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Probing the effects of some new physics models in $B_s \to D_s \tau \bar{\nu}$ decay

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Abstract

Recent measurements on the semi-leptonic $b \to c\ell\bar{\nu}_\ell$ and $b \to s\ell^+\ell^-$ decays imply possible hints of Lepton Flavor Universality (LFU) violation and therefore have attracted much attention. In this paper, motivated by these anomalies, we analyze the $B_s \to D_s \tau \bar{\nu}$ decay in some New Physics (NP) models, such as the Supersymmetry with R-parity violation (RPV SUSY) model, the W' model, the vector leptoquark model and the aligned two-Higgs-doublet model (A2HDM). Using the parameter spaces obtained from various flavor constraints, we calculate the branching fraction $\mathcal{B}(B_s \to D_s \tau \bar{\nu})$ and the ratio R_{D_s} of this decay process, and also show the effects of NP models on the differential branching fraction $\frac{d\mathcal{B}}{dq^2}$, the differential ratio $R_{D_s}(q^2)$, the lepton-side forward-backward asymmetry $A_{FB}(q^2)$, the convexity parameter $C_F^\tau(q^2)$ and the τ polarization fraction $P_L^\tau(q^2)$. We find that: (i) the effects of NP models are significant for $\mathcal{B}(B_s \to D_s \tau \bar{\nu})$, but R_{D_s} is only sensitive to the effect of A2HDM; (ii) both $\frac{d\mathcal{B}}{dq^2}$ and $R_{D_s}(q^2)$ in the four NP models show obvious deviations from the SM predictions; (iii) for $A_{FB}(q^2)$, $C_F^\tau(q^2)$ and $P_L^\tau(q^2)$, only the A2HDM presents obvious effect because the scalar type interactions can be generated in such model.

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1. Introduction

Although there is no direct evidences for physics beyond the Standard Model (SM) found for now, some possible hints of new physics (NP) have been observed in the semi-leptonic B meson decays [1,2]. For the lepton flavor universality (LFU) violation, the ratios $R_{D^{(*)}} = \frac{\mathcal{B}(B\to D^{(*)}\tau\bar{\nu})}{\mathcal{B}(B\to D^{(*)}\ell\bar{\nu})}$ with $\ell = e, \mu$ have been measured first by the BaBar Collaboration [3,4]. After that, the Belle [5–9] and LHCb [10,11] Collaborations also reported their results. The latest averaged results obtained by Heavy Flavor Averaging Group (HFAG) are [12]

$$R_D = 0.340 \pm 0.027 \pm 0.013, \quad R_{D^*} = 0.299 \pm 0.013 \pm 0.008\,. \tag{1}$$

Comparing with the SM predictions of $R_{D^{(*)}}$ [13,14]

$$R_D = 0.299 \pm 0.011,\ R_{D^*} = 0.252 \pm 0.003\,, \tag{2}$$

it can be found that the divergence is at the level of 3.1σ.

Besides $R_{D^{(*)}}$, the LFU ratio $R_{J/\psi}$ has also been measured by LHC [15],

$$R_{J/\psi} = \frac{\mathcal{B}(B_c \to J/\psi\tau\bar{\nu})}{\mathcal{B}(B_c \to J/\psi\ell\bar{\nu})} = 0.71 \pm 0.17 \pm 0.18\,, \tag{3}$$

which exceeds the SM prediction by 2σ.

In recent years, the anomalies mentioned above have been widely studies within the SM and its extensions [16–57], and it is found that the effects of some NP models are very significant for the observables of B meson semi-leptonic decays. In this paper, we would like to pay our attention to the $B_s \to D_s\tau\bar{\nu}$ decay, which is also induced by $b \to c\tau\bar{\nu}$ transition at quark level and has been discussed based on some model-independent approaches [58–73]. In this work, we will discuss the effects of some NP models, such as the RPV SUSY model [74,75], the W' model [76], the leptoquark model [77] and A2HDM [78] on some observables of this decay process.

Our paper is organized as follows. In the next section, we introduce the SM theoretical framework for $B_s \to D_s\tau\bar{\nu}$ transition. The four NP models have been discussed in sec. 3. In sec. 4, we present the numerical analyses. Finally, the main conclusions are summarized in sec. 5.

