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Quasinormal modes for heavy vector mesons in a finite density plasma

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ABSTRACT

There are strong indications that ultra-relativistic heavy ion collisions, produced in accelerators, lead to the formation of a new state of matter: the quark gluon plasma (QGP). This deconfined QCD matter is expected to exist just for very short times after the collision. All the information one can get about the plasma is obtained from the particles that reach the detectors. Among them, heavy vector mesons are particularly important. The abundance of $c\bar{c}$ and $b\bar{b}$ states produced in a heavy ion collision is a source of information about the plasma. In contrast to the light mesons, that completely dissociate when the plasma is formed, heavy mesons presumably undergo a partial thermal dissociation. The dissociation degree depends on the temperature and also on the presence of magnetic fields and on the density (chemical potential). So, in order to get information about the plasma out of the quarkonium abundance data, one needs to resort to models that provide the dependence of the dissociation degree on these factors. Holographic phenomenological models provide a nice description for charmonium and bottomonium quasi-states in a plasma. In particular, quasi-normal modes associated with quarkonia states have been studied recently for a plasma with magnetic fields. Here we extend this analysis of quasinormal models to the case when charmonium and bottomonium are inside a plasma with finite density. The complex frequencies obtained are then compared with a Breit Wigner approximation for the peaks of the corresponding thermal spectral functions, in order to investigate the quantitative agreement of the different descriptions of quarkonium quasi-states.

1. Introduction

One of the most fascinating challenges currently faced by physicists is to build a detailed picture of the quark gluon plasma (QGP) from the particles that reach the detectors in heavy ion collisions. Hadronic matter, at the very high energy densities produced in these processes, apparently produce the QGP, a state of matter where color is not confined inside bound states. It is not possible to observe the plasma directly. All the available information comes from particles created after this very short-lived state of matter hadronizes and disappears. For reviews about QGP, see for example [1–4].

An important source of indirect information about the QGP is the abundance of heavy vector mesons. In contrast to light mesons, that dissociate once the plasma is formed, mesons made of $c\bar{c}$ or $b\bar{b}$ quarks may survive in the thermal medium. They undergo a partial dissociation process that depends on the flavor (charm or bottom), on the temperature and density of the plasma and on the presence of magnetic fields produced by the motion of charged particles. It is important to understand the thermal behavior of heavy vector mesons in a plasma in order to relate their relative abundance with the properties of the medium.

In recent years, it has been shown that holographic phenomenological models are a fruitful framework for describing heavy vector mesons inside a thermal medium. In Ref. [5] a holographic AdS/QCD model was proposed in order to represent the spectra of masses and decay constants of charmonium and bottomonium states in the vacuum. This model has two energy parameters, fixed by the criteria of the best fit to the experimental data. The extension to finite temperature and density appeared in refs. [6,7] where the spectral functions representing the quasi-states of heavy vector mesons were calculated.

Then, it was shown that a better fit for the spectra is obtained from an improved model involving three energy parameters [8,9]. These parameters represent: the quark mass, the string tension and an ultraviolet (UV) energy scale. This UV energy parameter is related to the

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large mass change that happens in non-hadronic decays and is necessary in order to fit the decay constant spectra. A very nice picture of the dissociation process of charmonium and bottomonium states was obtained in these references by calculating the thermal spectral functions.

A complementary tool for studying the dissociation process using holographic models is the determination of the quasinormal modes. The spectral function is obtained from real frequency solutions of the equations of motion of gravity fields dual to the hadrons. In contrast, quasinormal modes are solutions with complex frequency and more restrictive boundary conditions. The real part of the frequency is related to the thermal mass and the imaginary part to the thermal width of the quasi-states. In Ref. [10], quasinormal modes for heavy vector mesons were studied using the holographic model of [8,9]. This analysis was then extended, for the case when a magnetic field is present, in Ref. [11]. The dissociation process corresponds to an increase in the imaginary part of the frequency of the quasinormal modes.

Here we present an extension of the analysis of heavy vector meson quasinormal modes to the case when the medium has a finite density. We investigate the dependence on the density of the complex frequencies for bottomonium and charmonium quasi-states at different temperatures. Then we compare the results obtained for the quasinormal frequencies with the results from the spectral function approach. The comparison is performed by considering a Breit Wigner approximation for the spectral function peaks. Such an approximation provides estimates for the real and imaginary parts of the quasinormal frequencies that are in a very nice agreement with the results obtained directly. This shows that the two approaches are consistent not only qualitatively but also from a quantitative point of view.

This letter is organized as follows: in section 2 we review the holographic model for heavy vector mesons. In section 3 we present the calculation of quasinormal modes at finite temperature and density. Then in section 4 we compare the results obtained with a Breit Wigner approximation of the first spectral function peaks. Section 5 is devoted to conclusions and final comments.

