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Quantum Chromodynamics and Colour Singlet Exchange in High Energy Interactions

BY

RIKARD ENBERG



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Abstract

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Quantum chromodynamics (QCD) is the fundamental theory in elementary particle physics that describes the strong interaction in terms of exchanges of force-carrying, *colour*-charged particles known as gluons. Although well-established through experimental verifications, there are fundamental unsolved problems in the theory.

In this thesis, some novel aspects of strong interaction dynamics are studied in the context of *colour singlet exchange* processes — interactions where complex systems of gluons with no net colour charge are exchanged. Both perturbative and non-perturbative QCD methods are used, as well as Monte Carlo computer simulations.

Soft colour interactions in the final state of a high energy collision can lead to effective colour singlet exchange. Non-perturbative models for such interactions are shown to give a good description of diffractive production of W, Z, $b\bar{b}$, J/ψ and jets in $p\bar{p}$ collisions at the Tevatron. Predictions are given for diffractive Higgs boson and prompt photon production at hadron colliders.

Rapidity gaps between jets is a new phenomenon which is studied with an improved perturbative calculation of hard colour singlet exchange using the BFKL equation, taking into account previously neglected contributions and non-leading logarithmic corrections. Including also underlying soft rescattering effects, the complete model reproduces well data from the Tevatron.

Diffractive vector meson production through hard colour singlet exchange in γp collisions is studied in the framework of the conformal invariant non-forward solution of the BFKL equation. Expressions for helicity-dependent amplitudes are derived, and the results show good agreement with data on J/ψ and ρ production from the ep collider HERA.

These studies lead to a deeper knowledge of complex gluon dynamics, and therefore advance our understanding of QCD.

Rikard Enberg, Department of Radiation Sciences, Uppsala University, Box 535, SE-751 21 Uppsala, Sweden

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It is a capital mistake
to theorize before one has data.
Insensibly one begins to
twist facts to suit theories,
instead of theories to suit facts.

Sherlock Holmes

He who loves practice without theory
is like the sailor who boards ship
without a rudder and compass
and never knows where he may cast.

Leonardo da Vinci

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I *Rapidity gaps at HERA and the Tevatron from soft colour exchanges*
R. Enberg, G. Ingelman and N. Tîmneanu
Journal of Physics G **26** (2000) 712, `hep-ph/0001016`

II *Soft color interactions and diffractive hard scattering at the Fermilab Tevatron*
R. Enberg, G. Ingelman and N. Tîmneanu
Physical Review D **64** (2001) 114015, `hep-ph/0106246`

III *Diffractive Higgs Boson Production at the Fermilab Tevatron and the CERN Large Hadron Collider*
R. Enberg, G. Ingelman, A. Kissavos and N. Tîmneanu
Physical Review Letters **89** (2002) 081801, `hep-ph/0203267`

IV *Diffractive Higgs bosons and prompt photons at hadron colliders*
R. Enberg, G. Ingelman and N. Tîmneanu
Physical Review D **67** (2003) 011301, `hep-ph/0210408`

V *Hard colour singlet exchange at the Tevatron from the BFKL equation*
R. Enberg, L. Motyka and G. Ingelman
In proc. DIS 2001, World Scientific, p. 820, `hep-ph/0106323`

VI *HARDCOL: Hard Colour Singlet exchange processes in PYTHIA*
R. Enberg
TSL/ISV–2003–0268, http://www3.tsl.uu.se/thep/hardcol/

VII *Hard colour singlet exchange and gaps between jets at the Tevatron*
R. Enberg, G. Ingelman and L. Motyka
Physics Letters B **524** (2002) 273, `hep-ph/0111090`

VIII *Diffractive heavy vector meson photoproduction from the BFKL equation*
R. Enberg, L. Motyka and G. Poludniowski
European Physical Journal C **26** (2002) 219, `hep-ph/0207027`

IX *Diffractive vector mesons at large momentum transfer from the BFKL equation*
R. Enberg, L. Motyka and G. Poludniowski
Acta Physica Polonica B **33** (2002) 3511, `hep-ph/0207034`

X *Vector Meson Photoproduction from the BFKL Equation. I: Theory*
R. Enberg, J.R. Forshaw, L. Motyka and G. Poludniowski
In manuscript, TSL/ISV–2003–0269

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Abbreviations and acronyms

BFKL	Balitsky, Fadin, Kuraev and Lipatov *(equation)*
CERN	European Laboratory for Particle Physics Research, originally Conseil Européen pour la Recherche Nucléaire *(accelerator laboratory in Geneva, Switzerland)*
DESY	Deutsches Elektronen Synchrotron *(accelerator laboratory in Hamburg, Germany)*
DGLAP	Dokshitzer, Gribov, Lipatov, Altarelli and Parisi *(equations)*
DIS	Deep Inelastic Scattering
DPE	Double Pomeron Exchange
GAL	Generalised Area Law
HERA	Hadron–Elektron–Ringanlage *(accelerator at DESY)*
LEP	Large Electron Positron Collider *(earlier accelerator at CERN, closed in 2000)*
LHC	Large Hadron Collider *(future accelerator at CERN, to start in 2007)*
LLA	Leading logarithmic approximation
LO	Leading order
NLLA	Next-to-leading logarithmic approximation
NLO	Next-to-leading order
PETRA	Positron–Elektron–Tandem–Ringanlage *(earlier accelerator at DESY)*
pdf	Parton distribution function
pQCD	Perturbative QCD
QCD	Quantum Chromodynamics
QED	Quantum Electrodynamics
SCI	Soft Colour Interaction
SLAC	Stanford Linear Accelerator Center *(accelerator laboratory in Stanford, California)*
SM	Standard Model
SU	Special Unitary

Preface

The fundamental theory that describes the interactions of the quarks and gluons building up the proton — and that ultimately describes the inner structure of the world — is firmly established, but is nevertheless far from being a closed case. There are still many challenging questions and problems. The theory is known as Quantum chromodynamics, but is often referred to by the acronym "QCD", and this is the subject of this thesis.

Ongoing research in QCD no longer primarily has the goal of testing the theory to see if it is relevant for the description of nature — this has already been achieved with tremendous success through the application of so-called perturbation theory and comparison with experiments. Rather, we now want to *understand* the theory and the wealth of non-trivial phenomena it can accommodate. Furthermore, we want to test *how well* we understand it. In order to do that, we have to be able to do calculations with the theory to get a quantitative understanding and not be content with a qualitative picture of the physical phenomena. But we are only able to calculate things in certain types of problems. In many situations the well-known standard methods of calculations do not work. This is especially true in cases when the above-mentioned perturbation theory is not applicable, in the realm of what is called "non-perturbative QCD". The most important unsolved problem of QCD, the problem of *confinement*, belongs to this category.

This thesis is mainly about QCD phenomenology. The jargon word "phenomenology" means that it is a theoretical study of QCD in close connection to experiment, and the subject is sometimes referred to as "collider physics". This means that one takes information from experi-

mental studies, formulates new models or theories, and develops methods to calculate quantities that can be compared to existing or future data.

All this will be explained and elaborated on in this thesis, but I will start in the Introduction with a brief overview, for the non-expert, of modern elementary particle physics and a look at the developments that lead to it. Then, in Chapter 2, the Standard Model of particle physics will be described, focusing on its theoretical structure. I will also review the phenomenological side; the different types of experiments that are performed, and what we learn from them. This will serve to introduce many of the important concepts used in this thesis. Chapter 3 treats QCD in some detail and can hopefully be used as a short introduction to the subject. Chapter 4 goes into the specific area of research carried out in this thesis, namely *diffraction* and *low-x physics*. Chapter 5 summarises the included papers, and finally in Chapter 6 conclusions are drawn.

Chapter 1

Introduction

Our modern view of the world is that it is built from a small number of fundamental constituents — the elementary particles. The term in itself makes it clear that when we talk about "elementary" particles, we implicitly mean that there is nothing smaller: these constituents cannot be divided into smaller pieces.

This is not a new idea, but what has been considered as the fundamental, elementary objects has changed with time. In modern science, the idea of indivisible building blocks goes back to 1803 and the atoms of Dalton, and later the periodic system of Mendeleev from 1869, which could explain regularities in the properties of the chemical elements. Today we know that the atoms are themselves composite objects, built out of nuclei and electrons. While the electrons are believed to be truly elementary, the nucleus is built out of protons and neutrons, that in turn are composed of elementary particles called quarks and gluons.

What we know today about the structure of matter is collected into the Standard Model (see e.g. [1]), which describes *what* the world is made of, namely the elementary particles, but it also describes *how* these particles affect each other when they collide or bind together; that is when they *interact*. The Standard Model is a *quantum field theory* and will be described at a technical level in Chapter 2. In the rest of this chapter, I will explain the "what" and the "how" and what "quantum field theory" means.

1.1 The particles

The road from "elementary" atoms to elementary particles required both experimental and theoretical discoveries.[1] One starting point was the discovery of the electron by Thomson in 1897 [2]. He proposed a model, in which the atom is made of some positively charged material with the negatively charged electrons spread evenly throughout. Experiments by Rutherford, Geiger and Marsden in 1911 [3] then led Rutherford to postulate a model in which almost all of the mass of the atom is concentrated in a minute, positively charged nucleus at the centre of the atom, circled by electrons. The size of the nucleus is many orders of magnitude smaller than the size of the atom, roughly 10^{-15} m, while the size of the atom is around 10^{-10} m. Bohr extended the work of Rutherford in his atomic model of 1913 [4], conflating the "planetary" atom with the revolutionary quantum hypotheses of Planck [5] and Einstein [6]. In Bohr's model the electrons revolve around the nucleus, according to the idea that there is only a discrete number of orbits allowed, each with a fixed energy. This brilliant idea could explain atomic spectra measured in laboratory experiments and gave way for quantum mechanics, developed in the 1920's starting with Schrödinger and Heisenberg [7]. The orbiting electrons are here replaced by a cloud of electrons in different "quantum states", corresponding to the different orbits, described by the *wave function* of the atom.

A very important consequence of quantum mechanics is that all particles can alternatively be seen as waves. This also holds for light, which is well known as an electromagnetic wave, but can also be seen as a particle. The particle nature of light was introduced by Einstein [6] following the idea by Planck that light is quantised. The particle of light is known as the *photon*, and was experimentally confirmed by Compton in 1923 [8].

To summarise, at this time the elementary particles building up the world were thought to be electrons and various species of atomic nuclei together with photons.

From the 1930's onward, physicists learnt of several new particles. The neutron, the electrically neutral cousin of the proton, was discovered in 1932, and it was then realised that the atomic nucleus consists of protons and neutrons. The positron, i.e., the positively charged antiparticle of the electron, was also discovered in 1932 but had actually been predicted a few years earlier by Dirac based on theoretical considerations. The neutrino was predicted by Pauli in 1930 but proved very elusive. It was finally discovered in 1956.

[1] For a good summary of the history of particle physics, see http://particleadventure.org/particleadventure/other/history/

Table 1.1: The six quarks of the Standard Model. There are three generations of quarks, with increasing masses, and in each generation there is one "down" type and one "up" type quark with a difference in electric charge of one unit. The charges are given in units of the fundamental charge e. All quarks have spin 1/2. In addition, each quark carries a colour charge (red, green or blue).

Charge	Generation		
	I	II	III
$+\frac{2}{3}$	u up	c charm	t top
$-\frac{1}{3}$	d down	s strange	b bottom

But this was not the end; in the following decades an enormous amount of "elementary" particles were discovered. There was the muon, the heavier version of the electron; there were the π mesons; and there were the antiparticles of the proton and the neutron. Many more particles, collectively known as *hadrons*, were produced in accelerator experiments in the fifties and sixties. But this begged the question: why these hundreds of particles? The situation did not look as simple and beautiful as people traditionally wanted physical theories — where was the guiding principle?

The answer came with the emergence of the quark model introduced independently by Gell-Mann and Zweig in 1964 [9]: the properties of this multitude of hadrons (to which the protons, neutrons and π mesons belong) can be described by a mathematical tool known as group theory, if one assumes that they are built up of smaller particles (quarks) — either three quarks or one quark and one *antiquark*. All known hadrons have charges that are integer multiples of the charge of the electron, so the quarks must have charges that are thirds of the electron charge. In addition they carry another type of charge called *colour*, which can take three different values: red, green or blue. There are six different quarks in nature, grouped into three generations, see Table 1.1.

More evidence for quarks in the proton came from a very different direction. In the end of the sixties, experiments were performed at the Stanford Linear Accelerator Center (SLAC) in California where protons were bombarded by a beam of electrons at high energy. The results of these experiments [10] were interpreted by Bjorken, Paschos and Feynman [11] as collisions of the electrons with particles *inside* the proton, dubbed *partons* by Feynman. Careful analyses showed that these partons had to have charges which were thirds of the proton charge, and that they had to have spin 1/2. Of course the temptation to identify the

Table 1.2: The six leptons of the Standard Model. Just as for the quarks, there are three generations of leptons. In each generation there is one electrically charged lepton with unit charge and one neutral neutrino, both with spin 1/2. The charges are given in units of the fundamental charge e.

Charge	Generation		
	I	II	III
-1	e^- electron	μ^- muon	τ^- tau lepton
0	ν_e electron neutrino	ν_μ muon neutrino	ν_τ tau neutrino

partons with the quarks was irresistible, and this turned out to be the correct thing to do.

The other particles, the electron, muon, tau lepton and neutrinos all belong to another group of six particles referred to as *leptons*, see Table 1.2. They too come in three families in a similar fashion to the quarks, although they have integer charges. The leptons and the quarks build up all known matter in the universe. They have one more thing in common, namely what is called the spin angular momentum. This is a quantum mechanical property that all particles have, and all of these matter particles have spin 1/2. They therefore belong to a class of particles known as *fermions*, i.e., particles with half-integer spin. A quick look at Tables 1.1 and 1.2 shows that there is a remarkable symmetry between the quark and lepton "sectors" of the Standard Model.

The remaining particles in the Standard Model, such as the photon, belong to the other class of particles known as *bosons*, i.e., particles with integer spin. These particles are not "matter" particles, but have to do with the forces between particles, as will be described in the next section.

