A SEARCH FOR MISSING BARYON STATES

By

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A Thesis Submitted to the Graduate Faculty of Rensselaer Polytechnic Institute in Partial Fulfillment of the Requirements for the Degree of DOCTOR OF PHILOSOPHY Major Subject: Physics

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Rensselaer Polytechnic Institute Troy, New York

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The original of the complete thesis is on file in the Rensselaer Polytechnic Institute Library

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ABSTRACT

We perform a partial wave analysis on the reaction $\gamma p \rightarrow p\pi^+\pi^-$ for photon energies of 0.5-2.4 GeV ($W = \sqrt{s} = 1.35 - 2.35 \text{ GeV/c}^2$). The data was collected using the CLAS detector located at Jefferson Laboratory in Newport News, VA. We are searching for baryon states produced in $\gamma p \rightarrow B$ and decaying by $B \rightarrow p\pi^+\pi^-$ through quasi-two body intermediate states such as $\Delta \pi$ and $p\rho$. Our partial wave decomposition allows us to accurately calculate the total and differential cross section. We also calculate the cross section for $\gamma p \rightarrow \Delta^{++}\pi^-$, $\gamma p \rightarrow \Delta^0 \pi^+$, and $\gamma p \rightarrow p\rho$. We identify the D₁₃(1520), P₃₃(1600) and F₁₅(1680) states in the decomposition. We do not see evidence for the baryon state decaying to $\Delta \pi$ at 1700 MeV/c² proposed by Ripani, et al. [?]. We see no strong evidence for the positive parity missing baryons, although there are signals in the data which warrant further investigation.

The constituent quark model does an excellent job of predicting the hadron spectrum. Capstick, Cutkosky, Forsythe, Isgur and Koniuk [?, ?, ?, ?] have augmented the quark model for baryons, including decays, with QCD-inspired corrections and get very good agreement with experiment. It has been known however, since the 1960's that there are many predicted baryons which are not observed experimentally [?, ?]. Many of the models use a harmonic oscillator basis, and it is found that these missing states all fall in the N=2, positive parity band. This prompted Lichtenberg[?] to propose the diquark model, where two of the three quarks become tightly bound. This constraint leads to a spectrum devoid of the missing resonances of the full model. There is nothing in QCD however, which would imply any sort of diquark coupling. Later calculations [?, ?, ?] suggest that these missing states may couple more strongly to $N\pi\pi$ final states than $N\pi$ final states. Previous analysis, such as those performed by Manley and Saleski [?], have focused on $N\pi$ scattering, where most of the experimental data lies.

CHAPTER 1 Theoretical Motivation

1.1 Standard Model

Particle physics is the study of the most fundamental building blocks of matter and the forces which govern their interaction. In the past, the atom was believed to be the most basic constituent of matter. but this was shown not to be the case. In fact the atom is composed of a central core, made up of of protons and neutrons, and a cloud of lighter electrons surrounding it. As time went on more and more sub-atomic particles were discovered. It is instructive to take a look at some of these characteristics these particles possess. This will give us an idea of how we classify them.

- Mass. How much inertia (or gravitational attraction) does the particle posses? No matter how fast the particle is moving, no matter its energy, this value can be measured from a particle's energy and momentum. For some very short lived particles, their mass may have some distribution. Two of the same particle may have slightly different masses.
- Charge. This is our every day quantity of electric charge. The basic unit is the magnitude of the charge of the electron, 1.602 × 10⁻¹⁹C. Almost all particles are some integer multiple of this charge (keeping in mind it may be 0). Quarks, the particles that make up protons and neutrons, possess ±1/3 or ±2/3 the electron's charge.
- Spin. All particles will have some intrinsic angular momentum associated with them. If they are composite particles, it will some combination of the spin of the particles that comprise them and the relative orbital angular momentum of these constituent particles. A particle can either have integer spin or half-integer spin. This spin also allows us to characterize particles by the statistics they obey.

- Fermi-Dirac (Fermions). These are particles which posses half-integer spin. Two identical fermions cannot occupy the same state. This is the basis for the Pauli exclusion which gives rise to electron energy levels in atoms.
- Bose-Einstein (Bosons). These particles posses integer spin. Any number of bosons can occupy the same state.
- Parity, charge conjugation, isospin, strangeness, charmness, etc. There are other properties that these particles possess which we may use to classify them. Some of these will be discussed in future chapters.

Particles are also classified by how they interact with each other. There are four forces in the universe and not all matter (or energy) interact via all of these forces. Let's look at these forces.

- **Gravity.** Affects the motion of all matter and energy. It is the most macroscopic of the forces. It influences the formation of stars and galaxies, keeps the planets in motion and ties us to the Earth. The most accurate theory we have to understand gravity is that of General Relativity. It is a large scale, geometric theory and does not lend itself to understanding phenomena on an atomic scale.
- Electromagnetism. Affects all charged objects (or neutral objects which are composed of individually charged particles). It is the force which drives electric currents as well as binds the electrons to the protons in atoms.
- Weak nuclear force. Primarily observed in various decays, most notably beta decay in certain nuclei and non-parity conserving decays in muons, kaons, pions and other particles. It has been demonstrated that both the weak nuclear and electromagnetic force, can be understood as manifestations of *one* force imaginatively called the electro-weak force.
- Strong nuclear force. Acts between quarks. In binding quarks into protons and neutrons, there is a residual effect which in turn binds the protons and neutrons together in atomic nuclei.

Leptons				
	Flavor			
Symbol	Name	Mass (MeV/c^2)	Charge	
e	electron	0.5	±	
$ u_e, \bar{ u}_e$	electron neutrino	$< 3 \times 10^{-6}$	0	
μ	muon	105.6	±	
$ u_{\mu}, ar{ u}_{\mu}$	muon neutrino	< 0.19	0	
au	tau	1777	±	
$ u_{ au}, ar{ u}_{ au}$	tau neutrino	< 18.2	0	

Table 1.1: Summary of lepton properties.[?]

With these traits and forces in mind we can begin to form some sort of order out of the most fundamental particles. The electrons belong to a family of structure-less particles called leptons. Leptons interact via the weak force and even though they have varying masses, they are believed to be structure-less. They are not composed of any more fundamental particles. The charged particles can also interact via the electromagnetic force. Leptons are all spin-1/2 and obey Fermi-Dirac statistics. A summary of leptons can be seen in Table 1.1 [?]. Electrons, muons and the tau's are positive, but have a negatively charged antimatter compliment. The various neutrino flavours are all neutral, and have a neutral antimatter compliment.

Unlike the electron, the proton and neutron do have structure. They are composed of quarks. Quarks (and the particles that they make up) interact via the electromagnetic, strong nuclear and weak nuclear forces (and gravity). Quarks are also spin-1/2. Quarks will be discussed in greater detail in the next section. A summary of quark properties is given in Table 1.2.

There is another class of particles known as gauge bosons. Photons are gauge bosons. In Einstein's photoelectric theory, they are quantized packets of light, or discrete amounts of electromagnetic radiation. With the acceptance of quantum electrodynamics, photons also became thought of as mediators of the electromagnetic force. These integer-spin mediators are known as gauge bosons. Applying

Quarks				
Flavor				
Symbol	Name	Mass (MeV/c^2)	Charge	
d	down	1.5 - 5	$-\frac{1}{3}$	
u	up	3 - 9	$+\frac{2}{3}$	
s	strange	60 - 170	$-\frac{1}{3}$	
с	charm	1100 - 1400	$+\frac{2}{3}$	
b	bottom	4100 - 4400	$-\frac{1}{3}$	
t	top	168,600 - 179,000	$+\frac{2}{3}$	

Table 1.2: Summary of quark properties.[?]

Table 1.3: Summary of gauge boson properties.[?]

Mediators - Gauge Bosons					
Symbol	Name	Mass (MeV/c^2)	Charge	Force	
g	gluon	0	0	Strong	
γ	photon	0	0	Electromagnetism	
W^{\pm}	W boson	80,419	\pm	(charged) Weak	
Z^0	Z boson	$91,\!188$	0	(neutral) Weak	

gauge invariance led Sheldon Glashow, Steven Weinberg and Abdus Salam to unify the weak and electromagnetic forces. Their theory gave rise to three massive mediators and one massless mediator. The massive bosons are the W^{\pm} and Z^{0} . The massless was the familiar photon. A similar approach led to quantum chromodynamics (QCD) with gluons as mediators. The properties of these particles are listed in Table 1.3.

The goal of physics is to be able to explain all these forces as various manifestations of one theory. There have been significant successes in this endevour. In the Glashow-Weinberg-Salaam theory, the electromagnetic and weak forces are united as different aspects of one force. Quantum Chromodynamics is believed to be a viable framework for understanding the interactions of the strong force. The group of theories which explain these forces is referred to as the Standard Model. The Standard Model also provides a framework to understanding the flavors and groupings of the elementary particles. There is still no viable framework for merging gravity and quantum theory, and so gravity remains outside the Standard Model.

1.2 Quarks

1.2.1 Constituent Quark Model

In the mid-1940's, thanks to the efforts of J.J. Thomson, E. Rutherford, J. Chadwick and many others, physicists knew of a handful of sub-atomic particles.

- Protons and neutrons. Found in the nuclei of atoms.
- Electrons. Found in a diffuse cloud around atoms.
- Pions and muons. First discovered in cosmic ray experiments.
- Photons. The quantized packets of light.

But by 1960, the picture had grown more complicated. Starting with the theoretical prediction and subsequent discovery of the positron in 1932, science became aware of antiparticles. The positron is the antimatter complement to the electron. In the mid 1950's, the Berkeley Bevatron observed the anti-proton and the antineutron. Adding to the confusion, the neutrino, a neutral particle initially thought to be massless, was thrown into the mix. The neutrino had initially been proposed by Pauli to account for the energy spectrum of the electron emitted in beta decay. Evidence for it's existence was later collected in photographs of the decays of cosmic ray muons.

But beyond these developments a whole family of particles seemed to be emerging. In 1949, C.F. Powell showed evidence for a charged particle similar to the pion but more massive. It was called the kaon. It was observed in photographs of cosmic ray decays. The decay observed was

$$K^+ \to \pi^+ \pi^+ \pi^-$$

This decay implies that the kaon possess integer-spin like the pion. Over the next few years more of these integer-spin particles were observed like the η, ϕ, ω and others. In keeping with early naming schemes involving the muon and pion, these particles were called *mesons*.

Another family of particles was also emerging around these times. These particles had a proton in the final state of their decays implying that they possessed half-integer spin. Some of these particles were the Σ 's, Ξ 's and Δ 's. These halfinteger spin particles became known as *baryons*. Both the mesons and baryons were being produced in a way that implied strong force interactions (though they did not always decay by the strong force). Particles which can interact via the strong force are called *hadrons*.

The picture was uncomfortably chaotic. There was no rigorous order applied to these particles in the way that the periodic table imposed order upon the elements. In 1961, M. Gell-Mann proposed an ordering scheme he called the Eightfold Way. Y. Ne'eman proposed essentially the same idea around the same time. The Eightfold Way arranged the hadrons into various geometrical patterns according to charge and hypercharge. Hypercharge is related to a quantity called strangeness. It was found that certain mesons like the kaon, and certain baryons like the Λ were easily produced in scattering experiments, but they would decay weakly. They would live for relatively long periods of time. Also, this group of particles would be produced in pairs. You could produce a final state with KK or $K\Lambda$, but never just the one. These strange characteristics led Gell-Mann and Nishijima to propose a quantity that Gell-Mann referred to as strangeness. This quantity was conserved in the production of these particles and so they possessed either S = +1 or S = -1. Hypercharge was defined as S for mesons and S+1 for baryons. Later it was shown that strangeness itself was a more useful way to group the baryons.

The eight lightest mesons can be arranged in the hexagonal pattern shown in Fig. 1.1. The eight lightest baryons can be arranged in a similar fashion and are

shown in the same figure. Other baryons can be arranged in the decuplet pattern shown in Figure 1.2. Note that particles of the same strangeness lie along the horizontal, and like charges fall on the diagonals.



Figure 1.1: The eight lightest baryon and eight lightest mesons grouped according to the Eightfold Way



Figure 1.2: The first decuplet of baryons grouped according to the Eightfold Way

But just like the original periodic table, there were gaps in these arrangements. The decuplet shown in Figure 1.2 did not originally have the Ω^- at the bottom. Gell-Mann noticed that the mass difference between the horizontal rows was about the same and wondered if this could predict the mass of the missing particle.

$$M_{\Delta} - M_{\Sigma^*} = M_{\Sigma^*} - M_{\Xi^*} = M_{\Xi^*} - M_{???}$$

$$M_{\Delta} - M_{\Sigma^{*}} = 1232 MeV - 1385 MeV$$

= -153MeV
$$M_{\Sigma^{*}} - M_{\Xi^{*}} = 1385 MeV - 1533 MeV$$

= -148MeV
$$M_{\Xi^{*}} - M_{???} = 1533 MeV - ??? MeV$$

 $\approx -150 MeV$

Gell-Mann used this information to predict a particle of strangeness -2 at a mass of about 1680 MeV/c². In 1964, the Ω^- was discovered with these characteristics.

This simple model, the Eightfold Way, was found to have powerful predictive ability.

But *why* does the Eightfold Way predict what it does? In 1964, Gell-Mann and G. Zweig independently put forth the idea that hadrons are composed of more basic constituent particles. Gell-Mann called these particles, quarks.

Gell-Mann proposed three types, or flavours, of quarks. Each quark (antiquark) possessed a fractional charge and either 0 or +1(-1) strangeness. These quantities are summarized in Table 1.4.

The name for the strange quark takes its name from the quantity, strangeness, which had been associated with the kaons, Σ 's and other particles. The up and down quarks took their names from the historical discussions of isospin. The idea was that because the masses of the proton and neutron were so similar, they could be viewed as one particle, the nucleon, which was an isospin- $\frac{1}{2}$ particle and had two projections in isospin space. The proton was the "up" projection, and the neutron

Quarks					
Flavor					
Symbol	Name	$\text{Isospin}(\mathbf{I}_z)$	Strangeness	Charge	
u	up	$\frac{1}{2}(+\frac{1}{2})$	0	$+\frac{2}{3}$	
d	down	$\tfrac{1}{2}(-\tfrac{1}{2})$	0	$-\frac{1}{3}$	
S	strange	0	-1	$-\frac{1}{3}$	

Table 1.4: Summary of Gell-Mann's quark properties.[?]

was the "down" projection. The quarks also have anti-quark complements with opposite charge and opposite strangeness. Gell-Mann had two rules that his quarks followed.

- Baryons are composed of **3** quarks or **3** anti-quarks.
- Mesons are composed of 1 quark and 1 anti-quark.

With these rules, you could now look at the geometrical arrangements of the Eightfold Way, in terms of the constituent quarks. Figure 1.3 shows how the quark configurations map onto the first baryon decuplet.

There was a problem with this picture, however. The quarks carry spin $\frac{1}{2}$. This means that they must obey Fermi-Dirac statistics. Just like the electron in atomic orbits, no two identical quarks could occupy the same state. Yet the quark model had the Δ^{++} consisting of 3 *u* quarks in the ground state. In 1964, O.W. Greenberg proposed a solution. In addition to flavour, the quarks belonged to a statistical group called para-Fermi. This suggestion led to the idea that quarks possessed an additional degree of freedom that became known as *colour*. In analogy with optics, each quark could come in *red*, *blue* or *green*, or for the anti-quarks, *antired*, *antiblue* or *antigreen*. Continuing with the optical analogy, a *red* – *blue* – *green* combination resulted in a colour-less object. As did a *red* – *antired*, *blue* – *antiblue* or *green* – *antigreen* combination. The idea is that all particles that we observe are **colour-less**. This solved two problems: that the same quarks in the ground state





could have an anti-symmetric wave function due to colour, and that no free quarks could be observed.

With the discovery of the J/ψ in 1974, a new flavour of quark (charm) was added to the chart. We now know of six flavours of quarks (summarized in Table 1.2. The constituent quark model (CQM) was able to handle each new discovery. There is a glaring problem though. There were whole multiplets of baryons which were predicted by the model, and never observed. It is this missing baryon problem which is at the heart of this research and will be elaborated upon in upcoming sections.

1.2.2 Quark Model with Corrections

While the CQM was able to arrange the hadrons into some sort of sensical pattern, the goal of many was to more accurately predict the masses and other characteristics of the particles. In this next section, I give an overview of some of the corrections to the CQM as applied to baryons.[?]

Baryons possesses half-integer spin, and so obeys Fermi-Dirac statistics. The

total wave function of the quarks must be anti-symmetric under the exchange of any two. The total wave function can be broken into *flavour, spatial, spin* and *colour*. Though it will not be elaborated upon here, it can be shown that the colour part of the wave function must be anti-symmetric [?]. This allows us to decompose the wave function as follows.[?]

$$|qqq\rangle_A = |colour\rangle_A \times |space, spin, flavour\rangle_S$$

So the product of the spatial, spin and flavour portions of the wave function must be symmetric. This constraint allows you to build up a more complicated model than in the previous picture. For example, if you have all the same flavour of quarks with spins aligned, they must be in a symmetric spatial state. We want to use this information to calculate the masses of the baryons. So let's start with a simple Hamiltonian picture.

$$H|\Psi\rangle = E|\Psi\rangle.$$

How is H defined in this model? In the simplest treatment of this model we can approximate the quarks as moving in a 3-D harmonic oscillator potential. There are various corrections that can be applied at this point. In the Isgur-Karl nonrelativistic model [?, ?, ?] a hyperfine spin-spin interaction is added.

$$H = H_{HO} + \sum_{i < j} H^{ij}_{hyperfine}$$

The hyperfine interaction term can be written out as

$$H_{hyp}^{ij} = \frac{2\alpha_s}{3m_i m_j} \left[\frac{8\pi}{3} \delta^3(\vec{r_{ij}}) \vec{S_i} \cdot \vec{S_j} + \frac{1}{r_{ij}^3} (3\vec{S_i} \cdot \hat{r_{ij}} \vec{S_j} \cdot \hat{r_{ij}} - \vec{S_i} \cdot \vec{S_j}) \right].$$

where α_s is the hyperfine structure constant for the strong force, and S_i and m_i are the spin operator and mass for the *i*th quark. It turns out that this factor is a significant contribution to the mass splittings. For example, the mass difference between the proton and the Δ^+ is primarily due to different spin-alignments of the quarks.

	Quark makeup	Spin	Quark spin alignment	Mass (MeV/c^2)
proton	uud	$\frac{1}{2}$	$\uparrow \uparrow \downarrow$	938
Δ^+	uud	$\frac{3}{2}$	$\uparrow \uparrow \uparrow$	1232

Contrast this with the hyperfine splittings in atomic spectra. For example the difference between the states of the hydrogen atom where the spins of the proton and electron are either aligned or anti-aligned. The ground state of the hydrogen atom has an energy of about 13.6 eV, but the hyperfine splitting is only 6×10^{-5} eV.

