \quad (4.3)$$

$S_0(q)$  is real, but  $\psi$  may be complex. Many models also involve a slowly varying exponential prefactor contributing at order  $m_p^2$ , but for the purposes of the present discussion it can be assumed that this is absorbed into  $C$ . Also, a possible phase at order  $m_p^0$  depending only on  $q$  may be absorbed into  $\psi$ , so  $C$  may be taken to be real.

Eq.(4.3), it must be stressed, is an *ansatz* for the solution, and many of the predictions subsequently derived depend on assuming this particular form. We will return later to the question of the validity and usefulness of this particular ansatz.

The Wheeler-DeWitt equation may now be solved perturbatively, by inserting the ansatz (4.3), and using the Planck mass  $m_p$  as a large parameter. Since this parameter

is not dimensionless, the expansion is meaningful only on length scales much greater than the Planck length. Note also that a double expansion of the full Wheeler-DeWitt equation is involved: a WKB expansion in the Planck mass, and a perturbation expansion in small inhomogeneities about minisuperspace.

Equating powers of the Planck mass, one obtains the following. At lowest order, one gets the Hamilton-Jacobi equation for  $S_0$ ,

$$\frac{1}{2}(\nabla S_0)^2 + U(q) = 0. \quad (4.4)$$

To next order one obtains a conservation equation for  $C$

$$2\nabla S_0 \cdot \nabla C - C\nabla^2 S_0 = 0 \quad (4.5)$$

and a Schrödinger equation for  $\psi$ ,

$$i\nabla S_0 \cdot \nabla \psi = H_2 \psi \quad (4.6)$$

Consider now (4.4). As indicated above, it may be argued that a wave function predominantly of the form  $e^{im_p S_0}$  indicates a strong correlation between coordinates and momenta of the form

$$p_\alpha = m_p^2 \frac{\partial S_0}{\partial q^\alpha} \quad (4.7)$$

Since  $\dot{q}^\alpha = p^\alpha$ , (4.7) is a set of  $n$  first order differential equations (where, recall,  $\alpha = 1, \dots, n$ ). Furthermore, since by (4.4),  $S_0$  is a solution to the Hamilton Jacobi equation, it may be shown that these equations define a first integral to the classical field equations. The first integral (4.7) may be solved to yield an  $n$ -parameter set of classical solutions. It is for this reason that one says that the wave function (4.3) to leading order, corresponds to an ensemble of classical solutions to the field equations.

In ordinary quantum mechanics, this interpretation may be substantiated by subjecting an initial wave function of the form (4.3) to a sequence of approximate position samplings, and showing that the resulting probability distribution for histories is peaked about the set of classical solutions satisfying the first integral (4.7). See Halliwell and Dowker (1992), for example, for efforts in this direction.

The tangent vector to this congruence of classical paths is

$$\nabla S_0 \cdot \nabla \equiv \frac{\partial}{\partial t} \quad (4.8)$$

(4.6) is therefore the functional Schrödinger equation for the perturbation modes along the family of minisuperspace trajectories. This indicates that the perturbation modes are described by quantum field theory (in the functional Schrödinger picture) along the family of classical backgrounds described by (4.7).

### D. The Measure on the Set of Classical Trajectories

The final stage is to put a measure on the congruence of classical paths, and to find an inner product for solutions to the Schrödinger equation (4.8). Suppose one chooses some  $(n-1)$ -dimensional surface in minisuperspace as the beginning of classical evolution. Through (4.7), the wave function then effectively fixes the initial velocities on that surface, in terms of the coordinates. However, the wave function should also provide a probability measure on the set of classical trajectories about which the wave function is peaked. How is this measure constructed from the wave function? The construction of a satisfactory non-negative measure remains an outstanding problem of quantum cosmology, but perhaps the most successful attempts so far involve the Klein-Gordon current,

$$J = \frac{i}{2} (\Psi \nabla \Psi^* - \Psi^* \nabla \Psi) \quad (4.9)$$

It is conserved by virtue of the Wheeler-DeWitt equation (4.2),

$$\nabla \cdot J = 0 \quad (4.10)$$

Choose a family of surfaces  $\{\Sigma_\lambda\}$ , parametrized by  $\lambda$ , cutting across the flow of  $J$ . Then (4.10) suggests that for each  $\lambda$ , a probability measure on the congruence of trajectories is the flux of  $J$  across the surface:

$$dP = J \cdot d\Sigma \quad (4.11)$$

This measure is conserved along the flow of  $J$ , as is readily shown from (4.10).

