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The Curvature Perturbation and Non-Minimal Coupling

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We review aspects of the curvature perturbation generated during inflation. In particular we focus on the time dependence and non-Gaussianity of the curvature perturbation, as a non-trivial time dependence and deviation from Gaussianity represent two key signatures of inflationary models beyond the canonical single-field scenario. After reviewing these features in the context of General Relativity, we consider how the situation is changed on introducing non-minimal coupling. We pay particular attention to how the δN formalism and its application in calculating the correlation functions of the curvature perturbation are modified.

1. Introduction

It is now widely accepted that the density perturbations seeding temperature fluctuations in the cosmic microwave background (CMB) and the formation of large scale structure (LSS) find their origin in the quantum fluctuations of one or more scalar fields, stretched to super-horizon scales during a period of quasi-de Sitter expansion dubbed inflation [1–5]. The relevant quantity in determining the exact nature of the initial conditions that these quantum fluctuations give rise to is the gauge-invariant curvature perturbation on hypersurfaces of constant energy density, ζ. In the context of simple, single-field inflationary models, there are two very robust predictions regarding the properties of the curvature perturbation. Firstly, it is known to be conserved on super-horizon scales, not just at linear order but fully non-linearly [6]. Secondly, it is known that deviations from Gaussianity are unobservably small [7]. As such, when looking for signatures of models beyond simple, single-field inflation, we are particularly interested in any possible violation of the above two predictions. A time-dependence of the curvature perturbation generically indicates the presence of non-adiabatic perturbations, which in the context of inflation is synonymous with the presence of multiple fields. A non-negligible deviation from Gaussianity may be another signature of the presence of multiple fields, but could also indicate that the action for the single scalar field is non-canonical.

A non-minimal coupling of one or more scalar fields to the Ricci scalar offers one example of a non-canonical feature in the inflationary action. Indeed, such a non-minimal coupling is well motivated in the context of quantum field theory in curved spacetime or higher-dimensional unifying theories. Moreover, there are whole classes of inflationary models with non-minimal coupling whose predictions are known to be in very good agreement with observational constraints coming from WMAP and *Planck*. In this article we will review aspects of inflation models with non-minimal coupling, focusing in particular on the calculation of the curvature perturbation on hypersurfaces of constant energy density, its time dependence and non-Gaussianity. In calculating the non-Gaussianity of ζ, one particularly powerful method is the δN formalism, which allows one to calculate the non-linear curvature perturbation on super-horizon scales using knowledge of only the background dynamics [8–13]. The formalism is essentially independent of the model of gravity under consideration, but as we will see, there are some subtleties that come to light when we try to use the formalism to calculate

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correlation functions of ζ in models with non-minimal coupling. One of the aims of this review is to clarify these subtleties. Another focus of this article will be the relation between the curvature perturbation as defined in the so-called Jordan and Einstein frames, denoted by ζ and $\tilde{\zeta}$ respectively. In the single-field case it is known that $\zeta = \tilde{\zeta}$ non-perturbatively, which allows us to freely use the somewhat simpler Einstein frame formulation of this class of models. However, this is no longer true in the presence of multiple fields, meaning that we have to be more careful in determining exactly how ζ and $\tilde{\zeta}$ are related to observables. At least classically, the Jordan and Einstein frames are simply related by a relabelling of the metric, so predictions for observables should be independent of the frame in which we calculate them. The physical interpretation in each frame, on the other hand, may be very different. The issue of "frame dependence" is a longstanding one, and is still very relevant today, for example in the context of Higgs inflation.

The rest of this article will be structured as follows: In Sec. 2 we begin by reviewing properties of the curvature perturbation on hypersurfaces of constant energy density in the context of General Relativity (GR). We start by defining the gauge-invariant curvature perturbation and considering its time-dependence at linear order in perturbation theory. Moving beyond linear perturbations we then give a summary of the δN formalism as applied to single- and multi-field models of inflation. In Sec. 3 we introduce non-minimal coupling. After firstly motivating the introduction of non-minimal coupling, we briefly review the general formulation of models that contain non-minimal coupling and their application to inflation. Moving on to the curvature perturbation, ζ, we then take a look at how the linear and non-linear analyses of Sec. 2 are modified in the presence of non-minimal coupling, focusing in particular on the subtleties that arise when using the δN formalism to calculate the correlation functions of ζ. We also consider the relation between ζ as defined in the Jordan frame and $\tilde{\zeta}$ as defined in the Einstein frame. Finally we conclude in Sec. 4.

