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A Simultaneous Analysis of Z′ and New Fermion Effects. Global Constraints in E_6 and SO(10) Models.

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Abstract

In grand unified theories based on extended (rank >4) gauge groups, the new gauge interactions require new fermions to insure anomaly cancellation. We analyse the two kinds of new physics effects that are naturally present in these models: *i*) the effects of the new neutral gauge bosons; *ii*) the effects of a mixing of the known fermions with the new ones. Concentrating in particular on E_6 and SO(10) models, we perform a global analysis of the electroweak data to constrain simultaneously these two new physics effects, and we pay particular attention to their reciprocal interplay. Our set of experimental results includes the LEP data on the Z decay widths and fermion asymmetries, low energy neutral current experiments (atomic parity violation, ν–scattering), the W boson mass M_W, as well as charged currents measurements like various tests of universality of the W–leptons couplings and the constraints on unitarity of the CKM matrix. We derive stringent bounds on the Z_0–Z_1 mixing ($|\phi| \lesssim 0.02$), on the fermion mixing parameters ($\sin^2\xi_i \lesssim 0.01$ in most cases) and on the mass of the new gauge boson ($M_{Z'} > 170$–350GeV, depending on the model). In many observables the different sources of new physics induce comparable effects that can compensate each other. We confront the results derived by considering only one effect at a time with the results of a joint analysis, and we point out which of the existing bounds are relaxed and which ones remain unaffected.

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1. Introduction

Grand unified theories (GUTs) are an attractive extension of the standard model (SM), allowing to understand the relative values of the gauge couplings, the quantization of the electric charge, as well as successfully predicting some fermion mass ratios. Furthermore, GUTs are a natural outcome of more fundamental theories such as superstrings. As soon as one considers unification groups beyond the 'simplest' SU(5), two general consequences result: *i*) the low energy gauge group often contains extra U(1) factors; *ii*) the fermionic sector is enlarged, since the matter multiplets are in larger representations (**16** for SO(10), **27** for E_6, etc.). Moreover, since with the fermion content of the SM no new anomaly free currents are possible beyond those of SU(5), the presence of new fermions in any extended unified gauge model is a necessary condition to ensure anomaly cancellation.

In many models the masses of the new fermions arise from the same vacuum expectation values (VEVs) that give mass to the extra gauge bosons, and hence are expected to be not much larger than $M_{Z'}$ itself. If new fermions are present, there are good reasons to believe that they will mix with the known states: for the neutral fermions the mixings naturally arise in see-saw models, that provide a nice explanation for the lightness of the known neutrinos. For the charged fermions, a mixing would provide a natural channel for the decay of the heavy ones, avoiding cosmological consequences that would be problematic if the heavy fermions were stable [1,2]. Hence, in the presence of a light (100 GeV$\div$1 TeV) Z' boson, one also expects some light ($\lesssim$ 1 TeV) fermions mixed with the known ones, and the modifications on the electroweak observables induced by the presence of both these kinds of new states may well compete, so that it is important to consider all these effects simultaneously.

From the phenomenological point of view, much effort has been devoted to constrain a Z' boson associated to an extra U(1) surviving below the TeV scale, either via its indirect effects [3–6], or via the limits on direct production [7], resulting in comparable bounds [8]. Besides the direct searches for new particles, strong bounds have been also set on the mixings between the known fermions and heavy new ones, that would affect the couplings of the light states to the standard gauge bosons [9–11]. While rather exhaustive analyses exist where either only the modifications due to an extra neutral boson, or only the mixing effects induced by the new fermions are considered, at present only a few steps have been done in trying to take into account these two effects simultaneously [12-14]. The aim of the present paper is to study in detail the interrelation between these two possible sources of deviations from the SM predictions and, through a global analysis of the present accurate data on electroweak observables, to constrain these effects and to see to what extent the bounds on the Z' parameters (both its mass and its mixing with the Z boson) could be affected by the presence of fermion mixing effects and vice-versa.

As experimental constraints, besides the W boson mass M_W and the low energy neutral current (NC) data (neutrino scattering, atomic parity violation (APV), polarized e-hadron scattering), we have used all the recent LEP data on Z resonance observables (Z partial decay widths, leptonic asymmetries at ± 1 GeV around resonance, b forward-backward asymmetry and the τ polarization asymmetry). Although the Z' couplings are universal, the fermion mixings are clearly not, so that in the present analysis it is necessary to rely on flavor dependent measurements rather than on the flavor averaged values as is usually done in constraining Z' effects alone. Charged currents (CC) measurements, such as $e-\mu-\tau$ universality and the precise test of Cabibbo-Kobayashi-Maskawa (CKM) unitarity, are of primary importance to constrain fermion mixings, and have been included in our analysis as well.

We have taken into account in the theoretical expressions all the relevant QED, QCD and electroweak radiative corrections, that are crucial to reach the present agreement between the SM predictions and the experimental results. Electroweak higher order effects depend on the two still unknown parameters of the SM, the top and Higgs boson masses. In particular, the large dependence on m_t allows to fit the top mass in the SM context, and with our comprehensive set of observables our fit gives $m_t = 116^{+28}_{-35}$ GeV for a Higgs mass $m_H = 100$ GeV. In extended models, the presence of new particles can also give rise to virtual effects that would introduce a dependence on additional unknown parameters (heavy masses, quantity and type of new fermions). As we justify in Section 3, these effects are either very suppressed or already roughly parametrized in terms of the two unknown masses m_t and M_H. In GUTs also the scalar sector is generally enlarged. We will assume that the Higgs bosons only appear in singlets or doublets of weak-isospin (as is the case e.g. in superstring-inspired E_6 models), so that the custodial $SU(2)$ symmetry is preserved at the tree level.

In Section 2 we outline a formalism that allows to describe simultaneously the effects of new gauge bosons and of new fermions mixed with the known ones. After a general discussion, we concentrate on E_6 theories, for which a class of new neutral gauge bosons (Z_1), that covers a wide range of possibilities – including as a particular case the $SO(10)$ model – can be easily defined. These models also involve a rich fermion content, being then well suited to analyse the combined effects of new gauge bosons and new fermions. In Section 3 we describe the procedure that we have followed to confront the theoretical expressions with the experimental data, and we also briefly discuss the effects of new higher order corrections from E_6. All the experimental inputs that we have used, as well as the theoretical expressions for the corresponding observables, are also collected in this section. In Section 4 we present the bounds resulting from our global analysis. We derive individual constraints on the Z' parameters and on each single fermion mixing angle, and we confront these bounds with the corresponding results of joint analyses where the two effects are simultaneously present and cancellations among the different fermion mixings are also allowed. We discuss the interplay among all these different effects in the various observables, and their implications for deriving limits on the different parameters. We identify the bounds that are considerably relaxed by mutual cancellations and those that are left unchanged. Finally, in Section 5, we draw our conclusions.

2. Z′ and new fermion effects in E_6. Formalism

In this section we will introduce the formalism to describe the combined effects due to the presence of a new neutral gauge boson, and of new fermions that could mix with the known ones, paying a particular attention to theories based on E_6 as a unifying gauge group, where both these new physics effects are naturally present. While the presence of a new $U(1)$ factor affects only the neutral current sector, the fermion mixings affect also the charged currents and then the formalism must include this sector as well. We will first focus on the consequences of assuming the presence of a new neutral gauge boson at a relatively low energy and then we will analyse the effects induced on the currents coupled to the vector bosons by mixings between the known fermions and possible new heavy states.

Effects of a new neutral gauge boson

We will restrict our analysis to the case when the group E_6 breaks to the direct product of the SM gauge group $\mathcal{G}_{\rm SM} = SU(2)_L \times U(1)_Y \times SU(3)_C$ and extra $U(1)$ factors defined through the chain:

$$
\begin{array}{l}
\mathrm{E}_6 \longrightarrow U(1)_\psi \times SO(10) \\
\qquad\qquad\qquad \hookrightarrow U(1)_\chi \times SU(5) \\
\qquad\qquad\qquad\qquad\qquad \hookrightarrow \mathcal{G}_{\mathrm{SM}}
\end{array}
\tag{2.1}
$$

Then, since the new charges obey an orthogonality relation with the usual weak isospin and hypercharge, the neutral current lagrangian has the form

$$
-\mathcal{L}_{\mathrm{NC}} = eJ^\mu_{\mathrm{em}} A_\mu + \sum_m g_m J^\mu_m Z_{m\mu} \tag{2.2}
$$

where the Z_m are the neutral massive vector bosons corresponding to the standard Z_0 and to the additional abelian generators ($m \geq 1$) . In the following we will assume that either E_6 breaks directly to rank 5, or that one of the two new bosons is heavy enough so that its effects on the low energy physics are negligible. Then, the additional Z_1 corresponds to some linear combination of the gauge bosons associated to the $U(1)_\psi$ and $U(1)_\chi$ generators in (2.1), that we will parametrize in terms of an angle β ($c_\beta = \cos\beta,\ s_\beta = \sin\beta$)

$$
Z_1 = s_\beta Z_\psi + c_\beta Z_\chi \tag{2.3}
$$

Particular cases that are commonly studied in the literature [5,6,15] correspond to $s_\beta = 1,\ \sqrt{5/8},\ 0$ and are respectively denoted Z_ψ, Z_η and Z_χ models. Z_ψ occurs in $\mathrm{E}_6 \to SO(10)$, while Z_η occurs in superstring models when E_6 directly breaks down to rank 5. A Z_χ boson occurs in $SO(10) \to SU(5)$ (see (2.1)) and, beyond representing a particular case in E_6, it is also present in SO(10) GUTs. The two cases are distinguished by the different content of new fermions (one additional singlet neutrino ν^c_L per family in $SO(10)$ and 12 new states in the E_6 case) and thus they will be treated separately. Another model that is usually analysed in the context of E_6 is the $SU(2)_L \times SU(2)_R \times U_{LR}(1)$ left–right model [5,6,15]. However, since in this case the presence of new $W^\pm_R$ gauge bosons would excessively complicate the formalism for the CC sector, we will not consider it.

If one additional Z_1 exists, in general it will mix with the standard Z_0. This is always the case when the Higgs fields responsible for the breaking of $\mathcal{G}_{\mathrm{SM}}$ down to $SU(3)_c \times U(1)_{\mathrm{em}}$ transforms non–trivially under the new $U(1)$. In the Z_0–Z_1 basis, the general form of the neutral gauge boson mass matrix is

$$
\mathcal{M}^2 = \begin{pmatrix} M^2_{Z_0} & \delta M^2 \\ \delta M^2 & M^2_{Z_1} \end{pmatrix} . \tag{2.4}
$$

This matrix is diagonalized via an orthogonal transformation parametrized by an angle ϕ, and $M^2_{Z_0}$ can then be written in terms of the physical boson masses as

$$M^2_{Z_0} = c^2_\phi M^2_Z + s^2_\phi M^2_{Z'} \tag{2.5}$$

Henceforth the subscripts "0" and "1" will refer to the gauge eigenstate bosons, while unprimed and primed quantities will refer to the physical (mass eigenstates) Z and Z' bosons. According to the breaking (2.1), $M^2_{Z_0}$ (and not the physical Z mass) enters the expression for the weak mixing angle θ_w, *i.e.* $s^2_w \equiv \sin^2\theta_w = 1 - M^2_W/M^2_{Z_0}$. In this paper we are considering models characterised by the fact that the Higgs bosons transform as doublets or singlets under $SU(2)_L$, that are usually referred to in the literature as "constrained" models. We will also follow the more phenomenological approach of treating $M_{Z'}$ and ϕ as independent parameters, so that the bounds derived will hold for any of these models. However, once the Higgs sector is specified for a given model, M^2_Z, $M^2_{Z'}$ and ϕ are not independent quantities and a relation between them can be obtained. Typically this relation is of the form $\phi \simeq kM^2_Z/M^2_{Z'}$, where k depends on the quantum numbers and VEVs of the different Higgs bosons. If a relation between ϕ and $M_{Z'}$ is specified, then the limits on ϕ translate into bounds for $M_{Z'}$ that are usually much stronger than in the general case.

We will normalize the new abelian Q_ψ and Q_χ generators to the hypercharge axis Y so that, assuming a similar renormalization group evolution of the abelian couplings down to the electroweak scale, the same coupling constant g_Y is associated to the hypercharge as well as to the new charges.[1]

Then the coupling constants g_m in (2.2) can be written as

$$\begin{aligned} g_0 &= (4\sqrt{2}G_F M^2_{Z_0})^{1/2} \\ g_1 &= g_0 s_w . \end{aligned} \tag{2.6}$$

The currents that couple with strength g_0 to the physical Z and Z' are:

$$\begin{pmatrix} J^\mu_Z \\ J^\mu_{Z'} \end{pmatrix} = \begin{pmatrix} c_\phi & s_\phi \\ -s_\phi & c_\phi \end{pmatrix} \begin{pmatrix} J^\mu_0 \\ s_w J^\mu_1 \end{pmatrix}, \tag{2.7}$$

where

$$J^\mu_0 = J^\mu_3 - s^2_w J^\mu_{em} \tag{2.8}$$

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[1] In first approximation, a deviation from 1 of the ratio $\lambda = g_1/g_Y$ can be taken into account by rescaling the $Z_0 - Z_1$ mixing angle by $\sqrt{\lambda}$, and the Z' mass by $\lambda^{-1/2}$ [4].

is the usual SM neutral current, written in terms of the neutral isospin and electromagnetic currents, and, according to (2.3), J_1^μ corresponds to the following combination of (spontaneously broken) generators:

$$Q_1 = s_\beta Q_\psi + c_\beta Q_\chi. \tag{2.9}$$

Equations (2.5) and (2.7) summarize the effects of Z_0–Z_1 mixing.

In the present case, the transformation (2.7) cannot be applied straightforwardly to the couplings of the fermions in $J^\mu_{0,1}$. In fact, due to fermion mixings, the couplings of the fermion mass eigenstates appearing in the non–conserved J_3^μ and J_1^μ currents are in general modified with respect to the corresponding couplings of the gauge eigenstates. The rest of this section is devoted to analyse these effects for the couplings of the known fermions.

Effects of new fermions

In the models under investigation each fermion family is assigned to a **27** fundamental representation of E_6, that beyond the 15 standard degrees of freedom, hereafter denoted as *known* fermions, contains 12 *new* additional degrees of freedom: a color–triplet weak–singlet vector quark D_L, D_L^c of electric charge $q_{em} = -1/3$, a vector doublet of leptons $(N\ E^-)_L^T$, $(E^+\ N^c)_L^T$ and two singlet neutrinos ν_L^c and S_L. In the present analysis we will assume three generations of fermions.

As it will become clear in the following, from a phenomenological point of view it is convenient to classify the fermions present in E_6 in terms of their transformation properties under $SU(2)_L$. According to the nomenclature in use [9,10], we denote the particles with unconventional isospin assignments (left–handed singlets or right–handed doublets) as *exotic* fermions. The weak singlets D_L quark, the weak doublets of L-leptons $(E^+ N_E^c)_L^T$, CP conjugate of $SU(2)$ R-doublets, and the singlet neutrinos ν_L^c and S_L, are exotic. All the remaining new fermions, as well as all the standard ones, that have conventional assignments, are referred to as *ordinary*.

Since no new fermions have been directly observed yet, if new states exist they should be rather heavy, ($m_{\rm new} \gtrsim M_Z/2$, with the possible exception of the singlet neutrinos, on which we will comment later). However, since the light mass eigenstates will in general correspond to superpositions of the known and new states, the new fermions could manifest themselves indirectly through a mixing with the known ones. Since $U(1)_{em}$ and $SU(3)_c$ are unbroken, different gauge eigenstates can mix only when they have the same electric and color charges, and hence the electromagnetic and color currents of the mass eigenstates are not modified by fermion mixings. However, since gauge eigenstates with different eigenvalues of the spontaneously broken generators T_3 and Q_1 can mix, the couplings of the mass eigenstates to the Z_0 and Z_1 bosons will in general be affected.

