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A method for model-independent measurement of the CKM angle β via time-dependent analysis of the $B^0 \to D\pi^+\pi^-$, $D \to K_s^0\pi^+\pi^-$ decays

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Abstract: A new method for model-independent measurement of the CKM angle β is proposed, that employs time-dependent analysis of flavour-tagged $B^0 \to D\pi^+\pi^-$ decays with D meson decays into $\mathcal{CP}$-specific and $K_s^0\pi^+\pi^-$ final states. This method can be used to measure the angle β with future data from the BelleII and LHCb experiments with the precision level of one degree.

Keywords: CP violation, Heavy Quark Physics

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Contents

1 Introduction

The B-factory experiments at SLAC [1] and KEK [2] have made impressive progress in studies of the $\mathcal{CP}$ symmetry breaking in B meson decays. The LHCb [3] experiment has been contributing significantly to this field since recently. The $\mathcal{CP}$-violating phenomena observed so far are in agreement with the KM mechanism of the $\mathcal{CP}$ symmetry breaking proposed by Cabibbo, Kobayashi and Maskawa [4, 5]. Nevertheless, theoretical estimates [6] claim that the KM mechanism cannot provide the value of $\mathcal{CP}$ violation large enough to generate the observed baryon asymmetry of the Universe [7]. Thus, searches for other mechanisms of $\mathcal{CP}$ violation and tests of the KM mechanism should be continued.

Comparison of the angle β values of the Unitarity Triangle (UT) [8] measured in different processes is a valuable test of the KM mechanism. The value of $\sin 2\beta$ obtained using the $b \to c\bar{c}s$ transitions [9–13] is currently the most precisely measured parameter related to the UT angles [14]:

$$\sin 2\beta^{(b\to c\bar{c}s)} = 0.691 \pm 0.017. \tag{1.1}$$

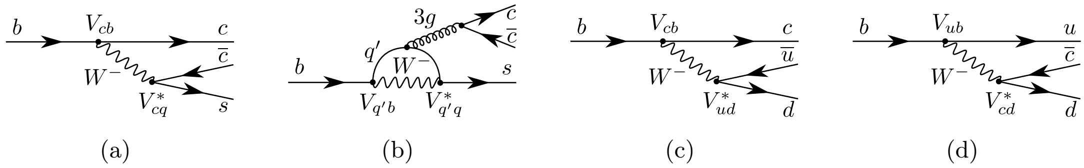

Figure 1. The tree (a) and loop (b) contributions to the $b \to c\bar{c}s$ transition, $b \to c\bar{u}d$ transition (c) and suppressed $b \to u\bar{c}d$ transition (d).

The value of $\sin 2\beta$ measured in the $b \to c\bar{u}d$ transitions [15] is consistent with the $b \to c\bar{c}s$ result though it is statistically limited:

$$\sin 2\beta^{(b\to c\bar{u}d)} = 0.66 \pm 0.10 \pm 0.06. \tag{1.2}$$

Within the Standard Model, the angle β measurements in $b \to c\bar{c}s$ and $b \to c\bar{u}d$ transitions should give the same result up to the hadronic corrections that are expected to be small. However, due to the difference of the $b \to c\bar{c}s$ and $b \to c\bar{u}d$ structure (see figure 1), the New Physics phenomena may manifest themselves differently in these transitions [16]. The doubly Cabibbo-suppressed loop contributions to the $b \to c\bar{c}s$ transitions, limiting the interpretation of measurements, can be controlled using the SU(3) flavor symmetry, as it is shown by De Bruyn and Fleischer in ref. [17]. Bias of the observable 2β value can be controlled at the level of $0.3°$ assuming 20% accuracy in U-symmetry approximation.

The obtained value of $\sin 2\beta$ leaves the ambiguity $\beta \to \pi/2-\beta$, which can be resolved by measuring $\cos 2\beta$. Several approaches to measure $\cos 2\beta$ in the $b \to c\bar{u}d$ transitions using the time-dependent Dalitz plot analysis were discussed: (1) the analysis of $B^0 \to Dh^0$, $D \to K_S^0\pi^+\pi^-$ decays was proposed in ref. [18], (2) the analysis of $B^0 \to D_{\mathcal{CP}}\pi^+\pi^-$ decays was mentioned in ref. [19] and considered in detail in ref. [20], (3) the analysis of $B^0 \to D\pi^+\pi^-$, $D \to K_S^0\pi^+\pi^-$ decays was mentioned in ref. [20]. Only the $B^0 \to Dh^0$, $D \to K_S^0\pi^+\pi^-$ decays analysis was implemented in practice providing the first [21] as well as the most precise at the moment measurements of $\cos 2\beta$ [22, 23].[1] These results indicate positiveness of the $\cos 2\beta$ as expected within the KM mechanism.

Measurements of $\cos 2\beta$ in $B^0 \to Dh^0$, $D \to K_S^0\pi^+\pi^-$ decays require knowledge of the phase difference $\Delta\delta_D$ between the amplitudes of $\overline{D}^0 \to K_S^0\pi^+\pi^-$ and $D^0 \to K_S^0\pi^+\pi^-$ decays that varies over the phase space and cannot be measured directly. The common workaround is to build a phenomenological decay amplitude model and obtain the D meson decay amplitude phase from the model. A model uncertainty is inherent in this approach.

The LHCb and Belle II [25] experiments are expected to collect samples of B meson decays much larger than those available today. Precision of model-dependent measurements of the angle β with that statistics will probably be limited by the model uncertainty. Indeed, currently the model uncertainty is assessed mostly from the statistical error of

[1] Results of the $\cos 2\beta$ measurement in $B^0 \to Dh^0$, $D \to K_S^0\pi^+\pi^-$ decays via joint analysis of the Belle and BaBar experiments data are being prepared for publication at the moment. It is expected to be the most precise measurement of $\cos 2\beta$ before the Belle II data is available. See the talk by M. Roehrken at the 52nd Rencontres de Moriond EW 2017 conference.

model parameters, assuming that the obtained value exceeds the uncertainty related to justification of the model approach. There is no reason to rely on this assumption in a percent-precision-level measurement.

The idea of binned Dalitz plot analysis proposed in ref. [26] was to overcome the limitations of model-dependent consideration of multibody decays. The initial idea is related to measuring the UT angle γ in $B^\pm \to DK^\pm$, $D \to K_S^0\pi^+\pi^-$ decays. It was developed further and extended to several other applications in refs. [27–34]. A measurement of $\cos 2\beta$ in ref. [23] has been performed in a model-independent way using these ideas.

In this work, the model-independent approach is considered in a context of the angle β measurement in time-dependent analysis of $B^0 \to D\pi^+\pi^-$ decays with D meson decaying into $\mathcal{CP}$-specific and $K_S^0\pi^+\pi^-$ states. It is shown the angle β and necessary hadronic parameters of the $B^0 \to \overline{D}{}^0\pi^+\pi^-$ decay can be obtained in a single measurement. Formalism of the time-dependent analysis of the $B^0 \to D\pi^+\pi^-$ decays is described in section 2. The method for model-independent measurement of the angle β with the $B^0 \to D\pi^+\pi^-$ decays is developed in section 3. The statistical precision with future data of the Belle II and LHCb experiments is evaluated in section 4. The measurement bias due to the neglect of $b \to c\bar{u}d$ transition and charm mixing is considered in appendices B), (C, and D.

2 Time-dependent analysis of $B^0 \to D\pi^+\pi^-$ decays

Phenomenology of time-dependent $\mathcal{CP}$ violation measurements at an asymmetric-energy e^+e^- B-factory is described elsewhere [35]. The decay probability density for a flavour-tagged B meson is expressed by

$$p(\Delta t) \propto e^{-\frac{|\Delta t|}{\tau_B}} \left[1 + q_B \left(\mathcal{D}_f \cos\left(\Delta m_B \Delta t\right) - \mathcal{F}_f \sin\left(\Delta m_B \Delta t\right)\right)\right], \tag{2.1}$$

where $\Delta t \in (-\infty, \infty)$ is the proper decay time of a tagged B meson counted from the moment of the tagging B meson decay,[2] $q_B = 1$ ($q_B = -1$) corresponds to B^0 ($\overline{B}{}^0$) flavour at $\Delta t = 0$, Δm_B is the mass difference between the B meson mass eigenstates, τ_B is the B^0 lifetime, and

$$\mathcal{D}_f = \frac{1 - |\lambda_f|^2}{1 + |\lambda_f|^2}, \quad \mathcal{F}_f = \frac{2\,\mathrm{Im}\,\lambda_f}{1 + |\lambda_f|^2}, \tag{2.2}$$

where f denotes the B meson final state and

$$\lambda_f = \frac{q}{p}\frac{\overline{\mathcal{A}}_f}{\mathcal{A}_f}, \tag{2.3}$$

where q and p are the parameters of B meson mixing and $\mathcal{A}_f$ ($\overline{\mathcal{A}}_f$) is the $B^0 \to f$ ($\overline{B}{}^0 \to f$) decay amplitude. Hereafter, absence of direct $\mathcal{CP}$ symmetry breaking in B and D meson decays as well as absence of $\mathcal{CP}$ symmetry breaking in B meson mixing are assumed[3] which

[2] Corresponding expressions for the time-dependent analysis at LHCb are obtained by the formal substitution of $\Delta t \to t$, where $t \in [0, \infty)$.

[3] The case of direct $\mathcal{CP}$ violation in B meson decay due to the $b \to u\bar{c}d$ quark transition is considered in appendix B. The effect of charm mixing is considered in appendix C.

implies

$$\frac{q}{p} = e^{-2i\beta}, \quad \mathcal{A}_f \equiv \overline{\mathcal{A}}_{\overline{f}}, \quad \overline{\mathcal{A}}_f \equiv \mathcal{A}_{\overline{f}}, \tag{2.4}$$

where $\overline{f}$ denotes the state obtained by $\mathcal{CP}$ conjugation of state f.

The amplitude of $B^0 \to \overline{D}^0\pi^+\pi^-$, $\overline{D}^0 \to f_D$ can be expressed as

$$\mathcal{A}_{\overline{D}^0\pi^+\pi^-} \propto \mathcal{A}_B\left(\mu_+^2, \mu_-^2\right)\overline{\mathcal{A}}_D, \tag{2.5}$$

where $\overline{\mathcal{A}}_D$ is the $\overline{D}^0$ meson decay amplitude and $\mathcal{A}_B$ depends on the Dalitz plot variables $\mu_\pm^2 \equiv m^2\left(D\pi^\pm\right)$.