2. Curvature Perturbation

In this section we briefly review the properties of the curvature perturbation on hypersurfaces of constant energy density in the context of GR. We start by defining the gauge-invariant curvature perturbation on hypersurfaces of constant energy density at linear order and considering its time dependence. We then discuss how the δN formalism allows us to determine the curvature perturbation non-perturbatively.

2.1 *Linear Theory and Time-Dependence*

Following the notation of [14], assuming a spatially flat Friedmann-Lemaître-Robertson-Walker (FLRW) background and including only scalar perturbations, we take our perturbed metric to be of the form

$$\begin{aligned} ds^2 &= -(1+2AY)dt^2 - 2aBY_i dt dx^i \\ &\quad + a^2\left[(1+2\mathcal{R})\,\delta_{ij} + 2H_T\frac{1}{k^2}Y_{,ij}\right]dx^i dx^j, \end{aligned} \tag{2.1}$$

where perturbations have already been decomposed into Fourier modes using the scalar harmonic functions Y (k-indices suppressed) satisfying $(\nabla^2 + k^2)Y = 0$, $Y_i = -k^{-1}Y_{,i}$ and $Y_{,i} = \partial_i Y$.

Turning to the energy-momentum tensor, at background level it is taken to be of the perfect fluid form

$$T_{\mu\nu} = pg_{\mu\nu} + (\rho+p)u_\mu u_\nu, \tag{2.2}$$

and the perturbations are decomposed as

$$\begin{aligned} \delta T_{00} &= -\rho\delta g_{00} + \delta\rho Y, \\ \delta T_{0i} = \delta T_{i0} &= p\delta g_{0i} - \delta q Y_{,i} \quad \text{and} \\ \delta T_{ij} = \delta T_{ji} &= p\delta g_{ij} + a^2\left(\delta p Y\delta_{ij} + p\Pi_T Y_{ij}\right), \end{aligned} \tag{2.3}$$

where Π_T is the anisotropic stress perturbation, $\delta q = -(\rho+p)\delta u/k$, δu is the fluid velocity potential perturbation and Y_{ij} is defined as $Y_{ij} =$

$k^{-2}Y_{,ij} + \delta_{ij}Y/3$. Note that at this stage we do not restrict ourselves to the energy-momentum tensor associated with single- or multi-field inflationary models.

With the above decompositions, the curvature perturbation on hypersurfaces of constant energy density, ζ, is defined as

$$\zeta \equiv \mathcal{R} - \frac{H}{\dot{\rho}}\delta\rho. \tag{2.4}$$

As discussed in the introduction, we are particularly interested in the time-dependence of this quantity, or the lack thereof. Taking the time derivative of (2.4), and making use of the energy-momentum constraint equations, we find [15]

$$\begin{aligned}\dot{\zeta} \;=\; & -\frac{H}{\rho+p}\delta p_{\rm nad} + \left(\frac{k}{aH}\right)^2 \\ & \frac{H}{3}\left[\zeta - \left(\mathcal{R} - \frac{aH}{k}\sigma_g\right)\right. \\ & \left.\left(1+\left(\frac{k}{aH}\right)^2\frac{2\rho}{9(\rho+p)}\right)\right], \end{aligned} \tag{2.5}$$

where $\sigma_g = a\dot{H}_T/k - B$ and $\delta p_{\rm nad}$ is the non-adiabatic pressure perturbation defined as

$$\delta p_{\rm nad} = \delta p - \frac{\dot{p}}{\dot{\rho}}\delta\rho. \tag{2.6}$$

On super-horizon scales ($k \ll aH$) the above expression reduces to

$$\dot{\zeta} \simeq -\frac{H}{\rho+p}\delta p_{\rm nad}. \tag{2.7}$$

As such, we see that on super-horizon scales the curvature perturbation on hypersurfaces of constant energy density is conserved in the absence of non-adiabatic perturbations, i.e. perturbations that do not satisfy $\delta p = (\dot{p}/\dot{\rho})\delta\rho$.

On super-horizon scales, the above curvature perturbation on hypersurfaces of constant energy density is closely related to the comoving curvature perturbation, defined as the curvature perturbation on hypersurfaces where $\delta T^0{}_i = 0 \leftrightarrow \delta q = 0 \leftrightarrow \delta u = 0$, namely

$$\mathcal{R}_c = \mathcal{R} + \frac{H\delta q}{\rho+p} \tag{2.8}$$

To see this we note that, using two of Einstein's equations, it is possible to show that

$$\begin{aligned}\zeta - \mathcal{R}_c \;&=\; -\frac{H}{\dot{\rho}}\left(\delta\rho - 3H\delta q\right) \\ &=\; \frac{2\rho}{9(\rho+p)}\left(\frac{k}{aH}\right)^2\left(\mathcal{R} - \frac{aH}{k}\sigma_g\right), \end{aligned} \tag{2.9}$$

so that on super-horizon scales we have $\zeta \simeq \mathcal{R}_c$.

