GLUON SPLITTING AND ITS IMPLICATION ON CONFINEMENT - A NON-PERTURBATIVE APPROACH*

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Abstract: We review briefly the perturbative treatment of the infrared problem and then argue that, for theories with self couplings, the situation in a non-perturbative approach is vastly different from that in QED. In particular, we explicitly demonstrate the impossibility to excite a hard gluon in $(\varphi^3)_6$ because of successive splitting. In our view, this is equivalent to hard gluon confinement.

Résumé: Nous donnons un compte-sendu bref de la méthode de perturbation du problème infrarouge. Ensuite nous constatons que, dans les théories avec self-couplage, la situation des méthodes non-perturbatifs est vastement differente que celle-la de QED. En particulier, nous montrons l'impossibilité de l'excitation d'un gluon dur dans $(\varphi^3)_6$, à cause de "splitting" consécutif. A notre avis, celui-ci est équivalent au bornage des gluons durs.

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I. INTRODUCTION

I would like to give a discussion of the infrared problem relevant to the effort in understanding the structure of non-abelian gauge theory,[1] which is being actively pursued with the promising outlook that it has to do with fundamental hadronic interaction.

According to the lore , the non-abelian gauge theory possesses certain peculiar features. For example, it is argued that while the fundamental fields introduced are quarks and gluons, they cannot become asymptotic. Various conjectures have been made to give theoretical justification for this confinement postulate.[2] They all are based on some assumed infrared behavior of the theory. I would like to give a summary of a detailed calculation of the infrared behavior of φ^3 in six dimensions done by Chang and myself[3] and then advocate a specific mechanism for gluon confinement.

To lead to a self contained exposition of this subject, I shall first give a brief description of the problem and what has been done in the past year or so, which basically showed that there is order by order cancellation of infrared singularities[4] as in quantum electrodynamics QED.

Then I shall spend most of the time to give you some of the results obtained by Chang and myself to show that for theories with self couplings, such as nonabelian gauge theory, there are distinct differences from QED. In particular, I shall explicitly show that the true infrared behavior cannot be obtained via naive perturbative approach. Finally, I will discuss the problem of gluon confinement.

II. ZERO MASS PROBLEM

Now, what is the infrared problem - a better word is probably the zero mass problem - and why is it important for us to deal with it? Let me first state the problem. Essentially, the zero mass problem is a study of degenerate systems. When we are given a system with massless particles, then there are two situations which can give rise to degenerate states:

(1) Infrared: if we have a particle with mass m and energy E, then a state with this same particle together with an indefinite number of low momentum massless particles is degenerate in energy with it.

(2) Collinear: groups of parallel moving massless particles in the same direction of total momentum $\vec{P}$ are degenerate in energy i.e. if

$$\vec{p}_1 \parallel \vec{p}_2 \; \cdots \; \cdots \parallel \vec{p}_n$$

such that

$$\sum_{i=1}^{n} \vec{p}_i = \vec{P}$$

then they all have the same energy

$$\sum_{i=1}^{n} E_i = E$$

for an arbitrary n.

The study of degenerate systems is not a novelty in quantum mechanics. The most relevent works in relation to our study here are done by Kinoshita, Lee, and Nauenberg in the form of a theorem.(5) What it states, in the field theory language, is that if the mass renormalization constant in a theory is not singular in the massless limit of certain mass parameters, the n there exist ensembles [i], [j] such that the transition rates

$$\sum_{\substack{i \in [i] \\ j \in [j]}} |\langle i|S|j\rangle|^2$$

are finite. What this means in QED is that if we imagine the photon to have a mass for a moment, we must make sure that the electron mass is finite when the photon mass is made to vanish. The emsembles here are the well known Block-Nordsieck states.(6) Here comes the importance of such study: As far as the theory is concerned, these emsembles are the only states of any physical interest to us. Therefore, it is necessary to construct them first to form the state space before we can extract out the relevant physics.

III. PERTURBATIVE APPROACH

There have been several demonstrative examples of infrared cancellations in low orders. Let me proceed to describe an example which is very similar to QED and thus should be familiar to most of us. This is the quark quark scattering.

