SLAC – PUB – 3593
March 1985
T/E

AMPLITUDE ANALYSIS FOR $D \to K\pi$ AND $K \to \pi\pi$ DECAYS AND A MEASURE OF 6-DOMINANCE*

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ABSTRACT

An amplitude analysis for $D \to K\pi$ and $K \to 2\pi$ decays is made and a measure of [6]-dominance in $D \to K\pi$ calculated. The analysis of $K \to 2\pi$ amplitudes determines the ratio A_2/A_0 and the difference of the two $\pi - \pi$ scattering phase shifts at the K-meson mass very precisely.

Submitted to *Physical Review Letters*

⋆ Work supported by the Department of Energy, contract DE – AC03 – 76SF00515.

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It is well known,[1] though not as well understood,[2] that $K \to 2\pi$ decay is dominated by the $\Delta I = \frac{1}{2}$ part of the weak Hamiltonian, H_w. In $SU(3)$ this implies that H_w, in the strange sector, transforms predominantly as an octet. Because of the boson symmetry in the final state, $\Delta I = \frac{1}{2}$ part of H_w results in an $I = 0$ two-pion final state while the $\Delta I = \frac{3}{2}$ part of H_w results in an $I = 2$ two-pion final state. It is also well known[3] that $A_2/A_0 \approx 0.045$, where A_0 and A_2 are the decay amplitudes resulting in the two pions in $I = 0$ and 2 states respectively. Octet dominance over the 27-plet of H_w is thus a well-established fact. In this work we have reanalyzed $K \to 2\pi$ decays and evaluated relevant parameters to a better accuracy.

In $SU(4)$ the [20] representation contains[4] a $\Delta C = 1$ [6] and the $\Delta C = 0$, $\Delta S = 1$ [8] of $SU(3)$, while the [84] representation of $SU(4)$ contains a $\Delta C = 1$ [15*] and the $\Delta C = 0$, $\Delta S = 1$ [27] of $SU(3)$. The assumption of [20] enhancement over [84] of $SU(4)$ would lead us to expect that [6] would be enhanced relative to [15*] in the charm sector as would be [8] over [27] in the strange sector. Though the hypothesis of [6]-dominance is widely assumed to be a working postulate no quantitative measure analogous to the ratio A_2/A_0 for K-decays appears to exist.[5] The aim of this work is to carry out an amplitude analysis for $D \to K\pi$ decays and evaluate such a measure.

Consider $D \to K\pi$ via [6] of H_w. Then $H_w |D\rangle$ transforms like [8] + [10] of $SU(3)$. Since the final state is made up of two identical octets the transition can only occur to an $[8_s]$ state which contains $I = \frac{1}{2}$. Thus [6] of H_w leads only to an $I = \frac{1}{2}$ $K\pi$ final state. On the other hand, [15*] of H_w can lead to a [27] representation since $[15^*] \otimes [3^*] = [8] + [10] + [27]$. As [27] of $SU(3)$ contains $I = \frac{3}{2}$ (in addition to $I = \frac{1}{2}$) [15*] of H_w can lead to an $I = \frac{3}{2}$ $K\pi$ final state.

A measure of [6]-dominance would be the ratio A_3/A_1 where A_1 and A_3 are the $D \to K\pi$ amplitudes for decays into $I = \frac{1}{2}$ and $\frac{3}{2}$ final state respectively. Though [6]-dominance would imply a small value for A_3/A_1, the converse need not necessarily be true since [15*] can lead to an $I = 1/2$ final state also.