 H_{HO} can be written as

$$H_{HO} = \frac{p_1^2}{2m} + \frac{p_2^2}{2m} + \frac{p_3^2}{2m'} + \frac{1}{2}K|\vec{r_1} - \vec{r_2}|^2 + \frac{1}{2}K|\vec{r_1} - \vec{r_3}|^2 + \frac{1}{2}K|\vec{r_2} - \vec{r_3}|^2.$$

It may be easier to work with different coordinates. So define a coordinate system based on the position of each *i*th quark and the center of mass.

$$\vec{\rho} \equiv \frac{1}{\sqrt{2}}(\vec{r_1} - \vec{r_2})$$
$$\vec{\lambda} \equiv \frac{1}{\sqrt{6}}(\vec{r_1} + \vec{r_2} - 2\vec{r_3})$$
$$R_{c.m.} \equiv \frac{m(\vec{r_1} + \vec{r_2}) + m'\vec{r_3}}{2m + m'}$$

2m+r

Define some mass combinations:

$$M = 2m + m'$$
$$m_{\rho} \equiv m$$
$$m_{\lambda} \equiv \frac{3mm'}{2m + m'}$$

 $m_{1,2}$ is the mass of quarks 1 and 2, and m' is the mass of quark 3. p_i and r_i are the momenta and position of the *i*th quark. We now have new momenta which follow



Figure 1.4: Representation of the angular momenta of this model.

from the previous definitions.

$$p_{c.m.} = M \frac{dR_{c.m.}}{dt}$$
$$p_{\rho} = m_{\rho} \frac{d\rho}{dt}$$
$$p_{\lambda} = m_{\lambda} \frac{d\lambda}{dt}$$

The harmonic oscillator Hamiltonian can now be rewritten as.

$$H_{HO} = \frac{p_{c.m.}^2}{2M} + \frac{p_{\rho}^2}{2m_{\rho}} + \frac{p_{\lambda}^2}{2m_{\lambda}} + \frac{3}{2}K\rho^2 + \frac{3}{2}K\lambda^2$$

This can be interpreted as the motion of the center-of-mass, plus two independent harmonic oscillators. Note that we have assumed the same spring constant. We can also interpret the angular momenta ℓ_{ρ} and ℓ_{λ} as the angular momentum of 2 of the quarks around their center of mass, and the angular momentum of the 3rd quark with respect to the other two as shown in Figure 1.4 [?].

The solutions to the harmonic oscillator part of the Hamiltonian are known and allow us to write the wave function in terms of N, L and M. Where M and m_i are the z-projections of L and L_i , respectively.

$$N = 2(n_{\rho} + n_{\lambda}) + \ell_{\rho} + \ell_{\lambda}$$
$$L = \ell_{\rho} + \ell_{\lambda}$$

$$M = m_{\rho} + m_{\lambda}$$

This now gives us a means to understand the various baryons in terms of spin couplings and H.O. excitations.[?]

- Positive parity ground states with N = 0. This includes the proton and the $\Delta(1232)$.
- Negative parity excited states with N = 1 have either $\ell_{\rho} = 1$ or $\ell_{\lambda} = 1$.
- Positive parity excited states with N = 2. This can mean either radial excitations in n_{ρ} or n_{λ} or orbital excitations with $\ell_{\rho} + \ell_{\lambda} = 2$ and L = 0, 1 or 2

With the harmonic oscillator wave functions as solutions we can classify states in terms of parity and varying degrees of excitation. The model can now be used to make predictions as to the masses and quantum states of the baryons, specifically, the total angular momentum and parity, J^P . If we represent the sum of the intrinsic spin of the particles by \vec{S} , and the relative orbital angular momentum by \vec{L} then $\vec{J} = \vec{S} + \vec{L}$. *P* is the parity of the state. $P = (-1)^{L+1}$

1.2.3 Relativistic Corrections

In the previous section we showed how the Hamiltonian can be written in terms of a harmonic oscillator term and a spin-spin interaction term. Capstick and Isgur [?] have worked on a more complete relativistic model of the interaction. The potential used is motivated by the one gluon exchange mechanism and an adiabatic Y-shaped string potential. But perhaps the best way to examine this potential is to look at it in the non-relativistic limit $p/m \to 0$. [?]

$$V \rightarrow V_{\text{string}} + V_{\text{Coul}} + V_{\text{hyp}} + V_{\text{so(cm)}} + V_{\text{so(Tp)}}$$

• $V_{\rm string}$ - This is an adiabatic, Y-shaped string potential which includes 3-body forces.



Figure 1.5: Representation of the diquark model.

- V_{Coul} A colour-Coulomb potential.
- $V_{\rm hyp}$ A colour-hyperfine (spin-spin) interaction.
- $V_{\rm so(cm)}$ A colour-magnetic spin-orbit interaction.
- $V_{\rm so(Tp)}$ A Thomas-precession spin-orbit interaction.

The non relativistic limit is not applicable to the light quark systems we will be looking at. The actual potentials used in the calculations are momentum dependent. I will not go into the details of this model, but I mention this to show that the model used is non-trivial. I will later show predictions based on this model and so I try to justify the validity of this approach. The solutions to this Hamiltonian are still given in terms of the familiar N, L and M.

Comparisons with experiment show that the spin-orbit terms are small in comparison to other terms.[?] This is not fully understood. It has been suggested that the Thomas-precession term may cancel out the colour-magnetic term.

1.2.4 Diquark Model

However, there are a group of resonances predicted by the quark model which are *not* observed in experiment. In 1969, Lichtenberg proposed a solution. He claimed that two of the quarks bind so tightly as to act as one and thus remove one of the degrees of freedom of the model as represented in Fig. 1.5.

This model explains these missing states quite well. But there are reasons to doubt the validity of this reasoning as explained in the next section.
1.3 Quantum Chromodynamics (QCD)

Many of the relativized corrections to the CQM have their motivations in quantum chromodynamics (QCD). It is therefore useful to spend some time discussing the salient points of this theory.

In the CQM, quarks are the only players on the stage. It is their interactions and characteristics which give rise to hadronic features. QCD is a gauge theory mediated by massless, colour-carrying bosons called *gluons*. I will not go into detail here, but the prescription to derive QCD has its roots in the same gauge invariance arguments that led to the marriage of electrodynamics and the weak interaction.

QCD, has its coupling constant α_s . Like the more well-known α of electromagnetic interactions, α_s is a "running" coupling constant. Its strength changes as the distance between interacting particles changes. Unlike electromagnetism, α_s decreases with decreasing distance. At high enough energies, α_s becomes small enough that you can treat calculations perturbatively.

At the energies we are looking at, perturbation theory cannot be used. Higher order terms play a significant role in the calculation of matrix elements. Some problems can be treated numerically, and this is the branch of study known as lattice-QCD.

The quarks of QCD have a much smaller mass than the quarks of the CQM. Sometimes the QCD quarks are referred to as *bare* quarks. The higher order terms can be thought of as contributing to the *dressed* quarks of the CQM.

QCD is believed to be an accurate theory of hadronic interaction. Recent lattice-QCD calculations have shown marked improvement in predicting baryon masses [?]. We learn something else from QCD: there is nothing in QCD which would support the diquark model. QCD places all three constituent quarks on equal footing with no preference for any coupling between two of them.

1.4 Predictions of Constituent Quark Model

In 1980 Koniuk and Isgur[?] used the QCD-inspired quark model discussed earlier to predict decay properties of the baryon spectrum. It was found that the missing baryons couple very weakly to $N\pi$ final states. These resonances fall in the positive parity N = 2 states. Since most partial wave analysis have focused on elastic $N\pi \to N\pi$, it is not surprising that these resonances have not been observed. The model suggests that these states may preferentially decay to other final states. If they have a sufficient branching ratio to $N\pi\pi$, then they could be seen in photoproduction experiments.

Photon-nucleon scattering is sometimes described in terms of vector-meson dominance (VMD). The idea is that the photon manifests itself as a quark-antiquark pair at the point of interaction. This pair then interacts strongly with the nucleon. The quantum numbers (J^{PC}) for the photon are 1⁻⁻. The quark-anti-quark pair can be thought of as a meson with these quantum numbers. A meson with J = 1 is a vector meson. The most common vector mesons are the $\rho(770), \omega(782)$ and $\phi(1020)$. Photoproduction of these mesons has been very successfully described using the VMD model [?]. The ρ primarily decays to $\pi\pi$ so a resonance with a strong photocoupling should be seen in $N\pi\pi$ final states.

Some results of Isgur, Karl, Capstick and Roberts are shown in Table 1.4. For the N = 2, positive-parity, missing baryons I show the predicted spins, mass and width along with branching fractions to $N\pi\pi, \Delta\pi$ and $N\rho$. These latter two will show up in $N\pi\pi$ analysis given the high branching fractions for $\Delta \to N\pi$ and $\rho \to \pi\pi$. I also plot the theoretical calculations for the photoproduction amplitudes. There are two amplitudes that can be calculated. The photon has spin 1 and has only two z-projections, -1 and +1. The proton is a spin- $\frac{1}{2}$ particle and has two projections as well, $+\frac{1}{2}$ and $-\frac{1}{2}$. From the rules for adding orbital angular momenta, we see that our resonance can have $M = +\frac{3}{2}, +\frac{1}{2}, -\frac{1}{2}, -\frac{3}{2}$. If we apply parity, the amplitude to produce our intermediate resonance in an $M = +\frac{1}{2}$ state is the same as to produce it in an $M = -\frac{1}{2}$ state, so we label the appropriate amplitudes by



Figure 1.6: Representations of spin-aligned and spin-anti-aligned scattering and the amplitudes associated with these processes.

 $A_{1/2}$ and $A_{3/2}$.

As a comparison we can look at the same quantities for known states. We list these properties in Table 1.4 and Table 1.4. For N^* 's we list the $p\gamma$ decay amplitudes which are proportional to the photoproduction amplitudes. For the Δ 's we list the $N\gamma$ amplitudes which still gives us a handle on the $p\gamma$ coupling.

1.5 Summary

Even if we ignore the missing baryons problem, we see that there are still known resonances which have significant photoproduction amplitudes and $N\pi\pi$ couplings. These states should provide "anchors" for our analysis. That is, in order to trust any identification of previously unidentified states, we should be able to pick out some of the better known states.

			Branching fractions			γp amplitudes	
J^P	mass	width	$N\pi$	$\Delta \pi$	$N\rho$	$A_{1/2}$	$A_{3/2}$
$\frac{1}{2}^+$	1880	150	0.05	0.49	0.03	0	
$\frac{1}{2}^{+}$	1975	50	0.08	0.47	0.14	-12	
$\frac{3}{2}^{+}$	1870	190	0.20	0.12	0.02	-2	-15
$\frac{3}{2}^{+}$	1910	390	0.00	0.75	0.03	-21	-27
$\frac{3}{2}^{+}$	1950	140	0.12	0.43	0.11	-5	2
$\frac{3}{2}^{+}$	2030	90	0.04	0.57	0.15	-9	15
$\frac{5}{2}^{+}$	1980	270	0.01	0.89	0.02	-11	-6
$\frac{5}{2}^{+}$	1995	190	0.00	0.51	0.33	-18	1
$\frac{7}{2}^{+}$	2000	50	0.13	0.53	0.03		
Δ 's							
State	mass	width	$N\pi$	$\Delta \pi$	$N\rho$	$A_{1/2}$	$A_{3/2}$
$\frac{1}{2}^+$	1835	310	0.05	0.63	0.20	-31	
$\frac{3}{2}^{+}$	1985	220	0.05	0.44	0.25	6	3
$\frac{5}{2}^{+}$	1990	350	0.05	0.56	0.10	-10	-20

 N^* 's

Table 1.5: Properties of missing baryons from the N = 2 band.[?, ?, ?] Masses, widths and branching fractions are calculated from the relativized CQM. The same model is used to predict the γp photoproduction amplitudes for spin-aligned ($A_{3/2}$) or spinanti-aligned ($A_{1/2}$). Masses and widths are in MeV/c². $p\gamma$ amplitudes are in 10^{-3} GeV^{$\frac{1}{2}$}.

				Branching fractions			$p\gamma$ decay	amplitudes
J^P		mass	width	$N\pi$	$\Delta \pi$	N ho	$A_{1/2}$	$A_{3/2}$
$\frac{1}{2}^{-}$	S_{11}	1535	150	0.35-0.55	< 0.01	< 0.04	$90{\pm}30$	
$\frac{1}{2}^{-}$	S_{11}	1650	150	0.55-0.90	0.01 - 0.07	0.04-0.12	53 ± 16	
$\frac{1}{2}^{+}$	P_{11}	1440	350	0.60-0.70	0.20-0.30	< 0.08	-65 ± 4	
$\frac{1}{2}^+$	P_{11}	1710	100	0.10-0.20	0.15-0.40	0.05-0.25	-9 ± 22	
$\frac{3}{2}^{-}$	D_{13}	1520	120	0.50-0.60	0.15 - 0.25	0.15-0.25	-24 ± 9	166 ± 5
$\frac{3}{2}^{-}$	D_{13}	1700	100	0.05-0.15	0.50-0.60	< 0.35	-18±13	-2 ± 24
$\frac{3}{2}^{+}$	P_{13}	1720	150	0.10-0.20	< 0.15	0.70-0.85	18 ± 30	-19 ± 20
$\frac{5}{2}^{-}$	D_{15}	1675	150	0.40-0.50	0.50-0.60	0.01-0.03	19 ± 8	15 ± 9
$\frac{5}{2}^{+}$	F_{15}	1680	120	0.60-0.70	0.05-0.15	0.03-0.15	15 ± 6	133 ± 12
$\frac{7}{2}^{-}$	G_{17}	2190	450	???	???	???	\approx -42±12	$\approx -130 \pm 50$
$\frac{9}{2}^+$	H_{19}	2220	400	0.10-0.20	???	???	???	???
$\frac{9}{2}^{-}$	G_{19}	2250	400	???	???	???	???	???

 $N^*{\rm 's}$

Table 1.6: Properties of known three and four star N^* 's. Values are taken from the 2002 PDG.[?] Masses and widths are in MeV/c². $p\gamma$ decay amplitudes are in $10^{-3} \text{ GeV}^{\frac{1}{2}}$.

Δ 's

				Branching fractions		$N\gamma$ decay amps		
J^P		mass	width	$N\pi$	$\Delta \pi$	$N\rho$	$A_{1/2}$	$A_{3/2}$
$\frac{1}{2}^{-}$	S_{31}	1620	150	0.20-0.30	0.30-0.60	0.07-0.25	27±11	
$\frac{1}{2}^+$	P_{31}	1910	250	???	???	???	3 ± 14	
$\frac{3}{2}^{-}$	D_{33}	1700	300	0.10-0.20	0.30-0.60	0.30-0.55	104 ± 15	85 ± 22
$\frac{3}{2}^{+}$	P_{33}	1232	120	>0.99			-135 ± 6	-255 ± 8
$\frac{3}{2}^{+}$	P_{33}	1920	200	0.05-0.20	???	???	$\approx 40 \pm 14$	$\approx 23 \pm 17$
$\frac{3}{2}^{+}$	P_{33}	1600	350	0.10-0.25	0.40-0.70	< 0.25	-23 ± 20	-9 ± 21
$\frac{5}{2}^{-}$	D_{35}	1930	350	0.10-0.20	???	???	$\approx 40 \pm 14$	$\approx 23 \pm 17$
$\frac{5}{2}^{+}$	F_{35}	1905	280	0.05-0.15	< 0.25	>0.60	$26{\pm}11$	45 ± 20
$\frac{7}{2}^{+}$	F_{37}	1950	300	0.35-0.40	0.20-0.30	< 0.10	76 ± 12	97 ± 10
$\frac{11}{2}^{+}$	H _{3,11}	2420	400	0.05-0.15	???	???	???	???

Table 1.7: Properties of known three and four star Δ 's. Values are taken from the 2002 PDG.[?] Masses and widths are in MeV/c². $p\gamma$ amplitudes are in $10^{-3} \text{ GeV}^{\frac{1}{2}}$.

CHAPTER 2 The Experiment

2.1 Thomas Jefferson National Accelerator Facility

The experiment was conducted at Thomas Jefferson National Accelerator Facility (JLab) in Newport News, VA. Construction began in 1987, and the first experiments were performed in 1994. JLab was built with the express purpose of probing the nucleus, although much of the physics performed is outside the realm of traditional nuclear physics. The lab provides a continuous, high intensity, low emittance electron beam with an energy of up to 6 GeV. The beam can be split into 3 different halls ("A", "B", "C") which house different detectors. All three halls can receive beam at the same time. Hall B houses the CLAS detector where this experiment was performed.



Figure 2.1: The Continuous Electron Beam Accelerator Facility at JLab in Newport News, VA.[?]



Figure 2.2: Side view of Hall B, the tagger, the CLAS and other components of the beam line. [?]

2.2 Accelerator

Jefferson Lab operates a continuous electron beam. The accelerator consists of superconducting accelerator cavities and bending magnets. The path that the electrons follow looks like a race track. The long straightaways are filled with superconducting cryomodule cavities. Electrons are accelerated through these cavities by means of an RF modulated current. The electron's energy increase over the straight portions of the accelerator. Steering magnets bend them around the curved section. After each loop, the beam has a higher energy, and so must go through a different set of steering magnets. The available energy is therefore limited by the number of steering magnet tracks the lab has built. The current (2003) conditions allow for 5 complete loops. The electrons can be directed to any hall after any number of loops so each hall can have a different beam energy.

The electron beam is continuously supplied in 2.004 ns pulses. For this experiment the electron beam current was 10 nA with an energy of 2.445 GeV.

The electrons are accelerated by means of superconducting radio frequency (RF) cavities. A current flows through the surfaces of the cavities and is modulated by means of a radio frequency. The frequency can be tuned so that a beam bunch entering a cavity always sees a accelerating electric field.

2.3 Tagger

[?] For this experiment we use a photon beam. As the electron beam enters Hall B, it is directed onto a thin-foil radiator. As some of the electrons pass through the radiator, they electromagnetically scatter off the nuclei of the radiator and emit a photon. This process is called bremsstrahlung. The angle with respect to the original beam that the photon and electron come out at is given by the following.

$$\theta_{\gamma} = \frac{mc^2}{E_0}$$
$$\theta_e = \theta_{\gamma} \frac{E_{\gamma}}{E_e}$$

m is the electron mass and E_0 is the energy of the incident electron beam. We are

able to "tag" photons up to 95% of the electron beam energy. If I use the beam energy of 2.445 GeV we get

$$\theta_{\gamma} = 0.00021$$
$$\theta_{e} = 0.0040$$

So to first order, neglecting multiple scattering, both the recoil electron and the photon come out in the same direction as the incident electron beam.

The photons continue on toward the target. To determine the energy of the photon, we must measure the energy of the recoil electron. Energy conservation will then allow us to determine the photon energy.

$$E_{\gamma} = E_0 - E_e$$

As the both scattered and non-interacting electrons leave the radiator, they encounter the tagger proper, shown in Fig. 2.3. Upon entering the tagger they encounter a magnetic field produced by a uniform-field dipole. The electrons are bend downward and the photons pas through unaffected. For a constant magnetic field, the radius of curvature for the electrons is proportional to their momentum. The electrons are bent onto a series of scintillators. There are two sets of scintillators. Each electron should hit at least one paddle in each set. While the momentum will determine which paddle is hit for either set, one set is primarily designed to give momentum/energy information, and the other set is designed to give timing information.