Specifically, I take a set of quarks interacting with a set of non-abelian gauge fields governed by a Lagrangian

$$\mathcal{L} = -\frac{1}{4}\left(\partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g\epsilon^a_{bc}A^b_\mu A^c_\nu\right)^2$$

$$- \bar{\psi}\left\{\gamma^\mu\left(\frac{1}{i}\partial_\mu - g\frac{\tau_a}{2}A^a_\mu\right) + m\right\}\psi$$

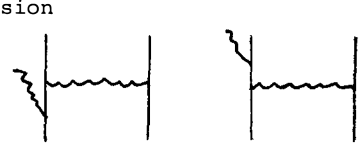
$$\left[\frac{\tau_a}{2}, \frac{\tau_b}{2}\right] = i\,\epsilon_{abc}\frac{\tau^c}{2}$$

The ϵ's are real and totally antisymmetric. The symmetry here is the color symmetry. The dynamics will be called quantum chromodynamics QCD.

For quark quark scattering, the situation is quite like that in QED. Recall that there the second order correction to electron electron scattering cross section is finite,[7] if we don't just consider the effects due to virtual correction

etc.

but also soft emission



etc.

In words, what it means is that since it is very easy to emit soft photons, the probability of emitting no photon is zero; whereas if we allow soft photons to be emitted then we obtain a finite probability.

Note that in QED, because the photons don't carry any charge, the initial and the final states have definite charge.

We now turn to QCD. It turns out that aside from a few technical details, we can perform the same calculation for quark quark scattering. However, in order to have a finite cross section we need to make an observation, namely that the color static charge is gauge dependent and in fact an infrared divergent quantity. Besides, gluons carry colors as well. Thus, for instance if we assume the color symmetry to be SU(2), then the quark quark scattering cross section has both I = 0 and I = 1 contents

$$\sigma_{qq} = \sigma_{I=0} + \sigma_{I=1}$$

We do not obtain finite $\sigma_{I=o}$ or $\sigma_{I=1}$. It is only the statistical sum which is finite.

We have here a curious situation that the combination

$$\sigma(pp\to pp) + \sigma(pp\to pp\rho^{\circ}) + \sigma(pp\to pn\rho^{+}) + \sigma(pp\to np\rho^{+})$$

which has the channel quantum number I = I, $I_z=1$, is infinite, but the sum

$$\begin{aligned}&\sigma(pp\to pp) + \sigma(pp\to pp\rho^{\circ}) + \sigma(pp\to pn\rho^{+}) + \sigma(pp\to np\rho^{+})\\ &+\sigma(nn\to nn) + \sigma(nn\to nn\rho^{\circ}) + \sigma(nn\to np\rho^{-}) + \sigma(nn\to pn\rho^{-})\\ &+\sigma(np\to np) + \sigma(np\to np\rho^{\circ}) + \sigma(np\to nn\rho^{+}) + \sigma(np\to pp\rho^{-})\\ &+\sigma(pn\to pn) + \sigma(pn\to pn\rho^{\circ}) + \sigma(pn\to nn\rho^{+}) + \sigma(pn\to pp\rho^{-})\\ &+\sigma(np\to pn\rho^{\circ}) + \sigma(pn\to np\rho^{\circ})\end{aligned}$$

which has mixed channel quantum numbers, is finite.

From such simple examples, we can establish, among other things, some results:

(1) Coherent states do not have definite color contents and only color blind rates are finite.

(2) Coherent states for a 'quark' consist of the quark and an indefinite number of soft gluons. (Block-Nordsieck)

(3) Coherent states for a 'hard gluon' consist of an indefinite number of parrallel moving hard gluons and an indefinite number of soft gluons.

(4) Static quantities, except for quark mass, such as color magnetic moment, color charge, etc. cannot be defined on shell. In particular, renormalization subtractions have to be done off shell.

This program of perturbative finiteness to all orders have been pushed forward much further.

IV. NON PERTURBATIVE APPROACH

Now I would like to describe some of the work being carried out by Chang and myself[3] in the past few months.

We have seen that one can construct states such that infrared and collinear singularities cancel out order by order in QCD. In other words, there does not seem to be anything improper to assert that mathematically it is consistent to have these states.

What about dynamically? Can we produce such states? In particular, can we prepare a hard gluon?