2. Theoretical framework

2.1. Effective Lagrangian

Considering the contributions of NP and neglecting the contributions of the right-handed neutrinos and the tensor operator, the effective Lagrangian for $b \to c\tau\bar{\nu}$ transition can be written as [79,80]

$$\begin{aligned}\mathcal{L}_{eff} = -\frac{4G_F}{\sqrt{2}}V_{cb}\big\{&(1+V_L)\bar{\tau}_L\gamma_\mu\nu_L\bar{c}_L\gamma^\mu b_L + V_R\bar{\tau}_L\gamma_\mu\nu_L\bar{c}_R\gamma^\mu b_R \\ &+ S_L\bar{\tau}_R\nu_L\bar{c}_R b_L + S_R\bar{\tau}_R\nu_L\bar{c}_L b_R\big\} + h.c.,\end{aligned} \tag{4}$$

where, G_F is the Fermi constant, and V_{cb} is the Cabibbo-Kobayashi-Maskawa (CKM) matrix element.

Assuming all of NP Wilson coefficients (WCs) are real, eq. (4) can be further written as [81]

$$\mathcal{L}_{eff} = -\frac{G_F}{\sqrt{2}} V_{cb}\big\{G_V \bar{\tau}\gamma_\mu(1-\gamma_5)\nu_\tau \bar{c}\gamma^\mu b - G_A \bar{\tau}\gamma_\mu(1-\gamma_5)\nu_\tau \bar{c}\gamma^\mu\gamma_5 b$$
$$+G_S \bar{\tau}(1-\gamma_5)\nu_\tau \bar{c} b - G_P \bar{\tau}(1-\gamma_5)\nu_\tau \bar{c}\gamma_5 b\big\} + H.c., \tag{5}$$

where, $G_V = 1 + V_L + V_R$, $G_A = 1 + V_L - V_R$, $G_S = S_L + S_R$ and $G_P = S_L - S_R$.

2.2. Transition form factors

For the semi-leptonic $B_s \to D_s\tau\bar{\nu}$ decay, the hadronic matrix elements of vector and scalar currents can be parameterized in terms of two form factors as [82]

$$\langle D_s(P_{D_s}) \mid \bar{c}\gamma^\mu b \mid B_s(P_{B_s})\rangle = f_+(q^2)\big[p^\mu_{B_s} + p^\mu_{D_s} - \frac{m^2_{B_s} - m^2_{D_s}}{q^2} q^\mu\big]$$
$$+ f_0(q^2)\frac{m^2_{B_s} - m^2_{D_s}}{q^2} q^\mu,$$
$$\langle D_s(P_{D_s}) \mid \bar{c} b \mid B_s(P_{B_s})\rangle = f_0(q^2)\frac{m^2_{B_s} - m^2_{D_s}}{m_b(\mu) - m_c(\mu)}, \tag{6}$$

where, $q^\mu = p^\mu_{B_s} - p^\mu_{D_s}$ is the four momentum transfer.

Using the BCL parameterization approach, the expressions of form factors can be written as [83]

$$P_0(q^2) f_0(q^2) = \sum_{j=0}^{J-1} a_j^{(0)} z^j, \quad P_+(q^2) f_+(q^2) = \sum_{j=0}^{J-1} a_j^{(+)} [z^j - (-1)^{j-J}\frac{j}{J} z^J], \tag{7}$$

where,

$$z(q^2) = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - t_0}}, \quad t_+ = (M_{B_s} + M_{D_s})^2,$$
$$t_0 = (M_{B_s} - M_{D_s})^2, \quad P_{0,+} = 1 - \frac{q^2}{M^2_{0,+}}. \tag{8}$$

Here, $P_{0,+}$ are Blaschke factors, $M_0 = 6.42 \pm 0.10$ and $M_+ = 6.330 \pm 0.009$ are the resonance masses. The values of the coefficients a_j have been given in ref. [82].

2.3. Observables

The differential angular distribution for $B_s \to D_s\tau\bar{\nu}$ decay is [81]

$$\frac{d^2\Gamma(B_s \to D_s\tau\bar{\nu})}{dq^2 d\cos\theta} = \frac{G_F^2 |V_{cb}|^2 q^2 |\vec{p}_{D_s}|}{128\pi^3 m^2_{B_s}} (1 - \frac{m^2_\tau}{q^2})^2$$
$$\times \big[G_V^2 H_0^2 \sin^2\theta + \frac{m^2_\tau}{q^2}[H_0 G_V \cos\theta - H_{tS}]^2\big], \tag{9}$$

where $|\vec{p}_{D_s}| = \sqrt{\lambda(m_{B_s}^2, m_{D_s}^2, q^2)}/2m_{B_s}$ is the D_s momentum in the B_s rest frame, $\lambda(a,b,c) = a^2+b^2+c^2-2(ab+bc+ca)$, and θ is the angle between the three momentum of τ and that of D_s in the $\tau - \bar{\nu}_\tau$ rest frame. The auxiliary functions, H_i, are written as

$$H_0 = \frac{2m_{B_s}|\vec{p}_{D_s}|}{\sqrt{q^2}} f_+(q^2), \quad H_t = \frac{m_{B_s}^2 - m_{D_s}^2}{\sqrt{q^2}} f_0(q^2),$$
$$H_S = \frac{m_{B_s}^2 - m_{D_s}^2}{m_b(\mu) - m_c(\mu)} f_0(q^2), \quad H_{tS} = H_t G_V + \frac{\sqrt{q^2}}{m_\tau} H_S G_S. \tag{10}$$

