# Uncertainties in nuclear transition matrix elements for neutrinoless $\beta\beta$ decay

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Abstract. To ascertain the uncertainties associated with the nuclear transition matrix elements  $M^{(0\nu)}$  for the neutrinoless  $\beta\beta$  decay of  $^{94,96}$ Zr,  $^{98,100}$ Mo,  $^{104}$ Ru,  $^{110}$ Pd,  $^{128,130}$ Te and  $^{150}$ Nd isotopes in the case of  $0^+ \rightarrow 0^+$  transition, a statistical analysis has been performed by calculating eight (twelve) different nuclear transition matrix elements  $M^{(0\nu)}$  in the projected Hartree-Fock-Bogoliobov (PHFB) model using four different parameterizations of a Hamiltonian with pairing plus multipolar effective two-body interaction and two (three) different parameterizations of Jastrow-type short range correlations. The averages in conjunction with their standard deviations provide an estimate of the uncertainties associated the nuclear transition matrix elements  $M^{(0\nu)}$  calculated within the PHFB model.

#### 1. Introduction

The observation of lepton number violating neutrinoless double beta  $(\beta\beta)_{0\nu}$  decay is a convenient tool for establishing the Majorana nature of neutrinos. Once observed, it can provide information on many a gauge theoretical parameters, namely effective mass of light as well as heavy neutrinos, the effective coupling constants of left-right and right-right handed currents in the left-right symmetric models, the intergeneration Yukawa coupling constants in the  $R_p$ -violating minimal super-symmetric (SUSY) standard model, leptoquark-Higgs coupling constant, mixing parameters of heavy sterile neutrinos with light Majorana neutrinos, the compositeness scale, the brain-shift parameter of extradimensional models and Majoron-neutrino coupling constants in addition to the violation of Lorentz invariance and weak equivalence principle. According to the "black box theorem" [1], the observation of  $(\beta\beta)_{0\nu}$  decay implies the non-zero mass of Majorona neutrinos at the weak scale in any gauge theoretical model with spontaneous symmetry breaking independent of underlying mechanisms. The extension of this "black box theorem" to SUSY models [2], implies the existence of massive sneutrinos provided the  $(\beta\beta)_{0\nu}$  decay is observed.

The reliability of extracted gauge theoretical parameters depends on the accuracy of calculated model dependent nuclear transition matrix elements (NTMEs). The nuclear manybody theory is quite complex as it deals with the nonperturbative region of quantum chromo dynamics (QCD). Hopefully, the present attempts to develop the effective field theory (EFT) as an alternative method for low energy QCD will emerge as a powerful technique for solving nuclear many-body theory in the near future. The basic problem in the conventional nuclear structure calculations is to solve the nuclear many body problem perturbatively to all orders and the nuclear shell model is a prototypical nuclear many-body theory. However, most of the  $\beta\beta$  emitters are medium and heavy mass nuclei for which there is a necessity of large scale configuration mixing and the presently available computational facilities are limited for this purpose. Therefore, one is forced to look for alternative models such as quasiparticle random-phase approximation (QRPA) and their extensions, projected-Hartree-Fock-Bogoliobov model (PHFB), deformed Hartree-Fock (DHF) and generator coordinate method plus particle number and angular momentum projection (GCM+PNAMP), single state dominance hypothesis (SSDH) as well as group theoretical methods.

In general, there are four main sources of uncertainties, namely the inclusion of pseudoscalar and weak magnetism terms in the Fermi, Gamow-Teller (GT) and tensorial NTMEs [3, 4], the effects due to finite size of nucleons (FNS) as well as short range correlations (SRC) [5, 6, 7, 8, 9, 10, 11, 12], deformation and the use of two effective values of the axial-vector coupling constant  $g_A$ . In addition, different nuclear models produce different NTMEs even for a given transition depending on the approximations involved and for a given model, NTMEs also depend on the model space and effective two-body interaction selected. Based on these observations, Vogel proposed that the spread between the calculated NTMEs can provide a measure of the theoretical uncertainty [13]. In the case of the well studied <sup>76</sup>Ge isotope, it was observed that the calculated decay rates  $T_{1/2}^{0\nu}$  differ by a factor of 6–7 and hence, the uncertainty in the effective neutrino mass  $\langle m_{\nu} \rangle$  is about 2–3. Experimental limit  $T_{1/2}^{0\nu} > 1.6 \times 10^{25}$  yr [14] imply upper limits on  $\langle m_{\nu} \rangle$  between 0.4 eV and 1.0 eV, depending on the NTMEs [15, 16, 17].