In general, the decay amplitudes $D \to K\pi$, in different charged states are defined as

$$A(D^0 \to \overline{K}^0\pi^0) = \frac{1}{\sqrt{3}} \left(\sqrt{2}A_3e^{i\delta_3} + A_1e^{i\delta_1}\right) \tag{1}$$

$$A(D^0 \to K^-\pi^+) = \frac{1}{\sqrt{3}} \left(A_3e^{i\delta_3} - \sqrt{2}\,A_1e^{i\delta_1}\right) \tag{2}$$

$$A(D^+ \to \overline{K}^0\pi^+) = \sqrt{3}\,A_3e^{i\delta_3} \tag{3}$$

δ_1 and δ_3 are the phases of the two amplitudes A_1 and A_3. Define next the following ratios,[6]

$$R_{00} = \frac{\Gamma(D^0 \to \overline{K}^0\pi^0)}{\Gamma(D^0 \to K^-\pi^+)} = \frac{1 + 2\sqrt{2}\,r\cos\delta + 2r^2}{2 - 2\sqrt{2}\,r\cos\delta + r^2} \tag{4}$$

$$R_{0+} = \frac{\Gamma(D^0 \to K^-\pi^+)}{\Gamma(D^+ \to \overline{K}^0\pi^+)} = \frac{1}{9}\left(1 - \frac{2\sqrt{2}}{r}\cos\delta + \frac{2}{r^2}\right) \tag{5}$$

where $r = A_3/A_1$ and $\delta = \delta_1 - \delta_3$. Experimentally[6,7]

$$R_{00} = 0.35 \pm 0.07 \pm 0.07 \tag{6}$$

$$R_{0+} = 3.7 \pm 1.0 \pm 0.7 \tag{7}$$

In evaluating (7) we have used $\tau(D^+)/\tau(D^0) = 2.5 \pm 0.6$ (statistical only).

In principle, given good enough data, would could solve for δ and r from (4) and (5). However error propagation makes this procedure hazardous, particularly for $\cos\delta$ which is bounded by unity. Notice that (4) and (5) have a mirror symmetry under $(r,\delta) \to (-r, \pi-\delta)$; theoretical prejudice would have to be invoked to pick one of the pair of solutions.

In Figs. (1) and (2) we have plotted R_{00} and R_{0+} as functions of r for fixed values of δ. Numerically it is found that simultaneous solutions exist for $35° \leq \delta \leq 80°$, $r < 0$ and it's mirror image $100° \leq \delta \leq 145°$, $r > 0$. *It is important to note that an analysis with real amplitudes will not be able to satisfy the experimental constraints on R_{00} and R_{0+} and the triangular relation implied by (1)-(3),*

$$A(D^0 \to \overline{K}^0\pi^0) + \sqrt{2}\, A(D^0 \to K^-\pi^+) = A(D^+ \to \overline{K}^0\pi^+) \tag{8}$$

To pick one of the two possible solutions we note that the phase of a two body weak decay amplitude is equal to the scattering phase shift provided that the scattering in the final state is elastic. A fairly reliable analysis of the $K\pi$ scattering in 0^+ state is available[8,9] which is known to resonate in $I = \frac{1}{2}$ state at 1.35 GeV (Kappa meson[10]). Thus δ_1 crosses 90° at 1.35 GeV.[9] The phase shift δ_3 is about $-30°$ at 1.35 GeV.[9] Thus δ is about 120° at 1.35 GeV. One would expect it to be larger at D-mass and lie in the second quadrant. Theoretical prejudice would therefore select the solutions with $100° \leq \delta \leq 145°$, $r > 0$. In Table I we list the values of the ratio A_3/A_1 for different values of δ in the range $100° \leq \delta \leq 145°$. We conclude that A_3/A_1 is about 0.25. In contrast A_2/A_0 in $K \to 2\pi$ decays is 0.045. Thus though [6] does dominate over [15*] in the charm sector, octet-dominance in the strange sector is much more striking.

A similar analysis done with the preliminary MARK III data[11] for $D \to K\rho$ and $D \to K^*\pi$ leads to $r \approx 0.35$ for the vector-pseudoscalar decays. This result is to be expected on the basis of [6]-dominance since the final state does not have to belong to $[8_s]$ as was the case in $D \to K\pi$. One therefore expects a larger admixture of $I = 3/2$ final state.