1.2 The interactions

To have a useful theory of the world, we do not only need to know what objects the theory contains, but also how these objects affect each other. That is, what happens if, for example, two electrons collide, or if we place a quark close to an antiquark? Forces between particles have since long been described by *fields*. A field is simply a physical quantity that has a certain value at each point in space, the most well-known examples being the electric and magnetic fields. Just as the electric charge of an electron is the source of an electric field, all other fields we consider have other kinds of charges as sources, charges being carried by different types of particles.

In quantum field theory, all *particles* are represented by fields. The electromagnetic field of Maxwell's equations is quantised, meaning that it can have fluctuations which only take certain discrete values. These fluctuations, or *quanta*, correspond to photons, so the field also describes particles. Field theory describes both the particle and wave nature of the photons, thus unifying the two descriptions. Electrons are also quanta of a field, the electron field, that specifies the properties of the electron.

In the Standard Model all interactions between different particles are described by quantum fields. The interaction of two particles is mediated by other, force carrying, particles exchanged between the particles. Because the force between electrons is described by the electromagnetic field, the interaction takes place by exchange of photons — the photons are the force carriers of the electromagnetic interaction. This is frequently depicted in a so-called Feynman diagram, see Fig. 1.1.

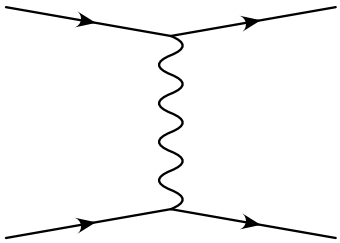


Figure 1.1: A Feynman diagram showing a collision between two electrons, represented by the solid lines, through the exchange of a photon, represented by the wavy line. The diagram is not intended to give a picture of the collision in space, but is rather a schematic drawing, with a precise mathematical meaning related to the theoretical calculation of the process. One can, however, imagine a horizontal time-axis and a vertical space axis.

There are four different forces, or interactions, in nature. The electromagnetic and gravitational interactions are known from daily life and have very long ranges. The electromagnetic force holds together atoms and solid materials and affects everything that has electric charge (or magnetic moments). The gravitational force binds us to the ground and holds together planetary systems and galaxies; it affects everything that has mass or energy.

The other two forces are known as the strong and weak interactions. They have very small ranges, at the order of the size of an atomic nucleus, and are not important in larger systems. But in the world of elementary particles they are extremely important. The strong force binds the quarks together into hadrons such as protons and neutrons. Its exchange particles are called *gluons* and are quite similar to photons. They affect

Table 1.3: The five gauge bosons, or force carriers, of the Standard Model. The photons and gluons are massless while the weak bosons are very heavy, with masses about 80–90 times heavier than the proton. The table shows what charges are carried by the bosons and what charges they couple to, i.e., with what kind of charged particles they interact.

	Electromagnetic interaction	Strong interaction	Weak interaction		
Boson	γ photon	g gluon	Z^0	W^+	W^-
El. charge	0	0	0	$+1$	-1
Other charge	none	colour	weak isospin		
Couples to	el. charge	colour	weak isospin		

everything that has colour charge, which is carried by quarks, and by the gluons themselves. The strong force also keeps the atomic nuclei together. Weak interactions are responsible for nuclear beta decay and for many other nuclear reactions, such as the processes producing energy in the sun. It is mediated by three different exchange particles called Z^0, W^+ and W^- that interact with all particles that have a type of charge known as weak isospin. This includes all the quarks and leptons and also the Z^0 and $W^\pm$ themselves.

The Standard Model accounts for the strong, weak and electromagnetic interactions but does not incorporate gravity, which is the most important force at macroscopic distances. It can, however, be neglected in particle physics. As mentioned above, the exchange particles are all bosons, and are referred to as *gauge bosons* for reasons that will become clear in Chapter 2. Table 1.3 shows the gauge bosons of the Standard Model.

All the particles described above, fermions as well as bosons, have been discovered in experiments. The last particle to be found was the top quark, which was discovered in 1994 [12]. There is one complication with the quarks and gluons, however, namely that all particles with colour charge are *confined* in hadrons, i.e., they have to combine with other colour charged particles into colour neutral ones. This phenomenon is referred to as *confinement* and has profound consequences for the physical world. It will be discussed in more detail in later chapters, but here we note that it means that the quarks and gluons cannot be seen directly in experiments like the other particles, rather they have to be inferred indirectly, for example from so-called jets of particles in particle collisions.

But there is one piece missing from the Standard Model. We have the matter particles, quarks and leptons, which are fermions, and we have

the interaction particles, the gauge bosons. The third major component of the Standard Model is the Higgs mechanism [13], which is responsible for giving masses to the other particles. This relies on a field known as the Higgs field, that fills all of space. The particle corresponding to this field is called the *Higgs boson* and is an integral part of the theoretical framework of the Standard Model. Unfortunately it has not been found yet, maybe because it is very heavy. This is one of the most pressing issues in particle physics today, and one which receives a lot of attention, both experimentally and theoretically.

For example, the Large Hadron Collider (LHC) accelerator which is presently being constructed at CERN is to a large part devoted to searching for the Higgs boson, as was the previous accelerator at CERN, the Large Electron Positron Collider (LEP). At LEP, one found an indication of the Higgs particle, but could not claim a discovery [14]. There is also a constant stream of theoretical papers devoted to finding new ways of looking for it.

It should be emphasised that the Standard Model is a fantastically successful theory. It has been thoroughly tested, perhaps more than any other scientific theory, and it has been able to predict a lot of things that have later been confirmed by experiment. There is not a single measurement that contradicts it, although some hints of deviations exist. And yet, physicists believe that there is something beyond it; that there is a yet deeper theory behind it all. The reason is that there are many unresolved questions that cannot be answered by the Standard Model. For example, why are there three generations of quarks and leptons? And why is the charge of the electron exactly the opposite of the charge of the proton, so that atoms are electrically neutral? This has no explanation in our current theories.

Chapter 2

Particle physics & the Standard Model

In this chapter I give a concise overview of the Standard Model of particle physics from a theoretical and phenomenological viewpoint, leaving the historical track of the last chapter. I refer the interested reader to the books in [1] and reviews in [15] for details on its astonishing agreement with experimental measurements and the experiments giving such important clues in its formulation. The theoretical material covered in the first section of this chapter is described in greater detail in the books in [16], and for the second section describing the phenomenology, the book [17] is recommended.

2.1 The Standard Model

The Standard Model is a quantum field theory that describes the interactions between the fermionic quark and lepton fields through the exchange of gauge boson fields. It consists of two main parts: Quantum chromodynamics (QCD) describing the strong interaction, and the Electroweak theory (EW) unifying the description of the electromagnetic and the weak interactions. The gravitational interaction is left out of the description, because it has not yet been possible to successfully formulate a theory of quantum gravity, and furthermore, at present energies in high energy physics it is not important.

There are three important keywords to the Standard Model. It is of course a *quantum* theory, it is a *gauge* theory, and very importantly, it is a *renormalisable* theory. The quantum nature of the theory is the basis of the whole description in terms of a quantum field theory. The gauge

invariance of the theory constrains the terms in the Lagrangian and tells us what kind of interactions there can be. And the renormalisability of the theory guarantees that we can handle divergences that appear when calculating certain observables, and still make well-defined predictions.

2.1.1 Perturbation theory

The standard way of calculating cross sections, decay rates and other observables in a quantum field theory is by perturbation theory. For a theory without interactions, i.e. a free theory, one can find exact solutions to the equations of motion. For example, for the free electron field $\psi(x)$ (or generally any free fermion) we have the Dirac equation [18][1]

$$(i\gamma^\mu\partial_\mu - m)\psi(x) = 0, \tag{2.1}$$

where γ^μ are the Dirac matrices obeying the anti-commutation relation $\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}$, the derivative is $\partial_\mu = \partial/\partial x^\mu$, and m is the fermion mass. Similarly, for the electromagnetic fields we have Maxwell's equations. These equations can be solved exactly separately. However, if we introduce interactions of the electron field with the electromagnetic field A_μ, as will be described below, Eq. (2.1) is modified to

$$(i\gamma^\mu\partial_\mu - m)\psi(x) = -e\gamma^\mu A_\mu(x)\psi(x), \tag{2.2}$$

and because of the interaction term on the right hand side that couples the fields it is no longer possible to solve the equations exactly.

What one can do now is to hope that the interaction term is sufficiently small that it is possible to treat it as a small perturbation of the free-field solution. Indeed, the term is multiplied by the electron charge e, which is related to the fine structure constant, or electromagnetic coupling strength, by $\alpha = e^2/4\pi \simeq 1/137$. In any scattering amplitude involving electrons there will be factors of e, which can be considered as small. There is thus a systematic way of making more and more refined calculations: one makes a power series expansion in α of a calculated observable A,

$$A = A^{(n)}\alpha^n + A^{(n+1)}\alpha^{n+1} + A^{(n+2)}\alpha^{n+2} + \cdots, \tag{2.3}$$

where n is the lowest order contributing in perturbation theory. Then, in the quantised theory there exists a well-defined method of defining the calculations at each order of α to compute the $A^{(n)}$. This is the method of Feynman diagrams, which assigns to each possible interaction and

[1] Throughout this thesis, "natural" units are used, such that $\hbar = c = 1$. This means that masses and momenta are given in GeV, lengths in GeV^{-1}, and e is dimensionless.

partaking field a certain mathematical factor derived from the action of the theory, see Fig. 1.1 for an example. Each such factor is represented graphically by a vertex for an interaction and by a line for a particle. Virtual particles that connect vertices contribute a propagator, derived from the free-field Lagrangian, to the overall expression, while external spin-1/2 fermion lines contribute a spinor factor and vector bosons a polarisation vector.

2.1.2 Renormalisation

At higher orders in the coupling constant, there will be Feynman diagrams containing loops. One such diagram is shown in Fig. 2.1; this is one of the possible next order corrections to the diagram shown in Fig. 1.1.

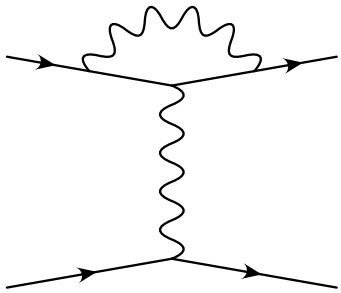


Figure 2.1: Vertex correction to $ee \to ee$ scattering.

Loop diagrams do, however, in general give rise to divergent integrals over momenta in the loops. This is an essential feature of quantum field theory, and was in the beginning taken as evidence that this was not the way to go. But with the advent of renormalisation and its successful application to Quantum Electrodynamics (QED) by Tomonaga, Schwinger and Feynman [19], the belief in field theory was established. Nowadays renormalisation is seen as one of the great successes of theoretical physics, rather than a salvation from annoying divergences. The divergences should not be seen as an undesired feature of the theory; rather they reflect the infinite number of degrees of freedom of quantum field theory. The main point is that the divergences can be absorbed into shifts of the parameters of the theory, i.e., the coupling strengths and masses.

The electron charge used when writing down the Lagrangian of QED is not the physical charge that can be measured in experiments, but the bare charge corresponding to the bare electron–photon vertex. When

higher-order diagrams are included, the vertex gets corrections such as the one in Fig. 2.1. The physical charge should include all those corrections, that is, the bare charge should be "dressed". In a renormalisable theory, it is possible to arrange the calculation so that all divergences can be absorbed into redefinitions of the parameters of the original Lagrangian.

For this purpose the divergent integrals first have to be regularised, for example by a method known as dimensional regularisation [20]. The divergence is isolated and subtracted and the parameters redefined. How this is done is a matter of convention, and several renormalisation schemes exist. In any case, the renormalisation introduces an energy scale, on which the parameters will depend. That is, the coupling strengths and masses depend on the energy scale.

This is the reason for the oxymoronic term "the running coupling constant", which is not a constant at all. In QED the electromagnetic coupling α increases with increasing energy scale, so that interactions become stronger at larger energies. For example, at energies relevant for atomic physics α has the valued quoted above, $\alpha \simeq 1/137$, but in electron–positron collisions at the LEP accelerator (at a centre-of-mass energy of 90 GeV), it has increased to $\alpha \simeq 1/128$. In QCD the coupling strength α_s instead *decreases* with increasing energy scale. This leads to the important concept of asymptotic freedom — at larger energies interactions become weak and particles become quasi-free. This is a feature of all non-Abelian gauge theories where the gauge symmetry is not spontaneously broken (see below).

2.1.3 Gauge theory

Gauge invariance of a theory means that the Lagrangian is invariant under a certain type of symmetry transformations. This was first studied in classical electrodynamics, where there is a freedom to add derivatives of arbitrary scalar functions to the potentials. In quantum field theory, gauge invariance is now a very general principle on which realistic theories are built, and was proposed as such by Weyl [21] (see [22] for a history of gauge theory).

