

On representations of Yangian of Lie Superalgebra $A(n, n)$ type

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Abstract. The finite-dimensional irreducible representations of Yangian of Lie Superalgebra of $A(n, n)$ type is described in terms of Drinfel'd polynomials. The necessary and sufficient conditions of finite-dimensionality of irreducible representation are formulated and proved. The Poincare-Birkhoff-Witt theorem for Yangian of $A(n, n)$ Lie Superalgebra is proved also.

1. Introduction

The description of irreducible representations of Yangians of Lie Superalgebras be an important problem for the theory of exactly solvable models of Statistical Mechanics and Quantum Field Theory. For example, construction of the transfer-matrix based on the determination of image of universal R -matrix of quantum double of Yangian by the action of tensor product of irreducible and identical representations. Computation of spectrum of Hamiltonian and correlation functions is based also on the representation theory of Yangians by using universal R -matrix formula. Therefore representations theory of Yangians of simple and reductive Lie algebras is a developed theory, which began to develop until to appearance of term "Yangian"(see [1], [10], [3], [4], [15], [25], [26]). In contrast with these theory, representation theory of Yangians of Lie Superalgebras is a young discipline, which appear at the beginning of 90-th years of 20 century. In the last years the number of applications of Representation Theory of Yangians of Lie Superalgebras rise, to began research problems of Yang-Mills fields and Quantum Superstring Theory using Yangian Theory technique (see [2], [6], [19]).

In this paper we research the finite-dimensional irreducible representations of Yangian of Lie Superalgebra of $A(n, n)$ type (see [14], [11]). The main result of this work is a theorem on classification of such representations. Let's note that classification of the finite-dimensional irreducible representations of Yangian of Lie Superalgebra of $\mathfrak{gl}(m+1, n+1)$ was received in the middle of 90-th by R.B. Zhang (see [27], [28]) and classification of the the finite-dimensional irreducible representations of Yangian of Lie Superalgebra of $A(m, n)$, $m \neq n$ was received in [24] (see also, [23]). The case $m = n$ is a more difficult, because in these case Cartan generators aren't linearly independent.

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Let's note that Yangians together with quantized enveloping algebras and elliptic quantum algebras are one of the three most important examples of quantum algebras. The notion Yangian was introduced by V. Drinfel'd in honour to C.N. Yang. But algebras isomorphic to Yangian were used for investigation exactly solvable models by means of Quantum Inverse Scattering Method in the 80-th years by V. Tarasov (see, for example, [25], [26]).

It should be mentioned that representation theory of Lie Superalgebra $\mathfrak{sl}(m+1, n+1)$ differs from representation theory of simple Lie algebra $\mathfrak{sl}(n+1)$ that Lie Superalgebra has together typical representations also so called atypical representations. This pathology reveals itself in the representation theory of Yangian of Lie superalgebra $A(m, n)$ also.

Let's note that Yangian with quantized enveloping algebras and quantum elliptic algebras are most important examples of quantum algebras. The notion of quantum algebra was introduced by V. Drinfel'd in the middle of 80-th years of 20 century (see [7], [8]). But earlier algebras isomorphic to Yangian are used for investigation exactly solvable models by Quantum Inverse Scattering Method (QISM) (see [25], [26]). Yangians of simple and reductive Lie algebras are investigated in detail in present time (see books [3], [15]). Yangians of Lie Superalgebras began investigate at the 90-th years of last century (see [16], [20]). In the papers [21], [22] was proved formula for universal R -matrix of Yangin Double of Yangin for special linear Lie Superalgebra. Let us note that using of results papers [21], [22], [24] and this work it can be systematically describe all possible quantum R -matrices and transfer matrices connecting with Yangian $Y(A(m, n))$ including examples important for exactly solvable models of Quantum Field Theory.

Some words about structure of this works. In the second section we recall definition of the Yangian of Lie Superalgebra $A(n, n)$ in terms of generators and defining relations. We define it in terms of current system of generators following the author's work [20]. In the third section we recall construction of PBW bases and give a sketch of the proof of this theorem. In the forth section we recall the main definitions from representation theory of Yangians of Lie Superalgebras and formulate and prove the main result of work the theorem on classification of finite dimensional irreducible representations of Yangian of Lie Superalgebra $A(n, n)$.

We'll use the following notations. We denote by C the field of complex numbers, by $M_n(K)$ the ring of $N \times N$ - matrices with elements from ring K ; by $K[u]$, $K[[u]]$ the ring of polynomials and ring of formal power series, correspondingly, with coefficients from ring K ; by $Z_+ = \{0, 1, 2, \dots\}$ the set of non negative integers, which is union of zero and set of natural numbers N . The end of proof we denoted by symbol $\square$.

2. Definition of Yangian of Lie Superalgebra $A(n, n)$

2.1. Lie Superalgebra $A(n, n)$

Let me recall basic definitions from Lie Superalgebra theory, related to the Lie Superalgebra $A(n, n)$. This Lie superalgebra has rank equals $2n - 2$ and dimension equals $4n^2 - 2$, number of simple roots equals $2n - 1$ and differs from rank. Lie Superalgebra $A(n, n)$ is generated the generators $h_i, x_i^\pm, i \in \{1, 2, \dots, n, n+1, \dots, 2n+1\}$. The Cartan matrix (distinguished Cartan matrix) determining the system of defining relations has following form:

$$A = (a_{i,j})_{i,j=1}^{2n+1} = \begin{pmatrix} 2 & -1 & \dots & 0 & 0 & \dots & 0 & 0 \\ -1 & 2 & \dots & 0 & 0 & \dots & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & 0 & 1 & \dots & 0 & 0 \\ 0 & 0 & \dots & -1 & 2 & \dots & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & 0 & 0 & \dots & -1 & 2 \end{pmatrix}.$$

The symmetric Cartan matrix is follows:

$a_{ii} = 2, i < n + 1, a_{n+1,n+1} = 0, a_{i,i} = -2, n + 1 < i \leq 2n + 1, a_{i,i+1} = a_{i+1,i} = -1$ for $i < n + 1$, and $a_{i,i+1} = a_{i+1,i} = 1$ for $i \geq n + 1$. The all other matrix elements are equal zero. As a rule we'll use the symmetric Cartan matrix.