2.3.1 E-counters

The first set of scintillators are designed to give the most accurate momentum measurement. These are referred to as E-counters. To get the most precise measurement of momentum, the position should be measured as accurately as possible. There are 384 overlapping E-counters. Each one consists of a plastic scintillator attached to a light guide and a photomultiplier tube (PMT).

The plastic scintillators are 20 cm long, 4 mm thick and range from 6 to 8 mm wide. They are overlapped so that coincidence between adjacent scintillators can be used to even more accurately measure position. This effectively gives us 767 photon energy bins giving an energy resolution of 0.001 E_0 . The E-counters are set up to effectively tag photons from 20% to 95% of the incident electron beam energy.

As these are used for spatial measurements, the only information needed from the PMT's is whether or not a charged particle passed through the scintillator. No pulse information is required beyond this. The signal from the PMT's are fed to an amplifier, discriminator and then to a FASTBUS TDC. This TDC runs in a common-stop mode with a resolution of 500 ps/channel. The stop signal comes from an event in the the CLAS detector.

2.3.2 T-counters

The other set of scintillators is used for timing information. These are the T-counters. There are 61 T-counters arranged in the same overlapping fashion as the E-counters. This gives us 121 timing bins.

The design of the T-counters is different from the E-counters. They are much thicker at 2 cm. Any electrons passing through this will produce more electrons than a thinner piece of plastic and so we get much more robust light pulse to analyze. The width varies over the range to maintain a relatively constant counting rate from counter to counter. The bremsstrahlung spectrum goes as $1/E_{\gamma}$ and so the counters that count higher energy photons (lower energy electrons) would have a lower counting rate than the counters measuring low energy photons (high energy electrons) if they were all the same size.

The scintillators have a PMT at each end. The output of each PMT is fed to a fast discriminator and from there to a FASTBUS TDC. The TDC has a resolution of 50 ps. This is the best timing information for a physics event in the CLAS. The TDC's are operated in a common stop mode. That is, when a signal comes in from the CLAS detector that a possible physics event has been detected, all the TDC's will start recording. The stop for each TDC comes from the discriminator. For each physics event a T-counter will only record the first photon that strikes it. The



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Figure 2.3: Schematic of the electron tagger located in Hall B. [?]

flux during the g1c run was such that there were usually 2-3 photons in the tagger per event. The difference in arrival time between these photons easily allowed us to distinguish which photon caused the event.

A separate signal is run from each PMT on a T-counter to a fast AND logic unit. A left-right coincidence is used to determine that a photon has been tagged in that energy range. All 61 AND gates are connected to a master OR. This OR provides a signal that a photon has been tagged somewhere in the tagger and this is used in the trigger as will be discussed later.

2.4 Target

For the g1c running period a target of liquid H_2 was used. The target was kept at a temperature of 20 K and a pressure of 1095 mbar. The target is a cylinder 18 cm long with a 2 cm radius. The material used is Kapton. The thickness of the target walls and inlet and exit windows is 127 microns.

2.5 CEBAF Large Acceptance Spectrometer (CLAS)

Hall B houses the CLAS, the CEBAF Large Acceptance Spectrometer. It was designed to cover as much of 4π solid angle as possible. It consists of a variety of detectors.

The CLAS is divided into 6 sectors by a superconducting, toroidal magnet. That is, if you define the beam direction to be the z-axis, the field lines point in ϕ . For this experiment the field lines are such that positively charged particles are bent out away from the beam and negatively charged particles are bent in toward the beam. In this region are the drift chambers which tell us the path the particle takes. From this we can get curvature and then momentum information.

Immediately surrounding the target is the start counter, a set of scintillators divided into 3 regions. Their purpose is to provide a time for the start of the reaction. This time can then be matched to a tagged photon.

After the drift chambers is another set of scintillators arranged. These are the time-of-flight walls. The timing information they provide for charged tracks is used in conjunction with the start counter to measure velocities and energies.

There are calorimeters also located in the forward region. They are primarily used to detect neutrons and other neutral particles.

Cerenkov counters are used to trigger on electrons for electron beam experiments, as well as distinguish high momenta electrons from pions.

For this experiment, we made use of the drift chambers, start counter and TOF wall.

2.5.1 Torus magnet

A magnetic field is generated by a superconducting toroidal magnet shown in Fig. 2.4 and consisting of six separate coils. Each coil is made of aluminum-stabilized NbTi/Cu conductor. Cooling tubes at the edges of the windings allow super-critical helium to be cool the torus down to 4.5 K. The maximum current is 3860 A, though for this experiment only half the max current was run. Negatively charged tracks bend inward and if they bend too much, they will exit the detector through the same hole that the beam leaves and will not be detected. By decreasing the magnetic



Figure 2.4: The CEBAF Large Acceptance Spectrometer



Figure 2.5: Cut away side view of the CLAS.



Figure 2.6: Cut away forward view of the CLAS.

field we do not bend the negative tracks as much and increase acceptance for these particles.

The field varies over θ . Contour maps of the field are shown in Fig. 2.7 and Fig. 2.8. To give an idea of the strength of the magnets, we can look at how much field different paths map out: $\int B \times dl$. For a max field, this varies from 2.5 T·m for forward angle, high momentum tracks, to about .6 T·m for tracks greater than 90°.

Part of the support structure for the magnet consists of five carbon-fiber rods per sector. These are arrayed toward the edges of the coils and maintain the distance between them. The forward most rod will absorb or scatter forward going protons. This is modeled in the Monte Carlo as will be shown in a later chapter.

2.5.2 Drift Chambers

The drift chambers were designed to measure particles with momenta of greater than 200 MeV/c and cover a a polar angle between 8° and 142°. Each of the 6 sectors houses an identical set of drift chambers. There are 3 Regions, with R1 being the Region closest to the target. These can be seen in Figs. 2.5, 2.6 and 2.9. Each



Figure 2.7: Contours of magnetic field, equidistant between two coils. The projection of the coils is shown. [?]

Region contains 2 sets of drift wires referred to as em superlayers.

The wires in the drift chamber are stretched between two end plates parallel to the magnet coils. A superlayer is composed of quasi-hexagonal cells of wires as can be seen in Fig. 2.10. Each cell has a sense wire at the center maintained at a positive potential, and the vertices of the cell are field wires which are maintained at a negative potential, half that of the sense wires. The average distance between the field and sense wires is 0.7 cm in R1, 1.5 cm in R2 and 2.0 cm in R3. There are 35,148 sense wires in the drift chamber system. The drift chambers are filled with an 88-12% mixture of argon and CO₂.

When a charged particle passes through the drift chamber it ionizes the argon. The ions will move in the field created by the sense and field wires. They will drift to the the sense wires where they will create a current which is fed out to an amplifier, discriminator and then to a common-stop TDC. Timing information from the TDC's can be used to give a more accurate positional measurement by making use of the



Figure 2.8: Magnetic field vectors transverse to the beam. The length of each line is proportional to the field strength. Note the cross-sections of the six coils. [?]

drift velocity of the ions. On average, the spatial resolution for a cell is 310, 315 and 380 μ m for R1, R2 and R3 respectively. For particles with a momentum of 1 GeV/c, the design should give a momentum resolution of $\delta p/p \leq 0.5\%$ and $\delta \theta, \delta \phi \leq 2$ mrad.

2.5.2.1 Tracking

To identify the track of a charged particle in the drift chambers, a 4-step procedure is outlined here.[?]

- Cluster finding. A contiguous group of hits in each superlayer is identified. An example of this can be seen in Fig. 2.10.
- A lookup table is used to identify clusters or portions of cluster which are consistent with a track passing through that superlayer.



Figure 2.9: Schematic of an R2 and R3 drift chamber attached to the torus cryostat. [?]

- Another lookup table is used to identify segments across superlayers which are consistent with tracks passing through the drift chamber region.
- Using these segments a track is fitted to these hits. Two different values are used for the fitting.
 - Hit-based-tracking (HBT). The position used in the fitting is the location of the wire.
 - Time-based-tracking (TBT). A position is derived using the drift times for the ions to the wire. This improves the resolution of the tracking procedure. For this analysis, TBT was used for the particle identification

and momentum information.

2.5.3 Start counter

The start counter is a set of scintillators that sit immediately outside the carbon fiber beam pipe surrounding the target. The purpose of this instrumentation is to register the start of an event in the CLAS.

The scintillators are 3 mm thick. The other dimensions are shown in Fig. 2.11. Note that the design of the counters necessitates joining two of the strips at one end. This configuration will cover two sectors. PMT's are placed upstream at the thicker ends of the paddles. This reduces unnecessary mass in the detector region. The start counter in its position around the target is shown in Fig. 2.13.

The start counter is used in the event trigger. To get a robust signal the analog signal from both PMT's is combined before being fed into a discriminator. Each PMT has an opportunity to register a hit, regardless of which leg the charged particle passes through. However one of them will register a smaller light pulse due to the attenuation from the longer path length.

A second signal is read out from each PMT and sent to individual discriminators and TDC's. With this timing information from two TDC's the location of the charged track can be better identified and the timing resolution improved. The resolution varies from 255 ps to 271 ps, depending upon where in the scintillator the particle passes.

2.5.4 Time of Flight (TOF) systems

Outside the Region 3 drift chambers are sets of scintillators which extend across the sector azimuthally. These are designed to provide time-of-flight (TOF) information about the particles. They extend over the polar angle form 8° to 142°. The TOF counters are after the drift chamber and Cerenkov counters, but in front of the calorimeters.

The design is similar to the T-counters in the tagger. A plastic (Bicron BC-408) scintillator with a thickness of 5.08 cm has a PMT attached to each end. The counters covers about 1.5° of scattering angle so the forward counters are 15 cm

	•	
Resolution	Momentum ($\theta \leq 30^{\circ}$)	$\sigma_p/p pprox 0.5\%$
	Momentum ($\theta \geq 30^{\circ}$)	$\sigma_p/p \approx (1-2)\%$
	Polar angle	$\sigma_{\theta} \approx 1 \mathrm{mrad}$
	Azimuthal angle	$\sigma_{\phi} \approx 4 \mathrm{mrad}$
	Time (charged particles)	$\sigma_t \approx (100\text{-}250)~\mathrm{ps}$
	Photon energy	$\sigma_E/E \approx 10\%/\sqrt{E}$
Data acquisition	Event rate	4 kHz
	Data rate	25 MB/s

Summary of the CLAS detector

Table 2.1: Summary of characteristics of the CLAS detector.

Details for glc running period - Oct. 2 - Nov 20, 1999					
Target	Liquid hydrogen				
Electron Beam Energy	2.445 GeV				
Photon Beam Range	0.5 - 2.325 GeV				
Magnetic Field	Half of max field				
Current	10 nA				

Table 2.2: Summary of g1c running period.

wide and the large angle counters are 22 cm wide. The lengths vary from counter to counter to provide full azimuthal coverage for each of the six sectors.

Signals from the PMT's are split. One signal is sent to the trigger. The other signal is sent to both a TDC and an ADC (analog-to-digital converter). The ADC measures pulse height and with the timing information can give information about the position on the scintillator where the particle went.

2.5.5 CLAS performance summary

2.6 Calibration

The off-line calibration for the g1c data set was performed by Luminita Todor. Her work is detailed in an internal CLAS note. [?]

2.7 Trigger

The hardware trigger to measure an event for this experiment was the Level 1 trigger. This required a photon to be tagged and at least one charged track in the

CLAS. The trigger was a coincidence between a start counter signal, a TOF signal and a T-counter signal from the tagger.

2.8 Particle Identification (PID)

The charged particle leaves the target. The first piece of instrumentation it hits is the start counter, located about 1 foot from the target. Call this time, t_{sc} .

The particle now travels through the drift chambers and hits the TOF wall outside of Region 3. Call this time, t_{tof} The information from the drift chambers and TOF paddle gives us a path length for this particle while traveling from the start counter to the TOF wall. Call this distance d.

Using the d, t_{sc} and t_{tof} a preliminary velocity can be calculated. This velocity is used to extrapolate the path from the start counter to the target. Using the timing information from the tagger, we can match up the event with the photon that caused it. We can now use the timing information from the T-counter for the start of the event. Call this time, t_{tag} . The resolution for the T-counter is 50 ps. This is the best timing information we have in the CLAS and allows us to calculate a new an more accurate velocity. It is this velocity that is used for the rest of the PID.

Particle identification in the CLAS for charged particles relies on two independent measurements: the momentum and the velocity.

$$p = m v \gamma$$

where $\gamma = 1/\sqrt{1-\beta^2}$ and $\beta = v/c$. This gives us

$$m = \frac{p}{c\beta}\sqrt{1-\beta^2}$$

Momentum information is provided by the drift chamber measurements. Note that p and β are determined independently. The different particles can be seen quite clearly if you plot β vs p for all charged tracks.

In Fig. 2.14 you can easily see the distinct bands of different particles. Particle

identification is made simply by placing cuts around these bands. Of the two heaviest bands, the uppermost is the pions. The misidentified electrons are seen as a line at $\beta = 1$. The lower of the heavy bands are protons. Between the two, the kaons are slightly visible. And some deuterons below the protons. There are also some structures which look like palm fronds peeling away from the main bands. These are due to mistakes in timing coincidences with the discret 2 ns photon bunches.

2.9 Normalization

For this analysis, the photon flux was calculated using the gflux method developed by Eugene Pasyuk [?]. When a electron is first registered in the tagger in a T-counter, a 200 ns window is opened to record data from all the other T-counters. The correct photon can be picked out from timing information from the start counter, but these "out-of-time" photons give us a measure of the photon flux.



Figure 2.10: Two superlayers in the R3 drift chamber are shown. The Cerenkov detector is seen in the upper right. The hexagonal arrangement indicates the cells formed by the wires. Note that the borders shown here do not actually exist. The vertices represent the field wires and the sense wire would be at the center of the cell. The highlighted cells are indicative of what would happen in a charged particle passed through this region. [?]



Figure 2.11: Schematic of the start counter scintillators. Dimensions are in millimeters unless otherwise specified. [?]



START COUNTER COUPLED-PADDLE

Figure 2.12: Schematic of the electron tagger located in Hall B. [?]



Figure 2.13: Layout of TOF counters in one sector. [?]



Figure 2.14: β vs. p for a sample of charged particles from the g1c run.

CHAPTER 3 Monte Carlo simulation

We used flat three-body phase space for our Monte Carlo data. The data was generated using ROOT's TGenPhaseSpace class. Fig. 3.1 is a Dalitz plot of the raw events around 1.7 GeV/c². showing the uniformity of the events except for some minor depletion at the edges. This is due to the overlapping W bins, which increases the number events in the overlap. The dotted lines around the data are the kinematic boundaries for the low and high edges of the W bin.



Figure 3.1: Dalitz plots for raw Monte Carlo data for $1.675 \leq W < 1.700 GeV/c^2$. The dotted lines outline the kinematic limits based on the low and high edges of the W bin.

We use a simulation of the detector called gsim. We wanted to be confident that it reproduced the CLAS response so we conducted a thorough study discussed in the following section. Details of the study can be found in the appropriate CLAS note [?].

3.1 Empirical acceptance method

The analysis involves three final state particles. If I detect any two, the third can always be reconstructed from missing momentum, whether or not it has been detected in CLAS. This principle can be used to map out an acceptance for CLAS using either real world or simulated data. We can then compare these two acceptance functions.

For example, suppose I detect the π^+ and π^- in CLAS. I calculate the missing mass off the two pions and see if this corresponds to the mass of a proton. I then loop over the particles detected in the CLAS to see if this proton was detected.



Figure 3.2: Representation of looking for a missing particle where I expect it to be based on my momentum reconstruction.

Using this information, I can now calculate some efficiency for detecting a particle based on it's momentum and angle.

$$\epsilon(\rho, \theta, \phi) = \frac{\text{\# of times CLAS found the particle}}{\text{\# of identified missing particles}}$$

where ρ, θ and ϕ are the spherical components of the momentum. Note that this efficiency combines both acceptance and detector response and *can be determined* separately for either real or simulated data..

I determine the efficiency as a function of some kinematic variable. I choose some narrow momentum bin, and I integrate over all ϕ . I then plot the efficiency over $\cos(\theta)$ as seen in Fig. 3.3. We found that our comparisons are sensitive to the differences in the real world momentum distributions and the phase space which we run through the simulation. For this reason it is important that we bin finely in both momentum and $\cos(\theta)$.



Figure 3.3: The first plot shows the distribution of π^+ lab angle $\cos(\theta)$ as reconstructed from missing mass for the momentum bin 0.24 GeV/c. The fine binning is important for obtaining a valid result. The second plot shows the distribution $of <math>\pi^+$ where they are detected in CLAS. The third plot is the second divided by the first plot. This gives us our efficiency. This particular example uses real world data.

3.1.1 Characteristics of the data and tuning gsim.

The Monte Carlo data was processed as follows. Three-body phase space events were generated using the TGenPhaseSpace class found in the ROOT package. They were generated roughly uniformly over the beam energy range $0.5-2.5 \text{ GeV/c}^2$. Following that, the gsim detector simulation package, gpp, a program designed to simulate inefficient TOF paddles and drift chamber wires, and alc, the cooking routine. Details of these programs and the flags used can be found in the CLAS note [?].

After this there were still some obvious discrepancies between the data and the Monte Carlo. When the Cerenkov detectors were installed, there was a support bar used that was not put into gsim at the time of this document. Protons that strike this bar are sometimes scattered or absorbed and are not detected. By plotting momentum vs. $\cos(\theta)$ for the protons I see a depletion region which is not visible in the Monte Carlo. This is shown in Fig. 3.4.

The other depletion structures are due to the carbon rods which support the main torus, or junctures between two time-of-flight walls and seem to be modeled properly in gsim. I impose a momentum cut to remove the Cerenkov support bar depletion region in both my data and Monte Carlo.



Figure 3.4: Plots showing the effect of the support bar in the Cerenkov detector on the distribution of protons. The dashed line in the data plot outlines the region that will be cut out of both real world and simulated data.



Figure 3.5: These plots show the mass of π^+ 's as calculated from timeof-flight measurements. The first plot is real data and the second plot is Monte Carlo. The sharp cutoff at 0.3 GeV is imposed by the CLAS software's particle identification routine.

If I look at the time-of-flight mass of the pions in Fig. 3.5, I see what looks to be misidentified electrons. I cut them out of both MC and data.

I also want to make sure that I have knocked out dead paddles in the Monte Carlo. After I have run it through gpp I can compare the distribution of hits for a particle in a particular sector over all the TOF paddles. If I look at Fig.3.6 I can see that paddle #44 is dead in the data, but has not been properly accounted for in the Monte Carlo.

Fig.3.7 shows this distribution after I have cut out what I refer to as misidentified electrons. Paddle #44 shows no reconstructed pions now in the Monte Carlo,



Figure 3.6: Hits in the time-of-flight paddles for π^+ 's in sector 1. The first plot is real data and the second plot is Monte Carlo.