A simple example — Abelian gauge theory

One of the simplest possible gauge theories is QED. It describes the interactions of charged spin-1/2 particles (such as the electron) and photons based on the Dirac equation, Eq. (2.1). This equation can be obtained from the Lagrangian

$$\mathscr{L}_\psi = \bar{\psi}(x)(i\gamma^\mu \partial_\mu - m)\psi(x), \tag{2.4}$$

where $\bar{\psi}$ is the adjoint fermion field $\bar{\psi} = \psi^\dagger\gamma^0$. $\mathscr{L}_\psi$ is invariant under (global) phase rotations of the fermion field, but this does not give us any new information. The magic of gauge theory enters when we require that the Lagrangian be invariant under *local* phase rotations that depend on the position four-vector x^μ. The transformations $U(x) = e^{i\theta(x)}$ are elements of the unitary group U(1), so the transformations are referred to as U(1) gauge transformations. They affect the fields as just a phase rotation,

$$\psi(x) \to \psi'(x) = U(x)\psi(x) = e^{i\theta(x)}\psi(x), \tag{2.5}$$

but the Lagrangian is no longer invariant when the fields are transformed this way; rather, the derivative term picks up an extra term:

$$\begin{aligned} \bar{\psi}(x)\partial_\mu\psi(x) &\to \bar{\psi}(x)\partial_\mu\psi(x) + \bar{\psi}(x)U^\dagger(x)(\partial_\mu U(x))\psi(x) \\ &= \bar{\psi}(x)\partial_\mu\psi(x) + \bar{\psi}(x)[i\partial_\mu\theta(x)]\psi(x). \end{aligned} \tag{2.6}$$

If we demand invariance of the Lagrangian, we have to add to it a term that transforms in such a way as to cancel the extra term from the derivative. This is done by replacing the derivative ∂_μ with the *covariant derivative* D_μ, which transforms covariantly under the gauge transformation, i.e., like the fermion field:

$$D_\mu\psi(x) \to (D_\mu\psi(x))' = U(x)D_\mu\psi(x), \tag{2.7}$$

implying that $\bar{\psi}(x)D_\mu\psi(x)$ is invariant. The covariant derivative includes the minimal substitution known from classical electrodynamics, involving the electromagnetic field vector potential A_μ,

$$D_\mu = \partial_\mu - ieA_\mu. \tag{2.8}$$

Our new gauge invariant Lagrangian reads

$$\mathscr{L}_\psi = \bar{\psi}(x)[i\gamma^\mu(\partial_\mu - ieA_\mu) - m]\psi(x). \tag{2.9}$$

Thus, we know how the gauge field A_μ couples to fermions, but if we regard it as representing the physical photons we have to add a kinetic term to the Lagrangian to exploit its dynamics. The only renormalisable term in QED is

$$\mathscr{L}_A = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu}, \tag{2.10}$$

where

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \tag{2.11}$$

is the electromagnetic field tensor. The Euler–Lagrange equations for this Lagrangian give Maxwell's equations. The tensor $F_{\mu\nu}$ is gauge invariant by itself, and so is $\mathscr{L}_A$. The complete QED Lagrangian is now

given by the sum of the fermion part and the photon part;

$$\mathscr{L}_{\mathrm{QED}} = \bar{\psi}(x)(i\gamma^\mu D_\mu - m)\psi(x) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}. \tag{2.12}$$

Note how the electron–photon coupling was completely determined by requiring invariance of $\mathscr{L}_{\mathrm{QED}}$ under U(1) gauge transformations, and in addition that all terms be renormalisable. One could in principle imagine other types of couplings, e.g. a $\bar{\psi}\sigma_{\mu\nu}\psi F^{\mu\nu}$ term, but this term is not renormalisable. In fact, $\mathscr{L}_{\mathrm{QED}}$ is uniquely determined by these requirements.

An important point is that gauge invariance forbids mass terms for the gauge field in the Lagrangian, such as $m^2 A_\mu A^\mu$. That is, gauge bosons are massless unless the gauge symmetry is broken.

Going further — non-Abelian gauge theory

It is of course a logical step to go to more complicated symmetries, and this was done by Yang and Mills in 1954 [23].[2] They constructed a theory invariant under gauge transformations belonging to the simplest non-Abelian Lie group SU(2), in which the transformations do not commute. Furthermore, SU(2) has dimension three (the transformations have three generators) which means that there will be three different gauge bosons. In general, an SU(N) gauge theory will have $N^2 - 1$ gauge bosons, so for example, QCD, which is based on SU(3) invariance, has 8 gluons. Let the generators of infinitesimal SU(N) transformations be denoted T^a, where $a = 1 \ldots N^2 - 1$. Then the generators form a Lie algebra

$$[T^a, T^b] = if^{abc}T^c, \tag{2.13}$$

where f^{abc} are the structure constants of the algebra. These are not all zero for SU(N) groups, so the algebra is non-Abelian (non-commutative).

The fermion field ψ transforms in the fundamental representation of the group, and therefore has N components ψ_i, $i = 1 \ldots N$. The gauge transformation is now generalised to

$$\psi_i \to \psi_i' = U_{ij}\psi_j, \tag{2.14}$$

where $U = \exp(-iT^a\theta^a)$ and θ^a are the x-dependent parameters of the transformation. Demanding invariance of the Lagrangian under this transformation, we are led to a generalisation of the covariant derivative

$$D_\mu = \partial_\mu - igT^a A_\mu^a, \tag{2.15}$$

[2] Their theory had actually been discovered in 1938 by O. Klein [24] in the context of Kaluza–Klein theories.

where g is the coupling strength between the fermion field and the gauge field, and A^a_μ with $a = 1 \ldots N^2 - 1$ are the gauge fields. Note that this is actually a matrix equation, where $(D_\mu)_{ij}$ and T^a_{ij} are $N \times N$ matrices in the fundamental representation.

The gauge field tensor $F^a_{\mu\nu}$ is modified in Yang–Mills theory — it now contains an extra term with far-reaching consequences; and it is no longer invariant under gauge transformations. The product $F^a_{\mu\nu}F^{a\mu\nu}$ appearing in the Lagrangian is, however, invariant. We now have

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - g f^{abc} A^b_\mu A^c_\nu. \tag{2.16}$$

The Lagrangian still contains two parts. The fermion part now has the form

$$\mathscr{L}_\psi = \bar{\psi}_i \left[i\gamma^\mu (D_\mu)_{ij} - m\delta_{ij} \right] \psi_j = \bar{\psi}(i\gamma^\mu D_\mu - m)\psi, \tag{2.17}$$

and the total Lagrangian invariant under SU(N) gauge transformations is

$$\mathscr{L} = \bar{\psi}(i\gamma^\mu D_\mu - m)\psi - \frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu}. \tag{2.18}$$

Mass terms for the gauge field are still forbidden by gauge invariance. There is one very important difference between the Abelian and non-Abelian cases, however, namely that in Yang–Mills theory there are *self-interactions* of the gauge field — it couples to itself. This occurs because the A^a_μ itself carries SU(N) charge. The new term in the field tensor gives, when expanded, terms like $g f^{abc} A^a_\mu A^b_\nu (\partial^\mu A^{c\nu})$ describing the three-boson interaction and $g^2 f^{abc} f^{ade} A^b_\mu A^c_\nu A^{d\mu} A^{e\nu}$ describing the four-boson interaction. Note that it is the same coupling strength g that characterises the coupling of the fermion field to the gauge field and the coupling of the bosons amongst themselves.

The Standard Model consists of three gauge theories: QCD is a gauge theory like the one just described, consisting of six quark fields and the SU(3) gauge field, and will be treated in Chapter 3. I have already mentioned QED with gauge group $\mathrm{U}(1)_{em}$, but this theory is combined with the weak interactions into the electroweak theory, with gauge group $\mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y$ (here L stands for the weak isospin symmetry of the left-handed fermions and Y stands for weak hypercharge).

The electroweak theory and the Higgs mechanism

The electroweak theory is based on the Glashow–Salam–Weinberg theory, introduced by Glashow [25] in 1961. It contains all Standard Model fermions, the massless photon, and the massive Z^0 and $W^\pm$ gauge bosons. The boson masses arise through spontaneous breaking of the $\mathrm{SU}(2)_L \otimes$

$\mathrm{U}(1)_Y$ symmetry into $\mathrm{U}(1)_{em}$. This is known as the Higgs[3] mechanism [13], and was introduced for the electroweak gauge theory by Weinberg and Salam independently [26]. The ensuing theory was later shown to be renormalisable by 't Hooft and Veltman [27].

The unbroken Yang–Mills theory contains three massless gauge bosons (W_1, W_2, W_3) corresponding to $\mathrm{SU}(2)_L$ and one (B) corresponding to $\mathrm{U}(1)_Y$. The masses of the physical bosons arise because of the introduction of the scalar Higgs field $\mathrm{SU}(2)_L$ doublet coupled to the gauge fields through a covariant derivative. The essential point is that the Higgs field has a non-zero vacuum expectation value, spontaneously breaking the gauge symmetry and creating mass terms for the $W^\pm$ and Z bosons. The photon and the Z are linear combinations of the B and W_3 bosons,

$$A^\mu = B^\mu \cos\theta_W + W_3^\mu \sin\theta_W, \tag{2.19}$$

$$Z^\mu = -B^\mu \sin\theta_W + W_3^\mu \cos\theta_W, \tag{2.20}$$

while the $W^\pm$ are combinations of W_1 and W_2,

$$(W^\pm)^\mu = \frac{1}{\sqrt{2}}(W_1^\mu \mp i W_2^\mu). \tag{2.21}$$

The masses of the gauge bosons are determined by two free parameters of the theory, the Weinberg angle θ_W, given by the ratio of the $\mathrm{U}(1)_Y$ and $\mathrm{SU}(2)_L$ couplings, $\tan\theta_W = g'_W/g_W$, and the Higgs field vacuum expectation value $v \approx 246$ GeV. The Weinberg angle is relatively large, $\sin^2\theta_W \approx 0.23$, so there is really a mixing of the two theories. The Higgs boson mass is determined by $M_H = \lambda v^2$, where λ is an additional free parameter. The fermions acquire masses by interacting with the Higgs field through a Yukawa coupling, i.e., a Higgs–fermion vertex. Each fermion mass is proportional to its Yukawa coupling strength, which is a free parameter. The gluon and photon have no couplings to the Higgs field and so remain massless.

The left-handed fermions form doublets under $\mathrm{SU}(2)_L$ (this is the reason for the L), while the right-handed fermions are singlets. In other words, right-handed fermions do not interact with the $\mathrm{SU}(2)_L$ gauge bosons.

There is one SU(2) doublet and two singlets for each generation, with the same pattern for quarks and leptons,

$$\begin{pmatrix} u \\ d \end{pmatrix}_L \; u_R \quad d_R \qquad \begin{pmatrix} c \\ s \end{pmatrix}_L \; c_R \quad s_R \qquad \begin{pmatrix} t \\ b \end{pmatrix}_L \; t_R \quad b_R$$

$$\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L \; \nu_{eR} \quad e_R^- \qquad \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}_L \; \nu_{\mu R} \quad \mu_R^- \qquad \begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix}_L \; \nu_{\tau R} \quad \tau_R^-$$

[3] It may be more correctly referred to as the Brout–Englert–Higgs mechanism, as it was discovered by F. Englert and R. Brout and by P.W. Higgs independently.

In addition, the weak interaction eigenstates of the left-handed down type quarks are mixtures of the mass eigenstates. This so-called Cabibbo-Kobayashi-Maskawa mixing [28] allows electroweak transitions between generations.

2.1.4 Quantisation and Feynman diagrams

Until now we have bypassed the issue of quantising the theory. In the last section a number of Lagrangians were displayed, but these Lagrangians only describe the classical theory. There are several methods of furnishing a quantum theory out of the classical theory, but the method that has become standard is the Feynman path integral formalism [29], which has a rather intuitive interpretation.

The fundamental object here is the generating functional, which is a functional integral of the action over all field configurations with an added source term. For a scalar field ϕ, it is given by

$$W[J] = \int [d\phi]\; e^{i(S+J(x)\phi(x))}, \tag{2.22}$$

where S is the action

$$S = \int d^4x\; \mathscr{L}(\phi(x)), \tag{2.23}$$

and $J(x)$ is an external source field. The integral measure $[d\phi]$ means that the integration is a *functional* integration over all admissible functions $\phi(x)$. The classical solution of the field equations obtained from the Euler–Lagrange equations minimises the action, but quantum effects allow fluctuations around the classical solution; this is encoded into (2.22).

Writing the correct expression for the generating functional means that the theory has been quantised. One can then in principle derive all Green functions, but as discussed previously this is usually not possible to do, and perturbation theory has to be used. The Feynman rules of perturbation theory are related to the Green functions by way of the Lehmann–Symanzik–Zimmermann formalism [30].

The external source J is used when deriving the Green functions of the theory by functional differentiation. For example, the Feynman rule for a propagator is obtained from the two-point Green function $G^{(2)}(x_1, x_2)$, given by

$$G^{(2)}(x_1, x_2) = \left.\frac{\delta^2 \ln W[J]}{\delta J(x_1)\delta J(x_2)}\right|_{J=0}. \tag{2.24}$$

For theories involving fermions and for gauge theories the simple expression (2.22) is modified. Fermions require fermionic sources and

integration over anti-commuting variables, so-called Grassman variables. But for gauge fields there is an even larger complication — the generating functional is not well-defined in the form given in Eq. (2.22). This has to do with the gauge invariance of the theory; the integration double-counts field configurations which are equivalent up to a gauge transformation. This is remedied by fixing the gauge, i.e, introducing a gauge-fixing term in the Lagrangian of the theory, and by restricting the integral measure so that the integration over the group space only counts each equivalent configuration once.

This means that the theory is no longer gauge invariant, but it is known already from classical electromagnetism that it is possible to impose certain gauge conditions. Common examples are the Lorentz gauge, $\partial_\mu A^\mu = 0$, or the Coulomb gauge $\nabla \cdot \boldsymbol{A} = 0$. The Feynman rules derived for the gauge theory depend on the choice of gauge, but are usually simpler in Lorentz covariant gauges. The price to pay in such gauges is that there will effectively be a new set of fields, known as *Faddeev–Popov ghosts* [31], that have to be included in calculations of amplitudes. These fields are unphysical, in the sense that they do not appear in any external states, only internally in loop diagrams.

2.2 Collider physics: phenomenology and experiment

Most of the experiments in particle physics are scattering experiments where two particles collide and new particles are produced. Such experiments are done at accelerators, where either a beam of particles collides with a fixed target of hydrogen, deuterium or some other nucleus; or two beams collide with each other. The latter type of experiments achieve the highest centre-of-mass energies, and are called collider experiments. In colliders the beams are usually electrons, positrons, protons or antiprotons, so there are three main types of experiments: e^+e^-, $e^\pm p$, and pp (or $p\bar{p}$) collisions.

Examples of such accelerators are LEP (e^+e^-) that recently ended its operation at CERN , HERA ($e^\pm p$) at DESY, the Tevatron ($p\bar{p}$) at Fermilab, and the LHC (pp) that is to start in 2007 at CERN.

