

## 2D gravity on $AdS_2$ with Chern-Simons corrections

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**ABSTRACT:** We study 2D Maxwell-dilaton gravity with higher order corrections given by the Chern-Simons term. The model admits three distinctive  $AdS_2$  vacuum solutions. By making use of the entropy function formalism we find the entropy of the solutions which is corrected due to the presence of the Chern-Simons term. We observe that the form of the correction depends not only on the coefficient of the Chern-Simons term, but also on the sign of the electric charge; pointing toward the chiral nature of the dual CFT. Using the asymptotic symmetry of the theory as well as requiring a consistent picture we can find the central charge and the level of U(1) current. Upon uplifting the solutions to three dimensions we get purely geometric solutions which will be either  $AdS_3$  or warped  $AdS_3$  with an identification.

**KEYWORDS:** AdS-CFT Correspondence, 2D Gravity, Black Holes.

*Dedicated to Hessamaddin Arfaei on the occasion of his 60th birthday*

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## 1. Introduction

2D quantum gravity on  $AdS_2$  geometry is important due to its essential role in the context of black hole physics. Indeed the  $AdS_2$  geometry is the factor which appears in the near horizon geometry of extremal black holes in any dimension. Therefore understanding quantum gravity on  $AdS_2$  might ultimately help us understand the origin of the black hole entropy in other dimensions.

The main problem which prevents us to explain quantum gravity on  $AdS_2$  geometry is the fact that it is not quite clear what it actually means. Indeed this is the case for any dimension. An attempt to understand, or in better words, to make sense of quantum gravity in three dimensions has been made by Witten in [1] where it was argued that 3D quantum gravity makes sense only on  $AdS_3$ . The main reason supporting the argument is due to the existence of non-trivial three dimensional black holes, BTZ solutions, which carry non-zero entropy [2]. Being an AdS background it is natural to define the quantum gravity in terms of the dual CFT via AdS/CFT correspondence [3].

In 2D Maxwell-dilaton gravity there are several classical solutions with non-zero entropy which may be interpreted as 2D extremal black holes. Therefore we would expect to have non-trivial 2D quantum gravity on  $AdS_2$  geometry. Following the idea explored in [1] one may suspect that quantum gravity on  $AdS_2$  can be defined via its CFT dual. We note, however, that although  $AdS_{d+1}/CFT_d$  correspondence has been understood for  $d \geq 2$  mainly due to explicit examples, little has been known for the case of  $d = 1$  and indeed it remains enigmatic. Nevertheless there are several attempts to explore  $AdS_2/CFT_1$  correspondence, including [4–23].

The aim of the present article is to further study 2D gravity on  $AdS_2$  along the recent studies [16, 18, 21] where 2D Maxwell-dilaton gravity has been considered. To have consistent boundary conditions it was shown in [16] that the asymptotic symmetry of the model is generated by a twisted energy momentum tensor whose central charge is non-zero. This

central charge along with eigenvalue of  $L_0$  of the dual CFT can be used to *consistently* reproduce the entropy of the bulk gravity via the Cardy formula. This was taken as an evidence that the CFT dual to gravity on  $AdS_2$  should be *a chiral half of a 2D CFT*.

To elaborate the above statement we will consider 2D Maxwell-dilaton gravity in the presence of higher order corrections given by 2D Maxwell-gravitational Chern-Simons term. To have consistent boundary conditions one needs to work with the twisted energy momentum tensor, though in this case due to the presence of the Chern-Simons term the corresponding central charge gets a correction. An important observation is that the correction not only depends on the coefficient of the Chern-Simons term, but also it is sensible to the sign of the electric charge. The sign dependent effect should, indeed, be associated with the fact that the dual theory should be a chiral half of a 2D CFT.

To study the vacuum solutions of the model we should solve the equations of motion with a constant dilaton. Equivalently, we may utilize the entropy function formalism [24] by which we are also able to find the entropy of the corresponding solutions. From the equations of motion we find three distinctive  $AdS_2$  vacuum solutions. Using the asymptotic symmetry of the theory together with requiring to have a consistent picture we will be able to read the central charge of the corresponding solutions as well as the level of  $U(1)$  current.

The 2D solutions may be uplifted to three dimensions. The obtained 3D solutions are purely geometric solutions that will be either  $AdS_3$  or warped  $AdS_3$  with an identification. The warped  $AdS_3$  solution has recently been studied in [25] (see also [26–29]).

The paper is organized as follows. In the next section we will introduce our model where we apply the entropy function formalism to find the vacuum solutions as well as their entropy. Re-writing the entropy in a suggestive form, we will give an expression for the corrected central charge. In section 3 requiring to have consistent boundary conditions we will find the asymptotic symmetry of the theory which can be used to read the central charge. In section 4 we uplift the 2D solutions to three dimensions which may be compared with 3D solutions in [25]. The last section is devoted to discussions.