We now discuss the mixing between the known and new states in a general context. In the gauge currents chirality is conserved, and it is then convenient to group the fermions with the same electric charge and chirality $\alpha = L, R$ in a column vector of the known and new gauge eigenstates $\Psi^o_\alpha = (\Psi^o_\mathcal{K}, \Psi^o_\mathcal{N})^T_\alpha$. The gauge eigenstates in Ψ^o_α can be mixed via the mass matrix, and their relation with the corresponding light and heavy mass eigenstates $\Psi_\alpha = (\Psi_l, \Psi_h)^T_\alpha$ is given by a unitary transformation

$$\begin{pmatrix} \Psi^o_\mathcal{K} \\ \Psi^o_\mathcal{N} \end{pmatrix}_\alpha = U_\alpha \begin{pmatrix} \Psi_l \\ \Psi_h \end{pmatrix}_\alpha \qquad \text{where} \qquad U_\alpha = \begin{pmatrix} A & G \\ F & H \end{pmatrix}_\alpha, \qquad \alpha = L, R. \tag{2.10}$$

The submatrices A and F describe the overlap of the light eigenstates with the known and the new states respectively. From the unitarity of U we have

$$A^\dagger A + F^\dagger F = AA^\dagger + GG^\dagger = I, \tag{2.11}$$

and so the matrix A deviates from a unitary one by small light–heavy mixing effects contained in F. Note that we have not introduced an extra index to label the electric charge, and in the following we will treat Ψ^o_α and Ψ_α as generic vectors corresponding to a definite value of q_{em}. In terms of the fermion mass eigenstates the neutral current corresponding to a (broken) generator $\mathcal{Q}$ reads

$$J^\mu_\mathcal{Q} = \sum_{\alpha=L,R} \bar{\Psi}_\alpha \gamma^\mu U^\dagger_\alpha \mathcal{Q}_\alpha U_\alpha \Psi_\alpha. \tag{2.12}$$

The generalization of (2.12) to charged currents, corresponding to non diagonal generators as $T_\pm$, is straightforward. In (2.12) $\mathcal{Q}_\alpha$ represents a generic diagonal matrix of the charges for the chiral fermions. In the present case, since the term proportional to J^μ_{em} in (2.8) is not affected by fermion mixing, we need to consider only the mixing effects in J^μ_3 and J^μ_1. Hence $\mathcal{Q} = T_3$, Q_1 and the elements of the corresponding matrices are given by the eigenvalues t_3 and $q_1 \equiv c_\beta q_\psi + s_\beta q_\chi$. The $q_{\psi,\chi}$ charges for the left handed fermions in the $\mathbf{27}_L$ of E_6 are listed in Table I, and we have $q_{\psi,\chi}(f_R) = -q_{\psi,\chi}(f^c_L)$.

From (2.12) one readily notes that if in one subspace of states with equal electric charge and chirality the matrix $\mathcal{Q}_\alpha$ is proportional to the identity, the current for these fermions is not modified in going to the base of the mass eigenstates, and the corresponding gauge couplings are not affected. This happens for example

TABLE I. Quantum numbers for the left–handed fermions of the fundamental **27** representation of E_6. Abelian charges are normalized to the hypercharge axis according to: $\sum_{f=1}^{27}(Q^f)^2 = \sum_{f=1}^{27}(\frac{Y^f}{2})^2 = 5$.

	S_L	$\binom{E^+}{N^c}_L$ D_L	$\binom{N}{E^-}_L$ D^c_L	ν^c_L	$\binom{\nu}{e}_L$ d^c_L	e^c_L u^c_L $\binom{u}{d}_L$
$6\sqrt{\frac{2}{5}}q_\psi$	4	−2			1	
$6\sqrt{\frac{2}{3}}q_\chi$	0	2	−2	−5	3	−1

in the SM, where for a given electric charge and chirality, the eigenvalues of T_3 are indeed the same, implying in particular the absence (at the tree level) of flavor changing neutral currents (GIM mechanism).

In models with new fermions, the diagonal matrices $\mathcal{Q}_\alpha$ have the general form $\mathcal{Q}_\alpha = \mathrm{diag}(\mathcal{Q}^{\mathcal{K}}_\alpha, \mathcal{Q}^{\mathcal{N}}_\alpha)$. Also, if the gauge group is generation independent all the known states appearing in one vector Ψ^o_α have the same eigenvalues with respect to the generators of the gauge symmetry, and hence $\mathcal{Q}^{\mathcal{K}}_\alpha = q^{\mathcal{K}}_\alpha I$ with $q^{\mathcal{K}}_\alpha = t_3(f^{\mathcal{K}}_\alpha),\, q_1(f^{\mathcal{K}}_\alpha)$. This also happens for the new charged states in E_6, *i.e.* $\mathcal{Q}^{\mathcal{N}}_\alpha = q^{\mathcal{N}}_\alpha I$. In contrast, since more different types of neutrino are present in E_6, for the neutral states appearing in $\Psi^o_{\mathcal{N}}$, $\mathcal{Q}^{\mathcal{N}}_\alpha$ is not proportional to the identity.

Since we are only interested in the indirect effects of fermion mixings in the couplings of the light mass eigenstates we now project (2.12) on Ψ_l, obtaining

$$J^\mu_{l\mathcal{Q}} = \sum_{\alpha=L,R} \bar{\Psi}_{l\alpha}\gamma^\mu \left[q^{\mathcal{K}}_\alpha A^\dagger_\alpha A_\alpha + F^\dagger_\alpha \mathcal{Q}^{\mathcal{N}}_\alpha F_\alpha\right] \Psi_{l\alpha} \tag{2.13}$$

$$= \sum_{\alpha=L,R} \bar{\Psi}_{l\alpha}\gamma^\mu \left[q^{\mathcal{K}}_\alpha + (q^{\mathcal{N}}_\alpha - q^{\mathcal{K}}_\alpha)F^\dagger_\alpha F_\alpha\right] \Psi_{l\alpha}. \tag{2.14}$$

The first form (2.13) is general, and describes the effects of fermion mixings in the neutral–currents of light–states for a wide class of models, while the second form (2.14), obtained via the unitarity relation (2.11), holds when the mixing is with only one type of new states that have the same $q^{\mathcal{N}}_\alpha$ charges, as it is the case for the charged fermions of E_6 and the neutrinos in SO(10).

Ordinary–exotic fermion mixing modifies the isospin currents, and hence affects the couplings to the Z_0 (and $W_\pm$), while, as it is clear from (2.13) and (2.14), a mixing between states of different $q_{\psi,\chi}$ charges will affect the couplings to the Z_1 bosons. It is also worth to stress that, being the $F^\dagger F$ terms in general non–diagonal, besides affecting the strength of the flavor diagonal couplings of the mass eigenstates, the fermion mixings will usually induce also flavor changing neutral currents (FCNC).

i) Charged fermions

We now consider in more detail the different fermions present in E_6. Starting with the charged ones, we denote as $\varepsilon_{m\alpha}(f)$ the *flavor diagonal* chiral ($\alpha = L, R$) couplings of the charged mass eigenstates to the Z_m bosons. From (2.8) and (2.14) and dropping from now on the index $\mathcal{K}$ for the known fermions, we have

$$\begin{aligned} \varepsilon_{0\alpha}(f) &= t_3(f_\alpha) - s_w^2 q_{\rm em}(f) + [t_3(f_\alpha^{\mathcal{N}}) - t_3(f_\alpha)](F_\alpha^\dagger F_\alpha)_{ff} \\ \varepsilon_{1\alpha}(f) &= q_1(f_\alpha) + [q_1(f_\alpha^{\mathcal{N}}) - q_1(f_\alpha)](F_\alpha^\dagger F_\alpha)_{ff}, \qquad \alpha = L, R. \end{aligned} \tag{2.15}$$

For the $q_{\rm em} = +\frac{2}{3}$ states, since there are no new fermions in E_6 with whom the u–quarks could be mixed, the gauge couplings are not modified, and we readily obtain

$$\begin{aligned} \varepsilon_{0L}(u) &= \frac{1}{2} - \frac{2}{3}s_w^2, \qquad & \varepsilon_{0R}(u) &= -\frac{2}{3}s_w^2 \\ \varepsilon_{1\alpha}(u) &= q_1(u_\alpha), & \alpha &= L, R. \end{aligned} \tag{2.16}$$

The new R–handed charged leptons and L–handed $q_{\rm em} = -\frac{1}{3}$ quarks have exotic SU(2) assignments, and since also $q_1^{\mathcal{N}} \neq q_1^{\mathcal{K}}$, the couplings to both Z_0 and Z_1 are affected by the mixing. Ordinary–exotic fermion mixing can induce flavor changing transitions in the interactions mediated by Z_0, through the off–diagonal terms $(F^\dagger F)_{ij}$ $(i \neq j)$ in (2.14). However, extremely stringent constraints exist on μe, sd, and bd transitions, and imply that the corresponding terms are at most $\mathcal{O}(10^{-4})$ [9]. Tight bounds $\sim \mathcal{O}(10^{-2})$ exist for bs, τe and $\tau\mu$ flavor changing parameters as well [9]. Hence, if flavor changing vertices exist in Z_0 interactions, most of them must be negligible, and it is then reasonable to concentrate in constraining possible deviations in the flavor diagonal couplings. As is shown in [9], assuming the absence of FCNC is equivalent to assume that different light mass eigenstates are not mixed with the same exotic partner, in which case the $F^\dagger F$ terms corresponding to ordinary–exotic mixings are diagonal. With this assumption, we can define the mixing angles $\theta^f_{L,R}$ that describe the mixing between L or R ordinary and exotic partners through

$$(F^\dagger_\alpha F_\alpha)_{ff'} = (s^f_\alpha)^2\ \delta_{ff'}\ , \qquad\qquad f_\alpha, f'_\alpha = e_R, \mu_R, \tau_R, d_L, s_L, b_L. \tag{2.17}$$

where $(s^f_{L,R})^2 \equiv 1-(c^f_{L,R})^2 \equiv \sin^2\theta^f_{L,R}$. The flavor diagonal chiral couplings to the Z_0 of the corresponding light mass eigenstates then read

$$\begin{aligned}\epsilon_{0R}(f) &= -\frac{1}{2}(s^f_R)^2 + s^2_w \qquad\qquad f = e,\mu,\tau,\\ \epsilon_{0L}(f) &= -\frac{1}{2}(c^f_L)^2 + \frac{1}{3}s^2_w \qquad\qquad f = d,s,b\end{aligned} \tag{2.18}$$

Similarly, according to (2.8) the chiral couplings in the J^μ_1 current are also flavor diagonal, and we find

$$\epsilon_{1\alpha}(f) = q_1(f_\alpha) + (s^f_\alpha)^2[q_1(f^{\mathcal{N}}_\alpha) - q_1(f_\alpha)] \qquad\qquad f_\alpha = e_R,\mu_R,\tau_R,d_L,s_L,b_L. \tag{2.19}$$

For the L–handed charged leptons and R–handed $q_{\rm em} = -\frac{1}{3}$ quarks, the effect of the mixing is more subtle. In this case the new states are ordinary and hence couple to the Z_0 boson with the same charges as the corresponding known fermions $t_3(f^{\mathcal{N}}) = t_3(f^{\mathcal{K}})$. Then, since the coefficient of the the $F^\dagger F$ term in (2.13) vanishes identically, the J^μ_0 current is not modified in going to the mass eigenstate basis, and the chiral couplings of the corresponding fermions conserve the standard form

$$\epsilon_{0L}(e) = -\frac{1}{2} + s^2_w, \qquad\qquad \epsilon_{0R}(d) = \frac{1}{3}s^2_w. \tag{2.20}$$

In contrast, we have in general $q_1(f^{\mathcal{N}}) \neq q_1(f^{\mathcal{K}})$, and then a mixing between the ordinary known and new fermions will indeed affect the ϵ_1 couplings.[2] Unfortunately, since Z_0 interactions cannot provide informations on the $F^\dagger F$ parameters, there is little hope to derive meaningful constraints on the ordinary–ordinary mixings. We have not attempted this, and since we will not present bounds on these mixings, we also avoid introducing new parameters to describe their effects. The consideration of these mixings will be mandatory once a Z' boson is found, since

[2] However, for the particular case of the η model ($s_\beta = \sqrt{5/8}$, $c_\beta = -\sqrt{3/8}$), for the ordinary fermions we have again $q_\eta(f^{\mathcal{N}}) = q_\eta(f^{\mathcal{K}})$ (see Table I). Then in this model ordinary–ordinary mixing effects are completely absent.

they may induce important FCNC effects as well as universality violations in the Z_1 couplings.[3]

At present energies however the effects of J_1^μ are already suppressed, and then the additional modifications due to ordinary–ordinary mixings can be expected to be negligible if the mixings are not particularly large. In our numerical analysis we have thus taken

$$\epsilon_{1L}(e) \simeq q_1(e_L), \qquad\qquad \epsilon_{1R}(d) \simeq q_1(d_R). \tag{2.21}$$

In the case in which some of these mixings were close to maximal our approximation would not be good, and it is reasonable to question if the limits derived on $M_{Z'}$ and ϕ are still reliable. We believe that this is the case, since the couplings of the remaining fermions in the J_1^μ current are known ($u_{L,R}$) or fairly well (and reliably) constrained (e_R, d_L, ν_L) by charged current processes and by their effects on J_0^μ.

Fermion mixings affect the charged current sector as well. For the hadrons, since the only exotic quarks present in E_6 are d_L–type, the general formalism developed in [9] acquires a much simpler form. In the standard base where the gauge and mass eigenstate up–quarks coincide, the charged current between "light" mass eigenstate quarks is

$$\frac{1}{2} J_W^\mu = \bar{\Psi}_{lL}^u \gamma^\mu A_L^d \Psi_{lL}^d \,, \tag{2.22}$$

where $\Psi_{lL}^u = (u, c, t)_L^T$, and $\Psi_{lL}^d = (d, s, b)_L^T$. A_L^d here plays the rôle of an apparent CKM mixing matrix, but clearly it is not unitary due to the mixing with the exotic quarks. It is useful to decompose it as

$$A_{L\,ij}^d = K_{ij} c_L^j, \tag{2.23}$$

where $i = u, c, t$, $j = d, s, b$ and K is unitary.

ii) Neutral fermions

For the neutral fermions the situation is more complex and a few specific assumptions have to be formulated as well. In first place, neutral fields with

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[3] We also note that in theoretical models in which one relates the fermion mixings to light (doublet VEVs) and heavy (singlet VEVs) masses, the ordinary-ordinary mixings, unlike the ordinary-exotic ones, are not suppressed as the ratio of the two mass scales, and hence are not necessarily expected to be very small [15].

three different weak–isospin assignments can mix simultaneously in the presence of Majorana mass terms. In fact, besides the known neutrinos in the standard $(\nu\ e^-)^T$ doublets, there are new *ordinary* neutrinos in the L–doublet $(N\ E^-)^T$. The *exotic* neutral states with $t_3 = -1/2$ appearing in $(E^+ N^c)^T_L$ can also mix with the known ν's through lepton number violating $\Delta L = \pm 2$ Majorana mass terms, and finally, for each fermion family, two $SU(2)$ exotic singlets ν^c_L and S_L are also present.

A second complication is due to the lack of experimental constraints on neutrino FCNC's so that, as for the ordinary–ordinary mixings, again we cannot make any assumption on the form of the $F^\dagger F$ term in (2.13). However, in all the measurements that we will consider the final state neutrinos are not detected, so that a sum over the flavor of the final mass eigenstates has to be taken. Under this condition, we can again account for the mixing effects in the neutral sector without introducing explicit FCNC parameters.

A further assumption has to be made regarding the number of "light" neutrinos. The LEP measurement of the effective number of light neutrino species $\mathcal{N}_\nu$, implies that if new neutrinos with a large non–singlet component exist, they must be heavier than $M_Z/2$. However, the new E_6 neutrino singlets could well be "light" ($m < M_Z/2$) and a mixing with the doublet neutrinos would allow them to couple to the Z boson and to contribute to the invisible width.[4] For simplicity we will not consider this case, assuming that only the three known neutrinos, that are mainly ordinary states, are light. This assumption in particular implies the bound $\mathcal{N}_\nu \leq 3$. In view of the existing experimental upper bounds on the different ν masses, we will also neglect all the related kinematical effects.

In analogy with the charged fermions, we introduce a vector $n^o_L = (\nu^o_\mathcal{K}, \nu^o_\mathcal{N})^T_L$ for the known and new neutral gauge eigenstates, and a vector $n_L = (n_l, n_h)^T_L$ for the light and heavy mass eigenstates, we will label the elements of the two basis with indices $a, b, \ldots$ and $i, j, \ldots$ respectively, and we will drop the index $\mathcal{K}$ when no confusion can arise. We will also not distinguish between left handed neutrinos and antineutrinos, they are all described by fields n_L. The right handed fields will be denoted as $n^c_R = C\bar{n}^T_L$, and clearly $n^{oc}_R = U_R n^c_R$ with $U_R = U^*_L$. Hence it is understood that (2.13) has to be restricted only to L–chirality states in this case.

Since the new neutral states have different t_3 (and q_1) assignments, it is useful to decompose the vector of new states and the matrix F_L in (2.10), relating

[4] the existence in E_6 of new singlet neutrinos lighter than a few MeV, would however conflict nucleosynthesis [16] and supernova SN 1987A [17] constraints, unless $M_{Z'} \gtrsim \mathcal{O}(\mathrm{TeV})$.

the new states with the light ones, as

$$\begin{aligned} \nu^o_{NL} &= (N,\, N^c,\, \nu^c,\, S)^T_L \\ F_L &= (O,\, E,\, S,\, S')^T_L. \end{aligned} \tag{2.24}$$

where each submatrix describes the overlap of the light states with the new ordinary, exotic doublet and exotic singlet neutrinos respectively. Since we have assumed only three light neutrino states, that are mainly the standard ν^o_κ neutrinos, the matrix A_L in (2.10), describing the overlap of the light neutrinos with the ordinary known ones, is 3×3 and deviates from a unitary one only by the small mixing effects in F_L.