Figure 3.7: Hits in the time-of-flight paddles for π^+ 's in sector 1 after I have cut out misidentified electrons. The first plot is real data and the second plot is Monte Carlo.

but paddle #29 has been depleted. The Monte Carlo data has been filled with a status for each TOF paddle. But it does not always act the same as data when it comes to reconstruction the time and so I get more misidentified electrons in the Monte Carlo distribution. My solution was to impose a strict requirement on the health of the TOF paddles. First I only take hits that have a completely healthy status. I also go through and identify what I will call bad paddles in the data and knock them out in my analysis code in both data and Monte Carlo.

There may also be slight discrepancies in the drift chamber positions between the real CLAS and in gsim. To deal with this I cut in very slightly in the forward region and in phi. I do not cut on the momentum of the track, but where in the detector the particle went. I chose one plane in each of the three Regions. Tracks that passed within 5, 20, or 22 cm of the edge of a sector for R1, R2 or R3 respectively were cut out. The minimum foward angle for tracks was effectively increased from 8° to about $14^\circ.$

3.1.2 Comparison

The data was binned in 20 MeV/c momentum bins and 0.02 bins in $\cos(\theta)$. I integrated over all ϕ . When I'm able to reconstruct a particle from missing mass, I'll refer to it as a *reconstructed* particle. When CLAS detects the particle, I'll refer to it as a *found* particle.

To calculate the error on the efficiency, I assumed a binomial distribution for the found particles and used that to give me the error for them.

$$\sigma_{\text{found}}^2 = N_{\text{found}} \times p_{\text{found}} \times p_{\text{missed}} \tag{3.1}$$

$$= N_{\text{reconstructed}} \times \frac{N_{\text{found}}}{N_{\text{reconstructed}}} \times \left(1 - \frac{N_{\text{missed}}}{N_{\text{reconstructed}}}\right)$$
(3.2)

$$= N_{\text{found}} \times \left(1 - \frac{N_{\text{missed}}}{N_{\text{reconstructed}}}\right)$$
(3.3)

I also assume zero error on the reconstructed particles. The error on the efficiency then just comes out to be

$$\sigma_{\rm efficiency} = \frac{\sigma_{\rm missed}}{N_{\rm reconstructed}}$$

I require that there be at least 30 reconstructed particles in a bin to calculate an efficiency. I can now look at the data (black) and Monte Carlo (red) overlaid on the same plot, or as a ratio. (See Fig.3.8.)

There is a huge amount of data, so I needed some way to determine the validity of my cuts. I used a χ^2 per degree of freedom to see how much my ratio deviates from 1. I calculate this for each momentum bin where the degrees of freedom are the number of $\cos(\theta)$ bins that have values for both data and Monte Carlo.

$$\chi^2 = \frac{(1-R)^2}{\sigma_R^2}$$
$$R = \frac{\epsilon_{\text{data}}}{\epsilon_{\text{MonteCarlo}}}$$



Figure 3.8: The first plot shows the comparison in efficiency for realworld data (black) and Monte Carlo data (red). The second plot shows the ratio of the two data sets.

I can look at this χ^2 for each momentum bin and see how the cuts improve the agreement, as seen in Figs. 3.9-3.11.



Figure 3.9: χ^2 per degree of freedom for the proton and pions before and after the cuts.

There is a separate document which contains plots of all the momentum bins generated for comparison in this analysis [?] This document also details of all the the cuts.



Figure 3.10: χ^2 per degree of freedom for the proton and pions before and after the cuts.



Figure 3.11: χ^2 per degree of freedom for the proton and pions before and after the cuts.

3.1.3 Conclusions

To summarize, here is the procedure we used to process and compare the data.

- Monte Carlo
 - gsim
 - gpp
 - a1c
- CLAS data and Monte Carlo

- Energy loss and momentum corrections.
- Vertex cuts.
- Misidentified electron cuts.
- Bad TOF paddles knocked out.
- Region in proton momentum distribution affected by the Cerenkov support bar is knocked out.
- Fiducial cuts applied to track position in the drift chambers.

The combination of all the cuts seem to improve the agreement in the empirical efficiency calculation. In the course of the study it was noted that the bulk of the improvement in the middle $\cos(\theta)$ regions comes from the TOF paddle knock outs. The fiducial cuts seem to primarily improve the agreement for forward going particles.

The biggest disagreement in this study was for lower energy protons. It may be possible that limitations in binning and the difference in flat phase space and real world data is the reason for this. In the end we decided to analyze particles where all three tracks were detected in the CLAS. The hope is that the extra constraints on the missing mass will give us the cleanest data sample.

3.2 Monte Carlo events

I generated 60 million events and ran them through the simulation with about 5-9% acceptance over the energy range after all cuts. Fig. 3.12 shows the distribution of the generated events, the accepted events and the integrated acceptance. Note that the raw and accepted MC seem discontinuous. When the flux was calculated by Eugene Pasyuk [?], some T-counters and E-counters in the tagger were found to not give consistent readings, and were cut out of the data. To maintain consistency, I cut out Monte Carlo events associated with these counters as well. I generated the data in two different periods and generated slightly fewer events above W = 1.8 GeV/c². There is no other significance to this.



Figure 3.12: The distribution of generated Monte Carlo events, accepted Monte Carlo Events and integrated acceptance.
CHAPTER 4 Characteristics of the data

4.1 Data Selection

At this point we have discussed a number of cuts. These cuts serve the following purposes.

- Cleanly identifying the photon which caused the reaction.
- Improving the particle identification.
- Improving the agreement between gsim and the CLAS detector.

These cuts have been described in greater detail in the previous chapter.

At this point we can select the data upon which we will perform our analysis. I will have three particles in the final state. I can select these particles in four different ways. I can detect all three in CLAS, or I can detect any two and reconstruct the third from missing momentum.

In the first case, I want to make sure I only choose events with a proton, π^+ and π^- . Fig. 4.1 shows the missing mass squared off of the three particles for different beam energies. The largest peak is centered on 0, and these are the events we want to choose. For higher energies a secondary peak can be seen. This peak is centered around 0.02 GeV²/c⁴ and is caused by π^0 's. These must be cut out. I make a cut of $-0.01 < MM^2(p\pi^+\pi^-) < 0.01 \text{ GeV}^2/c^4$.



Figure 4.1: Missing mass squared off the proton, π^+ and π^- for four different energy ranges. The cuts that were made for identifying exclusive events are shown at $\pm 0.01 \text{ GeV}^2/c^4$ and a reference line at 0 is also drawn.

After I make these cuts, there will still be some background leaking in from π^{0} 's. To try to tighten the cuts to exclude π^{0} events I make a cut on the missing z-component of momentum. In Fig. 4.2 I plot the z-component of missing momentum for different beam energies. The peak is centered at 0 with very little background. The plots also show the cut at $-0.05 < \text{Missing } p_{z}(p\pi^{+}\pi^{-}) < 0.05 \text{ Gev/c}$ that we place on the data.



Figure 4.2: Missing z-component of momentum off the proton, π^+ and π^- for four different energy ranges. The cuts that were made for identifying exclusive events are shown at \pm 0.05 GeV/c and a reference line at 0 is also drawn.



Figure 4.3: Missing mass squared off the proton, π^+ and π^- after cuts on missing mass and missing momentum. Note the change in *x*-axis scale from the previous plots.



Figure 4.4: Monte Carlo events. Missing mass squared off the proton, π^+ and π^- after cuts on missing mass and missing momentum.

Fig. 4.3 shows the missing mass squared after our cuts. We have reduced the background so that it is on the order of 1%. We make the same plots for the Monte Carlo data in Fig. 4.4. While the Monte Carlo data does not have π^0 events, we see that the resolution for detecting the missing mass is well reproduced by our simulation.

If we chose, we could also select the data by reconstructing any of the three particles from missing mass. In Fig. 4.5 we plot the missing mass squared off the proton and π^+ for four different beam energies. The peak is at the mass squared of the π^- and a line is drawn at this mass. However, we would be less able to separate out the contaminating π^{0} 's. The resolution on the missing mass is also not as good as when I detect all three particles. It was also shown that the agreement in acceptance between Monte Carlo and CLAS data is worse for low momenta protons. By requiring that all three particles be detected in CLAS, it is hoped that the extra constraints will give us the most well measured events. With a finite amount of time to study the detector and interpret the PWA results we chose to analyze exclusive events with all three being detected. The trade-off is that we are more limited by the detector acceptance and we lose some portions of phase space as will be shown in the next section. It is hoped that further analysis will be performed on the data set where the π^- is not detected in the CLAS but instead reconstructed from missing mass.



Figure 4.5: Missing mass squared off the proton and π^+ when these are the only two charged tracks detected. A reference line is drawn at the mass squared of the π^- .

4.2 Amount of data

After all the cuts are made I have 775,553 data events to analyze. I break the data up by binning in groups of 16 E-bins. This gives us a manageable amount of statistics in each bin. In Fig. 4.6 I plot the number of events in each bin the photon flux for these events and the flux corrected data yield. The data is rather jumpy due to the photon flux and bad E-counters that I have cut out of the data. Right at $W \approx 1.7$ I have completely discarded the events associated with the E-counters in that region due to problems during the flux calculation. In the third plot I've divided the data by the photon flux to show that removing the effect of cutting out the bad E-counters gives us a smoother distribution.



Figure 4.6: Data, photon flux and flux corrected data used in this analysis.

4.3 Characteristics of the data for different W

Before we begin the PWA in earnest, we show aspects of the data and draw some conclusions about what it tells us about the physics at hand as well as what our analysis limitations might be. In the following sections we look at the same variables for different W, starting at the lower energy range of our data sample, and working our way up to higher W. No acceptance correction has been applied.

I plot the three invariant two-body masses: $p\pi^+$, $p\pi^-$ and $\pi^+\pi^-$. If the reaction proceeds through quasi-two-body final states, unstable isobars should appear in plots of the two-body invariant masses. For the $p\pi$ plots a dotted line is drawn at the the mass of the $\Delta(1232)$. This is a dominant feature in portions of our energy range. For the $\pi^+\pi^-$ plot a dotted line is drawn at the mass of the $\rho(770)$, another dominant feature, though in a different range.

I plot the three final state particles $\cos(\theta)$ in the center of mass. Features of the CLAS affect these plots similarly. The hole in the forward region means that forward going tracks will not be detected. Negatively charged tracks bend inward and so we lose a greater portion of these tracks than the outward bending positively charged tracks. The CLAS also has no instrumentation for $\theta > 142^{\circ}$ so tracks that go too far backward are lost as well.

I show two Dalitz plots for $M^2(p\pi^+)$ vs. $M^2(p\pi^-)$ and $M^2(\pi^+\pi^-)$ vs. $M^2(p\pi^-)$. The information on these plots is redundant, but by plotting it differently some features may become more obvious. On this plot I draw dotted lines representing the mass squared of the $\Delta(1232)$ and the $\rho(770)$ for the appropriate axis. Isobars will be visible in these plots. On the Dalitz plots there are two dotted lines. For a particular W, there is a definite boundary to the Dalitz plot which is a function of W and the masses of the final state particles. These Dalitz plots are for a small range of W (25 MeV/c² wide) so we show the boundary for both the low and high W.

$4.3.1 \quad 1.4 \le W < 1.5 \,\, GeV/c^2$

This is at the lower end of our energy. Threshold for this reaction is $W \approx 1.22$ GeV/c². This reaction may proceed through the decay of an intermediate $\Delta(1232)$ resonance. The threshold for $\Delta \pi$ is $W \approx 1.46$ GeV/c². So for this W we are at the limit for this reaction and this is supported by the lack of structure in the $p\pi$ mass distributions.

The Dalitz plots are particularly interesting in this range. The dotted lines outline the allowable phase space for this W bin. For the $M^2(p\pi^+)$ vs. $M^2(\pi^-)$ plot it is noticeably depleted in the upper left and lower right portions of the plot. Is this physics? No. If we run Monte Carlo three-body phase space through our detector simulation, we get the same depletions. How does this happen? The events in the upper left correspond to those where the $p\pi^-$ mass is closer to threshold. There is less breakup momentum for this two body system and so in their center-of-mass frame, they have very little relative velocity. This region of the plot also implies a greater breakup momentum for the $p\pi^+$ system and so there must be more relative momentum between these two. In this lower W-bin there is a limited amount of energy and so the most favorable distribution is for the $p\pi^-$ system to continue more or less forward with little transverse momentum. Because of the structure of CLAS, we lose particles which travel too far forward due to the hole in the detector in the forward region. Because we are requiring that all three particles be detected, we do not detect those events which populate the upper left region of the plot. The same argument explains the depletion in the lower right. A similar effect is noticed if we plot the invariant mass squared of the $\pi^+\pi^-$ system on one of the axis.

For comparison I plot the $M^2(p\pi^+)$ vs. $M^2(\pi^-)$ for both raw and accepted phase space as shown in Fig. 4.8. The raw phase space is evenly populated, with slightly fewer events due to the fact that our plot is not just one W, but a range of W. The accepted phase space shows that our detector is insensitive to certain kinematic distributions and so we lose events in parts of our plot. This will have consequences for our PWA as will be discussed in later chapters.



Figure 4.7: Two Dalitz plots for $1.475 \leq W < 1.500 \text{ GeV/c}^2$. The dotted lines outline the kinematic limits based on the low and high edges of the W bin. The depleted regions are due to the acceptance of the CLAS.



Figure 4.8: Monte Carlo data. The same variables are plotted on these two Dalitz plots for $1.475 \leq W$; $1.500 \text{ GeV}/c^2$. The first plot is raw phase space and the second plot phase space run through our detector simulation. The dotted lines outline the kinematic limits based on the low and high edges of the Wbin. The depleted regions are due to the acceptance of the CLAS.



Figure 4.9: Details of the data for $1.4 \leq W < 1.5 \text{ GeV}/c^2$.

4.3.2 $1.7 \le W < 1.8 \text{ GeV}/c^2$



Figure 4.10: Two Dalitz plots for $1.775 \leq W < 1.800 \text{ GeV}/c^2$. The dotted lines outline the kinematic limits based on the low and high edges of the W bin. The depleted regions are due to the acceptance of the CLAS.

The Δ^{++} is clearly visible in the $p\pi^+$ mass spectrum in both the mass plot and Dalitz plot in Figs. 4.10 and 4.11. The Δ^0 is visible as well in the $p\pi^-$ mass spectrum, though it is not as prominent as the Δ^{++} . This does make sense if we look at the resonances in this region. Referring back to Table 1.4 and Table 1.4, I find in this mass range there more contributing N^* 's than Δ 's. Invoking isospin conservation we find the N^* favours the $\Delta^{++}\pi^-$ over the $\Delta^0\pi^+$ decay channel by almost 9:1. For a Δ the ratio is 9:4. Because there are more N^* in this region we see a greater strength of Δ^{++} than Δ^0 .

Threshold for $p\rho$ is $W = 1.708 \text{ GeV}/c^2$ so in this region we are just hitting the tail edge of the ρ and so there is very little structure in the $\pi^+\pi^-$ plot.



Even with the acceptance colouring the angular distributions we see that both the π 's prefer to go forward and the proton prefers to go backward.

Figure 4.11: Details of the data for $1.7 \leq W < 1.8 \text{ GeV}/c^2$.

4.3.3 $1.9 \le W < 2.0 \text{ GeV}/c^2$



Figure 4.12: Two Dalitz plots for $1.975 \leq W < 2.000 \text{ GeV}/c^2$. The dotted lines outline the kinematic limits based on the low and high edges of the W bin. The depleted regions are due to the acceptance of the CLAS.

The Δ 's are both still visible and we are now seeing a strong ρ . The π 's are more forward peaked and the proton is even more backward going.



Figure 4.13: Details of the data for $1.9 \leq W < 2.0 \text{ GeV}/c^2$.

4.3.4 $2.2 \le W < 2.3 \text{ GeV}/c^2$



Figure 4.14: Two Dalitz plots for $2.225 \leq W < 2.250 \text{ GeV}/c^2$. The dotted lines outline the kinematic limits based on the low and high edges of the W bin. The depleted regions are due to the acceptance of the CLAS.

In this region we are now almost entirely dominated by ρ production, though we cannot make any claims about whether it is *s*-channel or *t*-channel production. The Δ^{++} is still visible, though much less than at lower energies.



Figure 4.15: Details of the data for $2.2 \leq W < 2.3 \text{ GeV}/c^2$.

CHAPTER 5 Partial Wave Analysis (PWA)

In the simplest terms, I know the initial state is (γp) and the final state $(p\pi^+\pi^-)$ and I want to know what the intermediate states were and their relative strengths.

5.1 Formalism

5.1.1 Introduction

The amplitude for a scattering process can be written as some operator, T, which connects our initial and final states.

$$\Psi = \langle f | T | i \rangle$$

More rigorously, I write my operator T as a product of T_i and T_f which act on the initial and final states respectively. I can then expand over intermediate states in some basis. τ represents the kinematics for the reaction.

$$\begin{aligned} \langle f|T|i \rangle &= \langle p\pi^{+}\pi^{-}; \tau_{f}|T|\gamma p; \tau_{i} \rangle \\ &= \langle p\pi^{+}\pi^{-}; \tau_{f}|T_{\tau_{f}}T_{\tau_{i}}|\gamma p; \tau_{i} \rangle \\ &= \sum_{\alpha} \langle p\pi^{+}\pi^{-}; \tau_{f}|T_{\tau_{f}}|\alpha \rangle \langle \alpha|T_{\tau_{i}}|\gamma p; \tau_{i} \rangle \\ &= \sum_{\alpha} \psi_{\alpha}(\tau_{f}) V_{\alpha} \end{aligned}$$

I now have two amplitudes.

- $\psi_{\alpha}(\tau_f)$ This is the amplitude for the decay of the intermediate state into our final state.
- V_{α} This represents the production amplitude for our initial state.

The partial wave decomposition method allows us to *calculate* the decay amplitudes, and *fit* the production amplitudes. Note that I have discarded the τ dependence



Figure 5.1: Representation of decomposing the scattering amplitude in some known basis.

for the production amplitudes. The square of the amplitude gives me a probability density which is proportional to the intensity which we observe. Our analysis will bin in the mass of the final state particles. If we bin fine enough, the assumption is that the physics is energy independent within the bin.

5.1.2 Scattering amplitudes

We assume that the scattering process proceeds through two body decays. Write the intensity in terms of the scattering amplitude squared.

$$I(\tau) = \left| \sum_{\alpha} V_{\alpha} \psi_{\alpha}(\tau) \right|^{2}$$
$$= \sum_{\alpha, \alpha^{i}} V_{\alpha} V_{\alpha^{i}}^{*} \psi_{\alpha}(\tau) \psi_{\alpha^{i}}^{*}(\tau)$$

The α label, indicates the waves we are summing over: isobars, spin, parity, orbital angular momentum. The ψ 's are some set of known functions in which we want to expand our intensity. The V's are the relative strengths of each of these functions: a measure of how much each wave contributes to the total intensity.

For s-channel production, the assumption is the photon and proton interact to form some resonance with spin J and parity P. This resonance decays to some two-body state with some angular momentum L.

For most of this analysis, we confined our allowed J's to $\frac{1}{2}, \frac{3}{2}$ and $\frac{5}{2}$ with both + and - parity. The motivation being that these are the quantum numbers of the resonances that dominate below $W = 1.8 \text{ GeV/c}^2$. We later added $\frac{7}{2}^{\pm}$ states.



