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Transient oscillating modes in a relativistic expanding gas

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Abstract. We show that a transient effective theory for rapid anisotropically expanding systems based on the Boltzmann equation exhibits transient oscillatory behavior.

1. Introduction and setup

The many-body dynamics of a weakly interacting, dilute system where two-particle correlations can be neglected is governed by the Boltzmann equation,

$$k^\mu \partial_\mu f(x,k) = C[f](x,k). \tag{1}$$

The complete description of the microscopic dynamics involves an infinite number of nonhydrodynamic modes contained in the linearized collision term $C[f]$; the virtual infinite complexity is reduced by systematically truncating these. (In general the collision term is non-linear, but in this work we will only considered the linearized collision term.) Hydrodynamics is an effective theory for the evolution of the conserved macroscopic quantities where only the so-called hydrodynamic modes, those that satisfy the dispersion relation $\lim_{\boldsymbol{k}\to 0} \omega_n(\boldsymbol{k}) = 0$, are included. It has always been previously thought that the nonhydrodynamic modes of the Boltzmann equation are exponentially damped on microscopic time scales [1]. This is in stark contrast to strongly coupled systems [2, 3] where the nonhydrodynamic modes are damped in an oscillatory manner.

In this proceedings we show that the former only holds when all local momentum anisotropies are treated perturbatively by expanding the Boltzmann equation around its local equilibrium value. This assumption is relaxed by considering a system that expands stronger in the longitudinal rather than transverse directions, such as the quark-gluon plasma created during relativistic heavy-ion collisions [4], where these local momentum anisotropies can become large and must be treated nonperturbatively. Here we do so by expanding the distribution function around a local "quasi-equilibrium" state $f_{\boldsymbol{k}} = f_{\mathrm{a},\boldsymbol{k}} + \delta\tilde{f}_{\boldsymbol{k}}$ by introducing a non-hydrodynamic degree of freedom $\xi(x)$ describing the deviation from momentum isotropy [5]:

$$f_{\mathrm{a},\boldsymbol{k}} = f_{\mathrm{eq}}\left(\beta_{\mathrm{a}}\sqrt{k_\perp^2 + (1+\xi)k_z^2}\right). \tag{2}$$

$\beta_{\mathrm{a}}(x)$ is a temperature-like parameter that is varied on macroscopic time scales related to energy conservation while the anisotropy parameter ξ is not constrained by conservation laws, but controlled by the expansion of the system and by microscopic time scales associated with the

Boltzmann collision term. In a procedure that closely follows [6], but now resums and truncates the moment hierarchy according to a modified power counting scheme based on the Knudsen and *residual* inverse Reynolds numbers from which the largest contributions arising from the local momentum anisotropy have been eliminated, we derive a new type of transient relativistic fluid dynamics in which the slowest non-hydrodynamic mode (associated with the local momentum anisotropy) turns out to exhibit transient oscillations. It should not be surprising that the physical phenomena emerging from this situation is analogous to that of a plasma placed in an external field; here ξ plays the role, in this loose analogy, of a fictitious external (gravitational) field that causes momentum space inhomogeneities.

2. Main ingredients

For a gas of massless particles the only conserved current is the energy-momentum tensor which is decomposed for anisotropic systems as [7]

$$T^{\mu\nu} \equiv \mathcal{E} u^\mu u^\nu - \mathcal{P}_\perp \Delta^{\mu\nu} + (\mathcal{P}_L - \mathcal{P}_\perp) z^\mu z^\nu + \tilde{\pi}^{\mu\nu} \,, \tag{3}$$

where $\mathcal{E}$ is the LRF energy density, $\mathcal{P}_{L,\perp}$ are the longitudinal and transverse pressures, and $\tilde{\pi}^{\mu\nu}$ is the residual shear stress tensor arising from $\delta\tilde{f}$ effects. u^μ is the fluid velocity, z^μ is the z unit vector in the LRF, and $\Delta^{\mu\nu} \equiv g^{\mu\nu} - u^\mu u^\nu$. We now replace the Boltzmann equation by a hierarchy of moment equations. The solution to the Boltzmann equation is now equivalent to solving the dynamical equations for the moments $\tilde{\rho}_r^{\mu\nu}$ [6, 8]:

$$\dot{\tilde{\rho}}_r^{\mu\nu}(x) \equiv \Delta^{\mu\nu}_{\alpha\beta} D \tilde{\rho}_r^{\alpha\beta}(x) \equiv \Delta^{\mu\nu}_{\alpha\beta} D \int dK (u\cdot k)^r k^{\langle\alpha} k^{\beta\rangle} \delta\tilde{f}_{\boldsymbol{k}} \,, \tag{4}$$

where $D \equiv u\cdot\partial$ is the time derivative in the local rest frame (LRF). The angle brackets around any two Lorentz indices denote a tensor that is transverse to u and traceless. Linearizing the collilsion term and using the Boltzmann equation (1) in the form