We also tested octet-dominance in $K \to 2\pi$ decays using the same technique. For $K \to 2\pi$ decays we have,

$$A(K_S \to \pi^+\pi^-) = \sqrt{\frac{2}{3}}\, A_0 e^{i\delta_0} + \frac{1}{\sqrt{3}}\, A_2 e^{i\delta_2} \tag{9}$$

$$A(K_S \to \pi^0\pi^0) = \frac{1}{\sqrt{3}}\, A_0 e^{i\delta_0} - \sqrt{\frac{2}{3}}\, A_2 e^{i\delta_2} \tag{10}$$

$$A(K^+ \to \pi^+\pi^0) = \frac{\sqrt{3}}{2}\, A_2 e^{i\delta_2}\ . \tag{11}$$

A_0 is the amplitude for decay into an $I = 0$ state. This results entirely from the $\Delta I = \frac{1}{2}$ octet part of H_w. A_2, the amplitude for decay into an $I = 2$ state, results entirely from the $\Delta I = \frac{3}{2}$ part of H_w *if we assume that there is no* $\Delta I = \frac{5}{2}$ *part in* H_w. δ_0 and δ_2 are the $\pi - \pi$ scattering phase shifts in these two isospin states.

One can define R_{00} and R_{0+} analogously to $D \to K\pi$ as follows,

$$R_{00} = \frac{\Gamma(K_S \to \pi^0\pi^0)}{\Gamma(K_S \to \pi^+\pi^-)} \tag{12}$$

$$R_{0+} = \frac{\Gamma(K_S \to \pi^+\pi^-)}{\Gamma(K^+ \to \pi^+\pi^0)}\ . \tag{13}$$

The branching ratios involved are known[10] to better than 1% accuracy. The phase space can be calculated very precisely as the masses involved are known to better than 0.03% accuracy. By factoring out the phase space we define $\overline{R}_{00}$ and $\overline{R}_{0+}$ as follows,

$$\overline{R}_{00} = 0.985\, R_{00} = \frac{1 - 2\sqrt{2}\, r\cos\delta + 2r^2}{2 + 2\sqrt{2}\, r\cos\delta + r^2} \tag{14}$$

$$\overline{R}_{0+} = 1.012\, R_{0+} = \frac{4}{9}\left(1 + \frac{2\sqrt{2}}{r}\cos\delta + \frac{2}{r^2}\right) \tag{15}$$

where $r = A_2/A_0$ and $\delta = \delta_0 - \delta_2$.

From the Particle Data Group[10] listing we calculate

$$\overline{R}_{00} = 0.451 \pm 0.004 \tag{16}$$

$$\overline{R}_{0+} = 454.7 \pm 3.9 \tag{17}$$

In Fig. 3 and 4 we have plotted $\overline{R}_{00}$ and $\overline{R}_{0+}$ as functions of r for fixed values of δ. By eliminating $\cos\delta$ from (14) and (15) one obtains an equation in r^2 from which r^2 is determined to better than 1% accuracy (i.e. r is determined to better than 0.5% accuracy),

$$r = 0.045 \pm 0.0002\ . \tag{16}$$

By going back to (14) and (15) one can determine δ. Equation (14) determines

$$\delta = (56.5 \pm 3.0)^\circ\ . \tag{17}$$

The solutions can be seen from Fig. 3. Equation (15) is less selective and determines δ far less precisely, $\delta = (55 \pm 20)^\circ$. This is also seen from Fig. 4. From $\pi - \pi$ phase shift analysis Kleinknecht[12] determines

$$\delta = (53 \pm 5)^\circ \tag{18}$$

which is consistent with our determination (17).

Note that the mirror solution with $r < 0$ and $\delta \rightarrow (180^\circ - \delta)$ is excluded on grounds the $\pi - \pi$ phase shift analyses[12,13] suggest that δ_0 is in the first quadrant and δ_2 small and negative at the K-mass.

Our determination of r, Eq. (16), is consistent with previous determinations[14,15] of r though much more precise. Reference 14 quotes r with a 10% error while Ref. 15 determines it with a 4% error.