2. Holographic model for heavy vector mesons

In the phenomenological model developed in Refs. [8,9], vector mesons are described by a vector field $V_m = (V_\mu, V_z)(\mu = 0, 1, 2, 3)$, representing the gauge theory current $J_\mu = \bar{\psi}\sigma^\mu\psi$ in the gravity side of the duality. The fields live in a curved five dimensional space, that is anti-de Sitter space for the case when the mesons are in the vacuum. Additionally, there is a scalar background. The action for the vector field reads

$$I = \int d^4x dz \sqrt{-g}\, e^{-\phi(z)} \left\{ -\frac{1}{4g_5^2} F_{mn}F^{mn} \right\} , \tag{1}$$

where $F_{mn} = \partial_m V_n - \partial_n V_m$. The model contains three energy parameters that are introduced through the background scalar field $\phi(z)$:

$$\phi(z) = \kappa^2 z^2 + Mz + \tanh\left(\frac{1}{Mz} - \frac{\kappa}{\sqrt{\sigma}} \right) . \tag{2}$$

The parameter σ, with dimension of energy squared, represents effectively the string tension of the strong quark anti-quark interaction. The mass of the heavy quarks is represented by κ. The third parameter, M, has a more subtle interpretation. Heavy vector mesons undergo non hadronic decay processes, when the final state consists of light leptons, like an e^+e^- pair. In such transitions there is a very large mass change, of the order of the meson mass. The parameter M represents effectively the mass scale of such a transition, characterized by a matrix element $\langle 0|\, J_\mu(0)\, |n\rangle = \epsilon_\mu f_n m_n$, where f_n is the decay constant, $|n\rangle$ is a meson state at radial excitation level n with mass m_n, $|0\rangle$ is the hadronic vacuum and J_μ the hadronic current. The values that provide the best fit to charmonium and bottomonium spectra of masses and decay constants at zero temperature are respectively

$$\kappa_c = 1.2, \sqrt{\sigma_c} = 0.55, M_c = 2.2\,;\ \kappa_b = 2.45, \sqrt{\sigma_b} = 1.55, M_b = 6.2\,, \tag{3}$$

where all quantities are expressed in GeV. The geometry that corresponds to a thermal medium with a finite density μ was studied in refs. [12–14]. It is a 5-d anti-de Sitter charged black hole space-time with the metric

$$ds^2 \;=\; \frac{R^2}{z^2}\Big(- f(z)dt^2 + d\vec{x}\cdot d\vec{x} + \frac{dz^2}{f(z)}\Big), \tag{4}$$

where

$$f(z) = 1 - \frac{z^4}{z_h^4} - q^2 z_h^2 z^4 + q^2 z^6, \tag{5}$$

and z_h is the horizon position where $f(z_h) = 0$. The relation between z_h and the temperature T of the black hole, assumed to be the same as the gauge theory temperature, comes from the condition of absence of conical singularity at the horizon:

$$T = \frac{|f'(z)|_{(z=z_h)}}{4\pi} = \frac{1}{\pi z_h} - \frac{q^2 z_h^5}{2\pi}\,. \tag{6}$$

The parameter q is proportional to the black hole charge. In the dual gauge theory, q is related to the density of the medium, or quark chemical potential, μ. In the gauge theory Lagrangean, μ would appear multiplying the quark density $\bar{\psi}\gamma^0\psi$. So it would work as the source of correlators for this operator. In the dual supergravity description the time component V_0 of the vector field plays this role. So, one considers a particular solution for the vector field V_m with only one non-vanishing component: $V_0 = A_0(z)$ ($V_z = 0, V_i = 0$). Assuming that the relation between q and μ is the same as in the case of no background, that means $\phi(z) = 0$, the solution for the time component of the vector field is: $A_0(z) = c - qz^2$, where c is a constant. Imposing $A_0(0) = \mu$ and $A_0(z_h) = 0$ one finds:

$$\mu = q z_h^2 \,. \tag{7}$$

From eqs. (6) and (7) it becomes clear that specifying both z_h and q, the values of the temperature and the chemical potential are fixed and contained into the metric (4).

3. Quasinormal modes at finite density

In the context of gauge/gravity duality, quasinormal modes are normalizable gravity solutions representing the quasi-particle states in a thermal medium, with complex frequencies ω. The real part, $\mathrm{Re}(\omega)$, is related to the thermal mass and the imaginary part, $\mathrm{Im}(\omega)$, is related to the thermal width. In the zero temperature limit one recovers the vacuum hadronic states, corresponding to solutions that are called normal modes. The normalizability condition requires that, either in the zero temperature case or in the finite temperature one, the fields must vanish at the boundary z=0. The main difference in that at finite T there is an event horizon where one additionally has to impose infalling boundary conditions. These two types of boundary conditions are simultaneously satisfied in general for complex frequencies.

