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Bianchi type-*III*, V and VI_0 generalized ghost dark energy models with polytropic gas in Brans-Dicke theory of gravitation

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> **Abstract:** This paper elaborates the homogeneous and anisotropic Bianchi type-*III*, *V* and VI_0 space times filled with matter and generalized ghost dark energy components in the framework of Brans-Dicke (Phys. Rev. 124, 925 1961) theory of gravitation. The behavior of generalized ghost dark energy for Bianchi type-*III*, *V* and VI_0 background is discussed. The solutions of the field equations obtained using following two assumptions: (i) shear scalar of the model is proportional expansion scalar, which leads to a relation between the metric, (ii) scalar field in the Brans-Dicke theory is a function of average scale factor of the models. We have further established a correspondence between the generalized ghost dark energy models with the polytropic gas dark energy model. We also reconstructed the potential and dynamics of the scalar field. We discuss some physical and geometrical properties of the obtained models which are found to be consistent with recent observation.

Keywords: Bianchi type - III, V and VI_0 metrics, Ghost dark energy, Polytropic gas, Brans-Dicke theory

1. Introduction:

Recent observational cosmology sets up the evidence based on various measurements that universe is experiencing an accelerated expansion phase Perlmutter et al. [1]; Riess et al. [2];

Amanullah et al. [3]. This is attributed to an unknown exotic component with huge negative pressure causes repulsion among galaxies which is popularly known as dark energy (DE). The simplest model compatible with all cosmological observations is a ACDM model but it suffers issues like fine tuning and cosmic coincidence problems leading to some alternatives to investigate its description. There are two possible ways to describe the current status of the universe: dynamical DE models and the modification of gravity.

First approach deals with modifying the matter part with unchanged gravitational part as quintessence Barreiro et al. [4], phantom Caldwell [5], tachyon Bagla such et al. [6]; Padmanabhan and Choudhury [7], k-essence Armendariz et al. [8], dilatonic ghost condensate Gasperini et al. [9] together with interacting dark energy models such as holographic Horava and Minic [10]; Thomas [11]; Setare [12] and age graphic Wei and Cai [13]. Recently, a new model of dark energy called Veneziano ghost dark energy has been proposed, which supposed to solve the $U(1)_A$ problem in low-energy effective theory of QCD Urban and Zhitnitsky [14]; Ohta [15]; Sheykhi et al. [16]; Ebrahimi and Sheykhi [17]; Feng et al. [18] Indeed, the contribution of the ghosts field to the vacuum energy in curved space or timedependent background can be regarded as a possible candidate for the dark energy. Veneziano ghost exhibits non trivial physical effects in the expanding Universe and these effects give rise to a vacuum energy density $\rho_D \sim \Lambda_{OCD}^3 H \sim (10^{-3} eV)^4$. With $H \sim 10^{-33} eV$ and $\Lambda_{OCD} \sim 100 eV$ we have the right value for the force accelerating the universe today. Energy density of the ghost DE reads as $\rho_G = \alpha H$ where H is Hubble parameter and α is a constant parameter of the model, which should be determined. A generalization of the model Cai et al. [19] also was proposed for which energy density reads as $\rho_{G} = \alpha H + \beta H^{2}$, with α and β constant parameters of the model, which should be determined.

In stellar astrophysics, the polytropic gas model can explain the equation of state of degenerate white dwarfs, neutron stars and also the equation of state of main sequence stars Christensen-Dalsgard [20]. The polytropic gas is a phenomenological model of dark energy. In a phenomenological model, the pressure p is a function of energy density ρ , i.e., $p = -\rho - f(p)$. for $f(\rho) = 0$, the equation of state (EoS) of phenomenological models can cross w = -1, i.e., the cosmological constant model. The idea of dark energy with polytropic gas equation of state has been investigated by Mukhopadhyay and Ray [21] in cosmology. Recently, Adhav [22], Setare and Kamali [23], Taji and Malekjani [24] have investigated polytropic gas in different contexts.

Modified theories of gravity are based on generalized models that came into being by modifying gravitational part of the Einstein-Hilbert action. Scalar-tensor gravity, in particular, Brans-Dicke (BD) theory Brans and Dicke [25] is considered as a successful approach to investigate the cosmic behavior in the ultimate epochs of the universe. Various significant features of this theory include variable gravitational constant, direct coupling of scalar field and geometry, consistency with various physical laws as well as solar experiments bounds Weinberg

[26]; Bertotti et al. [27]; Felice et al. [28]. The Brans-Dicke field equations for combined scalar and tensor field are given by

$$G_{ij} = -8\pi\phi^{-1}T_{ij} - \omega\phi^{-2}\left(\phi_{,i}\phi_{,j} - \frac{1}{2}g_{ij}\phi_{,k}\phi^{,k}\right) - \phi^{-1}(\phi_{i;j} - g_{ij}\phi_{,k}^{,k})$$
(1.1)

and

$$\phi_{k}^{k} = 8\pi (3+2\omega)^{-1}T \tag{1.2}$$

where $G_{ij} = R_{ij} - \frac{1}{2}Rg_{ij}$ is an Einstein tensor, R is the scalar curvature, ω and n are constants, T_{ij} is the stress energy tensor of the matter and comma and semicolon denote partial and covariant differentiation respectively.

Also, we have energy - conservation equation

$$T_{;j}^{ij} = 0$$
 (1.3)

This equation is a consequence of the field equations (1.1) and (1.2).