From eq. (9), integrating out $\cos\theta$, one can obtain the differential decay rate written as

$$\frac{d\Gamma(B_s \to D_s \tau\bar{\nu})}{dq^2} = \frac{G_F^2|V_{cb}|^2 q^2 |\vec{p}_{D_s}|}{96\pi^3 m_{B_s}^2}(1-\frac{m_\tau^2}{q^2})^2\big[G_V^2 H_0^2(1+\frac{m_\tau^2}{2q^2}) + \frac{3m_\tau^2}{2q^2}H_{tS}^2\big]. \tag{11}$$

The corresponding differential branching fraction is defined as

$$\frac{d\mathcal{B}(B_s \to D_s \tau\bar{\nu})}{dq^2} = \tau_{B_s}\frac{d\Gamma(B_s \to D_s \tau\nu)}{dq^2}, \tag{12}$$

where τ_{B_s} is the lifetime of B_s meson.

Then, using the formulas given above, one can obtain the differential and integrated LFU ratios defined as

$$R_{D_s}(q^2) = \frac{d\Gamma(B_s \to D_s \tau\bar{\nu})/dq^2}{d\Gamma(B_s \to D_s \ell\bar{\nu})/dq^2}, \tag{13}$$

$$R_{D_s} = \frac{\mathcal{B}(B_s \to D_s \tau\bar{\nu})}{\mathcal{B}(B_s \to D_s \ell\bar{\nu})}. \tag{14}$$

The lepton-side forward-backward asymmetry is defined as

$$A_{FB}(q^2) = \frac{\int_{-1}^{0} d\cos\theta(d^2\Gamma/dq^2 d\cos\theta) - \int_0^1 d\cos\theta(d^2\Gamma/dq^2 d\cos\theta)}{\int_0^1 d\cos\theta(d^2\Gamma/dq^2 d\cos\theta) + \int_{-1}^{0} d\cos\theta(d^2\Gamma/dq^2 d\cos\theta)}. \tag{15}$$

The convexity parameter is defined as

$$C_F^\tau(q^2) = \frac{1}{d\Gamma/dq^2}\frac{d^2}{d\cos^2\theta}\Big(\frac{d^2\Gamma}{dq^2 d\cos\theta}\Big). \tag{16}$$

The τ polarization fraction is obtained via

$$P_L^\tau(q^2) = \frac{d\Gamma^{\lambda_\tau=1/2}/dq^2 - d\Gamma^{\lambda_\tau=-1/2}/dq^2}{d\Gamma^{\lambda_\tau=1/2}/dq^2 + d\Gamma^{\lambda_\tau=-1/2}/dq^2}, \tag{17}$$

where $d\Gamma^{\lambda_\tau=1/2}/dq^2$, $d\Gamma^{\lambda_\tau=-1/2}/dq^2$ are written as

$$\frac{d\Gamma^{\lambda_\tau=1/2}}{dq^2} = \frac{G_F^2|V_{cb}|^2|\vec{p}_{D_s}|m_\tau^2}{192\pi^3 m_{B_s}^2}(1-\frac{m_\tau^2}{q^2})^2\big[G_V^2 H_0^2 + 3H_{tS}^2\big],$$
$$\frac{d\Gamma^{\lambda_\tau=-1/2}}{dq^2} = \frac{G_F^2|V_{cb}|^2 q^2|\vec{p}_{D_s}|}{96\pi^3 m_{B_s}^2}(1-\frac{m_\tau^2}{q^2})^2 G_V^2 H_0^2. \tag{18}$$

3. Brief review of NP models

In this section, we review briefly four types of NP models, which are the RPV SUSY model [74,75], W' model [76], the vector leptoquark model [77] and A2HDM [78].