The system of defining relations of $A(n, n)$ has the following form:

$$[h_i, h_j] = 0,$$

$$i \in I = \{1, 2, \dots, n, n + 1, \dots, 2n + 1\}; \quad (1)$$

$$[h_i, x_j^\pm] = \pm a_{ij} x_j^\pm, i, j \in I; \quad (2)$$

$$[x_i^+, x_j^-] = \delta_{ij} h_i, i, j \in I; \quad (3)$$

$$[x_{n+1}^\pm, x_{n+1}^\pm] = 0; \quad (4)$$

$$[x_i^\pm, x_j^\pm] = 0; |i - j| > 1; \quad (5)$$

$$[x_i^\pm, [x_i^\pm, x_j^\pm]] = 0, |i - j| = 1, i, j \in I; \quad (6)$$

$$[[x_n^\pm, x_{n+1}^\pm], [x_{n+1}^\pm, x_{n+2}^\pm]] = 0. \quad (7)$$

Let's note that the generators $x_{n+1}^\pm$ are odd and other generators are even, or $p(x_{n+1}^\pm) = 1, p(x_i^\pm) = p(h_j) = 0$, where p be a parity function, $j \in I, i \in I \setminus \{n + 1\}$ (see [14]). Let be below $\mathfrak{g} = A(n, n) = \mathfrak{sl}(n + 1, n + 1)$.

2.2. Definition of Yangian

First, we recall the definition of Yangian $Y(\mathfrak{g}) = Y(A(n, n))$ (see also [20], [12]).

Yangian, as other quantum algebras, is defined as a result of deformation (in the case of Yangian) of universal enveloping (super)algebra of current Lie superalgebra (with values in Lie superalgebra) in the class of Hopf superalgebras. In the paper [20] was shown that this definition is equivalent other definition using so called current system of generators. This system of generators is convenient for representation theory of Yangians. Therefore we'll consider it further.

Definition 1 Let $Y(\mathfrak{g})_\hbar$ be a superalgebra (over ring of formal power series $C[[\hbar]]$), generated by generators $h_{i,k}, x_{i,k}^\pm, i \in I = \{1, 2, \dots, 2n + 1\}, k \in Z_+$ ($p(x_{n+1,k}^\pm) = 1, p(x_{i,k}^\pm) = p(h_{j,k}) = 0, i \in I \setminus \{n + 1\}, j \in I, k \in Z_+$), which satisfied the following defining relations (compare with relations from paper [18], definition 2 of paper [20], and also relations from proposition 9.1 of work [12]):

$$[h_{i,k}, h_{j,l}] = 0, i, j \in I, k, l \in Z_+; \quad (8)$$

$$[x_{i,k}^+, x_{j,l}^-] = \delta_{i,j} h_{i,k+l}, i, j \in I, k, l \in Z_+; \quad (9)$$

$$[h_{i,k+1}, x_{j,l}^\pm] = [h_{i,k}, x_{j,l+1}^\pm] \pm (a_{ij}/2)\hbar(h_{i,k}x_{j,l}^\pm + x_{j,l}^\pm h_{i,k}),$$

$$i, j \in I, k, l \in Z_+; i \neq n \quad \text{and} \quad j \neq n \quad (10)$$

$$[h_{n+1,k+1}, x_{n+1,l}^\pm] = 0; \quad (11)$$

$$[h_{i,0}, x_{j,l}^\pm] = \pm a_{ij} x_{j,l}^\pm, i, j \in I, l \in Z_+; \quad (12)$$

$$[x_{i,k+1}^\pm, x_{j,l}^\pm] = [x_{i,k}^\pm, x_{j,l+1}^\pm] \pm (a_{ij}/2)\hbar(x_{i,k}^\pm x_{j,l}^\pm + x_{j,l}^\pm x_{i,k}^\pm),$$

$$i, j \in I, k, l \in Z_+; i \neq n \quad \text{or} \quad j \neq n; \quad (13)$$

$$[x_{n+1,k}^\pm, x_{n+1,l}^\pm] = 0; \quad (14)$$

$$[x_{i,k_1}^\pm, [x_{i,k_2}^\pm, x_{j,k_3}^\pm]] + [x_{i,k_2}^\pm, [x_{i,k_1}^\pm, x_{j,k_3}^\pm]] = 0,$$

$$|i - j| = 1, i, j \in I, k_1, k_2, k_3 \in Z_+; \quad (15)$$

$$[x_{i,k}^{\pm}, x_{j,l}^{\pm}] = 0, |i - j| > 1; \quad (16)$$

$$[[x_{n,r}^{\pm}, x_{n+1,0}^{\pm}], [x_{n+1,0}^{\pm}, x_{n+2,s}^{\pm}]] = 0. \quad (17)$$

Easy to check that for different $\hbar$, (aren't equal zero), Hopf Superalgebras $Y(\mathfrak{g})_{\hbar}$ are isomorphic.

Yangian $Y(\mathfrak{g})$ is called Hopf Superalgebra $Y(\mathfrak{g})_1$ over field of complex numbers C .

Let us introduce the following generating functions of Yangian generators:

$$h_i(u) = 1 + \sum_{k=0}^{\infty} h_{i,k} \cdot u^{-k-1} = \sum_{k=-1}^{\infty} h_{i,k} \cdot u^{-k-1}, i = 1, 2; \quad (18)$$

$$x_i^{\pm} = \sum_{k=0}^{\infty} x_{i,k}^{\pm} \cdot u^{-k-1}, i = 1, 2. \quad (19)$$

Easy to check that relation (10) for Yangian generators is equivalent the following relation for generating functions:

$$[h_i(u), x_j^{\pm}(t)] = \mp \frac{a_{ij}}{2} \frac{(h_i(u)(x_j^{\pm}(t) - x_j^{\pm}(u)) + (x_j^{\pm}(t) - x_j^{\pm}(u))h_i(u))}{(u - t)}. \quad (20)$$

3. Root generators. Poincare-Birkhoff-Witt theorem.

3.1.

In this subsection we formulate Poincare-Birkhoff-Witt theorem. This result is a quantum analogue of classical Poincare-Birkhoff-Witt theorem for universal enveloping algebras (see [13], [3]).

Let below $\mathfrak{g} = A(n, n)$.

Let degree of generators $a_{i,k}, a_{i,k} \in \{x_{i,k}^+, x_{i,k}^-, h_{i,k}\}$ be its second index.

