Decay modes of the scalar exotic meson $T^{-}_{bs;\bar{u}\bar{d}}$

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We investigate the semileptonic decay of the scalar tetraquark $T^{-}_{bs;\bar{u}\bar{d}}$ to final state $T^{0}_{cs;\bar{u}\bar{d}}l\bar{\nu}_l$, which proceeds due to the weak transition $b\to cl\bar{\nu}_l$. For these purposes, we calculate the spectroscopic parameters of the final-state scalar tetraquark $T^{0}_{cs;\bar{u}\bar{d}}$. In calculations we use the QCD sum rule method by taking into account the quark, gluon, and mixed condensates up to dimension 10. The mass of the $T^{0}_{cs;\bar{u}\bar{d}}$ obtained in the present work (2878 ± 128) MeV indicates that it is unstable against the strong interactions, and can decay to the mesons $D^0\bar{K}^0$ and D^+K^-. Partial widths of these S-wave modes as well as the full width of the tetraquark $T^{0}_{cs;\bar{u}\bar{d}}$ are found by means of the QCD light-cone sum rule method and technical tools of the soft-meson approximation. The partial widths of the main semileptonic processes $T^{-}_{bs;\bar{u}\bar{d}}\to T^{0}_{cs;\bar{u}\bar{d}}l\bar{\nu}_l$, $l=e,\ \mu$, and τ are computed by employing the weak form factors $G_1(q^2)$ and $G_2(q^2)$, which are extracted from the QCD three-point sum rules. We also trace back the weak transformations of the stable tetraquark $T^{-}_{bb;\bar{u}\bar{d}}$ to conventional mesons. The obtained results for the full width $\Gamma_{\rm full}=(3.28\pm 0.60)\times 10^{-10}$ MeV and mean lifetime $\tau=2.01^{+0.44}_{-0.31}$ ps of $T^{-}_{bs;\bar{u}\bar{d}}$, as well as predictions for decay channels of the tetraquark $T^{-}_{bb;\bar{u}\bar{d}}$ can be used in experimental studies of these exotic states.

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I. INTRODUCTION

Investigation of exotic mesons composed of four valence quarks, i.e., tetraquarks is one of the interesting and intriguing problems on agenda of high energy physics. Experimental data collected by various collaborations and achievements in their theoretical explanations made these states an important part of hadron spectroscopy [1–5]. But the nonstandard mesons discovered till now and considered as candidates to exotics are wide resonances which decay strongly to conventional mesons. These circumstances obscure their four-quark bound-state nature and inspire appearance of alternative dynamical models to account for observed effects. Therefore, theoretical and experimental studies of 4-quark states, which are stable against the strong interactions can be decisive for distinguishing dynamical effects and genuine multiquark states from each another.

The problems of stability of 4-quark mesons were already addressed in the original papers [6–8]. The principal conclusion made in these works was that, if a mass ratio m_Q/m_q is large, then the heavy Q and light q quarks may constitute stable $QQ\bar{q}\bar{q}$ compounds. The stabile nature of the axial-vector tetraquark $T^{-}_{bb;\bar{u}\bar{d}}$ (briefly, T^{-}_{bb}) was predicted in Ref. [9], and confirmed by recent investigations [10–12]. The similar conclusions about the strong-interaction stability of the tetraquarks $T^{-}_{bb;\bar{u}\bar{s}}$, and $T^{0}_{bb;\bar{d}\bar{s}}$ were drawn in Ref. [12] as well. The spectroscopic parameters and semileptonic decays of the axial-vector tetraquark $T^{-}_{bb;\bar{u}\bar{d}}$ were analyzed in our work [13]. Our result for the mass of the T^{-}_{bb} state (10035 ± 260) MeV is below the $B^-\bar{B}^{*0}$ and $B^-\bar{B}^0\gamma$ thresholds, respectively, which means that it is strong- and electromagnetic-interaction stable particle and can decay only weakly. We evaluated the full width and mean lifetime of T^{-}_{bb} using its semileptonic decay channel $T^{-}_{bb}\to Z^{0}_{bc}l\bar{\nu}_l$ (for simplicity, $Z^0_{bc}\equiv Z^0_{bc;\bar{u}\bar{d}}$). The predictions $\Gamma=(7.17\pm 1.23)\times 10^{-8}$ MeV and $\tau=9.18^{+1.90}_{-1.34}$ fs obtained in Ref. [13] are useful for further experimental studies of this double-heavy exotic meson.

Because the tetraquark T^{-}_{bb} decays dominantly to the scalar state Z^0_{bc}, in Ref. [13] we calculated also the spectroscopic parameters of Z_{bc}^0. The mass of this state (6660 ± 150) MeV is considerably below 7145 MeV required for strong decays to heavy mesons B^-D^+ and $\bar{B}^0D^0$. The threshold for electromagnetic decays of Z_{bc}^0 exceeds 7600 MeV, and is also higher than its mass. The semileptonic and nonleptonic weak decays of the tetraquark Z_{bc}^0 were explored in Ref. [14]. The dominant weak decay modes of Z_{bc}^0 contain at the final state the scalar tetraquark $T_{bs;\bar{u}\bar{d}}^-$, which has the mass (5380 ± 170) MeV, and is strong- and electromagnetic-interaction stable particle.

The spectroscopic parameters and width of the axial-vector state T_{bc}^0 with the same quark content $bc\bar{u}\bar{d}$ were computed in Ref. [15]. The central value of its mass (7105 ± 155) MeV is lower than corresponding thresholds both for strong and electromagnetic decays. Both the semileptonic and nonleptonic weak decays of T_{bc}^0 create at the final state the scalar tetraquark $T_{cc;\bar{u}\bar{d}}^+$, which is strong-interactions unstable particle and decays to conventional mesons D^+D^0 [16].

The 4-quark compounds $bc\bar{u}\bar{d}$ were subjects of interesting theoretical studies [11,12,17–19]. Thus, an analysis performed in Ref. [11] showed that Z_{bc}^0 lies below the threshold for S-wave decays to conventional heavy mesons, whereas the authors of Ref. [12] predicted the masses of the scalar and axial-vector $bc\bar{u}\bar{d}$ states above the $B^-D^+/\bar{B}^0D^0$ and B^*D thresholds, respectively. Nevertheless, explorations conducted using the Bethe-Salpeter method [17], and recent lattice simulations proved the strong-interaction stability of the axial-vector exotic meson T_{bc}^0 [18]. An independent analysis of Ref. [19] also confirmed the stability of the tetraquarks $bc\bar{u}\bar{d}$; it was demonstrated there, that both the scalar and axial-vector states $bc\bar{u}\bar{d}$ are stable against the strong interactions.

Summing up one sees, that T_{bb}^- transforms due to chains of the decays $T_{bb}^- \to Z_{bc}^0 l\bar{\nu}_l \to T_{bs;\bar{u}\bar{d}}^- l\bar{\nu}_l \bar{l}' \nu_{l'}$ and $T_{bb}^- \to Z_{bc}^0 l\bar{\nu}_l \to T_{bs;\bar{u}\bar{d}}^- P l\bar{\nu}_l$, where P is one of the pseudoscalar mesons π^+ and K^+. At the last stage $T_{bs;\bar{u}\bar{d}}^-$ should also decay through weak processes and create a new tetraquark, which may be unstable or stable against the strong interactions. Therefore, semileptonic decays of $T_{bs;\bar{u}\bar{d}}^-$ to ordinary mesons through the intermediate 4-quark state are important for throughout analysis of the tetraquark T_{bb}^-.