3.1. RPV SUSY model

The most general R-parity violating superpotential are written as [74,75],

$$W_{RPV} = \mu_i L_i H_u + \frac{1}{2}\lambda_{ijk} L_i L_j E_k + \lambda'_{ijk} L_i Q_j D_k + \frac{1}{2}\lambda''_{ijk} U_i D_j D_k\,, \tag{19}$$

where L and Q are the $SU(2)$ doublet lepton and quark superfields, E and $U(D)$ denote the singlet lepton and quark superfields, respectively, i, j and k are generation indices. The λ''_{ijk} couplings break the proton stability. For the B meson semi-leptonic decays, the contribution of λ term is strongly suppressed relative to the one of λ' term [84], so only the contribution of λ' coupling is considered in this paper.

The λ' couplings can be expanded in terms of fermions and sfermions as [84]

$$\begin{aligned}\Delta\mathcal{L}_{RPV} = -\lambda'_{ijk}\big[&\tilde{\nu}^i_L \bar{d}^k_R d^j_L + \tilde{d}^j_L \bar{d}^k_R \nu^i_L + \tilde{d}^{k*}_R \bar{\nu}^{ci}_R d^j_L - V_{jl}(\tilde{\ell}^i_L \bar{d}^k_R u^l_L + \tilde{u}^l_L \bar{d}^k_R \ell^i_L + \tilde{d}^{k*}_R \bar{\ell}^{ci}_R u^l_L)\big] \\ &+ h.c.,\end{aligned} \tag{20}$$

where V_{jl} denotes the CKM matrix element. All the SM fermions $d_{L,R}$, $\ell_{L,R}$ and ν_L are in their mass eigenstates, and we assume that the sfermions are also in their mass eigenstates. Meanwhile, we adopt the setting that only the third family is effectively supersymmetrized [85]. As has been discussed in ref. [86,87], the first two generation sfermions can be decoupled from the low-energy spectrum. In addition, for simplicity, we take the assumption that all λ' couplings as real.

With the RPV SUSY contributions, the tree-level sbottom exchange gives

$$V_L = \frac{v^2}{4m^2_{\tilde{b}_R}} \lambda'_{333} \sum_{j=1}^{3} \lambda'^*_{3j3} (\frac{V_{ij}}{V_{i3}}), \tag{21}$$

where $v = 246$GeV is the Higgs vacuum expectation value. V_L in eq. (21) is calculated at the matching scale $\mu = m_{\tilde{b}_R}$, it does not need to be run down to the scale $\mu = m_b$ because of the conservation of vector currents.

Under the constraints form current measurements on the branching ratios of $B^+ \to \pi^+ \nu\bar{\nu}$ and $B^+ \to K^+ \nu\bar{\nu}$, the leptonic W (Z) couplings $g_{W_{\tau_L \nu_\tau}}/g_{W_{\ell_L \nu_\ell}}$ $(g_{Z_{\tau_L \nu_\tau}}/g_{Z_{\ell_L \nu_\ell}})$, $R_{D^{(*)}}$, $R_{J/\psi}$, $P^\tau_L(D^*)$ and $P^{D^*}_L$, the numerical results of λ' couplings have been obtained at the 2σ level, [88]

$$\begin{aligned}-0.082 < \lambda'_{313}\lambda'^*_{333}(\frac{m_{\tilde{b}_R}}{\text{TeV}})^2 < 0.087,\\ 0.018 < \lambda'_{323}\lambda'^*_{333}(\frac{m_{\tilde{b}_R}}{\text{TeV}})^2 < 0.057,\\ 0.033 < \lambda'_{333}\lambda'^*_{333}(\frac{m_{\tilde{b}_R}}{\text{TeV}})^2 < 0.928.\end{aligned} \tag{22}$$

3.2. W' model

The Lagrangian describing the couplings of a W' boson to quarks and leptons can be generally written as [76]

$$\mathcal{L}_{eff}^{W'} = \frac{W'}{\sqrt{2}}\left[\bar{u}_i \gamma^\mu (\epsilon^L_{u_i d_j} P_L + \epsilon^R_{u_i d_j} P_R) + \bar{\ell}_i \epsilon^L_{\ell_i \nu_j} \gamma^\mu P_L \nu_j\right] + h.c., \tag{23}$$

where $P_{L(R)} = \frac{1\mp\gamma_5}{2}$ is the left (right)-handed chirality projector; $u_i \in (u, c, t)$, $d_j \in (d, s, b)$ and $\ell_i, \ell_j \in (e, \mu, \tau)$; $\epsilon^L_{u_i d_j}$, $\epsilon^R_{u_i d_j}$ and $\epsilon^L_{\ell_i \nu_j}$ are the effective flavor-dependent coupling parameters. We neglect the contributions of right-handed neutrinos, and assume that NP is only sensitive to the third generation of leptons, *i.e.* $\epsilon^L_{e\nu_e} = \epsilon^L_{\mu\nu_\mu} = 0$. In addition, we also take the relevant parameters to be real.

