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Quark–gluon plasma fireball evolution with one-loop correction in the mean-field potential

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The study of the free energy evolution of quark–gluon plasma (QGP) with one-loop correction factor in the mean-field potential is discussed. The energy evolution with the effect of the correction factor in potential shows a higher transition temperature in the range of $T = 180$ to 250 MeV in comparison to the transition temperature without the one-loop correction factor. The transition temperature is also affected by the dynamical flow parameter of quark and gluon used in the potential and it results in decreasing observable QGP droplets of stable radius 2.5–4.5 fm.

Subject Index D31

1. Introduction

The study of phase transition [1,2] from a confined system of quarks in hadrons to a deconfined state has become an interesting topic in the last two decades. During the early stages of the formation of the universe, it is expected that matter was present as deconfined quarks and gluons, and in due course, cooling led to the formation of hadrons. The process of this early stage of the universe can be replicated as a complicated phenomenon in heavy-ion collider experiments. So, the study of the quark–gluon plasma (QGP) fireball in ultra-relativistic heavy-ion collisions has become an exciting field in current heavy-ion collider physics [3–5]. In this brief paper, we focus on QGP evolution through the free energy expansion of the system. To evaluate the free energy, we use the mean-field potential with one-loop correction factor to construct the density of states of particles in the system. Thus the free energy evolution is obtained through this density of state. Due to the correction factor in the potential through the coupling value [6–10], there are changes in the free energy expansion of the QGP fireball, and it also impacts the stability of droplets with the variation of dynamical quark and gluon flow parameters.

In brief, we construct the density of states with one-loop correction factor in the potential and study the free energy evolution affected by the loop correction. In conclusion, we give details of the evolution of the QGP fireball with different flow parametrization values. Preliminary results in this paper were reported earlier in Ref. [11]

2. Density of states for QGP with one-loop correction

The interacting mean-field potential $V_{\mathrm{conf}}(q)$ is now modified with the inclusion of a one-loop correction factor from the simple confining potential, which is obtained through the Hamiltonian of the system. The modified potential is therefore obtained through the expansion of strong coupling constants of the one-loop factor within the perturbation theory as [12–14]:

$$V_{\mathrm{conf}}(q) = \frac{8\pi}{q}\sqrt{(1/\gamma_g)^2 + (1/\gamma_q)^2}\,\alpha_s(q)T^2\left[1 + \frac{\alpha_s(q)}{4\pi}a_1\right] - \frac{m_0^2}{2q}, \tag{1}$$

where the quark and gluon parametrization factors are $\gamma_q = 1/8$ and $\gamma_g = (8{-}10)\gamma_q$. These factors determine the dynamics of QGP flow and enhance the transformation to hadrons. $\alpha_s(q)$ is the coupling value of quark and gluon with degree of freedom n_{f}, as

$$\alpha_s(q) = \frac{4\pi}{(33 - 2n_{\mathrm{f}})\ln(1 + q^2/\Lambda^2)}, \tag{2}$$

in which QCD parameter Λ is taken as 0.15 GeV. The coefficient a_1 in the confining potential is the correction factor of one-loop correction in their interactions and it is given as [15,16]:

$$a_1 = 2.5833 - 0.2778\,n_{\mathrm{l}}, \tag{3}$$

where n_{l} is considered as the number of light quark elements [15–18].

Now the density of states in phase space with loop correction in the interacting potential is obtained through a generalized Thomas–Fermi model as [19–22]:

$$\int \rho_{q,g}dq = \nu/\pi^2[-V_{\mathrm{conf}}(q)]^2\frac{dV_{\mathrm{conf}}}{dq}, \tag{4}$$

or

$$\rho_{q,g}(q) = \frac{\nu}{\pi^2}\left[\frac{\gamma_{q,g}^3T^2}{2}\right]^3 g^6(q)A, \tag{5}$$

where

$$A = \left\{1 + \frac{\alpha_s(q)a_1}{\pi}\right\}^2\left[\frac{(1 + \alpha_s(q)a_1/\pi)}{q^4} + \frac{2(1 + 2\alpha_s(q)a_1/\pi)}{q^2(q^2 + \Lambda^2)\ln\left(1 + \frac{q^2}{\Lambda^2}\right)}\right], \tag{6}$$

ν is the volume occupied by the QGP, q is the relativistic four-momentum in natural units, and $g^2(q) = 4\pi\alpha_s(q)$.