2.2 *Single- and Multi-Field Inflation*

In the case of single-field inflation, assuming a canonical kinetic term and potential V, one finds that the non-adiabatic pressure perturbation is given as $\delta p_{\rm nad} = -\frac{2V_\phi}{3H\dot{\phi}}\left(\delta\rho - 3H\delta q\right)$, where $V_\phi = \partial V/\partial\phi$. As such, using (2.9), we see that the comoving curvature perturbation is conserved on super-horizon scales. We also note that in the single-field case we have $\delta T^0{}_i \propto \delta q \propto \delta\phi$, meaning that the comoving curvature perturbation coincides with the curvature perturbation on hypersurfaces of constant ϕ. In the case of single-field inflation, this latter quantity in fact serves as a more appropriate definition of the comoving curvature perturbation when we wish to go beyond linear theory.

Moving on to multi-field inflation, let us consider a Lagrangian of the form

$$\mathcal{L}_\phi = -\frac{1}{2}h_{ab}(\vec{\phi})g^{\mu\nu}\partial_\mu\phi^a\partial_\nu\phi^b - V(\vec{\phi}), \tag{2.10}$$

where the symmetric matrix $h_{ab}(\vec{\phi})$ represents the field-space metric. Unlike in the single-field case, even on super-horizon scales the non-adiabatic pressure perturbation is non-vanishing, meaning that ζ is no longer conserved. Specifically, one finds

$$\delta p_{\rm nad} \simeq -\frac{2h_{ab}\dot{\phi}^a}{h_{ef}\dot{\phi}^e\dot{\phi}^f}h_{cg}\frac{D\dot{\phi}^g}{dt}\mathcal{K}^{bc}, \tag{2.11}$$

where D/dt denotes the covariant derivative with respect to the field-space metric $h_{ab}(\vec{\phi})$, $\simeq$ denotes the fact that the equality holds on super-horizon scales and $\mathcal{K}^{ab}$ is defined as

$$\mathcal{K}^{ab} \equiv \delta\phi^a\dot{\phi}^b - \delta\phi^b\dot{\phi}^a. \tag{2.12}$$

As such, we see that the curvature perturbation will only be conserved on super-horizon scales if the background trajectory follows a geodesic of the field-space, i.e. $D\dot{\phi}^a/dt \propto \dot{\phi}^a$. See e.g. [9, 11, 16–18] for more details on the covariant approach to multi-field inflationary dynamics.

From (2.11) we see that $\mathcal{K}^{ab}$ plays a key role in the non-conservation of ζ, and this quantity is closely related to the so-called isocurvature perturbations associated with multi-field inflationary models. As introduced by Sasaki and Tanaka [11], when considering a multi-field model of inflation with some background trajectory in field-space, one can decompose perturbations at any instant into components along and perpendicular to this background trajectory.[1] These are referred to as the instantaneous adiabatic and isocurvature perturbations respectively [20]. For adiabatic perturbations, which satisfy $\delta\phi^a \propto \dot{\phi}^a$, the quantity $\mathcal{K}^{ab}$ vanishes, meaning that it is only the isocurvature perturbations that source the evolution of ζ on super-horizon scales. In the single-field case the field-space is only one-dimensional, meaning that there are no such isocurvature perturbations.

2.3 *Non-Linear Extension and the δN Formalism*

In this section we present an overview of the so-called δN formalism, which allows one to calculate the non-linear curvature perturbation on super-horizon scales using knowledge of only the background dynamics [8–13]. There are essentially two important steps: the first is showing that $\delta N = \zeta$, where δN will be defined more precisely below, and the second is showing that under the separate-universe approximation Einstein's equations and the equations of motion for the fields take on exactly the same form as those for the background quantities. The important point will be to choose the correct time slicing.

[1]See [19] for an alternative decomposition recently suggested, where, in the context of the δN formalism, perturbations are decomposed into components along the trajectory and along hypersurfaces of constant e-folding number.