In experiments with an electron or positron beam it is also possible to make experiments with photons. This is possible because the charged leptons radiate photons, and in some cases these photons are real, or almost real, and collide with the other beam. So in addition to the lepton–proton collisions at HERA, there are also photon–proton collisions, and at LEP there were also photon–electron and photon–photon collisions. The more commonly studied process in lepton–hadron collisions is deep inelastic scattering, where the lepton probes the structure

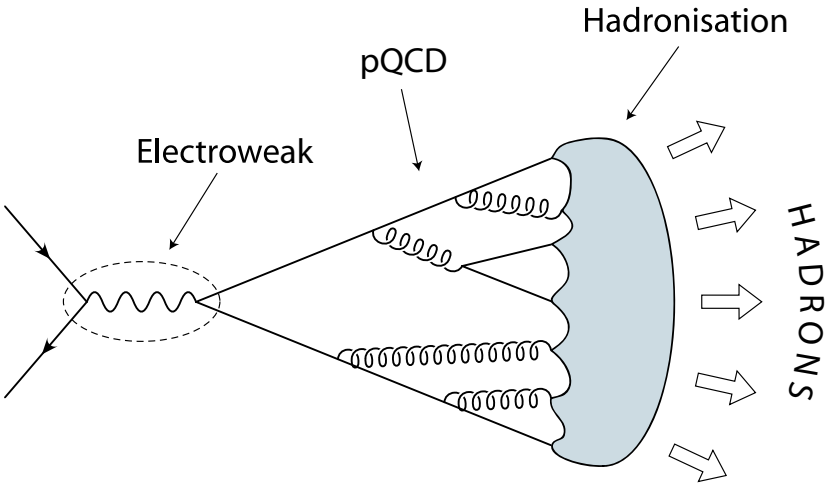


Figure 2.2: An electron–positron collision schematically showing parton radiation and hadronisation.

of the hadron by exchanging a virtual photon with large virtuality, i.e., large "mass" or four-momentum squared.

In this thesis, I have studied processes in proton–(anti)proton and photon–proton collisions, but it is helpful to briefly review all kinds of collisions to introduce some important concepts, with an emphasis on ideas relevant to this thesis. Many of the details will be discussed in Chapter 3.

2.2.1 e^+e^- collisions

The basic lowest order processes in e^+e^- collisions are $e^+e^- \to e^+e^-$, $e^+e^- \to$ *two gauge bosons*, or $e^+e^- \to f\bar{f}$ where f is any fermion other than the electron.

In the following we will concentrate on $e^+e^- \to q\bar{q}$ where q is any quark. Because of the confinement property of QCD what is seen in the detector is really $e^+e^- \to$ *hadrons*. The produced quark and antiquark can only propagate freely on a short distance scale, as allowed by asymptotic freedom, and eventually they have to transform into hadrons. This process is called *hadronisation*. The hadronisation mechanism is not known from first principles, because the problem takes place on a large distance scale and is therefore inherently non-perturbative, but several phenomenological models exist. The most comprehensive and most widely used of these is the Lund string model [32] (see Chapter 3).

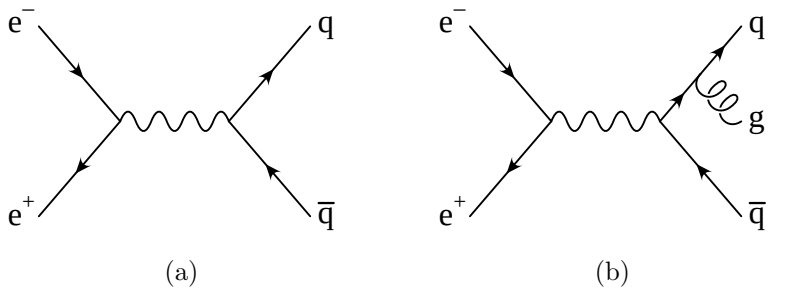


Figure 2.3: Basic lowest order QCD process in (a) a two-jet event and (b) a three-jet event.

The produced quark and antiquark will often radiate one or several gluons, which can in turn radiate more quarks or gluons, and the whole system of quarks and gluons (collectively referred to as *partons*, although they are here not constituent parts of the proton) will hadronise. This is illustrated in Fig. 2.2. It is natural to imagine that there is a time axis from left to right in Fig. 2.2, so that there is first an electroweak interaction where the quark and antiquark are produced, followed by radiation of gluons and quarks with lower energy, and finally the hadronisation of the whole system. This can be a useful fiction, but it is not strictly true, since the process is a quantum mechanical one. The different parts do, however, have different characteristic space-time scales that can provide such a "time-ordering". When referring to the original interaction one often talks about the "hard" process, which can then be followed by radiation of harder or softer particles.[4]

It is important to realise that although there will be radiation of softer partons from the primarily produced ones, and although the partonic system will hadronise, important features of the original hard process will survive to the hadron level. The most probable situation is that a quark and antiquark are produced in the hard interaction (see Fig. 2.3a), each radiating some amount of softer partons. The softer parton radiation will tend to be aligned with the original parton and the process of final state radiation can be described as an iterative cascade, or shower, of partons. The cascade gives rise to aligned sprays of partons, which, when hadronised, are known as *jets*. These jets roughly follow the directions of the original quark and antiquark, so the event is classified as a two-jet event.

[4]The words "soft" and "hard" are parts of the standard jargon, where "soft" simply refers to a particle with lower energy or momentum. Similarly "hard" refers to larger energy.

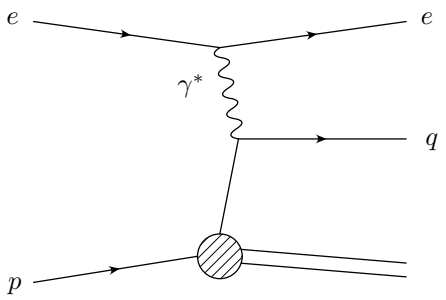


Figure 2.4: Leading order deep inelastic scattering process $ep \to eX$, where X means "anything." The star on γ^* denotes that the photon is virtual.

If one of the quarks emits a hard gluon at a larger angle the event may instead be a three-jet event, as shown in Fig. 2.3b. This, in fact, is how the gluon was discovered experimentally at the PETRA accelerator at DESY [33].

2.2.2 Deep Inelastic Scattering

In Deep Inelastic Scattering (DIS) experiments, leptons collide with protons, so there is an extra complication in that the proton is a composite object. In this context "deep inelastic" means that the lepton has high enough energy to interact with a constituent of the proton instead of with the proton as a whole. The interacting constituent parton is knocked out of the proton, which is usually dissociated in the process. The perturbative description of this interaction with a parton is made possible by asymptotic freedom, i.e., that the partons inside the proton are quasi-free at high energies when the strong coupling is small.

The interaction proceeds via photon or weak boson exchange between the lepton and the parton. The lowest order electroweak process is shown in Fig. 2.4.

Let p, p_e and q be the four-momenta of the incoming proton and electron and the exchanged photon, respectively. DIS is then described in terms of the Lorentz invariant variables $Q^2 = -q^2$ (the "virtuality" or "mass" of the photon), $x = Q^2/2p \cdot q$ (Bjorken-x), and $y = p \cdot q/p \cdot p_e$. If we for simplicity consider low enough Q^2 that only photon exchange is important ($Q^2 \ll M_Z^2$) the cross section is given in terms of structure functions F_1 and F_2:

$$\frac{d\sigma}{dx\,dQ^2} = \frac{4\pi\alpha^2}{xQ^2}\left[y^2 x\,F_1(x,Q^2) + (1-y)\,F_2(x,Q^2)\right], \tag{2.25}$$

that generally can depend on x and Q^2. If one assumes, like in the

original parton model [11], that the proton is made of point-like spin-1/2 particles that do not interact with each other, the structure functions are independent of Q^2, $F_{1,2}(x, Q^2) \to F_{1,2}(x)$, and furthermore Bjorken-x is equal to the longitudinal momentum fraction of the proton's momentum carried by the struck parton. This is known as Bjorken scaling [34]. In addition there is the Callan–Gross relation for the structure functions, $2xF_1 = F_2$ [35]. The parton model was confirmed by the early DIS experiments at SLAC [10] and provided strong evidence for the quark hypothesis — using partons with integer spin instead gives predictions that do not agree with experiments.

It was soon realised that Bjorken scaling is only approximate. At higher energies it was discovered that the structure functions depend on $\log Q^2$. This can be explained by QCD. The quark that is struck by the photon can radiate gluons before the hard interaction. This is known as initial state radiation. Furthermore, the proton does not only consist of the three *valence quarks*. About half of the momentum of the proton is actually carried by gluons, and there are in addition quark–antiquark pairs that arise because of quantum fluctuations. These quarks are known as *sea quarks*, and the photon can scatter on sea quarks just as well as on valence quarks, or it can scatter on gluons via a higher-order process where the photon and gluon produce a quark–antiquark pair. As the virtuality Q^2 of the photon increases, it can resolve more details of this quantum structure of the nucleon.

The parton structure of the proton is made manifest by introducing *parton distribution functions* (pdf's) that can be interpreted as momentum probability distributions of partons. At the lowest order of perturbation theory one can then write

$$F_2(x, Q^2) = \sum_f e_f^2 \left[x\, q_f(x, Q^2) + x\, \bar{q}_f(x, Q^2) \right], \tag{2.26}$$

where the sum is over all quark flavours f that can take part in the interaction, q_f and $\bar{q}_f$ are the quark and antiquark distributions, and e_f is the quark electric charge. There is also a gluon distribution $g(x, Q^2)$, but it does not contribute at lowest order.

The parton distributions cannot be computed in perturbation theory, because the characteristic momentum scale is small, and α_s is thus large. They should therefore be calculated using techniques of non-perturbative QCD. This has not yet been possible to do rigorously, but phenomenological models exist [36]. The pdf's are, however, unambiguously defined theoretically in terms of matrix elements of field operators, and are also *universal*, meaning that the same parton distributions are used in all types of processes for describing the parton content of the proton. There are also factorisation theorems (see [37]) stating that the description can

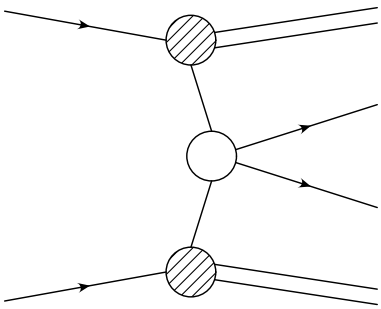


Figure 2.5: A hard interaction between two quarks in a hadron–hadron collision. The white blob in the middle represents the interaction, and the two shaded blobs represent the hadron vertices.

be factorised in this way. Because of factorisation and universality, the pdf's are extracted from experimental measurements and fitted to some simple functional form so that they can be used in calculations.

The Q^2-dependence of the pdf's, on the other hand, is predicted by QCD. It is described by the Dokshitzer–Gribov–Lipatov–Altarelli–Parisi (DGLAP) equations [38], and experimental DIS data are very precisely described by these calculations. This will be further described in Chapter 3.

2.2.3 Hadron–hadron collisions

When there are two colliding hadrons there are consequently two partonic systems involved. In an event with a hard interaction, the interaction will be between one parton from each hadron, as illustrated in Fig. 2.5. Each hadron is then described by its parton distribution functions, and because of factorisation it is possible to write the cross section on the form

$$\sigma_{h_1 h_2 \to X} = \sum_{i,j} \int dx_1 dx_2 \; f_{i/h_1}(x_1, \mu^2) \; f_{j/h_2}(x_2, \mu^2) \;\; \hat{\sigma}_{ij \to Y} \; , \quad (2.27)$$

where the total cross section is factorised into the cross section for the parton–parton interaction, convoluted with the pdf's giving the probabilities for partons in the hadrons. Here i and j denote the parton species taking part in the interaction, x_1 and x_2 are the momentum fractions in hadron h_1 and h_2, and f_{i/h_1} and f_{j/h_2} are the corresponding parton distribution functions. $\hat{\sigma}$ is the parton level cross section for scattering of the two partons i and j and is calculated in electroweak or QCD

perturbation theory. The energy scale μ where the parton distribution functions is evaluated is known as the factorisation scale.

Again, Feynman diagrams such as the one in Fig. 2.5 should not be taken too literally as a picture of a process in space-time; it is really a schematic picture showing flows of momentum. Because the momentum involved is large, a hard interaction (here represented by the blob in the middle) takes place on a very short space-time distance scale compared to the size of the proton. In order for the two partons to collide, there must be a considerable overlap of the two protons, so there is a large probability that there will be additional softer interactions between the remnants of the protons. There will also be radiation of partons from the incoming and outgoing partons, just as in DIS. One could imagine that these effects would ruin the simple formula (2.27), that can be interpreted in a probabilistic manner. Fortunately the above-mentioned factorisation theorems in QCD say that the cross section for inclusive observables can always be calculated in this way (see [37]).

This completes the survey of particle physics phenomenology. In the next chapter we turn to a description of Quantum chromodynamics.

Chapter 3

Quantum Chromodynamics

The theoretical basis of QCD is a non-Abelian gauge theory with gauge group SU(3), where the gauge field is coupled to fermions known as quarks. The quarks transform in the fundamental representation of the gauge group, which is three-dimensional, and the gauge charge carried by the quarks is called colour. The number of colours is therefore $N_c = 3$, and a quark can have any of three colours, usually called red, green and blue. The gauge sector consists of gluons — massless gauge bosons that transform in the adjoint representation of the gauge group. There are therefore $N_c^2 - 1 = 8$ gluons.

The classical Lagrangian of QCD is the Yang-Mills Lagrangian (2.18) coupled to the six fermion fields q_f,

$$\mathscr{L}_{\text{QCD}} = \sum_{f=1}^{6} \bar{q}_f (i\gamma^\mu D_\mu - m_f) q_f - \frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu}, \tag{3.1}$$

where the flavour index $f = 1 \ldots 6$ corresponds to the up, down, strange, charm, bottom and top quarks in order of increasing mass. The quark masses vary quite drastically from the very light up and down quarks that have masses of the order of a few MeV, to the very heavy top quark, whose mass is around 175 GeV, i.e., almost 200 times heavier than the proton! The up and down quarks can in many situations be treated as practically massless, while the three heaviest quarks usually can not.