Several aspects of Brans-Dicke cosmology have been extensively investigated by many authors. Rao and Santhi [29] have discussed Bianchi type-*II*, *VIII* and *IX* magnetized cosmological models in Brans – Dicke theory of gravitation. Rao and Sireesha [30] have studied a some string cosmological models in Brans-Dicke theory of gravitation. Rao et al. [31] have obtained LRS Bianchi type-I dark energy cosmological model in Brans-Dicke theory of gravitation. Reddy et al. [32] have studied Kantowski–Sachs bulk viscous string cosmological model in Brans–Dicke theory of gravitation. Recently Rao et al. [33] have studied FRW holographic dark energy cosmological model in Brans-Dicke Theory of gravitation. Rao and Jayasudha [34] & Santhi et al. [35] have discussed different aspects in this scalar-tensor theory of gravitation.

All mentioned above motivate us to investigate the interacting generalized ghost dark energy (GGDE) model and polytropic gas in the Bianchi type - III, V and VI_0 metric within Brans – Dicke theory of gravitation. The plan of the paper is follows: In sect. 2, we have presented metric and energy momentum tensor. In sect. 3, we find solutions of field equations and then we study the corresponding between the GGDE model and polytropic gas dark energy. In sect. 4, we obtained and discussed some important properties of the models. We have concluded our results in sect. 5.

2. Metric and Energy Momentum Tensor:

We consider a spatially homogeneous Bianchi type-III, V and VI_0 metrics of the form

$$ds^{2} = dt^{2} - A^{2}dx^{2} - B^{2}e^{-2x}dy^{2} - C^{2}e^{-2bx}dz^{2}$$
(2.1)

where A, B and C are functions of t only.

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It represents

Bianchi type-*III* if b = 0Bianchi type-*V* if b = 1Bianchi type- VI_0 if b = -1

The energy momentum tensors for matter and the ghost polytropic dark energy are defined as

$$T_{ij} = \rho_m u_i u_j \tag{2.2}$$

and
$$\overline{T}_{ij} = (\rho_G + p_G)u_iu_j - g_{ij}p_G$$
 (2.3)

Where ρ_m , ρ_G are energy densities of matter and ghost polytropic dark energy and p_G is the pressure of ghost dark energy.

In a co moving coordinate system, we get

$$T_1^1 = T_2^2 = T_3^3 = 0, T_4^4 = \rho_{m \text{ and }} \overline{T}_1^1 = \overline{T}_2^2 = \overline{T}_3^3 = -p_G^2, \overline{T}_4^4 = \rho_G^2$$
 (2.4)

where the quantities ρ_m , ρ_G and p_G are functions of 't' only.

3. Solutions of Field Equations:

Now with the help of (2.3) to (2.9), the field equations (1.1) for the metric (2.1) can be written as

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} + \frac{\omega\dot{\phi}^2}{2\phi^2} + \frac{\ddot{\phi}}{\phi} + \frac{\dot{B}\dot{\phi}}{B\phi} + \frac{\dot{C}\dot{\phi}}{C\phi} - \frac{b}{A^2} = -8\pi\phi^{-1}p_G$$
(3.1)

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{C}}{AC} + \frac{\omega\dot{\phi}^2}{2\phi^2} + \frac{\ddot{\phi}}{\phi} + \frac{\dot{A}\dot{\phi}}{A\phi} + \frac{\dot{C}\dot{\phi}}{C\phi} - \frac{b^2}{A^2} = -8\pi\phi^{-1}p_G$$
(3.2)

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} + \frac{\omega\dot{\phi}^2}{2\phi^2} + \frac{\ddot{\phi}}{\phi} + \frac{\dot{A}\dot{\phi}}{A\phi} + \frac{\dot{B}\dot{\phi}}{B\phi} - \frac{1}{A^2} = -8\pi\phi^{-1}p_G$$
(3.3)

$$\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{C}\dot{A}}{CA} - \frac{\omega\dot{\phi}^2}{2\phi^2} + \frac{\dot{A}\dot{\phi}}{A\phi} + \frac{\dot{B}\dot{\phi}}{B\phi} + \frac{\dot{C}\dot{\phi}}{C\phi} - \frac{(b^2 + b + 1)}{A^2} = 8\pi\phi^{-1}(\rho_m + \rho_G)$$
(3.4)

$$\frac{1}{A^2} \left(\frac{\dot{B}}{B} + b \frac{\dot{C}}{C} - (b+1) \frac{\dot{A}}{A} \right) = 0$$
(3.5)

$$\ddot{\phi} + \dot{\phi} \left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right) = 8\pi \left(3 + 2\omega\right)^{-1} \left(\rho_m + \rho_G - 3p_G\right)$$
(3.6)

$$\dot{\rho}_m + \dot{\rho}_G + 3H(\rho_m + \rho_G + p_G) = 0 \tag{3.7}$$

Here the over head dot denotes differentiation with respect to t.

Bianchi type-III (b = 0) cosmological model:

From equation (3.5), we get $B = c_1 A$

without loss of generality, by taking $c_1 = 1$ we get B = A (3.8)

The field equations (3.1) to (3.6) are four independent (in view of equation (3.8)) equations with six unknowns $B, C, \rho_m, \rho_G, p_G \& \phi$ which are functions of t. Since these equations are highly non-linear in nature, in order to get a deterministic solution we take the following plausible physical condition:

i. The relation between the scalar field ϕ and the scale factor of the universe a(t) given by Johri and Kalyani [36]

$$\phi = \phi_0 a^r \tag{3.9}$$

where ϕ_0 and r > 0 are constants.

ii. The shear scalar is proportional to expansion scalar, which leads to the relation between metric potentials Collins et al. [37]

$$B = C^n \tag{3.10}$$

where n is an arbitrary constant.