3. The free energy evolution

The free energy of quarks and gluons is defined in the following with the modified density of states as [23–25]:

$$F_i = \mp Tg_i\int dq\rho_{q,g}(q)\ln\left(1 \pm e^{-\left(\sqrt{m_i^2+q^2}\right)/T}\right), \tag{7}$$

with minimum energy cut-off as:

$$V(q_{\mathrm{min}}) = \left(\gamma_{g,q}N^{\frac{1}{3}}T^2\Lambda^4/2\right)^{1/4}, \tag{8}$$

where $N = (4/3)[12\pi/(33 - 2n_{\mathrm{f}})]$. The minimum cut off in the model leads the integral to a finite value by avoiding the infrared divergence while taking the magnitude of Λ and T as of the same order as in lattice QCD. g_i is the degeneracy factor (color and particle–antiparticle degeneracy) which is 6 for quarks, 8 for gluons, and 3 for pions. The interfacial energy obtained through a scalar Weyl surface in Ramanathan et al. [8,9,26] with a suitable modification to take care of the hydrodynamic effects is given as:

$$F_{\text{interface}} = \frac{1}{4}\gamma R^2 T^3. \tag{9}$$

This energy is used to replace the bag energy of the MIT model and it minimizes the drawback produced by MIT model. γ is the root mean square value of the quark and gluon parameters γ_q, γ_g. The pion free energy is [27]:

$$F_\pi = (3T/2\pi^2)\nu \int_0^\infty q^2 dq \ln(1 - e^{-\sqrt{m_\pi^2 + q^2}/T}). \tag{10}$$

To calculate the free energies, the particle masses are taken as: quark masses $m_u = m_d = 0$ MeV and $m_s = 0.15$ GeV [11], and pion mass $m_\pi = 0.14$ GeV. It is because the pions are assumed to be the dominant component of the hadronized phase which involves explicit chiral symmetry-breaking induced pion mass, but the total free energy only changes negligibly due to the introduction of pion mass. We can thus compute the total modified free energy F_{total} as,

$$F_{\text{total}} = \sum_i F_i + F_{\text{interface}} + F_\pi, \tag{11}$$

where i stands for u, d, s quarks and gluon.

4. Results

The free energy of the constituent particles of the QGP fireball with one-loop correction factor in the interacting mean-field potential is numerically calculated. The free energy of QGP–hadron fireball evolution with the modification in the density of states of each particle is replicated in Figs. 1–6. The free energies of the individual particles are shown in Fig. 1 at the particular temperature $T = 152$ MeV for the quark and gluon parametrization factors $\gamma_q = 1/8$, $\gamma_g = 12\gamma_q$ and the free