The problem with (4.11), however, is that it is not always positive. For example, if the surfaces  $\Sigma$  are taken to be surfaces of constant scale factor, the flow of  $J$  typically cuts these surfaces more than once, because of the possibility of expanding and collapsing universes, leading to negative values for (4.11). Furthermore,  $J$  vanishes when  $\Psi$  is real. Still, some sense may be made out of (4.11) by restricting to the WKB wave functions, (4.3). For these wave functions, the current is

$$J \approx m_p^2 |C|^2 |\psi|^2 \nabla S_0 + O(m_p^0) \quad (4.12)$$

For reasonably large regions of minisuperspace (but not globally), it is usually possible to choose a set of surfaces  $\{\Sigma_\lambda\}$  for which  $\nabla S_0 \cdot d\Sigma \geq 0$ , and thus the probability measure will be positive. Furthermore, this measure implies the standard  $|\psi|^2$  measure for the perturbation wave functions, completing the demonstration that quantum field theory for the perturbations emerges in the semiclassical limit.

This approach was described a long time ago by Misner (1972), and developed more recently by Vilenkin (1989). It is problematic for a number of reasons: one is the global problem of choosing the surfaces  $\{\Sigma_\lambda\}$ ; another is that is very tied to the WKB wave functions (4.3). More will be said about this in the next section. Still, it allows predictions to be made in a number of situations of interest.

The "naive" Schrödinger measure is also sometimes proposed in place of (4.11) (e.g., Hawking and Page, 1986, 1988). This is the assertion that the probability of finding the system in a region of superspace of volume  $dV$  is  $|\Psi|^2 dV$ , where  $\Psi$  is the wave function of the whole system. This does at least have the advantage that it is everywhere positive. Furthermore, it can be argued to reduce to (4.11) for WKB wave

functions, in the limit in which the volume of superspace  $dV$  is taken to be hypersurface of codimension one slightly thickened along the direction of the flow of  $J$ . As argued by Kuchař, however, this measure is problematic for other reasons (Kuchař, 1992).

At this stage, it is appropriate to comment on the problem of time. In the Wheeler-DeWitt equation (4.2) (or (2.1)), there is no distinguished variable that plays the role of time. This is the problem of time. In the scheme described above, however, a time parameter has emerged. It is the parameter  $\lambda$  labeling the family of surfaces  $\{\Sigma_\lambda\}$ , and may be chosen to be the same as the parameter  $t$  defined in Eq.(4.8), the affine parameter along the integral curves of  $\nabla S_0$ . The point to be made is that this parameter has emerged only in the region where the wave function is oscillatory, and in particular, as a consequence of the assumed semiclassical form of the wave function, (4.3). This is therefore an explicit illustration of the point of view, not uncommonly expressed, that time, and indeed spacetime, need not exist at the most fundamental level but may emerge as approximate features under some suitable set of conditions. It is in this sense that feature (QC5) of quantum cosmology may be reconciled with (C5) of ordinary quantum mechanics.