In order to analyse the leptonic charged currents between light states, one can chose the flavor basis such that the charged lepton flavor eigenstates coincide with the charged mass eigenstates up to light–heavy mixing effects. In this basis, the charged current between light states reads

$$\frac{1}{2} J^\mu_W = \bar{n}_L \gamma^\mu A^\dagger_L e_L + \bar{n}^c_R \gamma^\mu E^\dagger_R s^e_R e_R. \tag{2.25}$$

In the first term in this equation the overall strength of the left–handed current is reduced by the effect of light–heavy mixing appearing in the $A^\dagger_L$ neutrino projector, while the second term corresponds to an induced right–handed current that will produce neutrinos of the 'wrong' helicity in weak decays.

It is convenient to introduce the leptonic analog of the CKM matrix, K_ℓ, by writing $A^\dagger = K_\ell \mathcal{A}^{\nu\dagger}$. The matrix K_ℓ is unitary and is non–trivial if non–degenerate masses and mixings are present for the light neutrinos.

The exotic mixings appear only in $\mathcal{A}^\nu$, which can be chosen to be Hermitian, and deviates from the identity by terms of $\mathcal{O}(s^2)$. For instance, in a weak decay involving the $e_a \to n_i$ transition, the change with respect to the SM decay rate Γ_o induced by the corresponding mixings is

$$\frac{1}{\Gamma_o} \sum_i \Gamma(e_a \to n_i) = (A_L A^\dagger_L)_{aa} + (s^{e_a}_R)^2 (E_R E^\dagger_R)_{aa}. \tag{2.26}$$

The first term $(A_L A^\dagger_L)_{aa} = (\mathcal{A}^\nu)^2_{aa} \equiv (c^{\nu_a}_L)^2 = 1 - (s^{\nu_a}_L)^2$ accounts for the reduction in the light neutrinos coupling strength, and we see that the information in K_ℓ is lost when we sum over the unobserved final neutrino mass eigenstates. The second term, in which $(E_R E^\dagger_R)_{aa} \equiv (s^{\nu_a}_R)^2$, appears only when both the light neutrino and the R charged lepton mix with the components of an exotic doublet (as is required by helicity conservation in the W interaction), and is $\mathcal{O}(s^4)$ in the light–heavy mixing. Each $(s_R^{\nu_a})^2$ represents an additional mixing parameter, that is in principle unrelated to the corresponding $(s_L^{\nu_a})^2$. For $e_a = e, \mu$, the existing direct constraints on the right handed currents (RHC) [9] insure that it is safe to neglect the $\mathcal{O}(s^4)$ terms. However for the τ lepton, the existing direct limit is too weak to justify the same approximation (see Section 4). Nevertheless it is easy to show that $(s_R^{\nu_\tau})^2$ is bounded by $(s_R^{\nu_\tau})^2 \leq \sum_a (s_R^{\nu_a})^2 = \mathrm{Tr}(E_L^\dagger E_L) \leq \sum_a (s_L^{\nu_a})^2$, and this insures that it is safe to neglect the corresponding RHC contributions as well.

The normalized state produced in the weak decay described by (2.26) constitutes the initial neutrino state in neutrino scattering experiments, and is a coherent superposition of mass eigenstates

$$|n_{aL}\rangle = \frac{1}{c_L^{\nu_a}} \sum_i (A_L^{\nu\dagger})_{ia} |n_{iL}\rangle, \qquad \langle n_{aL}|n_{aL}\rangle = 1, \tag{2.27}$$

where a is the flavor of the associated charged lepton. Since for the baseline distances and neutrino energies relevant for neutrino neutral current scattering experiments the neutrino oscillations are fairly well constrained, we will assume that this state is unchanged (except for an overall phase) as the neutrino propagates, and for instance the ν_μ' s produced in π or K decays propagate essentially as flavor states up to the point where they interact with the target.

The light neutrino neutral currents that couple to the Z_m bosons are given in (2.13) where, to avoid double counting, only L–states have to be summed, and we have *e.g.* $T_3^{\mathcal{N}} = \mathrm{diag}(\frac{1}{2}, -\frac{1}{2}, 0, 0)$, where a 3×3 identity matrix multiplying each entry is understood.

These currents are in principle flavor changing, but owing to the fact that in the scattering process $n_a X \to n_i X'$ as well as in the $Z \to n_i n_j$ invisible decay width, a sum over the undetected light neutrinos $n_{i,j}$ has to be taken, the corresponding theoretical expressions can still be recasted in a simple form. However, unlike the CC case, additional parameters are needed here in order to describe the kind of neutrinos that mix with the light states. Using the unitarity relation (2.11), the unitarity of K_ℓ and $c_L^{\nu_a}$ from (2.26), we can introduce these parameters by writing

$$1 = \Big[K_\ell^\dagger (A_L^\dagger A_L + F_L^\dagger F_L) K_\ell\Big]_{aa} = (c_L^{\nu_a})^2 + (\lambda_N^a + \lambda_{N^c}^a + \lambda_{\nu^c}^a + \lambda_S^a)(s_L^{\nu_a})^2 \tag{2.28}$$

where *e.g.* $\lambda_N^a (s_L^{\nu_a})^2 \equiv (K_\ell^\dagger O_L^\dagger O_L K_\ell)_{aa}$ describes the amount of mixing with the heavy ordinaries N, and analogous expressions hold for the other λ_n^a parameters that describe the mixing with the exotics. These parameters satisfy $0 \leq \lambda_n^a \leq 1$ and from (2.28) we clearly have $\sum_n \lambda_n^a = 1$.

From (2.13), and neglecting for the moment possible mixing effects in the target as well as Z_1 effects, we can write the neutrino scattering cross section for the initial state (2.27), normalized to the standard cross section, as

$$\frac{1}{\sigma_o}\sum_i \sigma(n_a \to n_i) = \frac{1}{(c_L^{\nu_a})^2}\left[A_L(A_L^\dagger A_L + 2F_L^\dagger T_3^{\mathcal{N}} F_L)^2 A_L^\dagger\right]_{aa}. \tag{2.29}$$

By means of (2.28), and keeping only the $\mathcal{O}(s^2)$ terms, this becomes

$$\frac{1}{\sigma_o}\sum_i \sigma(n_a \to n_i) = 1 - 2(1 - \lambda_{\mathcal{N}}^a + \lambda_{\mathcal{N}^c}^a)(s_L^{\nu_a})^2 + O(s^4). \tag{2.30}$$

The decay rate of the Z boson into undetected neutrinos is proportional to the sum of the square of the neutrino NC couplings in (2.13). Focusing for the moment on the mixing effects in the Z_0 couplings, and using the same approximations as in the previous case, the corresponding modifications to the standard decay rate can be parametrized similarly

$$\Gamma_{Z_0 \to \mathrm{inv}} \propto \mathrm{Tr}(A_L^\dagger A_L + 2F_L^\dagger T_3^{\mathcal{N}} F_L)^2 = 3 - 2\sum_a (1 - \lambda_{\mathcal{N}}^a + \lambda_{\mathcal{N}^c}^a)(s_L^{\nu_a})^2 + O(s^4). \tag{2.31}$$

(In the complete expression for $\Gamma_{Z \to \mathrm{inv}}$, besides the effects of Z_0–Z_1 mixing, also additional indirect effects of the neutrino mixings are present – see Section 3). We see that to leading order in the neutrino mixing parameters, the modifications induced in both these processes can be described with the same mixing angles $s_L^{\nu_a}$ already introduced in (2.26) for the charged sector, with the addition, for each neutrino flavor, of an effective parameter $\Lambda_0^a \equiv 2(1 - \lambda_{\mathcal{N}}^a + \lambda_{\mathcal{N}^c}^a)$ that describes the particular admixture of heavy neutrinos involved in the mixing. If the light states are only mixed with heavy ordinary states, corresponding to $\lambda_{\mathcal{N}} = 1$, the NC processes are not affected ($\Lambda_0 = 0$) while a mixing only with neutrinos from exotic doublets corresponds to a maximal reduction in the coupling strength ($\Lambda_0 = 4$). For our numerical analysis we have used the intermediate value $\Lambda_0 = 2$. This corresponds to an equal amount of mixing with new states from ordinary and exotic doublets ($\lambda_{\mathcal{N}} = \lambda_{\mathcal{N}^c}$), but in particular describes also the interesting case of a mixing with only singlet neutrinos.

Although in the presence of mixings the couplings between the neutral mass eigenstates are no more flavor diagonal, it is useful to define the effective neutrino couplings as

$$\varepsilon_0(n) = \frac{1}{2} - \frac{\Lambda_0}{4}(s_L^\nu)^2 \tag{2.32}$$

since, when used naïvely, they allow to reproduce the results (2.30) and (2.31), and are easily handled in a numerical analysis.

Since the neutrino mixing parameters are fairly well constrained by charged and J_0^μ currents processes, their effects in the couplings to Z_1 are clearly higher order in the new physics, and are indeed negligible. However, an effective expression for $\varepsilon_1(n)$, that correctly accounts for the mixing effects also in Z–Z' interference, can be easily derived in a similar way, and is given here for completeness

$$\varepsilon_1(n) = q_1(\nu) - \frac{\Lambda_1}{4}(s_L^\nu)^2, \qquad \Lambda_1 = 4q_1(\nu) - 4\sum_n \lambda_n q_1(\nu_n^{\mathcal{N}}) \tag{2.33}$$

Now that all the couplings of the light mass eigenstates appearing in the $J_{0,1}^\mu$ currents have been derived, by applying the same transformation (2.7) that gives the J_Z^μ and $J_{Z'}^\mu$ currents, we can readily obtain the corresponding couplings $\varepsilon_\alpha(f)$ and $\varepsilon'_\alpha(f)$ to the physical Z and Z' bosons

$$\begin{aligned} \varepsilon_\alpha &= c_\phi\, \varepsilon_{0\alpha} + s_\phi s_w\, \varepsilon_{1\alpha} \\ \varepsilon'_\alpha &= -s_\phi\, \varepsilon_{0\alpha} + c_\phi s_w\, \varepsilon_{1\alpha} \qquad \alpha = L, R. \end{aligned} \tag{2.34}$$

We will use in the following also the vector and axial vector couplings, defined as

$$\begin{aligned} v_f &= \varepsilon_L(f) + \varepsilon_R(f) \\ a_f &= \varepsilon_L(f) - \varepsilon_R(f) \end{aligned} \tag{2.35}$$

with analogous definitions for v'_f and a'_f.

The $SO(10)$ Z_χ model can be easily obtained by retaining only the neutrino parameters ($s_L^{\nu_e}$, $s_L^{\nu_\mu}$, $s_L^{\nu_\tau}$) with $\lambda_{\nu^c}^a = 1$ and $\lambda_N^a = \lambda_{N^c}^a = \lambda_S^a = 0$ (corresponding to $\Lambda_0^a = 2$), and setting all the other fermion mixing angles to zero.

3. Experimental Constraints

In this section we describe the procedure that we have followed to derive limits on the parameters describing the new physics effects from E_6. We also present a brief discussion of the various experimental constraints that we have used in our analysis, and of some subtle indirect effects that depend on the particular experimental procedure used to extract the data. The experimental inputs used in the analysis are collected in Tables II to IV. The overall uncertainties have been evaluated by adding statistical and systematic errors in quadrature, and correlations among different quantities have been taken into account in all the relevant cases.

Input parameters

All the theoretical expressions have been numerically evaluated from a set of fundamental input parameters, consisting of the QED coupling constant α measured at $q^2 = 0$, the mass of the physical Z boson M_Z, and the Fermi coupling constant G_F. For the $U(1)'$ coupling constant g_1, that strictly speaking belongs to the same set, we have assumed $g_1 = g_Y \equiv e/c_w$, while $M_{Z'}$, ϕ and the various $(s^f_{L,R})^2$ that describe the fermion mixings, are treated as free parameters. As extracted from experiments, the numerical values of α, as well as the position of the resonance–pole in $e^+e^- \to f\bar{f}$ (*i.e.* the physical Z mass) are not affected by the new physics. We have fixed the Z–mass at the value $M_Z = 91.175$ GeV [18] since the uncertainties in the theoretical expressions induced by the present experimental error of ± 21 MeV are negligible. In contrast with the previous two parameters, the Fermi coupling constant, as extracted from the measured life–time of the μ-lepton, $G_\mu = 1.16637(2) \times 10^{-5}\mathrm{GeV}^{-2}$, is affected by fermion mixings. It is therefore useful to introduce a 'true' coupling G_F, formally independent of mixing effects, that is related to the effective μ-decay coupling through

$$G_\mu = G_F c_L^{\nu_e} c_L^{\nu_\mu} \tag{3.1}$$

where, as anticipated in section 2, we have neglected the effects of induced RHC that are higher order in the light–heavy mixing. Clearly the dependence on the ν_e and ν_μ mixing angles in (3.1) is propagated in all the expressions that contain G_F, but are numerically evaluated with G_μ.

Radiative corrections

A further remark concerns the higher order corrections. The inclusion of radiative corrections plays an important rôle in achieving the remarkable agreement between the SM predictions and the most accurate experimental results, and indeed loop effects must be taken into account also in deriving bounds on the parameters that describe possible effects due to new physics. However, to be fully consistent, any given model should be analysed by including its specific set of radiative corrections, that is in general larger than the SM set. A complete computation of higher order corrections in the frame of E_6 models is not available yet, but due to the large number of new unknown parameters (*e.g.* the masses of the additional fermions and bosons) the complete 1–loop expressions would not be very helpful anyhow. However, in particular for E_6 models, some work has been done in the direction of estimating the relevance of the new loop contributions, and at present we have enough informations to handle this problem. We will now briefly review the status of the art.

A general discussion of the renormalization of $SU(2)_L \times U(1)_Y \times U(1)'$ models has been given in [19]. The presence of a new Z' gives rise to a new set of two point functions that contribute to the neutral current amplitudes. In addition to the vacuum polarization diagram for the Z' boson, also γ–Z' and Z–Z' amplitudes are present. These new loop diagrams induce additional ultraviolet divergences, and new counterterms are required to render the expressions finite. In spite of these complications, a consistent procedure can be defined for which the 1–loop expressions are finite and, in the limit of vanishing Z–Z' mixing and large $M_{Z'}$, smoothly converge to the SM results. As discussed in [19] it is a sensible approximation to neglect the finite loop contributions due to the *new* two point functions, and restrict this set to the Z, γ and $W_\pm$ self energies. Since the Z_0–Z_1 mixing is severely constrained by $|\phi| \lesssim 0.01 \div 0.02$, it is safe to neglect the related effects in the standard 1–loop diagrams, and in view of the extremely stringent bounds existing on the fermion mixings [10], their effect on the couplings to the gauge bosons is also negligible in higher order expressions.

The presence of additional states in the self energy loops will however give rise to additional corrections to the standard vacuum polarization functions. Since for $s \ll m^2_{\rm new}$ heavy physics decouples from QED and does not contribute to the running of the electromagnetic coupling constant up to $s = M_Z^2$ (that is by far the largest SM loop correction), the leading effects of the new heavy states are expected mainly as contributions to the ρ parameter [20] that measures the difference between the $W_\pm$ and Z self energies at zero momentum. These effects have been analysed *e.g.* in [21]. In spite of the large number of new fermions, in the limit of small Z–Z' and fermion mixings only the new vector doublets of leptons can contribute to ρ, since all the other fermions are weak singlets,[5] and then the effects of the new fermions should not be dramatically large. New contributions to ρ are also expected from the enlarged scalar sector [23]. We note however that even in the SM the numerical corrections to ρ are largely undetermined, since the dominant contribution depends on the unknown value of the top mass, and a softer (logarithmic) dependence on the unknown mass of the SM Higgs boson is also present. Choosing a particular value for the two unknown masses m_t and m_H then fixes, within the SM, a particular value of ρ, and we will assume that this

[5] This insures that the contributions to the Peskin–Takeuchi S variable [22] is also small.

also parametrizes roughly the leading non standard higher order contributions. Fixing $m_H = 100$ GeV, we have performed a fit to the top mass in the SM context, corresponding to $M_{Z'} \to \infty$ with all the mixings set to zero. We obtain $m_t = 116^{+28}_{-35}$GeV, in good agreement with other recent analyses [24]. Our 1-σ limits are slightly relaxed due to the fact that we are using the LEP flavor-dependent measurements for leptons, that have larger experimental errors with respect to the measurements that assume lepton universality. Throughout the rest of our analysis we have then fixed the top mass at the value $m_t = 120$ GeV, that corresponds approximately to the minimum of the χ^2 function in the SM case, while m_H has been fixed at 100 GeV.

New effects could appear also in the vertex corrections to the Z partial decay widths, due to the presence of one additional Higgs doublet. These effects have been analysed in [25] for the cases in which the contributions are enhanced by large Yukawa couplings. They have found that even for extreme choices of the relevant parameters (VEV's and scalar masses) the corrections to $\Gamma_{Z\to b\bar{b}}$ and to $\Gamma_{Z\to\tau^+\tau^-}$ are well below the present experimental accuracy.