A degree of monom from generators is called the sum of degrees of multipliers. A degree of polynomial is called maximum of degrees of monoms of this polynomial. Let also degree of tensor products of monoms be the sum of degrees of tensor multipliers. A degree of tensor polynomial is called maximum of degrees of tensor monoms forming this polynomial.

Let's denote the space of elements of $Y(\mathfrak{g})$ of degree less or equal k by $Y_k = Y_k(\mathfrak{g})$. We get on $Y(\mathfrak{g})$ the following filtration:

$$Y_0 \subset Y_1 \subset Y_2 \subset \dots \subset Y_n \subset \dots$$

Let's note that on modulo of terms of lesser degree (that is on modulo $Y(\mathfrak{g})_{k+l-1}$) commutation relations in $Y(\mathfrak{g})$ has the following form:

$$[h_{\alpha, \bar{k}}, x_{\beta, \bar{l}}] = (\alpha, \beta) x_{\beta, \bar{k}+\bar{l}}; \quad [h_{\alpha, \bar{k}}, h_{\beta, \bar{l}}] = 0; \quad (21)$$

$$[x_{\alpha, \bar{k}}, x_{\beta, \bar{l}}] = \begin{cases} 0, & \alpha + \beta \notin \Delta \cup \{0\}, \\ h_{\alpha, \bar{k}+\bar{l}}, & \beta = -\alpha, \\ N(\alpha, \beta) x_{\alpha+\beta, \bar{k}+\bar{l}}, & \alpha + \beta \in \Delta \end{cases} \quad (22)$$

where $N(\alpha, \beta)$ is defined from the following relation in $U(\mathfrak{g})$:

$$[x_{\alpha}, x_{\beta}] = N(\alpha, \beta) x_{\alpha+\beta}, \text{ для } \alpha + \beta \in \Delta.$$

Let's construct root vectors for $Y(\mathfrak{g})$. Let α_{n+1} be an odd root and Δ_+ be a set of positive roots of Lie superalgebra $A(n, n)$. Let's define on Δ_+ so called convex order by induction. At first we fix ordering on the set of positive roots, corresponding its numeration: $\alpha_1 \prec \alpha_2 \prec \dots \prec \alpha_{2n+1}$. If $\alpha, \beta \in \Delta_+$, $\alpha \prec \beta$ and its sum is defined $\alpha + \beta$, then we define ordering by condition $\alpha \prec \alpha + \beta \prec \beta$. Let's fix monotonous mapping $\alpha : [1, 2, \dots, N] \rightarrow \Delta_+$ with respect to order: $\alpha(k) \prec \alpha(n)$ if $k < n$. (Let's note that $N = \frac{(2n+1)(2n+2)}{2}$.)

Let $\overline{(i, k)} = (i_1, k_1, i_2, k_2, \dots, i_{2n+1}, k_{2n+1})$ be a vector, $|\overline{(i, k)}| = \sum_{j=1}^{m+n+1} (i_j + m_j)$ be sum of components of its vector.

Let Y_- be an associative subalgebra of Yangian $Y(A(m, n))$, generated by generators $\{x_{i,k}^- = x_{\alpha(i),k}^- | 1 \leq i \leq N, k \in Z_+\}$. Let's describe ordering base in Yangian $Y(\mathfrak{g})$, as in vector space, i.e Poincare-Birkhoff-Witt base (PBW – base). Let $\overline{k} = (k_1, \dots, k_r), k_i \geq 0$, be a vector, $k = |\overline{k}| = \sum_{i=1}^r k_i$. Let also

$$x(\overline{k}, \overline{w}; \overline{s}, \overline{p}; \overline{r}, \overline{t}) = (x_{\alpha(1),k_1}^+)^{w_1} \cdot \dots \cdot (x_{\alpha(N),k_N}^+)^{w_N} \cdot (h_{1,s_1})^{p_1} \cdot \dots \cdot (h_{2n+1,s_{2n+1}})^{p_{2n+1}} \cdot (x_{\alpha(1),r_1}^-)^{t_1} \cdot \dots \cdot (x_{\alpha(N),r_N}^-)^{t_N}.$$

Let's define lexicographic ordering on the set of vectors $\{x(\overline{k}, \overline{w}; \overline{s}, \overline{p}; \overline{r}, \overline{t})\}$. Let $\alpha = \alpha_s + \alpha_{s+1} + \dots + \alpha_t; \alpha, \alpha_s, \alpha_{s+1}, \dots, \alpha_t \in \Delta_+$, and $\alpha_s, \alpha_{s+1}, \dots, \alpha_t$ be simple roots, $s < t, l = t - s + 1$. Let $x_{s,k_1}^\pm, \dots, x_{t,k_l}^\pm \in Y(G)$, $k = k_1 + k_2 + \dots + k_l$. Let's define root vectors by formulas:

$$x_{\pm\alpha,k} = [x_{s,k_1}^\pm, [x_{s+1,k_2}^\pm, [\dots, x_{t,k_l}^\pm] \dots]].$$

Easy to check that if $(k'_1, \dots, k'_l)$ be another decomposition of number k , then $x_{\alpha,k} = [x_{s,k'_1}^\pm, [x_{s+1,k'_2}^\pm, \dots, x_{t,k'_l}^\pm] \dots]$.

Let's note that onto modulo of terms of lesser degree (i.e. onto modulo $Y(\mathfrak{g})_{k-1}$) commutation relations in $\oplus_{k=1}^\infty Y(\mathfrak{g})_k / Y(\mathfrak{g})_{k-1} \oplus Y(\mathfrak{g})_0$ are coincide with commutation relations in universal enveloping superalgebra of current Lie superalgebra $U(\mathfrak{g}[t])$, i.e have the following form:

$$\begin{aligned} [h_{i,k}, x_{j,l}^\pm] &= \pm(\alpha_i, \alpha_j) x_{j,k+l}^\pm, \\ [x_{i,k}^+, x_{j,l}^-] &= \delta_{ij} h_{i,k+l}, \\ [h_{i,k}, h_{j,l}] &= 0, \\ [x_{i,l}^\pm, [x_{i,r}^\pm, x_{j,s}^\pm]] &= 0, \\ [[x_{n,r}^\pm, x_{n+1,0}^\pm], [x_{n+1,0}^\pm, x_{n+2,s}^\pm]] &= 0. \end{aligned}$$

It should be note that these relations enough for determination linear order on elements of base of polynomial current superalgebra $\mathfrak{g}[t] = A(n, n)[t]$, when odd root generators take part in relations. This can be checked by direct calculations. For every number k let's fix vector $(k_1, k_2, \dots, k_l), k = k_1 + \dots + k_l$, defines partition of its number.