Modulo the above difficulties, Eq.(4.11) is the desired probability measure on possible histories of the universe. It is commonly not normalizable over the entire surface  $\Sigma$ ; but this need not matter, because it is *conditional* probabilities that one is typically interested in. Suppose, for example, one is given that the history of the universe passed through a subset  $s_1$  of a surface  $\Sigma$ , and one wants the probability that the universe passed through a subset  $s_0$  of  $s_1$ . The relevant conditional probability is,

$$p(s_0|s_1) = \frac{\int_{s_0} J \cdot d\Sigma}{\int_{s_1} J \cdot d\Sigma} \quad (4.13)$$

The set of all histories intersecting  $\Sigma$  could include universes which recollapse after a very short time. A reasonable choice for  $s_1$ , therefore, might be universes that exist long enough for stars to form before recollapsing, as discussed earlier.  $s_0$  could then be taken to be the subset of such universes which possess certain features resembling our universe. If the resulting conditional probability turned out to be close to one or zero, this would then constitute a definite prediction.

Steps (A)–(D) above constitute the general interpretational scheme implicit or explicit in most attempts to extract predictions from the wave function of the universe. It is not by any means a consistent interpretational scheme, but is almost a list of rules of thumb inspired by the Everett interpretation, and built on analogies with ordinary quantum mechanics. It has many difficulties, some of which we now discuss.

## 5. Problems and Objections

We have argued that the WKB wave functions (4.3) correspond to an ensemble of classical paths defined by the first integral (4.7). Strictly speaking, what this means is that the WKB wave function is really some kind of *superposition* of wave functions, each of which corresponds to an individual classical history (like a superposition of coherent states).<sup>†</sup> A closely related point is the question of why one should be allowed

<sup>†</sup> For the explicit construction of wave packets in quantum cosmology, see Kazama and Nakayama (1985) and Kiefer (1988).

to study an interpretation based on the WKB form, (4.3): one would expect a more general wave function to be expressed as a *sum* of such terms. In each of these cases, we are acting as if we had a classical statistical ensemble, when what we really have is a superposition of interfering states. Why should it be permissible to treat each term in the sum separately, when strictly they are interfering?

This point concerns the general question of why or when it is permissible to ignore the interference terms in a superposition, and treat each term as if it were the member of a statistical ensemble. Technically, the destruction of interference is generally referred to as decoherence.

Two notions of decoherence have been employed. The most precise is that of the decoherent histories approach, where it enters at a very fundamental level: Interference is most generally and properly thought of as the failure of the probability sum rules for quantum-mechanical histories. Decoherence, as destruction of interference, is therefore best regarded as the recovery of these rules (Gell-Mann and Hartle, 1990).

By contrast, in the wave function approach to quantum cosmology, decoherence appears to have been added as an afterthought, using a different and somewhat vaguer definition: decoherence is held to be related to the tendency of the density matrix towards diagonality (Joos and Zeh, 1985). It is also associated with the establishment of correlations of a given system with its environment, and with the stability of certain states under evolution in the presence of an environment (Zurek, 1981, 1982; Unruh and Zurek, 1989). These definitions are problematic for a number of reasons. One is that (in ordinary quantum mechanics) the density matrix refers only to a single moment of time, yet the proper definition of interference – the effect one is trying to destroy – is in terms of histories. Another is the question of the basis in which the density matrix should be diagonal.<sup>†</sup>

Despite the difficulties, the density matrix approach has been the topic of a number of papers on decoherence in quantum cosmology, perhaps because it is technically much simpler (Fukuyama and Morikawa, 1989; Habib and Laflamme, 1990; Halliwell, 1989; Kiefer, 1987; Laflamme and Louko, 1991; Mellor, 1989; Morikawa, 1989; Padmanabhan, 1989; Paz, 1991; Paz and Sinha, 1992). Models are considered in which the variables of interest are coupled to a wider environment, and a coarse-graining is carried out, in which the states of the environment are traced over. In the case of the whole universe, which strictly has no environment, this is achieved quite simply by postulating a sufficiently complex universe with a suitably large number of subsystems, and ignoring some of the subsystems. The typical result of such models is that the interference terms are suppressed very effectively. Furthermore, one and the same mechanism of coarse-graining also causes decoherence in the histories-based definition of the process. It is therefore plausible that a more sophisticated analysis using the decoherent histories approach will lead to the same conclusions. One way or another, one finds some amount of justification for treating the terms in a superposition separately, and treating the set of paths to which a WKB wave function correspond as a statistical ensemble. These arguments have not gained universal acceptance, however (see for example, Kuchař (1992)).