By performing a statistical analysis, the spread between the calculated NTMEs can be translated into averages and standard deviations [18, 19]. According to Bilenky and Grifols [20], the comparison of calculated ratios of the corresponding NTMEs-squared and the ratios of half-lives could test the validity of nuclear structure calculations in a model independent way once the  $(\beta\beta)_{0\nu}$  decay would be observed in several nuclei. Rodin *et al.* [21] have estimated the theoretical uncertainty employing QRPA and renormalized QRPA (RQRPA), with three sets of basis states and three realistic two-body effective interactions. Different strategies to remove the sensitivity of QRPA calculations on the model parameters have been also proposed [22, 23]. Further studies on uncertainties in NTMEs due to SRC using the unitary correlation operator method (UCOM) [10] and self-consistent coupled cluster method (CCM) [11] have been carried out by Tuebingen group.

Specifically, the PHFB model is unique in allowing the description of the  $\beta\beta$  decay in medium and heavy mass nuclei by projecting a set of states with good angular momentum, while treating the pairing and deformation degrees of freedom simultaneously and on equal footing. The PHFB model, in conjunction with pairing plus quadrupole-quadrupole (PQQ) interaction [24] has been successful in the study of the  $0^+ \rightarrow 0^+$  transition of  $(\beta^-\beta^-)_{2\nu}$  decay, and it was possible to describe the lowest excited states of the parent and daughter nuclei along with their electromagnetic transition strengths, as well as to reproduce the measured  $\beta\beta$  decay rates [25, 26]. However, the structure of the intermediate odd Z-odd N nuclei and hence, the single  $\beta$ decay rates and the distribution of GT strength can not be studied in the present version of the PHFB model. In spite of this limitation, it is quite convenient for examining the explicit role of deformation on the NTMEs [27, 28]. The effects of pairing and quadrupolar correlations on the NTMEs of  $(\beta^-\beta^-)_{0\nu}$  decay has been also studied in the interacting shell model (ISM) [9, 29].

Presently, two different parameterizations of the QQ interaction have been employed, with and without the *HH* correlations. Further, the NTMEs  $M^{(0\nu)}$  are calculated with three different parametrizations of Jastrow SRC employing the four sets of wave functions. The twelve NTMEs provide a reasonable sample for estimating the associated uncertainties. In Sec. 2, the reliability of HFB wave functions has been discussed briefly. In Sec. 3, the effects due to the four different parameterizations of the pairing plus multipole Hamiltonian and three different parametrizations of SRC on the calculated NTMEs are analyzed, and in Sec. 4, their average values as well as standard deviations are estimated. Subsequently, the latter are employed to obtain upper limits on the effective mass of light Majorana neutrinos. Conclusions are given in Sec. 5.

#### 2. Reliability of HFB wave functions

The HFB wave functions were generated using an effective Hamiltonian [28] given by

