

EXPERIMENTAL STATUS OF QUANTUM ELECTRODYNAMICS

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It is a privilege for me to participate and speak at this symposium honoring the 60th birthday of Julian Schwinger. Although in the literal sense I am not a student of his, like all of us here I am, in the broad and true sense of the word, a student of his contributions and creations.

To all scientists educated in the era starting after World War II the fundamental achievements in quantum field theory of Julian Schwinger, together with Freeman Dyson, Richard Feynman, and Shin Ichirō Tomonaga, provided the concepts, the language, and the tools of our own growth and understanding. They might even be called the *Four Horsemen* of modern field theory who transformed the famine, the pestilence, the frustration, and the nightmare that was quantum field theory, with its divergence diseases and maladies, into today's remarkably flourishing and prodigiously successful quantum electrodynamics. At present QED has met all experimental challenges and serves as the literal model that guides our striving to understand the weak as well as the strong interactions.¹ In contrast to the *Four Horsemen* of the Apocalypse of dramatic lore, Dyson, Feynman, Schwinger, and Tomonaga were to modern field theory

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in 1948 what the legendary *Four Horsemen* of Notre Dame were to modern football, as Grantland Rice immortalized them following the 1924 Army-Notre Dame game when he wrote "*outlined against a blue-gray October sky, the four horsemen rode again*". Now, thirty years after the publication of their original papers² on modern field theory and their formulation of the renormalization program, we are here to honor Julian Schwinger on his 60th birthday.

Thus far, Julian has heard, and following my talk will continue to hear, richly deserved praise and accolades. From me, Julian, you will hear a complaint and, more than that, a plea to which I shall come shortly. As a green graduate student I first learned of your work shortly after the Shelter Island Conference when your prediction of $a_e \equiv \frac{1}{2}(g - 2) = \frac{\alpha}{2\pi}$ for the electron's magnetic moment anomaly was published and confirmed by experiment (see Figure 1). Now, thirty years later, the precision of both experiment and theory has advanced considerably to the point shown in Figure 2. The correction to the Dirac moment is now known to an accuracy of roughly two parts in ten million.

This experimental precision has been achieved by means of a radio-frequency resonance method.³ Circulating electrons in a magnetic field along which they are polarized will be spin-flipped and depolarized if they are subjected to a perturbing field of frequency $\omega_{\text{spin}} = \frac{1}{2} g \left(\frac{e}{mc} \right) B$. Their spin will also flip if the frequency $\omega_a = \frac{1}{2}(g - 2) \left(\frac{e}{mc} \right) B$ is applied because the electron in its rest frame sees ω_a combined with the cyclotron frequency $\omega_c = \left(\frac{e}{mc} \right) B$ which again gives the depolarizing frequency. One determines both resonance frequencies in the same magnetic field, and hence $a_e = \frac{1}{2}(g - 2)$ with very high precision. The

theoretical result shown is the result of a 6th order perturbation calculation.⁴ The indicated error in the 6th order contribution reflects the error both in the value of the fine structure constant, α , and in the integrations which have been carried through numerically. The numerical calculations involve 7-dimensional integrals performed by random variable sampling techniques. The numerical results in Figure 2 were obtained after 16 iterations, each involving 5 million points. The computer also does the trace algebra involved in these impressive calculations. The two error sources in the theoretical calculation are as shown. The dominant contribution comes from the uncertainty in α which, as of the present, has been determined most accurately from measurements based on the Josephson effect determination of $e/h = v/2V$, where V is the voltage at which steps appear in the dc current induced when the junction is radiated with microwaves of known frequency ν .

The error limits shown in Figure 2 on both the measured and calculated values of a_e are being reduced by work now under way. Measurements of a_e using line-splitting methods and stronger B fields, and thereby less obscured by line broadening, lead to an expectation of a factor of 10 to 20 reduction in the error limits in the very near future⁵ as indicated in Figure 3. On the theoretical side, there is anticipated progress to report both in the input value for the fine structure constant and in the evaluation of the 6th order radiative contribution. The limit on the accuracy of α via the Josephson junction relation^{1, 6} as shown in Figure 3 comes from the measurement of the proton's gyromagnetic ratio which is currently known to an accuracy of .42 parts per

million. An order of magnitude improvement in this is in the works based on a very accurate measurement of the dimensions of a precision solenoid.⁷ This will allow a more precise determination of \vec{B} and, thereby, of the gyromagnetic ratio $\gamma'_p = \frac{\omega'_p}{B}$ from the observed nuclear magnetic resonance absorption frequency of protons in water. At the same time, there are further accuracy improvements in progress⁴ in the numerical evaluation of the 6th order contribution to a_e , as well as in the development of analytic techniques for evaluating the integrals. Figure 3 summarizes the anticipated progress and indicates that we are coming to the point at which the 8th order calculations must be done.