In the radial gauge $V_z = 0$, and considering solutions of the type $V_\mu = e^{-i\omega t + ikx_3} V_\mu(z,\omega,k)$ that corresponds to plane waves propagating in the x_3 direction with wave vector $p_\mu = (-\omega, 0, 0, k)$. In terms of the electric field components $E_1 = \omega V_1, E_2 = \omega V_2$ and $E_3 = \omega V_3 + k V_t$ the equations of motion coming from action (1) with the metric (4) take the form:

$$E''_\alpha + \left(\frac{f'}{f} - \frac{1}{z} - \phi'\right) E'_\alpha + \frac{1}{f^2}\left(\omega^2 - k^2 f\right) E_\alpha = 0\,, \qquad (\alpha = 1, 2) \tag{8}$$

$$E''_3 + \left(\frac{\omega}{\omega^2 - k^2 f}\frac{f'}{f} - \frac{1}{z} - \phi'\right) E'_3 + \frac{1}{f^2}\left(\omega^2 - k^2 f\right) E_3 = 0, \tag{9}$$

where (') represents derivatives with respect to the radial z coordinate.

One has to impose the normalizability condition at $z = 0$ and the infalling condition at $z = z_h$. The idea, in order to impose the boundary conditions at the horizon, is to re-write the field equations in such a way that they separate into a combination of infalling and outgoing waves. It is convenient to introduce the Regge-Wheeler tortoise coordinate r_*. This coordinate is defined by the relation $\partial_{r_*} = -f(z)\partial_z$ with z in the interval $0 \le z \le zh$ and can be expressed explicitly by integrating the former relation and imposing $r_*(0) = 0$. In order to have a Schrödinger like equation, one can perform a Bogoliubov transformation on the electric fields, introducing ($\psi_j = e^{-\frac{B_j(z)}{2}} E_j$). Then, Eqs. (8) and (9) reduce to the form:

$$\partial^2_{r_*}\psi_j + \omega^2 \psi_j = U_j \psi_j \,, \tag{10}$$

with

$$B_\alpha(z) = \log(z) + \phi;\; B_3(z) = \log\left(\frac{w^2 - k^2 f}{\omega^2} z\right) + \phi. \tag{11}$$

For $j = (1, 2, 3)$ these potentials diverge at $z = 0$ so we must have $\psi_j(z = 0) = 0$. At the horizon we have for both potentials that $U_j(z = z_h) = 0$ in the limit $z \to z_h$ one expects to find *infalling* $\psi_j = e^{-i\omega r_*}$ and *outgoing* $\psi_j = e^{+i\omega r_*}$ wave solutions for equation (10). Only the first kind of solutions is physically allowed. The Schrödinger like equation can be expanded near the horizon leading to the following expansion the field solution:

$$\psi_j = e^{-i\omega r_*(z)}\left[1 + a_j^{(1)}(z - z_h) + a_j^{(2)}(z - z_h)^2 + \ldots\right]. \tag{12}$$

One can solve recursively for $a_j^{(n)}$. The first coefficients are:

$$a_\alpha^{(1)} = \frac{(2 - q^2 z_h^6)\left(z_h\left(\frac{k^2}{2 - q^2 z_h^6} + 2\kappa^2\right) - \frac{\operatorname{sech}^2\left(\frac{\kappa}{\sqrt{\sigma}} - \frac{1}{M z_h}\right)}{M z_h^2} + \frac{1}{z_h} + M\right)}{2\left(q^2 z_h^6 + i\omega z_h - 2\right)}\,, \tag{13}$$

$$a_3^{(1)} = \frac{(2 - q^2 z_h^6)\left(z_h\left(\frac{k^2}{2 - q^2 z_h^6} + 2\kappa^2\right) - \frac{\operatorname{sech}^2\left(\frac{\kappa}{\sqrt{\sigma}} - \frac{1}{M z_h}\right)}{M z_h^2} + \frac{4k^2 + \omega^2 - 2k^2 q^2 z_h^6}{\omega^2 z_h} + M\right)}{2\left(q^2 z_h^6 + i\omega z_h - 2\right)}\,. \tag{14}$$

Eqs. (8) and (9) are solved numerically for complex frequencies using a method that consists in imposing infalling boundary conditions for the electric field at the horizon. These conditions are obtained by expressing the expansion (12) in terms of the field E near the horizon:

$$\lim_{z \to z_h} E_j(z) \longrightarrow e^{-i\omega r_*(z) + \frac{B_j(z)}{2}}\left(1 + a_j^{(1)}(z - z_h) + a_j^{(2)}(z - z_h)^2 + \ldots\right). \tag{15}$$

This expansion leads to the following boundary conditions for the field and it's derivative at the horizon:

$$E_j(z_h) = e^{-i\omega r_*(z_h) + \frac{B_j(z)(z_h)}{2}}\,, \tag{16}$$

$$E'_j(z_h) = \left(-i\omega r'_*(z_h) + \frac{B'_j(z_h)}{2} + a_j^{(1)}\right) E_j(z_h)\,. \tag{17}$$