Comparing eq. (23) with eq. (5), one can obtain the contribution of this NP model to $b \to c\tau\bar{\nu}$ transition written as

$$V_L = \frac{\sqrt{2}}{4G_F V_{cb}} \frac{\epsilon^L_{cb}\epsilon^L_{\tau\nu_\tau}}{M^2_{W'}},$$
$$V_R = \frac{\sqrt{2}}{4G_F V_{cb}} \frac{\epsilon^R_{cb}\epsilon^L_{\tau\nu_\tau}}{M^2_{W'}}, \tag{24}$$

where $M_{W'}$ is the mass of the W' boson. Recent studies [79,89,90] show that the operator $\bar{\ell}_L \gamma_\mu \nu_L \bar{c}_R \gamma^\mu b_R$ does not contribute to LFU violation at leading order based on the effective field theory of SM (SMEFT) [91,92], therefore we do not consider the contribution of this operator in this paper.

The parameter $\epsilon^L_{cb}\epsilon^L_{\tau\nu_\tau}$ in eq. (24) has been discussed in many works [76,93–95]. Especially, in ref. [93], after considering the $R_{D^{(*)}}$ anomalies and the mono-tau signature at the LHC in the left-handed W' model, the authors obtain $\epsilon^L_{cb}\epsilon^L_{\tau\nu_\tau} = 0.14 \pm 0.03$ with $M_{W'} \in [0.5, 3.5]$TeV, which is consistent with the result of ref. [48]. Recently, the authors of ref. [76] carry out a χ^2 analysis with the seven latest experimental observables, such as $R_{D^{(*)}}$, $R_{J/\psi}$, $P^\tau_L(D^*)$, $P^{D^*}_L$, the branching fraction of $B_c \to \tau\nu$ and the ratio R_{X_c} of inclusive semileptonic B decays, and obtain the best fitted value $\epsilon^L_{cb}\epsilon^L_{\tau\nu_\tau} = 0.11$ with $M_{W'} = 1$TeV, which agrees with previous studies. In order to maximize the effects of NP, it is suggested in ref. [96] that

$$\epsilon^L_{cb}\epsilon^L_{\tau\nu_\tau} = (0.12 \pm 0.03)\left(\frac{M_{W'}}{\text{TeV}}\right)^2, \tag{25}$$

which is employed in our following discussions but with the allowed space at 2σ level.

3.3. Vector leptoquark model

In ref. [77], the authors extend the SM by a vector $SU(2)$ triplet leptoquark U_3, which can generate purely left handed currents with quarks and leptons. The vector triplet U_3, which transforms under the SM gauge group $(3, 3, 2/3)$, couples to a leptoquark current with $V - A$ structure,

$$\mathcal{L}_U = g_{ij} \bar{Q}_i \gamma_\mu \tau^A U^A_{3\mu} L_j + h.c., \tag{26}$$

where $i, j = 1, 2, 3$ are generation indices; the couplings g_{ij} are in general complex parameters, while we take the constraint that they are real for simplicity; $\tau^A (A = 1, 2, 3)$ are the Pauli matrices in the $SU(2)_L$ space; and $Q_i(L_j)$ are the left-handed quark and lepton doublets.

In the mass basis, the Lagrangian in eq. (26) is written with g_{ij} defined as the couplings of the $Q = 2/3$ component of the triplet $U^{2/3}_{3\mu}$ to $\bar{d}_{Li}$ and l_{Lj}. With the CKM matrix $\mathcal{V}$ from the left or PMNS (Pontecorvo-Maki-Nakagawa-Sakata) matrix $\mathcal{U}$ from the right, we obtain three types of vertices by rotating the g matrix,

$$\begin{aligned}\mathcal{L}_{U_3} = & U_{3\mu}^{2/3}[(\mathcal{V}g\mathcal{U}^\dagger)_{ij}\bar{u}_i\gamma^\mu P_L\nu_j - g_{ij}\bar{d}_i\gamma^\mu P_L l_j] \\ & + U_{3\mu}^{5/3}(\sqrt{2}\mathcal{V}g)_{ij}\bar{u}_i\gamma^\mu P_L l_j \\ & + U_{3\mu}^{-1/3}(\sqrt{2}g\mathcal{U}^\dagger)_{ij}\bar{d}_i\gamma^\mu P_L\nu_j. \end{aligned} \tag{27}$$

From such Lagrangian, the vector multiplet U_3^μ can mediate the $b \to c\tau\bar{\nu}$ transition at tree level. The contribution of this NP model can be written as

$$V_L = \frac{\sqrt{2}}{4G_F V_{cb}}\frac{g_{b\tau}^*(\mathcal{V}g)_{c\tau}}{M_U^2}. \tag{28}$$