$$H = H_{sp} + V(P) + V(QQ) + V(HH), \tag{1}$$

where  $H_{sp}$ , V(P), V(QQ) and V(HH) denote the single particle Hamiltonian, the pairing, quadrupole-quadrupole (QQ) and hexadecapole-hexadecapole (HH) parts of the effective twobody interaction, respectively. The model space and single particle energies (SPE's) have been already given in Refs. [25, 26, 27]. In the quadrupole-quadrupole part of the effective two-body interaction, the V(QQ) has three terms for proton-proton, neutron-neutron and the protonneutron interaction with coefficients  $\chi_{2pp}$ ,  $\chi_{2nn}$  and  $\chi_{2pn}$ , respectively. The strengths of the like particle components of the QQ interaction were taken as  $\chi_{2pp} = \chi_{2nn} = 0.0105$  MeV  $b^{-4}$ . By fitting the experimental excitation energy of the 2<sup>+</sup> state,  $E_{2^+}$ , the strength of protonneutron component of the QQ interaction  $\chi_{2pn}$  was fixed [25, 26, 27]. Alternatively, one can also employ an isoscalar parametrization of the quadrupole-quadrupole interaction, by taking  $\chi_{2pp} = \chi_{2nn} = \chi_{2pn}/2$ . and the three parameters were varied together to fit  $E_{2^+}$ . These two parameterizations of the quadrupole-quadrupole interaction are referred as PQQ1 and PQQ2with PQQ [24] type of effective two-body interaction.

The experimental excitation energies of  $2^+$  state  $E_{2^+}$  [30] can be reproduced within about 2% accuracy in both methods. By employing the PQQ1 parametrization, the maximum change in  $E_{4^+}$  and  $E_{6^+}$  energies with respect to PQQ1 interaction [25, 26] is about 5% and 18%, respectively. By employing either parametrizations, the reduced  $B(E2:0^+ \rightarrow 2^+)$  transition probabilities, deformation parameters  $\beta_2$ , static quadrupole moments  $Q(2^+)$  and gyromagnetic factors  $g(2^+)$  are in an overall agreement with the experimental data [31, 32]. With respect to PQQ1 parametrization, the maximum change in the calculated NTMEs  $M_{2\nu}$  for the  $0^+ \rightarrow 0^+$  transition in the case of PQQ2 parametrization, is about 21% but for <sup>94</sup>Zr isotope. By including the hexadecapolar term HH, we end up with four different parameterizations of the effective two-body interaction, namely PQQ1, PQQHH1, PQQ2 and PQQHH2.

#### 3. Short range correlations and NTMEs

Considering the finite size of nucleons, the inverse half-life of the  $(\beta^-\beta^-)_{0\nu}$  decay due to the exchange of light Majorana neutrinos for the  $0^+ \to 0^+$  transition is given by [5, 15, 33]

$$\left[T_{1/2}^{0\nu}(0^+ \to 0^+)\right]^{-1} = \left(\frac{\langle m_\nu \rangle}{m_e}\right)^2 G_{01} |M^{(0\nu)}|^2, \tag{2}$$

and the model dependent NTME  $M^{(0\nu)}$  is given by

$$M^{(0\nu)} = \sum_{n,m} \left\langle 0_F^+ \left\| \left[ \sigma_n \cdot \sigma_m - \left(\frac{g_V}{g_A}\right)^2 \right] H(r_{12}) \tau_n^+ \tau_m^+ \right\| 0_I^+ \right\rangle,$$
(3)

where

$$H(r_{12}) = \frac{4\pi R}{(2\pi)^3} \int d^3q \frac{\exp\left(i\mathbf{q}\cdot\mathbf{r_{12}}\right)}{q\left(q+\overline{A}\right)} \left(\frac{\Lambda^2}{\Lambda^2+q^2}\right)^4,\tag{4}$$

with  $\overline{A} = \langle E_N \rangle - \frac{1}{2} (E_I + E_F)$ . The cutoff momentum  $\Lambda = 850$  MeV [27].

Recently, Šimkovic *et al.* [11] have shown that it is possible to parametrize the SRC effects of Argonne V18 and CD-Bonn two nucleon potentials by the Jastrow type of correlations within a few percent accuracy. Explicitly, the effects due to the SRC can be incorporated in the calculation of  $M^{(0\nu)}$  through the prescription  $O_k \to fO_k f$ , where

$$f(r) = 1 - ce^{-ar^2}(1 - br^2),$$
(5)

with a = 1.1, 1.59 and 1.52 fm<sup>-2</sup>, b = 0.68, 1.45 and 1.88 fm<sup>-2</sup> and c = 1.0, 0.92 and 0.46 for Miller-Spencer [34], Argonne V18 and CD-Bonn NN potentials, respectively. Presently, the NTMEs  $M^{(0\nu)}$  are calculated in the PHFB model by employing these three sets of parameters for the SRC, denoted as SRC1, SRC2 and SRC3, respectively. The three functions f(r) have similar forms, but differ in their values at the origin, and at the position of their maximum, which lie at 1.54, 1.15 and 1.09 fm for SRC1, SRC2 and SRC3, respectively. As discussed below, they have clear influence on the calculated NTMEs and radial evolution of the  $(\beta^{-}\beta^{-})_{0\nu}$  decay matrix elements.