Let me describe the following picture. If we start out with one gluon of some energy $E > \Delta E$ = energy resolution, since it can cascade, we may end up with many many gluons. If this number is extremely large, then the energy of each one of them may be small enough to be below the energy resolution. They all become undetectable. If in fact the soft gluons always chew up all the hard energy this way, the mechanism is tantamount to gluon confinement if ΔE can be made as small as we please. In this view the solf gluons are acceptable since they are needed to construct the coherent states.

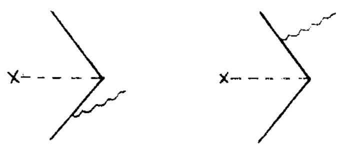
One further question we want to pursue is this. Lately, there has been quite a bit of activity in doing leading ℓn sum calculation[8] to investigate the infrared behavior of QCD. In such an approach, one expands a physical quantity A in power series of the coupling constant g

$$A = \sum_{n=0}^{\infty} (g^2)^n A_n$$

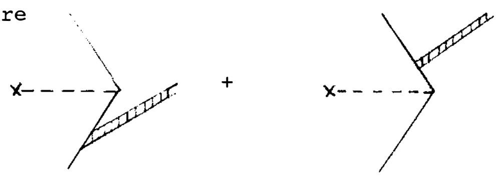
and for each A_n, one keeps only the most divergent term (usually it is a power of ℓn) as the soft cutoff is removed. We want to show that such an approach is misleading and does not reflect the true infrared behavior of the theory.

I want to use the following example to persuade you that we cannot use naive perturbation to obtain a meaningful answer.

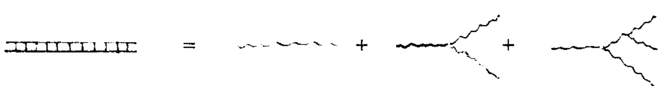
Let us consider the case when a quark passes by an external singlet field. Because of acceleration, gluons will be shed. Let $|M_0|^2$ be the probability of the quark emitting one gluon with energy $>\Delta E$, going into a color blind counter with angular resolution $\Delta\Omega$

$M_0 =$ 

We can also find out the inclusive probability of detecting an arbitrary number of gluons by this counter of energy resolution ΔE and angular resolution $\Delta\Omega$. The calculation was actually done by Chang and Tyburski[9] in the leading $\ln$ sum method. They concluded that the best way to do it is to go to the light cone gauge, where the diagrams which give the dominant contributions in each order are



where



In fact, to obtain the dominant term in the probability, we need only the diagonal terms when we square the amplitudes, which give

$$|M_0|^2 \rightarrow \sim |M_0|^2 \frac{1}{(1-c\, g^2 \ln\Delta\Omega\, \ln\frac{\Delta E}{P})^2} \qquad (1)$$

c here depends on the Casimir operator, and P is the momentum transfer.

This result is rather unphysical. As we let ΔE and $\Delta\Omega \to 0$, the inclusive probability actually blows up first, because of the pole, and then dies away. Physically it makes no sense whatsoever, since as ΔE is lowered, we should be able to detect gluons in a wider spectrum and as we decrease $\Delta\Omega$, we would be able to see the additional gluons which move closely to each other. Mathematically, it is also humbug because it involves such formal sum

$$\sum_{m=1}^{\infty} m\, x^{m-1} = \frac{1}{(1-x)^2}$$

in which the limit $x \to \infty$ is of interest to us.

This last example is certainly a strong enough motivation for us to perform a non-perturbative calculation to ascertain the true infrared behavior. Now, because of gauge invariance, it is quite impossible for us to truncate the non-abelian theory and obtain a closed set of equations which we can solve, yielding results which can be trusted. What we do instead is to look at a theory which has the general kinematics I described before: namely, we want a theory which is asymptotically free, which has cascading effects, and which has more or less the same singular structure in the soft emission limit. This is the φ^3 theory in six dimensions.(10)

Perhaps we should add a few words. It is not that we believe $(\varphi^3)_6$ is a candidate for fundamental hadronic interaction. The philosophy here is like using $(\varphi^3)_4$ to understand the mechanism for generating Regge behavior in the high energy limit. Thus, we place at the juncture more emphasis on finding a viable mechanism than on quantitative agreements. As for the six dimensional aspect, we may not find it so objectionable if we are reminded that in QCD there is a derivative for each trilinear vertex. The effective phase space volume is six dimensional.