In the present work we consider namely the processes $T_{bs;\bar{u}\bar{d}}^- \to T_{cs;\bar{u}\bar{d}}^0 l\bar{\nu}_l$, with $l = e, \mu$, and τ (in what follows we denote $T_{bs;\bar{u}\bar{d}}^- \Rightarrow T_{bs}^-$ and $T_{cs;\bar{u}\bar{d}}^0 \Rightarrow T_{cs}^0$, respectively), and calculate their partial widths. To this end, we first explore the properties of the scalar 4-quark state T_{cs}^0 and calculate its mass and coupling. Our prediction for the mass of this state $m_T = (2878 \pm 128)$ MeV demonstrates that T_{cs}^0 can decay strongly to the conventional mesons $D^0\bar{K}^0$ and D^+K^-, partial widths of which are computed as well. Using information on parameters of T_{cs}^0, we study the semileptonic decays of the tetraquark T_{bs}^- and find branching ratios of the processes $T_{bs}^- \to D^0\bar{K}^0 l\bar{\nu}_l$ and $T_{bs}^- \to D^+K^- l\bar{\nu}_l$. Results of the present work allow us also to analyze decays of the tetraquark T_{bb}^- and trace back its transformations to ordinary mesons.

This paper is organized in the following manner. In Sec. II we calculate the spectroscopic parameters of the scalar 4-quark state T_{cs}^0. Its strong decays are also analyzed in this section. Section III is devoted to semileptonic decays, where we calculate the weak form factors $G_{1(2)}(q^2)$ and partial widths of the processes $T_{bs}^- \to T_{cs}^0 l\bar{\nu}_l$. In Sec. IV we sum up information on T_{bs}^-, and analyze transformations of T_{bb}^- to conventional mesons.

II. SPECTROSCOPIC PARAMETERS AND STRONG DECAYS OF THE TETRAQUARK T_{cs}^0

It has been emphasized above that transformation of the T_{bs}^- to meson pairs $D^0\bar{K}^0$ and D^+K^- runs through creating and decaying of the intermediate scalar 4-quark state T_{cs}^0. Hence, parameters of this tetraquark are essential for our following analysis. In this section we calculate the mass and coupling of the tetraquark T_{cs}^0 by means of the QCD two-point sum rule method, which is an effective and powerful nonperturbative approach to investigate parameters of hadrons [20,21]. It can be used to determine masses, couplings, and decay widths not only of the conventional hadrons, but also of exotic states [22]. In calculations, we take into account effects of the vacuum condensates up to dimension 10.

Here, we also analyze decays of this exotic state to conventional mesons via strong interactions. For these purposes, we use the parameters of the tetraquark T_{cs}^0 and calculate the strong couplings $g_{TD^0\bar{K}^0}$ and $g_{TD^+K^-}$ corresponding to the vertices $T_{cs}^0D^0\bar{K}^0$ and $T_{cs}^0D^+K^-$, respectively. These couplings are necessary to find the widths of the S-wave decays $T_{cs}^0 \to D^0\bar{K}^0$ and $T_{cs}^0 \to D^+K^-$, and can be calculated by means the QCD light-cone sum rule (LCSR) approach [23]. Because the aforementioned vertices contain a tetraquark the LCSR method should be supplemented by a technique of the soft-meson approximation [24]. For investigation of the diquark-antidiquark states the soft-meson approximation was adjusted in Ref. [25], and successfully applied later to explore their strong decays (see, e.g., Refs. [26–28]).

A. Mass and coupling of the T_{cs}^0

The mass and coupling of the tetraquark T_{cs}^0 can be obtained from the QCD two-point sum rules. To this end, we start from the analysis of the two-point correlation function

$$\Pi(p) = i \int d^4x e^{ipx} \langle 0|\mathcal{T}\{J^T(x)J^{T\dagger}(0)\}|0\rangle, \qquad (1)$$

where

$$J^T(x) = \epsilon\tilde{\epsilon}[c_b^T(x)C\gamma_5 s_c(x)][\bar{u}_d(x)\gamma_5 C\bar{d}_e^T(x)] \quad (2)$$

is the interpolating current for the tetraquark T_{cs}^0. Here, $\epsilon\tilde{\epsilon} = \epsilon^{abc}\epsilon^{ade}$, and a, b, c, d, and e are color indices and C is the charge-conjugation operator.

We assume that T_{cs}^0 is composed of the scalar diquark $\epsilon^{abc}[c_b^T C\gamma_5 s_c]$ in the color antitriplet and flavor antisymmetric state, and the antidiquark $\epsilon^{ade}[\bar{u}_d\gamma_5 C\bar{d}_e^T]$ in the color triplet state. Because these diquark configurations are most attractive ones [29], the current (2) corresponds to the ground-state scalar particle T_{cs}^0 with lowest mass.

To find the phenomenological side of the sum rule $\Pi^{\rm Phys}(p)$, we use the "ground-state + continuum" scheme. Then, $\Pi^{\rm Phys}(p)$ contains a contribution of the ground-state particle which below is written down explicitly, and effects of higher resonances and continuum states denoted by dots

$$\Pi^{\rm Phys}(p) = \frac{\langle 0|J|T_{cs}^0(p)\rangle\langle T_{cs}^0(p)|J^\dagger|0\rangle}{m_T^2 - p^2} + \ldots \quad (3)$$

The QCD side of the sum rules is determined by the same correlation function $\Pi^{\rm OPE}(p)$ found using the perturbative QCD and expressed in terms of the quark propagators. Expressions for the invariant amplitudes $\Pi^{\rm Phys}(p^2)$ and $\Pi^{\rm OPE}(p^2)$, which are necessary to derive the required sum rules for the mass m_T and coupling f_T of the tetraquark T_{cs}^0, as well as manipulations with these functions are similar to ones presented in Ref. [14], therefore we do not repeat them here; required theoretical results can be obtained from corresponding expressions for the T_{bs}^- by a simple $b \to c$ replacement.

The sum rules for m_T and f_T contain the quark, gluon and mixed vacuum condensates, values of which are collected in Table I. This table contains also the masses of the b, c, and s quarks, as well as spectroscopic parameters of the mesons D and K, which will be utilized in the next subsection.

The sum rules also depend on two auxiliary parameters. First of them is M^2, which appears in expressions after applying the Borel transformation to sum rules to suppress contributions of the higher resonances and continuum states. The dependence on the continuum threshold parameter s_0 is an output of the continuum subtraction procedure. A choice of these parameters is controlled by constraints on the pole contribution (PC) and convergence of the operator product expansion (OPE), as well as by a minimum sensitivity of the extracted quantities on M^2 and s_0.

Thus, the maximum allowed M^2 should be fixed to obey the restriction imposed on PC

$$\mathrm{PC} = \frac{\Pi(M^2, s_0)}{\Pi(M^2, \infty)}, \quad (4)$$

TABLE I. Parameters used in calculations.