This now is my plea to Julian. This calculation is getting out of hand! We implore you to get to work, as you did in the original $\frac{\alpha}{2\pi}$, and tell us now what is the function $a_e = f(\alpha)$. The electron's $(g - 2)$ value is a fundamental QED prediction, in its most pure form, for a single electron. Thirty years ago this quantity provided the first direct, basic test of QED and also its most stunning triumph. In particular, it was the first quantity to demonstrate the consistency of the renormalization procedure in 4th order perturbation theory via the landmark calculation of Karplus and Kroll.⁸ Since one is dealing here with a system of light mass, that is the electron, the virtual momenta appearing in the virtual loops are effectively small $\sim m_e c$. Therefore small distance contributions from other more massive particles, such as the muons and hadrons, which contribute at high momenta via the photon's vacuum polarization corrections, are negligible even at the level of three parts in 10^{12} . Contributions from the weak currents in a standard renormalizable weak interaction gauge theory are another^{1, 4}

factor of 10^2 still smaller. On the other hand, the scattering of light by light insertion which is approximately $.4\left(\frac{\alpha}{\pi}\right)^3$, or 5000 parts in 10^{12} , has been determined to better than a 5% accuracy by the measurements already performed. A final comment on this beautiful process is that the measurements of the positron $g - 2$ value have given a precise check on the TCP mandated equality of electron and positron $g - 2$ values that is accurate to roughly a part per million.¹

Let us turn next to the muon's $g - 2$ value. Here we run into another class of interesting phenomena which are no longer direct checks of quantum electrodynamics. Rather they are probes of the theory at higher momenta, on the scale $m_\mu c \approx 200 m_e c$, and they include contributions of the hadrons to the photon's vacuum polarization. This is shown and summarized in Figures 4 and 5. The precise experimental results are obtained from the beautiful $g - 2$ experiment⁹ at CERN in which the rotation of the spins of the polarized muons from pion decay relative to their momenta as they circulate in a muon storage ring is observed. In order to achieve very high precision it is useful to work at the "magic" momentum of 3.094 GeV. For muons circulating at this momentum, i.e., for $a_\mu = \frac{1}{2}(g - 2) = \left(\frac{1}{\gamma^2} - 1\right)$, where $\gamma = E_\mu/m_\mu c^2$, there is no shift in the precession frequency due to the transverse electric field which traps them in the ring.¹⁰ This value of the momentum permits a very precise measurement independent of the electric field strength. The muon's spin precession is detected and analyzed from the pattern of its decay electrons. Figure 4 summarizes the individual contributions. The hadron vacuum polarization contribution to the photon propagator is related by a dispersion relation to the electron-positron

annihilation cross section as shown in Figure 5, which also identifies individual contributions.¹¹ We note that the threshold region in electron-positron annihilation below the $\rho - \omega$ resonance region, which has not been measured with great precision, leads to the greatest uncertainty in the comparison between theory and experiment at this point. The weak interaction contributions via a renormalizable gauge theory are definite and finite, and depend on details of the theory. They are given characteristically¹ by the dimensionless number indicated in Figure 6 and are smaller than the present sensitivity of the measurements. Also shown is the current status of the theoretical-experimental comparison. What we are describing here is, of course, no longer a test of pure QED, but is direct evidence of a hadronic contribution to QED via the photon's vacuum polarization insertion. However the consistency between theory and experiment shown in Figure 6 confirms the approximate correctness of the fourth order pure QED for muons and of the enormous scattering of light-by-light contributions.

The next test of QED which we come to is the renowned Lamb shift in hydrogen. An energy level diagram including both fine structure and hyperfine structure splittings is shown in Figure 7. The experimental value for the Lamb shift is compared with two theoretical numbers¹² which differ by the estimates made in evaluating higher order Coulomb contributions. This difference is indicative of the enormous difficulty in performing these calculations. Also shown is the accuracy with which the vacuum polarization and the finite proton size contributions to Lamb shift are determined within the experimental uncertainties. The conclusion from this is that, at present, there is full

agreement between theory and experiment for the Lamb shift and that further precision may probe finite proton size contributions, but is not expected to further illuminate fundamental properties of QED itself. I am sure that many of you recall the long standing discrepancies that have marked the history of the Lamb shift. These seem to be part of the past, and, indeed, the total vacuum polarization contribution is now confirmed to an accuracy of a few tenths of a percent.

