THREE-BODY LEPTONIC DECAYS OF K_1^0 AND K_2^0, $\Delta I = \frac{1}{2}$ RULE AND $\Delta S = \Delta Q$ RULE

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(presented by F. S. Crawford)

We first make a table showing ΔS, ΔQ, ΔI and ΔI_z of the strongly interacting particles K and π, in the three-body leptonic (L) decays. Here L stands for either muon or electron. Also, the notation will not distinguish between neutrinos and anti-neutrinos (or ν_e and ν_μ !)

Decay	ΔS	ΔQ	$(\lvert\Delta I\rvert, \lvert\Delta I_z\rvert)$
$K^+ \to \pi^0 L^+ \nu$	-1	-1	(1/2, 1/2) or (3/2, 1/2)
$K^0 \to \pi^- L^+ \nu$	-1	-1	(1/2, 1/2) or (3/2, 1/2)
$K^0 \to \pi^+ L^- \nu$	-1	$+1$	(3/2, 3/2)
$\bar{K}^0 \to \pi^+ L^- \nu$	$+1$	$+1$	(1/2, 1/2) or (3/2, 1/2)
$\bar{K}^0 \to \pi^- L^+ \nu$	$+1$	-1	(3/2, 3/2) .

Notice that $\Delta S = -\Delta Q$ comes only from (3/2, 3/2). If one assumes CP invariance and $K_1^0 = (K^0 + \bar{K}^0)/\sqrt{2}$ and $K_2^0 = (K^0 - \bar{K}^0)/\sqrt{2}$, one finds the following predictions by means of, for instance, a spurion-type calculation:

Spurion present:			$\dfrac{\Gamma_2(L^+ + L^-)}{\Gamma_+(L^+)}$	$\dfrac{\Gamma_1(L^+ + L^-)}{\Gamma_2(L^+ + L^-)}$
(1/2, 1/2)	(3/2, 1/2)	(3/2, 1/2)		
yes	no	no	2	1
yes	yes	no	$\neq 2$	1
yes	yes	yes	$\neq 2$	$\neq 1$

In our experiment [2] we produce K^0 through $\pi^- + p \to \Lambda + K^0$ in the 72-inch Alvarez chamber. We demand various fiducial criteria, and also use only neutral K decays between $t = 0.2$ and 20.0×10^{-10} sec. The mean decay distance for K_1^0 is about 5 cm. The mean decay distance for K_2^0 is about 3500 cm. The mean potential path for neutral K is about 100 cm.

We end up with the following numbers of events:

Number	Λ Decay	K Decay
~1800	$p\pi^-$	invisible
~900	$p\pi^-$	$\pi^+\pi^-$
23	$p\pi^-$	$\pi^\pm L^\mp \nu$
4	$p\pi^-$	$\pi^+\pi^-\pi^0$

Of the 23 leptonic decays, 14 occur beyond $4K_1^0$ mean lives. We calculate [2] that according to the prediction of the first line of the table and the known rates of $K^+ \to \pi^0 L^+ \nu$, we should have seen 25 counts, instead of the 14 actually seen. (We are sure that we are not missing K_2^0 decays; we second scan all single Λ^0 decays into $p\pi^-$, along the direction predicted for the neutral K; also, we find that three-body decays are not misidentified as normal K_1^0 decays into $\pi^+\pi^-$.) We calculate about one chance in 50 that this is a statistical fluke. We therefore do not believe that the $\Delta I = 1/2$ rule holds for leptonic K^0 decays. That is, we rule out the top line in the table of predictions, because we find $\Gamma_2(L^+ + L^-)/\Gamma_+(L^+)$ is about 1/1 instead of 2/1.

In order to distinguish between lines 2 and 3 in the table of predictions, we must measure

$$\Gamma_1(L^+ + L^-)/\Gamma_2(L^+ + L^-)$$

This is more difficult than measuring the absolute K_2^0 rate, since there are several ways in which a normal K_1^0-decay into $\pi^+\pi^-$ can, through some " anomaly ", fake a three-body decay. By imposing cut-offs, we believe we have eliminated such fake events. (See question by G. Snow and answer by Crawford after Crawford's talk in Plenary Session VII.)