## 2. 2D Maxwell-dilaton gravity with Chern-Simons term

Let us consider 2D Maxwell-dilaton gravity with the action

$$S = S_{EH} + S_{CS} \quad (2.1)$$

where  $S_{EH}$  is the Einstein-Hilbert action

$$S_{EH} = \frac{1}{8G} \int d^2x \sqrt{-g} e^\phi \left( R + 2\partial_\mu \phi \partial^\mu \phi + \frac{2}{l^2} e^{2\phi} - \frac{l^2}{4} F_{\mu\nu} F^{\mu\nu} \right), \quad (2.2)$$

and  $S_{CS}$  is the two dimensional Chern-Simons term given by

$$S_{cs} = -\frac{1}{32G\mu} \int d^2x \left( lR\epsilon^{\mu\nu} F_{\mu\nu} + l^3 \epsilon^{\mu\nu} F_{\mu\rho} F^{\rho\delta} F_{\delta\nu} \right). \quad (2.3)$$

The action  $S_{EH}$  can actually be obtained from the 3D pure gravity with cosmological constant by reducing to two dimensions along an  $S^1$  [4]. Similarly one may start from 3D

gravitational Chern-Simons term and reduce along an  $S^1$  to arrive at the 2D Chern-Simons  $S_{cs}$  [30, 31]. From three dimensional point of view these actions have been used to study the entropy of extremal black holes in the presence of higher order corrections (see for example [32]).

The aim of this section is to study the vacuum solutions of the model given by the action (2.1) which can be obtained by solving its equations of motion. In fact, setting  $F_{\mu\nu} = \sqrt{-g}\epsilon_{\mu\nu}F$ , the equations of motion are given by

$$g_{\mu\nu} \left( \nabla^2 e^\phi + \frac{1}{l^2} e^{3\phi} - \frac{l^2 F^2}{4} e^\phi + e^\phi \partial_\mu \phi \partial^\mu \phi \right) - \nabla_\mu \nabla_\nu e^\phi - 2e^\phi \partial_\mu \phi \partial_\nu \phi - \frac{l}{2\mu} \left[ g_{\mu\nu} \left( \nabla^2 F - l^2 F^3 - \frac{R}{2} F \right) - \nabla_\mu \nabla_\nu F \right] = 0 \quad (2.4)$$

$$R + \frac{6}{l^2} e^{2\phi} + \frac{l^2}{2} F^2 + 2e^\phi \partial_\alpha \phi \partial^\alpha \phi - 4\nabla^2 e^\phi = 0, \quad \epsilon^{\mu\nu} \partial_\mu \left( e^\phi F + \frac{1}{2\mu l} (R + 3l^2 F^2) \right) = 0 \quad (2.5)$$

It is useful to work with trace and traceless parts of the equation (2.4)

$$\nabla^2 e^\phi + \frac{2}{l^2} e^{3\phi} - \frac{l^2 F^2}{2} e^\phi = \frac{l}{2\mu} (\nabla^2 F - 2l^2 F^3 - RF) \quad (2.6)$$

$$g_{\mu\nu} (\nabla^2 e^\phi + 2e^\phi \partial_\alpha \phi \partial^\alpha \phi) - 2(\partial_\mu \phi \partial_\nu \phi + \nabla_\mu \nabla_\nu e^\phi) = \frac{l}{2\mu} (g_{\mu\nu} \nabla^2 F - 2\nabla_\mu \nabla_\nu F) \quad (2.7)$$

This model admits  $AdS_2$  vacuum solutions. To find them we should look for solutions with a constant dilaton. In this case one has

$$\frac{2}{l^2} e^{3\phi} - \frac{l^2 F^2}{2} e^\phi = -\frac{l}{2\mu} (2l^2 F^3 + RF), \quad R + \frac{6}{l^2} e^{2\phi} + \frac{l^2}{2} F^2 = 0, \quad (2.8)$$

which, for a given gauge field, can be solved to find the constant dilaton. Indeed, these equations reduce to the following equation for dilaton

$$\left( e^\phi - \frac{3l}{2\mu} F \right) \left( \frac{2}{l^2} e^{2\phi} - \frac{l^2}{2} F^2 \right) = 0. \quad (2.9)$$

Therefore, for arbitrary  $\mu$  and  $l$ , the model may have three different vacuum solutions with constant dilaton given by

$$e^\phi = \pm \frac{l^2}{2} F, \quad e^\phi = \frac{3}{\mu l} \frac{l^2}{2} F. \quad (2.10)$$

It is worth noting that, as it is evident from the above expressions, in the special case of  $\mu l = 3$  the third solution degenerates with the first one (the positive sign above). We will come back to this point later.

To find the whole solutions we need to plug these expressions for the dilaton into the equations of motion and solved for metric and gauge field. Equivalently, since the solutions we are looking for are  $AdS_2$ , one may utilize the entropy function formalism [24]. An advantage of the entropy function formalism is that with this method we can not only find the solutions, but also we can read the entropy of the corresponding solutions.

To proceed let us start from an ansatz preserving  $SO(1, 2)$  symmetry of the  $AdS_2$  solution

$$ds^2 = v \left( -r^2 dt^2 + \frac{dr^2}{r^2} \right), \quad e^\phi = u, \quad F_{01} = \frac{e}{l^2}. \quad (2.11)$$

The entropy function is given by

$$\mathcal{E} = 2\pi[qe - f(e, v, u)] \quad (2.12)$$

where  $f(e, v, u)$  is the Lagrangian density evaluated for the above ansatz. The parameters  $e, v$  and  $u$  can be obtained by extremizing the entropy function with respect to them. Then the entropy is given by the value of the entropy function evaluated at the extremum.