From equations (3.2), (3.3), (3.8) and (3.10), we get

$$\frac{\ddot{B}}{B} - \frac{\ddot{C}}{C} + \frac{\dot{B}}{B} \left(\frac{\dot{B}}{B} - \frac{\dot{C}}{C}\right) + \frac{\dot{\phi}}{\phi} \left(\frac{\dot{B}}{B} - \frac{\dot{C}}{C}\right) - \frac{1}{B^2} = 0$$
(3.11)

From equations (3.9)-(3.11), we get

$$A = B = n(k_2t + k_3)$$

$$C = [n(k_2t + k_3)]^{n/n}$$
(3.12)

From equations (3.9) and (3.12), we get

$$\phi = \phi_0 \left[\left[n \left(k_2 t + k_3 \right) \right]^{\frac{2n+1}{n}} e^{-x} \right]^{\frac{1}{3}}$$
(3.13)

where $k_1 = 2n\left(\frac{r}{3}+1\right) + \frac{r}{3}$ and $k_2 = \left(\frac{1}{(n-1)(k_1-n+1)}\right)^{\frac{1}{2}}$ and k_3 is an integrating constant.

The energy density of generalized ghost dark energy Cai et al. [19] is given by

$$\rho_G = \alpha H + \beta H^2 \tag{3.14}$$

where *H* is the Hubble parameter, α and β are constants. In the GGDE model, β is a free parameter and can be adjusted for better agreement with observations.

From equations (3.12)-(3.14), we get the generalized ghost dark energy density

$$\rho_G = \frac{\alpha k_2 (2n+1)}{3n(k_2 t + k_3)} + \frac{\beta k_2^2 (2n+1)^2}{9n^2 (k_2 t + k_3)^2}$$
(3.15)

For a universe where dark energy and dark matter are interacting to each other the total energy density ($\rho = \rho_m + \rho_G$) satisfies the continuity equation as following

$$\dot{\rho}_m + \dot{\rho}_G + 3H(\rho_m + \rho_G + p_G) = 0 \tag{3.16}$$

We consider the interaction between dark energy and dark matter. So, the energy densities of dark energy and dark matter do not conserve separately. The equation of continuity of matter is

$$\dot{\rho}_m + 3H\rho_m = Q \tag{3.17}$$

The equation of continuity of GGDE is

$$\dot{\rho}_G + 3H(\rho_G + p_G) = -Q \tag{3.18}$$

where Q represents the interaction between dark matter and GGDE.

In general Q should be a function with units of inverse of time. For convenience we choose $Q = 3H\delta\rho_m$ Wei and Cai [38] where δ is the coupling constant. On putting $\delta = 0$ the equation of continuity reduces to the non interacting case. The kinds of interaction are studied by Amendola [39], Chimento [40].

From equations (3.12) and (3.17), matter energy density is given by

$$\rho_m = k_4 \left(k_2 t + k_3 \right)^{\frac{(2n+1)(\delta-1)}{n}}$$
(3.19)

From equations (3.1) - (3.3), (3.12) & (3.13), we get

The pressure of the generalized ghost dark energy

$$8\pi p_{G} = \frac{\phi_{0}k_{2}^{2} \left[\left[n(k_{2}t+k_{3}) \right]^{\frac{2n+1}{n}} e^{-x} \right]^{\frac{1}{3}}}{\left[n(k_{2}t+k_{3}) \right]^{2}} \left\{ \frac{1}{3k_{2}^{2}} - c_{1}(1-n) - c_{2} \right\}$$
(3.20)
where $c_{1} = \frac{(2n+1)(r+6)}{9}, c_{2} = \left\{ \frac{n(5n-2)}{3} + \frac{r(2n+1)(4n+1)}{9} + \frac{r^{2}(2n+1)^{2}}{27} + \frac{\omega r(2n+1)}{6} \right\}$

Using the equation (3.14) and putting $Q = 3H\delta\rho_m$ in equation (3.18), we get

$$w_{G} = \frac{p_{G}}{\rho_{G}} = -1 - \frac{\alpha \dot{H} + 2\beta H \dot{H}}{3H(\alpha H + \beta H^{2})} - \frac{\delta \rho_{m}}{\rho_{G}}$$
$$= -1 + \frac{\frac{n}{(2n+1)} \left[\alpha + \frac{2\beta k_{2}(2n+1)}{3n(k_{2}t + k_{3})} \right] - \frac{3\delta_{n}k_{4}}{(2n+1)k_{2}} (k_{2}t + k_{3})^{\frac{(\delta-1)(2n+1)}{n}+1}}{\alpha + \frac{\beta k_{2}(2n+1)}{3n(k_{2}t + k_{3})}}$$
(3.21)

The metric (2.1), in this case can be written as

$$ds^{2} = dt^{2} - n^{2}(k_{2}t + k_{3})^{2}[dx^{2} + e^{-2x}dy^{2}] - [n(k_{2}t + k_{3})]^{\frac{2}{n}}dz^{2}$$
(3.22)

Thus the metric (3.22) together with (3.13), (3.15), (3.19), (3.20) and (3.21) constitutes a Bianchi type-*III* interacting GGDE model in Brans-Dicke [27] scalar-tensor theory of gravitation.



Fig.1: Plot of energy density of GGDE and matter versus time t (Gyr) for Bianchi type-III model.

Bianchi type-V(b=1) cosmological model:

From equation (3.5), we get $BC = c_2 A^2$

without loss of generality, by taking $c_2 = 1$ we get

$$A^2 = BC \tag{3.23}$$

From equations (3.1)-(3.3) & (3.23), we get

$$\frac{\dot{A}}{A} = \frac{\dot{B}}{B} = \frac{\dot{C}}{C}$$
(3.24)

From equation (3.24), we get

$$A = B = C = (k_5 t + k_6) \tag{3.25}$$

From equations (3.9) & (3.25) we get

$$\phi = \phi_0 \left[e^{-2x} (k_5 t + k_6)^3 \right]^{\frac{r}{3}}$$
(3.26)

where k_5 and k_6 are integrating constants.

From equations (3.25) & (3.14), we get

The GGDE energy density

$$\rho_G = \frac{\alpha k_5}{(k_5 t + k_6)} + \frac{\beta k_5^2}{(k_5 t + k_6)^2}$$
(3.27)

In this model also we consider the interaction between dark energy and dark matter. So from equation (3.17), the matter energy density

$$\rho_m = k_7 (k_5 t + k_6)^{3(\delta - 1)} \tag{3.28}$$

where k_6 in an integration constant.