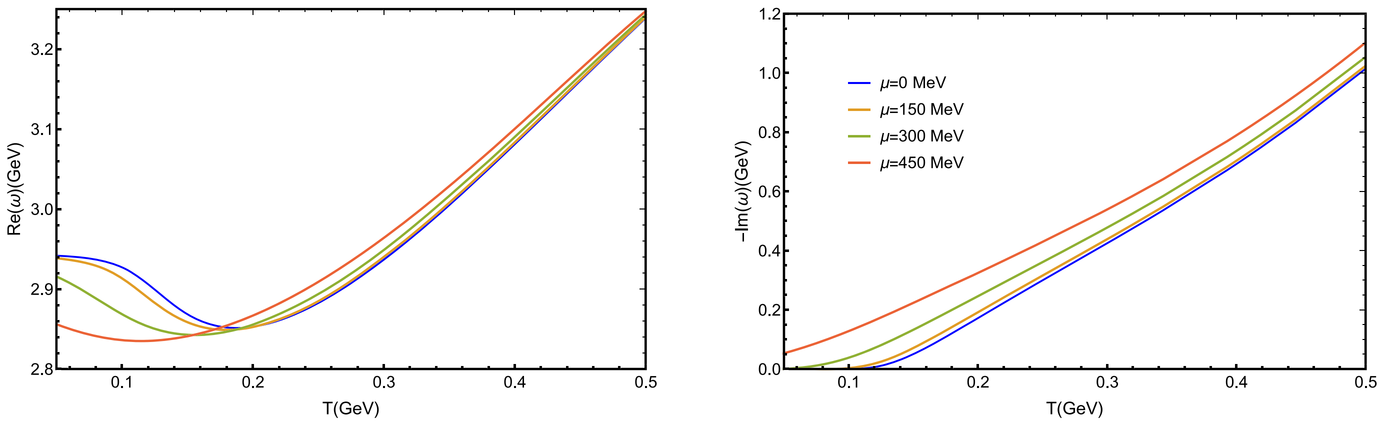

Fig. 1. Real (left panel) and imaginary (rigth panel) parts of the quasinormal mode frequency of J/ψ as a function of the temperature for four different values of the chemical potential: $\mu = 0, 150, 300, 450$ MeV.

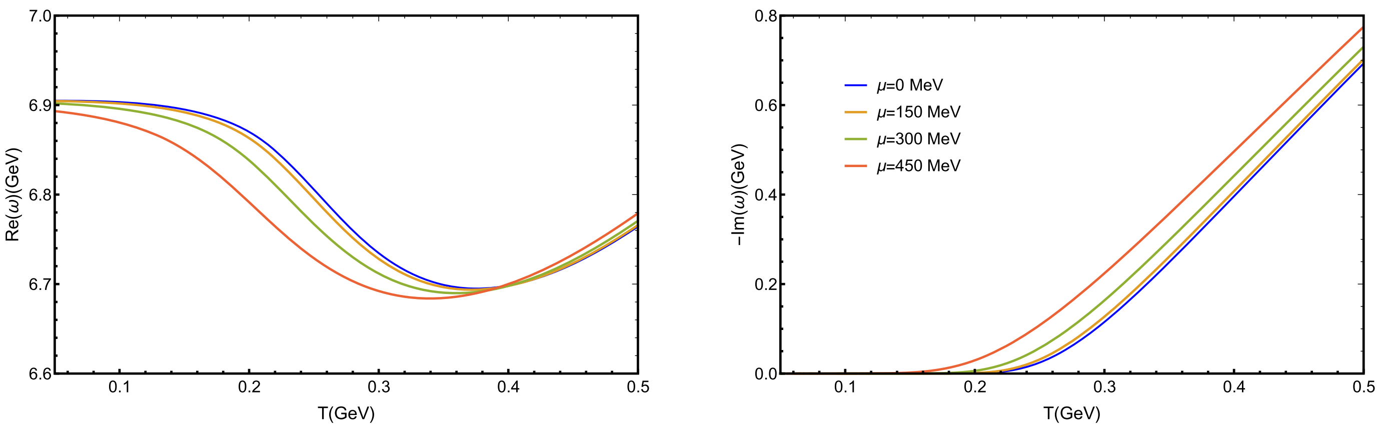

Fig. 2. Real (left panel) and imaginary (right panel) parts of the quasinormal mode frequency of Υ as a function of the temperature for four different values of the chemical potential: $\mu = 0, 150, 300, 450$ MeV.

Then one searches for the complex frequencies that provide solutions vanishing on the boundary: $E_j(z = 0) = 0$. These are the quasinormal frequencies and the corresponding – normalizable – solutions are the quasinormal modes, that represent the heavy meson quasi-states in the thermal medium.

Results

Let us start with heavy mesons at rest in the medium. Figs. 1 and 2 present, for the first states of charmonium J/ψ and bottomonium Υ, respectively, the dependence of the real and imaginary parts of the quasinormal frequencies on the temperature at four different values of the chemical potential μ.