Using the above ansatz, the entropy function for the action (2.1) reads

$$\mathcal{E} = 2\pi \left\{ qe - \frac{1}{8G} \left[ -2u + \frac{2u^3 v}{l^2} + \frac{e^2 u}{2vl^2} + \frac{1}{2\mu} \left( \frac{2e}{vl} - \frac{e^3}{v^2 l^3} \right) \right] \right\} \quad (2.13)$$

Extremizing the entropy function with respect to the parameters  $v, u$  and  $e$ , for generic  $\mu$  and  $l$  we find three different solutions

$$\begin{aligned} 1: \quad v &= \frac{1 + 1/\mu l}{-16Gq}, \quad e^{2\phi} = \frac{-4Gql^2}{1 + 1/\mu l}, \quad \frac{e}{l} = -\sqrt{\frac{1 + 1/\mu l}{-16qG}}, \quad q < 0, \\ 2: \quad v &= \frac{1 - 1/\mu l}{16Gq}, \quad e^{2\phi} = \frac{4Gql^2}{1 - 1/\mu l}, \quad \frac{e}{l} = \sqrt{\frac{1 - 1/\mu l}{16qG}}, \quad q > 0, \\ 3: \quad v &= \frac{1}{8Gq\mu l}, \quad e^{2\phi} = \frac{72Gq\mu l^3}{\mu^2 l^2 + 27}, \quad \frac{e}{l} = \sqrt{\frac{\mu l}{2Gq(\mu^2 l^2 + 27)}}, \quad q > 0. \end{aligned} \quad (2.14)$$

The entropy of the corresponding solutions written in a suggestive form is given by

$$\begin{aligned} 1: \quad S &= 2\pi \sqrt{\frac{-ql^2}{6} \frac{3}{2G} \left( 1 + \frac{1}{\mu l} \right)}, \\ 2: \quad S &= 2\pi \sqrt{\frac{ql^2}{6} \frac{3}{2G} \left( 1 - \frac{1}{\mu l} \right)}, \\ 3: \quad S &= 2\pi \sqrt{\frac{ql^2}{6} \frac{12\mu l}{G(\mu^2 l^2 + 27)}}, \end{aligned} \quad (2.15)$$

which may be compared with the Cardy formula for the entropy  $S = 2\pi \sqrt{\frac{L_0}{6}} c$ . Following the general philosophy of the AdS/CFT correspondence [3] if we assume that the 2D gravity on the  $AdS_2$  solutions (2.14) has a dual CFT, it is then natural to identify  $ql^2$  with the eigenvalue of  $L_0$  of the dual CFT. Then the central charges of the corresponding CFTs read

$$1: \quad c_R = \frac{3}{2G} \left( 1 + \frac{1}{\mu l} \right), \quad 2: \quad c_L = \frac{3}{2G} \left( 1 - \frac{1}{\mu l} \right), \quad 3: \quad c_L = \frac{12\mu l}{G(\mu^2 l^2 + 27)}. \quad (2.16)$$

If correct, this means that the 2D Maxwell-dilaton gravity on  $AdS_2$  background (2.14) is dual to a chiral half of a 2D CFT characterized by the above central charges. We note,

however, that since the identification of  $L_0 = ql^2$  was speculative, the above presentation cannot be considered as an argument supporting  $AdS_2/CFT_1$  correspondence. The best we can say is that as far as the entropy is concerned, with this identification, the picture seems self consistent. It is worth noting that for the case of  $\mu \rightarrow \infty$  where the effect of the Chern-Simons term is zero, we recover the known results in the literature (see for example [18, 21]) legitimating our identifications. In the next section we will present another calculation supporting self consistency of the above picture.

The indices  $L, R$  in equations (2.16) refer to the fact that, depending on the sign of  $q$ , the dual chiral CFT is left or right handed. Moreover, as we have already mentioned  $\mu l = 3$  is a special point. Indeed at this point the solution (3) degenerates with solution (2) where we get

$$c_R = \frac{2}{G}, \quad c_L = \frac{1}{G}. \quad (2.17)$$

Another interesting point is  $\mu l = \pm 1$  where we have two solutions with following central charges

$$c_R = \frac{3}{G}, \quad c_L = \frac{3}{7G}, \quad (2.18)$$

In section 4 we will compare these results with the solutions of 3D gravity coupled to Chern-Simons term.

### 3. Asymptotic symmetry and central charge

In this section we closely follow [16] to study 2D Maxwell-dilaton quantum gravity on the three different  $AdS_2$  backgrounds in (2.14). We will see in order to have consistent boundary conditions the usual conformal diffeomorphism, generated by the energy momentum tensor of (2.1), must be accompanied by a  $U(1)$  gauge transformation. As a result we will have to work with a twisted energy momentum whose central charge is non-zero [16]. We note, however, that although we would expect to get three different central charges for three solutions in (2.14), since all the solutions are obtained from the same action, (2.1), the procedure as well as the expressions for different quantities must be universal.