In order to measure

$$\Gamma_1/\Gamma_2 \equiv \Gamma_1(L^+ + L^-)/\Gamma_2(L^+ + L^-) \ ,$$

we must look at the time distribution of the events. Table I gives the details on each event. The events are arranged in increasing time order. From a likelihood analysis we find $\Gamma_1/\Gamma_2 = 6.6^{+6.0}_{-4.0}$. The meaning of this result, in terms of the probability that the relation $\Gamma_1 = \Gamma_2$ holds, can be expressed very simply. If we normalize our distribution to the 22 non-cut-off leptonic decay events of Table I, then if $\Gamma_1 = \Gamma_2$, we predict 2.96 counts in the first two K_1^0 mean lives. Instead we find 6 counts there. The probability for at least 6 counts is 6.5%. Thus there is a 6.5% chance that $\Delta S = \Delta Q$ holds and our results are a statistical fluke.

Notice also that (from Table I), six of the 22 leptonic decays also fit decay into $\pi^+\pi^-\gamma$, as well as into $\pi L\nu$. If we take them all to be $\pi\pi\gamma$ and remove them from the sample, the predicted number of counts in the first two K_1^0 mean lives is 2.15 (for $\Gamma_1 = \Gamma_2$); observed is 4. Clearly more statistics is needed. We have more data on film and hope to triple the number of counts within several months.

In summary, we are already convinced (by 50 to 1 odds) that $\Delta I = 1/2$ does not hold for K_2^0 three-body leptonic decays. We suggest (on the basis of 15 to 1 odds) that $\Delta S = -\Delta Q$ is at least partly responsible.

We do not believe the data can be fitted with (3/2, 1/2) and (3/2, 3/2) *alone*. (See remark by Crawford after talk by d'Espagnat in Plenary Session XII.)

Our result for Γ_1/Γ_2 can be compared with the previously published result of Ely *et al.* [3], who obtain

TABLE I

Summary of twenty-seven three-body decays. (The χ^2 probability is for the 1-constraint three-body decay, and corresponds to the probability for a χ^2 value at least as large as that actually obtained in the kinematical fit to the decay mode being tested. The χ^2 probability (prob) distribution for the « best interpretation » on each event agrees excellently with that expected for 1-constraint. For instance, 11/27 have prob <0.5, 2/27 have prob<0.1, 0/27 have prob <0.01.)

Serial No.	$P_{\rm Lab}(K^0)$ (MeV/c)	$l\,(K^0)$ cm	$t\,(K^0)$ 10^{-10}sec	χ^2 probabilities for decays					
				$\pi^-e^+\nu$	$\pi^+e^-\nu$	$\pi^-\mu^+\nu$	$\pi^+\mu^-\nu$	$\pi^+\pi^-\pi^0$	$\pi^+\pi^-\gamma$
781 208	530.8± 9.6	1.14	0.36	0.56	0.16	0.17	0.61	0	0.56
766 317	649.3± 4.9	2.24	0.57	0.0025	0	0.018	0.002	0	0
704 248	423.2± 7.2	2.12	0.83	0.068	0	0.20	0.95	0	0
735 269	556.8± 7.6	3.55	1.06	0.20	0	0	0	0	0
713 256	243.7± 4.8	2.12	1.44	0	0	0.96	0	0	0
707 247	399.9± 4.1	3.54	1.47	0.95	0.15	0.28	0.65	0	0.05
692 228	521.9± 5.8	7.71	2.45	0.95	0.28	0.054	0.001	0	0
521 330	575.7±12.0	9.17	2.64	0.61	0.015	0.99	0.42	0	0.70
564 309	468.3± 9.5	7.85	2.78	0.29	0.19	0.20	0.21	0	0.05
505 231	566.2± 8.0	11.84	3.47	0	0.0044	0.36	0	0	0
781 181	359.2± 4.6	8.38	3.87	0	0	0	0.23	0	0
773 547	452.8± 4.7	11.03	4.04	0.42	0.99	0.84	0.54	0	0
503 063	596.8± 7.0	20.95	5.82	0	0.17	0	0	0	0
853 275	486.0± 9.4	17.72	6.03	0	0	0	0	0.68	0
714 067	559.2± 8.8	21.80	6.46	0	0	0.15	0	0	0.44
722 026	481.7± 5.0	19.76	6.78	0.0024	0	0.59	0.27	0	0.05
559 553	633.8±10.9	26.02	6.81	0	0	0	0	0.084	0
522 599	597.1±13.6	24.59	6.83	0	0	0	0	0.61	0
565 027	442.1± 5.7	19.33	7.25	0.85	0.019	0.0017	0	0	0
819 009	602.1± 4.3	26.70	7.35	0.44	0.88	0.015	0.005	0	0
554 595	353.8± 3.1	22.76	10.69	0	0	0	0	0.25	0
568 280(*)	598.3±10.9	40.72	11.29	0	0.0014	0	0.76	0	0
525 293	291.2± 3.8	20.65	11.76	0	0	0.17	0.14	0	0
774 147	404.4± 5.0	32.42	13.29	0.77	0.024	0	0	0	0
532 310	621.1±14.1	56.93	15.20	0.014	0.47	0.022	0	0	0
756 453	273.9± 3.1	26.46	16.02	0	0.36	0.99	0.086	0	0
844 335	606.1±11.0	60.80	16.63	0	0.39	0.62	0	0	0