The behavior of charmonium and bottomonium is similar. As one can see on the left panels, the thermal mass represented by Re(ω) decreases with the chemical potential for low temperatures. For higher temperatures, $T \gtrsim 200$ MeV for J/ψ and $T \gtrsim 400$ MeV for Υ, the effect of the chemical potential is the opposite but much smaller.

The imaginary part of the frequency Im(ω) is related to the width of the associated peak of the spectral function. An increase in Im(ω) corresponds to a broadening of the peak. On the other hand, the broadening of a peak corresponds to a smaller probability of finding the quasiparticle in the medium because of the dissociation. The decrease in this probability is what we call as an increase in the "dissociation degree". For both charmonium and bottomonium ground states there is a monotonic increase in the absolute value of Im(ω) with temperature and chemical potential. This means that, the denser is the plasma, the higher is the dissociation degree of heavy vector mesons.

Now, let us consider mesons in motion in the plasma. Fig. 3 show the dependence of the charmonium J/ψ quasinormal frequencies at T = 125 MeV on the momentum k. The upper panels represent the case when the motion is transverse to the polarization and the lower panels show the case when the momentum is longitudinal to the polarization. Fig. 4 shows the analogue results for the bottomonium Υ quasi-state at the temperature T = 300 MeV. The reason for choosing these temperatures is that the corresponding spectral functions, for charmonium at T = 125 MeV and for bottomonium at T = 300 MeV, present clear peaks for the first quasi-state. So, at these temperatures it is simple to observe the effect of the density.

Both flavors present a similar behavior. The real parts of the QNM modes represent for $k = 0$ the thermal mass. The dependence on k is approximately equal to the trivial dispersion relation for a free relativistic particle: $w = \sqrt{m^2 + k^2}$. For the imaginary parts of the QNM, the dispersion relation is shown on the panels on the right-hand side of Figs. 3 and 4. For both flavors the dissociation considerably increases with the chemical potential but the effect of the motion of the meson inside the plasma depends on the direction of the momentum with respect to the polarization. For motion transverse to polarization the dissociation degree increases with the momentum k while for