Quantity	Value
$\langle\bar{q}q\rangle$	$-(0.24 \pm 0.01)^3$ GeV^3
$\langle\bar{s}s\rangle$	$0.8\langle\bar{q}q\rangle$
m_0^2	(0.8 ± 0.1) GeV^2
$\langle\bar{s}g_s\sigma Gs\rangle$	$m_0^2\langle\bar{s}s\rangle$
$\langle\frac{\alpha_s G^2}{\pi}\rangle$	(0.012 ± 0.004) GeV^4
$\langle g_s^3 G^3\rangle$	(0.57 ± 0.29) GeV^6
m_b	(4.18 ± 0.03) GeV
m_c	(1.275 ± 0.025) GeV
m_s	93_{-5}^{+11} MeV
m_{K^0}	(497.614 ± 0.024) MeV
m_{K^-}	(493.677 ± 0.016) MeV
m_D	(1864.84 ± 0.07) MeV
m_{D^+}	(1869.61 ± 0.10) MeV
$f_{K^-} = f_{K^0}$	(155.72 ± 0.51) MeV
$f_D = f_{D^+}$	(203.7 ± 4.7) MeV

where $\Pi(M^2, s_0)$ is the Borel-transformed and subtracted invariant amplitude $\Pi^{\rm OPE}(p^2)$. The lower bound of the window for the Borel parameter is determined from convergence of the OPE, which can be quantified by the ratio

$$R(M^2) = \frac{\Pi^{\rm DimN}(M^2, s_0)}{\Pi(M^2, s_0)}. \quad (5)$$

Here $\Pi^{\rm DimN}(M^2, s_0)$ denotes a contribution to the correlation function of the last term (or a sum of last few terms) in the operator product expansion. A stability of extracted quantities is among important requirements of the sum rule calculations.

In the present work, at the maximum of M^2 we apply the constraint PC > 0.2 which is typical for multiquark systems. To ensure convergence of the OPE, at the minimum limit of M^2 we use the restriction $R \leq 0.01$. Performed analysis demonstrates that the working regions

$$M^2 \in [1.8, 2.8]\ \mathrm{GeV}^2, \qquad s_0 \in [11, 12]\ \mathrm{GeV}^2, \quad (6)$$

obey the constraints imposed on the Borel and continuum threshold parameters. Indeed, the pole contribution at $M^2 = 2.8$ GeV^2 amounts to PC = 0.22, whereas at $M^2 = 1.8$ GeV^2 it reaches the maximum value 0.61. Numerical computations show that for DimN = Dim(8 + 9 + 10) the ratio $R(1.8\ \mathrm{GeV}^2)$ is equal to 0.007, which guarantees the convergence of the sum rules. These two values of M^2 determine the boundaries of the region within of which the Borel parameter can be varied.

In general, quantities extracted from sum rules should not depend on the auxiliary parameters used in calculations. In real computations, however, these quantities, i.e., m_T and

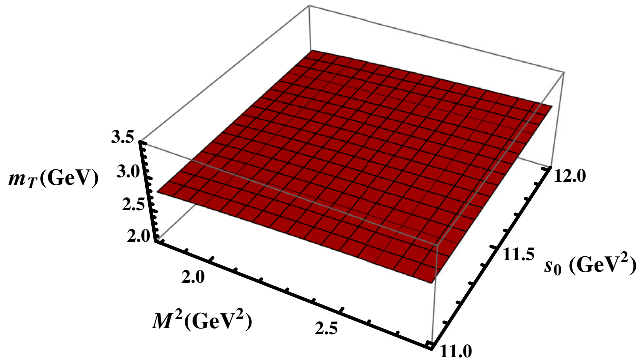

FIG. 1. The mass m_T of the tetraquark T_{cs}^0 as a function of the Borel and continuum threshold parameters.

f_T in the case under consideration, demonstrate a residual dependence on M^2 and s_0. Let us note that a dependence on the parameters M^2 and s_0 is a main source of unavoidable theoretical errors in the sum rule calculations, which however can be systematically taken into account.

In Figs. 1 and 2 we plot the predictions for the mass m_T and coupling f_T, in which one can see their dependence on the parameters M^2 and s_0.

Our results for the spectroscopic parameters of the tetraquark T_{cs}^0 read

$$m_T = (2878 \pm 128)\ \mathrm{MeV}, \quad f_T = (0.45 \pm 0.08) \times 10^{-2}\ \mathrm{GeV}^4. \tag{7}$$

These predictions will be used below to study the strong decays of T_{cs}^0.

B. Strong decays $T_{cs}^0 \to D^0\bar{K}^0$ and $T_{cs}^0 \to D^+K^-$

The spectroscopic parameters of the tetraquark T_{cs}^0 obtained in the previous subsection provide an information necessary to answer a question about its stability against the strong interactions. It is not difficult to see that the mass m_T makes kinematically allowed the strong decays $T_{cs}^0 \to D^0\bar{K}^0$ and $T_{cs}^0 \to D^+K^-$. There are other

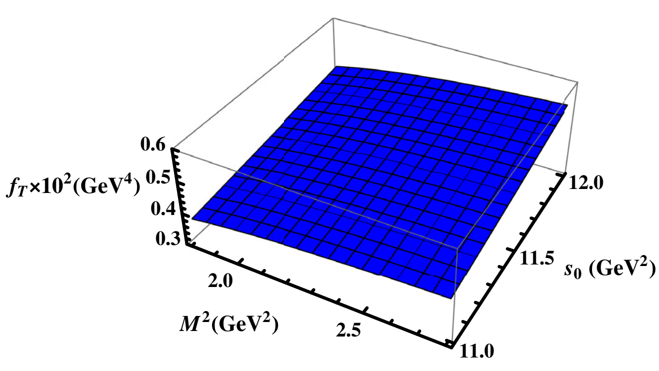

FIG. 2. The same as in Fig. 1, but for the coupling f_T.

strong decay modes of T_{cs}^0, but these two channels are S-wave processes. Here, we are going to consider in a detailed form the channel $T_{cs}^0 \to D^0\bar{K}^0$, and give final results for the second one.

The width of the decay $T_{cs}^0 \to D^0\bar{K}^0$, apart from other parameters, is determined by the strong coupling $g_{TD^0\bar{K}^0}$ corresponding to the vertex $T_{cs}^0 D^0\bar{K}^0$. Our aim is to calculate $g_{TD^0\bar{K}^0}$ which quantitatively describes strong interactions between the tetraquark and two conventional mesons. To this end, we use the LCSR method and begin from analysis of the correlation function

$$\Pi(p,q) = i\int d^4x e^{ipx}\langle \bar{K}^0(q)|\mathcal{T}\{J^{D^0}(x)J^{T\dagger}(0)\}|0\rangle, \tag{8}$$

where $J^{D^0}(x)$ is the interpolating current of the meson D^0; it has following form

$$J^{D^0}(x)(x) = \bar{u}(x)i\gamma_5 c(x). \tag{9}$$

Standard recipes require to write $\Pi(p,q)$ in terms of physical parameters of the particles T_{cs}^0, D^0, and $\bar{K}^0$

$$\Pi^{\mathrm{Phys}}(p,q) = \frac{\langle 0|J^{D^0}|D^0(p)\rangle}{p^2 - m_D^2}\langle D^0(p)\bar{K}^0(q)|T_{cs}^0(p')\rangle \times \frac{\langle T_{cs}^0(p')|J^{T\dagger}|0\rangle}{p'^2 - m_T^2} + \cdots, \tag{10}$$

where p' and p, q are 4-momenta of the initial and final particles, respectively. In the expression above by dots we note contributions of excited resonances and continuum states. The correlation function $\Pi^{\mathrm{Phys}}(p,q)$ can be simplified by introducing the matrix elements

$$\langle 0|J^{D^0}|D^0(p)\rangle = \frac{f_D m_D^2}{m_c + m_u},$$
$$\langle D^0(p)\bar{K}^0(q)|T_{cs}^0(p')\rangle = g_{TD^0\bar{K}^0}(p\cdot p'). \tag{11}$$