Above we introduce the linear order onto set of roots Δ and inducing by its order an order on root vectors. Let's note by Ω the set of ordering monoms from $x_\alpha, h_i, \alpha \in \Delta, i \in I$. We use notation $x_\alpha = x_\alpha^+, x_{-\alpha} = x_\alpha^-, \alpha \in \Delta_+$.

Theorem 1 $(\Omega, \prec)$ is Poincare-Birkhoff-Witt base in Yangian $Y(\mathfrak{g})$.

3.2.

Let's prove Poincare-Birkhoff-Witt theorem for Yangian $Y(A(n, n))$. We give a sketch of proof.

At first let's prove completeness of $\Omega(\prec)$.

For monom M belong to $\Omega(<)$ let's define its length $l(M)$ as number of multipliers or generators are contained in M . After reordering multipliers from (21), (22) it follows that we'll get additional summands either lesser degree or equal degree and lesser length. Using induction we get that $Y(\mathfrak{g})$ coincide with linear span of $\Omega(<)$. Note when we reording monoms contain odd root generators we use relation (17).

Let us prove that monoms from $\Omega(<)$ are linear independent. Our proving is based on existence of such representation ρ of Yangian $Y(\mathfrak{g})$ that $\rho(x_{i,0}^+)$, $\rho(x_{i,0}^-)$, $\rho(h_{i,0})$ are are linear independent.

After them we reduce to contradiction assumption that elements of $\Omega(<)$ are linear dependent. Suppose that monoms from $\Omega(<)$ aren't linear independent. Then exist such numbers $c_1, \dots, c_s \in C \setminus \{0\}$ and monoms $M_1, \dots, M_s \in \Omega(<)$, that

$$\sum_{1 \leq j \leq s} c_j M_j = 0 \quad (23)$$

It can be show that this assumption reduced to contradiction (see [24]).

□

4. Representations of Yangian $Y(A(n, n))$

4.1.

Let V be a module over Yangian $Y(A(n, n))$.

Definition 2 Let V be a module over Yangian $Y(\mathfrak{g})$ of Lie Superalgebra $\mathfrak{g} = A(m, n)$, $\bar{d} = \{d_{i,r} | i \in I, r \in Z_+\}$ be a collection of complex numbers. Let's denote by $V_{\bar{d}}$ and call weight subspace of module V the space

$$V_{\bar{d}} = \{v \in V : h_{i,r} v = d_{i,r} v\} \quad (24)$$

Collection of numbers $\bar{d} = \{d_{i,r}\}$ is called weight of Yangian module.

We are going to describe a structure of finite dimensional modules over Yangian $Y(\mathfrak{g})$ and to formulate necessary and sufficient conditions finite dimensionality of irreducible module.

The vector $v \in V$ is called *primitive*, if $v \in V_{\bar{d}}$ and $x_{i,r}^\pm \cdot v = 0$ for all $i \in I, r \in Z_+$.

The module V is called *highest weight module* if it is generated by primitive vector, that is $V = Y(A(n, n)) \cdot v$ for some primitive vector $v \in V_{\bar{d}}$.

At first we show that every finite dimensional representation of Yangian $Y(\mathfrak{g})$ has highest (primitive) vector.

Let's note that every highest weight module should be constructed as a factor of *Verma module*. Verma module should be constructed in usual manner as f factor module of Yangian $Y(\mathfrak{g})$ on ideal generated by vectors $x_{i,r}^+$ and $h_{i,k} - d_{i,k}$. In this case highest vector is a unite of Yangian 1. Let's consider weight $\sum_i \lambda_i \cdot h_{i,0}$, where λ is a fundamental weight of Lie Superalgebra $\mathfrak{g}$. Then weight subspace of Yangian Verma module with such weight is 1-dimensional. Easy to show that it follows Verma module has unique irreducible factor module is denoted as $V(\bar{d})$, i.e. $V(\bar{d}) = M(\bar{d})/N(\bar{d})$, where $N(\bar{d})$ be a maximal submodule of Verma module $M(\bar{d})$. Let $\pi : M(\bar{d}) \rightarrow V(\bar{d})$ be a canonical projection.

Theorem 2 Every finite dimensional representation of Yangian $Y(\mathfrak{g})$ (simple $Y(\mathfrak{g})$ -module) V contains unique (up to scalar multiplier) highest vector v .

The proof of this theorem is based on the following lemma. Let $V_0 = \{v \in V | x_{i,k}^+ v = 0, \forall k \in Z_+\}$.

Lemma 1 1) $h_{i,k} V_0 \subset V_0$.

2) $V_0 \neq 0$.

it Proof 1) Let $v \in V_0$. Then $x_{i,k}^+ h_{j,l} v = h_{j,l} x_{i,k}^+ v + [h_{i,l+1}, x_{i,k-1}^+] v + \frac{1}{2}(h_{i,k} x_{i,l}^+ + x_{i,l}^+ h_{i,k}) v$. Let rearrange generators $x_{i,s}^+$ и $h_{j,r}$ using commutation relations (defining relations of Yangian) we get $x_{i,k}^+ h_{j,l} v = 0$ or $h_{j,l} v \in V_0$ для $v \in V_0$, that is $h_{i,l} V_0 \in V_0$.

2) Let $v' \in V_0$. Let act on vector v' by elements of PBW-base. At first let consider the case $m = n = 1$, i.e. case of $Y(\mathfrak{sl}(1, 1))$ and after we consider a general case. In the case of $Y(\mathfrak{sl}(1, 1))$ exist such number $m \in \mathbb{N}$, that $(x_0^+)^m v' \neq 0$, $(x_0^+)^{m+1} v' = 0$. Let $v_2 = (x_0^+)^m v'$. We'll act by elements of PBW base in correspondence with order on the PBW base. Easy to check that exist such integer r that $v_r = (x_r^+)^{p_r} \cdot \dots \cdot (x_0^+)^{p_0} v \neq 0$, but $x_t^+ \cdot v_r = 0$ for $\forall t \in \mathbb{Z}_+$. Then $v_r \in V_0$. We get that $V_0 \neq \{0\}$.