We present the twelve NTMEs  $M^{(0\nu)}$  in Table 1, evaluated using the HFB wave functions in conjunction with PQQ1, PQQHH1, PQQ2, PQQHH2 interactions and three different parametrizations of the Jastrow type of SRC for the nuclei <sup>94,96</sup>Zr, <sup>98,100</sup>Mo, <sup>104</sup>Ru, <sup>110</sup>Pd, <sup>128,130</sup>Te and <sup>150</sup>Nd. Following Haxton's prescription [15], the average energy denominator  $\overline{A}$  has been taken as  $\overline{A} = 1.12A^{1/2}$  MeV. In Ref. [12], the NTMEs were calculated in the approximations of point nucleons (P), FNS, point nucleons with SRC (P+SRC), and finite size plus SRC (FNS+SRC). The NTMEs  $M^{(0\nu)}$  were also calculated for  $\overline{A}/2$  in the energy denominator in the case of point nucleons to obtain information on the validity of closure approximation. The following observations are worth mentioning [12].

- (i) It was observed that the relative change in NTMEs  $M^{(0\nu)}$  is of the order of 10 %, when the energy denominator was taken as  $\overline{A}/2$  instead of  $\overline{A}$ . It confirms that there is a weak dependence of NTMEs on average excitation energy  $\overline{A}$  for the  $(\beta^{-}\beta^{-})_{0\nu}$  decay and the closure approximation is quite valid.
- (ii) Due to the different parameterizations of the Hamiltonians, the variation in  $M^{(0\nu)}$  lies between 20–25%. In general, it was noticed that the NTMEs but for <sup>128</sup>Te isotope evaluated for both parameterizations of the quadrupolar interaction are quite close. The inclusion of the hexadecapolar term tends to reduce them by amounts which strongly depend on the specific nuclei.
- (iii) In the approximation of point nucleons, the inclusion of SRC induces an extra quenching in the NTMEs  $M^{(0\nu)}$ , which is of the order of 18–23% for SRC1 and negligible for SRC3.
- (iv) The dipole form factor always reduces the NTMEs by  $12{-}15\%$  in comparison to the point nucleon case.
- (v) Adding SRC can further reduce the transition matrix elements, for SRC1, or slightly enhance them, partially compensating the effect of the dipole form factor. It was interesting to note that the effect of FNS+SRC2 is almost negligible, i.e., nearly the same as FNS.

The radial evolution of  $M^{(0\nu)}$  has been studied by Šimkovic *et al.* [10] by defining

$$M^{(0\nu)} = \int C^{(0\nu)}(r) \, dr.$$
(6)

In the QRPA by Šimkovic *et al.* [10] and ISM by Menéndez *et al.* [35], it has been observed that the contributions of decaying pairs coupled to J = 0 and J > 0 almost cancel beyond  $r \approx 3$  fm and the magnitude of  $C^{(0\nu)}$  for all nuclei undergoing  $(\beta^{-}\beta^{-})_{0\nu}$  decay are the maximum about the internucleon distance  $r \approx 1$  fm.

The study of radial dependence of  $C^{(0\nu)}$  in the PHFB model due to PQQ1 parametrization of the effective two body interaction for six nuclei, namely <sup>96</sup>Zr, <sup>100</sup>Mo, <sup>110</sup>Pd, <sup>128,130</sup>Te and <sup>150</sup>Nd [12], provide following observations.