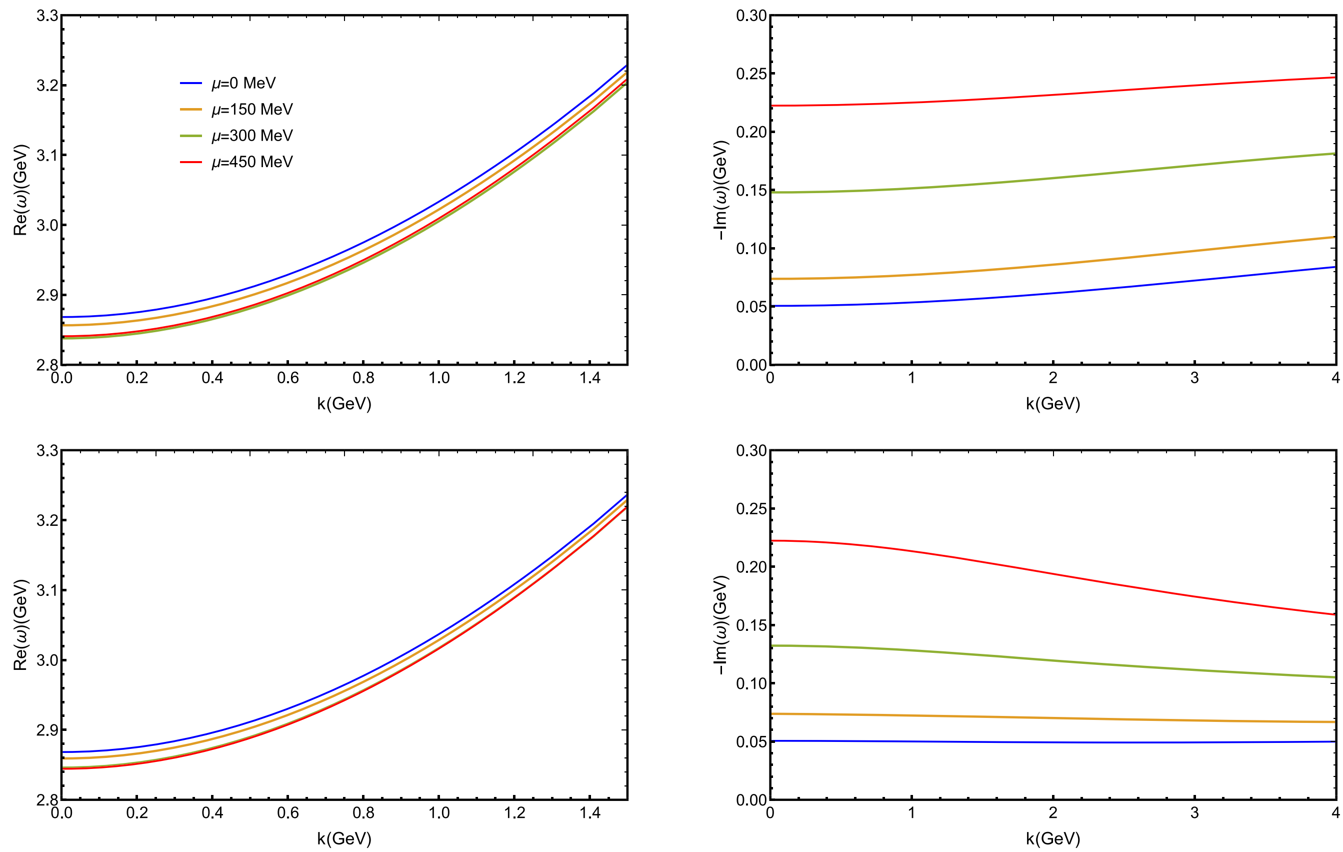

Fig. 3. Dispersion relations for J/ψ QNM at T = 125 MeV. The upper panels show the transverse ($j = 1, 2$) case and the bottom panels the longitudinal ($j = 3$) one. The real part of the frequencies are on the left panels and the imaginary part on the right panels.

longitudinal motion it decreases with the momentum. This effect is more noticeable for higher chemical potentials, when the dissociation caused by the density is partially compensated by longitudinal motion.

4. Comparison with spectral function results

Quasinormal modes (QNM) and spectral functions are complementary approaches for studying the thermal behavior of heavy vector mesons in a plasma. It is interesting to check if the results obtained here using QNM are consistent with the spectral function description, using the same holographic approach, presented in Ref. [8]. It is known that an increase in the imaginary part of the quasinormal frequencies corresponds to a broadening of the corresponding peak of the thermal spectral function. Such a qualitative analysis is trivial and indicates consistency between the two approaches. Much more interestingly, it is possible to make a quantitative test of consistency. In the vicinity of the peaks, the spectral function calculated from the imaginary part of the current-current retarded propagator has the approximate Breit Wigner (BW) form:

$$\frac{\rho(\omega)}{\omega} \approx \frac{a(\Gamma/2)^2}{(\omega - M)^2 + (\Gamma/2)^2}, \tag{18}$$

where the height of the peak a, the thermal mass M and the width Γ can be determined by the numerical adjust of the spectral function $\rho(\omega)$ (see for example [15] for a discussion).

Fig. 5 shows the spectral functions calculated from the model considered here for charmonium at $T = 125$ MeV (left panel) and bottomonium at $T = 300$ MeV (right panel). For details on how to determine these spectral functions, see Ref. [8].

It is clear from these plots that the height of the peaks decrease with the chemical potential. In order to illustrate how the BW approximation works, we show in Fig. 6 the BW fit and the actual peak for charmonium at T = 125 MeV and $\mu = 150$ MeV One notices that near the peak the BW approximation works very well. As mentioned in the previous section, the reason for choosing these temperatures is that the corresponding spectral functions, for charmonium at T = 125 MeV and for bottomonium at T = 300 MeV, present very clear peaks for the first quasi-states. So, at these temperatures it is simple to observe the effect of the density.

Performing a numerical fit of spectral functions, near the first peak, to the Breit Wigner (BW) form (18) one can test the agreement with the quasinormal modes approach. The comparison comes from the identification of the thermal mass M with the real part of the QNM frequency $\mathrm{Re}(\omega)$ and the resonance width Γ with twice the absolute value of the imaginary part: $2|\,\mathrm{Im}(\omega)|$.

Tables 1 and 2 summarize the results for QNM frequencies calculated using the method presented at section 3 and the results for the corresponding quantities obtained from a Breit-Wigner fit of the spectral functions peaks. One notices that the relative discrepancies are very small and then concludes that the results of quasinormal modes and spectral functions are consistent. They represent complementary approaches to study the thermal behavior of quarkonium inside a plasma.