Another source of corrections is of QED origin. At LEP energies, by far the largest deviations from the tree level formulæ for the partial widths, and especially for the leptonic asymmetries, are due to the bremsstrahlung of photons. The presence of additional Z' exchange diagrams with photons attached at the external legs could in principle alter the SM results, where only γ and Z exchange diagrams are present. For the specific case of a Z' from E_6, this problem has been systematically addressed in [26] for the partial widths, and in [27] for the leptonic asymmetries. As a general result, they have found that at the presently available energies, the QED corrections to diagrams involving the Z' boson are negligible. This ensures that the experimental values for the various $\Gamma_{e^+e^-\to f\bar{f}}$ widths, as extracted from the peak cross sections and from the Z line shape, are unmodified in the presence of a Z'. As regards the leptonic flavor dependent asymmetries, in comparing the theoretical expressions with the experimental data we have taken into account the bulk of the effects of QED initial–state radiation by convolving the various differential cross section distributions with the appropriate radiator kernels [28]. In order to minimize computing time, we have included in the convolution integrals only the γ and Z exchange terms, keeping the tree level expressions for the additional terms involving the Z' propagator. The analysis presented in [27] insures that this is a good approximation to the fully corrected results.

The conclusion of this brief discussion is that the procedure of including only the standard radiative corrections to the neutral current processes can be regarded as a safe and rather well motivated approximation.

A different set of new loop contributions appears in the CC sector, and originates from Z'–W box diagrams corrections to weak decays [29,19]. Surprisingly enough, this is the only Z' loop effect that is worth to take into account since, as is discussed in the next paragraph, it can contribute to bound $M_{Z'}$, and plays an interesting rôle in constraining the ν_μ mixing angle as well.

Charged currents

i) CKM unitarity

Tests of the unitarity of the CKM matrix provide strong constraints, at the level of $\simeq 0.1\%$, on possible new physics from E_6. Combining the experimental errors on the different matrix elements in quadrature, one in fact obtains for the first matrix row [30,31].

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.9981 \pm 0.0021 \tag{3.2}$$

in agreement with 3-generation unitarity. As is apparent from eq. (2.11–2.13), fermion mixings would induce violations of unitarity. Additional effects, due to the procedure adopted for extracting the data, will also modify (3.2). V_{ud} and V_{us} are obtained by dividing by G_μ the measured vector coupling in β decay and in K_{e3} and hyperon decays, respectively, after accounting for the $\mathcal{O}(\alpha)$ radiative corrections of the SM. Hence, using (2.21) and (3.1), in the present case we have

$$\begin{aligned} V_{ui} &= \frac{G_F}{G_\mu} c_L^{\nu_e} A^d_{L\,ui} \left[1 - \frac{\alpha}{\pi}\mathcal{F}(q_1, M^2_{Z'})\right] \qquad\qquad i = d, s. \\ \mathcal{F}(q_1, M^2_{Z'}) &= -\frac{3}{2}\frac{M^2_Z}{M^2_{Z'} - M^2_W} q_1(e_L)[q_1(e_L) - q_1(d_L)]\ln\frac{M^2_{Z'}}{M^2_W} \end{aligned} \tag{3.3}$$

The first factor in the r.h.s of (3.3) accounts for the direct and indirect effects of fermion mixings, while the second term in the square brackets (in which all mixing effects have been neglected) accounts for an additional loop correction originating from Z'–W box diagrams, that has to be taken into account in comparing the relative strengths of different weak amplitudes (see [29] for a detailed discussion). Since the value of $|V_{ub}|^2$, obtained from the analysis of semileptonic B decays, is negligibly small ($< 2 \cdot 10^{-4}$), we can write

$$\sum_{i=1}^{3} |V_{ui}|^2 \simeq \left(\frac{G_F}{G_\mu} c_L^{\nu_e}\right)^2 \left\{1 - \sum_{i=1}^{2} |V_{ui}|^2 (s_L^{d_i})^2\right\} - 2\frac{\alpha}{\pi}\mathcal{F}(q_1, M^2_{Z'}), \tag{3.4}$$

where the unitarity of the matrix K defined in (2.23) has been used, and we have approximated $|K_{ui}|^2$ with the experimental values $|V_{ui}|^2$ in the coefficients of the

$O(s^2)$ terms. From the expression of $\mathcal{F}(q_1, M^2_{Z'})$ in (3.3), we see that Z' loop effects vanish identically in the Z_ψ model ($\beta = 0$), since the q_ψ charges are the same for all the known fermions. In contrast, this effect is maximal for $\beta = \pi/2$, *i.e.* for a Z' from SO(10). Since in SO(10) only one additional neutrino for each generation is present (ν^c_L), and noting that the ν_e mixing effects cancel in the first factor in (3.4), in the absence of Z' loop effects, the CKM unitarity would directly measure (and constrain) the mixing of ν_μ [14]. However, for small values of the Z'_χ mass the two effects could be numerically comparable, and (3.4) then represents an interesting example of the interplay between different effects from SO(10) (and E_6) new physics.

For the $|V_{ui}|$'s we use the (conservative) values given in [30,31]. An analogous relation exists for the second row of the CKM matrix as well, but due to the large uncertainties affecting the corresponding matrix elements it does not give any relevant additional constraint.

ii) Lepton universality

Constraints from lepton universality are effectively expressed in terms of the measured ratios g_μ/g_e and g_τ/g_e of the leptonic couplings to the W boson, which in the SM are predicted to be unity. Due to the universality of the gauge interactions Z' box corrections cancel in the ratio, while the non universal fermion mixing effects modify the SM expressions according to

$$\left(\frac{g_i}{g_e}\right)^2 = \frac{(c_L^{\nu_i})^2}{(c_L^{\nu_e})^2}, \qquad i = \mu, \tau. \tag{3.5}$$

where the additional $\mathcal{O}(s_R^4)$ terms in (2.26) have been neglected. Experimentally, the ratios (3.5) are extracted from leptonic decays as discussed in [9,10]. In Table II we give the values [10] of $(g_i/g_e)^2$ measured from W-decays by UA1, UA2 and CDF [32], from τ and μ decays [30,33], and from the decays of K and π mesons [30].

TABLE II. Charged Current experimental constraints on lepton universality (g_i/g_e) and on the unitarity of the CKM quark mixing matrix V_{ij}.

Quantity	Experimental value	Correlation	Processes
$(g_\mu/g_e)^2$	1.00 ± 0.20		$W \to l\nu$
$(g_\tau/g_e)^2$	1.00 ± 0.08		
$(g_\mu/g_e)^2$	1.016 ± 0.026	0.40	$\tau \to l\nu\bar{\nu}$ and $\mu \to e\nu\bar{\nu}$
$(g_\tau/g_e)^2$	0.952 ± 0.031		
$(g_\mu/g_e)^2$	1.014 ± 0.011		$\pi \to l\nu$
"	1.013 ± 0.046		$K \to l\nu$
$\sum_{i=1}^{3} \lvert V_{ui}\rvert^2$	0.9981 ± 0.0021		hadrons decays

The W mass

The standard way of computing the value of the W mass is to compare the amplitude for W exchange at $q^2 \simeq 0$ in μ decay with the effective strength of the Fermi interaction, taking into account the large contribution of the radiative corrections [34]. In the present models, several new effects modify the standard formula. Z_0–Z_1 mixing affects the theoretical prediction through $M^2_{Z_0}$, which enters the expression for M_W in place of the physical Z mass. The ν_e and ν_μ mixings, entering indirectly via the G_μ/G_F ratio, also appear in the final expression, that reads

$$M_W^2 = \frac{\rho M^2_{Z_0}}{2}\left[1 + \sqrt{1 - \frac{G_\mu}{G_F}\frac{4\mathcal{A}}{\rho M^2_{Z_0}}\left(\frac{1}{1-\Delta\alpha} + \Delta r^{rem}\right)}\,\right], \tag{3.6}$$

where $\mathcal{A} = \pi\alpha/\sqrt{2}G_\mu$, $1/(1-\Delta\alpha)$ renormalizes the QED low energy coupling to the M_Z scale, and the leading top effects, quadratic in m_t, are included in $\rho \simeq 1 + 3G_\mu m_t^2/8\sqrt{2}\pi^2$ [20]. Other smaller corrections are collected in Δr^{rem}, and we refer to [35] for a detailed discussion of the various contributions. The Z'–W box corrections to μ–decay could be easily included in Δr^{rem} as well [19], however in the $M_{Z'}$ mass range we are interested in this contribution is always $\lesssim 10^{-3}$

[19], and since the prediction for M_W is already largely dependent on m_t, it is reasonable to neglect this additional effect.

We note that increasing values of all the parameters that describe the new physics in (3.6) tend to increase M_W, as do larger values of the top mass. A similar interdependence enters also the expression for the effective weak-mixing angle that defines the neutral-current couplings of the fermions, and then a sizeable anticorrelation among m_t, Z_0–Z_1 mixing [6], and s^{ν_e}, s^{ν_μ} [10] is to be expected, resulting in overall stronger constraints for larger values of m_t.

Experimentally, the value of the W mass measured by CDF is $M_W = 79.91 \pm 0.39$ GeV [36], while the UA2 collaboration has measured the ratio of the W and Z masses, for which many systematic errors cancel, obtaining $M_W/M_Z = 0.8813 \pm 0.0037$ [36]. Using the LEP value for M_Z and averaging the two results yields

$$M_W = 80.14 \pm 0.27 \mathrm{GeV}. \tag{3.7}$$

Physics at the Z–peak

The large amount of high precision data collected at LEP are extremely effective to constrain universal and non–universal new physics effects in the fermion couplings to the Z boson. In addition to the direct constraints on Z_0–Z_1 and fermion mixings effects in v_f and a_f, all the LEP measurement also provide a precise determination of the effective weak–mixing angle, that, modulo radiative corrections, reads $s^2_{eff} \simeq 1 - M^2_W/M^2_{Z_0}$ with M^2_W given in (3.4). Clearly, through s^2_{eff}, the LEP measurements also contribute to constrain indirectly the ν_e and ν_μ mixings, as well as $M_{Z'}$ and ϕ.

Besides the accurate determination of the value of the Z–mass, that completes the set of fundamental input parameters, also the total Z width and the partial decay widths into hadronic final states and into each of the three lepton flavors have been measured at LEP with very high precision. In order to constrain the non–universal fermion mixings together with the Z' parameters, we have used the experimental values of the five widths Γ_Z, Γ_h, Γ_e, Γ_μ and Γ_τ as obtained, with M_Z, from a 6 parameters fit to the corresponding hadronic and leptonic peak cross sections, and to the Z line shape [18]. These results do not assume universality, and then the experimental errors are larger than the errors obtained in the flavor–independent analyses, so that, while allowing to constrain effectively the lepton mixing angles, this also gives slightly relaxed limits on the Z' parameters.

The tree–level expressions for the Z partial decay width into f fermions reads

$$\Gamma_{Z\to ff} = N^f_c \frac{M_Z}{6\sqrt{2}\pi} G_F M^2_{Z_0}(v^2_f + a^2_f), \tag{3.8}$$

where $N_c^f = 3(1)$ for quarks (leptons), and the couplings v_f and a_f are given in (2.35). We see that besides the modifications in the vector and axial-vector couplings, that are specific for each fermion flavor, the overall strength of the Zff vertices is also affected by the gauge-boson and fermion mixings appearing in the factor $G_F M_{Z_0}^2$. The first factor of M_Z however, that comes from phase space, is the physical Z mass. All the relevant higher order corrections (universal and non-universal, as the $Zb\bar{b}$ vertex correction) that are not displayed in (3.8), have been taken into account in our numerical analysis. We have also included in our data set the measurements of the partial decay width into b quarks [37], that constrains the s_L^b mixing parameter. The corresponding experimental values are collected in Table III.

While the measurements of the different Γ_f's are sensitive to the particular combination of couplings $v_f^2 + a_f^2$, the forward-backward asymmetries $A_f^{\rm FB}$ are sensitive to the ratios v_f/a_f, and the combined measurement of these two sets of quantities allows for an independent determination of v_f and a_f. On resonance, the expression for the asymmetries reads

$$A_f^{\rm FB} = 3\,\frac{v_e a_e}{v_e^2 + a_e^2}\,\frac{v_f a_f}{v_f^2 + a_f^2}. \tag{3.9}$$

Forward-backward asymmetries have been measured for $f = e$, μ, τ and b final states, their values at the peak, averaged over the results of the four LEP collaborations, are given in Table III. For the leptonic asymmetries, in order to increase the statistics, we have also included in our analysis the data at ± 1 GeV around resonance. We refer to [10] for a more detailed discussion of these data.

The expression for the τ polarization asymmetry [38] reads

$$A_\tau^{pol} = \frac{-2 v_\tau a_\tau}{v_\tau^2 + a_\tau^2} \tag{3.10},$$

and its experimental value has been measured at LEP [39] by analysing the distributions of the τ decay products. A_τ^{pol} is very sensitive to the τ vector coupling to the Z, since unlike the forward-backward asymmetry, it is not suppressed by the small electron vector coupling, and then it provides an important direct constraint on s_R^τ.

On resonance measurements provide the strongest constraints for most of the fermion mixing angles, particularly for the heavy fermions (b, s and τ) mixings, that are poorly constrained by the low-energy and CC data. At the same time, LEP data have also remarkably strengthen the limits on the Z_0–Z_1 mixing [5,6], and the introduction of the forward-backward asymmetries in our data set has further improved the previous constraints. On resonance physics however tests essentially only the J_Z^μ current. In the case the Z_0–Z_1 mixing angle were vanishingly small, J_Z^μ would not be affected and all these measurements would be largely insensitive to the presence of a new neutral gauge boson, being then unable to effectively constrain its mass [40].

TABLE III. Results on Z–partial widths (in MeV) and on-resonance asymmetries. The values displayed for the leptonic asymmetries correspond to the peak–data and have been corrected only for angular acceptance.

Quantity	Experimental value	Correlation			
Γ_Z	2487 ± 10	0.36	0.48	0.26	0.22
Γ_h	1740 ± 12		−0.31	0.58	0.49
Γ_e	83.20 ± 0.55			−0.19	−0.16
Γ_μ	83.35 ± 0.86				0.33
Γ_τ	82.76 ± 1.02				
$A_e^{\mathrm{FB}}(peak)$	-0.019 ± 0.014				
$A_\mu^{\mathrm{FB}}(peak)$	0.0070 ± 0.0079				
$A_\tau^{\mathrm{FB}}(peak)$	0.099 ± 0.096				
A_τ^{pol}	-0.121 ± 0.040				
Γ_b	367 ± 19				
A_b^{FB}	0.123 ± 0.024				

Low Energy Neutral Currents

The results of NC experiments are conveniently given as fits to the parameters appearing in the effective Lagrangians that describe the corresponding four–fermion processes [30]. The form of these effective Lagrangians relies only on the assumption of spin–one gauge boson exchange and of massless left–handed neutrinos, and thus the experimental values of the phenomenological parameters are essentially model independent. We will treat separately the $\nu - q$, $\nu - e$ and the parity–violating $e-q$ sectors, and for clarity we will only display the tree level expressions, but the SM radiative corrections [41,42] have been always included in our numerical computations.

i) Neutrino–quark sector

The effective Lagrangian for the neutral current interaction of the light neutrinos with quarks is

$$-\mathcal{L}^{\nu q} = \frac{G_\mu}{\sqrt{2}}\,\bar{\nu}\gamma_\mu(1-\gamma_5)\nu\ \left[\epsilon_L(q)\bar{q}\gamma_\mu(1-\gamma_5)q + \epsilon_R(q)\bar{q}\gamma_\mu(1+\gamma_5)q\right]. \qquad (3.11)$$

The values of the quark couplings $\epsilon_{L,R}(q)$ are extracted from deep–inelastic scattering cross sections normalized to the CC cross sections, *e.g.* from the ratios $R_{\overset{(-)}{\nu}} = \sigma^{NC}(\overset{(-)}{\nu}_\mu N \to \overset{(-)}{\nu}_\mu X)/\sigma^{CC}(\overset{(-)}{\nu}_\mu N \to \overset{(-)}{\nu}_\mu X)$. Denoting as σ_o^{CC} the canonical CC cross section computed in terms of the apparent CKM angles (3.3), and taking into account also the modifications in the ν_μ couplings, it is easy to see that σ^{CC} is modified according to $\sigma^{CC}/\sigma_o^{CC} \simeq (G_F/G_\mu)^2(c_L^{\nu_\mu})^4$. In comparing the theoretical expressions for $R_{\overset{(-)}{\nu}}$ with the experimental data, the factors of G_F/G_μ cancel in the ratios, but an overall factor $(c_L^{\nu_\mu})^{-4}$, induced by the experimental normalization, has to be included. By properly taking into account the effect of Z' exchange, and using for the fermion couplings the expressions given in (2.34) that include all the other mixing effects, the experimental values for the $\epsilon_{L,R}$ 'couplings' obtained by fitting (3.11) to the data correspond to

$$\epsilon_\alpha(q) = \frac{2M_{Z_0}^2}{(c_L^{\nu_\mu})^2}\left[\frac{\varepsilon_L(\nu_\mu)\,\varepsilon_\alpha(q)}{M_Z^2} + \frac{\varepsilon'_L(\nu_\mu)\,\varepsilon'_\alpha(q)}{M_{Z'}^2}\right], \qquad q=u,d \qquad \alpha = L,R. \qquad (3.12)$$