There is one difference which I must point out, though. QCD has both infrared and collinear singularities, whereas $(\varphi^3)_6$ has only collinear singularities. As we shall see, this does not change the difficulty I described earlier if we do leading ℓn sum.

Now the problem we post for ourselves is: What is the inclusive probability for a virtual gluon of time like momentum p to dissociate into an arbitrary number of real physical gluons?

I would in the following establish a language which is distinct from naive perturbation, because it is in the way that we can free ourselves from the old way of doing things.

Let me consider only trees. Then, the gluon field satisfies the classical field equation (11)

$$(-\partial^2 + \mu^2)\chi = \xi + \frac{\lambda}{2} \chi^2 \qquad (2)$$

or

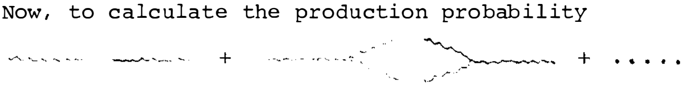
$$\chi = G\xi + \frac{\lambda}{2} G \chi^2 \qquad (3)$$

where

$$G = \frac{1}{(-\partial^2+\mu^2)} \qquad (4)$$

I have introduced a mass for the gluon field as a cutoff parameter. I could do it with dimensional regularization. However, this way is more suitable for later discussion.

Now, to calculate the production probability



we can regard χ as a field operator and replace

$$G\xi \rightarrow \varphi_0^{(-)} \text{ or } \varphi_0^{(+)}$$

$$\chi^{(\pm)} = \varphi_o^{(\pm)} + \frac{\lambda}{2} (\chi^{(\pm)})^2 \qquad (5)$$

$\varphi_0^{\pm}$ is a free creation or an annihilation operator. The inclusive dissociation probability is

$$F(x-y) = \langle \chi^{(+)}(x) \, \chi^{(-)}(y) \rangle$$
$$F(p^2) = \int d^6x \, e^{-ip\cdot x} F(x) \qquad p^2 < 0 \qquad (6)$$

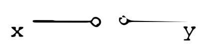
Using Eq(3) we have

$$F(x-y) = \langle \varphi_0^{(+)}(x) \, \varphi_0^{(-)}(y) \rangle$$
$$+ \left(\frac{\lambda}{2}\right)^2 \int dx_1 dy_1 G(x-x_1) G(y-y_1) \langle \chi^{(+)}(x_1)^2 \chi^{(-)}(y_1)^2 \rangle \qquad (7)$$

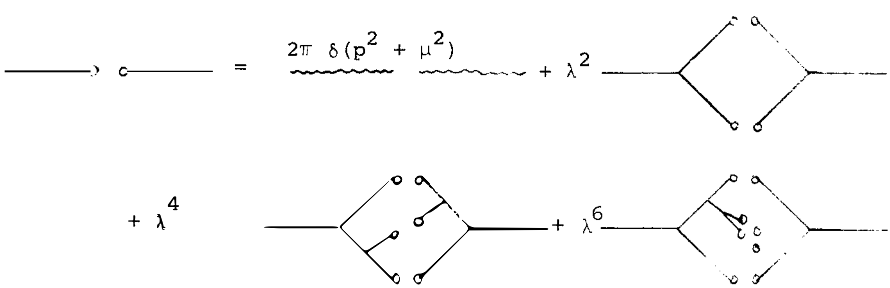
We are going to expand $\langle \chi^{(+)2} \chi^{(-)2} \rangle$ into a series of λGF the first few terms of which are