To proceed we note that the  $AdS_2$  vacuum solutions, setting  $r = \frac{1}{\sigma}$ , can be recast to the following form

$$ds^2 = -4v \frac{dt^+ dt^-}{(t^+ - t^-)^2}, \quad A_{\pm} = -\frac{e}{2\sigma l^2}, \quad u = \eta = \text{constant}, \quad (3.1)$$

where  $t^{\pm} = t \pm \sigma$  and  $v, e, u$  are given in (2.14).

Now the aim is to study 2D quantum gravity whose vacuum is given by either of the above solutions. To do so, we first need to understand the action of the conformal group on the theory. For this purpose, following the standard procedure in 2D CFT, we choose an appropriate gauge for the metric and the gauge field. For the metric we choose the conformal gauge

$$ds^2 = -e^{2\rho} dt^+ dt^- \quad (3.2)$$

and for the gauge field the Lorentz gauge

$$\partial_+ A_- + \partial_- A_+ = 0 \quad (3.3)$$

In this gauge the gauge field can be written as  $A_\pm = \pm \partial_\pm a$ , for a scalar field  $a$ , such that  $F_{+-} = -2\partial_+ \partial_- a$ . Our gauge choice fixes the coordinates and U(1) gauge field up to residual conformal and gauge transformations generated by

$$t^\pm \rightarrow t^\pm + \zeta^\pm(t^\pm), \quad a \rightarrow a + \theta(t^+) - \tilde{\theta}(t^-) \quad (3.4)$$

In this gauge the action (2.1) reads

$$S_{GF} = \frac{1}{4G} \int d^2t \left[ -2\partial_- \eta \partial_+ \rho + \frac{e^{2\rho}}{2l^2} \eta^3 - 2\frac{\partial_+ \eta \partial_- \eta}{\eta} + \frac{l^2}{2} e^{-2\rho} \eta (F_{+-})^2 \right] \\ - \frac{l}{4G\mu} \int d^2t [2e^{-2\rho} \partial_+ \partial_- \rho F_{+-} + l^2 e^{-4\rho} (F_{+-})^3] \quad (3.5)$$

This action should be accompanied by the equations of motion for the fields that have been fixed by the gauge choice. These show up as the following constraints

$$\frac{2}{\sqrt{-g}} \frac{\delta S}{\delta g^{\pm\pm}} \equiv T_{\pm\pm} = \frac{1}{4G} \left( -2\partial_\pm \rho \partial_\pm \eta + \partial_\pm \partial_\pm \eta + 2\frac{\partial_\pm \eta \partial_\pm \eta}{\eta} \right) \\ + \frac{l}{8G\mu} \left( -2\partial_\pm \rho \partial_\pm F + \partial_\pm \partial_\pm F \right) = 0 \quad (3.6)$$

$$- \frac{\delta S}{\delta A_\pm} \equiv G_\mp = \pm \frac{l^2}{8G} \partial_\mp (\eta F) + j_\mp = 0 \quad (3.7)$$

where

$$j_\pm = \mp \frac{l}{16G\mu} \partial_\pm (8e^{-2\rho} \partial_+ \partial_- \rho + 3l^2 F^2), \quad \text{with } \partial_- j_+ + \partial_+ j_- = 0. \quad (3.8)$$

On the other hand since we require no current flow out of the boundary one should impose the condition  $j_\sigma|_{\sigma=0} = 0$  which, using the equation (3.7), we find

$$j_\sigma = j_+ - j_- = -\frac{l^2}{8G} \partial_t (\eta F) = 0, \quad \text{at } \sigma = 0. \quad (3.9)$$

As a result, the boundary terms in the variation of the action will vanish if<sup>1</sup>

$$\partial_t a|_{\sigma=0} = A_\sigma|_{\sigma=0} = 0 \quad (3.10)$$

In general the boundary condition (3.10) is not preserved by the remaining allowed diffeomorphisms and hence the coordinate transformations should be accompanied by appropriate gauge transformations [16]

$$\theta(t^+) = \frac{e}{2l^2} \partial_+ \zeta^+, \quad \tilde{\theta}(t^-) = -\frac{e}{2l^2} \partial_- \zeta^-. \quad (3.11)$$

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<sup>1</sup>The constraints (3.7) together with (3.8) and the boundary condition  $j_\sigma|_{\sigma=0} = 0$ , completely determine  $j$ . Indeed from the variation of the action with respect to  $a$  we find a boundary term as  $\partial_+ (\eta F) \delta a$  which must be zero at the boundary. On the other hand due to (3.9) we are led to  $\delta a|_{\sigma=0} = 0$ . This forces a Dirichlet boundary condition for the field  $a$ .