The two most important concepts in QCD are *asymptotic freedom* and *confinement*. Asymptotic freedom means that at short distances — corresponding to high energies — quarks and gluons behave as quasi-free particles and their interactions can be computed in perturbative QCD

(pQCD). Confinement is the principle that only colour singlet particles (i.e., particles that transform as singlets under SU(3)) can be observed at a long space-time distance; quarks and gluons can no longer propagate freely but are bound into hadrons. The potential energy between two quarks grows linearly with increasing distance, so it is not possible to completely separate them without inserting an infinite amount of energy. While asymptotic freedom is completely understood in perturbation theory [39], and provided a crucial step in the formulation of QCD as the theory for strong interactions, confinement is only understood qualitatively. There are phenomenological models for the binding forces in hadrons and for the transition of partonic systems into hadrons, but there is poor understanding of soft processes from first principles.

This has to do with the general issue of non-perturbative QCD. The coupling strength α_s increases when the energy decreases, so at some point it is large enough that the series expansion does not converge and perturbation theory cannot be used. This means that scattering processes that involve large energy-momentum transfer or large masses ("hard processes") can be computed reliably in perturbation theory, while scattering at low momentum transfer cannot. Similarly, other low-energy properties, such as hadron masses and wave functions (and in particular parton distribution functions) are beyond the reach of pQCD.

This makes it necessary to use and develop models for such problems. For this purpose it is often convenient to study the interface between perturbative and non-perturbative QCD in problems where there are both hard and soft energy momentum scales involved.

3.1 Perturbative QCD

There are three basic vertices in QCD: quark–quark–gluon, three-gluon and four-gluon vertices. In covariant gauges there is also a ghost–ghost–gluon vertex. These are illustrated in Fig. 3.1. Using the expressions for the vertices together with the quark, gluon and ghost propagators for internal (virtual) particles, and the spinors and polarisation vectors for external (real) quarks and gluons, expressions for all amplitudes can be written down. In principle is it possible to compute cross sections and other observables to any order in perturbation theory. In real life, however, it turns out to be extremely difficult to even calculate diagrams with two internal loops, so normally calculations are done at the leading order (LO) or the next-to-leading order (NLO).

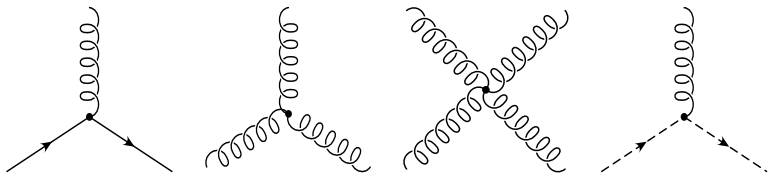


Figure 3.1: The fundamental vertices of QCD.

Renormalisation and running coupling

All calculations beyond LO have to be renormalised and defined in a certain renormalisation scheme. This introduces a dependence on the corresponding renormalisation scale μ, but any physical observable A should be independent of the renormalisation scheme and scale, which are just theoretical constructs. Therefore

$$\mu^2 \frac{d}{d\mu^2} A = 0. \tag{3.2}$$

But the observable depends on the coupling α_s and the renormalisation scale, so the above equation can be rewritten as

$$\begin{aligned} \mu^2 \frac{d}{d\mu^2} A(\mu, \alpha_s) &= \left(\mu^2 \frac{\partial}{\partial \mu^2} + \mu^2 \frac{\partial \alpha_s}{\partial \mu^2} \frac{\partial}{\partial \alpha_s} \right) A(\mu, \alpha_s) \\ &= \left(\mu^2 \frac{\partial}{\partial \mu^2} + \beta(\alpha_s) \frac{\partial}{\partial \alpha_s} \right) A(\mu, \alpha_s) = 0, \end{aligned} \tag{3.3}$$

where the β-function $\beta(\alpha_s) = \mu^2 \, \partial \alpha_s / \partial \mu^2$ has been defined. Eq. (3.3) is solved by introducing a running coupling $\alpha_s(\mu^2)$ that depends on μ.

The β-function can be calculated in perturbation theory, and is given by

$$\beta(\alpha_s) = -\beta_0 \alpha_s^2 - \beta_1 \alpha_s^3 - \ldots \tag{3.4}$$

where the leading order coefficient is

$$\beta_0 = \frac{33 - 2n_f}{12\pi} \tag{3.5}$$

and n_f is the number of quark flavours included in loop diagrams. The running coupling is thus at leading order determined by

$$\mu^2 \frac{\partial \alpha_s}{\partial \mu^2} = -\beta_0 \alpha_s^2, \tag{3.6}$$

and we therefore find, by rewriting the solution somewhat and introducing the integration constant Λ, that

$$\alpha_s(\mu^2) = \frac{1}{\beta_0 \ln(\mu^2/\Lambda^2)} = \frac{12\pi}{(33 - 2n_f) \ln(\mu^2/\Lambda^2)}. \tag{3.7}$$

Λ is a fundamental QCD parameter, and cannot be predicted by the theory but gives an estimate of where the QCD coupling gets strong and perturbation theory is not usable (formally the coupling diverges when $\mu \to \Lambda$). Its value depends on the renormalisation scheme, but is around 200 MeV, and to be sure that perturbation theory works well one has to consider scales larger than that, so that the coupling is not too large.

Eq. (3.7) shows clearly the asymptotic freedom of QCD: for $n_f < 16$ we see that $\alpha_s \to 0$ as $\mu \to \infty$. This conclusion is not modified by including higher orders in the β-function.

Infrared divergences, jets and parton radiation

The ultraviolet divergences that arise when momenta become infinite are taken care of by renormalisation, as discussed previously. In addition there are also infrared divergences. For example, when calculating the differential cross section for $e^+e^- \to q\bar{q}g$ corresponding to the diagrams in Fig. 2.3 there will be two kinds of such infrared divergences, soft and collinear. The soft divergences arise when the momentum of the emitted gluon tends to zero, and the collinear divergences arise when the momentum becomes parallel with the momentum of the quark from which it was emitted.

These divergences seemingly make it impossible to integrate the differential cross section in order to get the total cross section. But there is one thing missing. In order for the calculation to be consistent and gauge-invariant, all contributions at a certain order in the perturbative expansion must be included. In the present case there are diagrams with emission of one real gluon, that, when squared, contribute at order α_s (denoted as $O(\alpha_s)$). There are also diagrams with virtual gluon vertex corrections that have the same final state as the tree-level diagram without gluons, and must therefore be added before they are squared. This leads to interferences that are $O(\alpha_s)$. What is important here, is that these interference terms also contain infrared divergences that are exactly the same as those of the real emission diagrams but with the *opposite sign*. Therefore the divergences cancel, and the sum of amplitudes is finite.

This is not a coincidence, but there exist general theorems [40] stating that divergences stemming from real emissions and virtual loops exactly cancel each other at each order in the perturbative expansion, provided the observables are *infrared safe*, i.e., defined in a suitable way.

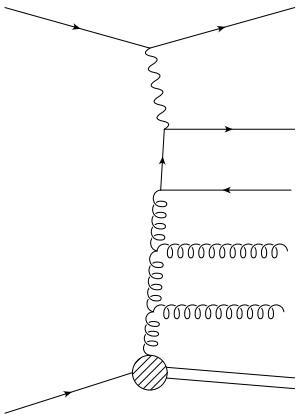


Figure 3.2: Deep inelastic scattering process by boson–gluon fusion with initial state radiation.

These cancellations occur for the total $e^+e^- \rightarrow$ *hadrons* cross section, but for less inclusive variables, such as kinematical distributions of jets, the divergences do not cancel unless the variable is carefully defined in such a way as to be infrared safe. Emitted gluons that are soft or collinear will contribute to the two-jet cross section where the leading order process has just a $q\bar{q}$ pair in the final state. Because of the divergence, gluons will tend to be emitted collinearly with the quark, contributing to build up the jet, so that also at higher orders in α_s the cross section will be dominated by two jets. On the other hand, to be able to experimentally detect a third jet in the detector it can neither be very soft nor very collinear, so the divergences are avoided. If a third jet is detected it must be well separated from the other two jets, and this represents an α_s correction to the two-jet cross section.

There can be many extra gluon emissions, which makes the division of events into two- or three-jet events ambiguous. It is therefore necessary to use various *jet algorithms* to define jets, see e.g. [17].

Scaling violations and factorisation

As discussed in Chapter 2, in collisions that involve hadrons there can also be radiation from the incoming partons taking part in the hard interaction. This initial state radiation makes the structure functions depend on the hard scale Q^2 and violate Bjorken scaling. The emitted partons will again tend to be radiated collinearly with the original parton and there will be collinear divergences. An example diagram is shown in Fig. 3.2.

At lowest order, with no QCD emissions from the incoming quark, the quark distribution q depends only on Bjorken-x,

$$F_2(x,Q^2) = \sum_{q,\bar{q}} e_q^2 x\, q_0(x), \tag{3.8}$$

and there is Bjorken scaling, $F_2(x,Q^2) \to F_2(x)$. But if $O(\alpha_s)$ QCD corrections are added, the structure function instead takes the form

$$F_2(x,Q^2) = \sum_{q,\bar{q}} e_q^2\, x \left[q_0(x) + \int_0^1 \frac{dy}{y} q_0(y) \frac{\alpha_s}{2\pi} \left\{ P_{qq}\left(\frac{x}{y}\right) \ln\frac{Q^2}{\lambda^2} + C\left(\frac{x}{y}\right) \right\} \right]. \tag{3.9}$$

Here P_{qq} is known as the Altarelli–Parisi or DGLAP splitting function for $q \to qg$ and C is a finite calculable function. The function $q_0(x)$ can be seen as the bare quark distribution function. The term involving $P_{qq}\ln(Q^2/\lambda^2)$ is a result of the collinear logarithmic [41] divergences in the gluon emissions, here regulated by introducing a small cut-off λ in the momentum integration. Comparing Eqs. (3.8) and (3.9) one can now define a new renormalised quark distribution function depending on Q^2,

$$q(x,Q^2) = q_0(x) + \frac{\alpha_s}{2\pi} \int_0^1 \frac{dy}{y} q_0(y) \left\{ P_{qq}\left(\frac{x}{y}\right) \ln\frac{Q^2}{\lambda^2} + C\left(\frac{x}{y}\right) \right\}, \tag{3.10}$$

which is finite and absorbs all collinear divergences (i.e. the regulator λ), in clear similarity with ultraviolet renormalisation. The same analysis can be made for the gluon distribution, taking into account the fact that gluons can split into quarks, and the structure function can now be expressed in terms of the renormalised pdf's. This process is called collinear factorisation, and the scale Q^2 is called the *factorisation scale*.[1] It can be shown that factorisation is universal, that is, the renormalised pdf can be used in all kinds of processes (see [37]).

From Eq. (3.10), modified by taking into account all $O(\alpha_s)$ corrections, and the corresponding equation for the gluon distribution, the leading order DGLAP equations follow [38]:

$$\begin{aligned} \frac{dq(x,Q^2)}{d\ln Q^2} &= \frac{\alpha_s(Q^2)}{2\pi} \int_x^1 \frac{dy}{y} \left[q(y,Q^2)\, P_{qq}\left(\frac{x}{y}\right) + g(y,Q^2)\, P_{qg}\left(\frac{x}{y}\right) \right] \\ \frac{dg(x,Q^2)}{d\ln Q^2} &= \frac{\alpha_s(Q^2)}{2\pi} \int_x^1 \frac{dy}{y} \left[\sum_{q,\bar{q}} q(y,Q^2)\, P_{gq}\left(\frac{x}{y}\right) + g(y,Q^2)\, P_{gg}\left(\frac{x}{y}\right) \right] \end{aligned} \tag{3.11}$$

[1] The factorisation scale μ_F^2 is actually distinct from the photon virtuality Q^2, but in practise this choice is usually made.

These integro-differential equations predict the Q^2-dependence of the parton distribution functions. They can be solved analytically by performing a Mellin transformation, which disentangles the convolution and turns the equations into algebraic ones, although the obtained expressions have to be evaluated numerically. To get pdf's at any scale one then needs them at some low starting scale Q_0^2 below which perturbation theory is not applicable. This cannot be predicted from perturbative QCD and is usually parametrised and fitted to experimental data.

Resummation

By a more careful derivation it can be shown that the DGLAP equations do not only embody the $O(\alpha_s)$ corrections corresponding to one gluon emission, but they actually resum, to all orders in α_s, leading terms proportional to $\alpha_s^n(Q^2)\ln^n(Q^2)$. This corresponds to *ladder diagrams* such as Fig. 3.2 with emission of n gluons. The terms resummed are the terms that, at each order in α_s, dominate in the region of phase space where the transverse momenta of emitted gluons are strongly ordered, i.e., $Q_0^2 \ll k_{T1}^2 \ll \cdots \ll k_{Tn}^2 \ll Q^2$. One says that the calculation is done in the leading logarithmic approximation (LLA).

Furthermore, the splitting functions also have a perturbative expansion,

$$P_{ij}(x) = P_{ij}^{(0)}(x) + \frac{\alpha_s}{2\pi} P_{ij}^{(1)}(x) + \left(\frac{\alpha_s}{2\pi}\right)^2 P_{ij}^{(2)}(x) + \ldots, \qquad (3.12)$$

where the P_{ij} used in Eqs. (3.9–3.11) are the leading order functions. If the next order in the expansion of the splitting functions is included, the DGLAP equations also resum leading terms in $\alpha_s^n(Q^2)\ln^{n-1}(Q^2)$, which is therefore called the next-to-leading logarithmic approximation (NLLA).

Notice that the logarithms that multiply the coupling constant are large, and that such logarithms appear at each order in α_s. The normal "fixed order" perturbative expansion will therefore not give a reliable result in this situation, and this is why the all-orders resummation of leading terms must be done.

The DGLAP approximation is valid when both x and Q^2 are not too small. If x is small there will be important terms proportional to $\ln(1/x)$, that must also be resummed. When x is small and Q^2 is still large, and if one imposes an additional strong ordering in x, the DGLAP equations can be solved analytically for the gluon distribution. The ensuing double leading logarithmic approximation is a subset of the DGLAP formalism. More interesting, however, is the case when Q^2 is not too large and $1/x$ is large. Then the cross section is dominated by terms proportional to

$\alpha_s^n(Q^2)\ln^n(1/x)$, and these terms are resummed by the Balitsky–Fadin–Kuraev–Lipatov (BFKL) equation [42]. The solution of this equation predicts a steep rise of the structure functions towards small x, and when this was discovered at HERA [43] the research field known as *low-x physics* received a lot of new interest. This will be discussed further in Chapter 4.