The matrix element $\langle 0|J^{D^0}|D^0(p)\rangle$ is expressed in terms of D^0 meson's mass m_D and its decay constant f_D, whereas $\langle D^0(p)\bar{K}^0(q)|T_{cs}^0(p')\rangle$ is written down using the strong coupling $g_{TD^0\bar{K}^0}$. In the soft-meson limit $q \to 0$ we get $p' = p$ [25], and must carry out the Borel transformation of $\Pi^{\mathrm{Phys}}(p, q = 0)$ over the variable p^2, which gives

$$\mathcal{B}\Pi^{\mathrm{Phys}}(p^2) = g_{TD^0\bar{K}^0}\frac{f_D m_D^2 f_T m_T \tilde{m}^2}{m_c + m_u}\frac{e^{-\tilde{m}^2/M^2}}{M^2}\ldots, \tag{12}$$

where

$$\tilde{m}^2 = \frac{m_T^2 + m_D^2}{2}. \tag{13}$$

The necessity to use the soft-meson approximation of the LCSR method and set $q = 0$ is connected with features of tetraquark-meson-meson strong vertices. Because a tetraquark is built of four valence quarks, calculations of the correlation function (8) by contracting quark fields from relevant interpolating currents lead to appearance of two quark fields at the same space-time position, which, sandwiched between the vacuum and $\bar{K}^0$ meson, generate the local matrix elements of $\bar{K}^0$. Then, to preserve the 4-momentum conservation at the vertex one has to set $q = 0$, and employ technical tools of soft-meson approach elaborated in the full LCSR method as the approximation to vertices containing only conventional mesons [24]. Let us emphasize that in the case of tetraquark-meson-meson vertices soft limit is an only way to calculate corresponding strong couplings in the framework of the LCSR method.

The soft approximation modifies the physical side of the sum rules. A problem is that in the soft limit some of contributions arising from the higher resonances and continuum states even after the Borel transformation remain unsuppressed. These terms correspond to vertices containing excited states of involved particles, and contaminate the physical side of sum rules. Therefore, before performing the continuum subtraction in the final sum rule they should be delated by means of some manipulations. This problem can be solved by acting on the physical side of the sum rule by the operator [24,30]

$$\mathcal{P}(M^2, \tilde{m}^2) = \left(1 - M^2 \frac{d}{dM^2}\right) M^2 e^{\tilde{m}^2/M^2}, \tag{14}$$

which keeps unchanged the ground-state term removing, at the same time, unsuppressed contributions. Naturally, the operator $\mathcal{P}(M^2, \tilde{m}^2)$ has to be applied to the QCD side of the sum rule as well, which has to be calculated in the soft-meson approximation and expressed in terms of the $\overline{K^0}$ meson's local matrix elements.

In the soft limit the correlation function $\Pi^{\mathrm{OPE}}(p)$ is determined by the expression

$$\Pi^{\mathrm{OPE}}(p) = i \int d^4x e^{ipx} \epsilon\tilde{\epsilon} [\gamma_5 \tilde{S}_c^{ib}(x) \gamma_5 S_u^{di}(-x) \gamma_5]_{\alpha\beta} \times \langle \bar{K}^0 | \bar{s}_\alpha^c(0) d_\beta^e(0) | 0 \rangle, \tag{15}$$

where

$$\tilde{S}(x) = C S_{c(q)}^T(x) C. \tag{16}$$

In Eqs. (15) and (16), $S_{c(q)}(x)$ are the c quark and light quark propagators explicit expressions of which can be found in Ref. [31]; for simplicity we do not provide these formulas here.

As is seen, the correlation function $\Pi^{\mathrm{OPE}}(p)$ depends on local matrix elements $\langle \bar{K}^0 | \bar{s}_\alpha^c(0) d_\beta^e(0) | 0 \rangle$, which should be recast to forms suitable for expressing them as standard matrix elements of $\bar{K}^0$. For these purposes, we employ the expansion

$$\bar{s}_\alpha^c d_\beta^e \to \frac{1}{12} \Gamma_{\beta\alpha}^j \delta^{ce} (\bar{s} \Gamma^j d), \tag{17}$$

where Γ^j is the full set of Dirac matrices

$$\Gamma^j = \mathbf{1}, \gamma_5, \gamma_\lambda, i\gamma_5\gamma_\lambda, \sigma_{\lambda\rho}/\sqrt{2}. \tag{18}$$

Then operators $\bar{s}\Gamma^j d$ and ones appeared due to G insertions from propagators $\tilde{S}$ and S, give rise to local matrix elements of the $\bar{K}^0$ meson. Substituting Eq. (17) into the correlation function and performing the color summation in accordance with prescriptions described in Ref. [25], we fix twist-3 local matrix element of $\bar{K}^0$

$$\langle 0 | \bar{d}(0) i\gamma_5 s(0) | \bar{K}^0 \rangle = \frac{f_{K^0} m_{K^0}^2}{m_s + m_d}, \tag{19}$$

that contributes to the correlation function.

The function $\Pi^{\mathrm{OPE}}(p)$ contains the trivial Lorentz structure which is proportional to I. The Borel transformed and subtracted expression of the corresponding invariant amplitude $\Pi^{\mathrm{OPE}}(p^2)$ reads

$$\Pi^{\mathrm{OPE}}(M^2, s_0) = \int_{(m_c+m_s)^2}^{s_0} ds \rho^{\mathrm{pert}}(s) e^{-s/M^2} + \frac{\mu_{K^0}}{6} e^{-m_c^2/M^2} \left\{ m_c \langle \bar{q}q \rangle + \frac{1}{8} \left\langle \frac{\alpha_s G^2}{\pi} \right\rangle \left[1 + \frac{m_c^2}{6M^2} \right] - \frac{m_c^3}{4M^4} \langle \bar{s} g_s \sigma G s \rangle - \frac{g_s^2 m_c^4}{81 M^6} \langle \bar{q}q \rangle^2 - \frac{m_c \pi^2}{18 M^6} \left\langle \frac{\alpha_s G^2}{\pi} \right\rangle \langle \bar{q}q \rangle (m_c^2 - 3M^2) \right\}, \tag{20}$$

where

$$\rho^{\mathrm{pert}}(s) = \frac{\mu_{K^0}}{24\pi^2} (3m_c^2 - s), \tag{21}$$

and $\mu_{K^0} = f_{K^0} m_{K^0}^2 / (m_s + m_d)$. Let us note that calculations of $\Pi^{\mathrm{OPE}}(M^2, s_0)$ are carried out by taking into account nonperturbative terms up to seventh dimension. Then, the sum rule for the strong coupling $g_{TD^0\bar{K}^0}$ takes the form

