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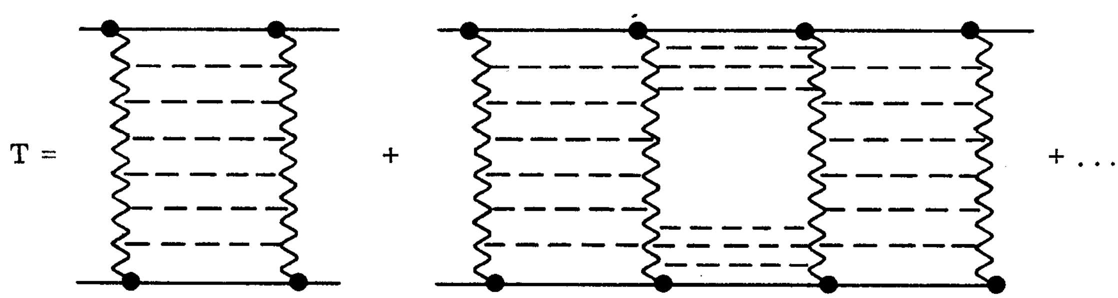
A NOTE ON REGGE CUTS

It is well known that the Reggeon calculus of Gribov[1,2] conflicts in detail with the interesting development of Abarbanel[3,4] who isolates N-Reggeon irreducible amplitudes using the unitarity relation and the multiperipheral model of production. For example, there are several opinions as to the correct sign of the two-Reggeon cut contribution to elastic scattering.[5] In this note a simple and probably unoriginal argument will be presented to show that the negative sign of the cut is correct and that the arguments of Refs. 3 and 4 omit a physically important contribution to the unitarity sum which is actually the dominant two Reggeon cut contribution. This term essentially changes the sign of the obvious contribution. The new graphs discussed here are a special case of the checkerboard graphs discussed in Refs. 6 and 7 and of the twin fireball contribution discussed in ancient times.[8]

Since our primary concern here is pedagogical, we shall be content to discuss only the leading contributions to the one- and two-Reggeon amplitudes. Consider the elastic scattering amplitude made up of a ladder graph and a window graph in the form



$$T = ZG_0Z + ZG_0ZG_1\,ZG_0Z + \ldots \quad ,$$

where Z is the sum of the vertical propagators in the two terms. The collection of propagators in the horizontal direction are termed G_0 and G_1, where G_0 has no gap in the rapidities of the secondaries as illustrated by the familiar ladder graphs, ZG_0Z. On the other hand, G_1 is required to have one large rapidity gap. A more precise definition of the gap will not be required here. There are further contributions to the second term involving four Z's; in particular there is a zero gap term in which G_0 replaces G_1 but such additional terms do not affect our arguments and can be easily discussed if desired.[9] Following Mandelstam's arguments[2] nonplanar twists are included along the top and bottom of the window diagram. These must be present in order to get a J-plane singularity but all terms contribute to our simple unitarity formulae.

The ladder graphs are known to lead to a Regge behavior for large energies and to that end let us define the Regge contribution R as $R \simeq ZG_0Z$. In the same large energy limit, several studies of such graphs[10] have shown that the leading terms in G_0 and G_1 are purely imaginary and this property will be of particular interest to our discussion.

The leading absorptive part of T is calculated by taking the discontinuity across the $G_{0,1}$ factors. The discontinuities involving the propagators in the Z factors are generally of a nonleading order and in any case can be grouped as generalized vertex-type corrections to the ladder graphs which we shall ignore for simplicity. The result for Im T is a sum of positive definite terms:

$$\mathrm{Im}\, T = (1 + ZG_0^*ZG_1^*)\, Z\, \mathrm{Im}\, G_0 Z\, (1 + G_1 Z G_0 Z) + (ZG_0^*Z)\, \mathrm{Im}\, G_1\, (ZG_0Z) \ .$$

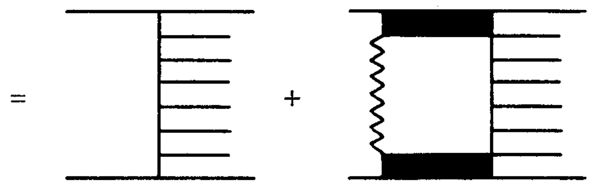
The last term is positive and it is clearly the two-Reggeon term isolated by Abarbanel.[3] However, it is not the only such term as is easily seen by examining the Z^4 contributions in the first term.

Since the imaginary parts of G_0 and G_1 should dominate their real parts, the total absorptive part can be written as

$$\begin{aligned} \mathrm{Im}\, T &= \mathrm{Im}\, R - R^* \mathrm{Im}\, G_1 R + 2\, (\mathrm{Im}\, RG_1)\, \mathrm{Re}\, R + \dots \\ &= \mathrm{Im}\, R - R^* \mathrm{Im}\, G_1 R \quad , \end{aligned}$$

where the explicitly neglected term involves two real parts and hence should be small. There are several other equivalent ways of writing this result. The physical reason for the change in sign of the dominant two-Reggeon contribution can be explored by examining the production amplitude for secondaries with no rapidity gap. This is given by the coefficients of the Im G_0 term:

$$T_0(2 \to N) = Z + ZG_1 \; ZG_0Z = Z(1 + G_1R) \quad .$$



The celebrated multiperipheral model[11] assumes that only the first term is important. It is now clear that a consistent treatment of the two-Reggeon contribution demands the inclusion of the second term. This term represents the physical process in which the Regge exchange (R) produces two fireballs (G_1) that in turn produce secondaries by the standard multiperipheral mechanism (Z). The elastic states are, of course, included in G_1. The importance of this term is underscored by noting that since the two fireball phase space is of order i/s, this process is the same order in s (except perhaps for log s) as Z if R is Pomeron exchange. This importance of this type of process has been emphasized in Ref. 6. This shadowing effect must be included if unitarity is to be implemented in a physically sensible way.

An extension of the approach used here, and inclusion of the correct shadowing in the multiperipheral production amplitudes, should allow one to carry through the interesting program of Abarbanel to isolate the various Regge singularities.

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4. G. F. Chew, invited talk, XVI International Conference on High Energy Physics, September 1972. See also D. Amati, S. Fubini, and A. Stanghellini, Nuovo Cimento 26, 896 (1962).
5. A. White gets a (-) sign. G. F. Chew argues on physical grounds for a (+) sign. F. E. Low advocates a (±) sign (public communication at the conference referenced in Ref. 4).
6. S. Auerbach, R. Aviv, R. Sugar and R. Blankenbecler, Phys. Rev. Letters 29, 522 (1972), and Phys. Rev. (to be published).
7. H. M. Fried and R. Blankenbecler, Stanford Linear Accelerator Center preprint (1971), to be published.
8. M. Baker and R. Blankenbecler, Phys. Rev. 128, 415 (1962).
9. The importance of insuring that the resultant force between two Pomeron trajectories is not sufficient to bind them has been shown in Ref. 6. We shall not bother to explicitly enforce this physical requirement on our model.
10. See, for example, H. Cheng and T. T. Wu, Phys. Rev. D3, 2394 (1971) where earlier references can be found to their work and S. J. Chang, T. M. Yan, and Y. P. Yao, Phys. Rev. D4, 3014 (1971).
11. See the classic paper of G. F. Chew and A. Pignotti, Phys. Rev. 176, 2121 (1968).