(i) In case of point nucleons, it was noticed that the  $C^{(0\nu)}$  are peaked at r = 1.0 fm and with the addition of SRC1, the peak shifts to 1.25 fm. However, the magnitude of  $C^{(0\nu)}$  are increased for SRC2 and SRC3 with unchanging peak position.

<b>Table 1.</b> Calculated NTMEs $M^{(0\nu)}$ in the PHFB model with four different parameterizations of
the effective two-body interaction, three different parameterizations of SRC along with average
NTMEs $\overline{M}^{(0\nu)}$ and uncertainties $\Delta \overline{M}^{(0\nu)}$ for the $(\beta^{-}\beta^{-})_{0\nu}$ decay of <sup>94,96</sup> Zr, <sup>98,100</sup> Mo, <sup>104</sup> Ru,
$^{110}$ Pd, $^{128,130}$ Te and $^{150}$ Nd isotopes.

Nucloi		$\Lambda I(0\nu)$		<i>a</i> .	$\overline{M}(0\nu)$		
muciel		SRC1	SRC2	SRC3	$\mathcal{G}A$	Case I	Case II
04-			5102	5100			
$^{94}$ Zr	PQQ1	4.0690	4.6639	4.8383	1.254	$4.2464 \pm 0.3883$	$4.4542 \pm 0.2536$
	PQQHH1	3.7315	4.2820	4.4441	1.0	$4.6382 \pm 0.4246$	$4.8668 \pm 0.2759$
	PQQ2	3.9802	4.4818	4.6259			
	PQQHH2	3.5424	4.0708	4.2266			
$^{96}$ Zr	PQQ1	2.9068	3.3590	3.4923	1.254	$3.1461 {\pm} 0.2778$	$3.3181 {\pm} 0.1243$
	PQQHH1	2.8507	3.3192	3.4578	1.0	$3.4481 {\pm} 0.3085$	$3.6376 {\pm} 0.1424$
	POO2	2.7758	3.2103	3.3385			
	PQQHH2	2.6745	3.1182	3.2497			
985.5	DOOI				1 051	- 100 ( 10 0010	
<sup>90</sup> Mo	PQQ1	6.7322	7.6297	7.8884	1.254	$7.1294 \pm 0.6013$	$7.4656 \pm 0.3635$
	PQQHH1	6.1984	7.0618	7.3114	1.0	$7.8398 \pm 0.6826$	$8.2099 \pm 0.4358$
	PQQ2	6.7630	7.6695	7.9307			
	PQQHH2	6.1344	6.9925	7.2406			
$^{100}Mo$	PQQ1	6.5036	7.4282	7.6920	1.254	$6.8749 {\pm} 0.6855$	$7.2163 {\pm} 0.4977$
	POOHH1	6.1597	7.0654	7.3248	1.0	$7.5660 \pm 0.7744$	$7.9419 \pm 0.5769$
	POO2	6.5534	7.4838	7.7493	-		
	PQQHH2	5.5520	6.3756	6.6113			
$^{104}$ Ru	PQQ1	4.6942	5.3989	5.5975	1.254	$4.8464 \pm 0.4840$	$5.1004 \pm 0.3280$
	PQQHH1	4.2809	4.9548	5.1454	1.0	$5.3599 {\pm} 0.5533$	$5.6396 {\pm} 0.3926$
	PQQ2	4.4137	5.0777	5.2647			
	PQQHH2	3.9648	4.5931	4.7708			
110							
<sup>110</sup> Pd	PQQ1	7.6982	8.7783	9.0850	1.254	$7.8413 \pm 0.8124$	$8.2273 \pm 0.6167$
	PQQHH1	6.3963	7.3535	7.6262	1.0	$8.6120 \pm 0.9184$	$9.0370 \pm 0.7128$
	PQQ2	7.3842	8.4187	8.7128			
	PQQHH2	6.7982	7.7816	8.0621			
$^{128}$ Te	PQQ1	3.2499	3.7258	3.8639	1.254	$4.0094 \pm 0.4194$	$4.2175 \pm 0.3074$
	PQOHH1	3.4994	4.0740	4.2401	1.0	$4.4281 \pm 0.4601$	$4.6571 \pm 0.3355$
	POO2	3 8893	4 4374	4 5956			
	PQQHH2	3.7336	4.3172	4.4860			
			-				
$^{130}\mathrm{Te}$	PQQ1	4.4319	5.0103	5.1753	1.254	$4.4458 {\pm} 0.5231$	$4.6633 {\pm} 0.4269$
	PQQHH1	3.6277	4.1964	4.3595	1.0	$4.9065 {\pm} 0.5837$	$5.1459 {\pm} 0.4802$
	PQQ2	4.3610	4.9320	5.0951			
	PQQHH2	3.6218	4.1879	4.3503			
150 n. j	$P \cap \cap 1$	2 9916	2 6975	2 7514	1 954	3 1048-0 4640	3 9431 + 0 4494
- 180	I QQI	0.2010 0.4000	0.0010 0.7447	0.7014 0.0050	1.204	$3.1040\pm0.4049$	$3.2431 \pm 0.4454$
	PQQHHI	2.4208	2.(44)	2.8359	1.0	3.4334±0.5181	$3.5850 \pm 0.4952$
	PQQ2	3.1574	3.5546	3.6661			
	PQQHH2	2.5031	2.8311	2.9234			