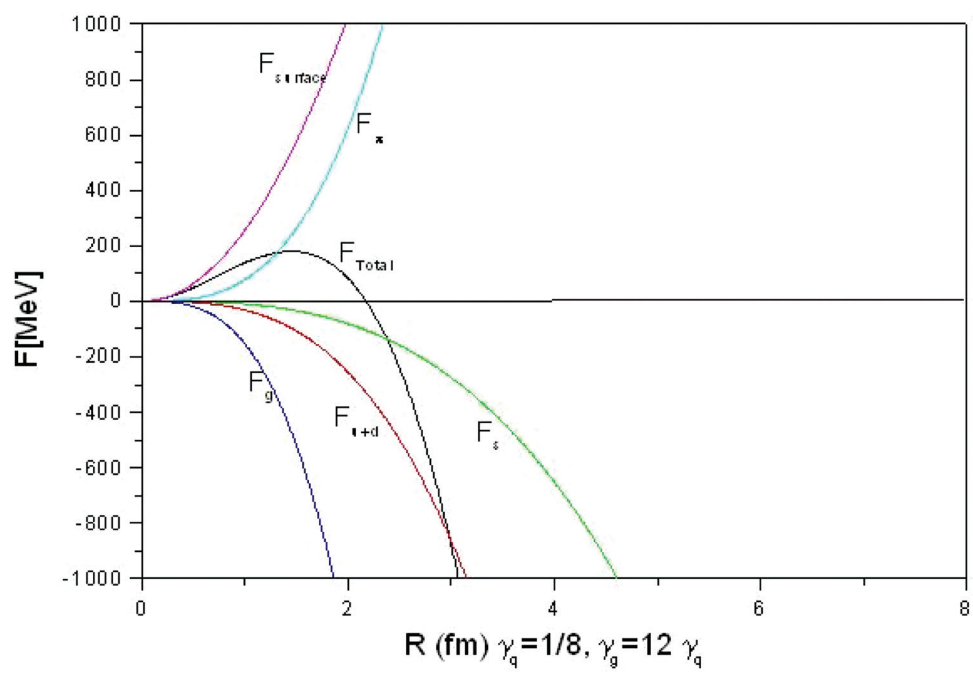

Fig. 1. Individual free energy contribution F_i vs. R at $\gamma_q = 1/8$, $\gamma_g = 12\gamma_q$ at the particular temperature $T = 152$ MeV.

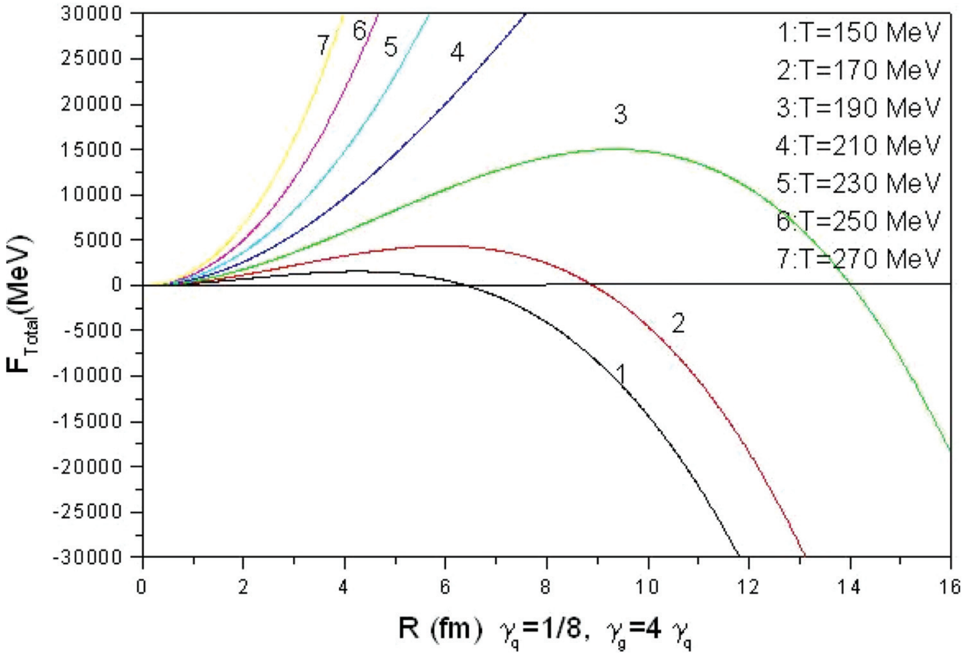

Fig. 2. The free energy vs. R at $\gamma_q = 1/8$, $\gamma_g = 4\gamma_q$ for various values of temperature.