Therefore the improved conformal transformations are generated by the twisted energy momentum tensor

$$\tilde{T}_{\pm\pm} = T_{\pm\pm} \mp \frac{e}{2l^2} \partial_{\pm} \mathcal{G}_{\pm}, \quad (3.12)$$

where  $\mathcal{G}_{\pm}$  is the current that generates the gauge transformations (3.11). Denoting by  $k$  the level of U(1) current which parameterizes the gauge anomaly due to the Schwinger term, the central charge of the model reads

$$c = 3k \frac{e^2}{l^4}. \quad (3.13)$$

The main challenge is to find the level of U(1),  $k$ . In general it can be obtained by making use of the anomaly calculations [33, 34]. We note, however, that it can be fixed using the known solutions. In particular for the case of  $\mu \rightarrow \infty$  where the theory is given by the first action in (2.1), the central charge is found to be  $\frac{3}{2G}$  [18, 21]. Equating this value with the central charge in (3.13) and using the first or second solution in (2.14) in the limit of  $\mu \rightarrow \infty$  one finds  $k = 8|q|l^2$ . Plugging this back into the equation (3.13) we get

$$1) \ c_R = \frac{3}{2G} \left(1 + \frac{1}{\mu l}\right), \quad 2) \ c_L = \frac{3}{2G} \left(1 - \frac{1}{\mu l}\right), \quad 3) \ c_L = \frac{12\mu l}{G(\mu^2 l^2 + 27)}, \quad (3.14)$$

in agreement with our consistent results in the previous section, (2.16).

#### 4. Relation to 3D gravity

In this section we would like to compare our 2D solutions with those in 3D Einstein-Chern-Simons gravity which have recently been studied in [25]. To do so, we note that the two dimensional  $AdS_2$  solutions (2.14) may be uplifted to three dimensions.

In general if we start from a 2D solution

$$ds_2^2 = g_{\mu\nu} dx^{\mu} dx^{\nu}, \quad e^{\phi}, \quad A_{\mu}, \quad (4.1)$$

which we assume to be symmetric under an isometry group  $\mathcal{G}$ , we can find a pure geometric 3D gravity solution

$$ds_3^2 = e^{2\phi} \left[ ds_2^2 + \left( dy + l A_{\mu} dx^{\mu} \right)^2 \right] \quad (4.2)$$

with isometry  $\mathcal{G} \times \text{U}(1)$ . Here  $y$  is a coordinate that parameterizes an  $S^1$  with period  $2\pi l$ .

In particular consider the case where the two dimensional solution is  $AdS_2$ . The isometry group of the solution is  $\text{SL}(2, R)$ .<sup>2</sup> Being symmetric under  $\text{SL}(2, R)$  group the solution has constant dilaton and  $F_{tr}$ . By uplifting the solution to three dimensions we find a pure geometric solution whose isometry is  $\text{SL}(2, R) \times \text{U}(1)$ ; the obtained solution will be  $S^1$  fibered over  $AdS_2$ . In other words, in light of the recent terminology, the solution may be thought of as *warped*  $AdS_3$  [25]. For particular values of the radius of the  $AdS_2$  space and field strength, the resultant three dimensional solution describes a locally  $AdS_3$  solution.

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<sup>2</sup>The solution may have extra symmetries. For example for the solutions (2.14) we have U(1) gauge symmetry as well.



However globally it is  $AdS_3$  with an identification. The effect of this identification is that the isometry group of  $AdS_3$ ,  $SL(2, R) \times SL(2, R)$ , breaks to  $SL(2, R) \times U(1)$ , as mentioned above.

Applying the above procedure to the solutions (2.14) we get

$$\begin{aligned}
 1: \quad ds^2 &= \frac{l^2}{4} \left( -r^2 dt^2 + \frac{dr^2}{r^2} + (dz - rdt)^2 \right), & q < 0, \\
 2: \quad ds^2 &= \frac{l^2}{4} \left( -r^2 dt^2 + \frac{dr^2}{r^2} + (dz + rdt)^2 \right), & q > 0, \\
 3: \quad ds^2 &= \frac{9l^2}{\mu^2 l^2 + 27} \left( -r^2 dt^2 + \frac{dr^2}{r^2} + \frac{4\mu^2 l^2}{\mu^2 l^2 + 27} (dz + rdt)^2 \right), & q > 0,
 \end{aligned} \tag{4.3}$$

where  $z = l\theta/|e|$  with the identification  $z \sim z + 2\pi l n \frac{l}{|e|}$ . Here  $n$  is an integer.

We note, however, that the above description must be considered with special care. It is known that the asymptotic symmetry of the  $AdS_2$  is a copy of the Virasoro algebra whose global part is an  $SL(2, R)$  [4]. This is, indeed, the generalization of  $AdS_3$  where the asymptotic symmetry is two copies of the Virasoro algebra with  $SL(2, R)_L \times SL(2, R)_R$  global part [35]. It is crucial to note that, in general, the global part of the Virasoro algebra of  $AdS_2$  geometry is not necessarily the  $SL(2, R)$  symmetry which only leaves the metric invariant. Indeed as we have seen in the previous section the asymptotic symmetry of  $AdS_2$  solutions of (2.14) is given by the twisted energy momentum tensor. Now uplifting the solutions to three dimensions the resultant  $SL(2, R)$  must be read from the twisted energy momentum tensor. In other words, if we denote the left/right handed energy momentum tensor of the three dimensional theory by  $T_{\pm\pm}^{(3)}$ , one should identify  $T_{\pm\pm}^{(3)} = \tilde{T}_{\pm\pm}$  [4].