From equations (3.1)-(3.3), (3.25) and (3.26), we get The pressure of the generalized ghost dark energy

$$8\pi p_G = \frac{\phi_0 \left[(k_5 t + k_6)^3 e^{-x} \right]_3^r}{(k_5 t + k_6)^2} \left\{ 1 - \left[1 + \frac{r^2}{3} \left(\frac{\omega}{2} + 1 \right) \right] k_5^2 \right\}$$
(3.29)

Using the equation (3.27) and putting $Q = 3H\delta\rho_m$ in equation (3.18), we get

$$w_{G} = \frac{p_{G}}{\rho_{G}} = -1 - \frac{\alpha \dot{H} + 2\beta H \dot{H}}{3H(\alpha H + \beta H^{2})} - \frac{\delta \rho_{m}}{\rho_{G}}$$
$$= -1 + \frac{\left\{\frac{1}{3}\left[\alpha + \frac{2\beta k_{5}}{(k_{5}t + k_{6})}\right] - \frac{\delta k_{7}}{k_{5}}(k_{5}t + k_{6})^{(3\delta - 2)}\right\}}{\left[\alpha + \frac{\beta k_{5}}{(k_{5}t + k_{6})}\right]}$$
(3.30)

The metric (2.1), in this case can be written as

$$ds^{2} = dt^{2} - (k_{5}t + k_{6})^{2} [dx^{2} + e^{-2x}(dy^{2} + dz^{2})]$$
(3.31)

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Thus the metric (3.31) together with (3.26)-(3.30) constitutes a Bianchi type-V interacting GGDE model in a scalar-tensor theory of gravitation proposed by Brans and Dicke [27].



Fig.2: Plot of energy density of GGDE and matter versus time t (Gyr) for Bianchi type-V model.

Bianchi type- VI_0 (b = -1) cosmological model:

From equation (3.5), we get $B = c_3 C$

Without loss of generality, by taking
$$c_3 = 1$$
 we get $B = C$ (3.32)

From equations (3.1) and (3.2), we get

$$\frac{\ddot{A}}{A} - \frac{\ddot{B}}{B} + \frac{\dot{C}}{C} \left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right) + \frac{\dot{\phi}}{\phi} \left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right) - \frac{2}{A^2} = 0$$
(3.33)

To solve the above equation we assume that the shear scalar is proportional to expansion scalar, which leads to the relation between metric potentials

$$A = B^k \tag{3.34}$$

where k is an arbitrary constant.

Using equations (3.9) and (3.34) in (3.33), we get

$$B = C = [k(k_9t + k_{10})]^{\frac{1}{k}}$$

$$A = [k(k_9t + k_{10})]$$
(3.35)

From equations (3.9) and (3.35), we get

$$\phi = \phi_0 \left\{ e^{-2x} \left[k \left(k_9 t + k_{10} \right) \right]^{\frac{k+2}{k}} \right\}^{\frac{k+2}{3}}$$
(3.36)

where $k_9 = \frac{2}{(k-1)(k_8 - k + 1)}$ & k_{10} is an integrating constant.

From equations (3.35) and (3.14), we get

The GGDE energy density as

$$\rho_G = \frac{\alpha (k+2)k_9}{3k(k_9t+k_{10})} + \frac{\beta (k+2)^2 k_9^2}{9k^2 (k_9t+k_{10})^2}$$
(3.37)

Here we consider the interaction between dark energy and dark matter. So from equations (3.17) and (3.35), the matter energy density

The matter energy density

$$\rho_m = k_{11} \left(k_9 t + k_{10} \right)^{\frac{(\delta - 1)(n+2)}{n}}$$
(3.38)

From equations (3.1)-(3.3), (3.35) and (3.36) we get The pressure of the generalized ghost dark energy

$$8\pi p_{G} = \frac{\phi_{0} \left[\left[k(k_{9}t + k_{10}) \right]^{\frac{k+2}{k}} e^{-x} \right]^{\frac{1}{3}}}{\left[k(k_{9}t + k_{10}) \right]^{2}} \left\{ 1 - \left[\frac{(k+2)(r+2)(1-k) + 2k(k-1) + 1}{3} + \frac{(k+2)^{2}}{9} \left[\left(\frac{\omega}{2} + 1 \right) r^{2} + r \right] - \frac{r(k+2)}{3} \right] k_{9}^{2} \right\}$$

$$(3.39)$$

Using the equation (3.37) and putting $Q = 3H\delta\rho_m$ in equation (3.18), we get

$$w_{G} = \frac{p_{G}}{\rho_{G}} = -1 - \frac{\alpha \dot{H} + 2\beta H \dot{H}}{3H(\alpha H + \beta H^{2})} - \frac{\delta \rho_{m}}{\rho_{G}}$$

$$= -1 + \frac{\left\{\frac{k}{(k+2)}\left[\alpha + \frac{2\beta (k+2)k_{9}}{3k(k_{9}t + k_{10})}\right] - \frac{3\delta k k_{11}}{(k+2)k_{9}}(k_{9}t + k_{10})^{\frac{(k+2)(\delta-1)}{k} + 1}\right\}}{\left[\alpha + \frac{\beta (k+2)k_{9}}{3k(k_{9}t + k_{10})}\right]}$$
(3.40)

The metric (2.1), in this case can be written as

$$ds^{2} = dt^{2} - [k(k_{9}t + k_{10})]^{2} dx^{2} - [k(k_{9}t + k_{10})]^{\frac{2}{k}} [e^{-2x} dy^{2} + e^{2x} dz^{2}]$$
(3.41)

Thus the metric (3.41) together with (3.36)-(3.41) constitutes a Bianchi type- VI_0 interacting generalized ghost dark energy in a scalar-tensor theory of gravitation proposed by Brans and Dicke [27].