The amplitude of the $\mathcal{CP}$-conjugated process, $\overline{B}^0 \to D^0\pi^+\pi^-$, $D^0 \to \overline{f}_D$, is

$$\overline{\mathcal{A}}_{D^0\pi^+\pi^-} \propto \overline{\mathcal{A}}_B\left(\mu_+^2, \mu_-^2\right)\mathcal{A}_D \equiv \mathcal{A}_B\left(\mu_-^2, \mu_+^2\right)\mathcal{A}_D. \tag{2.6}$$

The parameters $\mathcal{D}_f$ and $\mathcal{F}_f$ from eq. (2.1) take the form

$$\mathcal{D}_{D^0\pi^+\pi^-} = \frac{p_B\left(\mu_+^2, \mu_-^2\right)\overline{p}_D - p_B\left(\mu_-^2, \mu_+^2\right)p_D}{p_B\left(\mu_+^2, \mu_-^2\right)\overline{p}_D + p_B\left(\mu_-^2, \mu_+^2\right)p_D}, \tag{2.7a}$$

$$\mathcal{F}_{D^0\pi^+\pi^-} = \frac{2\sqrt{p_B\left(\mu_+^2, \mu_-^2\right)\overline{p}_D p_B\left(\mu_-^2, \mu_+^2\right)p_D}}{p_B\left(\mu_+^2, \mu_-^2\right)\overline{p}_D + p_B\left(\mu_-^2, \mu_+^2\right)p_D} \times \sin\left(\Delta\delta_f - 2\beta\right), \tag{2.7b}$$

where $p_B = |\mathcal{A}_B|^2$, $p_D = |\mathcal{A}_D|^2$, $\overline{p}_D = \left|\overline{\mathcal{A}}_D\right|^2$, $\Delta\delta_f = \Delta\delta_B - \Delta\delta_D$ and

$$\Delta\delta_B\left(\mu_+^2, \mu_-^2\right) = \arg\left(\frac{\mathcal{A}_B\left(\mu_-^2, \mu_+^2\right)}{\mathcal{A}_B\left(\mu_+^2, \mu_-^2\right)}\right), \; \Delta\delta_D = \arg\left(\frac{\overline{\mathcal{A}}_D}{\mathcal{A}_D}\right). \tag{2.8}$$

If the D meson is reconstructed in a flavour-specific final state, then $\mathcal{D}_{\text{flv}} = 1$ and $\mathcal{F}_{\text{flv}} = 0$.[4] A $\mathcal{CP}$-specific D meson final state with $\mathcal{CP}$ parity ξ_D results in

$$\mathcal{D}_{\mathcal{CP}} = \frac{p_B\left(\mu_+^2, \mu_-^2\right) - p_B\left(\mu_-^2, \mu_+^2\right)}{p_B\left(\mu_+^2, \mu_-^2\right) + p_B\left(\mu_-^2, \mu_+^2\right)}, \tag{2.9a}$$

$$\mathcal{F}_{\mathcal{CP}} = \frac{2\sqrt{p_B\left(\mu_+^2, \mu_-^2\right)p_B\left(\mu_-^2, \mu_+^2\right)}}{p_B\left(\mu_+^2, \mu_-^2\right) + p_B\left(\mu_-^2, \mu_+^2\right)} \times \xi_D \sin\left(\Delta\delta_B - 2\beta\right). \tag{2.9b}$$

The final state $K_S^0\pi^+\pi^-$ introduces the second Dalitz plot resulting in dependence of the D meson decay probability density and the phase difference between the $\overline{D}^0$ and D^0 decay amplitudes on the Dalitz plot variables $m_\pm^2 = m^2\left(K_S^0\pi^\pm\right)$:

$$\overline{p}_D\left(m_+^2, m_-^2\right) \equiv p_D\left(m_-^2, m_+^2\right), \quad \Delta\delta_D\left(m_+^2, m_-^2\right). \tag{2.10}$$

In this case, the B meson decay probability density from eq. (2.1) depends on time and four Dalitz plot variables.

[4] Hadronic decays like $D^0 \to K^-\pi^+$ are used in practice instead of flavour-specific decays. The relations $\mathcal{D} = 1$ and $\mathcal{F} = 0$ do not hold in this case because of suppressed decays $\overline{D}^0 \to K^-\pi^+$. The suppressed decays can be taken into account in a high-statistics measurement.

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In principle, any multibody self-conjugated final state, such as $K_S^0 K^+ K^-$, $\pi^+\pi^-\pi^0$ or $K^+K^-\pi^+\pi^-$ can be considered, but the $K_S^0\pi^+\pi^-$ state is the most experimentally clean and has rich resonance structure leading to significant variation of the phase difference $\Delta\delta_D$ over the Dalitz plot and good sensitivity to the $\mathcal{CP}$ violation parameters. Similar formalism can be developed for other multibody hadronic D meson final states, such as $K^-\pi^+\pi^0$. The D meson decay probability densities p_D and $\overline{p}_D$ would be independent in that case.

3 Binned Dalitz plot analysis

The decay probability densities derived in the previous section can be expressed in terms of the parameters of the binned Dalitz plot. We follow the notation introduced in ref. [29], where the $D^0 \to K_S^0\pi^+\pi^-$ Dalitz plot is divided into $2\mathcal{N}$ bins (we use $\mathcal{N} = 8$). The partitioning is done so that the bin index i ranges from $-\mathcal{N}$ to $\mathcal{N}$ excluding zero and the sign inversion $i \to -i$ corresponds to the Dalitz plot reflection $m_+^2 \leftrightarrow m_-^2$. The parameters K_i, $\overline{K}_i$, C_i and S_i are defined for the i^{th} bin:

$$K_i \equiv \frac{\int\limits_{\mathcal{D}_i} p_D \,\mathrm{d}m_+^2 \mathrm{d}m_-^2}{\sum\limits_i \int\limits_{\mathcal{D}_i} p_D \,\mathrm{d}m_+^2 \mathrm{d}m_-^2}, \quad \overline{K}_i \equiv \frac{\int\limits_{\mathcal{D}_i} \overline{p}_D \,\mathrm{d}m_+^2 \mathrm{d}m_-^2}{\sum\limits_i \int\limits_{\mathcal{D}_i} \overline{p}_D \,\mathrm{d}m_+^2 \mathrm{d}m_-^2}, \quad C_i \equiv \mathrm{Re}\, e_i, \quad S_i \equiv \mathrm{Im}\, e_i, \tag{3.1}$$

where integration is performed over the i^{th} bin and

$$e_i \equiv \frac{\int\limits_{\mathcal{D}_i} \mathcal{A}_D^*\left(m_+^2, m_-^2\right) \mathcal{A}_D\left(m_-^2, m_+^2\right) \,\mathrm{d}m_+^2 \mathrm{d}m_-^2}{\sqrt{\int\limits_{\mathcal{D}_i} p_D\left(m_+^2, m_-^2\right) \,\mathrm{d}m_+^2 \mathrm{d}m_-^2}\sqrt{\int\limits_{\mathcal{D}_i} p_D\left(m_-^2, m_+^2\right) \,\mathrm{d}m_+^2 \mathrm{d}m_-^2}}. \tag{3.2}$$

The relation (2.10) and symmetry of the Dalitz plot partitioning lead to the relations $C_i \equiv C_{-i}$, $S_i \equiv -S_{-i}$, and $\overline{K}_i \equiv K_{-i}$.

In a similar way, we divide the $B^0 \to D\pi^+\pi^-$ decay Dalitz plot into $2\mathcal{M} = 2 \times 8$ bins and define the parameters k_j, c_j and s_j for that Dalitz plot, where the bin index j ranges from $-\mathcal{M}$ to $\mathcal{M}$ excluding zero. A time-dependent $B^0 \to D\pi^+\pi^-$ decay probability density

$$N_j(\Delta t) \propto e^{-\frac{|\Delta t|}{\tau_B}} \left[1 + q_B \mathcal{D}_j \cos\left(\Delta m_B \Delta t\right) - q_B \mathcal{F}_j \sin\left(\Delta m_B \Delta t\right)\right], \tag{3.3}$$

is defined for the j^{th} bin. In the case of double Dalitz decay $B^0 \to D\pi^+\pi^-$, $D \to K_S^0\pi^+\pi^-$, the decay probability density is defined for each combination of B^0 Dalitz plot bin j and D^0 Dalitz plot bin i:

$$N_{ij}(\Delta t) \propto e^{-\frac{|\Delta t|}{\tau_B}} \left[1 + q_B \mathcal{D}_{ij} \cos\left(\Delta m_B \Delta t\right) - q_B \mathcal{F}_{ij} \sin\left(\Delta m_B \Delta t\right)\right]. \tag{3.4}$$

The following substitutions are used to express the coefficients $\mathcal{D}$ and $\mathcal{F}$ in the form suitable for the binned analysis:

$$p_B\left(\mu_+^2, \mu_-^2\right) \to k_j, \qquad p_B\left(\mu_-^2, \mu_+^2\right) \to k_{-j}, \tag{3.5a}$$

$$p_D\left(m_+^2, m_-^2\right) \to K_i, \qquad p_D\left(m_-^2, m_+^2\right) \to K_{-i}, \tag{3.5b}$$

$$\sin\Delta\delta_D \to S_i, \qquad \cos\Delta\delta_D \to C_i, \tag{3.5c}$$

$$\sin\Delta\delta_B \to s_j, \qquad \cos\Delta\delta_B \to c_j. \tag{3.5d}$$

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The expression eq. (2.9) for the $\mathcal{CP}$-specific D meson decays transforms into

$$\mathcal{D}_j^{\mathcal{CP}} = \frac{k_j - k_{-j}}{k_j + k_{-j}}, \tag{3.6a}$$

$$\mathcal{F}_j^{\mathcal{CP}} = 2\xi_D \frac{\sqrt{k_j k_{-j}}}{k_j + k_{-j}} \left(s_j \cos 2\beta - c_j \sin 2\beta\right). \tag{3.6b}$$

The double Dalitz plot case with the $D^0 \to K_S^0\pi^+\pi^-$ decay results in

$$\mathcal{D}_{ij} = \frac{K_i k_j - K_{-i}k_{-j}}{K_i k_j + K_{-i}k_{-j}}, \tag{3.7a}$$

$$\mathcal{F}_{ij} = 2\frac{\sqrt{K_i K_{-i} k_j k_{-j}}}{K_i k_j + K_{-i}k_{-j}} \times \left[(C_i s_j - S_i c_j)\cos 2\beta - (C_i c_j + S_i s_j)\sin 2\beta\right]. \tag{3.7b}$$

We consider the parameters K_i, C_i and S_i to be known because they can be measured in decays of coherent $D^0\overline{D}^0$ pairs [36]. The $2\mathcal{M}$ parameters k_j, $\mathcal{M}$ parameters c_j, $\mathcal{M}$ parameters s_j and the angle β constitute $4\mathcal{M}+1$ unknown parameters.