(*) This event is cut-off as a "Coulomb scatter" in the experiment to find $\Gamma_1\,(L^\pm)/\Gamma_2\,(L^\pm)$, but is accepted as a K_2 decay in the determination of $\Gamma_2\,(L^\pm)$.

$\Gamma_1(e^{\pm})/\Gamma_2(e^{\pm}) = 11.9^{+7.5}_{-5.6}$. The principle differences between their experiment and ours are the following:

(a) Their K^0 come from K^+ charge exchange in propane. Ours come from $\pi+p\to\Lambda+K^0$ in hydrogen. Their K^0 momentum is known typically to $\pm 25\%$, ours to $\pm 1.5\%$. The uncertainty in proper time is proportional to the uncertainty in K^0 momentum.

(b) Their events are all e^+ and e^-, selected by various criteria, especially range versus momentum. These selection criteria result in a detection efficiency of roughly 20%. Thus they cannot obtain an absolute decay rate. Our events are selected by kinematics alone (the stopping power of the liquid hydrogen is too small, or the 72-inch chamber too small, for us to determine the mass value, on most tracks). Consequently we often have an ambiguity among the four leptonic modes. As is evident from Table I, with a larger sample we could make a statistical separation of the four modes. At present we do not try this. On the other hand, our detection efficiency is essentially 100%, so that we obtain absolute rates as well as ratios.

(c) For a given value of Γ_1/Γ_2 one has two solutions $x = x'$ and $x = x'' = 1/x'$, where x is the ratio of the amplitude for $\Delta S = -\Delta Q$ to that for $\Delta S = +\Delta Q$. Because of our charge ambiguity we cannot decide between the two solutions. Ely *et al.* find $x = 0.55$ and exclude $x = 1.82$.

Decay of K_2^0 into $\pi^+\pi^-\pi^0$ and the $\Delta I = 1/2$ Rule for Non-Leptonic Decays [2)]

According to the $\Delta I = 1/2$ rule, the decay rate $\Gamma_2(+-0)$ of K_2^0 into $\pi^+\pi^-\pi^0$ is related to the decay rate $\Gamma_+(+00)$ of K^0 into $\pi^+\pi^-\pi^0$ by the relation [4)] $\Gamma_2(+-0) = 1.032\times 2\Gamma_+(+00)$. Here the factor 1.032 is due to phase space. The beauty of this formula is that it holds for all three of the $I = 1$ three-pion final states, so that one need not inquire into the separate question as to whether, for instance, the symmetric $I = 1$ state predominates, as is often assumed.

Using this formula and the known K^+ lifetime and branching ratios we calculate [2)] the predicted rate $\Gamma_2(+-0) = 2.87\pm 0.23\times 10^6$ sec^{-1}. From this we would predict 4.3 events $K_2\to\pi^+\pi^-\pi^0$ between $4K_1^0$ mean lives and 20.0×10^{-10} sec. We find 4 events (see Table I in paragraph 2), in perfect agreement with the prediction.

Another way to check this prediction, with better statistics on the number of $(+-0)$ decays, is to use the branching ratio of Luers *et al.* [5)] who find $\Gamma_2(+-0)/\Gamma_2(L^+ + L^-) = 0.155\pm 0.022$. If we combine this with our absolute decay rate for $\Gamma_2(L^+ + L^-)$, based on our 14 decays discussed in paragraph 2, we find a combined answer $\Gamma_2(+-0) = (1.44\pm 0.43)\times 10^6$ sec^{-1}. This is smaller than the predicted rate by a factor of 0.5, and differs from the predicted rate by 2.95 standard deviations. We thus have evidence that the $\Delta I = 1/2$ rule fails for $K\to 3\pi$. In this calculation we did not use the 4 decays into $(+-0)$ that we find in our own sample. The result using the branching ratio of Luers *et al.* would predict 2.2 decays $(+-0)$ in our sample, and this is not in disagreement with the 4 seen, (i.e. 4 ± 2).