Fig.3: Plot of energy density of GGDE and matter versus time (Gyr) for Bianchi type-VI₀ model.



Fig.4: Plot of equation of state parameter of GGDE versus time.

Figure 1, 2 and 3 are the plots of energy density of the ordinary matter (ρ_m) and polytropic gas (ρ_G) versus time in accelerating mode of the universe for the Bianchi type-*III*, *V*,

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 VI_0 models respectively. Here we observe that ρ_m and ρ_G are positive decreasing functions of time and are approaches to zero as $t \rightarrow \infty$.

Figure 4 shows that the EoS parameter of GGDE is increasing rapidly in the evolution of universeand varying in phantom region ($w_G < -1$), so we conclude that dark energy has phantom like behavior. Also, we observed that in three models EoS parameter approaches to cosmological constant ($\Lambda = -1$) in the future. So, our model represents a ΛCDM model for the future.

Corresponding between the GGDE and polytropic gas model of dark energy:

The equation of state parameter of polytropic gas is given by

$$p_{pg} = K \rho_{pg}^{1+\frac{1}{\eta}}$$
(3.42)

where *K* and η are the polytropic constant and the polytropic index respectively (Christensen 2004).

The energy density of polytropic gas is defined as

$$\rho_{pg} = (\Re a^{\frac{3}{\eta}} - K)^{-\eta}$$
(3.43)

where \Re is the positive constant of integration and *a* is the scale factor. It can be seen that the polytropic index should be even to obtain positive energy density.

Using the equations (3.42) and (3.43), we find the EoS parameter as

$$w_{pg} = \frac{P_{pg}}{\rho_{pg}} = -1 - \frac{\Re a^{\frac{5}{\eta}}}{(K - \Re a^{\frac{3}{\eta}})}$$
(3.44)

If polytropic gas is treated as an ordinary scalar field then the energy density and pressure of the scalar field are given by

$$\rho_{\Phi} = \frac{\dot{\Phi}^2}{2} + V(\Phi) \tag{3.45}$$

$$P_{\Phi} = \frac{\dot{\Phi}^2}{2} - V(\Phi) \tag{3.46}$$

where the over head dot denotes the derivative with respect to cosmic time 't' only.

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Thus using the equations (3.42)-(3.46), we obtain the scalar potential and the kinetic energy terms for the polytropic gas model as

$$V(\Phi) = \frac{\frac{1}{2}Ba^{\frac{3}{\eta}} - K}{(Ba^{\frac{3}{\eta}} - K)^{\eta + 1}}$$
(3.47)

$$\dot{\Phi}^{2} = \frac{Ba^{\frac{3}{\eta}}}{(Ba^{\frac{3}{\eta}} - K)^{\eta+1}}$$
(3.48)

To establish the correspondence between the GGDE and polytropic gas dark energy model, we compare the GGDE with the energy density of polytropic gas model and also equate the EoS parameters of both the models. In what follows, we assume that the GGDE density is equivalent to the energy density of polytropic gas.

Bianchi type-III (b = 0) cosmological model:

Equating the equations (3.15) and (3.43), we get

$$\rho_G = \frac{\alpha k_2 (2n+1)}{3n(k_2 t + k_3)} + \frac{\beta k_2^2 (2n+1)^2}{9n^2 (k_2 t + k_3)^2} = (\Re a^{\frac{3}{\eta}} - K)^{-\eta}$$
(3.49)

Also comparing equations (3.20) and (3.44), we have

$$w_{G} = -1 + \frac{\frac{n}{(2n+1)} \left[\alpha + \frac{2\beta k_{2}(2n+1)}{3n(k_{2}t+k_{3})} \right] - \frac{3\delta n k_{4}}{(2n+1)k_{2}} (k_{2}t+k_{3})^{\frac{(\delta-1)(2n+1)}{n}+1}}{(k_{2}t+k_{3})} = -1 - \frac{\Re a^{\frac{3}{\eta}}}{(K-\Re a^{\frac{3}{\eta}})}$$
(3.50)

From equations (3.49) and (3.50), we get

$$\Re = \left[\left(n(k_2t + k_3) \right)^{\frac{2n+1}{n}} e^{-x} \right]^{\frac{-1}{\eta}} \left[\alpha + \frac{\beta k_2(2n+1)}{3n(k_2t + k_3)} \right]^{\frac{-1}{\eta}} \left(\frac{(2n+1)k_2}{3n(k_2t + k_3)} \right)^{\frac{-1}{\eta}} \\ X \left\{ \frac{n}{(2n+1)} \left[\alpha + \frac{2\beta k_2(2n+1)}{3n(k_2t + k_3)} \right]^{-\frac{3\delta n k_4}{(2n+1)k_2}} (k_2t + k_3)^{\frac{(\delta-1)(2n+1)}{n}+1} \right\}$$
(3.51)

and

$$K = \left\{ \left[\alpha + \frac{\beta k_2 (2n+1)}{3n(k_2 t + k_3)} \right] \frac{(2n+1)k_2}{3n(k_2 t + k_3)} \right\}^{\frac{-1}{\eta}}$$

$$X \left\{ \frac{\frac{n}{(2n+1)} \left[\alpha + \frac{2\beta k_2 (2n+1)}{3n(k_2 t + k_3)} \right] - \frac{3\delta n k_4}{(2n+1)k_2} (k_2 t + k_3)^{\frac{(\delta-1)(2n+1)}{\eta}}}{\left[\alpha + \frac{\beta k_2 (2n+1)}{3n(k_2 t + k_3)} \right]} - 1 \right\}$$
(3.52)