The parameters k_j can be measured precisely in the time-integrated analysis of $\overline{B}^0 \to D^0\pi^+\pi^-$ decays with D^0 meson decaying into hadronic state $K^-\pi^+$. The expected fraction of events in the j^{th} Dalitz plot bin is

$$N_j \approx k_j - r_D^2 \frac{1-z}{1+z}\left(k_j - k_{-j}\right), \tag{3.8}$$

where

$$z \equiv \frac{1}{1+(\Delta m_B \tau_B)^2} \approx 0.6, \quad r_D^2 \equiv \frac{Br\left(D^0 \to K^+\pi^-\right)}{Br\left(D^0 \to K^-\pi^+\right)} \approx 3.5\times 10^{-3}. \tag{3.9}$$

The second term in eq. (3.8) is negligible even at the Belle II precision level.

The $B^0 \to D\pi^+\pi^-$ with $\mathcal{CP}$-specific D meson decays provide $2\mathcal{M}$ independent constraints (eq. (3.6)) and do not allow one to resolve the system. It should be noted that the above statement does not depend on $\mathcal{CP}$ parity of the D meson final state, particularly, final states with the same $\mathcal{CP}$ parities can be used and inclusion of a final state of the opposite $\mathcal{CP}$ parity would not increase the number of constraints.

The $B^0 \to D\pi^+\pi^-$ with $D \to K_S^0\pi^+\pi^-$ decay provide $2\mathcal{MN}$ additional constraints (eq. (3.7)) allowing to measure the parameters c_j and s_j together with the angle β in the joint analysis of the $B^0 \to D\pi^+\pi^-$ with $\mathcal{CP}$-specific and $D \to K_S^0\pi^+\pi^-$ decays for any $\mathcal{N}$ and $\mathcal{M}$.[5] The $B^0 \to D\pi^+\pi^-$, $D \to K_S^0\pi^+\pi^-$ decays alone provide enough constraints to measure the parameters c_j and s_j, and the angle β for $2\mathcal{M}(\mathcal{N}-1) \geq 1$.

[5] An important feature of the described setup is that the values of $\sin 2\beta$ and $\cos 2\beta$ cannot be considered as independent parameters. Indeed, the transformation

$$c_j \to \eta c_j, \quad s_j \to \eta s_j, \quad \sin 2\beta \to \frac{\sin 2\beta}{\eta}, \quad \cos 2\beta \to \frac{\cos 2\beta}{\eta} \tag{3.10}$$

with an arbitrary scale $\eta \neq 0$ does not change the expressions for decay probability densities and the scale η can not be determined.

3.1 Symmetrized $B^0 \to \overline{D}^0\pi^+\pi^-$ Dalitz plot binning

The number of parameters related to the $B^0 \to \overline{D}^0\pi^+\pi^-$ binned Dalitz plot can be reduced by a factor of 2 considering the j^{th} and $-j^{\text{th}}$ bins as a single bin. For the symmetrized in this way $B^0 \to \overline{D}^0\pi^+\pi^-$ decay Dalitz plot binning, the expressions eq. (3.6) and eq. (3.7) should be modified as follows:

$$\mathcal{D}^{\mathcal{CP}}_{|j|} = 0, \quad \mathcal{F}^{\mathcal{CP}}_{|j|} = d_j \sin 2\beta \tag{3.11}$$

and

$$\mathcal{D}_{i|j|} = \frac{K_i - K_{-i}}{K_i + K_{-i}}, \quad \mathcal{F}_{i|j|} = -2d_j \frac{\sqrt{K_i K_{-i}}}{K_i + K_{-i}} \left(S_i \cos 2\beta + C_i \sin 2\beta\right), \tag{3.12}$$

where the *dilution factor*

$$d_j = 2\frac{\sqrt{k_j k_{-j}}}{k_j + k_{-j}} c_j \tag{3.13}$$

is the single parameter for the j^{th} symmetric bin.

The analysis procedure is slightly different in the case of symmetrized binning of the $B^0 \to \overline{D}^0\pi^+\pi^-$ Dalitz plot. Flavour-specific D meson decays are not needed. A combined time-dependent fit of the $B^0 \to \overline{D}^0\pi^+\pi^-$ with D meson decays into $\mathcal{CP}$-specific and $K^0_S\pi^+\pi^-$ final states should be performed in order to measure the dilution factors d_j together with the angle β. The $K^0_S\pi^+\pi^-$ final state is still necessary since the $\mathcal{CP}$-specific final states provide $\mathcal{M}$ constraints while there are $\mathcal{M}+1$ unknown parameters.[6]

The symmetrization of binning leads to a certain loss of information. Particularly, the $B^0 \to D\pi^+\pi^-$ with $\mathcal{CP}$-specific D meson decays are not sensitive to the $\cos 2\beta$ (eq. (3.11)) in this case. A quantitative evaluation of the sensitivity decline related to the symmetrized $B^0 \to \overline{D}^0\pi^+\pi^-$ Dalitz plot partitioning is described in the next section.

4 Feasibility study

Sensitivity of the described method is assessed with a series of toy Monte Carlo (MC) experiments. The equal-phase $D^0 \to K^0_S\pi^+\pi^-$ decay Dalitz plot binning deduced from the decay model published in ref. [37] is used. The values of parameters K_i, C_i and S_i for that binning are taken from measurement in ref. [36].

A model-independent measurement of the angle β in $B^0 \to Dh^0$ decays is considered as a reference procedure. The coefficients $\mathcal{D}$ and $\mathcal{F}$ from eqs. (3.3) and (3.4) for the case of $B^0 \to Dh^0$ decays can be obtained using the formal substitutions

$$k_j \to \frac{1}{2\mathcal{M}}, \quad s_j \to 0, \quad c_j \to \xi^{\mathcal{CP}}_{h^0} (-1)^L, \tag{4.1}$$

where $\xi^{\mathcal{CP}}_{h^0}$ is the $\mathcal{CP}$ eigenvalue of h^0 meson and L is the angular moment of Dh^0 system.

The MC events are generated with probability density functions (PDFs) of the form

$$p(\Delta t) = (1 - f_{\text{bkg}}) \int_{-\infty}^{\infty} p^{\text{w}}_{\text{true}}(\Delta t') \mathcal{R}(\Delta t - \Delta t') \,\mathrm{d}\Delta t' + f_{\text{bkg}} \mathcal{R}(\Delta t) \,\mathrm{d}\Delta t', \tag{4.2}$$

[6] The continuous ambiguity defined in eq. (3.10) occurs for the case of symmetrized Dalitz plot binning too. In this case, instead of the phase parameters c_j and s_j, the dilution factors d_j should be scaled.

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Parameter	Belle & Belle II	LHCb
Time resolution σ_t (ps)	1.25	0.06
Tagging power $\varepsilon_{\rm tag}$ (%)	30	8
Background fraction (%)	30	5

Table 1. Experimental conditions adopted in numerical experiments.

Mode	Belle	Belle II	LHCb Run I	LHCb Run II	LHCb Upgr.
$B^0 \to D_{\mathcal{CP}}\pi^+\pi^-$	$1.0 \cdot 10^3$	$50 \cdot 10^3$	$2.0 \cdot 10^3$	$8 \cdot 10^3$	$140 \cdot 10^3$
$B^0 \to [K^0_S\pi^+\pi^-]_D\pi^+\pi^-$	$1.3 \cdot 10^3$	$65 \cdot 10^3$	$1.2 \cdot 10^3$	$5 \cdot 10^3$	$84 \cdot 10^3$
$B^0 \to D_{\mathcal{CP}}h^0$	$0.8 \cdot 10^3$	$40 \cdot 10^3$	—	—	—
$B^0 \to [K^0_S\pi^+\pi^-]_D h^0$	$1.0 \cdot 10^3$	$50 \cdot 10^3$	—	—	—

Table 2. Estimates of the signal yields for the $B^0 \to \overline{D}^0\{h^0,\ \pi^+\pi^-\}$, $\overline{D}^0 \to \{f_{\mathcal{CP}},\ K^0_S\pi^+\pi^-\}$ (and $\mathcal{C}$-conjugated) decays at the Belle, Belle II and LHCb experiments.

where the resolution function $\mathcal{R}$, employed also as the background PDF, is a Gaussian with zero mean and $f_{\rm bkg}$ is the background fraction. The function $p^{\rm w}_{\rm true}$ is a PDF from section 3 with the wrong B meson flavor tagging probability w factor

$$p^{\rm w}_{\rm true}(\Delta t) \propto e^{-\frac{|\Delta t|}{\tau_B}} \left[1 + q_B\,(1-2w)\,(\mathcal{D}\cos(\Delta m_B \Delta t) - \mathcal{F}\sin(\Delta m_B \Delta t))\right]. \tag{4.3}$$

The tagging power $\varepsilon_{\rm tag} \equiv (1-2w)^2$ characterizes effective reduction of data sample due to non-ideality of a B meson flavour tagging procedure. The tagging power $\varepsilon_{\rm tag} = 0.3$, typical for B factory experiments, is employed for the Belle and Belle II and $\varepsilon_{\rm tag} = 0.08$ is employed for the LHCb taking into account the recent progress in the flavour-tagging algorithms at hadronic machines [38].[7] The values of PDF parameters for the Belle (II) and LHCb are chosen based on results from refs. [15, 23, 39] and are shown in table 1.

Table 2 shows estimates of the signal yields for the Belle, Belle II and LHCb experiments. The estimates for Belle are obtained using the results from refs. [15, 23, 40]. The estimates for Belle II are obtained by extrapolating the Belle yields assuming the same experimental conditions and 50 times larger integrated luminosity. The estimate signal yields corresponding to the data collected by LHCb in 2010 – 2012 are based on the results from refs. [39, 41, 42]. This period of data taking is referred to as Run I. The estimates for the LHCb signal yields corresponding to the end of current data taking period (Run II) and to the period of data taking after the planned upgrade (Upgr.) [43] are roughly estimated to be, respectively, 4 and 70 times larger than the Run I values, assuming the corresponding luminosity integrals equal 8 fb^{-1} and 50 fb^{-1}.