In summary, we have computed a measure of [6]-dominance in the charm sector for $D \rightarrow K\pi$ decays. We have also shown that the data are precise enough to exclude real decay amplitudes. We have also presented a very precise calculation of the measure of octet dominance in the strange sector and the difference of the phases of the two isospin decay amplitudes in $K \rightarrow \pi\pi$. It is worth pointing out that an analysis using[16] $\tau(D^+)/\tau(D^0) = 2.3\ ^{+0.5+0.1}_{-0.4-0.1}$ makes little difference to the results. Table I remains unchanged to the accuracy used. I wish to thank F. Gilman, D. Hitlin, M. Scadron and R. Schindler for discussions at different times. This research was partly supported by a grant from the National Sciences and Engineering Research Council of Canada. The hospitality of the Theory Group at SLAC is gratefully acknowledged.

REFERENCES

1. See, for example, R. E. Marshak, Riazuddin and C. P. Ryan, *Theory of Weak Interactions in Particle Physics*, Wiley-Interscience, New York, 1966, Chapter 6.
2. M. K. Gaillard and B. W. Lee, Phys. Rev. Lett. 33, 108 (1974); G. Altarelli and L. Maiani, Phys. Lett. 52B, 351 (1974); M. Shifman, A. Vainshtein and V. Zakharov, JETP Lett. 22, 552 (1975); Nuc. Phys. B120, 316 (1977).
3. See, for example, C. S. Wu and T. D. Lee, Ann. Rev. Nuc. Sci. 16, 471 (1966); E. D. Commins and P. H. Bucksbaum, *Weak Interactions of Leptons and Quarks*, Cambridge University Press, New York, 1983, Chapter 6.
4. M. Einhorn and C. Quigg, Phys. Rev. D12, 2015 (1975).
5. M. S. Chanowitz, *A Review of D- and B-Meson Physics in High Energy e^+e^- Interactions,* AIP Conference Proceedings No. 62, Vanderbilt, 1980, AIP, New York (1980).
6. A. N. Kamal, Cabibbo-Angle Favored Two-Body Decays of D-Mesons, SLAC-PUB-3443, 1984 (unpublished).
7. R. H. Schindler, private communication, September 1984.
8. P. Estabrooks *et al.*, Nucl. Phys. B133, 490 (1978).
9. P. Estabrooks, Phys. Rev. D19, 2678 (1979).
10. Particle Data Group: *Review of Particle Properties*, Rev. Mod. Phys. 56, S1 (1984).
11. A. L. Duncan, private communication and APS Meeting at Santa Fe, October 31-November 3, 1984.
12. K. Kleinknecht, Ann. Rev. Nucl. Sci. 26, 1 (1976).

13. P. Estabrooks *et al.*, in $\pi - \pi$ *scattering*—(Tallahassee), AIP Conference Proceedings No. 13; Eds. P. K. Williams and V. Hagopian, AIP, New York (1973).

14. E. D. Commins and P. H. Bucksbaum in Ref. 3, p. 210.

15. B.H.J. McKellar and M. D. Scadron, Phys. Rev. D27, 157 (1983).

16. Mark III Collaboration, A Direct Measurement of Charmed D^+ and D^0 Semileptonic Branching Ratios, SLAC-PUB-3532 (1985).

FIGURE CAPTIONS

1. $R_{00}(K\pi)$ versus r for various values of $\delta_1 - \delta_3$.

2. $R_{0+}(K\pi)$ versus r for various values of $\delta_1 - \delta_3$.

3. $\overline{R}_{00}$ versus r for various values of $\delta_0 - \delta_2$.

4. $\overline{R}_{0+}$ versus r for various values of $\delta_0 - \delta_2$.