Figure 5.2: Decay of J^P into particles s_1 and s_2 with helicities λ_1 and λ_2 .

Fig. 5.2 represents some decay for which I want to calculate the amplitude. This decay is indexed by the following values:

- Parent state
 - J angular momentum
 - $-\eta$ parity
 - M z-component of angular momentum
 - $-\Lambda$ helicity
 - -p momentum
- s Daughters 1 and 2
 - $-s_1, s_2$ spins
 - $-\eta_1,\eta_2$ parity
 - $-\lambda_1, \lambda_2$ helicities
 - ℓ angular momentum between the daughters
 - τ kinematics of daughters

The formalism to calculate these amplitudes comes from Jacob and Wick [?] and has been applied to our analysis method by Chung [?]. The following comes from these two references.

We write our amplitudes in the helicity basis, where the helicity (λ) is the spinoperator for a particle projected onto it's momentum vector: $\lambda = \vec{s} \cdot \hat{p}$. We could calculate the amplitude in the total angular momentum basis, but as the orbital angular and intrinsic spin operators are defined in different reference frames, this is not the most straightforward calculation. The helicity operator is invariant under rotations and boosts along \hat{p} [?]. This allows us to construct basis vectors which are eigenstates of of total angular momentum and helicity, or linear momentum and helicity [?].

$$\psi = \langle \vec{p}\lambda_1, -\vec{p}, \lambda_2 | M | JM \rangle$$

= $4\pi \left(\frac{w}{p}\right)^{\frac{1}{2}} \langle \phi \theta \lambda_1 \lambda_2 | JM \lambda_1 \lambda_2 \rangle \langle JM \lambda_1 \lambda_2 | M | JM \rangle$
= $N_J F^J_{\lambda_1 \lambda_2} D^{J*}_{M\Lambda}(\phi, \theta, 0) \qquad \lambda = \lambda_1 - \lambda_2$

where p is the relative momentum and w is the effective mass of the final state two-particle system. F is the helicity decay amplitude. (Note that we use the $(j_1j_2m_1m_2|JM)$ convention to represent Clebsch-Gordon coefficients.)

$$F^{J}_{\lambda_{1}\lambda_{2}} = 4\pi \left(\frac{w}{p}\right)^{\frac{1}{2}} \langle JM\lambda_{1}\lambda_{2}|M|JM\rangle$$

$$N_J F^J_{\lambda_1 \lambda_2} = \sum_{\ell s} (2\ell + 1)^{\frac{1}{2}} (\ell 0 s \Lambda | JM) (s_1 \lambda_1 s_2 - \lambda_2 | s \lambda)$$

The intensity can now be written as

$$I(\Omega) = \left(\frac{2J+1}{4\pi}\right) \sum_{\Lambda\Lambda'\lambda_1\lambda_2} \rho^J_{\Lambda\Lambda'} D^{J*}_{\Lambda\lambda}(\phi,\theta,0) D^J_{\Lambda'\lambda}(\phi,\theta,0) g^J_{\lambda_1\lambda_2}$$

The spin-density matrix $\rho_{\Lambda\Lambda'}^J$ is defined as

$$\rho^{h} = \sum_{\Lambda\Lambda'} |J\Lambda\rangle \rho^{J}_{\Lambda\Lambda'} \langle J\Lambda'|$$

The term $g^J_{\lambda_1\lambda_2}$ is defined as

$$g^J_{\lambda_1\lambda_2} \sim \int dw K(w) \left| F^J_{\lambda_1\lambda_2} \right|^2$$

The term K(w) incorporates all the energy dependant functions for this reaction.

$$K(w) \sim f_{\ell}(p)\Delta_J(w)g_a$$

 $f_{\ell}(p)$ is the Blatt-Weisskopf centrifugal-barrier factor [?] and suppresses higher ℓ 's.

 $\Delta_J(w)$ is the Breit-Wigner form for the *w* dependence of the decaying state [?].

$$\Delta_J(w) = \frac{w_0 \Gamma_0}{w_0^2 - w^2 - i w_0 \Gamma_J(w)}$$

$$\Gamma_J(w) \propto \frac{q^{2J+1}}{w}$$

 g_a is a coupling term for this decay. In this procedure, this is rolled into the production amplitude as part of the fit procedure.

If one of the decay particles, s_1 or s_2 , decays, it's decay amplitude can be calculated and the entire process becomes the products of these two amplitudes.



Figure 5.3: Formation of J^P and subsequent decay to Δ or N^* and it's decay to $p\pi$.

It may be helpful to look at an example. Let's look at the intensity for the process shown in Fig. 5.3. Let the spin of the Δ or N^* be labeled as s. The helicities of the photon, initial proton and final state proton I label $\lambda_{\gamma}, \lambda_i$, and λ_f respectively. Also note that M for the intermediate process is the same as its helicity, which I label Λ . The intensity for this process may be written as follows [?]:

$$I(\Omega,\Omega_1) = \left(\frac{2J+1}{4\pi}\right) \left(\frac{2s+1}{4\pi}\right) \sum_{\Lambda\Lambda'\lambda\lambda'\lambda_1} \rho^J_{\Lambda\Lambda'} g^J_{\lambda\lambda'} g^s_{\lambda_1} D^J_{\Lambda\lambda}(\Omega) D^{J*}_{\Lambda'\lambda'}(\Omega) D^s_{\lambda\lambda_1}(\Omega_1) D^{s*}_{\lambda'\lambda_1}(\Omega_1)$$

Where we can write $\Lambda = \lambda_{\gamma} - \lambda_i$.

The intensity can be represented as the sum of 8 non-interfering terms which are indexed by the initial and final helicities, as are the decay amplitudes.

By applying parity at both the photon-proton vertex and the decay side, we are able to show that the production amplitudes need only be indexed by $|M|(|\Lambda|)$. Parity is conserved in this interaction. Applying parity to the vertex takes λ_{γ} to $-\lambda_{\gamma}$ and λ_i to $-\lambda_i$. The amplitudes for these two processes must be the same.

$$V_{\Lambda} = V_{\lambda_{\gamma} - \lambda_{i}}$$
$$= V_{-\lambda_{\gamma} - -\lambda_{i}}$$
$$= V_{-\Lambda}$$

I can apply parity to the decay side as well. The amplitude associated with λ_f will be the same as the amplitude associated with $-\lambda_f$.

Table 5.1 shows how the different helicities index the production and decay amplitudes. There are eight combinations of the helicities. These index both the non-interfering terms which comprise our intensity and the decay amplitudes. Actually, for ease's sake, the decay amplitudes are indexed by M, instead of λ_{γ} and λ_i , but this is the same thing. The production amplitudes are indexed by |M|. This means every *one* production amplitude is associated with *four* decay amplitudes.

	ψ			
λ_γ	λ_i	λ_{f}	M	V
+1	$+\frac{1}{2}$	$+\frac{1}{2}$	$+\frac{1}{2}$	$V_{1/2}$
+1	$+\frac{1}{2}$	$-\frac{1}{2}$	$+\frac{1}{2}$	$V_{1/2}$
+1	$-\frac{1}{2}$	$+\frac{1}{2}$	$+\frac{3}{2}$	$V_{3/2}$
+1	$-\frac{1}{2}$	$-\frac{1}{2}$	$+\frac{3}{2}$	$V_{3/2}$
-1	$+\frac{1}{2}$	$+\frac{1}{2}$	$-\frac{3}{2}$	$V_{3/2}$
-1	$+\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{3}{2}$	$V_{3/2}$
-1	$-\frac{1}{2}$	$+\frac{1}{2}$	$-\frac{1}{2}$	$V_{1/2}$
-1	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$V_{1/2}$

Table 5.1: Table representing the appropriate helicities for this reaction, the M of the intermediate process, and the constraints on the production amplitudes.

5.1.2.1 Waves

There were four decay paths we allowed for these states: $\Delta \pi$, $p\rho$, $p\sigma$ and $N^*(1440)\pi$. We were motivated by previous experimental evidence for the preferred decay modes of these resonances.

Only certain L's are allowed, based on the spin and parity of the initial quantum numbers and those of the decay products. At most there are two allowed L's for a given state and decay path.

For decays involving $p\rho$ the spins of the two can be aligned in a $\frac{1}{2}$ state or $\frac{3}{2}$ state with particular ℓ associated with each.

Because the coupling of the intermediate state and the isobar-bachelor state is wrapped up in the production amplitude, we are actually looking to fit an amplitude that involves the photo production of some J^P state in some M state and it's subsequent decay to some isobar-bachelor system. Decay amplitudes may be labeled in the following way:

$$\frac{3}{2}^{-}(M = +\frac{3}{2}) \to \Delta^{++}\pi^{-}(\ell = 0, \lambda_f = +\frac{1}{2})$$
$$\frac{1}{2}^{-}(M = -\frac{1}{2}) \to p\rho(s = \frac{1}{2}, \ell = 0, \lambda_f = -\frac{1}{2})$$

and production amplitudes may be labeled as:

$$\frac{3}{2}^{-}(M = \frac{3}{2}) \to \Delta^{++}\pi^{-}(\ell = 0)$$
$$\frac{1}{2}^{-}(M = \frac{1}{2}) \to p\rho(s = \frac{1}{2}, \ell = 0)$$

5.2 Unbinned extended maximum likelihood method

Given some n number of events where the probability of measuring each ith event is given by P_i , then the probability of measuring the entire data set is simply the product of all the individual probabilities.

$$P_{\text{data set}} = P_1 \cdot P_2 \cdot P_3 \cdots \P_n$$

This is the basis of the maximum likelihood method. If P_i is some function with some parameter that can be fit, we can vary this parameter until we reach the maximum probability of measuring this data set.

The intensity is interpreted as a probability density and so we use this to construct a likelihood function [?]. The intensity is calculated by summing over all waves, α . τ represents the kinematics of the events.

$$I = \sum_{\alpha \alpha'} V_{\alpha} V_{\alpha'}^* \psi_{\alpha}(\tau) \psi_{\alpha'}^*(\tau)$$

We also need to consider that our detector has a finite acceptance, $\eta(\tau)$. The likelihood function can now be written as:

$$\mathcal{L} \propto \left[\frac{\bar{n}^n}{n!}e^{-\bar{n}}\right] \prod_i^n \frac{I(\tau_i)}{\int I(\tau)\eta(\tau)d\tau}$$

The first term in the brackets is the Poisson probability for measuring n events. The expectation value, \bar{n} , is given by [?]:

$$\bar{n} = \int I(\tau) \eta(\tau) d\tau$$

This allows us to simplify the likelihood function:

$$\mathcal{L} = \frac{\bar{n}^n}{n!} e^{-\bar{n}} \prod_i^n \frac{I(\tau_i)}{\bar{n}^n}$$

$$= \frac{1}{n!} e^{-\bar{n}} \prod_i^n I(\tau_i)$$

$$\ln \mathcal{L} = -\ln n! - \bar{n} - \sum_i^n I(\tau_i)$$

$$= -\ln n! - \int I(\tau) \eta(\tau) d\tau + \sum_i^n I(\tau_i)$$

We have taken the log of the likelihood function as the sum of the log of the terms is much easier to deal with than the product of the terms. For purposes of the fit we will minimize the negative of the log likelihood function. Dropping the terms that are not dependent on the production amplitudes we are left with the following equation to minimize:

$$-\ln \mathcal{L} = -\sum_{i}^{n} I(\tau_{i}) + \int I(\tau) \eta(\tau) d\tau$$

I want to be able to numerically calculate my normalization integral. I can do this by using Monte Carlo data. Here I write out the expression using the raw Monte Carlo.

$$\int I(\tau)\eta(\tau)d\tau \Rightarrow \frac{1}{N_{RMC}} \sum_{i}^{RMC} I(\tau_i)\eta(\tau_i)$$

The $\eta(\tau_i)$ is our acceptance and can be thought of as a 1 or 0 depending on the acceptance for that region of phase space. So change the sum so that it runs over the accepted Monte Carlo, instead of the raw events.

$$\int I(\tau)\eta(\tau)d\tau \Rightarrow \frac{1}{N_{AMC}} \sum_{i}^{AMC} I(\tau_i)$$

$$\frac{1}{N_{AMC}} \sum_{i}^{AMC} I(\tau_i) = \frac{1}{N_{AMC}} \sum_{i} \sum_{\alpha\alpha'} V_\alpha V_{\alpha'}^* \Psi_\alpha(\tau_i) \Psi_{\alpha'}^*(\tau_i)$$
$$= \sum_{\alpha\alpha'} V_\alpha V_{\alpha'}^* \left[\frac{1}{N_{AMC}} \sum_{i} \Psi_\alpha(\tau_i) \Psi_{\alpha'}^*(\tau_i) \right]$$

The quantity in brackets is summed over the accepted Monte Carlo events and need only be calculated once for all the fits. The final likelihood function is then

$$\ln \mathcal{L} = \sum_{i}^{n} \ln \left[\sum_{\alpha \alpha'} V_{\alpha} V_{\alpha'}^* \psi_{\alpha}(\tau_i) \psi_{\alpha'}^*(\tau_i) \right] - n \sum_{\alpha \alpha'} V_{\alpha} V_{\alpha'}^* \left[\frac{1}{N_{AMC}} \sum_{i} \Psi_{\alpha}(\tau_i) \Psi_{\alpha'}^*(\tau_i) \right]$$

Where we have introduced a factor of n to the normalization integral to account for the an absolute normalization for the parameters, V's [?].

We perform the fits independently in W bins, the mass of the J^P state. By looking at how the intensity of waves changes from bin to bin, we can identify resonance behavior.

5.3 Isospin basis

It may be possible to use isospin conservation to extract information from the fit results, either at fit time, or in the mass-dependant analysis. When we write out our waves in terms of the J^P of the state, there is no isospin information. That is, we do not know if the state is an $N^*(I = \frac{1}{2})$ or a $\Delta(I = \frac{3}{2})$. Because the charge of our final state is +1 we know that it is $I_z = \frac{1}{2}$.

These two isospins have different branching ratios to $\Delta^{++}\pi^{-}$ and $\Delta^{0}\pi^{+}$ simply on the basis of isospin conservation. Using Clebsch-Gordon coefficients I can calculate these ratios. Reminding ourselves that Δ^{++} and Δ^{0} are the $+\frac{3}{2}$ and $-\frac{1}{2}$ projections of an $I = \frac{3}{2}$ state, the π^{+} and π^{-} are the +1 and -1 projections of an I = 1 state, and the proton is the $+\frac{1}{2}$ projection of an $I = \frac{1}{2}$ state.

First for $I = \frac{1}{2}$ going to $\Delta^{++}\pi^-$:

$$\langle j_1 j_2 m_1 m_2 | JM \rangle$$

$$\langle \frac{3}{2} 1 \frac{3}{2} - 1 | \frac{3}{2} 1 \frac{1}{2} \frac{1}{2} \rangle = \sqrt{\frac{1}{2}}$$
And for $\Delta^0 \pi^+$.
$$\langle \frac{3}{2} 1 - \frac{1}{2} 1 | \frac{3}{2} 1 \frac{1}{2} \frac{1}{2} \rangle = \sqrt{\frac{1}{6}}$$

We still need to take into account the decay of the Delta's:

$$\begin{array}{rcccc} \Delta^{++} & \to & p\pi^+ \\ \langle \frac{1}{2} 1 \frac{1}{2} 1 | \frac{1}{2} 1 \frac{3}{2} \frac{3}{2} \rangle & = & \sqrt{1} \\ \Delta^0 & \to & p\pi^- \\ \langle \frac{1}{2} 1 \frac{1}{2} - 1 | \frac{1}{2} 1 \frac{3}{2} - \frac{1}{2} \rangle & = & \sqrt{\frac{1}{3}} \end{array}$$

$$\begin{array}{l} \frac{|\frac{1}{2}\frac{1}{2}\rangle \to \Delta^0 \pi^+ \to p\pi + \pi -}{|\frac{1}{2}\frac{1}{2}\rangle \to \Delta^{++}\pi^- \to p\pi + \pi -} &=& \frac{\sqrt{\frac{1}{6}} \times \sqrt{\frac{1}{3}}}{\sqrt{\frac{1}{2}} \times \sqrt{1}} \\ &=& \sqrt{\frac{1}{9}} \end{array}$$

I follow the same procedure for the $I = \frac{3}{2}$ state and I get:

$$\frac{\left|\frac{3}{2}\frac{1}{2}\right\rangle \rightarrow \Delta^{0}\pi^{+} \rightarrow p\pi + \pi^{-}}{\left|\frac{3}{2}\frac{1}{2}\right\rangle \rightarrow \Delta^{++}\pi^{-} \rightarrow p\pi + \pi^{-}} = \frac{-\sqrt{\frac{8}{15}} \times \sqrt{\frac{1}{3}}}{\sqrt{\frac{2}{5}} \times \sqrt{1}}$$
$$= -\sqrt{\frac{4}{9}}$$

Note that these are the ratios for the *amplitudes*. I can use this information now in one of two ways.

I can rewrite my amplitudes involving a $\Delta \pi$ intermediate state in an isospin basis. I can take the amplitudes already generated and linearly combine them using the above ratios.

I also have the option of using the above as a guide to interpret the intensities. By squaring the above I get the ration for the *observed intensities*. Using the ratio of a J^P state decaying to each of the charged $\Delta \pi$ states should give me a handle on the isospin of that state. Interferences may make eyeballing the data a risky proposition. Interferences may affect the charge states differently and so this procedure would best be used when a full mass-dependant analysis is performed.

5.4 Non-resonant background terms

5.4.1 Contact term

The Born contact term can be represented as an isospin combination of the charged $\Delta \pi$ states [?]. Specifically, the ratio is almost the same as the N^* 's except that it is negative, $-\frac{1}{9}$.

The difficulty is that because we have only two charged states, when we move to the isospin basis we pick up an ambiguity when we have three states: $I = \frac{1}{2}, I = \frac{3}{2}$ and the contact term. Some fits were tried with in the isospin basis where we chose to leave out one of the isospin combinations when we included the contact term.

5.4.2 *t*-channel contributions



Figure 5.4: Representation of the amplitudes for *t*-channel production of the ρ, σ or π .

Threshold for the ρ production at the mass of the ρ is about $W = 1.7 \text{ GeV/c}^2$. This process can occur not only through the *s*-channel creation and subsequent decay of some resonance, but also through a *t*-channel process where some virtual process is created. This has the effect of pushing the ρ in the forward direction. Of course, the ρ is not the only particle that could be created through this process in our reaction. A σ could also be produced. We could also have π production where an N^* or Δ is left on the bottom resonance. Call whatever is produced on the top vertex, X. The amplitudes are represented in Fig. 5.4

We generated amplitudes for these processes and used them in the fit the same way we would any of the other waves. Because this is a different basis, we have the issue that we may be introducing mathematical ambiguities to the likelihood function. The hope is that the *t*-channel processes could only be mocked up by a large number of *s*-channel terms, and so by including the dominant *s*-channel waves and a minimum number of *t*-channel waves we would avoid any practical ambiguities.

We calculated these terms in two different ways.

In our first set of fits we generated an amplitude that was the product of two terms: an exponential, containing the t slope $(t = (\mathcal{P}_{\gamma} - \mathcal{P}_X)^2)$ and the Breit-Wigner amplitude.

$$\psi_t = e^{-\frac{\beta}{2}t} \Delta_X(p)$$

This procedure was only used for ρ production. When we tried this procedure, we did not let it interfere with the other terms and the one decay amplitude was associated with one production amplitude.