$(c_L^{\nu_\mu})^2$ in (3.12) comes from the experimental normalization, $M_{Z_0}^2$, given in (2.5), accounts for the modification in the overall coupling strength due to Z_0–Z_1 mixing, and the two terms inside the square bracket account respectively for Z and Z' exchange in the NC amplitude. The experimental values [30] are given in Table IV in terms of

$$g_\alpha^2 \equiv \epsilon_\alpha(u)^2 + \epsilon_\alpha(d)^2 \quad , \quad \theta_\alpha \equiv \tan^{-1}\left[\frac{\epsilon_\alpha(u)}{\epsilon_\alpha(d)}\right] \qquad \alpha = L,R \qquad (3.13)$$

that have negligible correlations.

ii) Neutrino–electron sector

The effective Lagrangian for the $\nu - e$ sector is

$$-\mathcal{L}^{\nu e} = \frac{G_\mu}{\sqrt{2}} \bar{\nu}\gamma_\mu(1-\gamma_5)\nu \, \bar{e}\gamma_\mu(g_V^e - g_A^e\gamma_5)e. \tag{3.14}$$

The electron vector and axial–vector couplings are extracted from $\nu_\mu - e$ scattering experiments that, as in the previous case, are normalized to ν_μ–hadron CC cross sections. Then the normalization factor $(c_L^{\nu_\mu})^2$ appears in this case as well. The relation between the parameters fitted through (3.14) and the theoretical couplings given in (2.35) is

$$\begin{aligned} g_V^e &= \frac{2M_{Z_0}^2}{(c_L^{\nu_\mu})^2}\left[\frac{\varepsilon_L(\nu_\mu)\,v_e}{M_Z^2} + \frac{\varepsilon'_L(\nu_\mu)\,v'_e}{M_{Z'}^2}\right] \\ g_A^e &= \frac{2M_{Z_0}^2}{(c_L^{\nu_\mu})^2}\left[\frac{\varepsilon_L(\nu_\mu)\,a_e}{M_Z^2} + \frac{\varepsilon'_L(\nu_\mu)\,a'_e}{M_{Z'}^2}\right] \end{aligned} \tag{3.15}$$

As experimental inputs we have used the determination of g_V^e and g_A^e from the CHARM I [43] and BNL [44] data on both ν_μ and $\bar{\nu}_\mu$ scattering off electrons, as well as the recent CHARM II results [45] for g_V^e/g_A^e, measured from the ratio of ν and $\bar{\nu}$ NC, in which the overall factor in (3.15) cancels out. All these data are separately listed in Table IV.

iii) Electron–quark sector

By interfering with the electromagnetic current, both J_Z^μ and $J_{Z'}^\mu$ contribute to induce parity violating transitions in atoms. The electron–quark parity violating coefficients $C_{1,2}$ are defined by the effective Lagrangian

$$-\mathcal{L}^{eq} = -\frac{G_\mu}{\sqrt{2}} \sum_i (C_{1i}\bar{e}\gamma_\mu\gamma^5 e \; \bar{q}^i\gamma^\mu q^i + C_{2i}\bar{e}\gamma_\mu e \; \bar{q}^i\gamma^\mu\gamma^5 q^i), \qquad i = u, d. \tag{3.16}$$

and the corresponding theoretical expressions are

$$\begin{aligned} C_{1i} &= 2\left(\frac{G_F}{G_\mu}\right) M_{Z_0}^2 \left[\frac{a_e v_i}{M_Z^2} + \frac{a'_e v'_i}{M_{Z'}^2}\right] \\ C_{2i} &= 2\left(\frac{G_F}{G_\mu}\right) M_{Z_0}^2 \left[\frac{v_e a_i}{M_Z^2} + \frac{v'_e a'_i}{M_{Z'}^2}\right] \end{aligned} \tag{3.17}$$

Parity violating transitions in Cs are quite effective for the determination of the coefficients C_1. The experimental results are expressed in terms of the weak charge $Q_W = -2[C_{1u}(2Z+N) + C_{1d}(Z+2N)]$ whose value is [46] $Q_W({}^{133}_{55}Cs) = -71.04 \pm 1.58 \pm 0.88$ (the second error comes from atomic theory [47]). The particular combination $C_{2u} - \frac{1}{2}C_{2d}$ has been also measured in the SLAC polarized $e - D$ scattering experiment [48]. The values of the parity–violating coefficients listed in Table IV have been derived from the quoted value of Q_W, and from the results given in Table 1 of ref. [48].

As it is apparent from (3.12), (3.15) and (3.17), low energy neutral–current experiments are directly sensitive to the $J^{\mu}_{Z'}$ current, and then are quite effective for testing the effects originating from the exchange of a possible new neutral gauge boson. As a general result, this sector constrains quite effectively $M_{Z'}$, even in the limit of vanishingly small Z_0–Z_1 mixing for which, as already stressed, Z' effects largely decouple from on resonance physics.

4. Results

We have collected all the theoretical predictions and the experimental results for the electroweak observables in a χ^2 function, that was analyzed using the MINUIT package. Our results for the various constraints on the Z' parameters, showing also the effects of fermion mixing on the limits on $M_{Z'}$ and ϕ for the various models, are depicted in figures 1–8. The limits on the fermion mixing parameters, with and without Z' effects, are collected in Tables V and VI.

In figures 1 and 2 we show the limits obtained by fitting only one parameter (respectively $M_{Z'}$ and ϕ) while minimizing with respect to all the other free variables. These bounds are given as a function of the angle β that parametrizes the general Z_1, defined in (2.3) as a combination of Z_ψ and Z_χ. The curves give the $\chi^2 = \chi^2_{\rm min} + 3.84$ contours, actually corresponding to the 95% c.l. for gaussian distributions, as it is approximately the case for ϕ. In contrast, the χ^2, as a function of $M_{Z'}$, is far from a parabolic one, and in fact it clearly does not produce any upper limit. As a result in this case the c.l. is actually larger than 95%. The dashed lines indicate the bounds in the absence of fermion mixing effects, while the solid line contours have been obtained by allowing for the simultaneous presence of all the fermion mixings that can appear in E_6 models. As it will become clear in the following, the main effect of the presence of new fermions mixed with the known light states, is to relax the bounds on the Z_0–Z_1 mixing angle ϕ, while the lower bounds on the Z' mass are essentially unaffected.

Figures 3 to 5 show, for the χ, η, and ψ models respectively, the $\chi^2 = \chi^2_{\rm min} + 4.61$ contours in the $M_{Z'}$–ϕ plane, corresponding, for gaussian distributions, to the 90% c.l. region in a two variable fit. Again, the dashed lines give the bounds

TABLE IV. Low energy Neutral Current experimental constraints.

Deep-inelastic ν-q			
g_L^2	0.2977 ± 0.0042		
g_R^2	0.0317 ± 0.0034		
θ_L	2.50 ± 0.03		
θ_R	$4.59 {+0.44 \atop -0.27}$		
ν-e scattering		experiment	
g_V^e/g_A^e	0.047 ± 0.046	CHARM II	
g_V^e	-0.06 ± 0.07	CHARM I	
g_A^e	-0.57 ± 0.07	"	
g_V^e	-0.10 ± 0.05	BNL	
g_A^e	-0.50 ± 0.04	"	
e-q parity violation		correlation	
C_{1u}	-0.249 ± 0.066	-0.99	-0.95
C_{1d}	0.391 ± 0.059		0.95
$C_{2u} - \frac{1}{2}C_{2d}$	0.21 ± 0.37		

in the absence of fermion mixings, while the solid lines show how the constraints are modified by minimizing the χ^2 at each boundary point, with respect to the additional fermion mixing parameters,

To better understand the physics involved in setting the constraints, we show in figs. 6–8 the deviations in the theoretical predictions O^{th} with respect to the experimental result $O^{exp} \pm \Delta O^{exp}$, for the χ, η, and ψ models respectively. We have plotted the normalized quantities $(O_i^{th} - O_i^{exp})/\Delta O_i^{exp}$ for the most relevant observables: M_W, Γ_Z, Γ_h, the sum of the three leptonic widths Γ_ℓ, the combined leptonic asymmetries $A_\ell^{\rm FB}$, $A_\tau^{\rm pol}$, the chiral couplings in deep–inelastic $\nu - q$ scattering $g_{L,R}$, the $\nu - e$ couplings $g^e_{V,A}$, the weak charge Q_W from APV in Cs, as well as the CC tests of unitarity of the CKM matrix, corresponding to the sum of the $|V_{ui}|^2$ elements, and of universality of the CC lepton couplings, given as the ratios g_μ/g_e and g_τ/g_e. The separation between the horizontal dashed lines in the figures corresponds, for each quantity, to one standard deviation of the theoretical prediction from the experimental values, in the different situations considered. The figures show, with a solid circle, the deviations of the SM predictions ($M_{Z'}=\infty$, $\phi=0$, $s^f_{L,R}=0$), with a star, the deviations at the minimum of the χ^2 function (best fit to the experimental results within each model), and with open and solid triangles respectively, the deviations at the two boundary points labeled A and B in figs. 3–5. These points correspond to the largest allowed values for ϕ, with $M_{Z'}$ at its lower bound. This shows in a clear way which observables play a major rôle in setting the limits. For example, the well known fact that a Z' can help to shift the SM prediction for the weak charge $Q_W^{SM} \simeq -73$, towards the value measured in Cs APV experiments $Q_W^{Cs} = -71.0 \pm 1.8$, is apparent in the figures, particularly for the χ model. The relevance of this measurement in setting the lower bounds on $M_{Z'}$ also appears in a clear way. Considering the points A and B in the different models, we see that in general the observables that determine the lower bound on $M_{Z'}$ are those involving $Z - Z'$ interference, such as $\nu - q$ and $\nu - e$ scattering, or $Z' - \gamma$ interference, such as APV, in which the Z' mass appears via a propagator. Instead, the observables that are mainly responsible for the bounds on ϕ are those that precisely measure the physical Z couplings, and hence that are most sensitive to $Z_0 - Z_1$ mixing, as e.g. the LEP measurements of the total and partial Z widths, and the on–resonance asymmetries. The $Z_0 - Z_1$ mixing affects the Z couplings to fermions in two different ways: *i*) a direct effect originates from the fact that the physical Z couples, proportionally to s_ϕ, to the J_1^μ current, as is shown in (2.34); *ii*) an indirect effect is due to the fact that gauge boson mixing lowers the value of M_Z with respect to the 'SU(2) mass' M_{Z_0} in (2.5), according to $M^2_{Z_0} = M^2_Z + s^2_\phi(M^2_{Z'} - M^2_Z)$. This affects the overall coupling strength g_0 (2.6) as well as the expression for the weak mixing angle s_{eff}. While the first effect does not depend on the Z' mass, the second becomes more important for heavier Z', explaining why the bound on ϕ improves continuously for asymptotically large values of $M_{Z'}$. At LEP, the measurements of the various Γ_ℓ's mainly constrain the first effect, while Γ_h and Γ_Z are particularly sensitive to the indirect effects on g_0 and s_{eff}. In the asymmetries, that are ratios of cross sections, the overall coupling strength g_0 cancels, and only a smaller dependence on M_{Z_0}, via s_{eff}, is left. However the asymmetries are well suited to observe the direct effects of the gauge boson mixing on the couplings, and as it is shown in figures 6 to 8, they give important contributions to constrain ϕ. As it is apparent from figures 3 to 5, for values of $M_{Z'}$ close to the lower bound, the limits on ϕ are somewhat relaxed. This is true also when fermion mixing effects are not included, and is due to the fact that in this region of the parameter space, both $M_{Z'}$ and ϕ may lead to comparable effects in several electroweak observables, allowing for possible cancellations as well as for compensating effects in the χ^2 function.

We turn next to discuss the effects induced on the Z' constraints, by the simultaneous presence of fermion mixings. The solid lines in figures 1 to 5, that depict the limits on the Z' parameters for this case, have been obtained by minimizing the χ^2 function with respect to all the fermion mixing parameters while searching for the $M_{Z'}$ and ϕ bounds, thus allowing for cancellations between fermion mixings and Z' effects.

The lower bounds on $M_{Z'}$ that, as already stated, are mainly set by observables that are sensitive to Z' exchange diagrams, are almost unaffected by the small fermion mixings in the couplings, and show that cancellations are not effective in this case. From fig. 1 we see that this feature is independent of the particular model. In contrast, the bounds on ϕ are generally modified in the presence of fermion mixings and, as it is shown in fig. 2, in some cases they can be relaxed by a factor 2 or 3. This happens because the limits on Z_0–Z_1 mixing mainly result from the constraints on the modifications it induces in the fermion couplings to the physical Z. Since additional modifications of comparable magnitude in these couplings can originate also from fermion mixings, large cancellations between the two effects are possible.

Our results suggest that in constraining $M_{Z'}$ in a 'model independent' way, fermion mixing effects can indeed be neglected. We stress however that when the Higgs representation is specified, and a relation exists between ϕ and $M_{Z'}$, the constraints on $M_{Z'}$ are generally driven by those on ϕ, and are often much tighter than in the general case. Clearly, in the presence of fermion mixings, the corresponding lower bounds on $M_{Z'}$ are largely relaxed as well.

Any functional relation between ϕ and $M_{Z'}$ corresponds to a particular curve in the ϕ–$M_{Z'}$ plane. In figures 3 to 5, for each model, we have plotted in dotted lines some cases corresponding to a *minimal* Higgs sector. For example in fig. 3 we present our bounds for the χ model. As usual, the solid and the dashed lines give, respectively, the limits with and without the E_6 fermion mixings, while the dot–dashed line corresponds to the limits obtained in the SO(10) model, *i.e.* allowing only for the additional mixings with the SO(10) singlet neutrinos (for a recent discussion of Z' and ν mixing effects in SO(10) see also [14]). The minimal Higgs content of SO(10) implies $\phi \simeq \sqrt{\frac{2}{3}} s_w M_Z^2/M_{Z'}^2$ (dotted line), and it is apparent that, with this constraint, the tight bounds on ϕ rise the lower limit on $M_{Z'}$ up to $\sim$ 700 GeV. However, allowing for cancellations between the Z'_χ and the neutrino mixings, the bound is weakened to $M_{Z'} \gtrsim 550$ GeV. For the η

and ψ models, the minimal Higgs content corresponds to the **27** representation. Assuming in addition that the scalar partner of the neutrinos do not acquire a VEV, the relation between ϕ and $M_{Z'}$ depends only on the ratio $\sigma \equiv |v_u/v_d|^2$, where the VEV $v_{u(d)}$ gives masses to the $u(d)$-type quarks (hence, since $m_t \gg m_b$, $\sigma > 1$ is theoretically preferred). In fig. 4 (η model) and fig. 5 (ψ model), the dotted lines enclose the region of the ϕ–$M_{Z'}$ plane corresponding to the minimal E_6 Higgs sector, with $2 \leq \sigma \leq \infty$. We see again that in both these models fermion mixing effects can relax the lower limit on $M_{Z'}$ by as much as 200 – 300 GeV.

Another very important consequence of fermion mixing is the indirect effect of $s_L^{\nu_e}$ and $s_L^{\nu_\mu}$ on both g_0 and s_{eff}. Actually, the relevant quantity appearing in these two parameters, once G_μ is used as numerical input, is the product $\rho M_{Z_0}^2(G_F/G_\mu)$. This term induces strong *anti*–correlations among the m_t loop effects, the Z' parameters, and the ν_e and ν_μ mixings, that appear respectively in the three different factors. In particular, both a non–vanishing ϕ and non–vanishing $s_L^{\nu_e}$ or $s_L^{\nu_\mu}$ lead to a negative shift on s_{eff} (corresponding to a positive shift in the W-boson mass) as do also increasing values of m_t. It follows that when s_{eff}, which is constrained by the combination of all the NC measurements, becomes relevant for establishing a bound on the Z' parameters, the presence of neutrino mixings can indeed result in an apparent improvement of the Z' bounds. This effect is seen for instance in the limit in A in fig. 4 as well as in fig. 2 for $-1 \leq \beta \leq -0.5$ and positive ϕ. We note however that, since $s_L^{\nu_e}$ and $s_L^{\nu_\mu}$ are both consistent with zero, one may conservatively take the (looser) dashed–line bounds as the more reliable. A similar interplay exists between m_t and the Z' bounds, as it has been exhaustively discussed in [6], and between m_t and the $\nu_{e,\mu}$ mixings, as was noted in [10]. Increasing values of m_t then result in improved constraints on all these parameters that describe the new physics. We have also checked that when the top mass is left free to vary in the fit, while the prediction for m_t is drawn towards the lower values allowed by direct searches (91 GeV [49]]), the bounds shown in our figures, as well as in Tables V and VI, (that correspond to m_t=120 GeV), are not sensibly relaxed.