Fitting to the measured ratios $R_{D^{(*)}}$ with acceptable q^2 spectra, the two best-fit solutions of $g_{b\tau}^*(\mathcal{V}g)_{c\tau}$ in eq. (28) have been obtained in ref. [97],

$$g_{b\tau}^*(\mathcal{V}g)_{c\tau} = (\frac{M_U}{\text{TeV}})^2\begin{cases} 0.18 \pm 0.04 & S_1 \\ -2.88 \pm 0.04 & S_2 \end{cases} \tag{29}$$

According to the forms of V_L in eq. (24) and eq. (28), it is obvious that the contributions of the two NP models depending on the values of $\epsilon_{cb}^L\epsilon_{\tau\nu_\tau}^L$ and $g_{b\tau}^*(\mathcal{V}g)_{c\tau}$, respectively. In order to obtain the maximum differences between them, we take solution S_2 of $g_{b\tau}^*(\mathcal{V}g)_{c\tau}$ at 2σ level in our discussion.

3.4. A2HDM

The most general Yukawa Lagrangian of the 2HDM is given as [78]

$$\begin{aligned}\mathcal{L}_Y = & \frac{\sqrt{2}}{v}\big[\bar{Q}_L'(M_d'\Phi_1 + Y_d'\Phi_2)d_R' + \bar{Q}_L'(M_u'\tilde{\Phi}_1 + Y_u'\tilde{\Phi}_2)u_R' \\ & + \bar{L}_L'(M_\ell'\Phi_1 + Y_\ell'\Phi_2)\ell_R' + h.c., \end{aligned} \tag{30}$$

where $\bar{Q}_L'$ and $\bar{L}_L'$ are the left-handed quark and lepton doublets, u_R', d_R' and ℓ_R' are the corresponding right-handed singlets, $\tilde{\phi}_a(x) \equiv i\tau_2\phi_a^*(x)$ are the charge-conjugated scalar doublets with hypercharge $Y = -\frac{1}{2}$. $M_f'(f = d, u, \ell)$ are the non-diagonal fermion mass matrices, and Y_f' are the couplings matrices to the scalar doublet with zero vacuum expectation value.

Because the Yukawa matrices M_f' and Y_f' couple to a given right-handed fermion field and can not be diagonalized simultaneously, the flavor-changing-neutral-current (FCNC) can be generated at tree-level. A simply way to avoid tree-level FCNC is to require the alignment in flavor space of the Yukawa matrices. The Yukawa alignment can make the matrices Y_f' proportional to M_f', thus they can be diagonalized simultaneously with the following results

$$Y_{d,\ell} = \varsigma_{d,\ell}M_{d,\ell}, \quad Y_u = \varsigma_u^* M_u, \tag{31}$$

where $\varsigma_f(u, d, \ell)$ are arbitrary complex parameters and thus can introduce new sources of CP violation beyond the SM. We assume ς_f is real in this paper.

In terms of the fermion mass-eigenstate fields, the interactions of the charged scalars can be written as

$$\mathcal{L}_{H^\pm} = -\frac{\sqrt{2}}{v}H^+\big\{\bar{u}[\varsigma_d V M_d P_R - \varsigma_u M_u^\dagger V P_L]d + \varsigma_\ell\bar{\nu}M_\ell P_R\ell\big\} + h.c., \tag{32}$$

where V is CKM matrix.

Table 1
Predictions for $\mathcal{B}(B_s \to D_s\tau\bar{\nu})$ and R_{D_s} in the SM, RPV SUSY model, W' model, vector leptoquark model and A2HDM.

Observables	SM	RPV SUSY	W'	Vector leptoquark	A2HDM
$\mathcal{B}(B_s \to D_s\tau\bar{\nu})\%$	$0.672^{+0.099}_{-0.094}$	[0.606,0.800]	[0.656,0.954]	[0.641,1.173]	[0.273,0.884]
R_{D_s}	$0.300^{+0.051}_{-0.045}$	[0.260,0.343]	[0.259,0.353]	[0.257,0.354]	[0.083,0.375]

According to eq. (5) and eq. (32), the contribution of A2HDM to $b \to c\tau\bar{\nu}$ can be written as

$$G_S = \frac{\varsigma_d m_b - \varsigma_c m_c}{M^2_{H^\pm}}(\varsigma^*_\tau m_\tau),$$
$$G_P = -\frac{\varsigma_d m_b + \varsigma_c m_c}{M^2_{H^\pm}}(\varsigma^*_\tau m_\tau), \tag{33}$$

which need to be run down from $\mu = M_{H^\pm}$ to $\mu = m_b$, the explicit evolution equations can be found in ref. [25].