The next step was to include a term for the angular distribution for the decay of the ρ or Δ or any isobar we used.

$$\psi_t = e^{-\frac{\beta}{2}t} \Delta_X(p) F_I(\Omega)$$

For different fits we would allow this to interfere in a variety of ways. Because the helicity of the isobar need not be tied to the initial helicity states the number of production amplitudes can quickly grow large. For most fits, we would truncate the number of terms by appealing to some physics justifications or in some arbitrary attempt to identify a stable fit. The results of these fits will be discussed in later chapters.

5.5 Mechanics of the fits

5.5.1 Computer tools

The code used for the bulk of the fitting procedure was written by John Cummings and Dennis Weygand [?] and the procedure is detailed in the references.

One change was made to the actual fitting routine. Instead of the original fit routine, I wrote a program I called fastfit. This routine is less general than the old code, but runs much faster. The core of the code is the same though in that the minimization is still performed by the CERN software, MINUIT [?]. The code was extensively checked to make sure it gives the same results as the original software.

5.5.2 Tracking fits

The code was set up so that we can easily do *tracking fits*. We perform the fits in W bins. The default procedure is to use random initial starting values for the complex fit parameters. However, if we so choose we can use the final values from a neighboring bin. The idea being that the physics has changed only slightly from bin to bin and so we may already be near a minima if we use the values from a nearby bin. We are then able to choose some starting bin and track out from there. We state when the results shown are from one of these fits.

5.5.3 Results

5.5.3.1 Weighting the Monte Carlo data

I can use the results of the fit to generate a weight for the accepted Monte Carlo data. In this way I compare some kinematic variable with CLAS data to get an idea of how good the fit describes the physics.

Fig. 5.5 shows the proton's $\cos(\theta)$ in the center of mass for 3 different sets of data. The first is just the data out of the CLAS. The error bars are just statistics. The next plot shows the phase space Monte Carlo after it has been run through the simulation routine, showing the effect of the CLAS acceptance. The last plot is the accepted Monte Carlo weighted by the results of the fit. If the fit is a good description of our physics, this plot should agree with the first one. In later chapters I will overlay these on one another to see the agreements or discrepancies. The error

bars on the weighted plot are also statistics.



Figure 5.5: The first plot shows the proton's $\cos(\theta)$ in the center of mass for data for a bin around $W = 1.70 \text{ GeV/c}^2$. The next plot shows the same quantity for the accepted phase space Monte Carlo. The last plot shows the same quantity for the accepted Monte Carlo weighted by the fit.

Fig. 5.6 shows the same quantities but for raw Monte Carlo. The first shows the flat distribution we expect. The next plot shows the raw Monte Carlo weighted by the results of the fit. This is what we refer to as the acceptance corrected distribution. In effect, this is what the fit is saying the physics looked like before it saw the detector. This is what we will provide to others in the community so that they may compare theoretical predictions with these results.



Figure 5.6: The first plot shows the proton's $\cos(\theta)$ in the center of mass for raw Monte Carlo. The next plot shows the same quantity weighted by the results of the fit. This is the acceptance corrected distribution for this quantity.

5.5.3.2 Calculating a cross section

To calculate a cross section I need to know how many events went into my detector, and so I must correct the detected number of events for the acceptance of the CLAS.

If CLAS was a perfect 4π detector and the physics was a purely flat distribution, I could just use the raw and accepted number of Monte Carlo events to get the number of events in a bin.

$$N = N_{\rm meas} \frac{N_{RMC}}{N_{AMC}}$$

But because we have physics and not flat phase space, this does not work. The fit results allow us to calculate the true number of events [?].

$$N = N_{\text{meas}} \frac{N_{RMC}}{N_{AMC}} \sum_{\alpha \alpha'} V_{\alpha} V_{\alpha'}^* \left[\frac{1}{N_{RMC}} \sum_i \Psi_{\alpha}(\tau_i) \Psi_{\alpha'}^*(\tau_i) \right]$$

where the normalization integral is over the raw Monte Carlo events. With the acceptance corrected yield in hand, I can now calculate a cross section [?].

$$\frac{N_{events}}{second} = \sigma \frac{\frac{N_{flux}}{second}}{F} = \sigma \times \text{Luminosity}$$

- $F \equiv \text{target constant}$
 - -[F] = barns
 - -1 barn $= 10^{-24}$ cm²

$$-F = \frac{A}{N_A \rho \ell}$$

- * $A \equiv$ atomic weight
- * $N_A \equiv$ Avogadro's number
- * $\rho \equiv \text{density}$
- * $\ell \equiv \text{length of target}$
- For the g1c liquid hydrogen target, these numbers are:

$$-A = 1$$

- $-~N_A = 6.022 \times 10^{23}$ particles per gram
- $-\rho = 0.071 \text{ grams cm}^{-3}$
- $-\ell = 18 \text{ cm}$
- $F = 1.3 \times 10^{-24} \text{ cm}^{-2}$
- F = 1.3 barns

$$\sigma = F \times \frac{N_{events}}{N_{flux}}$$
$$= 1.3 \times \frac{N_{events}}{N_{flux}}$$

5.5.4 The problem of local minima

Because we have finite statistics and finite computing time, there is no way for the fit to know if it has converged to a local minimum or global minimum. We conducted some studies to see how often we at least found the best minimum we could reasonably expect.

Fig. 5.7 shows the results of one of these studies. We ran this fit 21 times and plot the negative log likelihood function. Half of the time the fit finds the best minimum, or at least the best minimum out of these trials. Regardless of which minimum it converges to, we see that the acceptance corrected yield comes out roughly the same. There is some scatter however in the strengths of the individual waves.



Figure 5.7: We ran this fit 22 times in one bin. The plot in the upper left shows the distribution of the negative log likelihood function. We want to minimize this value so the points lower on the x-axis represent a better fit. The plot in the upper right shows the acceptance corrected yield calculated from each of these fits. The lower two plots show the yield from each of two individual waves.

Fig. 5.8 shows the results where we are only finding the best minimum less than 25% of the time. It is also possible that there is a better minimum which we have not seen yet. Because of these studies, the results that we show are either the best fits out of multiple trials or tracking fits.



Figure 5.8: We ran this fit 21 times in one bin. The plot in the upper left shows the distribution of the negative log likelihood function. We want to minimize this value so the points lower on the x-axis represent a better fit. The plot in the upper right shows the acceptance corrected yield calculated from each of these fits. The lower two plots show the yield from each of two individual waves.

CHAPTER 6 Results of PWA fits

6.1 Choices of fitting parameters

Over the course of this study, close to 200 fits were tried. Not all were tried over the full mass range. Some were the result of trying to find a minimal wave set by doing an iterative search of which waves could be added to give the best liklihood function. Some used physics justifications as a starting point. By this I mean, the quantum numbers and decay modes of expected resonances were put in the wave set. Some fits used t-channel waves and allowed them to interfere with the s-channel waves in different ways.

In this chapter I will focus primarily on six fits. You can think of these fits having increasing amounts of physics in the wave selection. It was found that if we put in all allowed quantum numbers for just $\Delta \pi$ and $p\rho$ decays, the approximately 100 waves needed rendered the fit results uninterpretable. The intensity distributions were not smooth and showed no evidence of any real resonance motion, even in regions where it should be exptected. Most of our focus was then on truncating the basis so we could still accurately describe the data, calculate a total cross section and still interpret the individual motion of the waves. To this end we referred to previous studies for branching ratios [?, ?] and photocouplings [?] to get a feel for what quantum numbers and isobars we could expect to contribute.

When choosing these waves I focused on resonaces below 1.8 GeV/c² as a starting point. There are questions about how to handle the *t*-channel ρ production, so we concentrated on this region for a time. This chapter is broken up into sections covering below and above this energy.

I will refer to these fits as fit 132, 163, 947, 121, 053 and 125, where I have listed them in what I think of as containing increasing amounts of physics. The reader will find these names somewhat arbitrary, but these were the numbers that were used during the course of this analysis and I choose to stick with them for the sake of consistency. In the next section I outline these six fits. Note that when I refer to $\Delta \pi$ I am referring to both charge states, $\Delta^{++}\pi^{-}$ and $\Delta^{0}\pi^{+}$.

6.1.1 Fit 132

At lowest energies, the leading contribution to the scattering process is a gauge invariant Born term [?] which results in a $\Delta \pi$ s-wave. The only quantum numbers which can decay this way is the $\frac{3}{2}^{-}$. This term can be represented as an isospin combination of the charge states [?]. The D₁₃(1520), D₁₃(1700) and D₃₃(1700) could also contribute to these waves. For this simple fit I try a $\frac{3}{2}^{-}$ decaying to $\Delta \pi$ in both allowed M values.

J^P	M	Isobars	# of waves
$\frac{3}{2}^{-}$	$\frac{1}{2}, \frac{3}{2}$	$(\Delta \pi)_{(\ell=0)}$	4
Total # of waves			4

Table 6.1: Fit 132

6.1.2 Fit 163

For this fit I try a limited amount of waves using the aforementioned references to guide me. The $\frac{3}{2}^{-}$ is motivated by the D₁₃(1520),D₁₃(1700) and D₃₃ and their respective observations in $\Delta \pi$ and $p\rho$ decays. I use the previously measured photon coupling amplitudes, $A_{1/2}$ and $A_{3/2}$, to try to identify the M value in which I should observe these decays. I also try to limit myself to particular ℓ 's or s's. It should be noted that the D₁₃(1700) prefers to decay to a $\Delta \pi$ in a D wave while the D₁₃(1700) prefers to decay to $\Delta \pi$ in a S wave.

Note that while the threshold for the producing $p\rho$ at the ρ peak is W = 1.7 GeV/c², the ρ has a width around 0.15 Gev/c². So ρ production could contribute significantly even at W = 1.6 GeV/c². In fact, the $D_{13}(1520)$ has a significant branching ratio to $p\rho$ according to the PDG [?].

The $\frac{3}{2}^+$ is motivated by the D₁₃(1720) which is reported to couple strongly to $p\rho$ final states and the D₃₃(1600) which has a stronger branching ratio to $\Delta \pi$.

The $\frac{5}{2}^+$ is motivated by the F₁₃(1680) and it's dominant $A_{3/2}$ amplitude.

J^P	M	Isobars	# of waves
$\frac{3}{2}^{-}$	$\frac{1}{2}, \frac{3}{2}$	$(\Delta \pi)_{(\ell=0,2)}$	8
	$\frac{1}{2}$	$(p\rho)_{(s=3/2);\ell=0,2)}$	2
	$\frac{3}{2}$	$(p\rho)_{(s=3/2);\ell=2)}$	1
$\frac{3}{2}^{+}$	$\frac{1}{2}, \frac{3}{2}$	$(\Delta \pi)_{(\ell=1)}$	4
	$\frac{1}{2}, \frac{3}{2}$	$(p\rho)_{(s=1/2);\ell=1)}$	2
$\frac{5}{2}^{+}$	$\frac{3}{2}$	$(\Delta \pi)_{(\ell=1)}$	2
Total # of waves			19

It is interesting to note that this is motivated by only a handful of resonance and yet we already have 19 waves which means 37 (2n - 1) parameters for our fit.

Table 6.2: Fit 163

The values I show for this fit are the best liklihood of 10 fits which used random starting values.

6.1.3 Fit 947

For fit 947 less care was taken in choosing the waves based on the photocoupling in order to open up more possibilities. The motivations for the $\frac{3}{2}^{-}$, $\frac{3}{2}^{+}$ and $\frac{5}{2}^{+}$ waves have been discussed in the previous fit. We also add $\frac{1}{2}^{-}$ (S₁₁(1535), S₁₁(1650) S₃₁(1620)), $\frac{1}{2}^{+}$ (P₁₁(1440), P₁₁(1710)) and $\frac{5}{2}^{-}$ (D₁₅(1675)).

Note that I am also allowing certain waves to decay to $p\sigma$ or $N^*(1440)\pi^+$.

J^P	M	Isobars	# of waves
$\frac{1}{2}^+$	$\frac{1}{2}$	$\Delta \pi$	2
$\frac{1}{2}^{-}$	$\frac{1}{2}$	$\Delta \pi$	2
	$\frac{1}{2}$	$(p\rho)_{(s=1/2;\ell=0)}$	1
$\frac{3}{2}^{+}$	$\frac{1}{2}, \frac{3}{2}$	$\left(\Delta\pi\right)_{(\ell=1)}$	4
	$\frac{1}{2}, \frac{3}{2}$	$(p\rho)_{(s=1/2;\ell=1)}$	2
	$\frac{1}{2}, \frac{3}{2}$	$(p\rho)_{(s=3/2;\ell=1,3)}$	4
	$\frac{1}{2}, \frac{3}{2}$	$N^{\star}(1440)\pi$	2
$\frac{3}{2}^{-}$	$\frac{1}{2}, \frac{3}{2}$	$(\Delta \pi)_{(\ell=0,2)}$	8
$\frac{5}{2}^{+}$	$\frac{1}{2}, \frac{3}{2}$	$(\Delta \pi)_{(\ell=1)}$	4
	$\frac{1}{2}, \frac{3}{2}$	$p\sigma$	2
$\frac{5}{2}^{-}$	$\frac{1}{2}, \frac{3}{2}$	$(\Delta \pi)_{(\ell=2)}$	4
Total # of waves			35

Table 6.3: Fit 947

The values I show for this fit are the best liklihood of 10 fits which used random starting values.

6.1.4 Fit 121

When I was putting together the fit 947, I made a mistake and neglected the coupling of the $\frac{3}{2}^-$ to $p\rho$ final states. For fit 121 I put these back into the wave set.
J^P	M	Isobars	# of waves
$\frac{1}{2}^{+}$	$\frac{1}{2}$	$\Delta \pi$	2
$\frac{1}{2}^{-}$	$\frac{1}{2}$	$\Delta \pi$	2
	$\frac{1}{2}$	$(p\rho)_{(s=1/2;\ell=0)}$	1
$\frac{3}{2}^{+}$	$\frac{1}{2}, \frac{3}{2}$	$(\Delta \pi)_{(\ell=1)}$	4
	$\frac{1}{2}, \frac{3}{2}$	$(p\rho)_{(s=1/2;\ell=1)}$	2
	$\frac{1}{2}, \frac{3}{2}$	$(p\rho)_{(s=3/2;\ell=1,3)}$	4
	$\frac{1}{2}, \frac{3}{2}$	$N^{\star}(1440)\pi$	2
$\frac{3}{2}^{-}$	$\frac{1}{2}, \frac{3}{2}$	$(\Delta \pi)_{(\ell=0,2)}$	8
	$\frac{1}{2}, \frac{3}{2}$	$(p\rho)_{(s=3/2;\ell=0,2)}$	4
$\frac{5}{2}^{+}$	$\frac{1}{2}, \frac{3}{2}$	$(\Delta \pi)_{(\ell=1)}$	4
	$\frac{1}{2}, \frac{3}{2}$	$p\sigma$	2
$\frac{5}{2}^{-}$	$\frac{1}{2}, \frac{3}{2}$	$(\Delta \pi)_{(\ell=2)}$	4
- -	Fotal	39	

Table 6.4: Fit 121

This is a tracking fit where I tracked outward from about 1.6 GeV/c^2 . The starting point was the best liklihood of 20 fits.

6.1.5 Fit 053

This fit was modeled on fit 947 so it is missing the $\frac{3}{2}^- \rightarrow p\rho$ waves. However, this is the first fit we tried with *t*-channel ρ production. To the waves from fit 947 we added one *non-interfering* ρ wave. We modeled it with an exponential slope of $\beta = 6.0$ and a Breit-Wigner for the ρ , but we *did not* include the angular decay of the ρ . This was the starting point for our *t*-channel studies.

J^P	M	Isobars	# of waves
$\frac{1}{2}^+$	$\frac{1}{2}$	$\Delta \pi$	2
$\frac{1}{2}^{-}$	$\frac{1}{2}$	$\Delta \pi$	2
	$\frac{1}{2}$	$(p\rho)_{(s=1/2)}$	1
$\frac{3}{2}^{+}$	$\frac{1}{2}, \frac{3}{2}$	$\left(\Delta\pi\right)_{(\ell=1)}$	4
	$\frac{1}{2}, \frac{3}{2}$	$(p\rho)_{(s=1/2)}$	2
	$\frac{1}{2}, \frac{3}{2}$	$(p\rho)_{(s=3/2;\ell=1,3)}$	4
	$\frac{1}{2}, \frac{3}{2}$	$N^{\star}(1440)\pi$	2
$\frac{3}{2}^{-}$	$\frac{1}{2}, \frac{3}{2}$	$\left(\Delta\pi\right)_{(\ell=0,2)}$	8
$\frac{5}{2}^{+}$	$\frac{1}{2}, \frac{3}{2}$	$(\Delta \pi)_{(\ell=1)}$	4
	$\frac{1}{2}, \frac{3}{2}$	$p\sigma$	2
$\frac{5}{2}^{-}$	$\frac{1}{2}, \frac{3}{2}$	$(\Delta \pi)_{(\ell=2)}$	4
$t\text{-channel}\ \rho,$ no Ω decay, no interfere			1
	36		

Table 6.5: Fit 053.

This is a tracking fit where I tracked outward from about 1.6 GeV/c^2 . The starting point was the best liklihood of 20 fits.

6.1.6 Fit 125

In this fit I added 4 *t*-channel waves and allowed them to interfere in a constrained way. I only allow the ρ to have helicity of ± 1 , like the incoming photon. But I allow for helicity flip. That is, a photon with $\lambda = +1$ may wind up as a ρ in either $\lambda = \pm 1$. I have a different production amplitude however for the different helicities of the incoming proton.

The decay amplitudes are only indexed by the helicity of the ρ (\pm 1) as the proton is just a spectator in this reaction. This gives us two decay amplitudes which appear *in all eight* non-interfering terms in the intensity calculation. Once we apply parity contraints to the system the production amplitudes are indexed by the helicity

of the photon and the absolute value of the M of the resonances. While there is no real resonance information in this wave, the M tells us which *s*-channel terms they interfere with. So there are two production amplitudes in each of the non-interfering terms in the intensity calculation and each of these production amplitudes appears twice.

In an effort to limit the number of waves in the fit and keep in manageable I removed a few isobars from the $\frac{3}{2}^+$ waves. The N* term had yielded negligible results in earlier fits and upon closer study of the literature, some of the $s\ell$ combinations for the $p\rho$ waves seemed extraneous.