The signal yields for $B^0 \to D\pi^+\pi^-$ with flavour-specific D meson decays are relatively large for both Belle and LHCb. Thus, the uncertainties related to the parameters k_j are neglected.

[7] The flavor tagging power $\varepsilon_{\rm tag}$ at LHCb strongly depends on the decay channel and the actual value for the $B^0 \to D\pi^+\pi^-$ decay may differ from the adopted in this work value 0.08.

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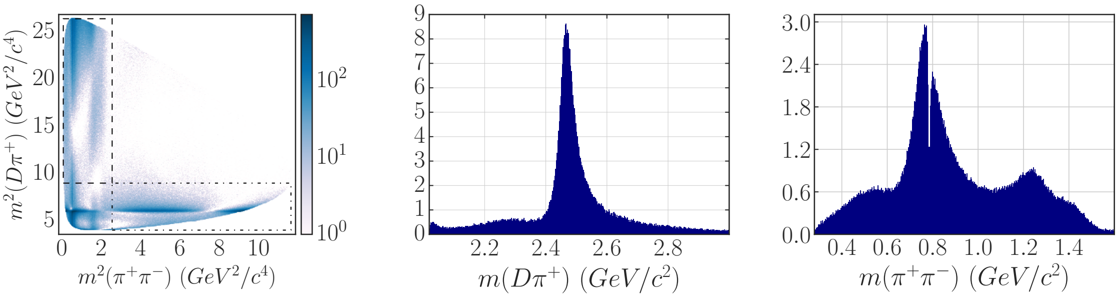
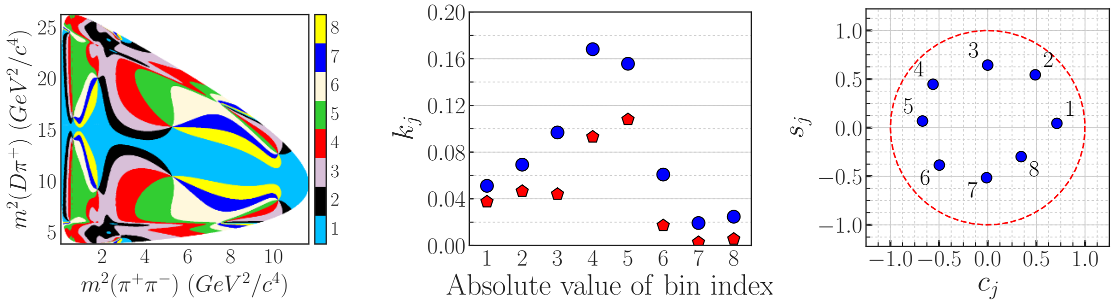


(a) (b) (c)

Figure 2. Dalitz plot distribution of the $B^0 \to \overline{D}^0\pi^+\pi^-$ decay (a), $m\left(D^0\pi^+\right)$ distribution below 3 GeV/c^2 for $m\left(\pi^+\pi^-\right) > 1.6$ GeV/c^2 (b), and $m\left(\pi^+\pi^-\right)$ distribution below 1.6 GeV/c^2 for $m\left(D^0\pi^+\right) > 3$ GeV/c^2 (c). The distributions are obtained with the $B^0 \to \overline{D}^0\pi^+\pi^-$ decay amplitude model described in appendix A. The dashed and dot-dashed regions on the Dalitz plot correspond to the distributions on the subplots (b) and (c), respectively.



(a) (b) (c)

Figure 3. Equal-phase Dalitz plot binning (a), values of the parameters k_j for $j > 0$ (blue circles) and $j < 0$ (red pentagons) (b), and values of the parameters c_j and s_j (blue circles) (c) obtained with the $B^0 \to \overline{D}^0\pi^+\pi^-$ decay amplitude model described in appendix A.

4.1 Parameters of the $B^0 \to \overline{D}^0\pi^+\pi^-$ decay binned Dalitz plot

Two models of the $B^0 \to \overline{D}^0\pi^+\pi^-$ decay amplitude are available in refs. [39, 40]. A simplified version of the model from ref. [40] is used in this study (see appendix A). The Dalitz distribution and distributions of the $D^0\pi^+$ and $\pi^+\pi^-$ invariant masses obtained with this model are shown in figure 2.

The equal-phase binning of the $B^0 \to \overline{D}^0\pi^+\pi^-$ decay Dalitz plot into 16 bins is performed using this model. The bin regions obtained and corresponding values of the parameters k_j, c_j and s_j are shown in figure 3.

4.2 Numerical experiments

Three approaches to measure the angle β are considered. Each approach implies the joint analysis of Δt distributions for the $B^0 \to D\pi^+\pi^-$ with D meson decays into $\mathcal{CP}$-specific and $K_S^0\pi^+\pi^-$ final states. These approaches are:

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Measuring scheme	Belle	Belle II	LHCb Run I	Run II	Upgr.
$B^0 \to D\pi^+\pi^-$	$\approx 10°$	$1.5°$	$\approx 15°$	$6°$	$1.5°$
Only $D \to K_S^0\pi^+\pi^-$	$\approx 15°$	$2°$	$\approx 20°$	$7°$	$2°$
$B^0 \to D\pi^+\pi^-$ (symm)	$\approx 15°$	$2°$	$\approx 20°$	$10°$	$2°$
Only $D \to K_S^0\pi^+\pi^-$	$\approx 20°$	$2.5°$	$\approx 25°$	$13°$	$3°$
$B^0 \to D^{(*)}h^0$	$5°$	$0.7°$	—	—	—
Only $D \to K_S^0\pi^+\pi^-$	$7°$	$1.1°$	—	—	—
Only $D \to f_{\mathcal{CP}}$	$6°$	$0.8°$	—	—	—

Table 3. Estimates of the angle β measurement statistical precision for the three schemes with the input value $\beta = 22°$.

1. The fit based on eqs. (3.6) and (3.7) with 17 free parameters: eight (c_j, s_j) pairs and the angle β.
2. The fit using symmetrized $B^0 \to D\pi^+\pi^-$ decay Dalitz plot binning with nine free parameters: eight dilution factors d_j, defined in eq. (3.13), and the angle β.
3. Model-independent measurement of the angle β in the $B^0 \to Dh^0$ decays as a reference. The angle β is the only free parameter in this case.

The statistical precision of the angle β measurement for the initial value $\beta = 22°$, obtained with each of the three approaches, is shown in table 3. The analysis of $B^0 \to D\pi^+\pi^-$ decays provides precision about 1.5 times worse than the analysis of $B^0 \to Dh^0$ decays. The prospects for the analysis of $B^0 \to Dh^0$ decays at LHCb are not clear since there are neutral particles in the final state. The Belle II and upgraded LHCb have comparable potential to measure the angle β in $B^0 \to D\pi^+\pi^-$ decays. A combination of the results from $B^0 \to Dh^0$ and $B^0 \to D\pi^+\pi^-$ analyses would yield the β precision in $b \to c\bar{u}d$ transitions below one degree.[8]

Figure 4 illustrates prospects for the Belle II experiment: a fit result for the dilution factors d_j (figure 4(a)) and for the parameters c_j and s_j (figure 4(b)) obtained with MC simulation for the input value $\beta = 22°$.

The results presented are obtained with a simple method of the Dalitz plot binning (the equal-phase binning). It is shown in refs. [28, 36] that the binning can be optimized to improve the statistical sensitivity by a factor of about 1.2.

5 Conclusions

A novel model-independent approach to measure the CKM angle β with time-dependent analysis of the $B^0 \to D\pi^+\pi^-$ decays dominated by the tree quark transition is proposed.

[8] At the moment, the uncertainty related to the C_i and S_i parameters measurement is about 1.1°, as it is stated in ref. [23]. The precision level below one degree can be achieved only if a more precise measurement of the parameters C_i and S_i appears. Such a measurement can be provided by the BESIII collaboration and by a future Super c-τ factory experiment.

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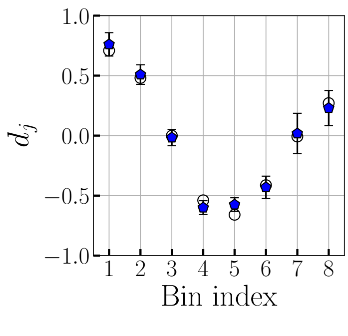

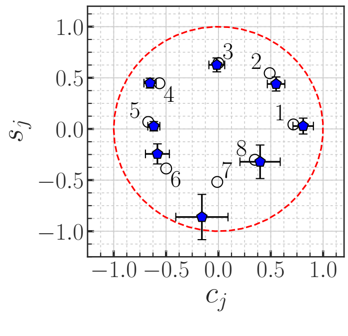

Figure 4. Results of MC simulation: dilution factors d_j (a) and phase parameters c_j and s_j (b). Empty circles show the input values, blue pentagons with error bars show the fit results obtained for the expected Belle II statistics and experimental conditions.

It is shown that the angle β and the parameters of binned $B^0 \to \overline{D}{}^0\pi^+\pi^-$ decay Dalitz plot can be obtained from the single measurement. Statistical precision of the method is comparable to that of the model-independent angle β measurement in $B^0 \to Dh^0$ decays.

The fact that only charged particles compose the final states of $B^0 \to D\pi^+\pi^-$, $D \to f_{\mathcal{CP}}$ and $D \to K_S^0\pi^+\pi^-$ decay chains for such $f_{\mathcal{CP}}$ as K^+K^-, $\pi^+\pi^-$, and ϕK_S^0 provides good experimental perspectives for LHCb.

The angle β can be measured with the one-degree precision level at the Belle II and LHCb experiments in $b \to c\bar{u}d$ transitions in a model-independent way, namely without the need to model neither the $D^0 \to K_S^0\pi^+\pi^-$ nor the $B^0 \to \overline{D}{}^0\pi^+\pi^-$ decay amplitudes. The combined time-dependent analysis of $B^0 \to Dh^0$ and $B^0 \to D\pi^+\pi^-$ decays with D meson decaying into a $f_{\mathcal{CP}}$ ($f_{\mathcal{CP}} = K^+K^-$, $K_S^0\pi^0$ etc.) and $K_S^0\pi^+\pi^-$ states should be performed in order to achieve such precision.

The measurement bias inherent in the proposed method due to the neglect of the suppressed transition $b \to u\bar{c}d$ and charm mixing is of order of 0.2° (see appendix D) and can be considered as a probably non-dominant systematic uncertainty.