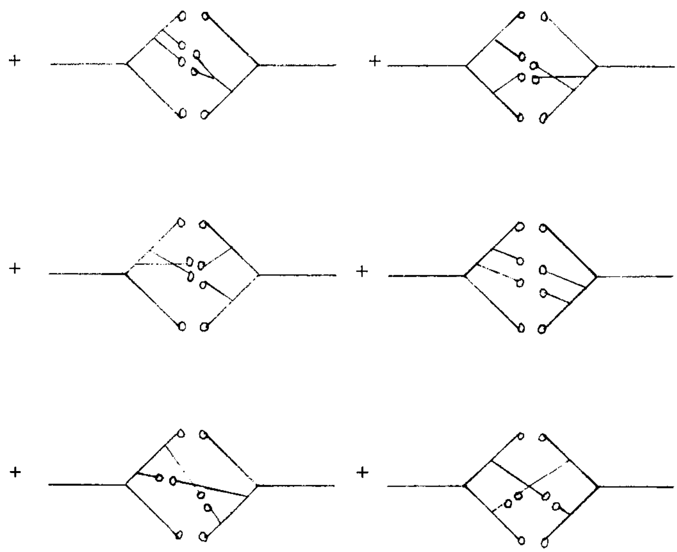
$$\langle \chi^{(+)}(x)^2 \chi^{(-)}(y)^2 \rangle = 2F(x-y) \, F(x-y)$$
$$+4\lambda^2 \int dx_1 dy_1 G(x-x_1) G(y-y_1) \cdot \qquad (8)$$
$$\cdot F(x_1-y_1) F(x-y_1) F(x_1-y) + O(\lambda^4)$$

It is best to represent this kind of expansion in graphs. Let me denote

$\langle \chi^{(+)}(x) \, \chi^{(-)}(y) \rangle =$ 

Then the integral equation is of the form





$+ \; 0 \; (\lambda^8)$

This set of graphs is the same as that we would obtain if we write down the proper self energy and make maximal unitarity cuts, i.e. retain nothing but the trees after the cuts. The vertices are to be the bare ones, but the propagators are fully dressed.(12)

If we keep only the first term in Eq. (8) and subsitute into Eq. (7) and then Eq. (6), we obtain an integral equation

$$F(p^2) = 2\pi \; \delta(p^2+\mu^2) + \frac{\lambda^2}{2} \; \frac{1}{(p^2 + \mu^2)^2} \cdot \qquad (9)$$
$$\cdot \int \frac{d^6r}{(2\pi)^6} \; F(r^2) \; F((p-r)^2)$$

which can be written in the invariant mass variables $s = -p^2$

$$sF(s) = \mu^2 2\pi\ \delta(s-\mu^2) + (\frac{g}{2\pi})^2 \int \frac{d\sigma_1}{\sigma_1} \frac{d\sigma_2}{\sigma_2} \cdot \tag{10}$$

$$\cdot \left(\frac{\lambda(s,\sigma_1,\sigma_2)}{s^2}\right)^{3/2} \sigma_1 F(\sigma_1)\ \sigma_2 F(\sigma_2)\ \theta(\sqrt{s}-\sqrt{\sigma_1}-\sqrt{\sigma_2})$$

where

$$g^2 = \lambda^2/384\pi^2$$

$$\lambda(s,\sigma_1,\sigma_2) = s^2+\sigma_1^2+\sigma_2^2 - 2s\sigma_1-2s\sigma_2-2\sigma_1\sigma_2 \tag{11}$$

σ_1 and σ_2 are the squares of the invariant masses of the cascaded blobs.

Note that because all the propagators are all time like in the tree approximation and we always have an even number of them in each term when we calculate the inclusive dissociation probability, our result due to truncation is a lower bound to the true probability.

We are interested in the limit of $\mu\to 0$, keeping s finite. However, because s and μ are the only two scales in the tree approximation, the limit is the same as $s\to\infty$ but μ finite. We should nevertheless keep the true limit in mind, which is essential when we deal with renormalization.

Let me define

$$p = \sqrt{s}\ ,\ q_1 = \sqrt{\sigma_1}\ ,\ q_2 = \sqrt{\sigma_2} \tag{12}$$

and

$$pf(p) = sF(s)$$

Then Eq. (10) becomes

$$pf(p) = \pi\mu\delta(p-\mu) + \frac{g^2}{\pi^2}\int_\mu^\infty dq_1\ dq_2\ f(q_1)\ f(q_2)\cdot \tag{13}$$

$$\cdot\left(\frac{\lambda}{s^2}\right)^{3/2} \theta(p-q_1-q_2)$$

We don't know of any way to solve this equation exactly. If we are to iterate this equation in the coupling constant g and retain only the leading term in $\ln(p/\mu)$, we obtain a result similar to Eq. (1) in QCD

$$sF(s) \cong \frac{1}{\left(1 - \frac{g^2}{2\pi} \ln(s/\mu^2)\right)^2} \qquad (14)$$

(leading $\ln$ sum)

the validity of which is being investigated, I will return to this point later. For all it is worth, this also shows that $(\varphi^3)_6$ has similar infrared structure as QCD in this approach.