After the presentation of this contribution, J. Ehlers asked about the observational

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<sup>†</sup> The basis issue is discussed, for example, in Barvinsky and Kamenshchik (1990), Deutsch (1985), and Markov and Mukhanov (1988). Also, see Zurek (1992) for a possible reconciliation of the above two differing views of decoherence.

status of quantum cosmology. Since quantum cosmology aspires to make observable predictions, this is obviously a very important question. Interest in quantum cosmology arose partly as a result of the realization that conventional classical cosmological scenarios relied on certain (possibly tacit) assumptions about initial conditions. As is well known, the inflationary universe scenario alleviates the hot big bang model of extreme dependence on initial conditions; but it does not release it from *all* dependence. The amount of inflation, the details of the density fluctuations generated, and indeed, the very occurrence of inflation are initial conditions-dependent. One of the main successes of quantum cosmology (modulo the objections described in this paper) has been to demonstrate that the desired initial conditions for inflation are implied by certain simple boundary condition proposals for the wave function of the universe. This might therefore be regarded as an observational prediction, although it is admittedly a very indirect one.

More direct tests will clearly be very difficult. The universe has gone through many stages of evolution, each of which is modeled separately. In observing the universe today, it is difficult to distinguish between effects largely due to initial conditions and those largely due to dynamical evolution or to the modeling of a particular stage. What is needed is an effect produced very early in the history of the universe, but that is largely insensitive to subsequent evolution. Grishchuk (1987) has argued that gravitational waves might be the sought-after probe of the very early universe. Boundary condition proposals for the wave function of the universe typically make definite predictions about the initial state of the graviton field (*e.g.*, Halliwell and Hawking, 1985). Memory of this initial state could well be preserved throughout the subsequent evolution of the universe, because gravitational waves interact so weakly. Parts of their spectra observable today might therefore contain signatures of the initial state, leading to the exciting possibility of distinguishing observationally between different boundary condition proposals. These ideas are of course rather speculative, and gravitational wave astronomy is still in a very primitive state. Still, quantum cosmology suffers from an acute lack of connections observational cosmology, and any potentially observable effect deserves further study.

L. Smolin commented that the scheme described here is very semiclassical in nature, and suggested that it is perhaps not much more than a classical statistical theory. It is certainly true that it is very semiclassical in nature, that its predictive output has the form of classical statistical theory, and perhaps causes one to wonder how much of it is really quantum-mechanical in nature. However, there *are* some genuinely quantum-mechanical aspects to this predictive scheme. Perhaps the principle one is the prediction of regions in which classical laws are not valid (the regions of superspace in which the wave function is exponential). Determination of the existence and location of these regions requires the quantum theory – their existence and location cannot be anticipated by inspection of the classical theory alone. Furthermore, the existence of these regions underscores the necessity of discussing the issue of initial or boundary conditions from within the context of the quantum theory. For in classical theories of initial conditions (including classical statistical ones), one might be attempting to impose classical initial conditions in a region in which classical laws are quite simply not valid.

Finally, a critical appraisal of the quantum cosmology program (which inspired some of the remarks made here) may be found in Isham (1991).

## 6. Conclusions

In this talk I have described the heuristic set of rules that have been used so far to make crude but plausible predictions in quantum cosmology. These rules are, however, rather heuristic and semiclassical in nature. The interpretation of the wave function seems to proceed on a case by case basis, and no satisfactory scheme for a completely general wave function is available. I am therefore forced to conclude that quantum cosmology does not yet possess an entirely satisfactory scheme for the extraction of predictions from the wave function.

At the present time, the decoherent histories approach appears to offer the most promising hope of improving the situation. A reasonable expectation is that the heuristic interpretational techniques described here will emerge from this more sophisticated approach. Detailed demonstration of this assertion is very much a matter of current research.

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