Let consider the general case of $Y(A(n, n))$. Consider linear order on vectors of PBW base from Y_+ . Let's denote these vectors by $\{x^+(j)\}_{j=0}^\infty$, and let $x^+(j) \prec x^+(l)$ for $j < l$. Then exist such integer $m \in \mathbb{Z}_+$, that $x^+(m) v' \neq 0$, but $x^+(m+1) v' = 0$ for $v' \in V_0$. $\square$

Lemma 2 V_0 be 1-dimensional space (that is every two vectors from V_0 are proportional).

Proof Let $v', v'' \in V_0, v' \neq v''$. Then acting on these vectors by elements of Yangian $Y(A(n, n))$, we get two submodules $Y(A(m, n))v'$ and $Y(A(m, n))v''$ of module V . These fact contradicts to irreducibility of module V in the case when v', v'' aren't proportional. $\square$

The theorem follows from these two lemmas.

Let us introduce class of highest weight modules are analogues of Verma modules. Let $V_0 = C v_+^\Lambda$ be 1-dimensional vector space, Y_0^+ be a subsuperalgebra of Yangian $Y(A(n, n))$, generated by generators $x_{i,k}^+, i \in \{1, 2, \dots, m+n+1\}, k \in \mathbb{Z}_+$; $Y_0^0 = \langle h_{i,k} | i \in I, k \in \mathbb{Z}_+ \rangle$ is a linear span of generators $\{h_{i,k} | i \in I, k \in \mathbb{Z}_+\}$; $Y^+ = Y_0^+ \cdot Y_0^0$. Let also $h_{i,k} v_+^\Lambda = d_{i,k} v_+^\Lambda, Y_0^0 \cdot v_+^\Lambda = 0$. (We'll use also notation $\Lambda = \bar{d}$ in order to stress analogy with highest weights of modules of Lie Superalgebras. We'll use also notation $v_+ = v_+^\Lambda$ for highest vector). Then V_0 became 1-dimensional $Y(A(n, n))$ -module. Let's define free module M_Λ with highest weight Λ by formula:

$$M_\Lambda = Y(\mathfrak{g}) \otimes_{Y^+} v_+^\Lambda. \quad (25)$$

Evidently, that M_Λ isomorphic to $Y_0^- \otimes v_+^\Lambda : M_\Lambda \cong Y_0^- \otimes v_+^\Lambda$ as a vector space. We'll write $x_{i,k}^- v_+^\Lambda$ instead of $x_{i,k}^- \otimes v_+^\Lambda$. Clearly, that module M_Λ is infinite dimensional. Standard arguments show that module M_Λ contains maximal submodule N_Λ . Then module $V_\Lambda = M_\Lambda / N_\Lambda$ is irreducible module with highest weight Λ . Using the standard methods of representation theory easy to check that every two modules with equal highest weights are isomorphic (see also [?]). Namely, let $V_1(\Lambda), V_2(\Lambda)$ be two irreducible modules with equal highest weights $\Lambda = \{\lambda_k\}_{k=0}^\infty$ and u_+^Λ, w_+^Λ be its highest vectors, correspondingly. Let $W = V_1(\Lambda) \oplus V_2(\Lambda)$, $v_+^\Lambda = (u_+^\Lambda, w_+^\Lambda)$ be a highest vector of some submodule $V(\Lambda)$ with highest weight Λ generated in W by vector v_+^Λ (under action of Yangian $Y(A(n, n))$). Let's define a projections $P_i : V(\Lambda) \rightarrow V_i(\Lambda), i = 1, 2 : P_1(v_1, v_2) = (v_1, 0), P_2(v_1, v_2) = (0, v_2), v_i \in V_i(\Lambda)$. Easy to check that $P_i, i = 1, 2$ are module homomorphisms and $P_1(v_+^\Lambda) = u_+^\Lambda, P_2(v_+^\Lambda) = w_+^\Lambda$. It follows that $Im P_i = V_i(\Lambda)$ from irreducibility of modules $V_i(\Lambda), i = 1, 2$. Then $Ker P_1 = \{0\}, Ker P_2 = \{0\}$ (or $Ker P_1 = V_2(\Lambda), Ker P_2 = V_1(\Lambda)$). But last equality is impossible because vectors $(0, w_+^\Lambda), (u_+^\Lambda, 0) \notin W$. Therefore $P_i, i = 1, 2$ are module isomorphisms and $V(\Lambda) \cong V_1(\Lambda) \cong V_2(\Lambda)$.

So then we prove the following theorem

Theorem 3 For every highest weight $\Lambda = \{\lambda_i\}_{i=0}^\infty$ exist unique irreducible $Y(\mathfrak{g})$ -module $V(\Lambda)$ with highest weight Λ .

4.2.

Let's define on module $M(\bar{d})$ the following two filtrations. As the module $M(\bar{d})$ as a vector space naturally isomorphic to $Y(\mathfrak{g})_-$, we can define at first filtrations on $Y(\mathfrak{g})_-$. After them we can define this filtrations on $M(\bar{d})$ using isomorphism between $Y(\mathfrak{g})_-$ and $M(\bar{d})$. Let $(Y(\mathfrak{g})_-)_k$ ($(Y(\mathfrak{g})_-)^k$) be a linear span of monoms of degree lesser or equal k . The degree of monom from generators, as usual, is sum of degrees of these generators form this monom. A degree of generators is a value of second index of this generators in first case and is number equals value of second index plus 1 in the second case. In the first case we'll denote degree of monom x by $d_1(x)$ and by $d_2(x)$ in the second case. Also as above we extend our definition on polynomials defining a degree of polynomial as a maximum of degrees of monoms form this polynomial. Let $Y(\mathfrak{g})_k = \{x \in Y(\mathfrak{g}) : d_1(x) \leq k\}$, a $Y(\mathfrak{g})^k = \{x \in Y(\mathfrak{g}) : d_2(x) \leq k\}$. Let's consider constriction of these filtrations on $Y(\mathfrak{g})_-$. Thus we get the following two filtrations on $Y(\mathfrak{g})_-$:

$$C \subset (Y(\mathfrak{g})_-)_0 \subset (Y(\mathfrak{g})_-)_1 \subset \dots \subset (Y(\mathfrak{g})_-)_k \subset \dots \quad (26)$$

$$\{0\} \subset C \subset (Y(\mathfrak{g})_-)^0 \subset (Y(\mathfrak{g})_-)^1 \subset \dots \subset (Y(\mathfrak{g})_-)^k \subset \dots \quad (27)$$

Let $M(\bar{d})_k = (Y(\mathfrak{g})_-)_k \cdot v^{\bar{d}}$, $M(\bar{d})^k = (Y(\mathfrak{g})_-)^k \cdot v^{\bar{d}}$. As irreducible module $V(\bar{d})$ is a factor-module of Verma module $M(\bar{d})_k$, the the above defined filtrations determine filtration on irreducible module $V(\bar{d})$. It should be mentioned that properties of these two filtrations are various. Let's note also that the every space with index k of the first filtration contains all spaces of second filtration with indices lesser or equals k .