We want to make the following observation: $f(p) \geq 0$ for $p \geq 0$ because it is a probability. Therefore, if we replace the kernel $(\lambda/s^2)^{3/2}$ in Eq. (13), which is a positive semi-definite quantity in the allowable kinematic region, by another positive semi-definite kernel $K(p,q_1,q_2)$ and call the corresponding solution $f'(p)$ then (13)

$$f'(p) \gtrless f(p) \text{ if } K(p,q_1,q_2) \gtrless (\lambda/s^2)^{3/2} \geq 0 \qquad (15)$$

We can easily derive the inequalities

$$1 \geq \frac{\lambda}{s^2} \geq \left(1 - \frac{q_1}{p} - \frac{q_2}{p}\right)^2 \qquad (16)$$

Hence the function $f(p)$ is bounded from above by $f^{ub}(p)$ which satisfies the equation

$$pf^{ub}(p) = \pi\mu(p-\mu) + \frac{g^2}{\pi^2} \int_\mu^\infty dq_1\, dq_2 \cdot f^{ub}(q_1)\, f^{ub}(q_2)\, \theta(p-q_1-q_2) \qquad (17)$$

We define the Laplace transform as

$$\tilde{f}^{ub}(\beta) = \int_\mu^\infty dp\, e^{-\beta p} f^{ub}(p) \qquad (18)$$

and

$$Z^{ub}(x) = \frac{g^2}{\pi^2} \tilde{f}^{ub}(\beta), \qquad x = \mu\beta \tag{19}$$

Then, we have a differential equation after the transform

$$-\frac{dZ^{ub}}{dx} = \frac{g^2}{\pi} e^{-x} + \frac{1}{x} (Z^{ub})^2 \tag{20}$$

The large p behavior is controlled by the small x region and vice versa. Because of this, the boundary condition for the function Z^{ub} is that

$$Z^{ub}(x) \rightarrow \frac{g^2}{\pi} e^{-x}, \qquad x \rightarrow \infty \tag{21}$$

which is a statement that we should recover the one particle pole for small p. When x is small Eq (20) is approximated by

$$-\frac{dZ^{ub}}{dx} \cong \frac{1}{x} (Z^{ub})^2 \tag{22}$$

which has the solution

$$Z^{ub} \cong \frac{1}{\ln x - \ln x_0} \tag{23}$$

We can show that the position of the pole

$$\ln x_0 = -\pi/g^2 \qquad \text{for small } g^2$$
$$x_o = \ln(g^2/\pi) - \ln(\ln(g^2/\pi)) \qquad \text{for large } g^2 \tag{24}$$

Then, using Eq. (23) we take the inverse Laplace transform to obtain for $p/\mu >> 1$

$$F^{ub}(s) \cong \frac{\pi^2}{g^2} \frac{x_0}{p\mu} e^{(x_o\, p/\mu)} \tag{25}$$

Note that x_0 is non analytic near to $g^2 = 0$.[14] Also, in this approach, we can recover the result of Eq. (14) provided $(g^2/\pi) \ln(p/\mu) < 1$ and $g^2/\pi << 1$.