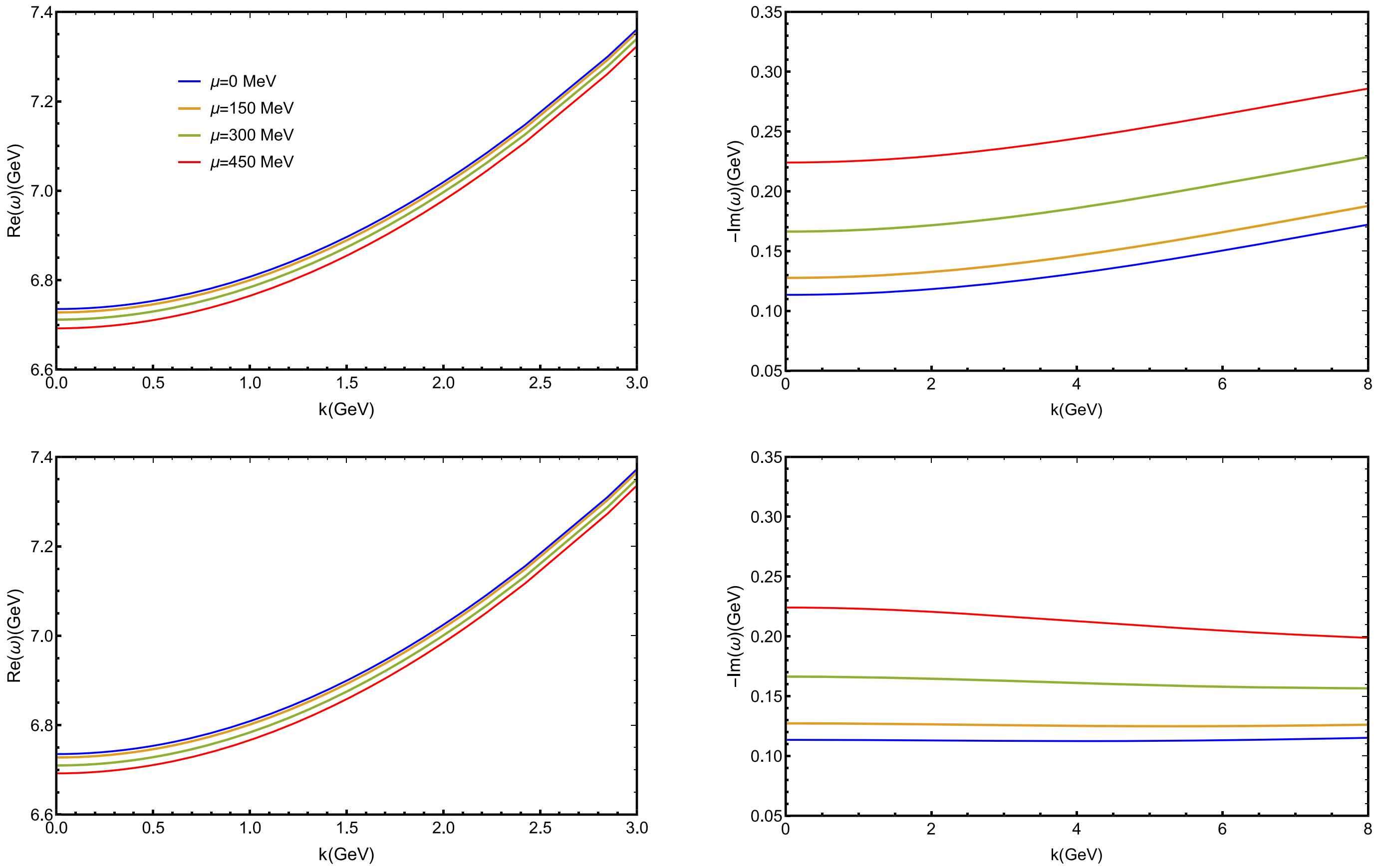

Fig. 4. Dispersion relations for Υ QNM at T = 300 MeV. The upper panels show the transverse ($j = 1, 2$) case and the bottom panels the longitudinal ($j = 3$) one. The real part of the frequencies are on the left panels and the imaginary part on the right panels.

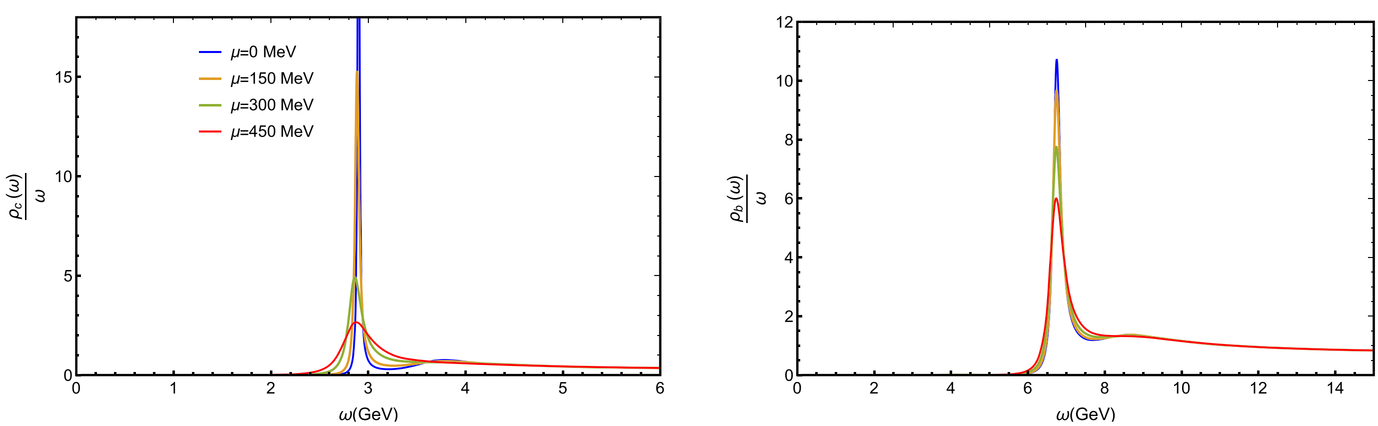

Fig. 5. Spectral functions for charmonium at T = 125 MeV and bottomonium at T = 300 MeV.