Let's describe conditions of finite dimensionality of irreducible highest weight Yangian module. Let note by $\bar{B}_{n+1,t}, \bar{B}_{i,k}, i \in I \setminus \{n+1\}$ the linear spans of following vectors

$$\begin{aligned} \bar{B}_{n+1,s} &:= \langle (x_{n+1,k_1}^-)^t \cdot \dots \cdot x_{n+1,k_r}^- \cdot v_+ | (k_1 + 1) + \dots + (k_r + 1) \leq s \rangle \\ \bar{B}_{i,s} &:= \langle (x_{i,k_1}^-)^{t_1} \cdot \dots \cdot (x_{i,k_r}^-)^{t_r} \cdot v_+ | t_1(k_1 + 1) + \dots + t_r(k_r + 1) \leq s \rangle, \quad i \in I \setminus \{n+1\} \end{aligned}$$

Lemma 3 *If $\bar{B}_{n+1,p} = \bar{B}_{n+1,p+1}$, then $\bar{B}_{n+1,p} = \bar{B}_{n+1,p+k}$ for every integer $k \in \mathbb{N}$.*

Proof. Let $a \in \bar{B}_{n+1,p}$. At first let show that every vector from $\bar{B}_{n+1,p+k}$ can be represent as image of $\bar{B}_{n+1,p}$ under action Cartan subalgebra $\mathfrak{h} = \langle h_{n+1,k}, h_{n+2,k} | k \in \mathbb{Z}_+ \rangle$, namely, $\bar{B}_{n+1,p+k} \subset \mathfrak{h} \cdot \bar{B}_{n+1,p}$. For $k = 1$ this fact it follows from condition of lemma. Let now $k = 2$. We need the following relation

$$\begin{aligned} h_{i,1} \cdot x_{j,l}^- \cdot v_+ &= x_{j,l}^- \cdot h_{i,1} \cdot v_+ + [h_{i,1}, x_{j,l}^-] \cdot v_+ = x_{j,l}^- \cdot h_{i,1} \cdot v_+ + [h_{i,0}, x_{j,l+1}^-] \cdot v_+ \\ + a_{i,j}/2(h_{i,0}x_{j,l}^- + x_{j,l}^-h_{i,0}) \cdot v_+ &= d_{i,1}x_{j,l}^- \cdot v_+ + a_{i,l}x_{j,l+1}^- \cdot v_+ + \\ a_{i,j}/2(h_{i,0}x_{j,l}^- + x_{j,l}^-h_{i,0}) \cdot v_+ &= a_{i,j}x_{j,l+1}^- \cdot v_+ + (d_{i,1} + (a_{i,j}/2)(a_{i,j} + 2d_{i,0}))x_{j,l}^- \cdot v_+. \end{aligned}$$

Using this relation let prove the lemma. Let $a \in \bar{B}_{m+1,p+2}$, then $a = \sum_{s=1}^r x_{i_s,k_s}^- b_s v_+$. Let represent an element a in the form

$$a = \sum_{s=1}^r a_{i_s-1,i_s}^{-1} [h_{i_s-1,1}, x_{i_s,k_s-1}^-] b_s v_+ \quad (28)$$

Repeating these arguments we can represent the element a as a sum of elements from $\bar{B}_{n+1,p+1}$ and product $x_{n+1,1}^-$ and elements from $\bar{B}_{n+1,p+1}$. As every element from $\bar{B}_{n+1,p+1}$ is contained in $\bar{B}_{n+1,p}$ we get that $a \in \bar{B}_{n+1,p+1}$ and after them it follows from condition of lemma that $a \in \bar{B}_{n+1,p}$. Lemma is proved. $\square$

Lemma 4 1) *There are the strict inclusions*

$$B_{i,k} \subset B_{i,k+1}, \forall k < n_i, k \geq 0, i \in I;$$

$$2) B_{i,n_i} = B_{i,n_i+1} = \dots$$

Proof The case $i \in I \setminus \{n+1\}$ it follows from results of representation theory of Yangian $Y(\mathfrak{sl}(2))$ (see [8], [25]). The point 1) it follows from definitions. The point 2) it follows from previous lemma in the case $i = n+1$. $\square$

4.3.

Let's formulate and prove the main result of our work – the following theorem. We'll suppose, that $n \geq 1$.

Theorem 4 1) Every irreducible finite dimensional $Y(A(n, n))$ -module V be a highest weight module with highest weight $d : V = V(d)$.

2) The module $V(d)$ be finite dimensional iff when exist the polynomials $P_i^d, i \in \{1, 2, \dots, n, n+2, \dots, 2n\} = I \setminus \{n+1\}$, and polynomials P_{n+1}^d, Q_{n+1}^d , such that:

a) all these polynomials with highest coefficients equal 1;

b)

$$\frac{P_i^d(u+1)}{P_i^d(u)} = 1 + \sum_{k=0}^{\infty} d_{i,k} \cdot u^{-k-1}, \quad i \in I \setminus \{n+1\}, \quad (29)$$

$$\frac{P_{n+1}^d(u)}{Q_{n+1}^d(u)} = 1 + \sum_{k=0}^{\infty} d_{n+1,k} \cdot u^{-k-1}. \quad (30)$$

Proof. Let's note that our proof is a slight modification analogous result from work [24].

The point 1) of theorem is proved already. Let deduce the point 2) from above proved lemmas. At first let's prove the necessary in theorem, that is let's prove that if the irreducible module $V(d)$ be finite dimensional then exist mentioned in point 2) of theorem the polynomials satisfying the conditions a), b) of theorem.