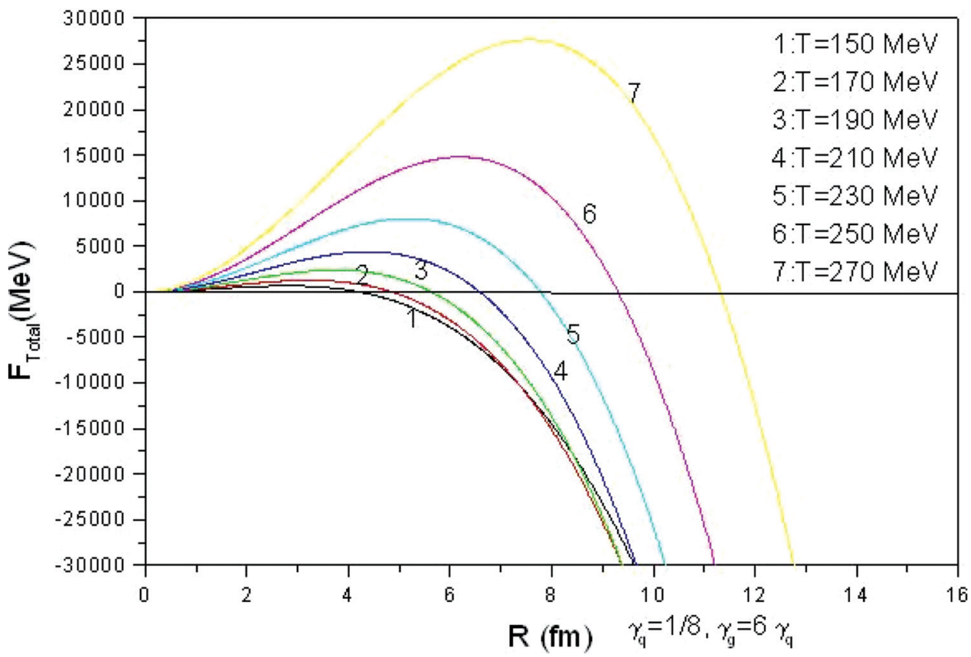

Fig. 3. The free energy vs. R at $\gamma_q = 1/8$, $\gamma_g = 6\gamma_q$ for various values of temperature.

energy shows the behavior of QGP–hadron droplet formation. It indicates that the free energy of the system is modified with changes in the amplitude by inclusion of one-loop correction in the interacting potential, and it may be compared to the transitions that are indicated in Figs. 1–8 of Ref. [8], where the same is computed without the inclusion of one-loop correction to the mean-field potential. In Figs. 2 and 3 the free energies show changes of the droplet formation in the range of