A different kind of interrelation between fermion mixings and the Z' bounds arises from the inclusion in our analysis of the CC data. The CC measurements in fact tend to be between 1 and 1.5 standard deviations away from their SM values, pushing some of the fermion mixings to non–zero values in order to account for the discrepancy. The CKM unitarity, for example, is better accounted for with a non–vanishing d_L mixing, the discrepancy in $\mu - e$ universality (a 1.3 σ excess in g_μ/g_e from π-decays), favours a non–vanishing ν_{eL} mixing ($(s_L^{\nu_e})^2 \sim 0.007$), while the longstanding problem of the disagreement between the computed and measured τ lifetime, points towards a significant ν_τ mixing $(s_L^{\nu_\tau})^2 \sim 0.03$ [10]. The effects of these non–vanishing fermion mixings propagate in all the NC observables, explaining why the best fits to the NC data (the 'stars' in figs. 6–8) still show a dispersion inside the $\pm 1\sigma$ region, similar to the SM fits (solid circles).

Turning now to discuss the constraints on the fermion mixings, we present in Table V the results of our global analysis, as 90 % c.l. upper bounds on the mixing parameters $(s^f_{L,R})^2$ that describe the fermion mixings in E_6. The first column in the Table V, labeled 'single', shows the bounds on $(s^f)^2$ when just one fermion mixing is allowed. The second column, labeled 'with Z'', shows how each single bound is relaxed in the presence of a new gauge boson. The third column ('complete') collects the bounds obtained by allowing the presence of all the different fermion mixings besides the Z' effects, leading in general to additional cancellations. Clearly the figures in column 3 give the most reliable limits, since all the new physics effects from E_6 are taken into account. The last column displays the observables that are most sensitive to the corresponding fermion mixing, and hence are the main responsibles for the bound. Since for different Z' models these results are qualitatively similar, we just show the limits for the ψ model, that are slightly more conservative than in the other cases.

From Table V we see that the limits on the $(s^f)^2$ parameters are very stringent, typically at the 1 % level, this is indeed due to the large number of precisely measured observables that contribute to them. We now discuss in detail these results.

The mixings of the R–handed charged leptons are mainly constrained by the corresponding Z–widths and asymmetries. By comparing the single bounds with the bounds obtained in the presence of Z' effects, we see that the limits on the $(s^\ell_R)^2$ factors are not significantly affected by Z_0–Z_1 mixing. This can be understood by noting that, via the indirect effects on g_0 and s_{eff}, non vanishing values of ϕ would induce a large effect on Γ_h and Γ_Z, and thus, due to the LEP constraints on the hadronic and total Z widths, Z_0–Z_1 mixing cannot compensate effectively the mixings of the charged leptons in the Γ_ℓ's. In contrast, in the 'complete' analysis, this mechanism is much less effective. This is due to the fact that in this case the effects of a non vanishing ϕ in Γ_h and Γ_Z can be well balanced by non vanishing mixings for the d_L–type quarks. As a result, due to this complicated mechanism of cancellations, the 'complete' limits for the charged lepton mixings are somewhat relaxed.

For the down–type quarks, the most important constraint on s^d_L arises from the CC bounds on the unitarity of the CKM matrix. The higher order effect of a Z' on this constraint (3.4) is generally very small (in particular it vanishes exactly in the ψ model – Table V), and then the presence of a Z' does not relax substantially the corresponding bound. However the limit is indeed relaxed in the complete analysis, due to the compensating effect of a non–zero $s^{\nu_\mu}_L$ (see (3.4)). For s^s_L the CKM constraint is not very effective, due to the Cabibbo suppression, while, due

TABLE V. 90 % c.l. upper bounds on the ordinary–exotic fermion mixing parameters for the ψ-model. The column labeled 'single' gives the limits obtained when only the corresponding single mixing is present. The column 'with Z'' shows how the single bounds are relaxed in the presence of the Z'_ψ. In the column 'complete', cancellations among the effects of all the different fermion mixings and of the new gauge boson are allowed. The bounds correspond to the value $\Lambda_0 = 2$. The last column displays which observables are more important to determine the limits. $s_{eff}^{\rm LEP}$ and $s_{eff}^{\rm NC}$ refer to the effective weak mixing angle, measured respectively in Z-peak and NC experiments. The last three lines collect the *indirect* bounds on the leptonic RHC parameters. Qualitatively similar bounds are obtained also in the χ and η models.

	Single	with Z'	Complete	Source
$(s_R^e)^2$	0.0062	0.0078	0.013	$\Gamma_e, A_e^{\rm FB}, A_\mu^{\rm FB}, \nu e$
$(s_R^\mu)^2$	0.0086	0.0087	0.011	$\Gamma_\mu, A_\mu^{\rm FB}$
$(s_R^\tau)^2$	0.011	0.011	0.013	$\Gamma_\tau, A_\tau^{pol}, A_\tau^{FB}$
$(s_L^d)^2$	0.0046	0.0051	0.0094	$\lvert V_{ui}\rvert^2, \Gamma_h, \Gamma_Z, \nu q$
$(s_L^s)^2$	0.011	0.020	0.020	$\Gamma_h, \Gamma_Z, V_{ui}^2$
$(s_L^b)^2$	0.011	0.019	0.020	$\Gamma_h, \Gamma_Z, \Gamma_b, A_b^{\rm FB}$
$(s_L^{\nu_e})^2$	0.0097	0.010	0.016	$s_{eff}^{\rm LEP}, g_e, s_{eff}^{\rm NC}, M_W$
$(s_L^{\nu_\mu})^2$	0.0019	0.0021	0.0074	$\lvert V_{ui}\rvert^2, g_\mu, \nu q, s_{eff}^{\rm LEP}, M_W$
$(s_L^{\nu_\tau})^2$	0.032	0.048	0.058	Γ_Z, g_τ
$(s_R^e s_R^{\nu_e})^2$	0.0003	0.0005	0.0011	
$(s_R^\mu s_R^{\nu_\mu})^2$	0.0004	0.0005	0.0009	$(s_R^\nu)^2 \le \sum_\ell (s_L^{\nu_\ell})^2$
$(s_R^\tau s_R^{\nu_\tau})^2$	0.0005	0.0007	0.0011	

to the relatively large experimental errors, Γ_b and $A_b^{\rm FB}$ are not so effective for constraining s_L^b. The most important constraints on the s_L and b_L mixings come from the hadronic and total Z-widths. However, as we have already pointed out, fermion mixing effects in these two observables can be efficiently compensated by Z' indirect effects. Hence, for both s_L^s and s_L^b, the bounds are largely relaxed in the presence of a new neutral boson. Finally, since all the d_L–quarks mixings modify Γ_h in the same direction, no relevant additional cancellations are possible in the 'complete' analysis, and the corresponding bounds are not further relaxed.

The ν_e and ν_μ mixings propagate to most of the observables by affecting the relation between G_μ and G_F. In the CKM unitarity constraint, the effect of the ν_e mixing induced by G_F/G_μ cancels against the direct effect on β decays, and only the indirect effect of $s_L^{\nu_\mu}$ in (3.4) is left. Contrary to s_L^d, the ν_μ mixing tends to increase the theoretical prediction for $\sum_i |V_{ui}|^2$ which, in the SM, is already almost one standard deviation above the very accurate experimental value. As a consequence, $s_L^{\nu_\mu}$ turns out to be the most strongly constrained parameter. G_μ/G_F is also tightly constrained by its effect on the NC variables s_{eff} and g_0, and to a less extent also by the measurement of M_W. These constraints are quite effective since, as already mentioned, in these quantities no cancellations between the ν_e and ν_μ mixings and the Z' effects are possible. Finally, also the CC tests of leptonic universality impose additional constraints on the neutrino mixings. From this discussion it is clear that the bounds on $s_L^{\nu_e}$ and $s_L^{\nu_\mu}$ cannot be significantly relaxed when allowing for a Z'. In the complete analysis however, we see that the cancellations against the remaining fermion mixings can give relevant effects.

For $s_L^{\nu_\tau}$ the most important constraint comes from the Z invisible width, that is bounded by the measurements of Γ_Z, Γ_h and Γ_ℓ. The effectiveness of this bound, however, depends crucially on the assumption that singlet neutrinos are heavier than $\simeq M_Z/2$. In fact, while a mixing between ν_τ and exotic states always results in a reduced invisible width (2.31), light singlet neutrinos mixed with non–singlet neutral states would open new invisible channels for the decay of the Z, allowing for a compensating effect. Also it should be noted that the effect of ν_τ mixing in $\Gamma_{\rm inv}$ depends on the isospin of the neutrino involved in the mixing, i.e. on the value of the parameter Λ_0^τ in (2.32). As a consequence, the bound would become stronger for larger Λ_0^τ, while the constraint would be ineffective for $\Lambda_0^\tau \simeq 0$ (see [10]). Clearly the constrain from $\Gamma_{\rm inv}$ is affected by the presence of a Z', and it is even further relaxed when allowing for the other fermion mixings. Another constraint on $s_L^{\nu_\tau}$ comes from τ decay. As is well known, the observed excess in the τ lifetime is better explained in the presence of a non–vanishing ν_τ mixing [50], and this fact is partially responsible for the large upper bounds on $s_L^{\nu_\tau}$.

A final comment concerns our approximation of neglecting, in the CC leptonic processes, the RHC terms $(s_R^\ell s_R^{\nu_\ell})^2$ that were defined in section 2 eq. (2.26). For $\ell = e, \mu$, the existing *direct* constraints on RHC are quite stringent. For example the results given in Table VI-*b* of [9] for the E_6 case imply $(s_R^e s_R^{\nu_e})^2 \le 0.0018$ and $(s_R^\mu s_R^{\nu_\mu})^2 \le 0.0015$ at 90% c.l. These limits were derived from measurements such as the muon decay parameters and the electron polarization in β decays, that are directly sensitive to the leptonic RHC, and at the same time are not affected by new neutral gauge bosons, thus they hold also in the present case. For the τ lepton, a much weaker *direct* constrain can be obtained from a recent measurement of the τ–decay Michel parameter $\rho_\tau = 0.725 \pm 0.031$ [51]. This gives $(s_R^\tau s_R^{\nu_\tau})^2 \lesssim 0.10$, and we note that, relying only on this limit, the τ RHC contribution to the decay rate (2.26) and to (3.6) could not be neglected. However, as already discussed, much tighter *indirect* limits on all the RHC terms can be derived via the relation $(s_R^\nu)^2 \leq \sum_\ell (s_L^{\nu_\ell})^2$ (see section 2). For completeness these limits are included in Table V as well. A comparison between the bounds on the L–mixings $(s_L^\ell)^2$ and on the corresponding $\mathcal{O}(s_R^4)$ terms then justifies our approximation.

In Table VI we show the bounds on the fermion mixings in SO(10) GUTs, where the new neutral boson corresponds to Z_χ, and only one additional singlet neutrino per generation is present. The 'single' bounds clearly coincide with those shown for the neutrinos in Table V. Then, in the first column, labeled 'all ν's' in Table VI, we list the bounds obtained in the presence of all the different neutrino mixings. The second column, labeled 'with Z'', corresponds to the case when also the Z'_χ effects are allowed. We see that only moderate cancellations take place among the different mixings, as is expected from our previous discussion about the origin of the bounds for the neutrinos, while again the presence of a Z' only affects the bounds on $s_L^{\nu_\tau}$.

TABLE VI. 90 % c.l. upper bounds on the mixings of the ordinary neutrinos with the three singlet neutrinos ν^c present in SO(10). The limits on each single parameter can be read from the 'single' column in Table V. The column 'all ν's' gives the bounds derived by allowing for cancellations among the different neutrino mixings. The bounds 'with Z', are obtained by allowing for the simultaneous effects of the neutrino mixings and of the new Z'_χ.

	all ν's	with Z'
$(s_L^{\nu_e})^2$	0.011	0.012
$(s_L^{\nu_\mu})^2$	0.0021	0.0020
$(s_L^{\nu_\tau})^2$	0.036	0.045

5. Conclusions

Many extensions of the SM based on gauge groups with rank larger than 4 allow for the presence of additional neutral gauge bosons, light enough to induce observable effects in precision experiments. As we have stressed, a general consequence of assuming an extended gauge symmetry is that additional fermionic degrees of freedom must be present to insure the consistency of the model.

In this paper we have analysed in detail the consequences of assuming the simultaneous presence of new, family–universal, Z_1 bosons together with new fermions. We have argued that the new fermions will naturally mix with the known light states and we have outlined a general formalism that allows to study the simultaneous effects of the new degrees of freedom on electroweak observables. We have shown that flavor changing neutral interactions could naturally arise in these models, since in general they are not equipped with a GIM mechanism. However, large masses for both the new fermions and the new gauge bosons lead to a natural suppression of the flavor changing low–energy couplings of the light states, and then the vanishingly small rates observed for the FCNC are easily accommodated in these models. However, if rates larger than the SM expectations were to be observed in future experiments, this could indeed be interpreted as a signal of new physics from this class of GUTs [11]. After a general discussion, we have specified our analysis to a class of models that can be easily embedded in the E_6 group, and we have investigated the consequences of the presence, at relatively low energy, of one additional Z_1, together with the 12 additional fermions per generation present in the **27** representation of the unifying group E_6.

We have identified a set of parameters that describe the new physics in these models. The effects of the new gauge boson have been parametrized in terms of a Z_0–Z_1 mixing angle ϕ and of its physical mass $M_{Z'}$. We have described the fermion mixings in the neutral sector with three mixing parameters $(s_L^\nu)^2$, and we have introduced the additional effective parameters $\Lambda^\nu_{0,1}$ to describe the kind of new states involved in the mixing. For the charged sector, relying on the very stringent experimental limits on FCNC, we have neglected possible flavor changing couplings of the light fermions. We have also argued that this restriction is not crucial to derive reliable limits for the Z' effects. To describe the remaining mixings, we have introduced the parameters $(s_R^e)^2$, $(s_R^\mu)^2$, $(s_R^\tau)^2$ for the charged leptons, and $(s_L^d)^2$, $(s_L^s)^2$, $(s_L^b)^2$ for the d–type quarks.

We have then performed a global analysis of the electroweak NC and CC data, obtaining very stringent constraints on all the parameters that describe the new effects, and we studied the interrelations among the different effects by confronting the bounds obtained in several different situations. We have first constrained the Z' effects alone (much in the spirit of previous analyses [5,6]), then each single fermion mixing, then we have analysed the interrelations of the Z' effects with each fermion mixing parameter, paying particular attention to the influence of the neutrino mixings, and finally we have simultaneously constrained all the parameters that describe the new physics from E_6, thus deriving a very reliable set of bounds. The particular case of the SO(10) GUT, where only three additional singlet neutrinos are present, was analysed as well. Finally, we have also commented on the possible correlations of our bounds with the unknown value of the top mass.

As a summary of our results, we have found that the limits on neutral gauge bosons mixings can indeed be affected by the simultaneous presence of fermion mixing effects. Even if somewhat relaxed by possible accidental cancellations and other compensating effects, the bounds on ϕ are still quite stringent, resulting in all cases in $|\phi| \leq 0.02$. The parameters that describe the mixings with new heavy states of the charged leptons, of the d quarks, and of the e and μ neutrinos, are tightly constrained at the 1% level, the s and b quark mixings do not exceed 2%, while the bounds on ν_τ mixing are looser, at the 5-6% level, and some indications of a possible non–zero mixing also exist in this case. As discussed above, the main source of the constraints for gauge boson mixing and fermion mixing effects is provided by the accurate LEP measurements. The limits on $M_{Z'}$, in contrast, arise mainly from Z–Z' and γ–Z' interference effects in low energy (ν-scattering and APV) experiments, and are quite insensitive to the presence of non vanishing (but small) fermion mixings. As a result, the $M_{Z'}$ mass is constrained to values larger than 170-400 GeV, depending on the model. In specific models for the Higgs structure where ϕ and $M_{Z'}$ are related, the bounds on $M_{Z'}$ are derived from those on ϕ, and are then much stronger (500-800 GeV) than in the general case. However, fermion mixing effects are again important in most of these cases, allowing to relax the lower limits by as much as 200-300 GeV.

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Figure captions

Fig. 1: Lower bound on $M_{Z'}$, corresponding to $\chi^2 = \chi^2_{\text{min}} + 3.84$, for the general E_6 neutral boson in eq. (2.3), as a function of $\cos\beta$. The dashed line gives the bound in the absence of fermion mixings. The solid line is obtained by allowing for all the fermion mixings that can be present in E_6.

Fig. 2: 95% c.l. limits ($\chi^2 = \chi^2_{\text{min}} + 3.84$) for the Z_0–Z_1 mixing angle ϕ, as a function of $\cos\beta$. The dashed lines enclose the allowed region when no additional effects due to the new fermions are present. The solid lines give the bounds when fermion mixing effects are taken into account.

Fig. 3: 90% c.l. contours ($\chi^2 = \chi^2_{\text{min}} + 4.61$) in the two parameter plane $M_{Z'}$–ϕ, for the model χ (corresponding to SO(10)/SU(5)). Dashed lines are in the absence of fermion mixings. Dot–dashed lines give the bounds when the neutrinos are mixed with the neutral singlets present in SO(10). Solid lines are obtained by allowing for all the mixings that could appear in E_6. The dotted curve depicts the theoretical relation between $M_{Z'}$ and ϕ, assuming a minimal SO(10) Higgs sector.