The hydrogen fine structure splitting between the $2p_{3/2}$ and $2p_{1/2}$ levels is insensitive to most of the quantum electrodynamic contributions and relativistic recoil effects since the p-wave electron wave functions vanish at the origin. This splitting is given accurately by the simple Dirac-Sommerfeld formula for the fine structure with reduced mass and anomalous magnetic moment contributions included. Indeed, the radiative self-energy effects, which are of order $\frac{\alpha}{\pi} (Z\alpha)^2 \ln \frac{1}{(Z\alpha)^2}$ contribute only ≈ 1.2 parts per million.⁶ The $2p_{3/2} - 2p_{1/2}$ splitting is excellent for determining the fine structure constant α to an accuracy of parts per million. As we already discussed, the Josephson junction relation gives a value of α that is an order of magnitude more precise, and so we only note here that there is full agreement between the values of α determined from these two methods.

In contrast to the hydrogen p-state fine structure, the hyperfine splitting in atomic hydrogen, which measures the interaction of the electron with the proton's magnetic field, is and remains an important historic link between the fields of high energy physics and precision atomic physics.⁶ This is because this interaction is of short range

and is sensitive to details of the proton structure and dynamics that are usually seen only in the high energy electron-proton scattering experiments. Historically, the primary significance of the hyperfine structure has been as a probe of QED behavior at small distances. The QED corrections to the original Fermi formula have been computed to an accuracy of parts per million including anomalous magnetic moment contributions, electron binding contributions, radiative corrections, and the vacuum polarization corrections.

At this level of precision the finite proton radius gives rise to an important contribution of -38.2 parts per million to the hyperfine splitting as first calculated by Zemach.¹³ There is an additional contribution at the parts per million level of accuracy due to the proton's internal dynamics. This dynamics and its spin dependence has been studied in hadron resonance excitation accompanying photon absorption, and in the spin dependence of deep inelastic electron scattering from a hydrogen target. The polarization of the proton structure by the electromagnetic field of a circulating electron in an s-orbit is also sensitive to this dynamics which causes a shift in the energy levels. This is akin to the A. Bohr effect¹⁴ in deuterium hyperfine structure which reflects the degree to which the atomic electron can adjust its orbit to follow the proton charge in the deuterium nucleus instead of circulating around the nuclear mass distribution. This effect is much smaller in magnitude in hydrogen since the proton is a much tighter structure than is a deuterium. For example a typical excitation energy for the proton is ≈ 300 MeV for the Δ -resonance, whereas a deuteron is bound by only 2.2 MeV. Estimates of the polarizability contribution¹⁵

come to but a few parts per million in accord with the agreement between theory and experiment which indicates

$$\frac{\nu_{\text{Hfs}}^{\text{th}} - \nu_{\text{Hfs}}^{\text{expt}}}{\nu_{\text{H}}^{\text{th}}} \sim (0.5 \pm 1.2) \times 10^{-6}.$$

As a result of the proton structure contributions the incredible experimental accuracy of 2 parts in 10^{12} as shown in Figure 7 cannot be exploited theoretically as a challenge to QED! However, we can avoid the complications of proton dynamics by studying the hyperfine structure in purely electromagnetic systems such as muonium and positronium. Here again, as the numbers¹⁶ in Figure 8 show, we find theory falling behind experiment.

Although the experimental precision exceeds the theoretical accuracy by an order of magnitude or more, there has been significant recent theoretical progress¹⁶ in the calculation of the hyperfine splitting. This has been achieved by progress in dealing with what has long been a *bête noir* of theoretical physicists: the relativistic bound state problem. This problem presents major practical calculational difficulty: the Coulomb interaction between two bound charged particles cannot be treated perturbatively, but must be given an exact treatment. Once the Coulomb bound state function is determined one must use it to compute, with a manageably finite amount of labor, the retardation and recoil corrections to a part per million accuracy in order to compare with the data that have achieved such accuracy. However the bound state equation for relativistic quantum field theory (called commonly the Bethe-Salpeter equation, but also originally proposed by Schwinger and Nambu)¹⁷ is