J^P	M	Isobars	# of waves
$\frac{1}{2}^+$	$\frac{1}{2}$	$\Delta \pi$	2
$\frac{1}{2}^{-}$	$\frac{1}{2}$	$\Delta \pi$	2
	$\frac{1}{2}$	$(p\rho)_{(s=1/2;\ell=0)}$	1
$\frac{3}{2}^{+}$	$\frac{1}{2}, \frac{3}{2}$	$(\Delta \pi)_{(\ell=1)}$	4
	$\frac{1}{2}, \frac{3}{2}$	$(p\rho)_{(s=1/2;\ell=1)}$	2
$\frac{3}{2}^{-}$	$\frac{1}{2}, \frac{3}{2}$	$(\Delta \pi)_{(\ell=0,2)}$	8
	$\frac{1}{2}, \frac{3}{2}$	$(p\rho)_{(s=3/2;\ell=0,2)}$	4
$\frac{5}{2}^{+}$	$\frac{1}{2}, \frac{3}{2}$	$(\Delta \pi)_{(\ell=1)}$	4
	$\frac{1}{2}, \frac{3}{2}$	$p\sigma$	2
$\frac{5}{2}^{-}$	$\frac{1}{2}, \frac{3}{2}$	$(\Delta \pi)_{(\ell=2)}$	4
$t\text{-channel}\ \rho$	$\frac{1}{2}, \frac{3}{2}$	$\lambda_{\rho} = \frac{1}{2}, \frac{1}{2}$	4
Tota	37		

Table 6.6: Fit 125

This is a tracking fit where I tracked outward from about 1.85 GeV/c^2 . The starting point was the best liklihood of 20 fits.

6.2 Total cross section

The reaction $\gamma p \rightarrow p \pi^+ \pi^-$ has been studied in other experiments. In 1968, the ABBHHM collaboration used an 85 cm bubble chamber and a photon beam with energies up to 5.8 GeV [?]. In 1966, the Cambridge Bubble Chamber Group used a 12 inch hydrogen chamber with a photon beam with energies up to 6.0 GeV [?, ?]. Both of these experiments had excellent acceptance but not a lot of events. The CBC group analyzed 3900 events with a photon energy from 0.5-2.5 GeV, the same energy range I am analyzing. The ABBHHM collaboration analyzed 16533 events over this same energy range. In 1994, the DAPHNE detector at Mainz was used to to make measurements of this reaction, but only for photon energies from 0.4-0.8 GeV ($W = 1.34 - 1.54 \text{ Gev}/c^2$).

Each of these three experiments was able to calculate a total cross section which I show in Fig. 6.1.



Figure 6.1: Total cross section for $\gamma p \rightarrow p\pi^+\pi^-$ as measured by three different experiments: ABBHHM, CBC and DAPHNE. Note the scale on the *x*-axis has changed in the plot on the right to show the improved resolution of the DAPHNE experiment.

The results of our fits allow us to acceptance correct our data and calculate a total cross section. Comparing our results to other experiments will give us a reference point for our fits. We have better statistics and resolution that the ABBHHM or CBC experiments and a greater range of energy than the more recent DAPHNE measurement, so we should be able to improve on these results.

6.3 $W < 1.8 \text{ GeV/c}^2$

In Chapter 5, we have discussed how we can weight the events in our accepted Monte Carlo and compare this to the CLAS data and use this as a diagnostic of how well the fit is describing the physics.

For the six fits I will show the comparison for four kinematic variables: the invariant mass of the $p\pi^-$ and $\pi^+\pi^-$ systems, the $\pi^- \cos(\theta)$ in the center-of-mass, and the $\pi^+ \cos(\theta)$ in the Δ^{++} helicity frame. I could show others, but I focus on these four as they are sensitive to the fits. I will show these for two different W-bins at 1.46 and 1.69 GeV/c².

As a reference, Fig. 6.2 shows the schematic of which plots are from which fit. They can be thought of in increasing physics as you go down.

Fit 132	Fit 132	Fit 132	Fit 132
Fit 163	Fit 163	Fit 163	Fit 163
Fit 947	Fit 947	Fit 947	Fit 947
Fit 121	Fit 121	Fit 121	Fit 121
Fit 053	Fit 053	Fit 053	Fit 053
Fit 125	Fit 125	Fit 125	Fit 125

Figure 6.2: Schematic of how the plots are laid out for which fit in the following pages.



Figure 6.3: Comparison of kinematic variables for different fits. The shaded region is CLAS data, the solid red line which is the same for each fit is accepted Monte Carlo data, and the dotted blue line is the accepted Monte Carlo data weighted by the fit results. $W \approx 1.47 \text{ GeV/c}^2$

In Fig. 6.3 we see that even fit 132, with very few waves, does a reasonable job fitting the data. However, the fit results differ very little from accepted phase space. In this lower energy region we are dominated by the acceptance of the detector and if we refer to the ????? plots of the Data chapter, we remind ourselves that we have heavily cut into the available phase space for this reaction by requiring all three particles. It still misses noticibly in the π^- angle around $\cos(\theta) = -0.2$ and in the helicity angle. But by the next fit, 163, the agreement is excellent and does not vary with increasing wave set.

One thing to point out is the consistent disagreement in the $\pi^{-} \cos(\theta)$ angle at $\cos(\theta) = 0.0$. The center-of-mass angle maps directly onto $\cos(\theta)$ in the lab frame and because this disagreement is so consistent from fit to fit it is likely the result of incorrect modeling in the gsim simulation. Perhaps an inefficient wire region or inefficient TOF paddle that was not perfectly modeled.



Figure 6.4: Comparison of kinematic variables for different fits. The shaded region is CLAS data, the solid red line which is the same for each fit is accepted Monte Carlo data, and the dotted blue line is the accepted Monte Carlo data weighted by the fit results. $W \approx 1.69 \text{ GeV/c}^2$

In this region the difference between our data and the flat Monte Carlo is bigger. Fit 132 does not do a good job at all and even fit 163 has a difficult time getting the $\pi^+\pi^-$ mass and both angles correct. All the fits have a discrepancy in the helicity angle at $\cos(\theta) > 0.5$.

6.3.3 Total cross section calculations

As a reference I acceptance correct my data using a flat correction and calculate the cross section. The correction is performed in each W bin by integrating over all variables and just getting a correction factor from the number of accepted Monte Carlo events divided by the number of raw Monte Carlo events. This cross section is shown in Fig. 6.5. The cross section is lower than previous measurements at W < 1.5GeV/c². and W > 1.7 GeV/c². But reasonably close for a simple correction.

Fig. 6.6 has more physics, but the agreement actually gets worse for these fits.

Fig. 6.7 shows the fits with >30 waves and the agreement seems to be good. Perhaps still low below $W < 1.5 \text{ GeV/c}^2$, but within error bars.



Figure 6.5: Total cross section. This PWA fit contains only a flat phase space correction.



Figure 6.6: Total cross section. These fits (132 and 163) contain 4 and 16 waves respectively and are considered to only have a minimal amount of physics in them.



Figure 6.7: Total cross section. These fits all contain >30 waves.

In an effort to understand how a flat acceptance correction is better than a fit with 31 parameters I take a look at the raw Monte Carlo weighted by the fits for the same kinematic variables, fits and W-bins that I did for the data. These plots are the physics that the fit has found for these bins: the acceptance corrected data. Looking at the lower bin in Fig. 6.8 I see that for fit 132, it has started pushing the π^- forward and backward slightly in the C.M. frame. In fit 163 this is even more pronounced but it has also depleted the region around $\cos(\theta) = 0.0$. Our interpretation is that the flat acceptance correction just happened to over estimate in some regions of the detector and underestimate others to get the correct cross section. For this bin and the higher bin shown in Fig. 6.9 the acceptance corrected data seems to approach a stable distribution as we add more parameters. This is important as one goal of this analysis is to be able to provide acceptance corrected distributions to the physics community. While we may not always be able to interpret the individual waves, it is good that we have demonstrated that the different fits converge to a common acceptance correction.



Figure 6.8: Raw MC. Comparison of kinematic variables for different fits. The shaded region is raw MC weighted by the fit results (acceptance corrected data) and the solid red line which is the same for each fit is raw Monte Carlo data. $W \approx 1.47$ GeV/c^2



Figure 6.9: Raw MC. Comparison of kinematic variables for different fits. The shaded region is raw MC weighted by the fit results (acceptance corrected data) and the solid red line which is the same for each fit is raw Monte Carlo data. $W \approx 1.69$ GeV/c^2

6.3.4 Individual waves

The results of the fits allow us to calculate the intensity distributions for individual waves or sum over any combination of these waves. For example, Fig. 6.10 shows the contributions of the individual waves for fit 132. Here I have plotted them all on the same scale. The strongest wave in the lowest W is the $\Delta^{++}\pi^{-}$ $(M = \frac{3}{2})$ wave. In later fits we we will see that this is always the case and is consistent with the dominance of the Born contact term.



Figure 6.10: Cross section for individual waves from fit 132.

In the following sections I will show some individual waves for the $\frac{3}{2}^{-}$, $\frac{3}{2}^{+}$ and $\frac{5}{2}^{+}$ quantum numbers and offer interpretations of the intensity distributions. For most of the waves I will focus on four fits: 947, 121, 053, 125. These fits all had >30 waves giving the fits significant freedom to describe the data. When applicable, I show a wave from fit 163, where there were much fewer waves, but a smoother motion.

To orient ourselves Fig. 6.11 shows how the waves will be consistently displayed in the following figures. Note that the labels for each wave start with some number: 1), 2), 11), etc. These were the numbers of the wave in that particular fit. Because there were different numbers of waves, these numbers may not be consistent. The scales may also be different within a group of plots.



Figure 6.11: Schematic of how the plots are laid out for which fit in the following pages.

6.3.4.1 $\frac{3}{2}^{-}$

The motivation for including the $\frac{3}{2}^{-}$ terms are the Born contact term at lower W, the D₁₃(1520), the D₁₃(1700) and the D₃₃(1700). Fig. 6.12 shows the $\frac{3}{2}^{-}$ decaying to a $\Delta^{++}\pi^{-}$ in an *s*-waves for the four fits we will focus on. Fig. 6.13 shows the same wave from fit 163. All fits seem to be consistent with each other for lower W. This term dominates the cross section in this region and the fact that it shows up primarily in $\Delta^{++}\pi^{-}$ as opposed to $\Delta^{0}\pi^{+}$ is consistent with predictions [?]. I have been unable to find any predictions about the strength of $A_{1/2}$ or $A_{3/2}$. We consistenly see this dominance in our $M = \frac{3}{2}$ term.



Figure 6.12: $\frac{3}{2}^{-}$ waves. The strength in the lower W bins can be interpreted as the Born contact term.



Figure 6.13: Fit 163. $\frac{3}{2}^{-}$ waves. The strength in the lower W bins can be interpreted as the Born contact term.

Figs. 6.14- 6.16 show the $\frac{3}{2}^- \Delta \pi$ S-waves in the two M states. The $\Delta^{++}\pi^-$ waves in Fig. 6.14 show an enhancement between 1.6 and 1.7 GeV/c². The $\Delta^0 pi^+$ waves in Figs. 6.15- 6.16 exhibit an enhancement between 1.5 and 1.6 GeV/c². These can be interpreted as the D₁₃(1700) or D₃₃(1700) and the D₁₃(1520) respectively. The PDG [?] states that previous experiments have constrained the Breit-Wigner

mass for the $D_{13}(1700)$ between 1.65 and 1.75 GeV/c² and the $D_{33}(1700)$ between 1.67 and 1.77 GeV/c², with the pole positions lying 20-40 MeV lower for each. While at first glance these observations may appear to inconsitent with previous experiments, the differences could be due to interference effects between overlapping states. It should also be noted that the $D_{13}(1700)$ is narrower at 100 MeV/c² than the wider $D_{33}(1700)$ at 200 MeV/c². The $D_{33}(1700)$ has also been found to have a higher photocoupling [?].



Figure 6.14: $\frac{3}{2}^- \Delta \pi$ *S*-waves.



Figure 6.15: $\frac{3}{2}^- \Delta \pi$ S-waves.



Figure 6.16: $\frac{3}{2}^- \Delta \pi$ S-waves.



Figs. 6.17- 6.20 show the D-waves. The observed motion is different from the S-waves, but is qualitatively consistent.

Figure 6.17: $\frac{3}{2}^- \Delta \pi$ *D*-waves.



Figure 6.18: $\frac{3}{2}^- \Delta \pi$ *D*-waves.



Figure 6.19: $\frac{3}{2}^- \Delta \pi$ *D*-waves.



Figure 6.20: $\frac{3}{2}^- \Delta \pi$ *D*-waves.

Figs. 6.21- 6.22 show the $p\rho$ decays that I allowed in fits 121 and 125. Note that I have maintained the layout for consistency. Fits 947 and 053 did not have these waves in them. Again we see enhancement around 1.7 GeV/c². Both the states here have been shown to couple to $p\rho$ final states. The D₁₃(1520) has been observed as well in $p\rho$ but we do not see evidence here.



Figure 6.21: $\frac{3}{2}^- p\rho$ waves.



Figure 6.22: $\frac{3}{2}^- p\rho$ waves.

Fig. 6.23 show the previous two waves from fit 163, the smaller fit. The pronounced enhancement at $1.7 \text{ GeV}/c^2$. is consistent with the four other fits.



Figure 6.23: Fit 163. $\frac{3}{2}^{-} p\rho$ waves.

In Figs. 6.24- 6.28 I have summed the $\frac{3}{2}^-$ waves over various subsets.

In Fig. 6.24 the Born contact term dominates the lower W in the $\Delta^{++}\pi^{-}$ waves, the effects of a resonance can be clearly seen above 1.6 GeV/c².

In Fig. 6.25 the $\Delta^0 \pi^+$ waves more clearly show evidence of the D₁₃(1520) and perhaps something at 1.7 GeV/c².



Figure 6.24: $\frac{3}{2}^-$ summed over all $\Delta^{++}\pi^-$ waves.



Figure 6.25: $\frac{3}{2}^{-}$ summed over all $\Delta^{0}\pi^{+}$ waves.



Figure 6.26: $\frac{3}{2}^{-}$ summed over all $\Delta \pi$ waves.

After summing over the $p\rho$ terms Fig. 6.27 shows the peak at 1.7 GeV/c² and perhaps evidence of the D₁₃(1520).



Figure 6.27: $\frac{3}{2}^-$ summed over all $p\rho$ waves.



Figure 6.28: $\frac{3}{2}^{-}$ summed over all waves.

It is a good test of the fit that when we sum over the waves the different fits remain consistent with one another.

6.3.4.2 $\frac{3}{2}^+$

The $\frac{3}{2}^+$ waves are motivated by the P₃₃(1600) and the P₁₃(1720). The $\Delta^{++}\pi^-$ waves in Fig. 6.29 and the $p\rho$ waves in Fig. 6.30 both show a strong enhancement at 1.6 GeV/c².



Figure 6.29: $\frac{3}{2}^+ \Delta \pi$ *P*-waves.



Figure 6.30: $\frac{3}{2}^+ p\rho$ waves.

The other spin-alignments of the $p\rho$ waves, Figs 6.31- 6.34, show more evidence of the P₁₃(1720). Other analysis [?] show this state coupling almost entirely to the $p\rho$ final state. They determined the $p\rho$ spin-alignment to be $\frac{1}{2}$. While we see this state in $s = \frac{3}{2}$ as well, the strongest wave seems to be our $s = \frac{1}{2}$ wave as well.

Recent observations in ep scattering at Jefferson Lab have posited a new N^* at 1.720 GeV/c² ??. This is based on the fact that in order to fit the cross section, they require a state at $\approx 1.7 \text{ GeV/c}^2$ which couples more strongly to $\Delta \pi$ final states than the reported values for the P₁₃(1720). We see no evidence for this coupling. This is not entirely inconsistent with these new observations as we use a real photon beam and they have a virtual photon from the electron scattering.



Figure 6.31: $\frac{3}{2}^+ p\rho$ waves.



 $(\widehat{\mathbf{q}}_{1}, \widehat{\mathbf{q}}_{2}, \widehat{\mathbf{q}}_{1}, \widehat{\mathbf{q}}_{2}, \widehat{\mathbf{q}}_{1}, \widehat{\mathbf{q}}_{2}, \widehat{$

Figure 6.32: $\frac{3}{2}^+ p\rho$ waves.



Figure 6.33: $\frac{3}{2}^+ p\rho$ waves.



Figure 6.34: $\frac{3}{2}^+ p\rho$ waves.

In Figs. 6.35- 6.37 I sum over the charged states for $\Delta \pi$ and then all of the $\Delta \pi$ waves. We see evidence for the D₃₃(1600). Previous measurements have measured the width between 200-400 MeV/c² which is not inconsistent with our observation. We see no obvious evidence for a state at 1.7 GeV/c². It is possible that this state destructively interferes with the D₃₃(1600) and so is not immediately visible in this stage of the analysis. A mass-dependant analysis of this data is necessary to determine this.

The structure in the $\Delta^0 \pi^-$ below 1.5 GeV/c² appears to be anomolous as it does not consistently appear in all four fits.



Figure 6.35: $\frac{3}{2}^+$ summed over $\Delta^{++}\pi^-$ waves.



Figure 6.37: $\frac{3}{2}^+$ summed over $\Delta \pi$ waves.

Fig. 6.38 shows the intensity summed over all $p\rho$ waves. The top two fits (947 and 121) do not have any *t*-channel ρ production and so any evidence of this process is rolled into the *s*-channel waves. The bottom two fits (053 and 125) do have *t*-channel ρ waves and show the show the 1.7 GeV/c² more clearly.



Figure 6.38: $\frac{3}{2}^+$ summed over $p\rho$ waves.



Figure 6.39: $\frac{3}{2}^+$ summed over all waves.

6.3.4.3 $\frac{5}{2}^+$

The $\frac{5}{2}^+$ waves are motivated by the F₁₅(1680). Figs. 6.40, 6.42 show the $\Delta 0\pi^+$ waves for the four fits. Fig. 6.41 shows one of the waves that was allowed in fit 163. All are consistent with a state between 1.6-1.7 GeV/², though perhaps narrower than the PDG value of ≈ 130 MeV [?].



Figure 6.40: $\frac{5}{2}^+ \Delta \pi$ *P*-waves.



Figure 6.41: Fit 163. $\frac{5}{2}^+ \Delta \pi$ *P*-waves.


Figure 6.42: $\frac{5}{2}^+ \Delta \pi$ *P*-waves.

In Figs. 6.30- 6.32 I sum over the charged states for $\Delta \pi$ and then all of the $\Delta \pi$ waves. The F₁₅(1680) is clearly visible in $\Delta^0 \pi^+$, but not in $\Delta^{++}\pi^-$, possibly due to interference effects.



Figure 6.43: $\frac{5}{2}^+$ summed over all $\Delta^{++}\pi^-$ waves.



Figure 6.44: $\frac{5}{2}^+$ summed over all $\Delta^0 \pi^+$ waves.



Figure 6.45: $\frac{5}{2}^+$ summed over all $\Delta \pi$ waves.

Some analysis observed the $F_{15}(1680)$ in $(\sigma N)_D$ decays and so I include this wave in the fits and Fig. 6.46 shows the sum over the two allowed M values. Fits 121 and 125 seem to show a bump around 1.67 GeV/c², while the others do not. These main difference is that these two fits have $\frac{3}{2}^-$ waves going to $p\rho$ while the others do not. It is possible that this freedom allows this $F_{15}(1680)$ decay to show up.



Figure 6.46: $\frac{5}{2}^+$ summed over all $p\sigma$ waves.

When we sum over all the waves in Fig. 6.47, the $F_{15}(1680)$ is still visible, though the shoulder at 1.5 GeV/c² is not understood.