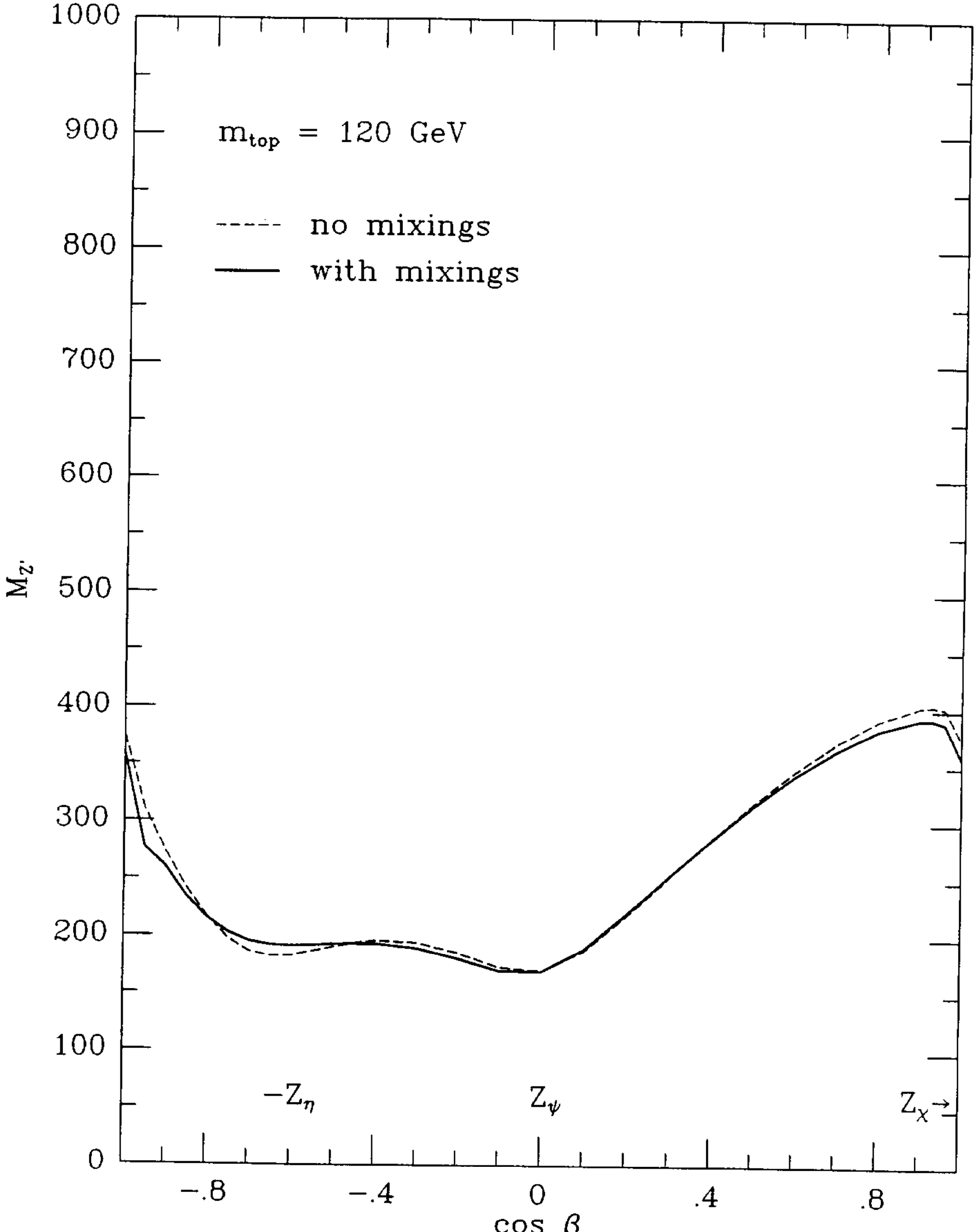
Fig. 4: Same as fig. 3, for the superstring–inspired η model, in which E_6 directly breaks to rank 5. The dotted curves enclose the region $2 \leq \sigma \leq \infty$, corresponding to a minimal E_6 Higgs sector, with $\sigma \equiv (v_u/v_d)^2$.

Fig. 5: Same as fig. 4, for the ψ model, that corresponds to E_6/SO(10).

Fig. 6: χ model. Normalized deviations $(O^{th} - O^{exp})/\Delta O^{exp}$ for the most accurate electroweak measurements. The solid circles give the deviations from the experimental values, for O^{th} computed within the SM. The stars give the best fit to the χ model, in the presence of fermion mixing effects. The triangles correspond to the deviations at the boundary points labeled as A and B in fig. 3, and show which observables are more relevant to constrain the Z'_χ parameters. CC constraints on the fermion mixing parameters are also displayed.

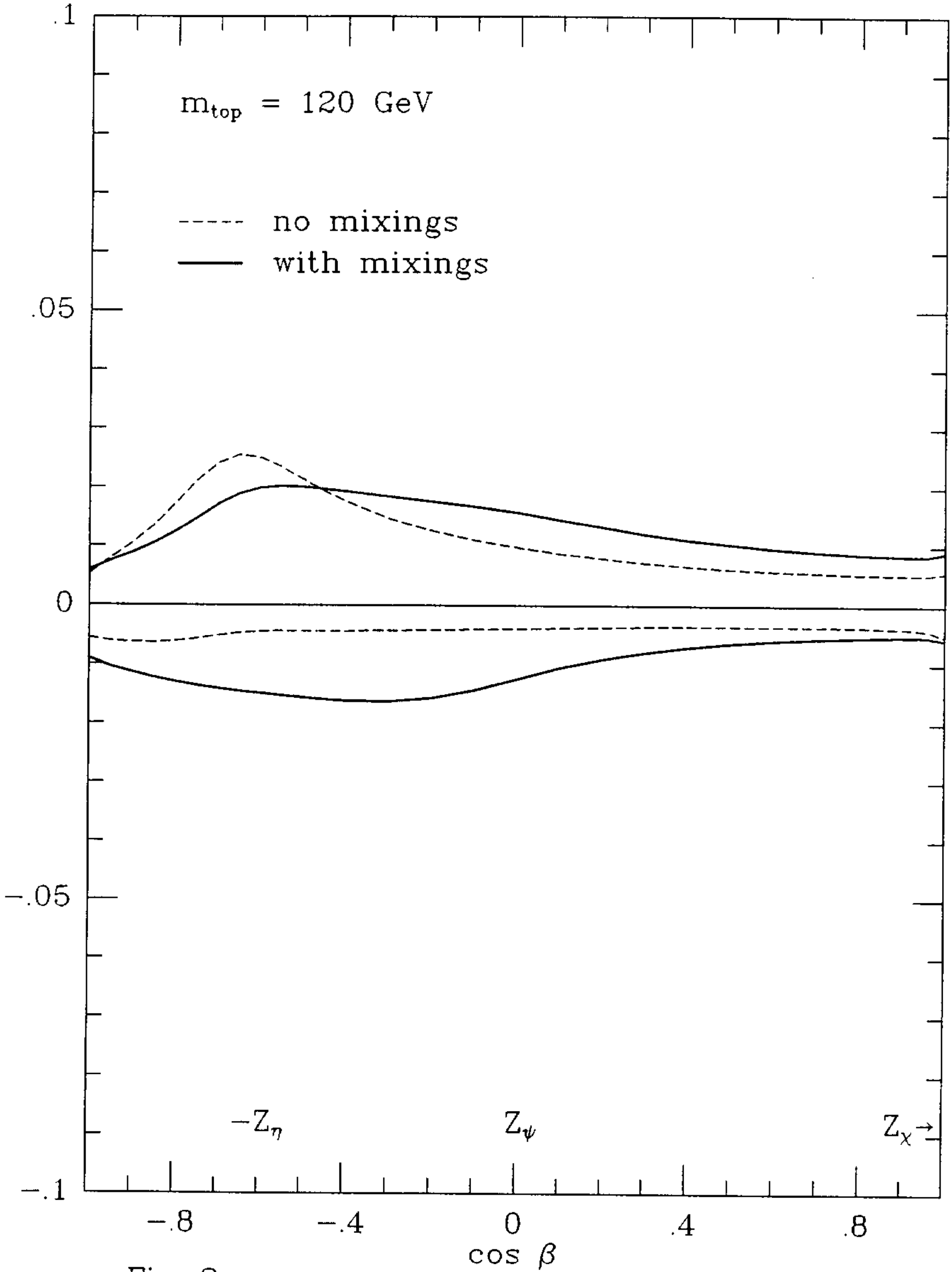
Fig. 7: Same as fig. 6, for the η model, with the boundary points A and B from fig. 4.

Fig. 8: Same as fig. 6, for the ψ model, with the boundary points A and B from fig. 5.



1000
900
800
700
600
500
400
300
200
100
0
$M_{Z'}$
m_{top} = 120 GeV
no mixings
with mixings
$-Z_\eta$
Z_ψ
Z_χ→
−.8
−.4
0
.4
.8
cos β

Fig. 1



.1
.05
0
−.05
−.1
m_{top} = 120 GeV
no mixings
with mixings
$-Z_\eta$
Z_ψ
$Z_\chi \rightarrow$
−.8
−.4
0
.4
.8
cos β

Fig. 2

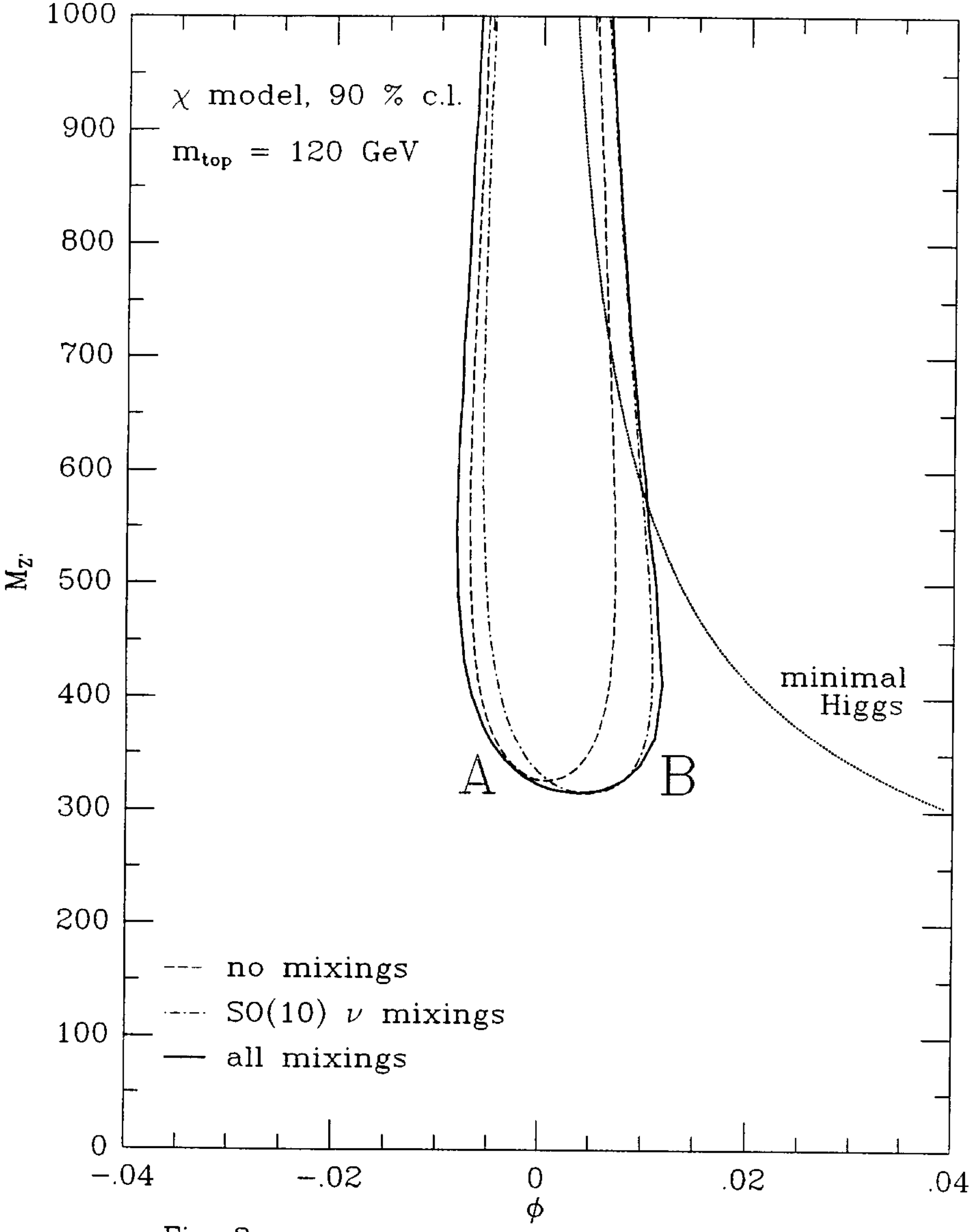

χ model, 90 % c.l.
m_{top} = 120 GeV
minimal Higgs
A
B
no mixings
SO(10) ν mixings
all mixings
$M_{Z'}$
φ
1000
900
800
700
600
500
400
300
200
100
0
−.04
−.02
0
.02
.04