Since in two dimensions the theory is chiral, upon uplifting the theory to three dimensions we only get non-zero excitations for one hand. In other words, depending on whether the two dimensional solution is left/right handed we will have left/right handed three dimensional energy momentum tensor. Actually from 3D point of view, as we have already mentioned, due to the identification the excitation states live purely in  $SL(2, R)_L$  or  $SL(2, R)_R$  factor of the isometry group.

On the other hand as we have seen in the previous section the 2D twisted energy momentum tensor has non-zero central charge given by (2.16). Therefore the corresponding central charge of the dual CFT of the three dimensional solutions is given by

$$1: \quad c_R = \frac{3l}{2G_3} \left( 1 + \frac{1}{\mu l} \right), \quad 2: \quad c_L = \frac{3l}{2G_3} \left( 1 - \frac{1}{\mu l} \right), \quad 3: \quad c_L = \frac{12\mu l^2}{G_3(\mu^2 l^2 + 27)}, \tag{4.4}$$

where  $G_3$  is 3D Newton constant. Of course, although the theory we get has excitations of only one hand, the other sector exists but has zero excitations. Thus the 2D CFT dual to the above 3D solutions has both  $c_L$  and  $c_R$ . Using the diffeomorphism anomaly by which we have  $c_L - c_R = -3/\mu G_3$  [36], one finds

$$1: \quad c_L = \frac{3l}{2G_3} \left( 1 - \frac{1}{\mu l} \right), \quad 2: \quad c_R = \frac{3l}{2G_3} \left( 1 + \frac{1}{\mu l} \right), \quad 3: \quad c_R = \frac{15\mu^2 l^2 + 81}{\mu G_3(\mu^2 l^2 + 27)}, \tag{4.5}$$

in agreement with [32] and [25].

As we have already mentioned the  $z$  coordinate in solutions (4.3) is periodic. Therefore one may interpret the solutions as 3D extremal black holes. Due to the identification the  $SL(2, R)_L/SL(2, R)_R$ -invariant  $AdS_3$  vacuum should give a thermal state for the left/right movers of the boundary CFT with zero right/left temperature and non-zero left/right temperature. On the other hand the  $t$  direction can be treated as the null coordinate of the boundary, while the  $z$  should be considered as a Rindler coordinate. Therefore the left/right temperature of the dual CFT is proportional to the magnitude of the shift in  $z$  direction. More precisely, one gets<sup>3</sup>

$$1 : T_R = \frac{2l}{\pi} \sqrt{\frac{-qG_3}{l(1 + \frac{1}{\mu l})}}, \quad 2 : T_L = \frac{2l}{\pi} \sqrt{\frac{qG_3}{l(1 - \frac{1}{\mu l})}}, \quad 3 : T_L = \frac{2l}{\pi} \sqrt{\frac{qG_3(\mu^2 l^2 + 27)}{8\mu l^2}}. \quad (4.6)$$

The corresponding entropy using the Cardy formula  $S = \frac{\pi^2}{3} c_{L/R} T_{L/R}$  reads

$$\begin{aligned} 1 : S &= 2\pi \sqrt{\frac{-ql^2}{6} \frac{3l}{2G_3} \left(1 + \frac{1}{\mu l}\right)}, \\ 2 : S &= 2\pi \sqrt{\frac{ql^2}{6} \frac{3l}{2G_3} \left(1 - \frac{1}{\mu l}\right)}, \\ 3 : S &= 2\pi \sqrt{\frac{ql^2}{6} \frac{12\mu l^2}{G_3(\mu^2 l^2 + 27)}}, \end{aligned} \quad (4.7)$$

which are compatible with those we have found in section two from 2D point of view.

## 5. Conclusions

In this paper we have studied 2D Maxwell-dilaton gravity on  $AdS_2$  geometry in the presence of higher order correction given by Chern-Simons term. The model admits three distinctive  $AdS_2$  vacuum solutions characterized by the sign of the electric field. Using the entropy function formalism we have evaluated the entropy of the solutions. Note that in the leading order when the action is given by the Einstein-Hilbert action the model has only one solution. Adding the Chern-Simons term the solution gets corrections which depend on the coefficient of the Chern-Simons term as well as the sign of the electric charge leading to three different solutions. The sign dependent nature of the corrections may be associated with the fact that the dual CFT is believed to be chiral half of a 2D CFT.

When the coefficient of the Chern-Simons term is set to zero the solution (1) degenerates with solution (2) while the third one disappears. In other words, the solutions (1) and (2) are Einstein solutions while the last one is not. Of course for particular values of  $\mu l$  the third one degenerates with the solution (2) as well.

Following [16] we have studied the action of the conformal group in the model where we have seen that in order to have consistent boundary conditions we will have to work

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<sup>3</sup>Note that such a treatment for warped  $AdS_3$  is tricky due to its boundary. Nevertheless in writing the expression for this case we are encouraged by the fact that in this case the picture fits nicely as well.

with a twisted energy momentum tensor. The twisted energy momentum tensor has non-zero central charge which should be associated with the central charge of the dual CFT. Requiring to have a consistent picture we have been able to read the corresponding central charge as well as the level of  $U(1)$  current.