3.2 Non-perturbative QCD

In perturbative QCD calculations can, at least in principle, be performed analytically from first principles. This is no longer the case when the momentum scale is small, so that α_s is large. There is then no generally applicable way of making calculations, and some amount of modelling of features that cannot be derived from the fundamental Lagrangian is usually needed.

One exception is *lattice gauge theory* [44], where space-time is discretised on a four-dimensional Euclidean lattice and the gauge field is represented by phases of exponentials. The theory is formulated fully from first principles, and it is possible to simulate small lattices by Monte Carlo methods and calculate certain static observables such as hadron masses. Agreement with experimental data is relatively good, but the computer simulations are extremely demanding and require very large computer resources. It is also problematic to include fermions in the simulations in a consistent way. Furthermore it is not possible to compute soft cross sections and related observables, although some progress is made in obtaining parton distribution functions at some starting scale.

Other, more "theoretical" approaches to non-perturbative QCD include QCD sum rules [45], instanton calculations [46], use of the large N_c limit [47], and methods borrowed from string theory such as the AdS/CFT correspondence [48] between string theory and gauge theory. These methods all include some modelling or extraction of information from other sources and it has not, for example, been possible to prove confinement in QCD.

Another approach is to make phenomenological models, based on theoretical and experimental input and intuition, and hope that a successful model may give some insight into the physical processes. A particularly efficient way to formulate such models is implementing them in a *Monte Carlo event generator* computer program. These programs simulate all aspects of a particle collision and include both perturbative calculations and non-perturbative models, as will be discussed in the next section.

3.2.1 Hadronisation and the Lund string model

Hadronisation, the process by which the partonic state produced in a collision is transformed into hadrons, is a particularly important subject, because it is an entirely non-perturbative process always present in collisions. To be able to make detailed analyses of collider data it is often necessary to have a good understanding of what is going on in the hadronisation process.

In some cases, for instance when calculating meson production, one has an explicit model for how mesons are formed in the hard process (this is discussed in Chapter 4). There are also various fragmentation functions describing how a hadron is formed from a quark or gluon in the partonic final state. But often one is interested in many-particle final states and correlations among particles. For example, Monte Carlo programs give final states with several partons from the hard process and parton radiation. In such cases it is desirable to have a description of the hadronisation of the parton system as a whole.

There are in effect three such general models: independent (Field–Feynman) fragmentation [49], the Lund string model [32], and the cluster fragmentation model [50], although independent fragmentation is not used much nowadays. A short review of the Lund model will be given here.

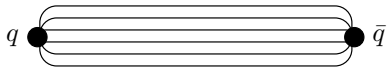


Figure 3.3: Schematic view of colour field lines between a quark and an antiquark.

The string model is a semi-classical model, starting from the observation that the colour field between two colour charges does not extend throughout space, like for two electromagnetic charges. Instead, because of the self-interaction of the gluons the field is concentrated into a colour flux tube, as illustrated in Fig. 3.3. The tube has a constant energy density $\kappa \approx 1$ GeV/fm per unit of length, i.e., the potential energy between the charges grows linearly with the separation distance. This gives a qualitative explanation of confinement: a colour charged particle will be the source of such a field and cannot be separated a long distance from other charges. If the distance becomes too large the tube will break by the production of a quark-antiquark pair from the field energy, so that there will be two new tubes. This picture of the confining potential energy is consistent with the spectrum of mesons, and is supported by simulations in lattice gauge theory.

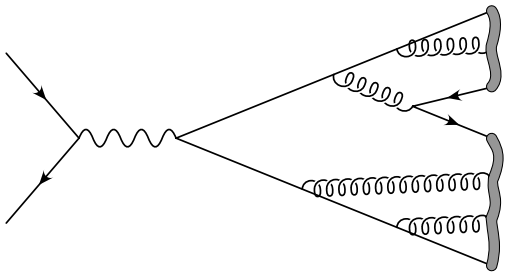


Figure 3.4: e^+e^- collisions with final state radiation and Lund strings stretched between final state partons.

The string model is based on approximating the tube with a one-dimensional massless relativistic string stretched between the charges. In the final state of a particle collision strings are stretched between partons, each string between a quark with colour **3** charge and an antiquark with $\overline{\mathbf{3}}$ charge. Gluons are represented by kinks on the string carrying energy and momentum. Each string system including the charges at the endpoints is colour singlet, i.e., colour neutral. An example of a string drawing in e^+e^- is shown in Fig. 3.4.

If initial hadrons are involved in the collision, the hadron remnant can no longer be colour neutral, as a colour charged parton has been removed. In case a quark was removed the remnant will have charge $\overline{\mathbf{3}}$, and in case of a gluon it will have charge **8**. In the former case the remnant valence quarks are represented as a diquark and the string is stretched from this diquark. In the latter case the remnant is split into a quark and a diquark, and there will be two strings connecting to the remnant. This is illustrated in the case of DIS in Fig. 3.5.

In simple events without many emissions, there is often only one possible string configuration, but in more complicated situations the way the string is stretched between partons is not unique. The strings are then stretched according to colour information from the hard scattering, which can be calculated in the *planar* approximation corresponding to the limit $N_c \to \infty$. In this limit the colour flow information is given by planar diagrams, and non-planar diagrams are suppressed.

Each string fragments into hadrons, successively breaking into shorter strings by creating quark–antiquark pairs or diquark–anti-diquark pairs through a quantum mechanical tunnelling process. Each new string will continue to break until only hadrons remain.

This breaking process explains several features that are observed in experiments. The multiplicity of hadrons naturally grows with the en-

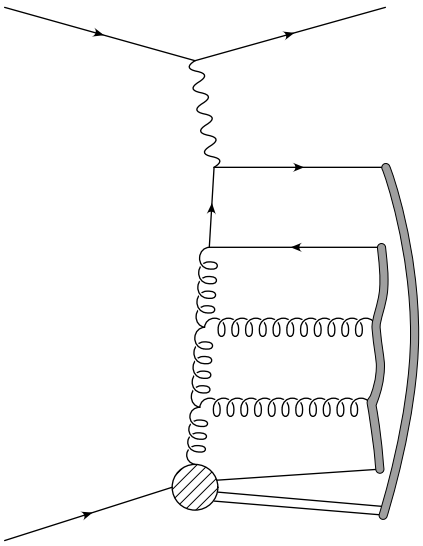


Figure 3.5: Deep inelastic scattering event with initial state radiation and Lund strings stretched in the final state. The lower incoming line is the proton, whose interaction is represented by the shaded blob. The double outgoing line represents a diquark.

ergy of the string system, and the distribution of the hadrons is flat in *rapidity*.[2] Particles are produced along the length of the strings, which normally span the whole rapidity region between the beam remnants according to the colour connections. Therefore particles will typically be produced in the whole rapidity range of available phase space.

3.3 Monte Carlo simulation

The fragmentation process discussed above can be formulated in an iterative manner, and as such the model is very suitable for implementation in a Monte Carlo program, whereas analytical calculations are difficult or impossible. This holds for many parts of a collision process. In particular, Monte Carlo simulation is in many cases the only way to get

[2] Rapidity is a very useful variable in analysing particle collisions, and is defined as $y = (1/2)\ln(E+p_z)/(E-p_z)$, where E is the energy and p_z is the momentum in the beam direction of a particle. This is a convenient variable as it transforms additively under Lorentz transformations in the z direction, and can be approximated by the pseudorapidity $\eta = -\ln\tan(\theta/2)$ which is just a transformation of the polar angle. For massless particles the approximation is exact.

detailed predictions from theories and models, such as when studying many-particle production, multiple interactions in collisions and so on.

Monte Carlo simulation is a general technique for, e.g., simulating stochastic processes and computing integrals by using random numbers (see [51] for an introduction) and has the advantage that integration of complicated functions over spaces with complicated boundaries is possible. In addition, it is possible to treat the integral being computed as a probability distribution and generate samples from it, distributed correctly in all degrees of freedom. This has an obvious application in integrating differential cross sections and at the same time generating physical scattering events from them.

This is exactly what is done in *event generator* programs; collision events are generated from the known, theoretically calculated, hard scattering cross sections, and the events are supplemented with other complex physical effects, such as parton radiation and hadronisation. For each event a list of produced particles with energies and momenta is obtained, making detailed analysis and comparison with experiment possible. There are several such general program packages available, like PYTHIA [52] and HERWIG [53] for all kinds of collisions and LEPTO [54] for DIS.

In the Monte Carlo each event is factorised into several parts that are treated separately. This is a necessary approximation, although not entirely physical. Fig. 3.6 shows an example, a proton–proton collision with electroweak boson production accompanied by several parton emissions. The various parts of the event generation process are shown in the figure.

In a proton–proton collision the subdivision of the event is as follows: The two particles collide and one parton from each proton, chosen according to the parton distribution functions, interacts in a hard scattering. The hard scattering is computed in perturbation theory and is the starting point of the whole event; kinematical variables are defined from the cross section. The interacting partons are allowed to radiate partons before and after the scattering in what are known as the initial and final state parton showers. The constitution of the remnants of the protons depends on whether a valence quark, gluon or sea quark initiated the interaction, and the remnant partons have to be given momenta according to some model. There can also be additional interactions between the protons, so-called multiple interactions, or there can be interactions between the final state partons, such as in the Soft Colour Interaction models [55, 56] to be described in the next chapter. Finally the whole partonic system is hadronised using the Lund string model (in e.g. PYTHIA and LEPTO) as was described in Section 3.2.1 or the cluster model (in e.g. HERWIG).

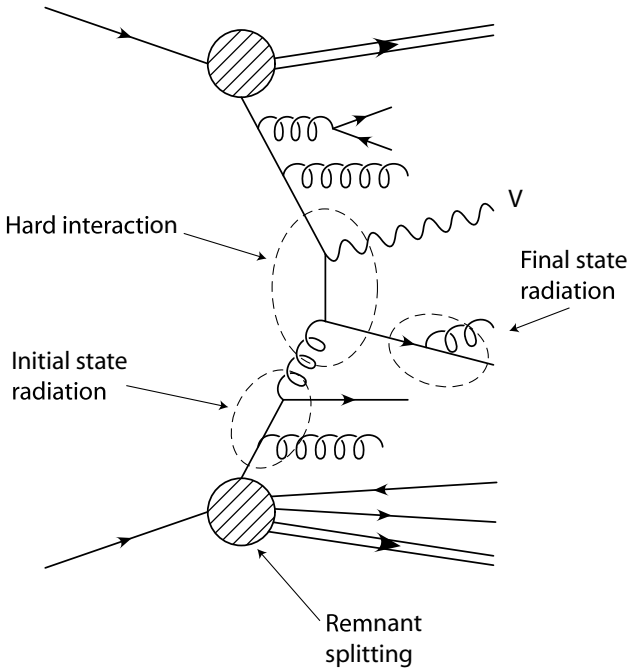


Figure 3.6: Example $pp \to VX$ ($V = \gamma, Z^0, W^\pm$) event with initial and final state radiation.

The parton showers are constructed to generate parton emissions distributed correctly according to the DGLAP formalism at leading logarithmic accuracy, and use the splitting functions discussed in Section 3.1. It can be shown that due to coherence effects the emission of gluons must obey angular ordering, i.e. that the angles must decrease along the chain of emissions. In the final state parton shower the emissions are generated sequentially and the evolution equation is effectively Monte Carlo-integrated. In the initial state parton shower, however, this would be extremely inefficient, as the end of the shower must match the kinematical variables generated for the hard interaction. It is therefore generated backwards [57], taking the DGLAP-evolved parton distributions as boundary conditions.

Chapter 4

Diffraction and low-x physics

DIFFRACTION AND LOW-x physics are the subjects of this thesis. They are quite closely related areas of research concerning complex dynamics of gluonic systems in high energy interactions. A main theme is exchange of *colour singlet* systems of gluons, i.e., with no net colour charge. One gluon is always a colour octet charge, but it follows from SU(3) group theory that two or more gluons can form systems whose colour charge is singlet, octet, or higher. This has interesting consequences for the production of particles in the interaction, as will be explained in this chapter.

4.1 What is diffraction?

Diffraction has a long history in particle physics and is nowadays a quite large area of research. The term diffraction originally referred to what is now called soft diffraction, see e.g. [58]. This is a process similar to elastic scattering, but one or both of the colliding particles are dissociated into a few particles, scattered at small angles with small momentum transfers.

This is traditionally explained by the Regge theory [59] of strong interactions as scattering by *pomeron* exchange. The pomeron is not a particle, but a so-called Regge trajectory, and has the quantum numbers of the vacuum. Regge theory is not a quantum field theory, but stems from the time before QCD when it was thought that a quantum field theoretical description of the strong interaction was not possible. It is rather a mathematical analysis of scattering amplitudes treating them as analytic functions of the kinematical variables, imposing certain general

conditions, and thus being able to predict the form of scattering cross sections.

Nowadays one also talks about *hard diffraction*, see [60, 61, 62, 63], where there is a hard interaction, such as jet or W production, involved in the process. The experimental signature is striking: an incoming proton is left intact by the interaction and keeps most of its initial momentum, and in addition there is a hard interaction similar to a normal hard interaction event. This was predicted for pp and ep collisions by Ingelman and Schlein [60] and later discovered experimentally by the UA8 collaboration at CERN [64].