Using the equations (3.47), (3.48), (3.51) and (3.52) we get the kinetic energy term and the potential of the polytropic gas dark energy model can be obtained as follows

$$\Phi = \int \left\{ \left[\alpha + \frac{\beta k_2 (2n+1)}{3n(k_2 t + k_3)} \right] \frac{(2n+1)k_2}{3n(k_2 t + k_3)} \right\}^{\frac{1}{2}} \left\{ \frac{n}{(2n+1)} \left[\alpha + \frac{2\beta k_2 (2n+1)}{3n(k_2 t + k_3)} \right] - \frac{3\delta n k_4}{(2n+1)k_2} (k_2 t + k_3) \frac{(\delta - 1)(2n+1)}{n}}{\left[\alpha + \frac{\beta k_2 (2n+1)}{3n(k_2 t + k_3)} \right]} \right\}^{\frac{1}{2}} dt$$

(3.53)

$$V(\Phi) = \left\{ \left[\alpha + \frac{\beta k_2 (2n+1)}{3n(k_2 t + k_3)} \right] \frac{(2n+1)k_2}{3n(k_2 t + k_3)} \right\} \left\{ 1 - \frac{\frac{1}{2} \left[\frac{n}{(2n+1)} \left[\alpha + \frac{2\beta k_2 (2n+1)}{3n(k_2 t + k_3)} \right] - \frac{3\delta n k_4}{(2n+1)k_2} (k_2 t + k_3) \frac{(\delta - 1)(2n+1)}{n} - 1}{\left[\alpha + \frac{\beta k_2 (2n+1)}{3n(k_2 t + k_3)} \right]} \right\} \right\}$$
(3.54)

Bianchi type-V(b=1) cosmological model:

Equating the equations (3.27) and (3.43), we get

$$\rho_G = \frac{\alpha k_5}{(k_5 t + k_6)} + \frac{\beta k_5^2}{(k_5 t + k_6)^2} = (\Re a^{\frac{3}{\eta}} - K)^{-\eta}$$
(3.55)

Also comparing equations (3.30) and (3.46), we have

$$w_{G} = -1 + \frac{\left\{\frac{1}{3}\left[\alpha + \frac{2\beta k_{5}}{(k_{5}t + k_{6})}\right] - \frac{\delta k_{7}}{k_{5}}(k_{5}t + k_{6})^{(3\delta - 2)}\right\}}{\left[\alpha + \frac{\beta k_{5}}{(k_{5}t + k_{6})}\right]} = -1 - \frac{\Re a^{\frac{3}{\eta}}}{(K - \Re a^{\frac{3}{\eta}})}$$
(3.56)

From equations (3.55) and (3.56), we get

$$\Re = \left[\left(k_5 t + k_6 \right)^3 e^{-2x} \right]^{\frac{1}{\eta}} \left[\alpha + \frac{\beta \, k_5}{\left(k_5 t + k_6 \right)} \right]^{\frac{-1}{\eta}} \left(\frac{k_5}{\left(k_5 t + k_6 \right)} \right)^{\frac{-1}{\eta}} \left\{ \frac{1}{3} \left[\alpha + \frac{2\beta \, k_5}{\left(k_5 t + k_6 \right)} \right] - \frac{\delta \, k_7}{k_5} \left(k_5 t + k_6 \right)^{(3\delta - 2)} \right\}$$

$$(3.57)$$

and

$$K = \left\{ \left[\alpha + \frac{\beta k_5}{(k_5 t + k_6)} \right] \frac{k_5}{(k_5 t + k_6)} \right\}^{-\frac{1}{\eta}} \left\{ \frac{\frac{1}{3} \left[\alpha + \frac{2\beta k_5}{(k_5 t + k_6)} \right] - \frac{\delta k_7}{k_5} (k_5 t + k_6)^{(3\delta - 2)}}{\left[\alpha + \frac{\beta k_5}{(k_5 t + k_6)} \right]} - 1 \right\}$$
(3.58)

Using the equations (3.47), (3.48), (3.57) and (3.58), we get the kinetic energy term and the potential of the polytropic gas dark energy model can be obtained as follows

$$\Phi = \int \left\{ \left[\alpha + \frac{\beta k_{5}}{(k_{5}t + k_{6})} \right] \frac{k_{5}}{(k_{5}t + k_{6})} \right\}^{\frac{1}{2}} \left\{ \frac{\frac{1}{3} \left[\alpha + \frac{2\beta k_{5}}{(k_{5}t + k_{6})} \right] - \frac{\delta k_{7}}{k_{5}} (k_{5}t + k_{6})^{(3\delta-2)}}{\left[\alpha + \frac{\beta k_{5}}{(k_{5}t + k_{6})} \right]} \right\}^{\frac{1}{2}} dt$$
(3.59)

$$V(\Phi) = \left\{ \left[\alpha + \frac{\beta k_{5}}{(k_{5}t + k_{6})} \right] \frac{k_{5}}{(k_{5}t + k_{6})} \right\} \left\{ 1 - \frac{\frac{1}{2} \left[\frac{1}{3} \left[\alpha + \frac{2\beta k_{5}}{(k_{5}t + k_{6})} \right] - \frac{\delta k_{7}}{k_{5}} (k_{5}t + k_{6})^{(3\delta-2)} \right] \right\} \left[\alpha + \frac{\beta k_{5}}{(k_{5}t + k_{6})} \right]$$
(3.60)

Bianchi type- VI_{θ} (b = -1) cosmological model:

Equating the equations (3.37) and (3.43), we get

$$\rho_G = \frac{\alpha \left(k+2\right)k_9}{3k\left(k_9t+k_{10}\right)} + \frac{\beta \left(k+2\right)^2 k_9^2}{9k^2 \left(k_9t+k_{10}\right)^2} = \left(\Re a^{\frac{3}{\eta}} - K\right)^{-\eta}$$
(3.61)