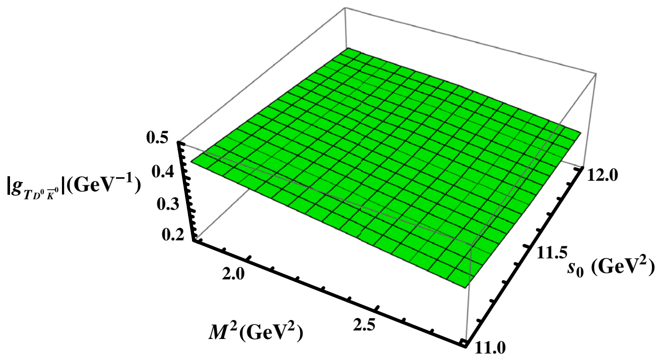

FIG. 3. The strong coupling $|g_{TD^0\bar{K}^0}|$ as a function of the Borel and continuum threshold parameters.

$$g_{TD^0\bar{K}^0} = \frac{m_c + m_u}{f_D m_D^2 f_T m_T \tilde{m}^2}\mathcal{P}(M^2, \tilde{m}^2)\Pi^{\mathrm{OPE}}(M^2, s_0). \quad (22)$$

The width of the decay $T_{cs}^0 \to D^0\bar{K}^0$ is given by the formula

$$\Gamma[T_{cs}^0 \to D^0\bar{K}^0] = \frac{g_{TD^0\bar{K}^0}^2 m_D^2}{8\pi}\lambda\left(1 + \frac{\lambda^2}{m_D^2}\right), \quad (23)$$

where

$$\begin{aligned} \lambda &= \lambda(m_T^2, m_D^2, m_{K^0}^2) \\ &= \frac{1}{2m_T}[m_T^4 + m_{K^0}^4 + m_D^4 \\ &\quad -2(m_T^2 m_D^2 + m_T^2 m_{K^0}^2 + m_{K^0}^2 m_D^2)]^{1/2}. \end{aligned} \quad (24)$$

In numerical computations of $g_{TD^0\bar{K}^0}$ the Borel and continuum threshold parameters are chosen as in Eq. (6). To visualize a sensitivity of the strong coupling on these parameters, in Fig. 3 we depict the dependence of $|g_{TD^0\bar{K}^0}|$ on M^2 and s_0; ambiguities generated by the choice of these parameters do not exceed $\pm 19\%$ of the central value.

For the strong coupling $g_{TD^0\bar{K}^0}$ our analysis yields

$$|g_{TD^0\bar{K}^0}| = (0.37 \pm 0.07)\ \mathrm{GeV}^{-1}. \quad (25)$$

Using the result obtained for $g_{TD^0\bar{K}^0}$, we can evaluate the partial width of the decay $T_{cs}^0 \to D^0\bar{K}^0$:

$$\Gamma(T_{cs}^0 \to D^0\bar{K}^0) = (15.35 \pm 4.11)\ \mathrm{MeV}. \quad (26)$$

The decay $T_{cs}^0 \to D^+K^-$ can be analyzed by the same manner. The difference is connected with quark contents of the mesons D^+ and K^- that generate small modifications, e.g., $\tilde{\Pi}^{\mathrm{OPE}}(p)$ takes the form

$$\begin{aligned} \tilde{\Pi}^{\mathrm{OPE}}(p) = i\int d^4x e^{ipx}\epsilon\tilde{\epsilon}[\gamma_5\tilde{S}_c^{ib}(x)\gamma_5 S_d^{di}(-x)\,\gamma_5]_{\alpha\beta} \\ \times \langle K^-|\bar{u}_\alpha^c(0)s_\beta^e(0)|0\rangle. \end{aligned} \quad (27)$$

Therefore, we write down the final results for the strong coupling $g_{TD^+K^-}$ and corresponding decay width

$$\begin{aligned} |g_{TD^+K^-}| &= (0.38 \pm 0.06)\ \mathrm{GeV}^{-1}, \\ \Gamma(T_{cs}^0 \to D^+K^-) &= (15.40 \pm 3.44)\ \mathrm{MeV}. \end{aligned} \quad (28)$$

These dominant decay channels allow us to estimate the full width of the tetraquark T_{cs}^0

$$\Gamma = (30.8 \pm 5.4)\ \mathrm{MeV}. \quad (29)$$

In light of obtained prediction for the full width of T_{cs}^0, we classify it as a relatively narrow unstable tetraquark.

III. SEMILEPTONIC DECAY $T_{bs}^- \to T_{cs}^0 l\bar{\nu}_l$

The semileptonic decay $T_{bs}^- \to T_{cs}^0 l\bar{\nu}_l$ runs through the transitions $b \to W^-c$ and $W^- \to l\bar{\nu}_l$. It is not difficult to see that decays with all lepton species $l = e,\ \mu$ and τ are kinematically allowed processes.

The transition $b \to c$ at the tree-level can be described using the effective Hamiltonian

$$\mathcal{H}^{\mathrm{eff}} = \frac{G_F}{\sqrt{2}}V_{bc}\bar{c}\gamma_\mu(1-\gamma_5)b\bar{l}\gamma^\mu(1-\gamma_5)\nu_l, \quad (30)$$

where G_F is the Fermi coupling constant, and V_{bc} is the relevant Cabibbo-Kobayashi-Maskawa (CKM) matrix element. After placing the effective Hamiltonian $\mathcal{H}^{\mathrm{eff}}$ between the initial and final tetraquarks and factoring out the lepton fields one gets the matrix element of the current

$$J_\mu^{\mathrm{W}} = \bar{c}\gamma_\mu(1-\gamma_5)b. \quad (31)$$

The matrix element $\langle T_{cs}^0(p')|J_\mu^{\mathrm{W}}|T_{bs}^-(p)\rangle$ can be expressed in terms of the form factors $G_i(q^2)$ that parametrize the long-distance dynamics of the weak transition. In the case of scalar tetraquarks it has the rather simple form

$$\langle T_{cs}^0(p')|J_\mu^{\mathrm{W}}|T_{bs}^-(p)\rangle = G_1(q^2)P_\mu + G_2(q^2)q_\mu, \quad (32)$$

where p and p' are the momenta of the tetraquarks T_{bs}^- and T_{cs}^0, respectively. Here, we use the shorthand notations $P_\mu = p'_\mu + p_\mu$ and $q_\mu = p_\mu - p'_\mu$. The q_μ is the momentum transferred to the leptons, and q^2 changes within the limits $m_l^2 \leq q^2 \leq (m - m_T)^2$, where m_l is the mass of a lepton l.

The sum rules for the form factors $G_i(q^2), i=1$, 2 can be derived from the three-point correlation function

$$\Pi_\mu(p,p') = i^2 \int d^4x d^4y e^{i(p'y-px)} \times \langle 0|\mathcal{T}\{J^T(y)J_\mu^{\mathrm{W}}(0)J^\dagger(x)\}|0\rangle, \quad (33)$$

where $J^T(y)$ and $J(x)$ are the interpolating currents for the states T_{cs}^0 and T_{bs}^-, respectively. The current $J^T(y)$ is given by Eq. (2), whereas for $J(x)$ we use the expression

$$J(x) = \epsilon\tilde{\epsilon}[b_b^T(x)C\gamma_5 s_c(x)][\bar{u}_d(x)\gamma_5 C\bar{d}_e^T(x)]. \quad (34)$$

First, we express the correlation function $\Pi_\mu(p,p')$ in terms of the spectroscopic parameters of the tetraquark and mesons, and fix the physical side of the sum rule, i.e., find the function $\Pi_\mu^{\mathrm{Phys}}(p,p')$. It can be easily written down in the form

$$\Pi_\mu^{\mathrm{Phys}}(p,p') = \frac{\langle 0|J^T|T_{cs}^0(p')\rangle\langle T_{cs}^0(p')|J_\mu^{\mathrm{W}}|T_{bs}^-(p)\rangle}{(p^2-m^2)(p'^2-m_T^2)} \times \langle T_{bs}^-(p)|J^\dagger|0\rangle + \ldots, \quad (35)$$

where we take explicitly into account a contribution of the ground-state particles, and denote by dots effects due to excited and continuum states.