Using the constraints from leptonic B, D and D_s decays, the allowed ranges of $\frac{\varsigma^*_\ell \varsigma_d}{M^2_{H^\pm}}$ and $\frac{\varsigma^*_\ell \varsigma_u}{M^2_{H^\pm}}$ have been obtained in ref. [98],

$$\frac{\varsigma^*_\ell \varsigma_d}{M^2_{H^\pm}} \in \left\{ \begin{array}{ll} [-0.036, 0.008] & \mathrm{GeV}^{-2} \\ [0.064, 0.108] & \mathrm{GeV}^{-2} \end{array} \right., \quad \frac{\varsigma^*_\ell \varsigma_u}{M^2_{H^\pm}} \in \left\{ \begin{array}{ll} [-0.006, 0.037] & \mathrm{GeV}^{-2} \\ [0.511, 0.535] & \mathrm{GeV}^{-2} \end{array} \right. . \tag{34}$$

Recently, using the model-independent approach, the authors of ref. [99] performed a χ^2 fit of the NP WCs with the latest experimental observables, such as $R_{D^{(*)}}$, $R_{J/\psi}$, $P^\tau_L(D^*)$, $P^{D^*}_L$ and the branching fraction of $B_c \to \tau\nu$. Their results indicate that, if considering the contributions of two different NP couplings (S_L, S_R) simultaneously, the NP model are disfavored since $\mathcal{B}(B_c \to \tau\nu)$ obtained in this case is much larger than 10%. Considering the contributions related to S_L and S_R, respectively, the best fit values of them are obtained. However, the $\chi^2_{\rm min}$ of the best fit results are rather large still. In this paper, combining their best fit values, we take reasonably one of these four combinations in eq. (34) in our calculation, which are

$$\frac{\varsigma^*_\ell \varsigma_d}{M^2_{H^\pm}} \in [-0.036, 0.008]\,\mathrm{GeV}^{-2}, \quad \frac{\varsigma^*_\ell \varsigma_u}{\mathrm{M}^2_{\mathrm{H}^\pm}} \in [-0.006, 0.037]\,\mathrm{GeV}^{-2}. \tag{35}$$

4. Numerical analyses

In the numerical calculations, except for the transition form factors and the NP coupling parameters given above, the values of the other input parameters are taken from the Particle Data Group (PDG) [100].

Using the above formulas and constrained NP parameter spaces, we calculate $\mathcal{B}(B_s \to D_s\tau\bar{\nu})$ and R_{D_s}, and collect their numerical results in Table 1. From these results it can be found that:

- For $\mathcal{B}(B_s \to D_s\tau\bar{\nu})$ and R_{D_s}, their SM results are consistent with ones of ref. [70], but they are generally smaller than the PQCD predictions as given in ref. [73].
- $\mathcal{B}(B_s \to D_s\tau\bar{\nu})$ is quite sensitive to the contributions of these four NP models. The vector leptoquark model effects improve it maximally by about 5σ level compared with the SM prediction, while the A2HDM effects can also reduce its SM prediction by about 4σ level.

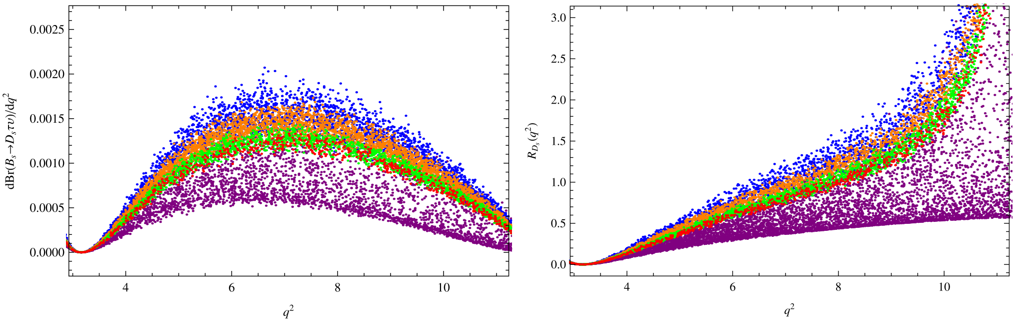

Fig. 1. The dependence of $\frac{d\mathcal{B}}{dq^2}$ (left) and $R_{D_s}(q^2)$ (right) on q^2. The red, green, orange, blue and purple dots correspond to the SM, RPV SUSY model, W' model, vector leptoquark model and A2HDM, respectively. (For interpretation of the colors in the figure(s), the reader is referred to the web version of this article.)