Fig. 3

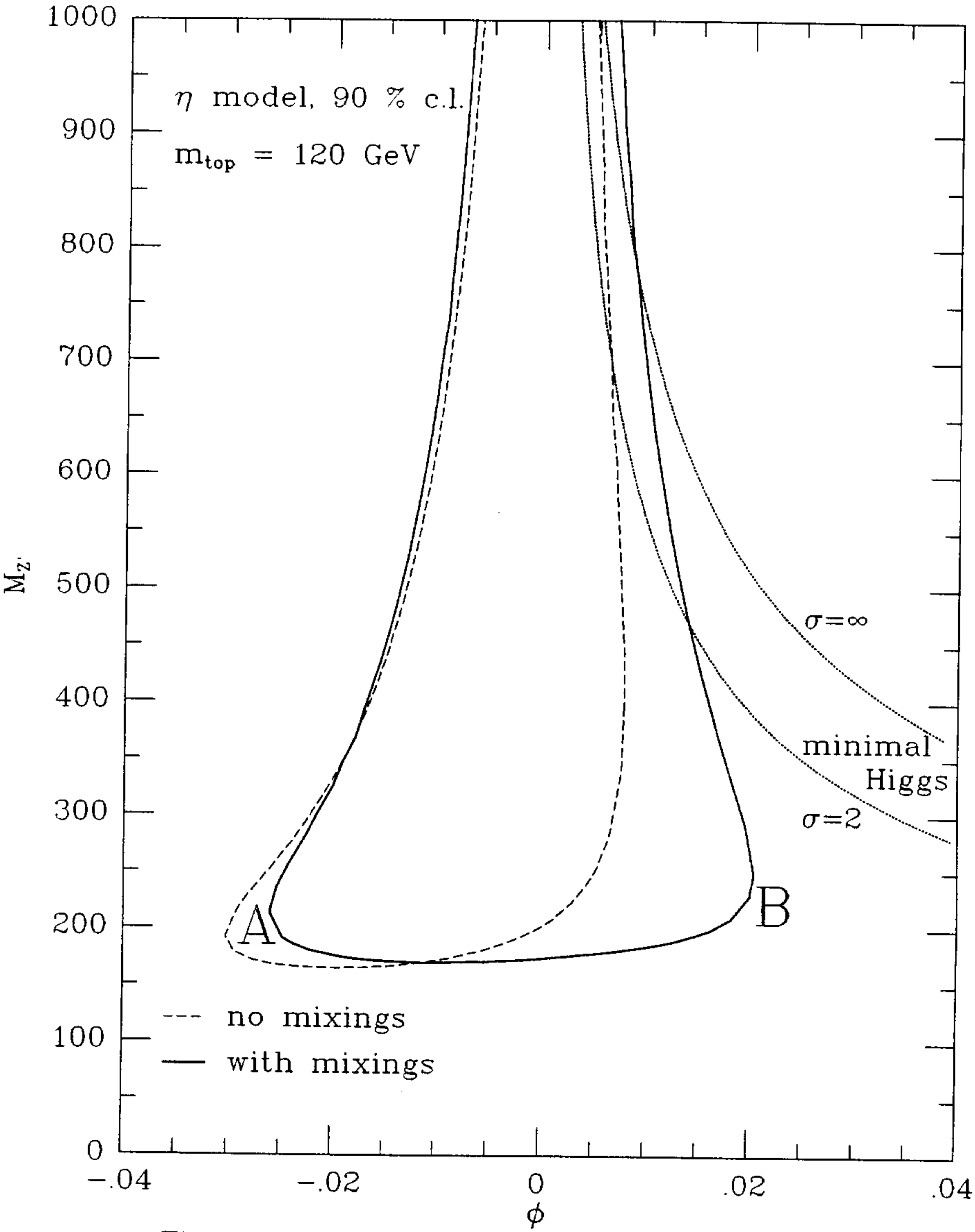
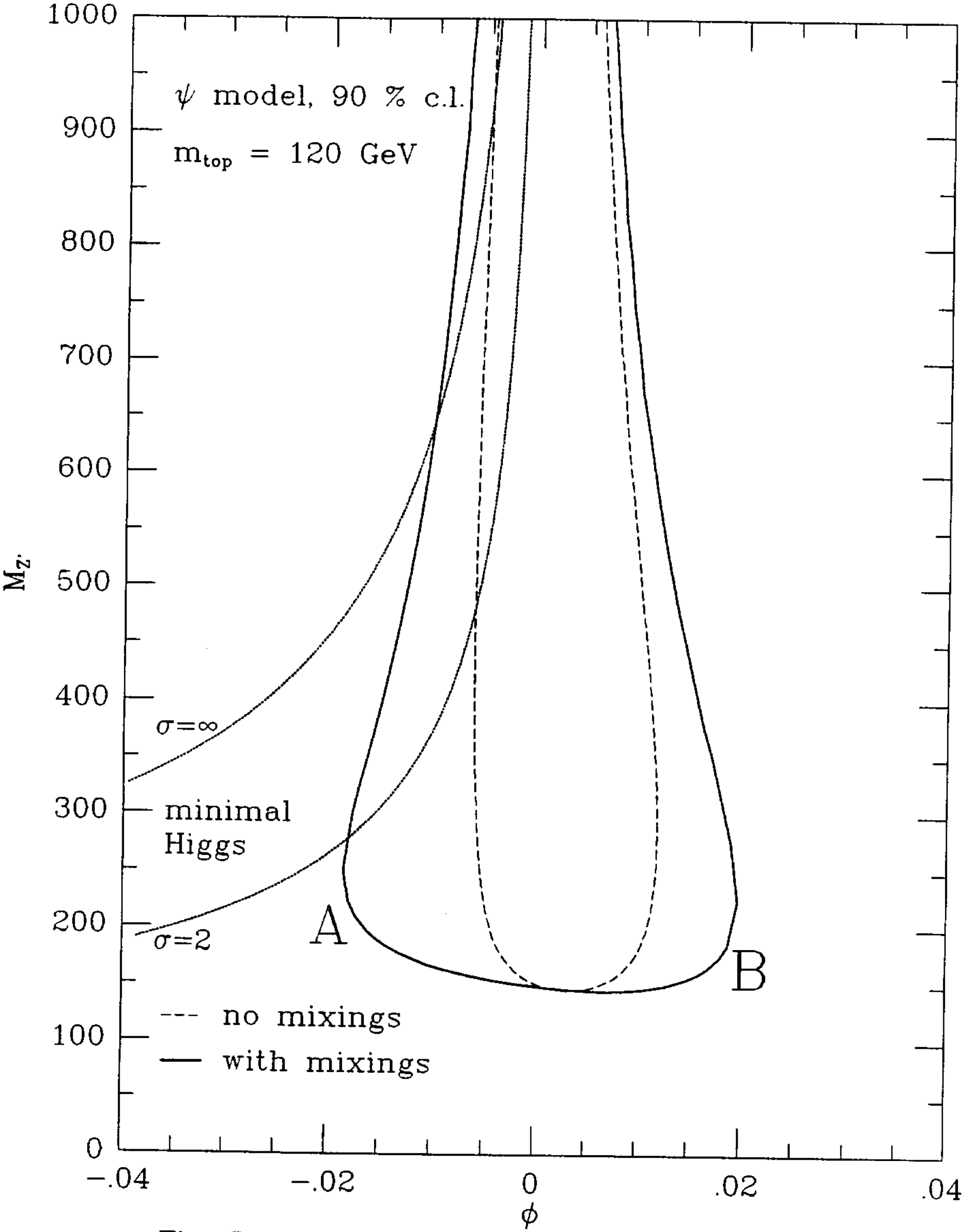
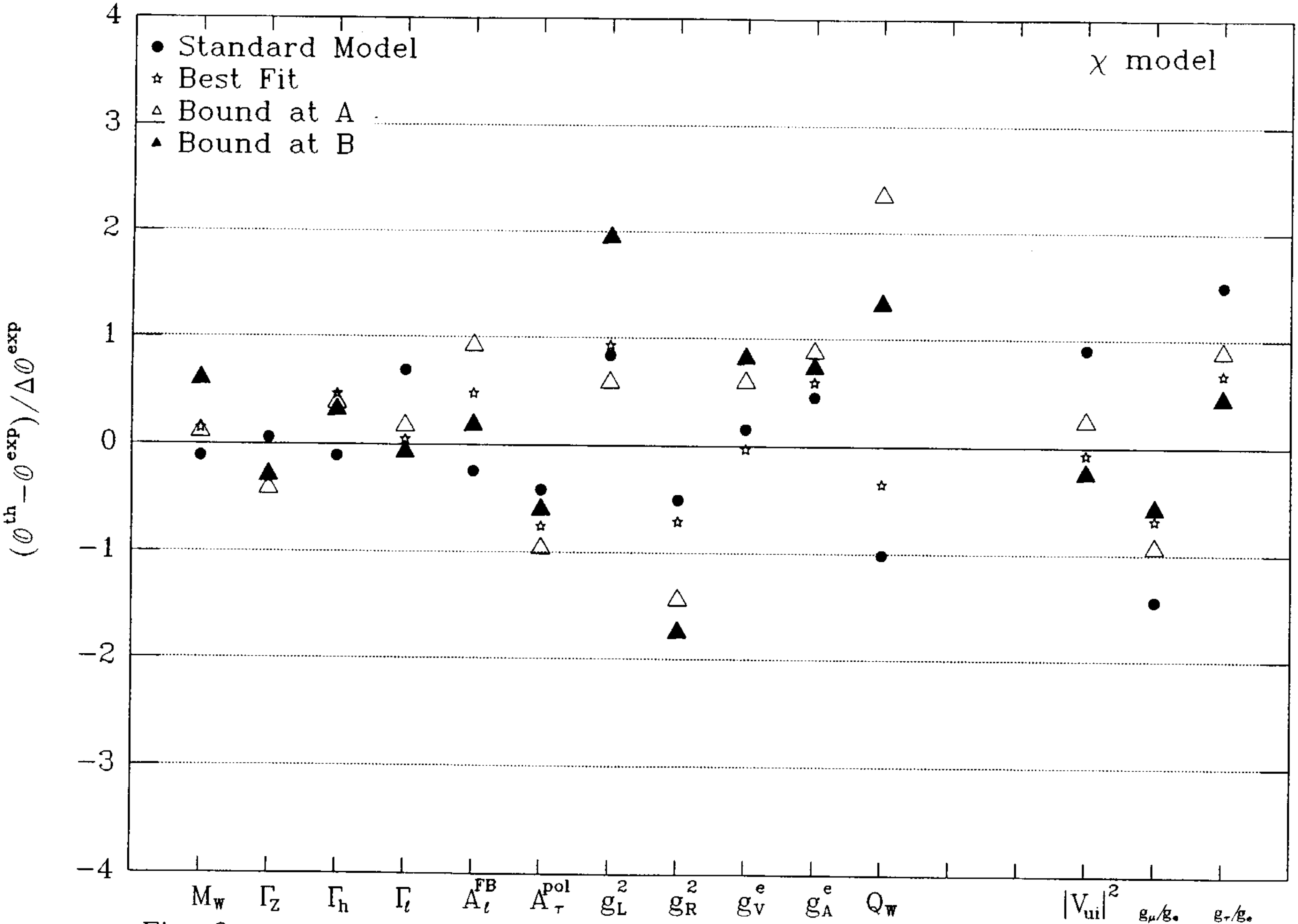

η model, 90 % c.l.
m_top = 120 GeV
1000
900
800
700
600
500
400
300
200
100
0
M_Z'
−.04
−.02
0
.02
.04
φ
A
B
σ=∞
minimal
Higgs
σ=2
no mixings
with mixings

Fig. 4



ψ model, 90 % c.l.
m_{top} = 120 GeV
σ=∞
minimal
Higgs
σ=2
A
B
no mixings
with mixings
$M_{Z'}$
φ
1000
900
800
700
600
500
400
300
200
100
0
−.04
−.02
0
.02
.04

Fig. 5



Standard Model
Best Fit
Bound at A
Bound at B
χ model
4
3
2
1
0
−1
−2
−3
−4
$(\mathcal{O}^{th}-\mathcal{O}^{exp})/\Delta\mathcal{O}^{exp}$
M_W
Γ_Z
Γ_h
Γ_ℓ
A_ℓ^{FB}
A_τ^{pol}
g_L^2
g_R^2
g_V^e
g_A^e
Q_W
$|V_{ui}|^2$
g_μ/g_e
g_τ/g_e

Fig. 6

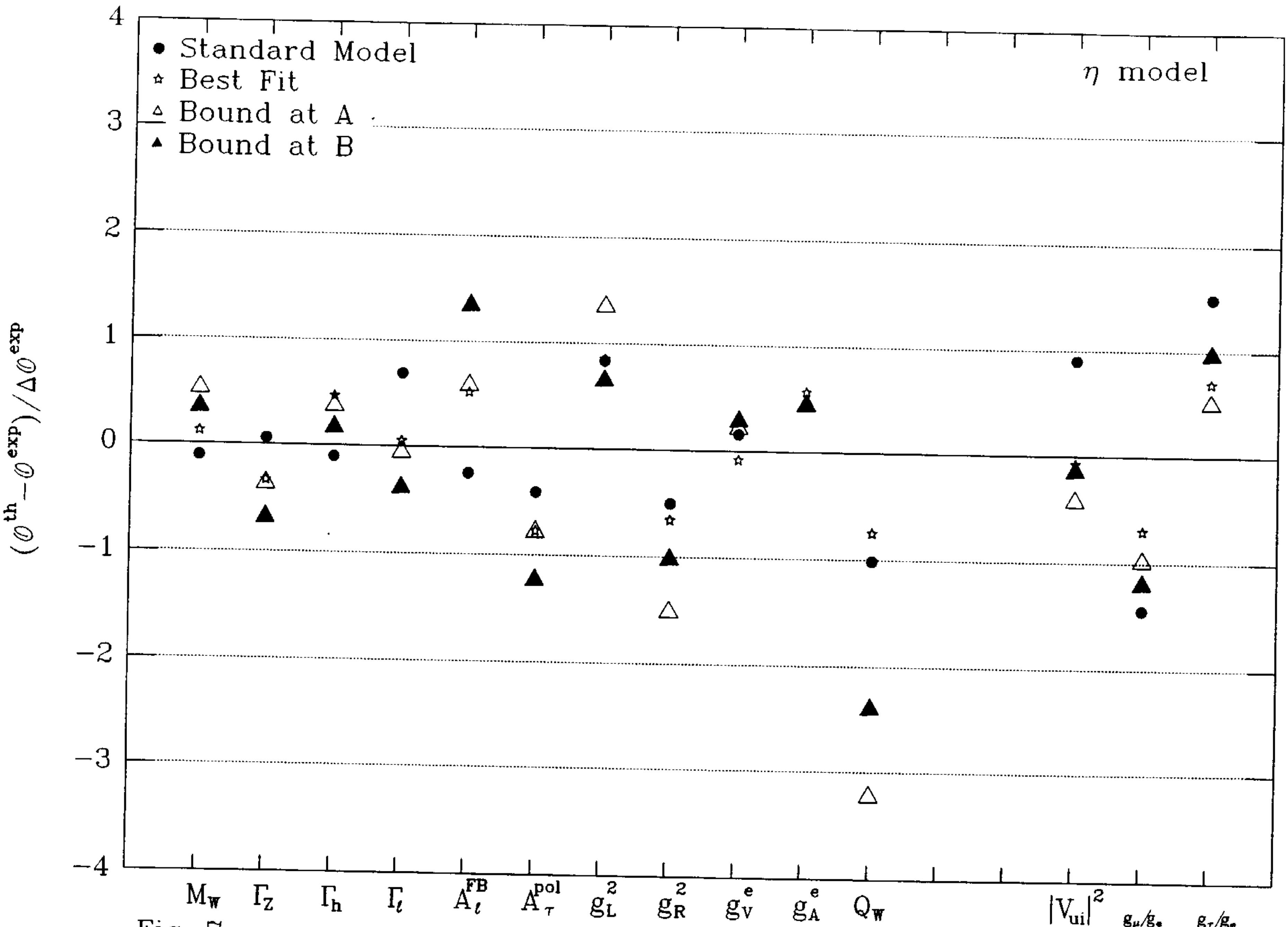
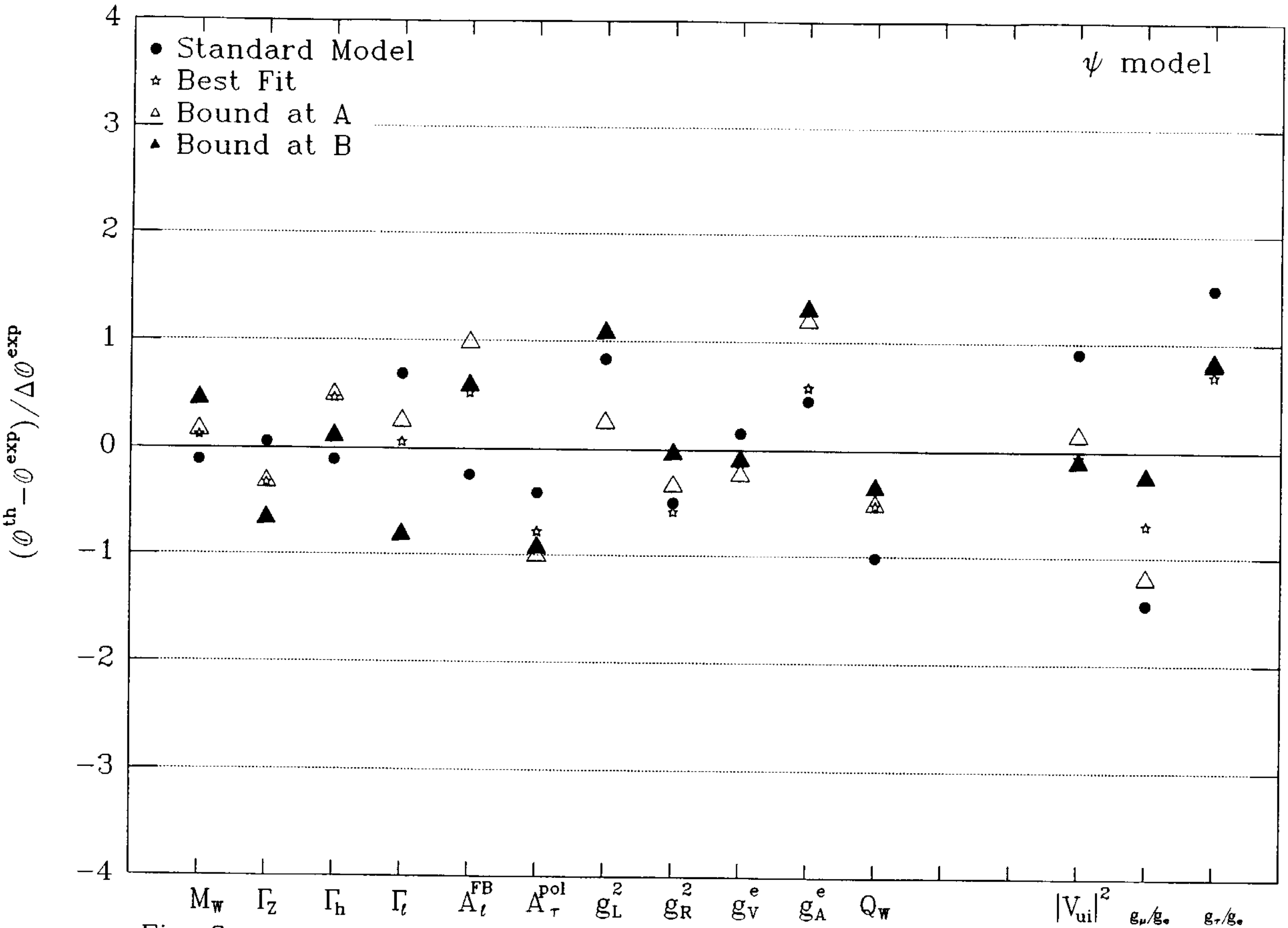

Standard Model
Best Fit
Bound at A
Bound at B
η model
$(\mathcal{O}^{th} - \mathcal{O}^{exp})/\Delta\mathcal{O}^{exp}$
4
3
2
1
0
−1
−2
−3
−4
M_W
Γ_Z
Γ_h
Γ_ℓ
A_ℓ^{FB}
A_τ^{pol}
g_L^2
g_R^2
g_V^e
g_A^e
Q_W
$|V_{ui}|^2$
g_μ/g_e
g_τ/g_e

Fig. 7



Standard Model
Best Fit
Bound at A
Bound at B
ψ model
$(\mathcal{O}^{th}-\mathcal{O}^{exp})/\Delta\mathcal{O}^{exp}$
4
3
2
1
0
−1
−2
−3
−4
M_W
Γ_Z
Γ_h
Γ_ℓ
A_ℓ^{FB}
A_τ^{pol}
g_L^2
g_R^2
g_V^e
g_A^e
Q_W
$|V_{ui}|^2$
g_μ/g_e
g_τ/g_e

Fig. 8