(ii) In the case of FNS, the  $C^{(0\nu)}$  are peaked at r = 1.25 fm, which remains unchanged with the inclusion of SRC1, SRC2 and SRC3. However, the magnitudes of  $C^{(0\nu)}$  change in the latter three cases.

The above observations also remain valid with the other three parametrizations of the effective two-body interaction.

#### 4. Uncertainties in NTMEs

We have performed a statistical analysis for estimating the uncertainties associated with the NTMEs  $M^{(0\nu)}$  for  $(\beta^{-}\beta^{-})_{0\nu}$  decay calculated in the PHFB model by evaluating the mean and the standard deviation, defined as

$$\overline{M}^{(0\nu)} = \frac{\sum_{i=1}^{N} M_i^{(0\nu)}}{N}$$
(7)

and

$$\Delta \overline{M}^{(0\nu)} = \frac{1}{\sqrt{N-1}} \left[ \sum_{i=1}^{N} \left( \overline{M}^{(0\nu)} - M_i^{(0\nu)} \right)^2 \right]^{1/2}.$$
 (8)

Based on the observation by Šimkovic *et al.* [11] that the Miller-Spenser parametrization of the Jastrow correlation is a major source of uncertainty and it is better to consider SRC2 or SRC3 due to the Argonne V18 and CD-Bonn NN potentials, respectively, we have calculated the averages and variances for two cases. In case I, the statistical analysis has been performed for twelve NTMEs listed in Table 1, for the bare and quenched values of axial vector coupling constant  $g_A = 1.254$  and  $g_A = 1.0$ , respectively. Similarly, the averages and standard deviations are calculated for eight NTMEs  $M^{(0\nu)}$  due to SRC2 and SRC3 in case II. In Table 1, the averages  $\overline{M}^{(0\nu)}$  and uncertainties  $\Delta \overline{M}^{(0\nu)}$  have been displayed. It is noticed that the exclusion of Miller-Spencer parametrization reduces the uncertainty by about 55% in <sup>96</sup>Zr to 4% in <sup>150</sup>Nd.