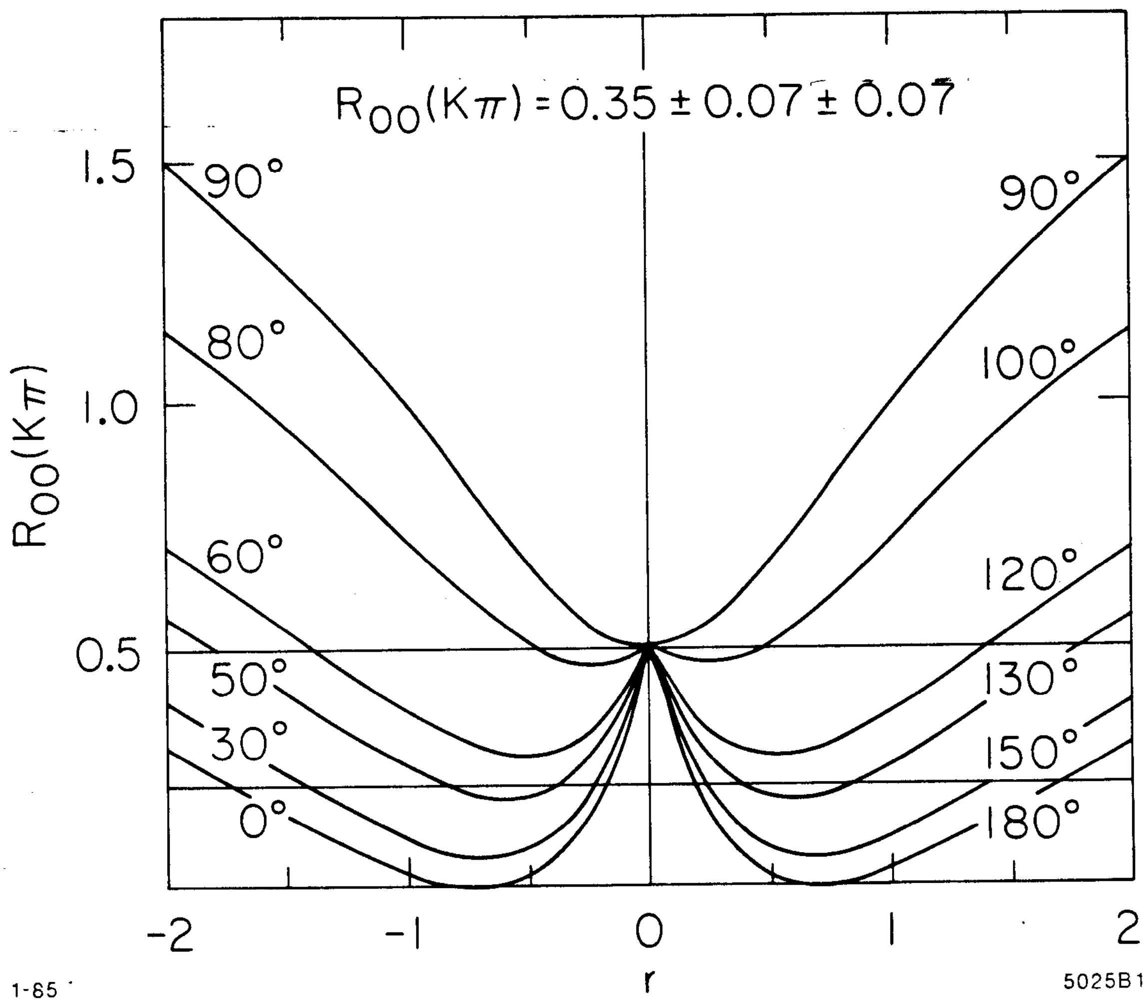
Table I

$\delta = \delta_1 - \delta_3$	r	Source
100°	$0.26 \pm {}^{0.11}_{0.04}$	(a)
110°	$0.26 \pm {}^{0.12}_{0.04}$	(a)
110°	$0.26 \pm {}^{0.12}_{0.04}$	(a)
120°	$0.27 \pm {}^{0.13}_{0.04}$	(a)
130°	$0.27 \pm {}^{0.11}_{0.05}$	(b)
140°	0.25 ± 0.02	(c)
145°	0.24 ± 0.00	(c)

(a) Central value and errors determined by R_{0+}. R_{00} is less selective than R_{0+} for this value of δ.