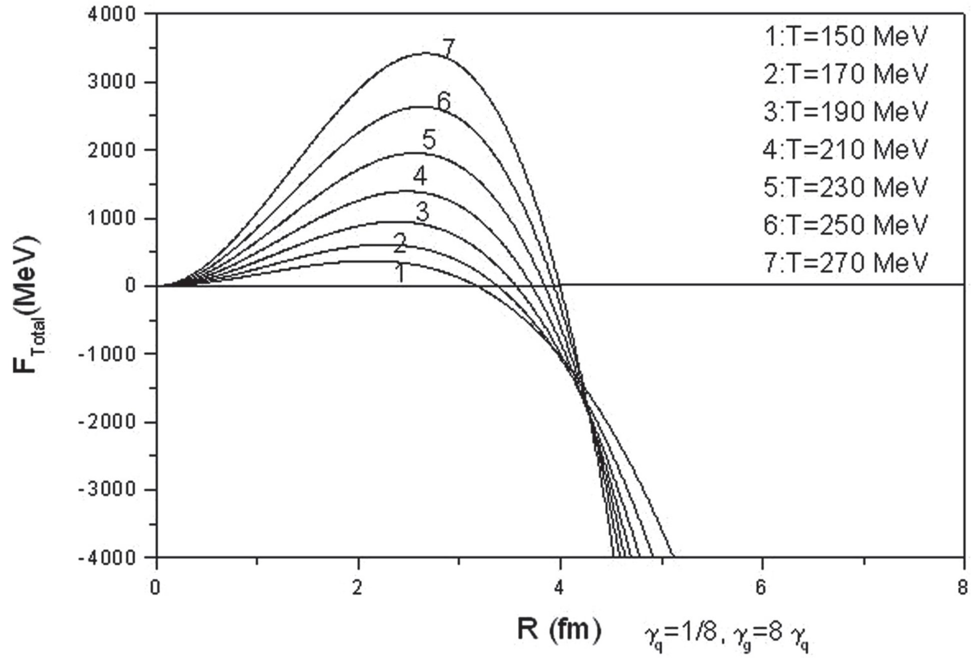

Fig. 4. The free energy vs. R at $\gamma_q = 1/8$, $\gamma_g = 8\gamma_q$ for various values of temperature.

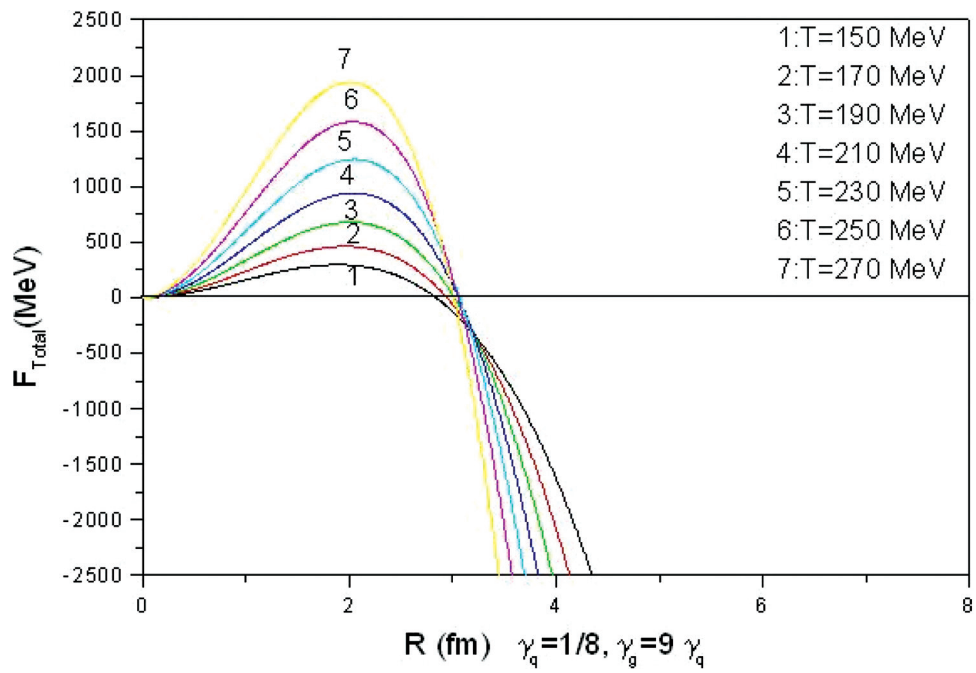

Fig. 5. The free energy vs. R at $\gamma_q = 1/8$, $\gamma_g = 9\gamma_q$ for various values of temperature.

parametrization factors $4\gamma_q \leq \gamma_g \leq 6\gamma_q$. In these figures, there is no stable droplet formation in this range of parameters. The figures show phase transition at the temperatures $T = 180\,\mathrm{MeV}$ with flow parameters $\gamma_q = 1/8$, $\gamma_g = 4\gamma_q$, and the changes of transition temperature are also found up to the temperature $T = 250\,\mathrm{MeV}$ with the increase of the gluon parameter $\gamma_g \leq 6\gamma_q$. As the x-axis represents the phase-separating line in which the total free energy is zero, the quarks are in the confined