Using the tetraquarks' matrix elements and expressing the vertex $\langle T(p')|J_\mu^{\mathrm{W}}|T_{bs}^-(p)\rangle$ in terms of the weak transition form factors $G_i(q^2)$ it is not difficult to find that

$$\Pi_\mu^{\mathrm{Phys}}(p,p') = \frac{f_T m_T f m}{(p^2-m^2)(p'^2-m_T^2)} \times [G_1(q^2)P_\mu + G_2(q^2)q_\mu], \quad (36)$$

where the matrix element of the state T_{bs}^- is defined by

$$\langle T_{bs}^-(p)|J^\dagger|0\rangle = fm. \quad (37)$$

To calculate $\Pi_\mu(p,p')$, we employ the interpolating currents and quark propagators, and find

$$\Pi_\mu^{\mathrm{OPE}}(p,p') = i^2 \int d^4x d^4y e^{i(p'y-px)} \epsilon\tilde{\epsilon}\epsilon'\tilde{\epsilon}' \times \mathrm{Tr}[\gamma_5 \tilde{S}_d^{e'e}(x-y)\gamma_5 S_u^{d'd}(x-y)] \times \mathrm{Tr}[\gamma_\mu(1-\gamma_5) \times S_b^{ib}(-x)\gamma_5 \tilde{S}_s^{cc'}(y-x)\gamma_5 S_c^{b'i}(y)]. \quad (38)$$

Then, the sum rules for the form factors $G_i(q^2)$ can be derived by equating the invariant amplitudes corresponding to the same Lorentz structures in $\Pi_\mu^{\mathrm{Phys}}(p,p')$ and $\Pi_\mu^{\mathrm{OPE}}(p,p')$. Afterwards, we carry out the double Borel transformation over p'^2 and p^2 which is required to suppress contributions of the higher excited and continuum states, and perform the continuum subtraction. These operations lead to the sum rules

$$G_i(\mathbf{M}^2, \mathbf{s}_0, q^2) = \frac{1}{f_T m_T f m} \int_{(m_b+m_s)^2}^{s_0} ds \times \int_{(m_c+m_s)^2}^{s_0'} ds' \rho_i(s,s',q^2) e^{(m^2-s)/M_1^2} e^{(m_T^2-s')/M_2^2}, \quad (39)$$

where $\rho_{1(2)}(s,s',q^2)$ are the spectral densities calculated as the imaginary parts of the correlation function $\Pi_\mu^{\mathrm{OPE}}(p,p')$ with dimension-five accuracy. In Eq. (39) $\mathbf{M}^2$ and $\mathbf{s}_0$ are a couple of the Borel and continuum threshold parameters, respectively; the set $(M_1^2,\ s_0)$ corresponds to the initial state T_{bs}^-, and the second set $(M_2^2,\ s_0')$ describes the tetraquark T_{cs}^0.

Parameters for numerical computations of $G_i(\mathbf{M}^2, \mathbf{s}_0, q^2)$ are listed in Table I. The mass and coupling of the tetraquark T_{bs}^-

$$m = (5380 \pm 170)\ \mathrm{MeV}, \quad f = (2.1 \pm 0.5) \times 10^{-3}\ \mathrm{GeV}^4, \quad (40)$$

and working windows for the parameters $(M_1^2,\ s_0)$

$$M_1^2 \in [3.4, 4.8]\ \mathrm{GeV}^2, \qquad s_0 \in [35, 37]\ \mathrm{GeV}^2 \quad (41)$$

are borrowed from Ref. [14]. The regions for $(M_2^2,\ s_0')$ and spectroscopic parameters of T_{cs}^0 are given by Eqs. (6) and (7), respectively. In numerical computations we also use the Fermi coupling constant $G_F = 1.16637 \times 10^{-5}\ \mathrm{GeV}^{-2}$ and CKM matrix element $|V_{bc}| = (41.2 \pm 1.01) \times 10^{-3}$. Like all quantities extracted from sum rule computations, the weak form factors $G_{1(2)}(q^2)$ depend on the Borel and continuum threshold parameters $\mathbf{M}^2$ and $\mathbf{s}_0$. Ambiguities connected with the choice of $(\mathbf{M}^2,\ \mathbf{s}_0)$ and ones due to other input parameters form theoretical errors of the sum rule analysis, which will be taken into account in the fit functions.

To obtain the width of the decay $T_{bs}^- \to T_{cs}^0 l\bar{\nu}_l$ one must integrate the differential decay rate $d\Gamma/dq^2$ (see, explanation below) of this process in the kinematical limits $m_l^2 \le q^2 \le (m-m_T)^2$. In the interval $m_l^2 \le q^2 \le 5\ \mathrm{GeV}^2$ the QCD sum rules lead to reliable predictions for the form factors $G_i(q^2)$, which do not cover the whole integration

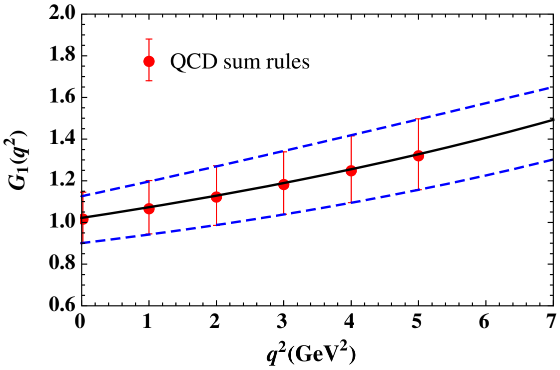

FIG. 4. Dependence of the weak form factor $G_1(q^2)$ on q^2: the QCD sum rule predictions and the fit function $\mathcal{G}_1(q^2)$. The solid line corresponds to the central values of the parameters $\mathcal{G}_1^0$, g_1^1, g_2^1, for the upper dashed curve $\mathcal{G}_1^0 = 1.126$, $g_1^1 = 1.792$, $g_2^1 = -0.875$, whereas for the lower dashed line $\mathcal{G}_1^0 = 0.901$, $g_1^1 = 1.255$, $g_2^1 = 1.106$.

region $m_l^2 \leq q^2 \leq 6.26\ \mathrm{GeV}^2$. Therefore, we replace the weak form factors $G_i(q^2)$ by the fit functions $\mathcal{G}_i(q^2)$, which at q^2 accessible for the sum rule computations coincide with $G_i(q^2)$, but can be easily extrapolated to the full integration region.