Now, let me construct a lower bound. Call the solution by the replacement

$$\left(\frac{\lambda}{s^2}\right)^{3/2} \rightarrow \left(1 - \frac{q_1 + q_2}{p}\right)^3 \tag{26}$$

F^{1b}. Its Laplace transform satisfies the equation

$$\frac{d^4}{d^4x} Z^{1b} = \frac{g^2}{\pi} e^{-x} + \frac{6}{x^4} (Z^{1b})^2 \qquad (27)$$

We can show the following:

$$x \cong 0: \qquad Z^{1b} \cong 140(1/(\ln(x/x_0))^4) \qquad (28)$$

and

$$\ln x_0 = -\pi/g^2 \qquad \text{for small } g^2$$

$$x_0 = \ln(g^2/\pi) - 4\ln(\ln(g^2/\pi)) \qquad \text{for large } g^2 \qquad (29)$$

From Eq. (28), we obtain for $p/\mu >> 1$

$$F^{1b}(s) \cong \frac{\pi^2}{g^2} \frac{x_o^4}{\mu^2} \frac{70}{3} (p/\mu)^2 e^{(x_0 p/\mu)} \qquad (30)$$

By assuming

$$F(s) \cong A p^n e^{(x_o p/\mu)}, \quad p/\mu >> 1 \qquad (31)$$

and substituting this into Eq.(13), we can determine A and n for a given x_0. We have in fact

$$A = \frac{1}{g^2} \frac{35\pi^2}{3\sqrt{2\pi}} (x_o/\mu)^{5/2}, \qquad n = 1/2 \qquad (32)$$

Because of Eq.(24) and Eq.(29), we know that

$$x_0 \cong \exp(-\pi/g^2) \qquad \text{for small } g^2$$

$$\ln(g^2/\pi) - c \ln(\ln(g^2/\pi)) \qquad \text{for large } g^2$$

$$1 < c < 4 \qquad (33)$$

It is obvious that the true behavior (Eq.(31)) bears no resemblance to Eq.(14), a result obtained by leading $\ln$ sum method.

V. CONJECTURE ON CONFINEMENT

I want to make an observation and a conjecture based on results of Eq. (31) and Eq. (33). From F(s), we can calculate the average multiplicity, i.e. the average number of gluons which the original gluon dissociates into

$$\begin{aligned} \langle n \rangle &= g^2 \frac{\partial}{\partial g^2} \ln F(s) \\ &\cong g^2 \frac{\partial}{\partial g^2} (x_o\, p/\mu) \end{aligned} \tag{34}$$

Since

$$\begin{aligned} \frac{\partial}{\partial g^2} x_o &\cong \frac{\pi}{g^4} e^{-\pi/g^2} & g^2 \ll 1 \\ &\cong \frac{1}{g^2} & g^2 \gg 1 \end{aligned} \tag{35}$$

we have

$$\begin{aligned} \langle n \rangle &\cong \frac{\pi}{g^2} e^{-\pi/g^2} \left(\frac{s}{\mu^2}\right)^{1/2} & g^2 \ll 1 \\ &\cong \left(\frac{s}{\mu^2}\right)^{1/2} & g^2 \gg 1 \end{aligned} \tag{36}$$

We can ask a crucial question: What is the energy carried away by all the soft gluons with energy $\sim\mu$? The answer is that it must be greater than

$$\begin{aligned} \lim_{\mu\to 0} \mu \langle n \rangle &\cong \frac{\pi}{g^2} e^{-\pi/g^2} \sqrt{s} & g^2 \ll 1 \\ &\cong \sqrt{s} & g^2 \gg 1 \end{aligned} \tag{37}$$

In both cases, it is a finite fraction of the total energy. In particular, we are interested in the large coupling limit, because we are dealing with an infrared unstable theory. In this case, all of the energy of a hard gluon dissipates into soft ones, which in fact are not moving at all. We can also calculate the multiplicity distribution, and a Gaussian with almost zero width results. This implies that the probability of having soft gluons to chew up the hard energy is unity. If we agree that the soft gluons are necessary components in constructing coherent states, then we have provided a mechanism for confinement.

What about the quarks? I have not dealt with them so far, but we may envision a mechanism for their confinement. In order to knock a quark out, we must kick it with something, which means acceleration. The quark will then radiate gluons, which are all soft and at rest relative to the quark because of the previous cascading processes. It is possible that the effective mass of this quark and the gigantic cloud of soft gluons around it becomes practically infinite. The quark just can't come out!

I should contrast this with the situation in QED. There, the Bremsstrahlung mean multiplicity is $\langle n \rangle \sim e^2 \ln(p/\mu)$, where p is the momentum transfer and μ is the infrared cutoff. Therefore, the mean energy taken by the soft photons is $\mu \langle n \rangle \sim \mu\ e^2 \ln(p/\mu)$, which can be made as small as we please by decreasing the cutoff μ. This, of course, agrees with our notion that electrons can have well defined energy.