In Fig. 1, we have plotted the average NTMEs  $\overline{M}^{(0\nu)}$  with the uncertainties  $\Delta \overline{M}^{(0\nu)}$ along with the recently available results obtained by employing the ISM [9], QRPA, RQRPA [11], IBM [36] and GCM+PNAMP [37]. It is observed that in all the models, the NTMEs  $M^{(0\nu)}$  have a weak or no dependence on the mass number A. Further, upper limits on the effective mass of light neutrinos  $\langle m_{\nu} \rangle$  have been extracted from the largest observed limits on half-lives  $T_{1/2}^{0\nu}$  of  $(\beta^{-}\beta^{-})_{0\nu}$  decay [38, 39, 40, 41, 42, 43, 44] using the phase space factors of Boehm and Vogel [45]. The extracted limits on  $\langle m_{\nu} \rangle$  for <sup>100</sup>Mo and <sup>130</sup>Te nuclei are  $0.51^{+0.06}_{-0.05} - 0.73^{+0.08}_{-0.07}$  eV and  $0.31^{+0.04}_{-0.03} - 0.45^{+0.06}_{-0.05}$  eV, respectively. Assuming  $\langle m_{\nu} \rangle = 50$ meV, the predicted half-lives of  $(\beta^{-}\beta^{-})_{0\nu}$  decay of <sup>96</sup>Zr, <sup>100</sup>Mo, <sup>128,130</sup>Te and <sup>150</sup>Nd isotopes for  $g_A = 1.254(1.0)$  are  $1.60^{+0.13}_{-0.11} \times 10^{26}$  ( $3.29^{+0.27}_{-0.24} \times 10^{26}$ ) yr,  $4.32^{+0.66}_{-0.54} \times 10^{25}$  ( $8.83^{+1.44}_{-1.15} \times 10^{25}$ ) yr,  $3.18^{+0.52}_{-0.42} \times 10^{27}$  ( $6.44^{+1.04}_{-0.84} \times 10^{27}$ ) yr,  $1.07^{+0.23}_{-0.17} \times 10^{26}$  ( $2.17^{+0.47}_{-0.35} \times 10^{26}$ ) yr and  $4.69^{+1.60}_{-1.06} \times 10^{25}$ ( $9.49^{+3.29}_{-2.16} \times 10^{25}$ ) yr, respectively.

#### 5. Conclusions

In our previous works [25, 26, 28], the reliability of wave functions generated with PQQ1 and PQQHH1 interactions was tested by calculating the yrast spectra, reduced  $B(E2:0^+ \rightarrow 2^+)$  transition probabilities, static quadrupole moments  $Q(2^+)$  and g-factors  $g(2^+)$  of participating nuclei in  $(\beta^-\beta^-)_{2\nu}$  decay as well as  $M_{2\nu}$  and comparing them with the available experimental data. It was observed that the PHFB wave functions generated by fixing  $\chi_{pn}$  or  $\chi_{pp}$  to reproduce the  $E_{2^+}$  are reasonably reliable in reproducing an overall agreement between the calculated and observed spectroscopic properties as well as  $M_{2\nu}$ .



**Figure 1.** NTMEs calculated in ISM [9], QRPA, RQRPA [11], PHFB [12], IBM [36] and GCM+PNAMP [37] in the mass range A = 90 - 150.

Presently, eight (twelve) NTMEs  $M^{(0\nu)}$  for  $(\beta^-\beta^-)_{0\nu}$  decay of <sup>94,96</sup>Zr, <sup>98,100</sup>Mo, <sup>104</sup>Ru, <sup>110</sup>Pd, <sup>128,130</sup>Te and <sup>150</sup>Nd isotopes were calculated by employing four different parameterizations of the pairing plus multipolar type of effective two body interaction, FNS and two (three) different parameterizations of Jastrow SRC. The mean and standard deviations were evaluated for the eight (twelve) NTMEs  $M^{(0\nu)}$  for both  $g_A = 1.254$  and  $g_A = 1.0$  to estimate the uncertainties in the NTMEs  $M^{(0\nu)}$  calculated in the PHFB model. The largest theoretical uncertainty, turns out to be of the order of 15% in the case of <sup>150</sup>Nd isotope. Also, limits have been extracted on the effective mass of light Majorana neutrinos  $\langle m_{\nu} \rangle$  from the available limits on experimental half-lives  $T_{1/2}^{0\nu}$  using average NTMEs  $\overline{M}^{(0\nu)}$ . In the case of <sup>130</sup>Te isotope, one obtains the best limit on the effective neutrino mass  $\langle m_{\nu} \rangle < 0.31^{+0.04}_{-0.03} - 0.45^{+0.06}_{-0.05}$  eV from the observed limit on the half-lives  $T_{1/2}^{0\nu} > 3.0 \times 10^{24}$  yr of  $(\beta^-\beta^-)_{0\nu}$  decay [43].

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