For the fit functions we choose the following analytic expressions

$$\mathcal{G}_i(q^2) = \mathcal{G}_i^0 \exp\left[g_1^i \frac{q^2}{m^2} + g_2^i \left(\frac{q^2}{m^2}\right)^2\right]. \tag{42}$$

In Figs. 4 and 5 one can see the QCD sum rule predictions for the form factors $G_1(q^2)$ and $|G_2(q^2)|$, in which ambiguities of computations are shown as error bars. Using the central values of the form factors and a standard fitting procedure, for the parameters of the functions $\mathcal{G}_1(q^2)$ and $\mathcal{G}_2(q^2)$ we get

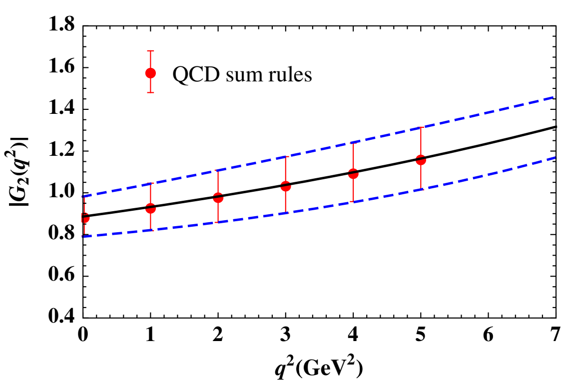

FIG. 5. The form factor $|G_2(q^2)|$. The solid line describes the central function. Parameters of the upper and lower dashed curves are $\mathcal{G}_1^0 = -0.982$, $g_1^1 = 1.771$, $g_2^1 = -0.545$, and $\mathcal{G}_1^0 = -0.790$, $g_1^1 = 1.039$, $g_2^1 = 2.414$, respectively.

TABLE II. The decay channels of the tetraquarks T_{bs}^- and T_{bb}^-, and their branching ratios. Above we have used $L = e^-e^+e^-$.

Channels	$\mathcal{BR}$
$T_{bs}^- \to D^0\bar{K}^0 e^-\bar{\nu}_e$	0.24
$T_{bs}^- \to D^+K^- e^-\bar{\nu}_e$	0.24
$T_{bs}^- \to D^0\bar{K}^0 \mu^-\bar{\nu}_\mu$	0.23
$T_{bs}^- \to D^+K^- \mu^-\bar{\nu}_\mu$	0.23
$T_{bs}^- \to D^0\bar{K}^0 \tau^-\bar{\nu}_\tau$	0.03
$T_{bs}^- \to D^+K^- \tau^-\bar{\nu}_\tau$	0.03
$T_{bb}^- \to D^0\bar{K}^0 L$	1.7×10^{-2}
$T_{bb}^- \to D^+K^- L$	1.6×10^{-2}
$T_{bb}^- \to D^0\bar{K}^0 \pi^+e^-e^-$	9.8×10^{-3}
$T_{bb}^- \to D^0\bar{K}^0 K^+e^-e^-$	1.3×10^{-3}
$T_{bb}^- \to D^+K^- \pi^+e^-e^-$	9.4×10^{-3}
$T_{bb}^- \to D^+K^- K^+e^-e^-$	1.3×10^{-3}

$$\mathcal{G}_1^0 = 1.022, \quad g_1^1 = 1.383, \quad g_2^1 = 0.756,$$
$$\mathcal{G}_2^0 = -0.886, \quad g_1^2 = 1.440, \quad g_2^2 = 0.813. \tag{43}$$

The upper and lower limits of the sum rule results are employed to find corresponding extrapolating functions, plotted in the figures in the form of dashed curves. Various combinations of these functions are used to estimate theoretical errors of the semileptonic processes' partial widths.

The differential decay rate $d\Gamma/dq^2$ of the process $T_{bs}^- \to T_{cs}^0 l\bar{\nu}_l$ can be calculated using the expression derived in Ref. [14], where one needs to replace parameters of the tetraquarks and weak form factors. Calculations yield the following predictions

$$\Gamma(T_{bs}^- \to T_{cs}^0 e^-\bar{\nu}_e) = (1.55 \pm 0.42) \times 10^{-10}\ \mathrm{MeV},$$
$$\Gamma(T_{bs}^- \to T_{cs}^0 \mu^-\bar{\nu}_\mu) = (1.54 \pm 0.42) \times 10^{-10}\ \mathrm{MeV},$$
$$\Gamma(T_{bs}^- \to T_{cs}^0 \tau^-\bar{\nu}_\tau) = (1.91 \pm 0.54) \times 10^{-11}\ \mathrm{MeV}. \tag{44}$$

Then, for the full width and mean lifetime of the tetraquark T_{bs}^- we find

$$\Gamma_{\mathrm{full}} = (3.28 \pm 0.60) \times 10^{-10}\ \mathrm{MeV},$$
$$\tau = 2.01^{+0.44}_{-0.31} \times 10^{-12}\ \mathrm{s}. \tag{45}$$

Branching ratios of the processes $T_{bs}^- \to D^0\bar{K}^0 l\bar{\nu}_l$ and $T_{bs}^- \to D^+K^- l\bar{\nu}_l$ can be found using $\mathcal{BR}(T_{bs}^- \to T_{cs}^0 l\bar{\nu}_l)$ and $\mathcal{BR}(T_{cs}^0 \to D^0\bar{K}^0) \simeq \mathcal{BR}(T_{cs}^0 \to D^+K^-) \simeq 0.5$. Results of these computations are collected in Table II.

IV. ANALYSIS AND CONCLUSIONS

In the present work we have calculated width and mean lifetime of the tetraquark T_{bs}^-, which is stable against the strong and electromagnetic decays. To this end, we have computed partial widths of its dominant semileptonic

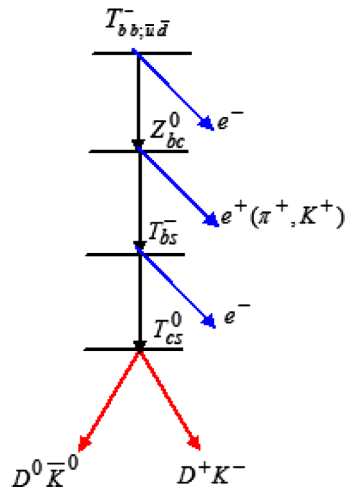

FIG. 6. Some of the decay modes of the tetraquark $T^-_{bb;\bar{u}\bar{d}}$.

decays $T^-_{bs} \to T^0_{cs} l\bar{\nu}_l$, where l is one of e, μ and τ leptons. The tetraquark T^0_{cs} appeared at the final state of this process is the strong-interaction unstable particle and decays to conventional mesons $D^0\bar{K}^0$ and D^+K^-. We have also evaluated the spectroscopic parameters of T^0_{cs} and computed the partial widths of its strong decays, which allowed us to find the branching ratios of the processes $T^-_{bs} \to D^0\bar{K}^0 l\bar{\nu}_l$ and $T^-_{bs} \to D^+K^- l\bar{\nu}_l$. Predictions for the mass of T^-_{bs} obtained in our previous work [14], and results for the full widths and mean lifetimes of the tetraquarks T^-_{bs} and T^0_{cs} provide a basis for their experimental investigations.