Acknowledgments

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A The $B^0 \to \overline{D}{}^0\pi^+\pi^-$ decay amplitude model

A simple isobar model of the $B^0 \to \overline{D}{}^0\pi^+\pi^-$ decay amplitude, inspired by the result from ref. [40], is used in numerical experiments. The resonances constituting the model are listed in table 4. Each resonance is described by a relativistic Breit-Wigner function [44]. Energy-dependent resonance width and Blatt-Weisskopf barrier factors [45, 46] are used.

Name	M (GeV/c^2)	Γ (MeV)	J	$\mathcal{F}$ (%)	φ (deg)
$D_2^*(2460)$	2.4657	49.6	2	29.9	0
D_v^*	2.01	10^{-4}	1	7.6	-145.0
$D_0^*(2400)$	2.308	276.11	0	6.5	-165.0
$\rho^0(770)$	0.7756	144	1	36.3	103.7
$\omega(782)$	0.7826	8.49	1	0.5	-88.4
$\rho(1450)$	1.465	310	1	0.4	-76.3
$f_2(1270)$	1.275	185	2	7.5	-97.6
$f_0(500)$	0.513	335	0	10.0	80.8
$f_0(1370)$	1.434	173	0	1.8	-139.2

Table 4. List of resonances included in the $B^0 \to \overline{D}{}^0\pi^+\pi^-$ decay amplitude model. The resonance fit fraction is denoted by $\mathcal{F}$ and the resonance amplitude phase is denoted by φ.

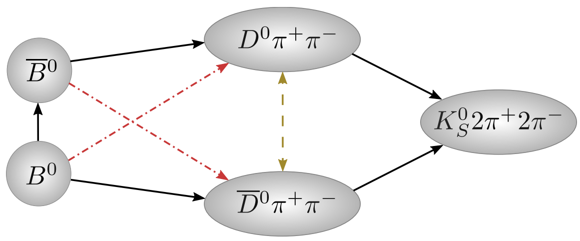

Figure 5. Transitions scheme of the $B^0 \to D\pi^+\pi^-$, $D \to K_S^0\pi^+\pi^-$ process. Black solid arrows denote dominant transitions, red dash-dotted arrows denote suppressed $B \to D$ and $\overline{B} \to \overline{D}$ transitions, brown dashed arrows denote D^0-$\overline{D}{}^0$ oscillations.

The model describes two main channels $B^0 \to D^0\rho^0(770)$ and $B^0 \to D_2^*(2460)\pi$. The scalar $D_0^*(2400)$ and virtual vector D_v^* resonances describe the remaining $D^0\pi$ structure. Following ref. [40] we call D_v^* virtual since the veto $|m(D\pi) - m(D^*)| > 3$ MeV/c^2 is imposed and only the tail of D_v^* resonance contributes the amplitude.

The remaining $\pi^+\pi^-$ structure is described by the wide scalar $f_0(500)$, narrow vector ω interfering destructively with $\rho^0(770)$ and resonances $\rho(1450)$, $f_2(1270)$ and $f_0(1370)$ responsible for the $\pi^+\pi^-$ mass spectrum above 1 GeV/c^2.

B Formalism accounting for the $b \to u\bar{c}d$ transition

A precise measurement of the angle β in the $b \to c\bar{u}d$ transitions requires understanding the bias due to the neglect of the suppressed decay $B^0 \to D^0\pi^+\pi^-$ and charm mixing. Both processes produce additional interfering amplitudes for the $B^0 \to \overline{D}{}^0\pi^+\pi^-$, $\overline{D}{}^0 \to K_S^0\pi^+\pi^-$ decay shown on the scheme at figure 5.

This appendix extends the formalism presented in sections 2 and 3 and accounts for the $B^0 \to D^0\pi^+\pi^-$ decay. Corrections due to the charm mixing are considered in appendix C.

Quantitative estimates of the bias due to the neglect of these processes are described in appendix D.

The $B^0 \to D\pi^+\pi^-$, $D \to K_S^0\pi^+\pi^-$ decay amplitude including the $b \to u\bar{c}d$ transition (without charm mixing) reads

$$\begin{aligned}
&\mathcal{A}_{B\to f}\left(\Delta t,\mu_+^2,\mu_-^2,m_+^2,m_-^2\right)\\
&\quad= \mathcal{A}_{B\to\overline{D}}\left(\mu_+^2,\mu_-^2\right)\overline{\mathcal{A}}_D\left(m_+^2,m_-^2\right)\cos\left(\frac{\Delta m_B\Delta t}{2}\right)\\
&\qquad+\mathcal{A}_{B\to D}\left(\mu_+^2,\mu_-^2\right)\mathcal{A}_D\left(m_+^2,m_-^2\right)e^{i\gamma}\cos\left(\frac{\Delta m_B\Delta t}{2}\right)\\
&\qquad+i\mathcal{A}_{\overline{B}\to D}\left(\mu_+^2,\mu_-^2\right)\mathcal{A}_D\left(m_+^2,m_-^2\right)e^{-2i\beta}\sin\left(\frac{\Delta m_B\Delta t}{2}\right)\\
&\qquad+i\mathcal{A}_{\overline{B}\to\overline{D}}\left(\mu_+^2,\mu_-^2\right)\overline{\mathcal{A}}_D\left(m_+^2,m_-^2\right)e^{-i(2\beta+\gamma)}\sin\left(\frac{\Delta m_B\Delta t}{2}\right),
\end{aligned}\tag{B.1}$$

where γ is the CKM phase. The corresponding $\overline{B}^0 \to D\pi^+\pi^-$, $D \to K_S^0\pi^+\pi^-$ decay amplitude is

$$\begin{aligned}
&\mathcal{A}_{\overline{B}\to f}\left(\Delta t,\mu_+^2,\mu_-^2,m_+^2,m_-^2\right)\\
&\quad= \mathcal{A}_{\overline{B}\to D}\left(\mu_+^2,\mu_-^2\right)\mathcal{A}_D\left(m_+^2,m_-^2\right)\cos\left(\frac{\Delta m_B\Delta t}{2}\right)\\
&\qquad+\mathcal{A}_{\overline{B}\to\overline{D}}\left(\mu_+^2,\mu_-^2\right)\overline{\mathcal{A}}_D\left(m_+^2,m_-^2\right)e^{-i\gamma}\cos\left(\frac{\Delta m_B\Delta t}{2}\right)\\
&\qquad+i\mathcal{A}_{B\to\overline{D}}\left(\mu_+^2,\mu_-^2\right)\overline{\mathcal{A}}_D\left(m_+^2,m_-^2\right)e^{2i\beta}\sin\left(\frac{\Delta m_B\Delta t}{2}\right)\\
&\qquad+i\mathcal{A}_{B\to D}\left(\mu_+^2,\mu_-^2\right)\mathcal{A}_D\left(m_+^2,m_-^2\right)e^{i(2\beta+\gamma)}\sin\left(\frac{\Delta m_B\Delta t}{2}\right),
\end{aligned}\tag{B.2}$$

The decay probability densities corresponding to the amplitudes (B.1) and (B.2) are

$$p\left(\Delta t,\mu_+^2,\mu_-^2,m_+^2,m_-^2\right) = \mathcal{U} + q_B\left[\mathcal{D}\cos\left(\Delta m_B\Delta t\right)+\mathcal{F}\sin\left(\Delta m_B\Delta t\right)\right],\tag{B.3}$$

where

$$\begin{aligned}
\mathcal{U} &= \frac{1}{2}p_D\left(m_+^2,m_-^2\right)\left[p_{B\to D}\left(\mu_+^2,\mu_-^2\right)+p_{B\to\overline{D}}\left(\mu_-^2,\mu_+^2\right)\right]\\
&\quad+\frac{1}{2}p_D\left(m_-^2,m_+^2\right)\left[p_{B\to\overline{D}}\left(\mu_+^2,\mu_-^2\right)+p_{B\to D}\left(\mu_-^2,\mu_+^2\right)\right]\\
&\quad+\sqrt{p_{B\to\overline{D}}\left(\mu_+^2,\mu_-^2\right)p_{B\to D}\left(\mu_+^2,\mu_-^2\right)p_D\left(m_+^2,m_-^2\right)p_D\left(m_-^2,m_+^2\right)}\\
&\quad\times\cos\left(\Delta\delta_B+\gamma-\Delta\delta_D\right)\\
&\quad+\sqrt{p_{B\to\overline{D}}\left(\mu_-^2,\mu_+^2\right)p_{B\to D}\left(\mu_-^2,\mu_+^2\right)p_D\left(m_+^2,m_-^2\right)p_D\left(m_-^2,m_+^2\right)}\\
&\quad\times\cos\left(\Delta\delta_B-\gamma+\Delta\delta_D+\psi_{\overline{B}}-\psi_B\right),
\end{aligned}\tag{B.4}$$

$$\begin{aligned}
\mathcal{D} &= \frac{1}{2}p_D\left(m_+^2,m_-^2\right)\left[p_{B\to D}\left(\mu_+^2,\mu_-^2\right)-p_{B\to\overline{D}}\left(\mu_-^2,\mu_+^2\right)\right]\\
&\quad+\frac{1}{2}p_D\left(m_-^2,m_+^2\right)\left[p_{B\to\overline{D}}\left(\mu_+^2,\mu_-^2\right)-p_{B\to D}\left(\mu_-^2,\mu_+^2\right)\right]
\end{aligned}$$

$$
\begin{aligned}
&+\sqrt{p_{B\to\overline{D}}\left(\mu_+^2,\mu_-^2\right)p_{B\to D}\left(\mu_+^2,\mu_-^2\right)p_D\left(m_+^2,m_-^2\right)p_D\left(m_-^2,m_+^2\right)}\\
&\times\cos\left(\Delta\delta_B+\gamma-\Delta\delta_D\right)\\
&-\sqrt{p_{B\to\overline{D}}\left(\mu_-^2,\mu_+^2\right)p_{B\to D}\left(\mu_-^2,\mu_+^2\right)p_D\left(m_+^2,m_-^2\right)p_D\left(m_-^2,m_+^2\right)}\\
&\times\cos\left(\Delta\delta_B-\gamma+\Delta\delta_D+\psi_{\overline{B}}-\psi_B\right),
\end{aligned}
\tag{B.5}
$$

$$
\begin{aligned}
\mathcal{F}=&-\sqrt{p_{B\to\overline{D}}\left(\mu_+^2,\mu_-^2\right)p_{B\to\overline{D}}\left(\mu_-^2,\mu_+^2\right)p_D\left(m_+^2,m_-^2\right)p_D\left(m_-^2,m_+^2\right)}\\
&\times\sin\left(\psi_B-2\beta-\Delta\delta_D\right)\\
&-\sqrt{p_{B\to D}\left(\mu_+^2,\mu_-^2\right)p_{B\to D}\left(\mu_-^2,\mu_+^2\right)p_D\left(m_+^2,m_-^2\right)p_D\left(m_-^2,m_+^2\right)}\\
&\times\sin\left(\psi_{\overline{B}}-2\beta+\Delta\delta_D-2\gamma\right)\\
&-p_D\left(m_-^2,m_+^2\right)\sqrt{p_{B\to\overline{D}}\left(\mu_+^2,\mu_-^2\right)p_{B\to D}\left(\mu_-^2,\mu_+^2\right)}\\
&\times\sin\left(\psi_{\overline{B}}-2\beta+\Delta\delta_B-\gamma\right)\\
&-p_D\left(m_+^2,m_-^2\right)\sqrt{p_{B\to\overline{D}}\left(\mu_-^2,\mu_+^2\right)p_{B\to D}\left(\mu_+^2,\mu_-^2\right)}\\
&\times\sin\left(\psi_B-2\beta-\Delta\delta_B-\gamma\right).
\end{aligned}
\tag{B.6}
$$