extremely difficult to work with because no exact Coulomb bound state solutions exist for two relativistic particles with spin. A formalism pioneered by Franz Gross in 1969 offers a way of doing an end run around the worst of these difficulties.¹⁸ Its development during the last two years by Lepage,¹⁶ working with Brodsky and Caswell,¹⁹ has now led to a much more tractable scheme of calculation. The procedure used involves studying the bound state equation and interaction kernel with one of the two charged particles restricted to its mass shell. The resulting formalism is effectively that of a one-time bound state equation. It contains less information than the full Bethe-Salpeter equation; but, as has been shown in these works, it has all that is needed for determining the energy spectrum. Moreover the equation with one of the particles restricted to the mass shell has a known exact simple Coulomb solution in terms of which all contributions to the s-state energies have been calculated through order $\alpha^2 \log \alpha$, for positronium, and through $\alpha^2 \frac{m_e}{m_\mu} \log \alpha$ for muonium. The corrections to the Fermi energy of order α^2 , without the $\log \alpha$, remain still to be evaluated for theory to catch up with experiment as shown in Figure 8.

The recent experimental progress²⁰ in muonium has resulted from precise microwave magnetic resonance measurements of the linear and quadratic Zeeman splittings in strong magnetic fields and over a range of pressures, permitting thereby an accurate extrapolation to remove the pressure-dependent contributions. These beautiful results provide a precision confirmation of muon-electron universality in the QED scheme of things.

Turning next to positronium, one has in addition to the Fermi interaction, the electron-positron annihilation channel. This has the effect of pushing the $3S_1$ level, which is odd under charge conjugation and can annihilate to a single virtual gamma, above the $1S_0$ level. Their energy splitting is shown in Figure 9. The experimental technique is to induce microwave magnetic resonance transitions between Zeeman levels and observe the frequencies at which the 3γ yield is quenched relative to the 2γ yield in the positronium annihilation. Again the theorists must struggle for another order of magnitude improvement in order to catch up with our experimental colleagues!

Figure 9 also indicates the precision with which the splitting of the first excited states of positronium has now been measured and agrees with theory. As a final comment, and one may view this as a special birthday present to Julian, I show in Figure 9 the decay rate for the annihilation to 3γ 's of orthopositronium, i.e., of the $n = 1$ $3S_1$ level of positronium. This is the only annihilation rate of a purely quantum electrodynamic system which has been measured to better than a 1% accuracy. As one sees in Figure 10 prepared by Peter Lepage, theory and experiment agree to 1 standard deviation, although the figure shows that a residual discrepancy of as much as 10 standard deviations was only very recently removed by efforts on the part of both the theorists and experimentalists.²¹ There are other precision measurements that one can also recount, all of which find theory and experiment in accord. These include helium fine structure¹ ($n = 2$ $3p_1 - 3p_0$ splitting) which provides another determination of the fine structure constant to an accuracy of roughly one part in a million, and muonic X rays from $\mu^- \text{Bi}$

and μ^- Pb as well as other high Z muonic atoms which have now been measured^{1, 22} to better than 40 parts per million accuracy. These results confirm vacuum polarization corrections due to electron loops to a precision of a few percent. There are also accurate Lamb shift measurements in μ -proton as well as measurements of the $2S_{1/2} - 2P_{3/2}$ separation in μ -helium.^{1, 23}

Finally we also refer to the high energy tests which probe the structure of QED with less precision, but with great sensitivity on tiny distance scales due to the high momentum transfers involved. A summary of the limits on modifications of QED based on the most sensitive experiments with colliding electron-positron beams²⁴ indicates that any such modifications are restricted to momenta greater than 15-30 GeV, or to distances less than 10^{-15} cm.

Armed with all these beautiful results we can step back on this 30th anniversary of QED's momentous initial triumphs and reflect with pleasure on all that has been accomplished since then. Indeed today our confidence in the validity of QED, on the tiny distance scales and at the great precisions that we have been describing, is so high that we rely on it as the standard with which to probe new and detailed predictions of renormalizable gauge theories of the weak interactions. There is an interesting difference between this and the original Rutherford discovery of the nuclear atom by scattering particles emerging from naturally radioactive substances from target atoms. In his analysis Rutherford assumed the validity of Coulomb's Law, as later found to be correct on the atomic scale of distances. In our current applications we have proceeded much more cautiously, first validating QED on the energy and distance scales that we are using in experiments

to probe the weak interaction theories. Most recently the observation²⁵ of asymmetries in the deep inelastic scattering of polarized electrons from deuterium to a precision of parts in 10^5 has shown evidence of a parity violating weak neutral current in confirmation of basic ideas in the renormalizable gauge theories of weak interactions, due primarily to Weinberg and Salam.²⁶ And, indeed, our theoretical efforts to understand such basic problems as quark confinement as well as the unification of weak, electromagnetic, and strong forces are based on a direct imitation of quantum electrodynamics. Current analyses start with local, current conserving renormalizable gauge theories, of which QED is the original model.