But, information gained in the present article is important also to trace back transformations of the state T^-_{bb}. Stable nature of the T^-_{bb} was explored and confirmed by different methods and authors. This state transforms in accordance with the chains of decays $T^-_{bb} \to Z^0_{bc} l\bar{\nu}_l \to T^-_{bs} l\bar{\nu}_l \bar{l}' \nu_{l'}$ and $T^-_{bb} \to Z^0_{bc} l\bar{\nu}_l \to T^-_{bs} P l\bar{\nu}_l$, where we take into account both the semileptonic and nonleptonic decays of the scalar tetraquark Z^0_{bc} [14]. Now with information on decays of the tetraquark T^-_{bs} at hands, we can fix some of decay channels of T^-_{bb} to conventional mesons. It is not difficult to see, that $T^-_{bb} \to D^0\bar{K}^0 l\bar{\nu}_l \bar{l}' \nu_{l'} l'' \bar{\nu}_{l''}$, $T^-_{bb} \to D^+K^- l\bar{\nu}_l \bar{l}' \nu_{l'} l'' \bar{\nu}_{l''}$, $T^-_{bb} \to D^0\bar{K}^0 P l\bar{\nu}_l l'' \bar{\nu}_{l''}$, and $T^-_{bb} \to D^+K^- P l\bar{\nu}_l l'' \bar{\nu}_{l''}$ are among important modes of such transformations. In Fig. 6 we depict some of such channels, which at the second leg of weak transformations contain products of semileptonic and nonleptonic decays of Z^0_{bc}. Their branching ratios can be found using results of Refs. [13,14] and information obtained in the present work. For the decay mode $T^-_{bb} \to D^0\bar{K}^0 L$ these computations yield

$$\mathcal{BR}(T^-_{bb} \to D^0\bar{K}^0 L) = \mathcal{BR}(T^-_{bb} \to T^-_{bs} e^- e^+) \times \mathcal{BR}(T^-_{bs} \to D^0\bar{K}^0 e^-) = 1.7 \times 10^{-2}. \quad (46)$$

For simplicity, we have denoted $L = e^- e^+ e^-$ and omitted final-state neutrinos. The branching ratios of other processes [shown in Fig. 6] are also collected in Table II. The decay channels of T^-_{bb} containing μ and τ leptons, and mixed modes with $e\mu$, $e\tau$, $\mu\tau$, and $e\mu\tau$ leptons at the final state can be analyzed by the same manner.

The results for the width and lifetime of the tetraquark T^-_{bs}, and predictions for branching ratios of T^-_{bs} and T^-_{bb} have been obtained using their dominant semileptonic decays. In the case of weak transformations of the tetraquark T^-_{bb} we took into account the nonleptonic decays of the scalar state Z^0_{bc}. During the present analysis we have neglected nonleptonic decay modes of T^-_{bs} and T^-_{bb}. Our investigations show that branching ratios of nonleptonic channels are suppressed relative to semileptonic ones [14,15], nevertheless, by including into consideration these modes one can refine the prediction (46) and ones presented in Table II.

We also ignored subdominant decay channels which may be generated by weak decays of heavy quarks, and which are suppressed due to smallness of the relevant CKM matrix elements. At earlier levels of the weak cascade some of these modes might create unstable 4-quarks that dissociate to other than D and K mesons.

Finally, in the present work the exotic meson T^-_{bs} has been treated as a scalar particle. But in the decay $Z^0_{bc} \to T^-_{bs} \bar{l} \nu_l$ the final-state tetraquark may bear also other quantum numbers. By including into analysis these options one may reveal new decay modes of Z^0_{bc}, and, hence of T^-_{bb}. Investigation of these alternative decays can add a valuable new information on features of the exotic mesons T^-_{bb} and T^-_{bs}.

[1] H. X. Chen, W. Chen, X. Liu, and S. L. Zhu, Phys. Rep. **639**, 1 (2016).
[2] H. X. Chen, W. Chen, X. Liu, Y. R. Liu, and S. L. Zhu, Rep. Prog. Phys. **80**, 076201 (2017).
[3] A. Esposito, A. Pilloni, and A. D. Polosa, Phys. Rep. **668**, 1 (2017).
[4] A. Ali, J. S. Lange, and S. Stone, Prog. Part. Nucl. Phys. **97**, 123 (2017).
[5] S. L. Olsen, T. Skwarnicki, and D. Zieminska, Rev. Mod. Phys. **90**, 015003 (2018).
[6] J. P. Ader, J. M. Richard, and P. Taxil, Phys. Rev. D **25**, 2370 (1982).

[7] H. J. Lipkin, Phys. Lett. B **172**, 242 (1986).
[8] S. Zouzou, B. Silvestre-Brac, C. Gignoux, and J. M. Richard, Z. Phys. C **30**, 457 (1986).
[9] J. Carlson, L. Heller, and J. A. Tjon, Phys. Rev. D **37**, 744 (1988).
[10] F. S. Navarra, M. Nielsen, and S. H. Lee, Phys. Lett. B **649**, 166 (2007).
[11] M. Karliner and J. L. Rosner, Phys. Rev. Lett. **119**, 202001 (2017).
[12] E. J. Eichten and C. Quigg, Phys. Rev. Lett. **119**, 202002 (2017).
[13] S. S. Agaev, K. Azizi, B. Barsbay, and H. Sundu, Phys. Rev. D **99**, 033002 (2019).
[14] H. Sundu, S. S. Agaev, and K. Azizi, Eur. Phys. J. C **79**, 753 (2019).
[15] S. S. Agaev, K. Azizi, and H. Sundu, arXiv:1905.07591.
[16] S. S. Agaev, K. Azizi, and H. Sundu, Phys. Rev. D **99**, 114016 (2019).
[17] G.-Q. Feng, X.-H. Guo, and B.-S. Zou, arXiv:1309.7813.
[18] A. Francis, R. J. Hudspith, R. Lewis, and K. Maltman, Phys. Rev. D **99**, 054505 (2019).
[19] T. F. Caramees, J. Vijande, and A. Valcarce, Phys. Rev. D **99**, 014006 (2019).
[20] M. A. Shifman, A. I. Vainshtein, and V. I. Zakharov, Nucl. Phys. **B147**, 385 (1979).
[21] M. A. Shifman, A. I. Vainshtein, and V. I. Zakharov, Nucl. Phys. **B147**, 448 (1979).
[22] R. M. Albuquerque, J. M. Dias, K. P. Khemchandani, A. M. Torres, F. S. Navarra, M. Nielsen, and C. M. Zanetti, J. Phys. G **46**, 093002 (2019).
[23] I. I. Balitsky, V. M. Braun, and A. V. Kolesnichenko, Nucl. Phys. **B312**, 509 (1989).
[24] V. M. Belyaev, V. M. Braun, A. Khodjamirian, and R. Ruckl, Phys. Rev. D **51**, 6177 (1995).
[25] S. S. Agaev, K. Azizi, and H. Sundu, Phys. Rev. D **93**, 074002 (2016).
[26] S. S. Agaev, K. Azizi, and H. Sundu, Phys. Rev. D **93**, 114007 (2016).
[27] H. Sundu, S. S. Agaev, and K. Azizi, Phys. Rev. D **97**, 054001 (2018).
[28] H. Sundu, S. S. Agaev, and K. Azizi, Eur. Phys. J. C **79**, 215 (2019).
[29] R. L. Jaffe, Phys. Rep. **409**, 1 (2005).
[30] B. L. Ioffe and A. V. Smilga, Nucl. Phys. **B232**, 109 (1984).
[31] H. Sundu, B. Barsbay, S. S. Agaev, and K. Azizi, Eur. Phys. J. A **54**, 124 (2018).