Figure 6.47: $\frac{5}{2}^+$ summed over all $\Delta^{++}\pi^-$ waves.

6.3.5 Unexplained intensities

There were a few waves that showed a consistent enhancement around 1.6 GeV/c^2 . Two waves in $\frac{1}{2}^+$ and one in $\frac{1}{2}^+$. These are shown in Figs 6.48- 6.50. This could be an indication that we are missing some required waves to explain the physics in this region, and so strength is put in these waves in an attempt to fit the data. It is also possible we have too many waves and these waves are destructively interfering with other unnecessary waves. Further study is required.



Figure 6.48: $\frac{1}{2}^+ \Delta \pi$ *P*-waves.



Figure 6.49: $\frac{1}{2}^+ \Delta \pi$ *P*-waves.



Figure 6.50: $\frac{5}{2}^+ \Delta \pi$ *P*-waves.

6.4 $W > 1.8 \text{ GeV}/^2$

Here I begin looking at the fit results for $W > 1.8 \text{ GeV}/^2$. We have learned that we a large amount of waves to describe the data in this region. I will only focus on the fits with >30 waves: 947, 121, 053 and 125.

6.4.1 Total cross section

We approach this section in the same way as the lower W range. First I look at how the fits describe the data. Fig. 6.51 shows the layout of the fits for the pages to follow. I show the kinematic variables for two W bins at $W = 2.00 \text{ GeV/c}^2$ and $W = 2.24 \text{ GeV/c}^2$. This region is more difficult to fit so in addition to the 4 kinematic variables I showed before I show four others as well: invariant mass of the $p\pi^+$ system, the proton and π^+ C.M. $\cos(\theta)$, and the $\pi^- \cos(\theta)$ in the Δ^0 helicity angle. These plots are shown in Figs. 6.52- 6.55.

Fit 947	Fit 947	Fit 947	Fit 947
Fit 121	Fit 121	Fit 121	Fit 121
Fit 053	Fit 053	Fit 053	Fit 053
Fit 125	Fit 125	Fit 125	Fit 125

Figure 6.51: Schematic of how the plots are laid out for which fit in the following pages.



Figure 6.52: Comparison of kinematic variables for different fits. The shaded region is CLAS data, the solid red line which is the same for each fit is accepted Monte Carlo data, and the dotted blue line is the accepted Monte Carlo data weighted by the fit results. $W \approx 2.00 \text{ GeV/c}^2$



Figure 6.53: Comparison of kinematic variables for different fits. The shaded region is CLAS data, the solid red line which is the same for each fit is accepted Monte Carlo data, and the dotted blue line is the accepted Monte Carlo data weighted by the fit results. $W \approx 2.00 \text{ GeV/c}^2$



Figure 6.54: Comparison of kinematic variables for different fits. The shaded region is CLAS data, the solid red line which is the same for each fit is accepted Monte Carlo data, and the dotted blue line is the accepted Monte Carlo data weighted by the fit results. $W \approx 2.24 \text{ GeV/c}^2$



Figure 6.55: Comparison of kinematic variables for different fits. The shaded region is CLAS data, the solid red line which is the same for each fit is accepted Monte Carlo data, and the dotted blue line is the accepted Monte Carlo data weighted by the fit results. $W \approx 2.24 \text{ GeV/c}^2$

Fits 947 and 121, which do not contain any t-channel terms fit the data poorly in this region. Fits 053 and 125, which do contain t-channel terms, do a much better job, though still not perfect. The disagreement is most noticable in the C.M. angles. Even with the disagreement in these angles the total cross section calculations shown in Fig. 6.56 demonstrate the agreement with previous calculations is much better when we do include t-channel terms.



Figure 6.56: $W \approx 2.24 \text{ GeV/c}^2$

This is also reflected in the acceptance corrected distributions shown in Figs.6.57-6.60. In the lower W bins we found that the distributions converged to a common shape with >30 waves, regardless of the inclusion of t-channel terms. Here we see



that these terms are necessary to get a stable shape.

Figure 6.57: Raw MC. Comparison of kinematic variables for different fits. The shaded region is raw MC weighted by the fit results (acceptance corrected data) and the solid red line which is the same for each fit is raw Monte Carlo data. $W \approx 2.00$ GeV/c^2



Figure 6.58: Raw MC. Comparison of kinematic variables for different fits. The shaded region is raw MC weighted by the fit results (acceptance corrected data) and the solid red line which is the same for each fit is raw Monte Carlo data. $W \approx 2.00$ GeV/c^2



Figure 6.59: Raw MC. Comparison of kinematic variables for different fits. The shaded region is raw MC weighted by the fit results (acceptance corrected data) and the solid red line which is the same for each fit is raw Monte Carlo data. $W \approx 2.24$ GeV/c^2



Figure 6.60: Raw MC. Comparison of kinematic variables for different fits. The shaded region is raw MC weighted by the fit results (acceptance corrected data) and the solid red line which is the same for each fit is raw Monte Carlo data. $W \approx 2.24$ GeV/c^2

6.4.2 Summed intensities

The results of these fits allow me to calulate cross sections for the different isobars. I show these for the larger wave sets. Fig. 6.61 shows the layout I will use for the plots which follow.



Figure 6.61: Schematic of how the plots are laid out for which fit in the following pages.

Figs. 6.62- 6.63 show my calculated cross sections for $\gamma p \to \Delta^{++}\pi^-$ and $\gamma p \to \Delta^0 \pi^+$ respectively. The dip in the $\Delta^{++}\pi^-$ cross section at 1.5 GeV/² is at the same spot as the enhancement in the $\Delta^0 \pi^+$ cross section. The combined cross section for both charge states is shown in Fig. 6.64. All these plots are consistent between the fits.



Figure 6.62: σ for $\gamma p \rightarrow \Delta^{++} \pi^{-}$



Figure 6.63: σ for $\gamma p \rightarrow \Delta^0 \pi^+$



Figure 6.64: σ for $\gamma p \rightarrow \Delta \pi$

Figs. 6.65-6.66 show the cross section for $\gamma p \rightarrow p\rho$ for the *s*-channel and *t*-channel processes respectively, reminding ourselves that only fits 053 and 125 contained *t*-channel terms. The *s*-channel waves differ when these terms are included



Figure 6.65: σ for $\gamma p \to p \rho$ s-channel waves



Figure 6.66: σ for $\gamma p \rightarrow p\rho$ t-channel waves



Figure 6.67: σ for $\gamma p \rightarrow p\rho$

6.5 $\frac{7}{2}$ and higher terms

6.5.1 Fit 080

During the course of this analysis, some very large wave sets were tried in the fits. The intensites of the individual waves became difficult to interpred due to jumpy waves and very large error bars. The fits also took on the order of a day. However we tried a few with a large number to see if it improved the agreement or changed the summed intensities.

Fit 080 is fit 947 with some waves added. I added all possible $\Delta \pi$ decays for $\frac{7}{2}^+$ and $\frac{7}{2}^-$ waves. I also added *t*-channel ρ,σ and π^- production with a variety of baryons produced in the bottom vertex. A summary of the waves is given in Table 6.7. All *t*-channel terms used $\beta = 6.0$.

J^P	M	Isobars	# of waves
# of waves fit 947			35
$\frac{7}{2}^{+}$	$\frac{1}{2}, \frac{3}{2}$	$(\Delta \pi)_{(\ell=1,5)}$	8
$\frac{7}{2}^{-}$	$\frac{1}{2}, \frac{3}{2}$	$(\Delta \pi)_{(\ell=2,4)}$	8
$t\text{-channel}\ \rho$	$\frac{1}{2}, \frac{3}{2}$	$\lambda_{\rho} = \pm 1.0$	6
$t\text{-channel}\ \sigma$	$\frac{1}{2}, \frac{3}{2}$		2
t-channel π^-	$\frac{1}{2}, \frac{3}{2}$	Δ^{++}	2
π^-	$\frac{1}{2}, \frac{3}{2}$	$N^{*}(1440)$	2
π^-	$\frac{1}{2}, \frac{3}{2}$	$N^{*}(1520)$	2
<i>t</i> -channel π^+	$\frac{1}{2}, \frac{3}{2}$	Δ^0	2
Total $\#$ of waves			67

Table 6.7: Fit 080

In the following sections I show some of the same quantities for this fit that I did for the previous fits. I will compare to fit 125, which seemed to be the best fit of the previous fits. First I look at how well the fit describes the data in the same W bins for the higher region, $W = 2.00 \text{ GeV}/^2$ and $W = 2.24 \text{ GeV}/^2$. Fig. 6.68 shows the layout for these plots.

Fit 125	Fit 125	Fit 125	Fit 125
Fit 080	Fit 080	Fit 080	Fit 080

Figure 6.68: Schematic of how the plots are laid out for which fit in the following pages.

Figs. 6.69- 6.72 show the comparison of the data and the fit results for these eight variables in two higher mass W bins. Fit 080 does a better job, especially in the agreement between the angles.



Figure 6.69: Comparison of kinematic variables for different fits. The shaded region is CLAS data, the solid red line which is the same for each fit is accepted Monte Carlo data, and the dotted blue line is the accepted Monte Carlo data weighted by the fit results. $W \approx 2.00 \text{ GeV/c}^2$



Figure 6.70: Comparison of kinematic variables for different fits. The shaded region is CLAS data, the solid red line which is the same for each fit is accepted Monte Carlo data, and the dotted blue line is the accepted Monte Carlo data weighted by the fit results. $W \approx 2.00 \text{ GeV/c}^2$



Figure 6.71: Comparison of kinematic variables for different fits. The shaded region is CLAS data, the solid red line which is the same for each fit is accepted Monte Carlo data, and the dotted blue line is the accepted Monte Carlo data weighted by the fit results. $W \approx 2.24 \text{ GeV/c}^2$



Figure 6.72: Comparison of kinematic variables for different fits. The shaded region is CLAS data, the solid red line which is the same for each fit is accepted Monte Carlo data, and the dotted blue line is the accepted Monte Carlo data weighted by the fit results. $W \approx 2.24 \text{ GeV/c}^2$

The total cross section is shown for these two fits in Fig. 6.73 and is fairly consistent above $W = 1.7 \text{ GeV/c}^2$. Below this value, it differs from our previous calculations, perhaps as a result of ambiguities in the waves which cannot be resolved in the lower mass.



Figure 6.73: Total cross sections for fit 125 and 080.

The acceptance corrected data distributions are shown in Figs. 6.74- 6.77. The distributions seem much the same except the C.M. angles are noticibly different. The π^- and π^+ have a shoulder in the forward region that is indicative of the competing ρ and π t-channel processes.



Figure 6.74: Raw MC. Comparison of kinematic variables for different fits. The shaded region is raw MC weighted by the fit results (acceptance corrected data) and the solid red line which is the same for each fit is raw Monte Carlo data. $W \approx 2.00$ GeV/c^2



Figure 6.75: Raw MC. Comparison of kinematic variables for different fits. The shaded region is raw MC weighted by the fit results (acceptance corrected data) and the solid red line which is the same for each fit is raw Monte Carlo data. $W \approx 2.00$ GeV/c^2



Figure 6.76: Raw MC. Comparison of kinematic variables for different fits. The shaded region is raw MC weighted by the fit results (acceptance corrected data) and the solid red line which is the same for each fit is raw Monte Carlo data. $W \approx 2.24$ GeV/c^2



Figure 6.77: Raw MC. Comparison of kinematic variables for different fits. The shaded region is raw MC weighted by the fit results (acceptance corrected data) and the solid red line which is the same for each fit is raw Monte Carlo data. $W \approx 2.24$ GeV/c^2

In the end this fit perhaps shows us the difficulty of exactly describing the higher energy region with the 30 waves we were using. But qualitatively there is not a great difference when we add this many new higher order terms. The individual wave intensities are rendered meaningless however.

CHAPTER 7 Results and conclusions

In this chapter I summarize our results. All plots shown here are from fit 125. See Table 6.6.

7.1 Total cross section

We calculate the total cross section for $\gamma p \rightarrow p\pi^+\pi^-$. This is shown in Fig. 7.1. The error bars are indicative of the shape of the liklihood function and are some convolution of the CLAS resolution, statistics and uncertainty in the wave selection. Other fits give the same cross section to within 1-4%.



Figure 7.1: Fit 125. Total cross section.

7.2 Cross section for isobars

We are able to pull out the cross section for the isobars we use. These are shown in Figs. 7.2-7.7. The $\gamma p \to p\rho$ cross section shown in Fig. 7.6 is summed over both *s*- and *t*-channel processes. In Fig. 7.3 we compare our results for the $\Delta^{++}\pi^{-}$ cross section with values extracted by the Cambridge Bubble chamber analysis [?]. Our results disagree with previous measurements, but have less model dependence.


Figure 7.2: Fit 125. Cross section for $\gamma p \rightarrow \Delta^{++} \pi^{-}$



Figure 7.3: Fit 125. Cross section for $\gamma p \rightarrow \Delta^{++}\pi^{-}$. Compare with CEA results.



Figure 7.4: Fit 125. Cross section for $\gamma p \rightarrow \Delta^0 \pi^+$



Figure 7.5: Fit 125. Cross section for $\gamma p \rightarrow \Delta \pi$



Figure 7.6: Fit 125. Cross section for $\gamma p \rightarrow p\rho$, including both *s*-channel and *t*-channel contributions.



Figure 7.7: Fit 125. Cross section for $\gamma p \rightarrow p\rho$. Compare with ABBHHM results.

Fig. 7.7 shows our $p\rho$ cross section plotted with results from the ABBHHM analysis [?]. Three different methods were used to extract their results and are all plotted. Our mass independent fit vastly improves upon the previous results.

7.3 Strengths of individual waves

We find the dominant wave below 1.5 GeV/c² to be the $\frac{3}{2}^- \Delta^{++}\pi^-$ in an *S*-wave. We make the new measurement that this is strongest in an $M = \frac{3}{2}$ state. This is shown in Fig. 7.8.



Figure 7.8: Fit 125. $\frac{3}{2}^{-}$ waves. The strength in the lower W bins can be interpreted as the Born contact term.

The $D_{13}(1520)$ is seen, as shown in Fig. 7.9. A peak is seen around 1.7 GeV/c² consistent with the $D_{13}(1700)$ or $D_{33}(1700)$ in Fig. 7.10.



Figure 7.9: Fit 125. $\frac{3}{2}^- \Delta \pi$ S-waves.



Figure 7.10: Fit 125. $\frac{3}{2}^- p\rho$ waves.

Evidence for the $P_{33}(1600)$ is shown in Fig. 7.11. Evidence for the $P_{13}(1720)$ is shown in Fig. 7.12 with all strength coming from $p\rho$ in support of the PDG values.



Figure 7.11: Fit 125. $\frac{3}{2}^+ \Delta \pi$ *P*-waves.



Figure 7.12: Fit 125. $\frac{3}{2}^+$ $p\rho$ waves.

A peak is seen in $\frac{5}{2}^+$ consistent with the $F_{15}(1680)$ and is shown in Fig. 7.18.



Figure 7.13: Fit 125. $\frac{5}{2}^+ \Delta \pi$ *P*-waves.

A full mass dependant fit is required to extract the parameters of these resonances and establish the isospin. However, we can do a simple fit to a Breit-Wigner and look at the phase difference.



Figure 7.14: Fit 125. Intensity of a $\frac{5}{2}^+$, a $\frac{3}{2}^+$ and the phase difference between the amplitudes.

Fig. 7.14 shows a very simple fit. I fit a Breit-Wigner to the $\frac{5}{2}^+$ wave and $\frac{3}{2}^+$ wave. I show the results of the fit overlaid on the intensities and the results of the fit plotted on the phase difference for these waves. The phase difference *is not* a constraint in the fit.

The fit returns a mass and width for the $\frac{5}{2}^+$ wave of 1650 MeV/c² and 115 MeV/c². The PDG value for the width of the F₁₅(1680) is 130 MeV/c². For the $\frac{3}{2}^+$ I get 1770 MeV/c² and 85 MeV/c². The width of the P₁₃(1720) is commonly given as 150 MeV/c².



Figure 7.15: Fit 125. Intensity of a $\frac{5}{2}^+$, a $\frac{3}{2}^+$ and the phase difference between the amplitudes.

Fig. 7.15 shows the same $\frac{5}{2}^+$ with a different $\frac{3}{2}^+$ wave. The fit returns a mass and width of 1580 MeV/c² and 70 MeV/c² for this $\frac{3}{2}^+$. The width of the P₃₃(1600) is much wider at 350 MeV/c², though some analysis have placed it as low as 250 MeV/c².

The phase difference from these fits qualitatively agrees with the data and our masses and widths are close to book values.

7.4 Differential cross sections

We are able to acceptance correct the data and calculate differential cross sections. We have begun providing this data to the community so that there is a standard to compare with theory. A dialogue has already begun with Victor Mokeev at Jefferson Lab. He has begun checking his model against our data set. He calculates the differential cross section in four variables: invariant mass of the $p\pi^+, p\pi^-$ and $\pi^+\pi^-$ systems and the $\pi^- \cos(\theta)$ in the center of mass. One bin of his comparisons is shown in Fig. 7.16



Figure 7.16: Fit 125. $\frac{5^+}{2} \Delta \pi$ *P*-waves.

7.5 Missing resonances

We see no evidence for the a P₁₃ state at 1.7 GeV/c² which strongly couples to $\Delta \pi$ final states, as reported by Ripani, et al [?].

There is strength in some of the $\frac{3}{2}^+$ waves around 2.0 GeV/c². The PDG lists the P₁₃(1900), a 2-star state, and the P₃₃(1940), a 3-star state. It is possible that this is the $\frac{3}{2}^+$ state at 2030 predicted by Capstick and Isgur [?] but the data is inconclusive at best and a full mass dependent fit is necessary to extract this information.



Figure 7.17: Fit 125. $\frac{3}{2}^+ \Delta \pi$ *P*-waves.



Figure 7.18: Fit 125. $\frac{3}{2}^+ \Delta \pi$ *P*-waves.

The mass dependant fit will also help characterize other resonances which are not part of the positive parity missing baryons, but simply poorly understood. Fig. 7.19 shows strength in the $\frac{3}{2}^{-}$ wave around 1.9 GeV/c². There is a D₃₃(1940) and a D₁₃(2080) which are 1- and 2-star states respectively [?].



Figure 7.19: Fit 125. $\frac{3}{2}^- \Delta \pi$ *P*-waves.

7.6 Future work

The mass independant partial wave decomposition holds great promise. A mass dependant fit using intensity and phase difference is the next step in this analysis. It may be possible to further constrain the fits by analyzing the data sets where one of the final state particles is not detected in the CLAS, but reconstructed from missing mass. This will allow us to double check our differential cross sections as we make this data available to the scientific community.

APPENDIX A Data

A.1 Invariant 2-body masses

A.1.1 $p\pi^+$



Figure A.1: Different W bins.

A.1.2 $p\pi^{-}$



Figure A.2: Different W bins.

A.1.3 $\pi^+\pi^-$



Figure A.3: Different W bins.

A.2.1 proton $cos(\theta)$



Figure A.4: Different W bins.



Figure A.5: Different W bins.





Figure A.6: Different W bins.

A.3 Dalitz plot



Figure A.7: Different W bins.