We have compared our solutions with those in 3D gravity by uplifting the solutions to three dimensions. The solutions (1) and (2) have been uplifted to a solution which is locally  $AdS_3$  though globally it is  $AdS_3$  with an identification. The third one has been uplifted to a solution which is known as warped  $AdS_3$  [25] with an identification. Due to the identification, the resultant 3D solutions may be thought of as 3D extremal black holes.

We have also determined the entropy of the extremal black holes which are given by the Cardy formula using the obtained central charges. The consistency of the results points toward the conjecture made in [25] where the authors proposed that the 3D gravity on the warped  $AdS_3$  geometry is dual to a 2D CFT with left and right hand central charges given by the third central charges in equations (4.4) and (4.5).

The  $AdS_3$  and warped  $AdS_3$  solutions in 3D gravity are believed to be dual to 2D CFTs with  $c_L$  and  $c_R$  given by equations (4.4) and (4.5). Therefore in general one might expect that from two dimensional point of view we should have got four solutions corresponding to four different sectors which are obtained from 3D  $AdS_3$  and warped  $AdS_3$ . But as we have seen in two dimensions only three of them can be realized. The missing one is the right handed sector of the warped  $AdS_3$  solution with central charge given by the third one of (4.5). This means that if we consider an extremal black hole in warped  $AdS_3$  there is only one possibility in which the left movers will survive. This is unlike an extremal black hole in  $AdS_3$  where it could be either left or right handed with non-zero excitations of left or right mover states, respectively.

It is worth mentioning that, as it was observed by the authors of [26], when we study asymptotic symmetry of the warped  $AdS_3$  solution one gets a copy of Virasoro algebra with central charge given by the third one of (4.5). This is exactly the one which cannot be realized from 2D point of view. It would be interesting to illustrate the physics behind this special behavior of the warped  $AdS_3$ .

It was shown in [37] that the TMG quantum mechanically makes sense only at  $\mu l = 1$  where we get chiral gravity. From 2D point of view although we have observed that  $\mu l = 1$  is a special point, it is not a priori clear why we should set  $\mu l = 1$  from 2D point of view. Indeed, there are several examples in string theory where we have extremal black holes which upon reduction to two dimensions we get an action very similar to that in (2.2). In these cases the coefficient of the Chern-Simons term usually is fixed by a topological number and the charges of black holes. As far as the black holes are concerned there are no conditions on the coefficient of the Chern-Simons term. It would be interesting to understand this point better

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## References

- [1] E. Witten, *Three-dimensional gravity revisited*, [arXiv:0706.3359](#).
- [2] M. Bañados, C. Teitelboim and J. Zanelli, *The Black hole in three-dimensional space-time*, *Phys. Rev. Lett.* **69** (1992) 1849 [[hep-th/9204099](#)].
- [3] J.M. Maldacena, *The large- $N$  limit of superconformal field theories and supergravity*, *Adv. Theor. Math. Phys.* **2** (1998) 231 [*Int. J. Theor. Phys.* **38** (1999) 1113] [[hep-th/9711200](#)].
- [4] A. Strominger, *AdS<sub>2</sub> quantum gravity and string theory*, *JHEP* **01** (1999) 007 [[hep-th/9809027](#)].
- [5] M. Cadoni and S. Mignemi, *Asymptotic symmetries of AdS<sub>2</sub> and conformal group in D = 1*, *Nucl. Phys. B* **557** (1999) 165 [[hep-th/9902040](#)].
- [6] J. Navarro-Salas and P. Navarro, *AdS<sub>2</sub>/CFT<sub>1</sub> correspondence and near-extremal black hole entropy*, *Nucl. Phys. B* **579** (2000) 250 [[hep-th/9910076](#)].
- [7] M.-I. Park and J.H. Yee, *Comments on 'Entropy of 2D black holes from counting microstates'*, *Phys. Rev. D* **61** (2000) 088501 [[hep-th/9910213](#)].
- [8] M. Cadoni and S. Mignemi, *Symmetry breaking, central charges and the AdS<sub>2</sub>/CFT<sub>1</sub> correspondence*, *Phys. Lett. B* **490** (2000) 131 [[hep-th/0002256](#)].
- [9] M. Cadoni, P. Carta, D. Klemm and S. Mignemi, *AdS<sub>2</sub> gravity as conformally invariant mechanical system*, *Phys. Rev. D* **63** (2001) 125021 [[hep-th/0009185](#)].
- [10] M. Astorino, S. Cacciatori, D. Klemm and D. Zanon, *AdS<sub>2</sub> supergravity and superconformal quantum mechanics*, *Ann. Phys. (NY)* **304** (2003) 128 [[hep-th/0212096](#)].
- [11] W.T. Kim, *AdS<sub>2</sub> and quantum stability in the CGHS model*, *Phys. Rev. D* **60** (1999) 024011 [[hep-th/9810055](#)].
- [12] C. Leiva and M.S. Plyushchay, *Conformal symmetry of relativistic and nonrelativistic systems and AdS/CFT correspondence*, *Ann. Phys. (NY)* **307** (2003) 372 [[hep-th/0301244](#)].
- [13] S. Hyun, W. Kim, J.J. Oh and E.J. Son, *Entropy function and universal entropy of two-dimensional extremal black holes*, *JHEP* **04** (2007) 057 [[hep-th/0702170](#)].
- [14] F. Correa, V. Jakubsky and M.S. Plyushchay, *Aharonov-Bohm effect on AdS<sub>2</sub> and nonlinear supersymmetry of reflectionless Poschl-Teller system*, [arXiv:0809.2854](#).
- [15] G. Kang, J.-i. Koga and M.-I. Park, *Near-horizon conformal symmetry and black hole entropy in any dimension*, *Phys. Rev. D* **70** (2004) 024005 [[hep-th/0402113](#)].
- [16] T. Hartman and A. Strominger, *Central charge for AdS<sub>2</sub> quantum gravity*, [arXiv:0803.3621](#).
- [17] A. Sen, *Entropy function and AdS<sub>2</sub>/CFT<sub>1</sub> correspondence*, *JHEP* **11** (2008) 075 [[arXiv:0805.0095](#)].
- [18] M. Alishahiha and F. Ardalan, *Central charge for 2D gravity on AdS<sub>2</sub> and AdS<sub>2</sub>/CFT<sub>1</sub> correspondence*, *JHEP* **08** (2008) 079 [[arXiv:0805.1861](#)].
- [19] R.K. Gupta and A. Sen, *AdS<sub>3</sub>/CFT<sub>2</sub> to AdS<sub>2</sub>/CFT<sub>1</sub>*, [arXiv:0806.0053](#).
- [20] M. Cadoni and M.R. Setare, *Near-horizon limit of the charged BTZ black hole and AdS<sub>2</sub> quantum gravity*, [arXiv:0806.2754](#).