The following notation is used (compare with eq. (2.8)):

$$
\psi_B=\arg\left(\frac{\mathcal{A}_{B\to\overline{D}}\left(\mu_-^2,\mu_+^2\right)}{\mathcal{A}_{B\to\overline{D}}\left(\mu_+^2,\mu_-^2\right)}\right),
\tag{B.7a}
$$

$$
\psi_{\overline{B}}=\arg\left(\frac{\mathcal{A}_{B\to D}\left(\mu_-^2,\mu_+^2\right)}{\mathcal{A}_{B\to D}\left(\mu_+^2,\mu_-^2\right)}\right),
\tag{B.7b}
$$

$$
\Delta\delta_B=\arg\left(\frac{\mathcal{A}_{B\to D}\left(\mu_+^2,\mu_-^2\right)}{\mathcal{A}_{B\to\overline{D}}\left(\mu_+^2,\mu_-^2\right)}\right),
\tag{B.7c}
$$

$$
\Delta\delta_D=\arg\left(\frac{\mathcal{A}_D\left(m_-^2,m_+^2\right)}{\mathcal{A}_D\left(m_+^2,m_-^2\right)}\right).
\tag{B.7d}
$$

Integration of eqs. (B.4), (B.5), (B.6) over the i^{th} bin of D Dalitz plot and the j^{th} bin of B Dalitz plot leads to

$$
\begin{aligned}
\mathcal{U}_{ij}=&\frac{1}{2}\left(K_{-i}k_j+K_ik_{-j}\right)\\
&+r_B\sqrt{K_iK_{-i}k_j\overline{k}_j}\left[\left(\widetilde{c}_jC_i+\widetilde{s}_jS_i\right)\cos\gamma-\left(\widetilde{s}_jC_i-\widetilde{c}_jS_i\right)\sin\gamma\right]\\
&+r_B\sqrt{K_iK_{-i}k_{-j}\overline{k}_{-j}}\left[\left(\widetilde{c}_{-j}C_i-\widetilde{s}_{-j}S_i\right)\cos\gamma+\left(\widetilde{s}_{-j}C_i+\widetilde{c}_{-j}S_i\right)\sin\gamma\right]\\
&+\frac{1}{2}r_B^2\left(K_i\overline{k}_j+K_{-i}\overline{k}_{-j}\right),
\end{aligned}
\tag{B.8}
$$

$$
\begin{aligned}
\mathcal{D}_{ij} &= \frac{1}{2}\left(K_{-i}k_j - K_i k_{-j}\right) \\
&+ r_B\sqrt{K_iK_{-i}k_j\overline{k}_j}\,[(\widetilde{c}_jC_i + \widetilde{s}_jS_i)\cos\gamma - (\widetilde{s}_jC_i - \widetilde{c}_jS_i)\sin\gamma] \\
&- r_B\sqrt{K_iK_{-i}k_{-j}\overline{k}_{-j}}\,[(\widetilde{c}_{-j}C_i - \widetilde{s}_{-j}S_i)\cos\gamma + (\widetilde{s}_{-j}C_i + \widetilde{c}_{-j}S_i)\sin\gamma] \\
&+ \frac{1}{2}r_B^2\left(K_i\overline{k}_j - K_{-i}\overline{k}_{-j}\right),
\end{aligned} \tag{B.9}
$$

$$
\begin{aligned}
\mathcal{F}_{ij} &= \sqrt{K_iK_{-i}k_jk_{-j}}\,[(c_jC_i + s_jS_i)\sin 2\beta - (s_jC_i - c_jS_i)\cos 2\beta] \\
&+ r_BK_{-i}\sqrt{k_j\overline{k}_{-j}}\left[c'_j\sin(2\beta+\gamma) - s'_j\cos(2\beta+\gamma)\right] \\
&+ r_BK_i\sqrt{k_{-j}\overline{k}_j}\left[c'_{-j}\sin(2\beta+\gamma) + s'_{-j}\cos(2\beta+\gamma)\right] \\
&+ r_B^2\sqrt{K_iK_{-i}\overline{k}_j\overline{k}_{-j}} \\
&\times [(\overline{c}_jC_i - \overline{s}_jS_i)\sin(2\beta+2\gamma) - (\overline{s}_jC_i + \overline{c}_jS_i)\cos(2\beta+2\gamma)],
\end{aligned} \tag{B.10}
$$

where

$$
k_j = \int_{\mathcal{B}_j} p_{B\to\overline{D}}\left(\mu_+^2,\mu_-^2\right)\mathrm{d}\mu_+^2\mu_-^2, \quad \overline{k}_j = \int_{\mathcal{B}_j} p_{B\to D}\left(\mu_+^2,\mu_-^2\right)\mathrm{d}\mu_+^2\mu_-^2, \tag{B.11}
$$

$$
c_j + is_j = \frac{1}{\sqrt{k_jk_{-j}}}\int_{\mathcal{B}_j}\mathcal{A}^*_{B\to\overline{D}}\left(\mu_+^2,\mu_-^2\right)\mathcal{A}_{B\to\overline{D}}\left(\mu_-^2,\mu_+^2\right)\mathrm{d}\mu_+^2\mu_-^2, \tag{B.12a}
$$

$$
\overline{c}_j + i\overline{s}_j = \frac{1}{\sqrt{\overline{k}_j\overline{k}_{-j}}}\int_{\mathcal{B}_j}\mathcal{A}^*_{B\to D}\left(\mu_+^2,\mu_-^2\right)\mathcal{A}_{B\to D}\left(\mu_-^2,\mu_+^2\right)\mathrm{d}\mu_+^2\mu_-^2, \tag{B.12b}
$$

$$
\widetilde{c}_j + i\widetilde{s}_j = \frac{1}{\sqrt{k_j\overline{k}_j}}\int_{\mathcal{B}_j}\mathcal{A}^*_{B\to\overline{D}}\left(\mu_+^2,\mu_-^2\right)\mathcal{A}_{B\to D}\left(\mu_+^2,\mu_-^2\right)\mathrm{d}\mu_+^2\mu_-^2, \tag{B.12c}
$$

$$
c'_j + is'_j = \frac{1}{\sqrt{k_j\overline{k}_{-j}}}\int_{\mathcal{B}_j}\mathcal{A}^*_{B\to\overline{D}}\left(\mu_+^2,\mu_-^2\right)\mathcal{A}_{B\to D}\left(\mu_-^2,\mu_+^2\right)\mathrm{d}\mu_+^2\mu_-^2 \tag{B.12d}
$$

Definitions in eq. (B.12) imply

$$
c_{-j} + is_{-j} \equiv c_j - is_j, \quad \overline{c}_{-j} + i\overline{s}_{-j} \equiv \overline{c}_j - i\overline{s}_j. \tag{B.13}
$$

The expressions for $\mathcal{CP}$ specific D meson decays and $B^0 \to Dh^0$ decay can be obtained as a particular cases of eqs. (B.4), (B.5) and (B.6):

- $B^0 \to D_{\mathcal{CP}}\pi^+\pi^-$

$$
\begin{aligned}
\mathcal{U}_j &= \frac{1}{2}\left(k_j + k_{-j}\right) + \frac{1}{2}r_B^2\left(\overline{k}_j + \overline{k}_{-j}\right) \\
&+ r_B\xi_D\left[\sqrt{k_j\overline{k}_j}\left(\widetilde{c}_j\cos\gamma - \widetilde{s}_j\sin\gamma\right) + \sqrt{k_{-j}\overline{k}_{-j}}\left(\widetilde{c}_{-j}\cos\gamma + \widetilde{s}_{-j}\sin\gamma\right)\right],
\end{aligned} \tag{B.14}
$$

$$
\begin{aligned}
\mathcal{D}_j &= \frac{1}{2}\left(k_j - k_{-j}\right) + \frac{1}{2}r_B^2\left(\overline{k}_j - \overline{k}_{-j}\right) \\
&+ r_B\xi_D\left[\sqrt{k_j\overline{k}_j}\left(\widetilde{c}_j\cos\gamma - \widetilde{s}_j\sin\gamma\right) - \sqrt{k_{-j}\overline{k}_{-j}}\left(\widetilde{c}_{-j}\cos\gamma + \widetilde{s}_{-j}\sin\gamma\right)\right],
\end{aligned} \tag{B.15}
$$

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$$
\begin{aligned}
\mathcal{F}_j = \xi_D \sqrt{k_j k_{-j}}\, (c_j \sin 2\beta - s_j \cos 2\beta) \\
+ r_B \sqrt{k_j \overline{k}_{-j}}\, \left[c'_j \sin(2\beta+\gamma) - s'_j \cos(2\beta+\gamma)\right] \\
+ r_B \sqrt{k_{-j} \overline{k}_j}\, \left[c'_{-j} \sin(2\beta+\gamma) + s'_{-j} \cos(2\beta+\gamma)\right] \\
+ r_B^2 \xi_D \sqrt{\overline{k}_j \overline{k}_{-j}}\, \left[\overline{c}_j \sin(2\beta+2\gamma) - \overline{s}_j \cos(2\beta+2\gamma)\right].
\end{aligned} \tag{B.16}
$$

- $B^0 \to Dh^0$, $D \to K_S^0 \pi^+ \pi^-$

$$
\mathcal{U}_i = \frac{1+r_B^2}{2}\,(K_{-i}+K_i) + 2r_B \cos\Delta\delta_B \sqrt{K_i K_{-i}}\,(C_i \cos\gamma + S_i \sin\gamma)\,, \tag{B.17a}
$$

$$
\mathcal{D}_i = \frac{1-r_B^2}{2}\,(K_{-i}-K_i) + 2r_B \sin\Delta\delta_B \sqrt{K_i K_{-i}}\,(S_i \cos\gamma - C_i \sin\gamma)\,, \tag{B.17b}
$$

$$
\begin{aligned}
\xi_{h^0} \mathcal{F}_i = \sqrt{K_i K_{-i}}\,(C_i \sin 2\beta + S_i \cos 2\beta) \\
+ r_B \left[K_i \sin(2\beta+\gamma+\Delta\delta_B) + K_{-i} \sin(2\beta+\gamma-\Delta\delta_B)\right] \\
+ r_B^2 \left[C_i \sin(2\beta+2\gamma) - S_i \cos(2\beta+2\gamma)\right],
\end{aligned} \tag{B.18}
$$

where the coefficient $\xi_{h^0} \equiv (-1)^L\, \xi_{\mathcal{CP}}^{h^0}$ accounts for the $\mathcal{CP}$ parity of h^0 meson and the angular moment L of the Dh^0 system.