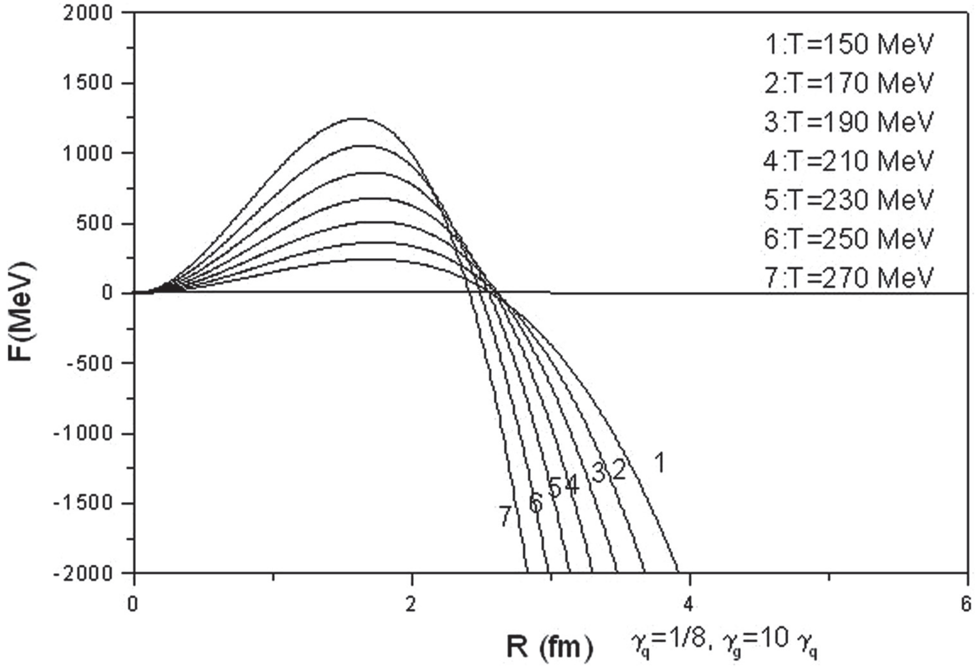

Fig. 6. The free energy vs. R at $\gamma_q = 1/8$, $\gamma_g = 10\gamma_q$ for various values of temperature.

phase below the line and deconfined above. So only the total free energy curves that make a crossover represent a QGP transition. Moreover, it is indicated by lattice QCD that formation of QGP droplets takes place under the condition of rollover phase rather than a sharp jump as temperature varies [28]. Here, in this model of one-loop correction it shows a weakly first-order phase to the ambiguity of crossover phase transition at low temperature. Besides, the parameters used in the work result in an increase in the interaction between the constituent particles and affect the formation of stable droplets, decreasing the amplitude of the free energy.

In Figs. 4 to 6 the various stable droplet formation for parameters ranging from $8\gamma_q \leq \gamma_g \leq 10\gamma_q$ are shown. In these figures we can easily observe the stability of droplet formation with the flow parameters of $\gamma_q = 1/8$ and $8\gamma_q \leq \gamma_g \leq 10\gamma_q$. The stability is obtained with the different size of droplet and the stable droplets are found in the range of radius 2.5–4.5 fm, which is around half the droplet size without one-loop correction and its size decreases as the value of the gluon parameter increases. This is obtained with the decrease in the quark and gluon flow parameters.

5. Conclusion

We can conclude from these results that due to the presence of loop correction in the mean-field potential, the stability of droplets increases while their size decreases in comparison with the result with uncorrected potential. So, we can further study the velocity of sound and thermodynamic properties of QGP on the basis of these smaller droplets. In our further work, we plan to compare the results with available data on fireball radius and possible experimental tests.

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