Lastly I will calculate the total K_2 decay rate and thus obtain the K_2 lifetime, which will then be compared with the only direct measurement so far obtained (by attenuation of K_2's with distance) by Bardon *et al.* [6)]. They find $\tau_2 = 8.1^{+3.3}_{-2.4}\times 10^{-8}$ sec.

Our rate for K_2^0 decay into $\pi^-e^+\nu$, $\pi^+e^-\nu$, $\pi^-\mu^+\nu$, $\pi^+\mu^-\nu$, and $\pi^+\pi^-\gamma$ is 9.31×10^6 sec^{-1}, based on 14 events. Using this absolute rate and the branching ratio of Luers *et al.* [5)] we have $\Gamma_2(+-0) = 0.155\times 9.31\times 10^6$ sec^{-1}. We still need the rate $\Gamma_2(000)$ for K_2^0 decay into $3\pi^0$. Here we will use the branching ratio result reported at this conference by M. Anikina *et al.* [7)] (see p. 452). They find (from 28 Dalitz pairs and 4100 "usual" K_2^0 decays in a cloud chamber with magnetic field) $\Gamma_2(000)/\Gamma_2$ (all charged modes) $= 0.38\pm 0.07$. Using our absolute rate and also Luers' branching ratio value 0.155, (see above.), we find $\Gamma_2(000) = 0.38\times 1.55\times 9.31\times 10^6$ sec^{-1}. If we add all these rates we get

$$\Gamma_2\,(\text{total}) = (1+0.155+0.38\times 1.155)\times 9.31\times 10^6\ \text{sec}^{-1}$$

$$= (14.8\pm 4.1)\times 10^6\ \text{sec}^{-1}.$$

This gives a lifetime $\tau_2 = 6.8^{+2.6}_{-1.5}\times 10^{-8}$ sec., which is in good agreement with that of Bardon *et al.* Notice that if in our experiment [2)] we had found twice as many K_2^0 decays then we would agree with $\Delta I = 1/2$, but would have obtained $\tau_2 = 3.4\times 10^{-8}$ sec from the above calculation, in poor agreement with the lifetime of Bardon *et al.*

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DISCUSSION

d'ESPAGNAT: Now that we are convinced that the leptonic $\Delta I = {}^1/_2$ rule is not valid, what we would like to try is the assumption that we have only $\Delta I = {}^3/_2$ and no $\Delta I = {}^1/_2$ vector currents. What are the odds against having such a situation?

SACHS: I think that you can say that if you combine the results on $\Delta S/\Delta Q$ of the Berkeley/Wisconsin/Padua collaboration, with this result and, say, take the ratio of K_1 to K_2 partial rate to be about 9, as a nice round number, that you cannot possibly fit the result that you would like, that is that the $\Delta I = {}^1/_2$ amplitude is zero. You can fit, I believe, the possibility that the $\Delta I = {}^3/_2$, $\Delta I_3 = {}^1/_2$ amplitude is zero, but that is not much help.

CRAWFORD: I want to agree with the last statement about our data agreeing with $\Delta I = {}^3/_2$, ${}^1/_2$ component being zero. Using the two results, the ratio Γ_1 to Γ_2, and the Γ_2 absolute rate I believe we get a best value of about 0.2 for $({}^3/_2, {}^1/_2)$ over $({}^1/_2, {}^1/_2)$. There is no possibility of fitting with $({}^1/_2, {}^1/_2) = 0$.

SNOW: What about the plus to minus ratio in the small forward interval, near the production origin?

CRAWFORD: Our statistics are small. In order to use the plus to minus ratio you have to use events only in the first few mean lives otherwise the interference wiggles start to get too many radians ahead of you. In that time interval we only have about six events; also, if we use only the decay kinematics, each event is only about "60% unique". That is, unless the particle actually comes to rest or is so slow that it almost comes to rest, the events will usually fit more than one hypothesis. Therefore we have not tried to resolve the plus to minus ratio.

LAGARRIGUE: We have tried to measure directly the mean life of the K_2^0 in a propane-freon chamber, observing all types of decay of K_2^0 and we have got $(5.1^{+2.4}_{-1.3}) \times 10^{-8}$ sec.