- $B^0 \to D_{\mathcal{CP}} h^0$

$$
\mathcal{U} = 1 + r_B^2 + 2\xi_D r_B \cos\Delta\delta_B \cos\gamma, \tag{B.19a}
$$

$$
\mathcal{D} = -2\xi_D r_B \sin\Delta\delta_B \sin\gamma, \tag{B.19b}
$$

$$
\xi_{h^0} \mathcal{F} = \xi_D \sin 2\beta + 2r_B \cos\Delta\delta_B \sin(2\beta+\gamma) + r_B^2 \xi_D \sin(2\beta+2\gamma). \tag{B.19c}
$$

As discussed in ref. [47], the expressions (B.17), (B.18), and (B.19) describe also the time-dependent analysis of tagged $B^0 \to DK_S^0$ decays. The CKM angles β and γ, phase $\Delta\delta_B$ and parameter r_B can be simultaneously measured in a such analysis. In contrast with the $B^0 \to Dh^0$ decay, the r_B value corresponding to the $B^0 \to DK_S^0$ decay can be as large as 0.2, improving sensitivity to the $\mathcal{CP}$ violation parameters. However, the expected number of reconstructed at a B factory $B^0 \to DK_S^0$ decays is about the order of magnitude less then the number of reconstructed $B^0 \to Dh^0$ decays. Numerical experiments have been performed to estimate the statistical precision one may expect with the Belle II data. The results obtained with $r_B = 0.2$ are

$$
\sigma^{(B^0\to DK_S^0)}(\beta) \approx 5^\circ, \quad \sigma^{(B^0\to DK_S^0)}(\gamma) \approx 8^\circ. \tag{B.20}
$$

These values are only marginally dependent on $\Delta\delta_B$. The angle γ precision doesn't improve much if the β value is considered as known.

C Formalism accounting for the charm mixing

We assume conservation of $\mathcal{CP}$ symmetry in charm mixing. The $B^0 \to D\pi^+\pi^-$, $D \to K_S^0\pi^+\pi^-$ decay amplitude taking into account charm mixing can be written as follows:

$$\begin{aligned}
&\mathcal{A}_{B\to f}\left(\Delta t, t_D, \mu_+^2, \mu_-^2, m_+^2, m_-^2\right) \\
&\quad= \left[\mathcal{A}_D\left(m_-^2, m_+^2\right)\varkappa\left(t_D\right) + \mathcal{A}_D\left(m_+^2, m_-^2\right) i\sigma\left(t_D\right)\right] \\
&\qquad\qquad\qquad \times \mathcal{A}_{B\to\overline{D}}\left(\mu_+^2, \mu_-^2\right)\cos\left(\frac{\Delta m_B \Delta t}{2}\right) \\
&\quad\quad + \left[\mathcal{A}_D\left(m_+^2, m_-^2\right)\varkappa\left(t_D\right) + \mathcal{A}_D\left(m_-^2, m_+^2\right) i\sigma\left(t_D\right)\right] \\
&\qquad\qquad\qquad \times \mathcal{A}_{B\to\overline{D}}\left(\mu_-^2, \mu_+^2\right) i\sin\left(\frac{\Delta m_B \Delta t}{2}\right) e^{-2i\beta},
\end{aligned} \tag{C.1}$$

where t_D is the D meson proper decay time and functions

$$\varkappa\left(t_D\right) = e^{-\frac{t_D}{2\tau_D}}\cos\left[\frac{t_D\left(x - iy\right)}{2\tau_D}\right] \quad \text{and} \quad \sigma\left(t_D\right) = e^{-\frac{t_D}{2\tau_D}}\sin\left[\frac{t_D\left(x - iy\right)}{2\tau_D}\right] \tag{C.2}$$

describe the D meson time evolution. Here x and y are the charm mixing parameters and τ_D is the D^0 lifetime. The corresponding amplitude of the $\overline{B}^0 \to D\pi^+\pi^-$, $D \to K_S^0\pi^+\pi^-$ decay is

$$\begin{aligned}
&\mathcal{A}_{\overline{B}\to f}\left(\Delta t, t_D, \mu_+^2, \mu_-^2, m_+^2, m_-^2\right) \\
&\quad= \left[\mathcal{A}_D\left(m_+^2, m_-^2\right)\varkappa\left(t_D\right) + \mathcal{A}_D\left(m_-^2, m_+^2\right) i\sigma\left(t_D\right)\right] \\
&\qquad\qquad\qquad \times \mathcal{A}_{B\to\overline{D}}\left(\mu_-^2, \mu_+^2\right)\cos\left(\frac{\Delta m_B \Delta t}{2}\right) \\
&\quad\quad + \left[\mathcal{A}_D\left(m_-^2, m_+^2\right)\varkappa\left(t_D\right) + \mathcal{A}_D\left(m_+^2, m_-^2\right) i\sigma\left(t_D\right)\right] \\
&\qquad\qquad\qquad \times \mathcal{A}_{B\to\overline{D}}\left(\mu_+^2, \mu_-^2\right) i\sin\left(\frac{\Delta m_B \Delta t}{2}\right) e^{2i\beta},
\end{aligned} \tag{C.3}$$

The coefficients $\mathcal{U}$, $\mathcal{D}$, and $\mathcal{F}$, defined in eq. (B.3) corresponding to amplitudes in eqs. (C.1) and (C.3), integrated over the D meson proper decay time t_D, are

$$\begin{aligned}
\mathcal{U} = &\frac{1}{4}\left(\frac{1}{1-y^2} + \frac{1}{1+x^2}\right)\left[p_B\left(\mu_+^2, \mu_-^2\right)p_D\left(m_-^2, m_+^2\right) + p_B\left(\mu_-^2, \mu_+^2\right)p_D\left(m_+^2, m_-^2\right)\right] \\
&+ \frac{1}{4}\left(\frac{1}{1-y^2} - \frac{1}{1+x^2}\right)\left[p_B\left(\mu_+^2, \mu_-^2\right)p_D\left(m_+^2, m_-^2\right) + p_B\left(\mu_-^2, \mu_+^2\right)p_D\left(m_-^2, m_+^2\right)\right] \\
&+ \sqrt{p_D\left(m_+^2, m_-^2\right)p_D\left(m_-^2, m_+^2\right)} \\
&\times\left(\frac{1}{2}\frac{x}{1+x^2}\sin\Delta\delta_D\left[p_B\left(\mu_+^2, \mu_-^2\right) - p_B\left(\mu_-^2, \mu_+^2\right)\right]\right. \\
&\qquad\left. + \frac{1}{2}\frac{y}{1-y^2}\cos\Delta\delta_D\left[p_B\left(\mu_+^2, \mu_-^2\right) + p_B\left(\mu_-^2, \mu_+^2\right)\right]\right),
\end{aligned} \tag{C.4}$$

$$\begin{aligned}
\mathcal{D} = &\frac{1}{4}\left(\frac{1}{1-y^2} + \frac{1}{1+x^2}\right)\left[p_B\left(\mu_+^2, \mu_-^2\right)p_D\left(m_-^2, m_+^2\right) - p_B\left(\mu_-^2, \mu_+^2\right)p_D\left(m_+^2, m_-^2\right)\right] \\
&+ \frac{1}{4}\left(\frac{1}{1-y^2} - \frac{1}{1+x^2}\right)\left[p_B\left(\mu_+^2, \mu_-^2\right)p_D\left(m_+^2, m_-^2\right) - p_B\left(\mu_-^2, \mu_+^2\right)p_D\left(m_-^2, m_+^2\right)\right]
\end{aligned}$$

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$$
\begin{aligned}
&+\sqrt{p_D\left(m_+^2,m_-^2\right)p_D\left(m_-^2,m_+^2\right)}\\
&\times\left(\frac{1}{2}\frac{x}{1+x^2}\sin\Delta\delta_D\left[p_B\left(\mu_+^2,\mu_-^2\right)+p_B\left(\mu_-^2,\mu_+^2\right)\right]\right.\\
&\qquad\left.+\frac{1}{2}\frac{y}{1-y^2}\cos\Delta\delta_D\left[p_B\left(\mu_+^2,\mu_-^2\right)-p_B\left(\mu_-^2,\mu_+^2\right)\right]\right),
\end{aligned}
\tag{C.5}
$$

$$
\begin{aligned}
\mathcal{F}=&\sqrt{p_D\left(m_+^2,m_-^2\right)p_D\left(m_-^2,m_+^2\right)p_B\left(\mu_+^2,\mu_-^2\right)p_B\left(\mu_-^2,\mu_+^2\right)}\\
&\times\left[\frac{1}{2}\left(\frac{1}{1-y^2}+\frac{1}{1+x^2}\right)\sin\left(2\beta-\Delta\delta_D+\Delta\delta_B\right)\right.\\
&+\left.\frac{1}{2}\left(\frac{1}{1-y^2}-\frac{1}{1+x^2}\right)\sin\left(2\beta-\Delta\delta_D-\Delta\delta_B\right)\right]\\
&-\sqrt{p_B\left(\mu_+^2,\mu_-^2\right)p_B\left(\mu_-^2,\mu_+^2\right)}\\
&\times\left(\frac{1}{2}\frac{y}{1-y^2}\sin\left(2\beta-\Delta\delta_B\right)\left[p_D\left(m_+^2,m_-^2\right)+p_D\left(m_-^2,m_+^2\right)\right]\right.\\
&\qquad\left.-\frac{1}{2}\frac{x}{1+x^2}\cos\left(2\beta-\Delta\delta_B\right)\left[p_D\left(m_+^2,m_-^2\right)-p_D\left(m_-^2,m_+^2\right)\right]\right).
\end{aligned}
\tag{C.6}
$$