In the case when each gluon can split into r components, that the multiplicity is not logarithmic can be seen from the following simple argument[15]. Let ε be the fraction of the original mass which resides with each of the daughters in the form of rest mass after each decay. Then, often n steps, each daughter has a mass $\sim \varepsilon^n p$, where p is the original mass. The process ends when $\varepsilon^n p \cong \mu$. The total number of particles in the final state is $\sim r^n \cong (p/\mu)^{(-\ln r/\ln \varepsilon)}$. What we have shown is that in the strong coupling limit, $\varepsilon \cong 1/r$. All final particles are then at rest with respect to the parent.

VI. REMARKS AND CONCLUSION

We have given an explicit calculation, which shows that the leading ℓn sum method does not reflect the true infrared behavior of $(\varphi^3)_6$ or QCD. From the nonperturbative results, we have also unravelled a dynamical feature, namely the impossibility to excite any gluons of energy $> \mu$, an infrared cutoff which can be made as small as we please. In our view, this mechanism is tantamount to hard gluon confinement. We also believe that soft gluons, while experimentally undetectable, are nevertheless necessary ingredients to construct physical states.

To be sure, we need many more investigations to find out whether this is the mechanism Nature chooses for confinement. Let me point out two: (1) What about radiative effects? The folklore in that one consequence is to change the coupling constant into a running effective one. In the infrared limit, this becomes large. We have to do an analysis to see how it works in the time-like region. Let me point out one aspect here. We know from low order calculation the reason that the effective coupling constant is large for small momentum (actually it has only been shown to be small for large space-like momentum) in $(\varphi^3)_6$ is due to vertex correction. Thus, to account for radiative effects, we need to look into equations of higher non-linearity than the one we have used so far, which contains self energy cuts only. (2) What about the proposition that only color singlets can be excited? We don't know how this is to come about, because color symmetry is not a natural symmetry in $(\varphi^3)_6$. We need to turn to QCD for an answer.

In any event, the mechanism we propose here for confinement is a physical one. We hope that further progress can be made in the near future to ascertain its truth.

Acknowledgments:

It has been my privilege to collaborate with S.-J. Chang on this subject. I have relied heavily on the results obtained by us in preparing this talk. Encouragement from G.L. Kane is appreciated.

FOOTNOTES AND REFERENCES

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12. This kind of equations was also looked into by A.M. Polyakov, Zh. Eksp. Teor. Fiz. 59, 542 (1970) [Sov. Phys.-JETP 32, 296 (1971)].

13. We are not invoking any deep mathematical theorem in stating this observation. We can prove it by simple iteration. For example, if $3\mu > p \geq 2\mu$, the solution to Eq. (13) is

$$pf(p) = \frac{g^2}{\pi^2}\int_\mu^\infty dq_1 dq_2 \pi\mu\ \delta(q_1-\mu)\pi\mu\delta(q_2-\mu)\cdot$$

$$\cdot\left(\frac{\lambda(p^2,\ q_1^2,\ q_2^2)}{s^2}\right)^{3/2} \quad \theta(p-q_1-q_2)$$

What we have claimed is simply that

$$pf(p) \leq \frac{g^2}{\pi^2}\int_\mu^\infty dq_1 dq_2 \pi\mu\ \delta(q_1-\mu)\pi\mu\ \delta(q_2-\mu) \quad \theta(p-q_1-q_2)$$

(see Eq.(16)). Now, the right hand side is the solution to Eq. (17) for this range of energy. It follows that

$$f(p) \leq f^{ub}(p)$$

We can construct a proof along this line for any finite μ and p. The main point here is that we are considering a series of sequential decays. By increasing or decreasing the phase space for each decay, we certainly should expect to obtain higher or lower rate, respectively.

14. It is amusing to note the instanton-like dependence on the coupling constant. We presumably have included some classical solutions in our approach, in a way which is not completely understood by me. I thank H.-S. Tsao for a discussion on this.

15. This is also known to A.H. Mueller.