At first, we consider particular case when $\mathfrak{g} = A(0, 1)$. More precisely, we consider inclusion $A(0, 1) \rightarrow A(n, n)$, inducing by the mapping of roots $\alpha_1 \rightarrow \alpha_{n+1}, \alpha_2 \rightarrow \alpha_{n+2}$. These inclusion unduces embedding $Y(A(0, 1))$ into $Y(A(n, n))$: $Y(A(0, 1)) \rightarrow Y(A(n, n))$ (in category of topological superalgebras) defined by mapping of root generators (which is induced by above mentioned mapping of roots).

In the first part of the proof we'll deal with particular case of irreducible representation of Yangian $Y(A(0, 1))$ with weight $\bar{d}(u) = (d_{n+1}(u), d_{n+2}(u))$. After them we'll reduce the general case to considering particular case.

Let as above $x_i^-(u) = \sum_{k=0}^{\infty} x_{i,k}^- \cdot u^{-k-1}, i \in I$. From proved above lemmas it follows that $x_i^-(u) \cdot v_+ = \sum_{s=0}^N \beta_s^i(u) \cdot v_{i,s}$, где $\{v_{i,s}\}$ form a base in $\bar{B}_{i,N}, i \in I$. Further we'll get explicit form for $\beta_s^i(u)$. Her it should be note that we can get all results for even root generators considering inclusions of $\mathfrak{sl}(2)$ -triples into $Y(A(n, n))$. These results are known as they are can be reduced to well known results on representations of Yangian $\mathfrak{sl}(2)$ (see [25] and also [26], [8], [9], [4]). Therefore we prove in detail relation (30) relating to odd part of Yangian $Y(A(n, n))$.

We need the relations (20) in the following particular case:

$$[h_{n+1}(u), x_{n+2}^-(t)] = \frac{1}{2(u-t)}(h_{n+1}(u)(x_{n+2}^-(t) - x_{n+2}^-(u)) + (x_{n+2}^-(t) - x_{n+2}^-(u))h_{n+1}(u)); \quad (31)$$

$$[h_{n+1}(u), x_{n+1}^-(t)] = 0. \quad (32)$$

Let $v_{i,k} = x_{i,k}^- v_+$. Then from lemma 4 it follows following equality

$$x_k^-(u)v_+ = \sum_{i=0}^N \beta_i^k(u)v_{k,i}. \quad (33)$$

For brevity we'll use the notation:

$$\beta_i(u) = \beta_i^{n+2}(u). \quad (34)$$

Note that

$$\begin{aligned} \sum_{k=0}^{\infty} x_{n+2,k}^- u^{-k-1} v_+ &= \sum_{k=0}^N x_{n+2,k}^- u^{-k-1} v_+ + \sum_{k=N+1}^{\infty} x_{n+2,k}^- u^{-k-1} v_+ = \\ &= \sum_{k=0}^N x_{n+2,k}^- u^{-k-1} v_+ + \sum_{k=0}^N u^{-N-1} \sum_{i=0}^{\infty} \varphi_k^i u^{-i-1} v_{n+2,k}. \end{aligned}$$

Let's denote

$$\varphi_k(u) = \sum_{i=0}^{\infty} \varphi_k^i u^{-i-1}, \quad (35)$$

$$\tilde{\varphi}_k(u) = u^{-k-1} + \varphi_k(u). \quad (36)$$

Let's rewrite relation (33) in the following form:

$$x_{n+2}^-(u) v_+ = \sum_{i=0}^N \beta_i(u) v_{n+2,i} = \sum_{i=0}^N (u^{-i-1} + u^{-N-1} \varphi_i(u)) v_{n+2,i}. \quad (37)$$

Then from equality (37) it follows that

$$x_{n+2,N+k+1}^- v_+ = \sum_{i=0}^k \varphi_i^k x_{n+2,i}^- v_+. \quad (38)$$

Let's act on the left and right sides of equality (38) by element $h_{n+2,1}$.

We get the following relations equating coefficients of equal linear independent vectors:

$$\varphi_k^{i+1} = \varphi_{k-1}^i + \varphi_N^i \cdot \varphi_k^0 \quad (39)$$

$$\varphi_0^{k+1} = 0. \quad (40)$$

Let's introduce the following notations:

$$\delta_i(u) = 1 + u^{i-N} \varphi_i(u). \quad (41)$$

$$\bar{d}_i(u) = \sum_{k=0}^{\infty} d_{i,k} u^{-k-1}, i \in \{1, \dots, 2n+1\} \quad (42)$$

Easy to see that from relations (39), (40), (41) it follows the equality :

$$\delta_i(u) = \delta_N(u) \left(1 - \sum_{j=i}^N \varphi_j^0 u^{j-N-1}\right). \quad (43)$$

Let

$$\beta_i(u) = u^{-i-1} \delta_i(u), i = 1, 2, \dots, N. \quad (44)$$

Then for some matrices $A^k(u) = (A_{i,j}^k)_{i,j=0}^N(u) \in M_{N+1}(C[[u^{-1}]])$, $k, r = n+1, n+2$ is fulfilled equality:

$$h_r(u) x_k^-(t) v_+ = \sum_{0 \leq i, j \leq N} A_{i,j}^r(u) \beta_j^r(t) v_{k,i}. \quad (45)$$

Considering relations (45) и (31-32) jointly we get the following relations

$$\sum_{0 \leq i, j \leq N} A_{i,j}^{n+1}(u) \beta_j(t) v_{n+2,i} = x_{n+2}^-(t) \bar{d}_{n+1}(u) v_+ + \frac{1}{2(u-t)} (h_{n+1}(u) (x_{n+2}^-(t) - x_{n+2}^-(u)) + (x_{n+2}^-(t) - x_{n+2}^-(u)) h_{n+1}(u)) v_+; \quad (46)$$

$$\sum_{0 \leq i, j \leq N} A_{i,j}^{n+1}(u) \beta_j(t) v_{n+1,i} = x_{n+1}^-(t) \bar{d}_{n+1}(u) v_+. \quad (47)$$