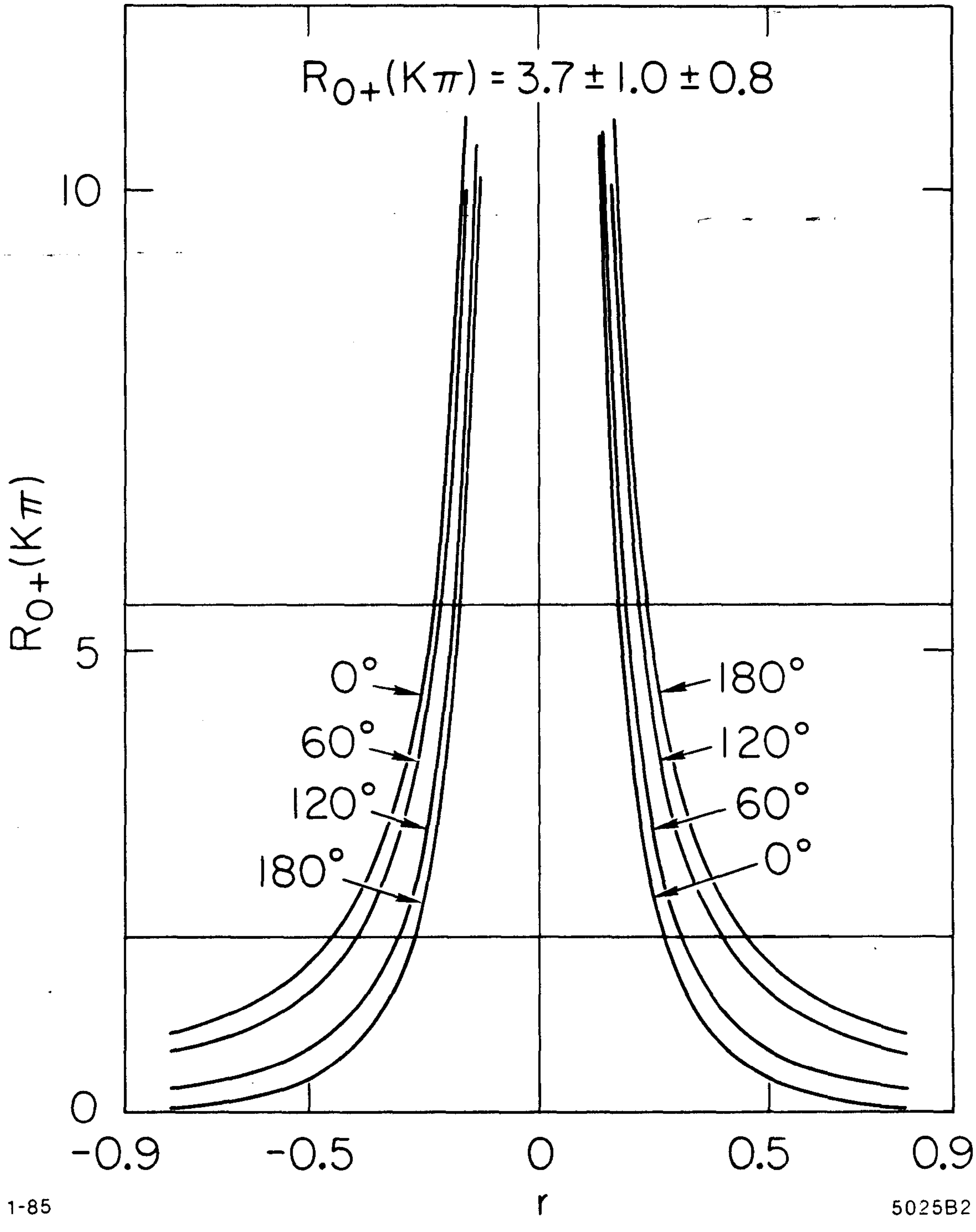
(b) Central value determined by R_{0+}. Upper limit determined by R_{00} which is more stringent than that determined by R_{0+} for this value of δ.