$$\delta\dot{\tilde{f}}_{\boldsymbol{k}} = -\dot{f}_{\mathrm{a},\boldsymbol{k}} - \frac{1}{k\cdot u}\Big(k\cdot\nabla\big(f_{\mathrm{a},\boldsymbol{k}} + \delta\tilde{f}_{\boldsymbol{k}}\big) - C_{\boldsymbol{k}}[f]\Big) \,, \tag{5}$$

results in

$$\begin{aligned} \Delta^{\mu\nu}_{\alpha\beta}\dot{\tilde{\rho}}_r^{\alpha\beta} + \sum_{n=0}^{N_2} (\mathcal{A}^{(2)}_{rn})^{\mu\nu}_{\alpha\beta} \tilde{\rho}_n^{\alpha\beta} \;&=\; \mathcal{L}_r^{\mu\nu}\dot{\xi} + \mathcal{M}_r^{\mu\nu\lambda}\dot{z}_\lambda + \big(\alpha^{(2)}_{\theta r}\big)^{\mu\nu}\theta + \big(\alpha^{(2)}_{\sigma r}\big)^{\mu\nu\lambda\rho}\sigma_{\lambda\rho} + \big(\alpha^{(2)}_{\omega r}\big)^{\mu\nu\lambda\rho}\omega_{\lambda\rho} \\ &+ \frac{\mathcal{B}_r^{\mu\nu}}{\tilde{\mathcal{J}}_{2,0,-1}}\tilde{\pi}^{\alpha\beta}\sigma_{\alpha\beta} - \frac{2}{7}(2r+5)\tilde{\rho}_r^{\lambda\langle\mu}\sigma^{\nu\rangle}_\lambda + 2\tilde{\rho}_r^{\lambda\langle\mu}\omega^{\nu\rangle}_\lambda - \frac{1}{3}(r+4)\tilde{\rho}_r^{\mu\nu}\theta. \end{aligned} \tag{6}$$

Here $\sigma_{\mu\nu} \equiv \nabla_{\langle\mu}u_{\nu\rangle}$ is the velocity shear tensor, $\omega_{\mu\nu} \equiv (\nabla_\mu u_\nu - \nabla_\nu u_\mu)/2$ is the vorticity tensor, and $\theta \equiv \partial\cdot u$ the scalar expansion rate. The matrix $\mathcal{A}^{(2)}$ contains all of the microscopic information of the linearized Boltzmann equation. Appearance of time derivatives (in the LRF) of the anisotropy parameter ξ and z^μ is due to the fact that $\dot{f}_\mathrm{a} \supset \dot{\beta}_a$, $\dot{u}^\mu$, $\dot{\xi}$, $\dot{z}^\mu$. $\dot{\beta}_a$ and $\dot{u}^\mu$ are replaced with the conservation laws in terms of only spatial (in the LRF) gradients of the local thermodynamic fields. However, the anisotropy parameter ξ does not evolve on purely macroscopic time scales and thus is not constrained by the conservation laws. We refer the reader to Ref. [9] for further details.

In order to close the system of equations (6), we need to identify and separate the microscopic times scales by determining the eigenmodes of $\mathcal{A}^{(2)}$ by introducing the matrix $\Omega^{(2)}$ that diagonalizes $\mathcal{A}^{(2)}$. This process results in an asymptotic "Navier-Stokes" limit for all microscopic eigenmodes except for the slowest one ($r = 0$):

$$\tilde{\rho}_i^{\mu\nu} = (\Omega_{i0}^{(2)})^{\mu\nu}_{\alpha\beta}\tilde{\pi}^{\alpha\beta} + \hat{\ell}_i^{\mu\nu}\dot{\xi} + \hat{m}_i^{\mu\nu\lambda}\dot{z}_\lambda + (\hat{\eta}_{\theta i}^{(2)})^{\mu\nu}\theta + (\hat{\eta}_{\sigma i}^{(2)})^{\mu\nu\lambda\rho}\sigma_{\lambda\rho} + (\hat{\eta}_{\omega i}^{(2)})^{\mu\nu\lambda\rho}\omega_{\lambda\rho} + \text{h.-o. terms.} \tag{7}$$

The tensors $\hat{\ell}$, $\hat{m}$, $\hat{\eta}_\theta^{(2)}$, $\hat{\eta}_\sigma^{(2)}$ and $\hat{\eta}_\omega^{(2)}$ are built from u^μ, z^μ, $\Delta^{\mu\nu}$ and the microscopic relaxation scales. The fact that all higher order tensor moments are expressed in terms of the lowest-order one that appears in $T^{\mu\nu}$, $\tilde{\rho}_0^{\mu\nu} \equiv \tilde{\pi}^{\mu\nu}$, Eq. (6) can be completely decoupled from higher order moments. This is achieved by multiplying Eq. (6) by the microscopic relaxation times $(\tau_{nr}^{(2)})^{\mu\nu}_{\alpha\beta} = [(\mathcal{A}^{(2)})^{-1}_{nr}]^{\mu\nu}_{\alpha\beta}$ and then using Eq. (7). But the asymptotic "Navier-Stokes" limit (7) involves time derivatives (in the LRF) of the anisotropy parameter. This is the main ingredient. Since $\xi \sim \mathcal{O}(1)$ in our power counting scheme and does not evolve according to the conservation laws it cannot be neglected as a higher order term thus leading to second order comoving time derivatives $\ddot{\xi}$. (When $\dot{\tilde{\rho}}_r^{\mu\nu}$ in Eq. (6) is approximated according to (7) the $\hat{\ell}_i^{\mu\nu}\dot{\xi}$ term generates $\ddot{\xi}$ terms.) The result is a set of equations that take the following form:

$$\begin{aligned}(\tau_\pi^{(n)})^{\mu\nu}_{\alpha\beta}\dot{\tilde{\pi}}^{\alpha\beta} \quad &+ \quad (\Omega_{n0}^{(2)})^{\mu\nu}_{\alpha\beta}\tilde{\pi}^{\alpha\beta} + (\tau_\xi^{(n)})^{\mu\nu}\dot{\xi} + (\lambda_z^{(n)})^{\mu\nu\lambda}\dot{z}_\lambda + (\mathcal{D}_2^{(n)})^{\mu\nu} + (\mathcal{D}_1^{(n)})^{\mu\nu} \\ &= \quad (\eta_\theta^{(2,n)})^{\mu\nu}\theta + (\eta_\sigma^{(2,n)})^{\mu\nu\alpha\beta}\sigma_{\alpha\beta} + (\eta_\omega^{(2,n)})^{\mu\nu\alpha\beta}\omega_{\alpha\beta} + (\mathcal{J}^{(n)})^{\mu\nu} + (\mathcal{K}^{(n)})^{\mu\nu}\ .\end{aligned} \tag{8}$$

See Ref. [9] for the explicit expressions for the coefficients. The main result is that the tensors $(\mathcal{D}_2^{(n)})^{\mu\nu}$ contain second-order time derivatives $\sim \ddot{\xi}$. Fig. 1 shows the result of the dispersion relations in the long wavelength limit $k \to 0$ for the eigenmodes of the linearized equations of motion (i.e. linear perturbations around a static background). The equal and opposite real and the negative imaginary parts of the frequencies of the nonhydrodynamic modes indicate that the momentum anisotropy parameter ξ undergoes transient (damped) oscillations.

3. Conclusions

We showed here that transient oscillations of the slowest nonhydrodynamic modes arise generically in the weakly coupled Boltzmann equation from large deviations from local momentum isotropy in rapidly expanding anisotropically expanding systems. If the distribution function is instead expanded around its local equilibrium value when deriving the macroscopic equations of motion from the Boltzmann equation, all nonhydrodynamic modes are found to be exponentially damped [1].

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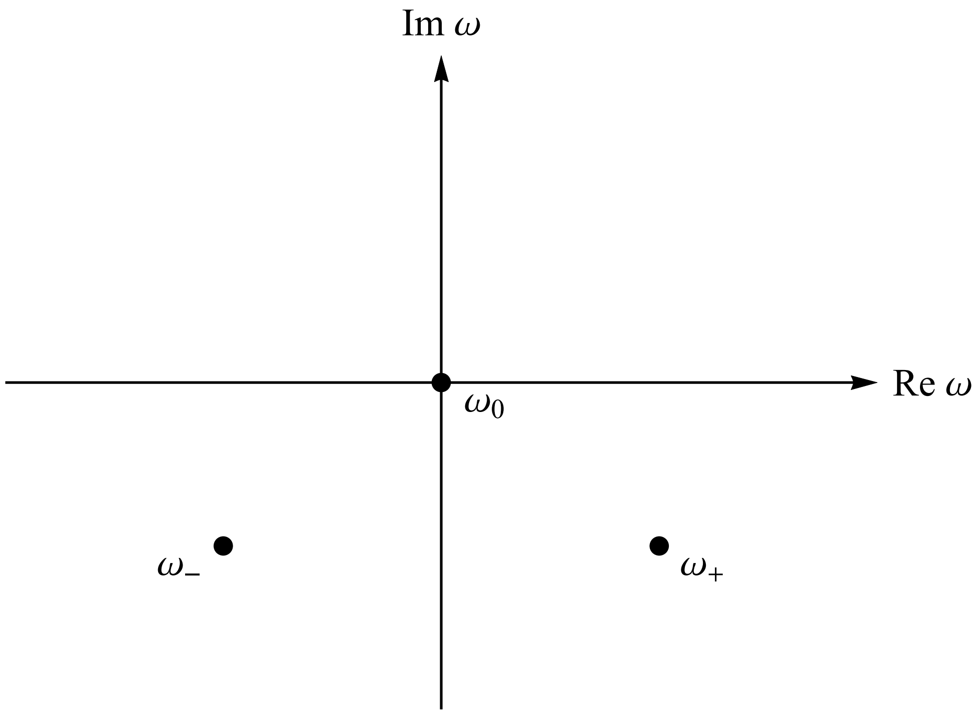

Figure 1. Shear channel eigenmodes in the zero wavenumber limit. Shown are the two degenerate hydrodynamic modes at $\omega_0 = 0$ and the first two non-hydrodynamic modes at $\omega_\pm$ that exhibit transient oscillatory behavior.