- [21] A. Castro, D. Grumiller, F. Larsen and R. McNees, *Holographic description of  $AdS_2$  black holes*, *JHEP* **11** (2008) 052 [[arXiv:0809.4264](#)].
- [22] A. Sen, *Quantum entropy function from  $AdS_2/CFT_1$  correspondence*, [arXiv:0809.3304](#).
- [23] T. Morita, *Hawking radiation and quantum anomaly in  $AdS_2/CFT_1$  correspondence*, [arXiv:0811.1741](#).
- [24] A. Sen, *Black hole entropy function and the attractor mechanism in higher derivative gravity*, *JHEP* **09** (2005) 038 [[hep-th/0506177](#)].
- [25] D. Anninos, W. Li, M. Padi, W. Song and A. Strominger, *Warped  $AdS_3$  black holes*, [arXiv:0807.3040](#).
- [26] G. Compere and S. Detournay, *Semi-classical central charge in topologically massive gravity*, [arXiv:0808.1911](#).
- [27] D. Anninos, *Hopping and puffing warped Anti-de Sitter space*, [arXiv:0809.2433](#).
- [28] S. Carlip, S. Deser, A. Waldron and D.K. Wise, *Topologically massive AdS gravity*, *Phys. Lett. B* **666** (2008) 272 [[arXiv:0807.0486](#)].
- [29] G.W. Gibbons, C.N. Pope and E. Sezgin, *The general supersymmetric solution of topologically massive supergravity*, *Class. and Quant. Grav.* **25** (2008) 205005 [[arXiv:0807.2613](#)].
- [30] G. Guralnik, A. Iorio, R. Jackiw and S.Y. Pi, *Dimensionally reduced gravitational Chern-Simons term and its kink*, *Ann. Phys. (NY)* **308** (2003) 222 [[hep-th/0305117](#)].
- [31] D. Grumiller and W. Kummer, *The classical solutions of the dimensionally reduced gravitational Chern-Simons theory*, *Ann. Phys. (NY)* **308** (2003) 211 [[hep-th/0306036](#)].
- [32] B. Sahoo and A. Sen, *BTZ black hole with Chern-Simons and higher derivative terms*, *JHEP* **07** (2006) 008 [[hep-th/0601228](#)].
- [33] N.S. Manton, *The Schwinger model and its axial anomaly*, *Ann. Phys. (NY)* **159** (1985) 220.
- [34] T. Heinzl, S. Krusche and E. Werner, *Nontrivial vacuum structure in light cone quantum field theory*, *Phys. Lett. B* **256** (1991) 55 [*Nucl. Phys. A* **532** (1991) 429C].
- [35] J.D. Brown and M. Henneaux, *Central charges in the canonical realization of asymptotic symmetries: an example from three-dimensional gravity*, *Commun. Math. Phys.* **104** (1986) 207.
- [36] P. Kraus and F. Larsen, *Holographic gravitational anomalies*, *JHEP* **01** (2006) 022 [[hep-th/0508218](#)].
- [37] W. Li, W. Song and A. Strominger, *Chiral gravity in three dimensions*, *JHEP* **04** (2008) 082 [[arXiv:0801.4566](#)].