Summarizing, Julian, QED says happy birthday to you with its beautiful and unblemished record of successes. This theory which describes interacting electrons and photons in a space-time free of granularity and flat in the ideal sense of the Einstein theory of special relativity, has surmounted all experimental challenges and provides the basic model for further work on both the weak and strong forces. There are, however, several clouds on the horizon. One is the humbling fact that the experimentalists by and large are ahead of us in the precision of their measurements. Surely that will be only a temporary lead! More seriously we now realize ours is a world of proliferating spin 1/2 charged leptons: electron, muon, tau... each with its own neutrino. To grapple with and understand the significance of the growing lepton family - and also of proliferating, confined quarks - is a major challenge ahead of us.

I am happy to acknowledge and thank Stanley Brodsky and Peter Lepage for valuable discussions during my preparation of this talk.

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$$\vec{\omega}_a = \frac{-e}{m_\mu c} \left[a_\mu \left\{ \vec{B} - \vec{\beta} \times \vec{E} \right\} + \frac{1}{\gamma^2 - 1} \vec{\beta} \times \vec{E} \right]$$

The term proportional to E vanishes when $\gamma = \sqrt{1 + 1/a_\mu} \sim 29.3$ corresponding to $p_\mu = 3.094$ GeV/c. (See Ref. 1, 9.)

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FIGURE CAPTIONS

1. Theoretical and experimental values of the electron's magnetic moment anomaly, a_e , in 1948.
2. Comparison of today's theoretical and experimental values of a_e . Sources of error in the theoretical value are as indicated.
3. Anticipated improvements in the accuracy of a_e are indicated. The most precise value of the fine structure constant is determined as shown in terms of the Rydberg, Ry_∞ ; the proton gyromagnetic ratio in water, γ_p ; the magnetic moment of the proton in a water sample in units of the electron Bohr magneton, μ_p/μ_B^e ; the ratio e/h as determined from the Josephson junction relation; and the ratio of the absolute to the NBS ohm, $c\Omega_{\text{abs}}/\Omega_{\text{NBS}}$.
4. Comparison of contributions to the muon and electron moment anomalies with numerical estimates as shown. The muonic, hadronic, and weak contributions to a_e are negligible as indicated because of the small mass $m_e \ll m_\mu, m_H, m_W$. The experimental values for a_μ are from the most recent CERN experiments.
5. Hadronic contributions to a_μ as related, via the dispersion integral for the photon propagator, to the total cross section for e^+e^- annihilation to hadrons. Contributions from different energy regions are indicated.
6. Comparison of theory and experiment for a_μ , with the (negligible) contribution characteristic of renormalizable gauge theories of the weak interactions indicated.
7. Hydrogen atom fine and hyperfine structure with the experimental values and accuracies as shown. Also indicated are two different

theoretical values and the values of the 2nd and 4th order vacuum polarization and finite proton size contributions.

8. Contributions to the muonium and positronium hyperfine structure and comparison with experiment.
9. Spectrum of positronium and the comparison of measured and calculated radiative decay rates of orthopositronium.
10. Comparison of decay rates of orthopositronium (in inverse microseconds) as a function of the year of the measurement or calculation.

1948

Theory:

$$\frac{1}{2} (g_0 - 2) \equiv a_0 = \frac{a}{2\pi}$$
$$= \underline{1.162 \times 10^{-3}}$$

Experiment:

$$a_0 = \underline{1.18(3) \times 10^{-3}}$$

10-78

3486A1

Fig. 1

1978

Experiment:

$$\alpha_e = 1159652410(\underline{200}) \times 10^{-12}$$

[0.19 ppm]

Theory:

$$\alpha_e = 1159652359(\underline{282}) \times 10^{-12}$$

$$= \frac{\alpha}{2\pi} - 0.328478445 \left(\frac{\alpha}{\pi}\right)^2 + 1.183(11) \left(\frac{\alpha}{\pi}\right)^3$$

sources of error:

$$\alpha_{\text{Josephson}}^{-1} = 137.035987(29) \quad [0.21 \text{ ppm}]$$

(246) of (282)

accuracy of numerical

integrals in $\left(\frac{\alpha}{\pi}\right)^3$:

(138) of (282)

10-78

3486A2

Fig. 2

What is happening to the errors :

$$[a_e]^{\text{expt}} : (200) \rightarrow (10) \times 10^{-12} \quad [0.01 \text{ ppm}]$$