These relations taking into account (33) can be rewrite in the following form

$$\begin{aligned} \sum_{0 \leq i, j \leq N} A_{i,j}^{n+1}(u) \beta_j(t) v_{n+2,i} &= \bar{d}_{n+1}(u) (\sum_{i=0}^N \beta_i(u) v_{n+2,i}) + \\ &\frac{1}{2(u-t)} (\bar{d}_{n+1}(u) (\sum_{i=0}^N (\beta_i(t) - \beta_i(u)) v_{n+2,i}) + \\ &\sum_{0 \leq i, j \leq N} A_{i,j}^{n+1}(u) (\beta_j(t) - \beta_j(u)) v_{n+2,i}); \end{aligned} \quad (48)$$

$$\sum_{0 \leq i, j \leq N} A_{i,j}^{n+1}(u) \beta_j(t) v_{n+1,i} = \bar{d}_{n+1}(u) (\sum_{i=0}^N \beta_i(t) v_{n+1,i}). \quad (49)$$

Let's equate coefficients by linear independent vectors $v_{n+2,i}$ and we get that from relation (49) it follows the following equality:

$$\begin{aligned} \sum_{j=0}^N A_{i,j}^{n+1}(u) (\beta_j(t) - \frac{1}{2(u-t)} (\beta_j(t) - \beta_j(u))) v_{n+2,i} = \\ \bar{d}_{n+1}(u) ((\beta_i(u) v_{n+2,i}) + \frac{1}{2(u-t)} (\beta_i(t) - \beta_i(u)) v_{n+2,i}) \end{aligned} \quad (50)$$

Let's note the following easy checking fact. We can to select such constants $u_0, \mu_0, \dots, \mu_N$, such that the sum $\sum_{i=0}^N \mu_i \beta_i(u_0)$ isn't equals zero. Let's put in equality (50) $u = u_0$ and multiply equality on μ_i . After addition getting equalities (for $0 \leq i \leq N$ we get equality. From this last equality we can explicitly express $d_{n+1}(u)$. Namely, we get that fulfilled equality

$$\bar{d}_{n+1}(u) = \frac{\sum_{i=0}^N (a_i u + b_i) \beta_i(u)}{\sum_{j=0}^N (c_j u + d_j) \beta_j(u)} \quad (51)$$

for some constants $a_i, b_i, c_i, d_i, i = 0, \dots, N$.

From relation (51), taking into account relations (43), (44), it follows proving equality (30).

As above we can deduce that relation (29) it follows from relations (46) for other indexes k , corresponding to other simple even roots of Lie Superalgebra $A(n, n)$. But, as above mentioned, the relation 29) follows from results of papers [25], [9]. Now, we finish the proof of theorem by standard arguments founding on the fact that inclusions of $Y(\mathfrak{sl}(1, 2)) \rightarrow Y(\mathfrak{sl}(2))$ in to $Y(A(n, n))$ are morphisms of Hopf Superalgebras. Thus, necessary condition is proved.

Let's prove sufficient condition. The second part of proof can be get conversing above given proving. Really, let $\bar{d}_{n+1}(u)$ satisfies to relations (29 – 30). Let's prove that the simple module $V(d)$ be finite dimensional. We, using theorem 1, get estimate of dimension of this module. We'll act on highest vector v_+ by monoms in the following form $x^-(t) = x_{1,k_1}^- \dots x_{1,k_r}^-, k_1 + \dots + k_r = t, k_r < \dots < k_1$. Let' note that from (30), $d_{n+1}(u)$ be a ratio of two polynomials. Let us show that from commutation relations it follows that only finite number of vectors $x^-(n) v_+$ are linearly independent. We get more strong result, namely, we prove that dimension of module $V(\bar{d})$ is limited (from above) by number $2^N, N = n_1 + \dots + n_k$, where $n_i = \deg(P_i(u))$ are the degrees of polynomials determine the highest weight. Let $\bar{d}_{n+1}(u) = 1 + \sum_{k=0}^{\infty} d_{n+1,k} \cdot u^{-k-1}$, and $\bar{d}_{n+1}(u) = \frac{P_{n+1}^d(u)}{Q_{n+1}^d(u)}$ be polynomials in a numerator and denominator, correspondingly, and these polynomials haven't common multipliers. Let us construct simple highest weight $Y(\mathfrak{sl}(1, 1))$ -module $V(\bar{d})$ with highest weight $\bar{d}(u)$. Let us show that dimension of this module is limited

(from above) by number 2^M , where $M = m_{d+1} = \deg(P_{n+1}(u)) = \deg(Q_{n+1}(u))$. Let us consider the linear span

$$M_1 = \langle \{v_+; x^-(r_1) \dots x^-(r_s) \cdot v_+ | 1 \leq r_s \leq \dots \leq r_1 \leq N\} \rangle \quad (52)$$

Let us show that

$$V(\bar{d}) \subseteq M_1 \quad (53)$$

Sufficiently to show that

$$x_k^+(u) \cdot x^-(n) = 0, \forall n > N. \quad (54)$$

Let's note that (54) it follows from following commutation relation in Yangian, which it follows from relations (9), (30):

$$\begin{aligned} [x_k^+(u), x_{m+1}^-(v)]v_+ &= \delta_{k,m+1} \frac{h_{m+1}(u) - h_{m+1}(v)}{u-v} v_+ = \\ &= \delta_{k,n+1} \frac{P_{n+1}(u)Q_{n+1}(v) - P_{n+1}(v)Q_{n+1}(u)}{Q_{n+1}(u)Q_{n+1}(v)(u-v)} v_+. \end{aligned} \quad (55)$$

Like, we can prove that dimensions of corresponding $Y(\mathfrak{sl}(2))$ -modules $V(d_k)$ with highest weights $d_k(u)$ are limited by numbers 2^{m_k} , where $m_k = \deg(P_k(u))$ is a degree of polynomial $P_k(u)$.

As embeddings of Yangians $Y(\mathfrak{sl}(2))$, $Y(\mathfrak{sl}(1,1))$ in to Yangian $Y(A(n,n))$ are morphisms of Hopf superalgebras they induce structure of $Y(A(n,n))$ -module on every modules $V(d_k)$ and also on they tensor products: $V(d_1) \otimes \dots \otimes V(d_{m+n})$. Easy to check that $V(\bar{d}) \subseteq V(d_1) \otimes \dots \otimes V(d_{m+n})$ and it is submodule of tensor product of modules. From last assertion it follows that module $V(\bar{d})$ be a finite dimensional and we get estimate of its dimension. Sufficient condition is proved. Theorem is proved also.

□

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