Integrating eqs. (C.4), (C.5) and (C.6) over i^{th} bin of the D Dalitz plot and j^{th} bin of the B Dalitz plot we obtain the expressions for the binned analysis:

$$
\mathcal{U}_{ij}=\frac{1}{2}k_jK_{-i}+\frac{1}{2}k_{-j}K_i+\frac{1}{2}\sqrt{K_iK_{-i}}\left[yC_i\left(k_j+k_{-j}\right)+xS_i\left(k_j-k_{-j}\right)\right],
\tag{C.7a}
$$

$$
\mathcal{D}_{ij}=\frac{1}{2}k_jK_{-i}-\frac{1}{2}k_{-j}K_i+\frac{1}{2}\sqrt{K_iK_{-i}}\left[yC_i\left(k_j-k_{-j}\right)+xS_i\left(k_j+k_{-j}\right)\right],
\tag{C.7b}
$$

$$
\begin{aligned}
\mathcal{F}_{ij}=&\sqrt{k_jk_{-j}K_iK_{-i}}\left[\left(C_ic_j+S_is_j\right)\sin 2\beta-\left(C_is_j-S_ic_j\right)\cos 2\beta\right]\\
&+\frac{1}{2}\sqrt{k_jk_{-j}}\Big[y\left(s_j\cos 2\beta-c_j\sin 2\beta\right)\left(K_i+K_{-i}\right)\\
&\qquad+x\left(c_j\cos 2\beta+s_j\sin 2\beta\right)\left(K_i-K_{-i}\right)\Big].
\end{aligned}
\tag{C.8}
$$

The expressions for $\mathcal{CP}$ specific D meson decays and $B^0\to Dh^0$ decay can be obtained as a particular cases of eqs. (C.7) and (C.8):

- $B^0\to D_{\mathcal{CP}}\pi^+\pi^-$

$$
\mathcal{U}_j=\frac{1}{2}\left(k_j+k_{-j}\right)\left(1+\xi_Dy\right),
\tag{C.9a}
$$

$$
\mathcal{D}_j=\frac{1}{2}\left(k_j-k_{-j}\right)\left(1+\xi_Dy\right),
\tag{C.9b}
$$

$$
\mathcal{F}_j=\xi_D\sqrt{k_jk_{-j}}\left(c_j\sin 2\beta-s_j\cos 2\beta\right)\left(1-\xi_Dy\right).
\tag{C.9c}
$$

- $B^0\to Dh^0$, $D\to K_S^0\pi^+\pi^-$

$$
\mathcal{U}_i=\frac{1}{2}\left(K_{-i}+K_i\right)+yC_i\sqrt{K_iK_{-i}},
\tag{C.10a}
$$

$$
\mathcal{D}_i=\frac{1}{2}\left(K_{-i}-K_i\right)+xS_i\sqrt{K_iK_{-i}},
\tag{C.10b}
$$

Process	(n_D, n_+, n_-)	$\delta\beta_{b\to u\bar{c}d}$	$\delta\beta_{\text{mix}}$
$B^0 \to D\pi^+\pi^-$	$(2,1,1)$	$0.17^\circ \times \frac{r_B}{0.02}$	$0.05^\circ \times \frac{\sqrt{x^2+y^2}}{0.01}$
	$(1,0,0)$	$0.15^\circ \times \frac{r_B}{0.02}$	$0.04^\circ \times \frac{\sqrt{x^2+y^2}}{0.01}$
	$(1,1,0)$	$0.14^\circ \times \frac{r_B}{0.02}$	$0.04^\circ \times \frac{\sqrt{x^2+y^2}}{0.01}$
	$(1,0,1)$	$0.23^\circ \times \frac{r_B}{0.02}$	$0.05^\circ \times \frac{\sqrt{x^2+y^2}}{0.01}$
$B^0 \to Dh^0$	$(1,0,0)$	$-0.2^\circ \times \cos\Delta\delta_B$	$0.02^\circ \times \frac{\sqrt{x^2+y^2}}{0.01}$
	$(0,1,0)$	$-1.9^\circ \times \cos\Delta\delta_B$	$-0.6^\circ \times \frac{y}{0.01}$
	$(0,0,1)$	$1.9^\circ \times \cos\Delta\delta_B$	$0.6^\circ \times \frac{y}{0.01}$

Table 5. Estimates for the angle β measurement bias due to the neglect of $b \to u\bar{c}d$ transition (3rd column) and charm mixing (4th column). The second column shows the D^0 decays combination used in the fit: n_D, n_+, n_- are relative fractions of $D^0 \to K_S^0\pi^+\pi^-$, $D^0 \to f_{\mathcal{CP}+}$ and $D^0 \to f_{\mathcal{CP}-}$ decays yields, respectively.

$$\xi_{h^0}\mathcal{F}_i = \sqrt{K_i K_{-i}}\,(C_i \sin 2\beta - S_i \cos 2\beta) + \frac{1}{2}\left[x \cos 2\beta\,(K_i - K_{-i}) - y \sin 2\beta\,(K_i + K_{-i})\right]. \tag{C.11}$$

- $B^0 \to D_{\mathcal{CP}}h^0$

$$\mathcal{U} = 1 + \xi_D y, \tag{C.12a}$$
$$\mathcal{D} = 0, \tag{C.12b}$$
$$\xi_{h^0}\mathcal{F} = \xi_D \sin 2\beta\,(1 - \xi_D y)\,. \tag{C.12c}$$

D Estimate of the bias due to neglect of $b \to u\bar{c}d$ transition and charm mixing

The neglect of $b \to u\bar{c}d$ transition and charm mixing leads to a bias of the observed value of the angle β. Numerical experiments have been performed to assess the bias value. Data samples for the numerical experiments are generated using the expressions from appendices B and C. The values of angle β and hadronic parameters c_j and s_j are extracted from the generated samples with the maximum likelihood method. The fit procedure uses equations from section 3 (i.e. neglects the $b \to u\bar{c}d$ transition and charm mixing). The results obtained are summarized in the table 5.

A model of the suppressed $B^0 \to D^0\pi^+\pi^-$ decay is needed to obtain the values of parameters $\overline{k}_j$, $\overline{c}_j$, $\overline{s}_j$, $\widetilde{c}_j$, $\widetilde{s}_j$, c'_j and s'_j defined in eqs. (B.11) and (B.12). We use the factorization assumption[9] to construct an ensemble of the $B^0 \to D^0\pi^+\pi^-$ decay models.

[9]The factorization assumption is not applicable to the $B^0 \to D^0\pi^+\pi^-$ decay, but it gives a qualitative arguments to construct the $B^0 \to D^0\pi^+\pi^-$ decay model as described in text.

The $B^0 \to \overline{D}^0\pi^+\pi^-$ decay model described in appendix A is taken as a basis and the following modifications are applied:

- The $B^0 \to D_2^{*-}(2460)\pi^+$ transition amplitude is reduced by a factor of 10 since it cannot proceed through a tree weak diagram.
- The $B^0 \to R\pi^-$, $R \to \overline{D}^0\pi^+$, $R \in \{D_2^*, D_*^v, D_0^*\}$ amplitudes are increased by factor $f_D/f_\pi \approx 1.6$, where $f_D \approx 207$ MeV and $f_\pi \approx 133$ MeV are the decay constants.
- The amplitudes of $B^0 \to \overline{D}^0 R$, $R \to \pi^+\pi^-$ transitions are taken from the $B^0 \to \overline{D}^0\pi^+\pi^-$ decay model since the production mechanisms of the $\pi^+\pi^-$ resonances in $B^0 \to D^0\pi^+\pi^-$ and $B^0 \to \overline{D}^0\pi^+\pi^-$ decays are similar.

An ensemble of 100 $B^0 \to D^0\pi^+\pi^-$ decay models is constructed with 100 random triples of phases $\varphi(D_2^*), \varphi(D_*^v), \varphi(D_0^*)$ corresponding to the D_2^*, D_*^v and D_0^* amplitudes, respectively. The values quoted in the third column of table 5 for the $B^0 \to \overline{D}^0\pi^+\pi^-$ decay are the maximal biases over the ensemble of models.

The results of numerical experiments and formalism described in appendices B and C lead to the following conclusions:

1. The bias due to neglect of the charm mixing is $3 \div 4$ times smaller than the bias due to neglect of the $b \to u\bar{c}d$ transition.
2. The biases corresponding $\mathcal{CP}$ specific D meson final states with $\xi_D = +1$ and $\xi_D = -1$ have equal absolute values and opposite signs. This feature was previously pointed out in ref. [48]. Eqs. (B.16), (B.19c), (C.9c) and (C.12c) show that the main terms are proportional to the ξ_D while the first order corrections do not depend on ξ_D.
3. Biases for the processes involving $D \to K_S^0\pi^+\pi^-$ decay are of the order of $0.1°$. Relative smallness of this value can be qualitatively explained by the pairwise reduction of bias in bins of the Dalitz plot. This effect generalizes the feature described in the previous item. The same reduction is takes place in the binned analysis of $B^0 \to \overline{D}^0\pi^+\pi^-$ decay.
4. The biases for the $B^0 \to D_{\mathcal{CP}}h^0$ decays are large enough to be observed with the Belle II statistics. However, assuming the statistics ratio 2/1/1 of the $K_S^0\pi^+\pi^-$, $\xi_D = +1$ and $\xi_D = -1$ events, respectively (which is close to reality), the residual bias is about $0.1°$.
5. Most of the D^0 decays to $\mathcal{CP}$ eigenstates collected by LHCb have negative $\mathcal{CP}$ parity ($D^0 \to K^+K^-, \pi^+\pi^-$). This $\mathcal{CP}$ parity imbalance does not lead to a significant bias in the case of analysis of the $B^0 \to \overline{D}^0\pi^+\pi^-$ decays, in contrast with the $B^0 \to Dh^0$ case, as it is shown in the third and fourth rows of the table. 5. The resulting bias due to neglect of the $b \to u\bar{c}d$ amplitude is at level of $0.2°$.

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