$[a_e]^{\text{theory}} :$

$$\alpha_J^{-2} = \frac{1}{4Ry_\infty} \cdot \frac{1}{\gamma_{p'}} \cdot \frac{\mu_{p'}}{\mu_B^e} \cdot \frac{2e}{h} \cdot \frac{c\Omega_{\text{Abs}}}{\Omega_{\text{NBS}}}$$

$$\alpha_J^{-1} : (246) \rightarrow (20) \times 10^{-12} \quad [0.02 \text{ ppm}]$$

Reduce numerical errors :

$$(138) \rightarrow (15) \times 10^{-12}$$

Remaining error source :

$$\sim \left(\frac{\alpha}{\pi}\right)^4 \approx (30) \times 10^{-12}$$

Fig. 3

Muon g-2

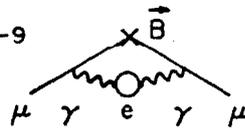
$$a_e = a_1 + a_2 \left(\frac{m_e}{m_\mu} \right) + a_H \left(\frac{m_e}{m_H} \right) + a_W \left(\frac{m_e}{m_W} \right)$$

$$2.8 \times 10^{-12} \quad \sim 2 \times 10^{-12} \quad \approx 0.05 \times 10^{-12}$$

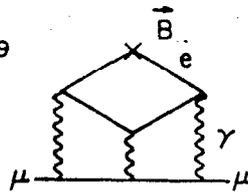
$$a_\mu = a_1 + a_2 \left(\frac{m_\mu}{m_e} \right) + a_H \left(\frac{m_\mu}{m_H} \right) + a_W \left(\frac{m_\mu}{m_W} \right)$$

a_2 contains purely electromagnetic contributions:

$$(a_\mu - a_e)^{a^2} \approx +1 \left(\frac{\alpha}{\pi} \right)^2 \approx 6000 \times 10^{-9}$$



$$(a_\mu - a_e)^{a^3} \approx +20 \left(\frac{\alpha}{\pi} \right)^3 \approx 300 \times 10^{-9}$$



$$(a_\mu - a_e)^{a^4} \approx +150 (70) \left(\frac{\alpha}{\pi} \right)^4 \approx 5 \times 10^{-9}$$

Experiment:

$$a_{\mu^+} = 1165910 (12) \times 10^{-9}$$

$$a_{\mu^-} = 1165936 (12) \times 10^{-9}$$

[10 ppm]

Fig. 4

a_W : Weak Interaction
Renormalizable Gauge Theories

$$a_W \sim \frac{G_F m_\mu^2}{6\sqrt{s} \pi^2} \sim 10^{-9}$$

$$a_\mu^{\text{theory}} = 1165926 (10) \times 10^{-9}$$

[9 ppm]

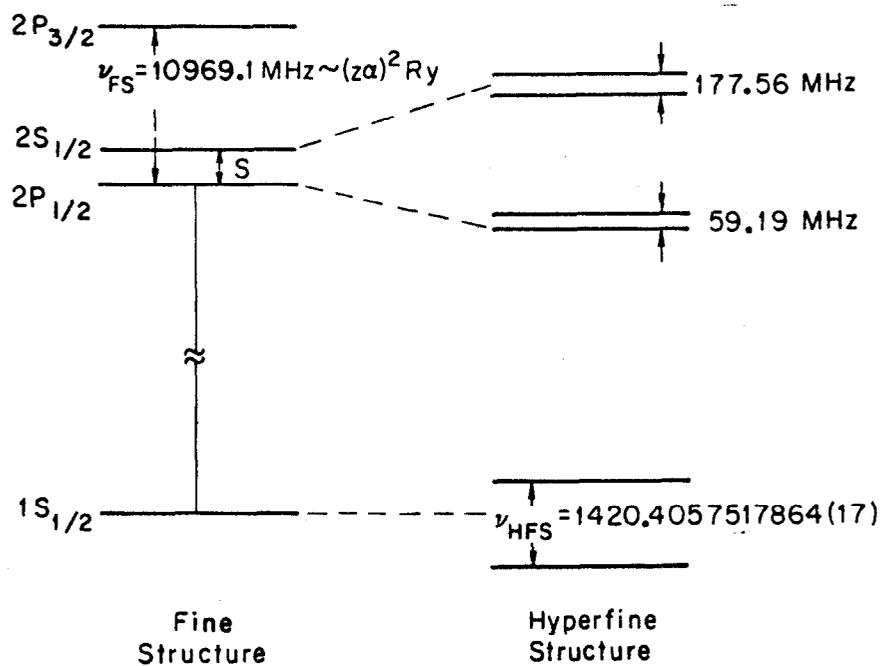
$$a_{\mu^\pm}^{\text{expt}} - a_{\mu^\pm}^{\text{th}} = \begin{cases} -16 \\ +10 \end{cases} (16) \times 10^{-9}$$