The traditional explanation for this phenomenon, suggested in [60], is that an initial proton emits a pomeron with a small longitudinal momentum fraction, and there is a subsequent hard interaction of this pomeron with the other beam particle; the beam particle is said to be diffractively dissociated. The pomeron is seen as a partonic system with a parton distribution function, and there is a flux of pomerons from the proton. In diffractive DIS, the pomeron is emitted from the proton, which survives, and there is a deep inelastic scattering on the pomeron. One can express the cross section analogously to the DIS cross section, introducing the *diffractive* structure function F_2^D [65]

$$\frac{d\sigma}{dx\,dQ^2\,dx_{I\!P}\,dt} = \frac{4\pi\alpha^2}{xQ^4}\left(1 - y + \frac{y^2}{2}\right) F_2^D(x, Q^2, x_{I\!P}, t). \tag{4.1}$$

Here x is the normal Bjorken-x, $x_{I\!P}$ is the pomeron longitudinal momentum fraction, and t is the momentum transfer squared from the proton. As it stands, this does not assume anything about the mechanism behind the process, but in the above picture of pomeron exchange, F_2^D is assumed to be *Regge-factorised* as

$$F_2^D(x, Q^2, x_{I\!P}, t) = f_{I\!P/p}(x_{I\!P}, t)\, F_2^{I\!P}(\beta, Q^2), \tag{4.2}$$

where $F_2^{I\!P}$ is the structure function of the pomeron, $f_{I\!P/p}$ is the pomeron flux [66], and $\beta = x/x_{I\!P}$ is the Bjorken-x of DIS on the pomeron. This is illustrated in Fig. 4.1.

The reaction can be written as $ep \to eXp$, instead of the normal DIS $ep \to eX$. The surviving proton is referred to as a leading proton, and DIS on the pomeron produces the system X. Because of kinematics the X-system and the leading proton will be separated by a large *rapidity gap*, that is, a large region in rapidity where no particles are produced. One often talks about rapidity gaps in connection with diffraction, as one of the experimental signatures. Another complementary signature is the direct measurement of the leading proton.

Diffraction is fundamentally connected to colour singlet systems. The pomeron is an exchanged colour singlet, but there are other ways of ob-

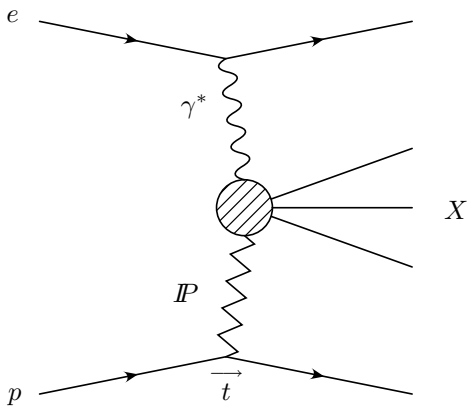


Figure 4.1: Diffractive deep inelastic scattering by pomeron exchange. The zigzag line represents the pomeron $I\!P$ emitted from the proton, and the blob represents the photon–pomeron interaction producing the system X.

taining effective colour singlets, such as exchange of two or more gluons. The singlet nature of the exchange is necessary because when a colour charged particle is exchanged, the proton remnant is no longer in a singlet state and cannot form a proton. Or seen alternatively, non-singlet colour exchange leads to a colour connection between the interacting particles, and they will therefore be connected with colour fields and the hadronisation process will fill the region between them with particles, so there will be no rapidity gap.

4.2 What is low-x physics?

The original interest in low-x physics came from the observation at HERA [43] that the structure function F_2 shows a steep rise as x decreases to very small values, as had been predicted [67].

The BFKL equation, mentioned in Chapter 3, in the leading logarithmic approximation (LLA) resums large terms $\alpha_s^n \ln^n(1/x)$ and is written in terms of the *unintegrated* gluon distribution $f(x, k_T^2)$, from which the "normal" integrated gluon distribution is obtained as

$$xg(x, Q^2) = \int^{Q^2} \frac{dk_T^2}{k_T^2} f(x, k_T^2). \tag{4.3}$$

The LLA BFKL equation can be solved analytically (see e.g. [42, 68, 69])

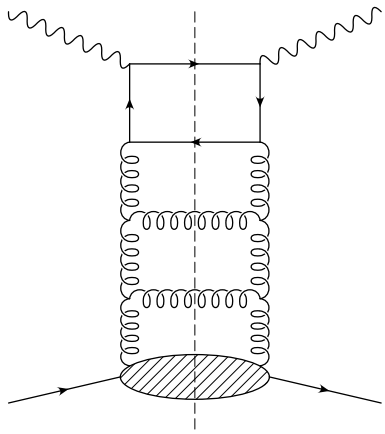


Figure 4.2: Cross section for deep inelastic scattering with gluon ladder, or equivalently, the amplitude for exchanging a colour singlet gluon ladder between the virtual photon and the proton. This amplitude at zero momentum transfer is related by the optical theorem to the total cross section for small-x DIS with gluon radiation, indicated by the cut line.

if one assumes a fixed α_s, giving the behaviour

$$f(x, k_T^2) \sim x^{-\lambda}, \quad \lambda = \frac{N_c\,\alpha_s}{\pi} 4 \ln 2. \tag{4.4}$$

For $N_c = 3$ and a reasonable α_s, $\lambda \approx 0.5$, which is rather large, but the value of λ decreases if the calculation is made in the next-to-leading logarithmic approximation (NLLA), see e.g. [70]. The BFKL prediction contains several uncertainties [68, 69]) but is not inconsistent with the data. The DGLAP prediction, on the other hand, fits the data down to surprisingly small x, so it is hard to say whether any effects of the BFKL resummation can be seen in the data.

Other observables such as forward jet and pion production, large rapidity gaps between two high-p_T jets, and diffractive production of vector mesons in photoproduction have been suggested as better ways of investigating BFKL dynamics. There are also many other issues involved, such as saturation of partons in the proton at small x, and the high energy behaviour of QCD amplitudes.

As a side remark, the field of research is usually referred to as "low-x physics", although it is not at all only concerned with the situation of small Bjorken-x in Deep Inelastic Scattering. Perhaps a more suitable description is that it treats the more general problem of QCD in the high energy limit — that is the limit $s/s_0 \gg 1$ when the centre-of-mass

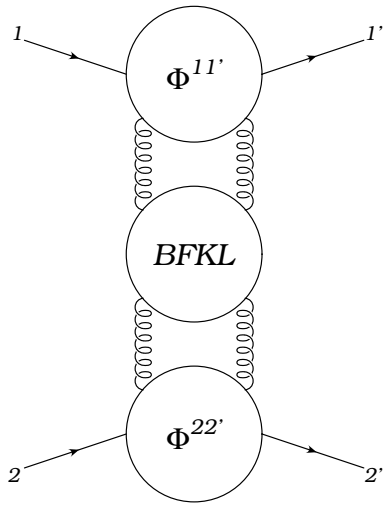


Figure 4.3: Amplitude for scattering of two particles $12 \to 1'2'$ by colour singlet BFKL pomeron exchange.

energy $\sqrt{s}$ is much larger than any other scale $\sqrt{s_0}$ in the problem. This is the case for small x: in DIS one can write $W^2/Q^2 \approx 1/x - 1 \to 1/x$ for very small x.

4.3 BFKL physics

The BFKL equation is more general than just calculating the gluon distribution function; it is really a way of calculating scattering of two objects by exchange of a colour singlet gluon ladder (also called the BFKL pomeron because it has vacuum quantum numbers and gives rise to Regge behaviour of the amplitude). The diagram in Fig. 4.2 is used for calculating Deep Inelastic Scattering at low x, where gluons dominate in the proton. It shows the amplitude for the elastic process $\gamma p \to \gamma p$ with exchange of a ladder of gluons as described by the BFKL equation.[1] Via the optical theorem, this amplitude in the forward direction is related to the total cross section, as indicated by the "cut" line. That is, the diagram can also be regarded as the amplitude squared for the diagram obtained by cutting along this line.

[1] A technical point here is that the vertical gluons in the ladder are so-called Reggeised gluons, and each three-gluon vertex is an effective vertex ("Lipatov vertex"). This effectively includes virtual corrections and crossed horizontal gluons.

The BFKL amplitude for scattering of two particles 1 and 2 is factorised into three parts, as shown in Fig. 4.3, and can symbolically be written

$$\mathcal{A} = \Phi^{11'} \otimes K_{\mathrm{BFKL}} \otimes \Phi^{22'}, \tag{4.5}$$

where $\Phi^{11'}$ is the $1 \to 1'$ *impact factor*, i.e., a function calculated from Feynman diagrams describing the transition $1 \to 1'$ with two gluons attached, K_{BFKL} is a function containing the BFKL evolution of the gluon ladder, and $\otimes$ denotes a convolution integral. K_{BFKL} is obtained by solving the BFKL equation.

The equation was solved for zero momentum transfer directly after its conception, leading to the result (4.4). The solution for non-zero momentum transfer was more elusive but was finally obtained by Lipatov [71], eight years later, by using the 2-dimensional conformal invariance of the equation in transverse coordinate space. The solution for the amplitude is an expansion in the basis of conformal eigenfunctions and reads

$$\mathcal{A} = \frac{1}{(2\pi)^2} \sum_{n=-\infty}^{\infty} \int_{-\infty}^{\infty} d\nu \, \frac{\nu^2 + n^2/4}{[\nu^2 + (n-1)^2/4][\nu^2 + (n+1)^2/4]} \times \exp[\omega_n(\nu) y] \, I_{n,\nu}^{11'}(q) \, (I_{n,\nu}^{22'}(q))^*, \tag{4.6}$$

where

$$\omega_n(\nu) = \frac{2N_c \, \alpha_s}{\pi} \, \mathrm{Re} \, (\psi(1) - \psi(1/2 + |n|/2 + i\nu)) \tag{4.7}$$

gives the eigenvalues of the BFKL kernel, $I_{n,\nu}^{11'}$ is a projection of the impact factor $\Phi^{11'}$ onto the conformal eigenfunction with eigenvalue $\omega_n(\nu)$, q is the momentum of the BFKL pomeron, y is the rapidity, and the integer n is referred to as the conformal spin. ψ is the digamma function, $\psi(x) = \Gamma'(x)/\Gamma(x)$.

In the case of elastic quark–quark scattering, the impact factors can be taken as $\Phi^{qq} = 1$ and in the approximation that only the $n = 0$ term is included[2], the amplitude (4.6) takes a particularly simple form known as the Mueller–Tang approximation [72]. This amplitude can be used to calculate the cross section for gaps between jets, but the result does not agree with Tevatron data. This problem is solved in Papers V–VII of this thesis, where it is shown that a full numerical solution of the BFKL equation exhibits a rather different behaviour of the amplitude, and can reproduce the data, see also [73, 74].

The formula (4.6) has also been used in Papers VIII–X of this thesis to compute production of vector mesons V at large momentum transfer,

[2]The function $\omega_n(\nu)$ is negative for $n \neq 0$ and $e^{\omega_n(\nu)} \to 0$ as $n \to \infty$, so the higher n terms in the series are suppressed for large y.

$\gamma p \to VX$, keeping all n in the summation. The impact factor $\Phi^{\gamma V}$ contains a model for the transition from photon to vector meson, which is factorised into a perturbative part, describing the photon splitting into a quark–antiquark pair that couples to the two gluons, and a non-perturbative part, where the wave function of the meson enters. This wave function has previously been computed using QCD sum rules, see the references of Paper X. In other words, the hadronisation is included already in the calculation of the hard interaction, in contrast to the hadronisation described in Chapter 3.

4.4 Models for diffraction

Existing HERA data on diffractive DIS are rather well described by fitting the diffractive structure function to data and performing DGLAP evolution, i.e., the pomeron model works well. But there are some problems with this picture, both conceptual and phenomenological.

To start with, there is no universality, in the sense that the diffractive structure function extracted from the data cannot be used to predict diffractive cross sections at hadron colliders; calculated cross sections are 5–10 times too large. This can be attributed to the breakdown of the factorisation formula (4.2) — neither the pomeron structure function nor the pomeron flux are universal or can be measured separately. In hadron–hadron collisions there are also soft underlying interactions that contribute to destroying rapidity gaps — for diffractive processes the normal collinear factorisation formula (2.27) is not valid, because its proof relies on summing over all possible final states to achieve certain cancellations. When one requires a rapidity gap in the final state such cancellations do not occur. In diffractive DIS, however, factorisation holds [75], because the exchanged photon does not provide a colour charged system with which there can be soft interactions.

Another problem is that it may not be appropriate to consider the emission of the pomeron, a system of partons, from the proton as a separate entity that has separate evolution. Instead there should be numerous soft interactions between the proton and the pomeron.

Finally, although the pomeron model may be an effective description and initially was a large step forward in the understanding of diffraction, it does not give a proper insight into the QCD processes underlying diffraction.

Therefore several alternative models for hard diffraction have been proposed, based on perturbative or non-perturbative QCD. It is clear that there is an interplay between the perturbative and non-perturb-

ative regimes, because the process at the proton vertex is soft, with a small momentum transfer, while there is also a hard scattering calculable in pQCD.

4.4.1 Dipole models

Any space-time picture of a collision process is frame dependent. The view of deep inelastic scattering described earlier, that a virtual photon penetrates the proton and scatters on a parton, is formulated in a Lorentz frame where the proton has very large momentum. The description of the scattering process in the rest frame of the proton is very different: the photon now fluctuates (at lowest order) into a quark–antiquark pair a long time before the collision, and it is this *colour dipole* that scatters off the proton. The lifetime of the fluctuation is inversely proportional to Bjorken-x, so this is a useful way of treating small-x processes. The transverse distance between the quark and antiquark changes very slowly, meaning that the scattering can be treated as the scattering of a frozen dipole on the proton. This is shown schematically in Fig. 4.4.

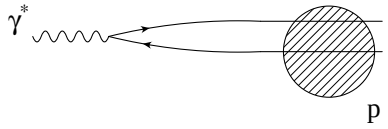


Figure 4.4: Photon fluctuating into a quark–antiquark colour dipole, which scatters off a proton.

The cross section can be written as a convolution of the photon wave function squared, i.e., the probability for the photon to fluctuate into the quark–antiquark pair, and the cross section for the scattering of the dipole on the proton. The wave function is calculated in QED, and the dipole cross section can be derived from various models.

This approach has been rather successful for both inclusive DIS at small x and for diffractive DIS. One possible model for the dipole cross section is two-gluon exchange (see e.g. [63]), as shown in Fig. 4.5. Differential cross sections for diffractive two-jet production as well as diffractive vector meson production $\gamma p \to Vp$ can be described by two-gluon exchange.