Also comparing equations (3.44) and (3.50), we have

$$w_{G} = -1 + \frac{\left\{\frac{\left(k+2\right)}{k}\left[\alpha + \frac{2\beta\left(k+2\right)k_{9}}{3k\left(k_{9}t+k_{10}\right)}\right] - \frac{3\delta\,k\,k_{11}}{\left(k+2\right)k_{9}}\left(k_{9}t+k_{10}\right)^{\frac{\left(k+2\right)\left(\delta-1\right)}{k}+1}\right\}\right\}}{\left[\alpha + \frac{\beta\left(k+2\right)k_{9}}{3k\left(k_{9}t+k_{10}\right)}\right]} = -1 - \frac{\Re\,a^{\frac{3}{\eta}}}{\left(K - \Re\,a^{\frac{3}{\eta}}\right)} \tag{3.62}$$

From equations (3.67) and (3.68), we get

$$\Re = \left[\left(k(k_9t + k_{10}) \right)^{\frac{k+2}{k}} e^{-2x} \right]^{\frac{-1}{\eta}} \left[\alpha + \frac{\beta k_9(k+2)}{3k(k_9t + k_{10})} \right]^{\frac{-1}{\eta}} \left(\frac{(k+2)k_9}{3k(k_9t + k_{10})} \right)^{\frac{-1}{\eta}} \left\{ \frac{k}{(k+2)} \left[\alpha + \frac{2\beta k_9(k+2)}{3k(k_9t + k_{10})} \right]^{-\frac{3\delta k k_{11}}{(k+2)k_9}} (k_9t + k_{10})^{\frac{(\delta-1)(k+2)}{k} + 1} \right\}$$

$$(3.63)$$

$$K = \left\{ \left[\alpha + \frac{\beta k_9(k+2)}{3k(k_9t+k_{10})} \right] \frac{(k+2)k_9}{3k(k_9t+k_{10})} \right\}^{-\frac{1}{\eta}} \left\{ \frac{\frac{k}{(k+2)} \left[\alpha + \frac{2\beta k_9(k+2)}{3k(k_9t+k_{10})} \right] - \frac{3\delta k k_{11}}{(k+2)k_9} (k_9t+k_{10}) \frac{(\delta-1)(k+2)}{k} + 1}{\left[\alpha + \frac{\beta k_9(k+2)}{3k(k_9t+k_{10})} \right]} - 1 \right\}$$
(3.64)

Using the equations (3.47), (3.48), (3.63) and (3.64), we get the kinetic energy term and the potential of the polytropic gas dark energy model can be obtained as follows

$$\Phi = \int \left\{ \left[\alpha + \frac{\beta k_9 (n+2)}{3n (k_9 t + k_{10})} \right] \frac{(n+2)k_9}{3n (k_9 t + k_{10})} \right\}^{\frac{1}{2}} \left\{ \frac{n}{(n+2)} \left[\alpha + \frac{2\beta k_9 (n+2)}{3n (k_9 t + k_{10})} \right] - \frac{3\delta n k_{11}}{(n+2)k_9} (k_9 t + k_{10}) \frac{(\delta - 1)(n+2)}{n} + 1}{\left[\alpha + \frac{\beta k_9 (n+2)}{3n (k_9 t + k_{10})} \right]} \right\}^{\frac{1}{2}} dt$$

$$V(\Phi) = \left\{ \left[\alpha + \frac{\beta k_9 (n+2)}{3n(k_9 t + k_{10})} \right] \frac{(n+2)k_9}{3n(k_9 t + k_{10})} \right\} \left\{ 1 - \frac{\left[\frac{n}{(n+2)} \left[\alpha + \frac{2\beta k_9 (n+2)}{3n(k_9 t + k_{10})} \right] - \frac{3\delta n k_{11}}{(n+2)k_9} (k_9 t + k_{10}) \frac{(\delta-1)(n+2)}{n} + 1}{\left[\alpha + \frac{\beta k_9 (n+2)}{3n(k_9 t + k_{10})} \right]} \right\}$$

$$(3.65)$$

(3.66)

This type of potential can produce an accelerating expansion of the universe. Thus one can establish a correspondence between the GGDE and polytropic gas and describe GGDE by making use of polytropic gas.

4. Some other important properties of the models:

Bianchi type-III (b = 0) cosmological model:

The spatial volume for the model is

$$V = (-g)^{\frac{1}{2}} = [n(k_2t + k_3)]^{\frac{(2n+1)}{n}} e^{-x}$$
(4.1)

The average scale factor for the model is

$$a(t) = V^{\frac{1}{3}} = \left[[n(k_2 t + k_3)]^{\frac{(2n+1)}{n}} e^{-x} \right]^{\frac{1}{3}}$$
(4.2)

The expression for expansion scalar θ calculated for the flow vector u^i is given by

1

$$\theta = u^{i}_{,i} = \frac{(2n+1)}{n} \frac{k_2}{(k_2 t + k_3)}$$
(4.3)

and the shear σ is given by

$$\sigma^{2} = \frac{1}{2}\sigma^{ij}\sigma_{ij} = \frac{7}{18}\frac{(2n+1)^{2}}{n^{2}}\frac{k_{2}^{2}}{(k_{2}t+k_{3})^{2}}$$
(4.4)

The deceleration parameter q is given by

$$q = (-3\theta^{-2})(\theta_{,i}u^{i} + \frac{1}{3}\theta^{2}) = \frac{(n-1)}{(2n+1)}$$
(4.5)

The Hubble's parameter H is given by

$$H = \frac{(2n+1)k_2}{3n(k_2t+k_3)}$$
(4.6)

The mean anisotropy parameter A_m is given by

$$A_{h} = \frac{1}{3} \sum_{i=1}^{3} \left(\frac{H_{i} - H}{H} \right)^{2} = \frac{2(n-1)^{2}}{(2n+1)^{2}}, \text{ where } \Delta H_{i} = H_{i} - H \ (i = 1, 2, 3)$$

$$(4.7)$$

Bianchi type-V(b=1) cosmological model:

Thespatial volume for the model is

$$V = (-g)^{\frac{1}{2}} = (k_5 t + k_6)^3 e^{-2x}$$
(4.8)

The average scale factor for the model is

$$a(t) = V^{\frac{1}{3}} = \left[\left(k_5 t + k_6 \right)^3 e^{-2x} \right]^{\frac{1}{3}}$$
(4.9)

The expression for expansion scalar θ calculated for the flow vector u^i is given by

$$\theta = u^{i}_{,i} = \frac{3k_5}{(k_5 t + k_6)} \tag{4.10}$$

and the shear σ is given by

$$\sigma^2 = \frac{1}{2} \sigma^{ij} \sigma_{ij} = \frac{7}{2} \frac{k_5^2}{(k_5 t + k_6)^2}$$
(4.11)

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The deceleration parameter q is given by

$$q = (-3\theta^{-2})(\theta_{,i}u^{i} + \frac{1}{3}\theta^{2}) = 0$$
(4.12)

The Hubble's parameter H is given by

$$H = \frac{k_5}{(k_5 t + k_6)} \tag{4.13}$$

The mean anisotropy parameter A_h is given by

$$A_{h} = \frac{1}{3} \sum_{i=1}^{3} \left(\frac{H_{i} - H}{H} \right)^{2} = 0 \text{ where } \Delta H_{i} = H_{i} - H \ (i = 1, 2, 3)$$
(4.14)

Bianchi type- VI_0 (b = -1) cosmological model:

Thespatial volume for the model is

$$V = (-g)^{\frac{1}{2}} = [k(k_9t + k_{10})]^{\frac{(k+2)}{k}}$$
(4.15)

The average scale factor for the model is

$$a(t) = V^{\frac{1}{3}} = [k(k_9t + k_{10})]^{\frac{(k+2)}{3k}}$$
(4.16)

The expression for expansion scalar θ calculated for the flow vector u^i is given by

$$\theta = u^{i}_{,i} = \frac{(k+2)k_9}{(k_9t+k_{10})} \tag{4.17}$$

and the shear σ is given by

$$\sigma^2 = \frac{1}{2}\sigma^{ij}\sigma_{ij} = \frac{7}{18} \frac{(k+2)^2 k_9^2}{(k_9 t + k_{10})^2}$$
(4.18)

The deceleration parameter q is given by

$$q = (-3\theta^{-2})(\theta_{i}u^{i} + \frac{1}{3}\theta^{2}) = \frac{2(n-1)}{(n+2)}$$
(4.19)

The Hubble's parameter *H* is given by

$$H = \frac{(k+2)k_9}{3(k_9t+k_{10})} \tag{4.20}$$

The mean anisotropy parameter A_h is given by

$$A_{h} = \frac{1}{3} \sum_{i=1}^{3} \left(\frac{H_{i} - H}{H} \right)^{2} = \frac{2(k-1)^{2}}{(k+2)^{2}}, \text{ where } \Delta H_{i} = H_{i} - H \quad (i = 1, 2, 3)$$
(4.21)

5. Discussion and Conclusions:

Among many possible candidates to play the role of dark energy, the GGDE and polytropic gas are considered as a possible source of dark energy that can explain the cosmic acceleration of the universe. Thus, in this work, we have investigated Bianchi type-*III*, *V* and *VI*₀ GGDE models with polytropic gas in Brans-Dicke scalar-tensor theory of gravitation. The solutions of the field equations are obtained by considering a relation between scalar field ϕ and the scale factor of the universe a(t), and some basic geometrical and physical properties of the models are discussed. We have further established a correspondence between the GGDE and the polytropic gas model of dark energy. We have also reconstructed the potential of the polytropic scalar field as well as the dynamics of the scalar field according to the evolution of the ghost dark energy.

We observe that at $t = \frac{-k_3}{k_2}$, $t = \frac{-k_6}{k_5}$, $t = \frac{-k_{10}}{k_9}$ the spatial volume vanishes for Bianchi

type-*III*, Bianchi type-*V*, Bianchi type-*VI*₀ cosmological models for positive values of *n* and *k* and increases with time respectively. The Bianchi type-*III* cosmological model (3.22) has a point type singularity for n > 0 and cigar type singularity for n < 0 at $t = \frac{-k_3}{k_2}$, Bianchi type-*VI*₀

cosmological model (3.41) has point type singularity for k > 0 and cigar type singularity for k < 0 at $t = \frac{-k_{10}}{k_9}$, whereas the Bianchi type-V cosmological model (3.31) has no singularity.

In our work for all the three models, the expansion scalar θ , shear scalar σ and the Hubble parameter *H* vanish for large values of *t*. The energy density of GGDE and matter are positive functions of time throughout the evolution of the universe and will become vanish for large values of *t*. The EoS parameter is varying in phantom region in the evolution of universe in all the three models and finally approaches to cosmological constant, i.e., the models are approaches to Λ CDM model for late times. This is consistent with recent astronomical observations.

From (4.14), one can observe that $A_h = 0$ and this indicates that Bianchi type-*V* cosmological model (3.31) always represents isotropic universe. From (4.7) & (4.21), we can observe that $A_h \neq 0$ for $n \& k \neq 1$ and this indicates that Bianchi type-*III* and VI_0 cosmological models are anisotropic, recent experiments show that there is a certain amount of anisotropy in the universe, hence anisotropic space-times are important. Also, we observe that the deceleration parameter q = 0 for Bianchi type-V cosmological model. Hence the expansion of the isotropic universe proceeds at a constant rate. For Bianchi type-*III* and VI_0 cosmological models, the deceleration parameter q is negative for n < 1 & k < 1. Hence they represent accelerated expansion of the universe, which is consistent with the present day observations.

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