10-78

3486A6

Fig. 6

THE HYDROGEN ATOM



Lamb Shift:

$$S^{\text{expt}} = 1057.893 (20) \text{ MHz}$$

$$S^{\text{theory}} = \begin{cases} 1057.864 (14) \\ 1057.916 (10) \end{cases}$$

Vacuum Polarization:

$$-27.14 - 0.24$$

Proton Size:

10-78

$$+ 0.13$$

3486A7

Fig. 7

MUONIUM AND POSITRONIUM HFS

$$E_F = \frac{2}{3} \frac{(z\alpha)^4}{m_1 m_2} \left(\frac{m_1 m_2}{m_1 + m_2} \right)^3 (1+a_1)(1+a_2) \left\langle \vec{\sigma}_1 \cdot \vec{\sigma}_2 \right\rangle \Bigg|_{j=1}^{-j=2}$$

MUONIUM

Theory:

$$E_F + O\left(\alpha \frac{m_e}{m_\mu}, \alpha^2, \alpha^3\right) E_F \quad 4463.293(6) \text{ MHz}$$

$$\left(2\alpha^2 \frac{m_e}{m_\mu} \ln \frac{1}{\alpha} \right) E_F \quad \frac{0.011 \text{ MHz}}{4463.304(6) \text{ MHz}}$$

$$\text{Not computed} \approx \left(\alpha^2 \frac{m_e}{m_\mu} \ln \frac{m_\mu}{m_e} \right) E_F \sim 0.01 \text{ MHz}$$

Experiment:

$$4463.30235(52) \text{ MHz}$$

$$[0.12 \text{ ppm}]$$

POSITRONIUM

Theory:

$$\frac{7}{4} E_F + O(\alpha) E_F \quad 203.3812 \text{ GHz}$$

$$\frac{5}{24} \alpha^6 \left(\ln \frac{1}{\alpha} \right) m_e \quad \frac{0.0191 \text{ GHz}}{203.4003 \text{ GHz}}$$

$$\text{Not computed} \approx \alpha^2 E_F \sim 0.01 \text{ GHz}$$

Experiment

$$203.3849(12) \text{ GHz}$$

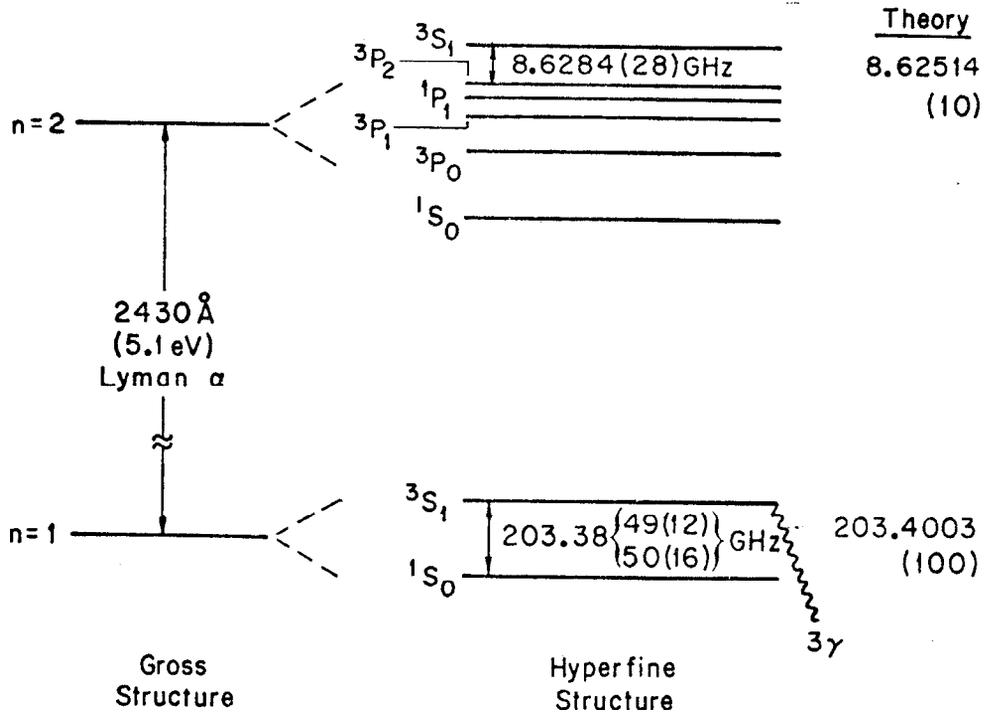
$$203.3870(16) \text{ GHz}$$

$$[6-8 \text{ ppm}]$$

10-78
3468A8

Fig. 8

POSITRONIUM



Theory
8.62514
(10)

$$\Gamma(1^3S_1) = \begin{cases} 7.058(15) \mu s^{-1} & \text{Experiment} \\ 7.0386(40) \mu s^{-1} & \text{Theory} \end{cases}$$