Another model is the saturation model [76] of Golec-Biernat and Wüsthoff, which phenomenologically implements the very important saturation of the gluon density at small x, when gluons become densely packed and therefore start recombining, and multi-gluon exchanges can occur. This model is able to describe well not only the HERA diffractive

DIS data on e.g. the diffractive structure function, but also the structure function F_2, i.e., the total cross section, down to very small x and Q^2.

A third example is the semi-classical model of Buchmüller and Hebecker (see [62] and references therein) that treats the scattering of the dipole off the soft, non-perturbative, field of the proton through many-gluon exchanges.

One thing these dipole models have in common, however, is that they are formulated for diffraction in ep or γp collisions, while diffraction in proton–proton collisions cannot be treated. The question of non-universality of the pomeron can therefore not be addressed.

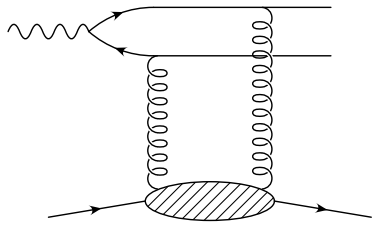


Figure 4.5: Explicit realisation of dipole cross section in terms of two-gluon exchange.

4.4.2 Soft Colour Interactions

An alternative, the Soft Colour Interaction (SCI) model, was proposed by Edin, Ingelman and Rathsman [55] as a different mechanism for rapidity gaps in DIS. Subsequently another model, the Generalised Area Law (GAL), similar in spirit but with a different basic assumption, was introduced by Rathsman [56].

The basic idea of these models is that the hard interaction of a diffractive event is of the same type as in a normal event; the colour singlet exchange that leads to rapidity gaps (and leading protons) is effected by additional soft, non-perturbative, interactions of the partons produced in the hard interaction with the soft colour field of the proton. As discussed in Chapter 2, a hard interaction takes place on a very small space-time scale compared to the size of the proton, and can be viewed as taking place *inside* the proton. The partons produced are therefore surrounded by a soft colour field, and the coupling is large, so one may expect abundant interactions.

The models are implemented in the Monte Carlo programs LEPTO and PYTHIA, modelling the interactions as non-perturbative gluon exchanges with negligible momentum transfer, whose only effect is ex-

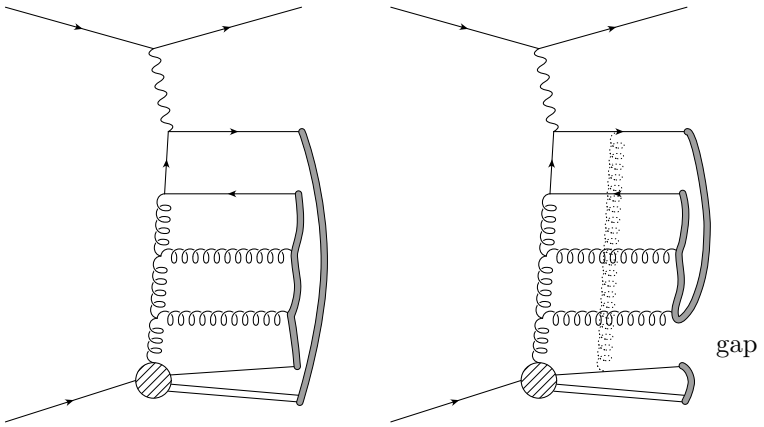


Figure 4.6: String drawing in a DIS event in the original configuration (left), and after one soft colour interaction, shown as a dotted gluon (right). In the right diagram there is a possibility for a rapidity gap.

changing the colours of the interacting partons. The probability for such a non-perturbative exchange cannot be computed and is therefore in the SCI model taken as a constant probability P, whose value is adjusted to fit the data on F_2^D. The important consequence of these exchanges is that such a switch of colours of two partons leads to reconnections of Lund model string pieces, changing the way the event is hadronised. This is illustrated in Fig. 4.6, where the string configuration in an example DIS event is shown before and after such an exchange, leading to a possible rapidity gap.

In the GAL model the interactions are instead pictured as soft exchanges between the strings themselves, leading to the same type of reconnections. This is founded on the notion that a parton picture of very soft interactions is not appropriate because the exchanged gluons are so soft that they cannot resolve the individual partons. The probability for an exchange between two string pieces is given by a generalisation of the string area law [32], which says that string configurations with large "area" are exponentially suppressed. The area of a string means the area swept out in energy-momentum space, and for a string between partons i and j it is here defined as $A_{ij} = 2(p_i \cdot p_j - m_i\, m_j)$. In the ordinary Lund model strings can decrease their area by breaking and producing hadrons; this is generalised so that the area can also be decreased by interactions between two strings, reconnecting into a configuration with

smaller total area. This gives the probability for an interaction

$$P = P_0 \left(1 - \exp(-b\Delta A)\right), \tag{4.8}$$

where ΔA is the change in area of the two strings introduced by the corresponding reconnection, and P_0 and b are parameters fitted to F_2^D and e^+e^- data. Thus, in this model, only reconnections that decrease the string area can occur.

The SCI and GAL models have been successful in reproducing diffractive data from HERA, and also the quite different process of quarkonium production [77], where the soft interactions turn a colour octet quark–antiquark pair into a colour singlet, allowing the formation of a meson.

In Papers I and II these models have been applied to hard diffraction at the Tevatron, and it has been shown that *all* available data from the CDF and DØ collaborations (see Paper II and references therein) can be described with the same parameter values as for the HERA data. This is in contrast to all other models for hard diffraction in DIS, that either cannot be formulated for hadron collisions, or fail to reproduce the data unless modifications tailored to fit the data are introduced. As an example, the pomeron model gives correct cross sections if the pomeron flux is renormalised so that its integral cannot exceed unity [78].

A new development is the idea that diffraction can be used in the search for the Higgs boson at hadron colliders, because diffractive events have rapidity gaps that make them "cleaner", with less hadronic activity, and thus makes it easier to see the decay products of the Higgs. A particularly clean type of event is where there are *two* leading protons, or equivalently, two rapidity gaps, known as double pomeron exchange (DPE). In Papers III and IV the SCI and GAL models have been further applied to predicting cross sections for diffractive Higgs boson production at the Tevatron and the LHC. The cross section predictions in the literature vary by orders of magnitude (see Papers III and IV and references therein), with our predictions at the low end of the scale. We predict that Higgs will be unobservable in diffractive events at the Tevatron, but many single diffractive and a few DPE Higgs events may be seen at the LHC.

Chapter 5

Summary of papers

Paper I

The overall ratios of cross sections of single diffractive to non-diffractive production of W bosons and dijets, as well as the ratio of DPE to single diffractive dijet cross sections were calculated using the soft colour interaction models SCI and GAL. The results were compared to Tevatron data and good agreement was found.

Paper II

A complete study of hard diffraction at the Tevatron was made, and all available data from CDF and DØ were reproduced, using the SCI and GAL models. This included single diffractive W, Z, $b\bar{b}$, dijet and J/ψ production and DPE production of dijets. Comparison was also made to Pomeron model results confirming that such models, tuned to data from diffractive DIS at HERA, do not work at the Tevatron without major modifications, in contrast to the SCI and GAL models that *were* tuned to HERA data *and* work at the Tevatron. The novel phenomenon of apparently different branching ratios of W boson decays in a diffractive sample was predicted.

Paper III

The models of Papers I and II were used to predict cross sections for diffractive Higgs boson production at the Tevatron and the LHC. This has been suggested as a good candidate for a Higgs discovery channel

because of the cleaner events with less hadronic activity, making it easier to reconstruct the Higgs decay products. We found unobservably small cross sections at the Tevatron, both for single diffraction and DPE, but at the LHC a substantial number of single diffractive and a handful of DPE events may be observed.

Paper IV

Comparison of SCI and GAL model results to the data on dijet mass fractions was made, and diffractive Higgs production was further investigated. It was found that for the clean Higgs decay channel $H \to \gamma\gamma$ the background from prompt photon production is always dominating, so other decay modes must be used. Prompt photon production is, however, an interesting way of testing the models for Higgs production because of the similar production mechanism, and predictions that can be tested in future experiments were made.

Paper V and VII

A complete numerical solution of the BFKL equation for elastic parton–parton scattering at large momentum transfer from hard colour singlet exchange was obtained. The equation was solved both in the leading logarithmic approximation and with added non-leading corrections in the form of a running coupling in the gluon ladder and the inclusion of the so-called consistency constraint.

The obtained solution was implemented in the PYTHIA event generator in order to get a complete description of events, including multiple interactions and soft colour interactions for the description of the underlying event and the survival probability of rapidity gaps.

Simulations were made, reproducing the Tevatron data on large rapidity gaps between two high-p_T jets. These data were previously believed to not be possible to explain by BFKL dynamics, based on a comparison with the Mueller–Tang approximation. We show here that this is not a good approximation in the experimental regime of non-asymptotically large rapidity difference between the jets.

Paper VI

This paper describes the program package with the implementation of hard colour singlet exchange in PYTHIA from Paper VII. The theoretical background is reviewed and detailed instructions for program usage are given.

Paper VIII

An analytical solution to the BFKL equation in the leading logarithmic approximation was obtained for the process $\gamma q \to Vq$ where q is a parton and V is a heavy vector meson (i.e. J/ψ or Υ). The solution includes all conformal spins, thus going beyond the earlier Mueller–Tang approximation. The necessary conformal impact factors were computed. Furthermore a numerical solution including the same non-leading corrections as in Paper VII was obtained.

The influence of non-zero conformal spins was studied, and it was shown that in order to get the correct two-gluon exchange result in the zero rapidity limit many conformal spins need to be included in the calculations.

These solutions were used to calculate diffractive J/ψ production at large momentum transfer in γp collisions, and good agreement with ZEUS data was obtained.

Paper IX

Helicity amplitudes for $\gamma q \to Vq$ for light vector mesons (i.e. $V = \rho, \phi, \omega$) of different polarisation were computed for chiral-even wave functions in the massless quark limit using the leading logarithmic BFKL equation for all conformal spins. The results were used to successfully reproduce ZEUS data on the cross section and spin density matrix elements for diffractive ρ meson photoproduction at large momentum transfer. The results are, however, sensitive to large endpoint contributions, when one of the quarks in the vector mesons carries most of the longitudinal momentum, and therefore the integration over the momentum fraction has to be arbitrarily cut off.

Paper X

In this paper the light vector meson helicity amplitudes were calculated for non-zero quark mass using the BFKL equation. This provides a complete leading logarithmic QCD analysis of the $\gamma q \to Vq$ process at large momentum transfer. Chiral-odd amplitudes, where the photon couples non-perturbatively to a quark–antiquark pair in a chiral-odd configuration, are now included in addition to the chiral-even amplitudes. We suggest that a perturbative treatment of the chiral-odd contribution using the constituent quark mass reasonably approximates the effects of chiral symmetry breaking. The obtained analytical results are used in a forthcoming paper were numerical results will be compared to experimental data. This analysis should provide critical tests of both the chiral-odd photon wave function and the BFKL dynamics.

Chapter 6

Conclusions and outlook

We have seen that there is a rich variety of interesting, intricate and important problems in Quantum chromodynamics, including the most general question in QCD — the question about confinement and non-perturbative QCD. The solution to this question is worth at least a million dollars [79], and is one of the great unsolved problems in particle physics.

There are other outstanding problems: The search for the Higgs boson, predicted by the Standard Model but not seen experimentally, is both an experimental and theoretical challenge, although the main issue is higher energy accelerators to achieve high enough centre-of-mass energies. Another problem is a question about the Standard Model itself: is it the low-energy limit of some other, more general theory? There are many free parameters in the theory that cannot be predicted, and the general structures are postulated rather than explained — the left-handed fermions, the $V - A$ structure of weak interactions, the three generations of fermions, and the SU(3) and SU(2)$\otimes$U(1) gauge groups. There is a widespread belief in the scientific community that the answer to the above question is "yes", and that if we are lucky we may start to see some signs of this when the LHC starts its operation in 2007.

But in the meantime it is important to gain a thorough understanding of the Standard Model, so that any deviations from its predictions can be clearly identified. This holds in particular for QCD, because at the high energies of the LHC there will be an enormous amount of strong interactions, and a high degree of control of the theory is required.

The LHC will also provide an excellent testing ground for QCD — it will be able to go to higher energies, higher momentum transfers, and

smaller x than before, providing new interesting opportunities for testing the theory.

In this thesis several aspects of QCD have been studied in the context of colour singlet exchange processes. Papers I–IV regard models for soft, non-perturbative interactions following a hard interaction, leading to effective colour singlet exchange that describes diffractive hard scattering at hadron colliders. The phenomenological success of these models for all kinds of hard diffractive data, as well as other types of observables such as quarkonium production, gives a hint to what kind of processes are responsible for the observed phenomena, and points to what kind of phenomena that have common explanations.

Another topic has been studied in Papers V–VII, namely the novel type of events with a rapidity gap between jets. There are now large momentum scales on both sides of the gap, so one might expect this process to be described by perturbation theory. This turns out to be the case, in the form of BFKL calculations, and these events are therefore said to be the result of hard colour singlet exchange. But also in this context there are soft processes, underlying the hard interaction, that cannot be described by perturbation theory. These processes can potentially destroy rapidity gaps and have to be treated properly, here in the form of multiple interactions between the colliding protons and soft colour interactions in the final state.

The third topic, studied in Papers VIII–X, is diffractive production of vector mesons at large momentum transfer. This is different from ordinary diffraction in that the momentum transfer between the proton and the diffractive system is large, so the proton is dissociated in the interaction. Therefore this process too is attributed to hard colour singlet exchange, i.e., exchange of a gluon ladder according to the BFKL equation. And also here there is a soft, non-perturbative component; the formation of the meson from the photonic fluctuation into a quark–antiquark pair.

This interplay between soft and hard interactions in QCD is possible because of the factorisation properties of the theory that underlie all calculations. It is therefore possible to isolate different parts of a problem, to gain understanding of the pieces that are not understood.

The studies in this thesis have contributed to new understanding of the interactions of systems of gluons in different physical processes, advancing our understanding of QCD as a whole.

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