It is important to remark that similar results are obtained if one chooses different temperatures. The choice of T = 125 MeV for charmonium and T = 300 MeV for bottomonium was motivated by the fact that at these temperatures the effect of the density is more evident. However the same kind of agreement between the spectral function and quasinormal mode approaches is observed at different temperatures.

5. Conclusions

In this letter the behavior of heavy vector mesons inside a plasma with finite chemical potential was studied through the determination of the quasinormal modes. A holographic model was used, in order to describe the quasi-states of $c\bar{c}$ and $b\bar{b}$ in terms of normalizable field solutions in a five dimensional dual space. This dual geometry contains a black hole with a Hawking temperature assumed to be the same as the gauge theory temperature. The chemical potential, or density, of the medium is represented in the holographic description, by the charge of the black hole.

The results obtained show how the density affects the partial thermal dissociation and the thermal mass of the quarkonia in the medium. In particular, the higher the density, the higher the dissociation degree. This fact holds for heavy mesons at rest or in motion

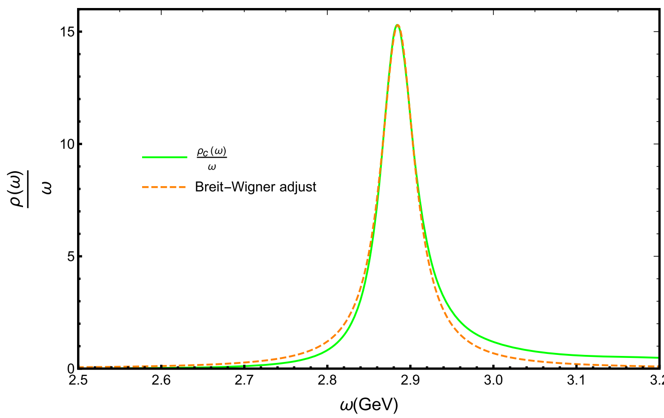

Fig. 6. Breit-Wigner fit compared with the actual spectral function peak for charmonium at T = 125 MeV and μ = 150 MeV.

Table 1
Real part of the quasinormal frequency Re(ω), compared with the thermal mass M obtained from Breit-Wigner fit of the spectral function and their relative discrepancy.

Quasi-state	μ (MeV)	T (MeV)	Re(ω) (MeV)	M (MeV)	Rel. discr. (%)
J/ψ	0	125	2.89931	2.88558	1.8
J/ψ	150	125	2.88270	2.86565	0.54
J/ψ	300	125	2.85034	2.88491	0.076
J/ψ	450	125	2.83546	2.89992	0.021
Υ	0	300	6.73452	6.75621	0.32
Υ	150	300	6.72812	6.75463	0.39
Υ	300	300	6.71170	6.74551	0.50
Υ	450	300	6.69218	6.74412	0.78

Table 2
Absolute value of the imaginary part of the quasinormal frequency $|\mathrm{Im}(\omega)|$, compared with one half of the thermal width Γ obtained from Breit-Wigner fit of the spectral function and their relative discrepancy.

Quasi-state	μ (MeV)	T (MeV)	$|\mathrm{Im}(\omega)|$ (MeV)	$\Gamma/2$ (MeV)	Rel. discr. (%)
J/ψ	0	125	0.0108540	0.0108923	0.35
J/ψ	150	125	0.0246513	0.0248886	0.96
J/ψ	300	125	0.0805289	0.0804919	0.046
J/ψ	450	125	0.173714	0.173152	0.32
Υ	0	300	0.115284	0.115169	0.10
Υ	150	300	0.127202	0.13211	3.8
Υ	300	300	0.163493	0.164364	0.53
Υ	450	300	0.224074	0.229888	2.6

relative to the medium. However, there is a non trivial aspect. For motion in the direction perpendicular to the polarization (transverse motion) the dissociation degree increases with momentum. In contrast, for motion in the direction of polarization (longitudinal motion), the dissociation degree decreases with momentum. So, for this longitudinal case, motion and density have opposite effects.

Regarding the thermal mass, there is also a non trivial behavior. For low temperatures ($T \lesssim 200$ MeV for J/ψ and $T \lesssim 400$ MeV for Υ) the thermal mass decreases with the chemical potential. For higher temperatures, the mass increases slightly with the chemical potential. It is important to note that the plasma at T = 125 MeV considered here would be a super-cooled plasma phase if the critical deconfinement temperature is above this value.

There are many interesting previous studies of mesons using holography to describe thermal effects and heavy flavors like for example [16–35]. An alternative approach to describe heavy meson spectra was proposed very recently in Ref. [36].

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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