$O(\alpha^2)$

Fig. 9

DECAY RATE OF θ -POSITRONIUM

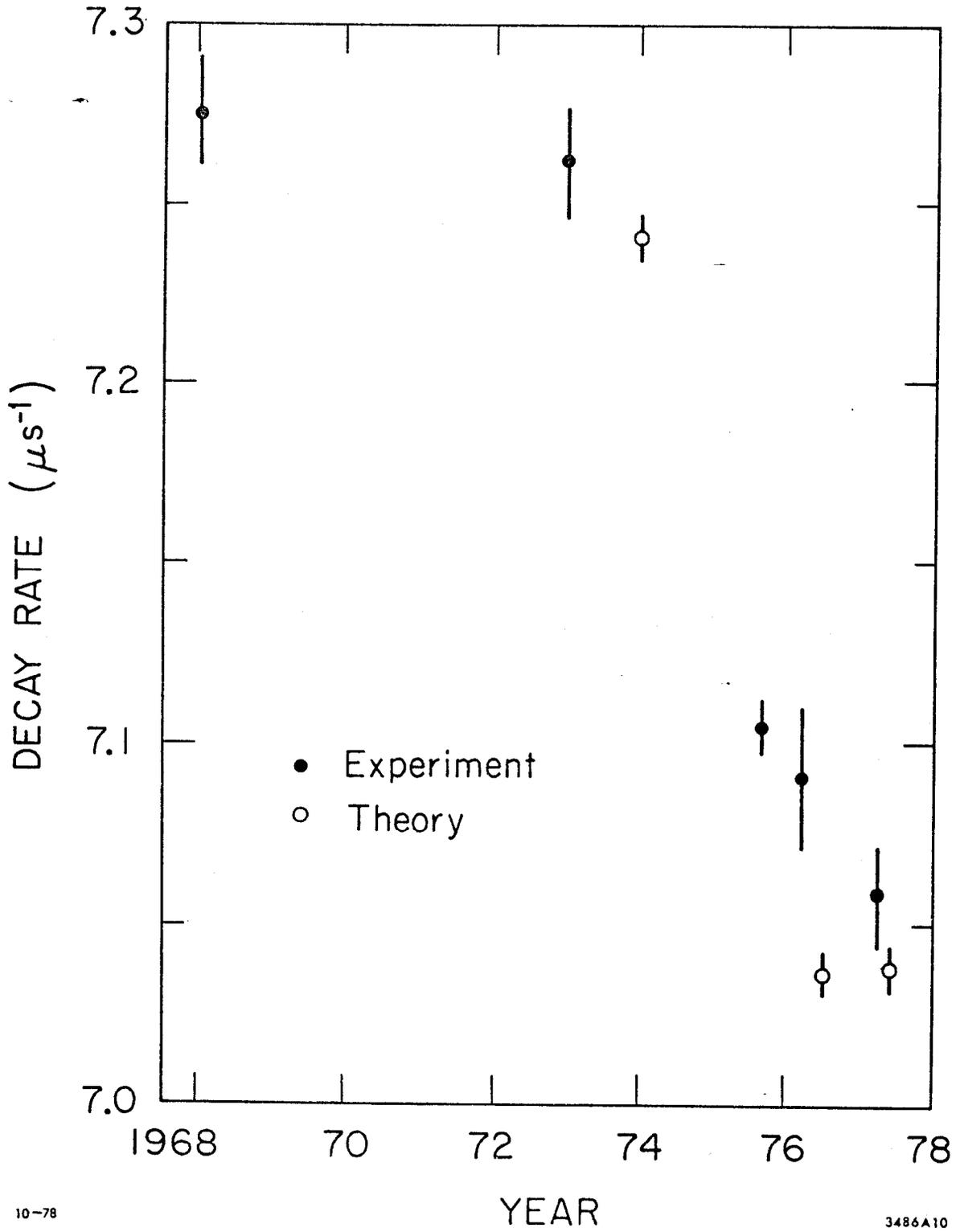


Fig. 10