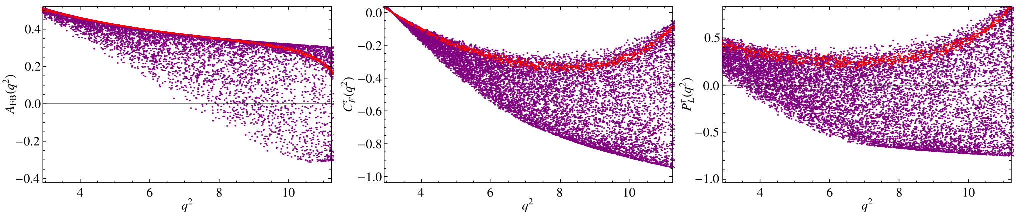

Fig. 2. The q^2-dependence of $A_{FB}(q^2)$ (left), $C_F^\tau(q^2)$ (middle) and $P_L^\tau(q^2)$ (right). The red and purple dots correspond to the SM and A2HDM, respectively.

- Except for the A2HDM, R_{D_s} is insensitive to the contributions of the other three NP models, the deviations from the SM prediction lie within the SM 1σ error band, which implies that the RPV SUSY model, W' model and the vector leptoquark model can not explain R_{D_s} and $R_{D^{(*)}}$ anomalies simultaneously. Interestingly, the A2HDM effects reduce R_{D_s} by about 4σ level relative to the SM prediction. Therefore, future measurements on $\mathcal{B}(B_s \to D_s \tau \bar{\nu})$ and R_{D_s} can be used to distinguish the hints of these NP models.

Considering the contributions of these four NP models, we also show the dependences of $\frac{d\mathcal{B}}{dq^2}$ and $R_{D_s}(q^2)$ on q^2 in Fig. 1. For $A_{FB}(q^2)$, $C_F^\tau(q^2)$ and $P_L^\tau(q^2)$, we show only the A2HDM effects in Fig. 2 because that new operators are not generated in the other three NP models and their effects in the numerator and the denominator of these ratios are canceled out exactly. Based on these figures, we have the following remarks:

- In Fig. 1, both $\frac{d\mathcal{B}}{dq^2}$ and $R_{D_s}(q^2)$ are sensitive to these four NP models. With the exception of the A2HDM, the SM results of $\frac{d\mathcal{B}}{dq^2}$ and $R_{D_s}(q^2)$ can be enhanced variously by the contributions of the rest three NP models in the whole q^2 region. In general, the contributions of W' model to $\frac{d\mathcal{B}}{dq^2}$ and $R_{D_s}(q^2)$ are larger than those of the RPV SUSY model, and the contributions of the vector leptoquark model are larger than those of the W' model. While, the A2HDM effects reduce their theoretical predictions at each q^2 points.
- In Fig. 2, for $A_{FB}(q^2)$, $C_F^\tau(q^2)$ and $P_L^\tau(q^2)$, the A2HDM effects are obvious. Comparing with the SM prediction, all of them can be considerably reduced by the A2HDM effects, and

for each observables, the deviation from the SM prediction becomes more significant with the increasing of q^2.

5. Conclusions

In this paper, using the constraints from $R_{D^{(*)}}$ and some other experimental data, we study the $B_s \to D_s \tau \bar{\nu}$ decay in the RPV SUSY model, the W' model, the vector leptoquark model and the A2HDM. The observables $\mathcal{B}(B_s \to D_s \tau \bar{\nu})$ and R_{D_s} are calculated within the SM and these four NP models. It is found that $\mathcal{B}(B_s \to D_s \tau \bar{\nu})$ is quite sensitive to the contributions of these four NP models, but R_{D_s} is only sensitive to the A2HDM effects. In addition, we also have discussed the contributions of these four NP models to $\frac{d\mathcal{B}}{dq^2}$, $R_{D_s}(q^2)$, $A_{FB}(q^2)$, $C_F^\tau(q^2)$ and $P_L^\tau(q^2)$. It is found that these four NP models have obvious effects on $\frac{d\mathcal{B}}{dq^2}$ and $R_{D_s}(q^2)$. However, for $A_{FB}(q^2)$, $C_F^\tau(q^2)$ and $P_L^\tau(q^2)$, the RPV SUSY model, the W' model and the vector leptoquark model present the same predictions as the SM since no new operators are generated within these three NP model, while the A2HDM effects are very significant. The study of these observables can serve as a good tool for probing and distinguishing the hints of these NP models with more precise measurements in the near future.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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