(c) Lower limit determined by R_{0+} and upper limit by R_{00}. Central value so chosen as to connect with the limits with a symmetrical error.



R00(Kπ) = 0.35 ± 0.07 ± 0.07
1.5
1.0
0.5
R00(Kπ)
90°
80°
60°
50°
30°
0°
90°
100°
120°
130°
150°
180°
-2
-1
0
1
2
r
1-85
5025B1

Fig. 1



$R_{0+}(K\pi) = 3.7 \pm 1.0 \pm 0.8$
10
5
0
$R_{0+}(K\pi)$
0°
60°
120°
180°
180°
120°
60°
0°
-0.9
-0.5
0
0.5
0.9
r
1-85
5025B2

Fig. 2

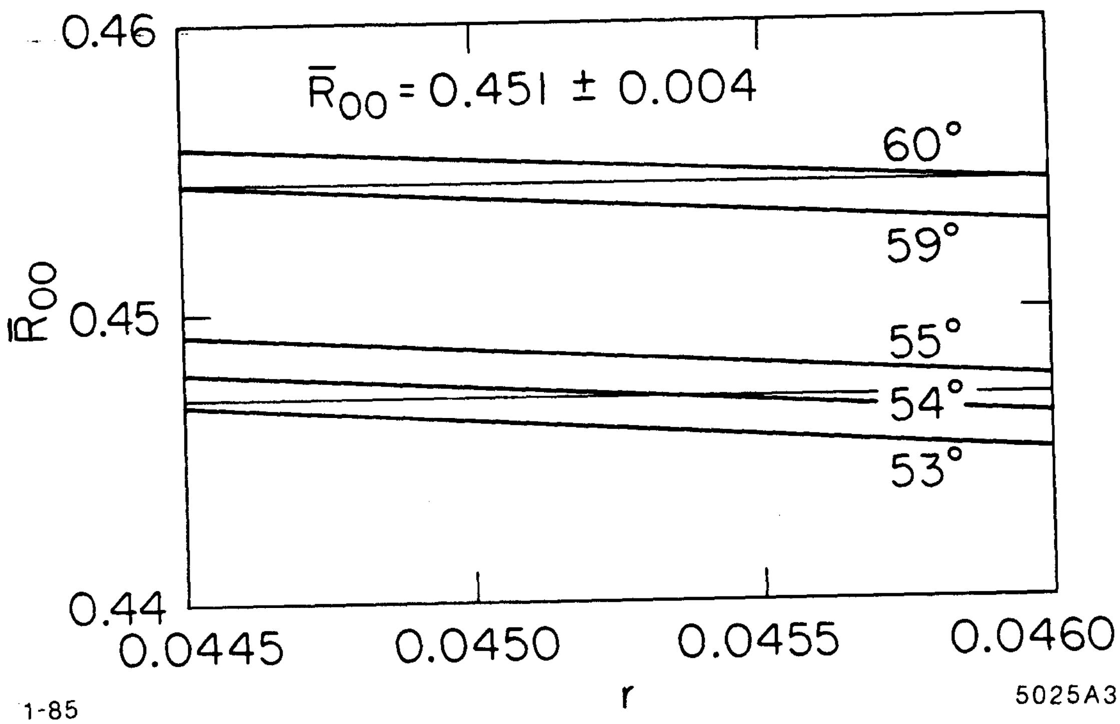
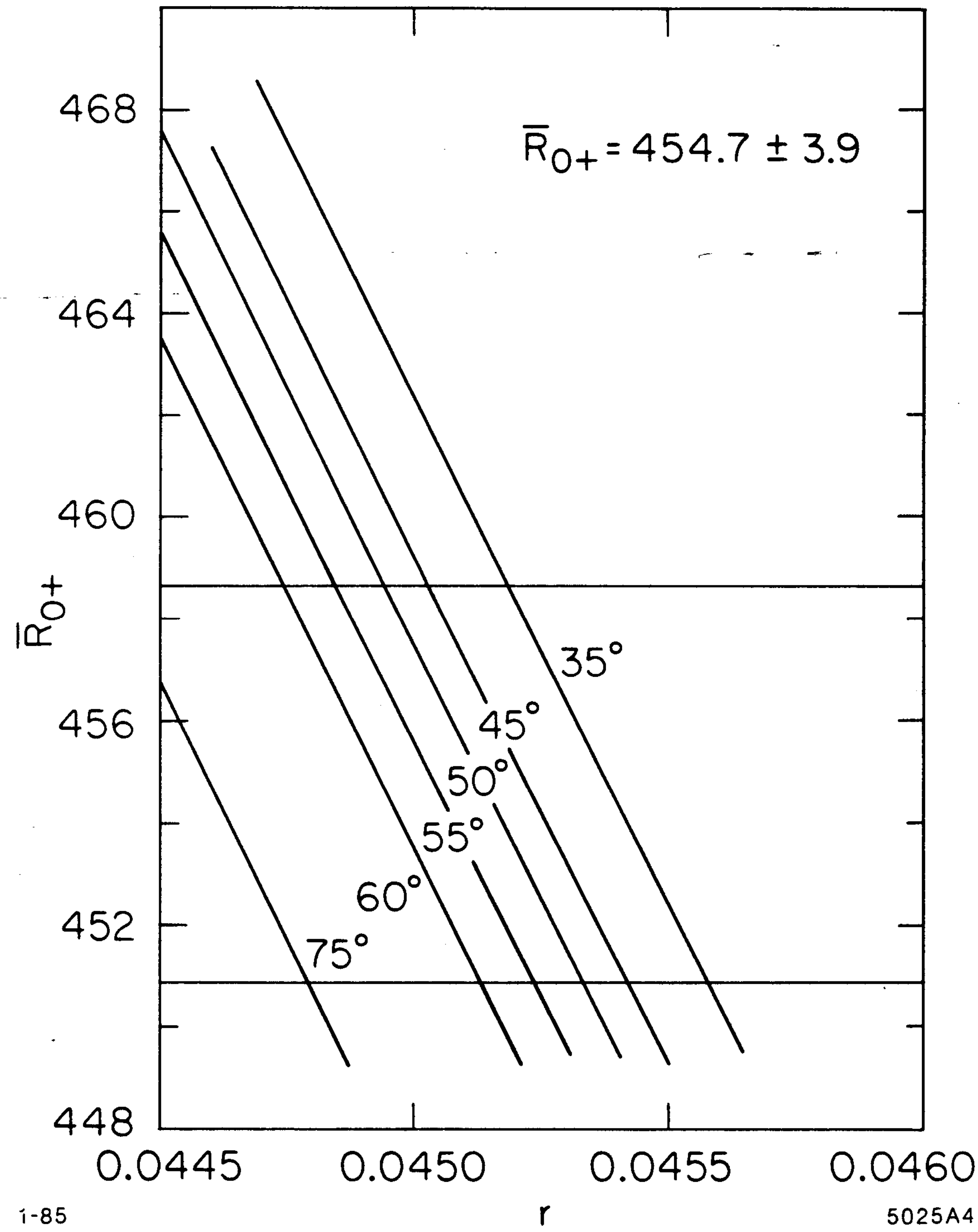

0.46
$\bar{R}_{00} = 0.451 \pm 0.004$
60°
59°
$\bar{R}_{00}$
0.45
55°
54°
53°
0.44
0.0445
0.0450
0.0455
0.0460
r
1-85
5025A3

Fig. 3



$\bar{R}_{0+} = 454.7 \pm 3.9$
468
464
460
456
452
448
$\bar{R}_{0+}$
35°
45°
50°
55°
60°
75°
0.0445
0.0450
0.0455
0.0460
r
1-85
5025A4

Fig. 4