Proving that $\delta N = \zeta$

In order to prove the relation $\delta N = \zeta$ we do not need to consider any specific model of gravity or action. Let us write our metric in the ADM form

$$ds^2 = -\alpha^2 dt^2 + a^2 e^{2\mathcal{R}} \gamma_{ij}(dx^i + \beta^i dt)(dx^j + \beta^j dt), \tag{2.13}$$

where the determinant of γ_{ij} is unity. We introduce an expansion parameter $\epsilon = k/aH$, which we will attach to any spatial gradient of a quantity ∂_i. Taking $\epsilon \ll 1$ corresponds to smoothing out small-scale inhomogeneities, and we assume that in the limit $\epsilon \to 0$ our Universe becomes locally described by the FLRW metric

$$ds^2 = -dt^2 + a^2(t)\delta_{ij} dx^i dx^j, \tag{2.14}$$

where here we have already assumed our universe to be flat. In order for the metric (2.13) to reduce to the FLRW form in the limit $\epsilon \to 0$ we require $\beta^i \to 0$ and $\gamma_{ij} \to \delta_{ij}$. The second condition can be achieved by performing a coordinate transformation in the case that γ_{ij} is time-independent, but not in the case that it is time-dependent. This leads us to the conclusion $\dot{\gamma}_{ij} \sim \mathcal{O}(\epsilon)$. The first condition similarly leads us to the conclusion $\beta^i \sim \mathcal{O}(\epsilon)$.

We define an effective local Hubble rate in terms of the expansion of the unit time-like vector normal to surfaces of constant t, n^μ. Namely, we define $H \equiv \theta_n/3$, where $\theta_n \equiv \nabla_\mu n^\mu$. Using the above form of metric this gives us

$$\theta_n = \nabla_\mu n^\mu = \frac{3}{\alpha}(\bar{H} + \partial_t \mathcal{R}) + \mathcal{O}(\epsilon^2). \tag{2.15}$$

Note that we attach bars to all background quantities in this section, so $\bar{H} = \dot{a}/a$. This in turn allows us to define the local number of e-foldings along the integrated curve of the normal vector n^μ

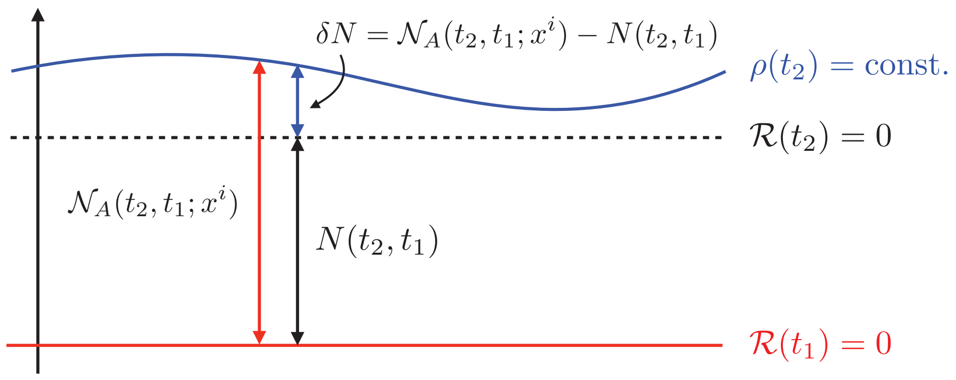

Fig. 1. Figure adapted from [21] depicting the method of the δN formalism.

$$\begin{aligned}\mathcal{N}(t_2, t_1; x^i) &\equiv \int_{t_1}^{t_2} H\alpha dt \\ &= N(t_2, t_1) + \mathcal{R}(t_2; x^i) - \mathcal{R}(t_1; x^i),\end{aligned} \tag{2.16}$$

where $\alpha dt = d\tau$ is the local time parameter and $N(t_1, t_2)$ is the background number of e-foldings between the times t_1 and t_2. The first thing that we notice is that if we take the flat slicing, where $\mathcal{R}(t; x^i) = 0$, then we recover the background number of e-foldings, namely $\mathcal{N}(t_2, t_1; x^i) = N(t_2, t_1)$.

Next consider taking two sets of initial and final slices labelled by A and B. We then get the relation

$$\begin{aligned}&\mathcal{N}_A(t_2, t_1; x^i) - \mathcal{N}_B(t_2, t_1; x^i) \\ &= \left(\mathcal{R}_A(t_2; x^i) - \mathcal{R}_B(t_2; x^i)\right) - \left(\mathcal{R}_A(t_1; x^i)\right. \\ &\quad \left. - \mathcal{R}_B(t_1; x^i)\right).\end{aligned} \tag{2.17}$$

If we specify that for A the initial slice is flat and the final slice is one of uniform energy density, and that for B both the initial and final slices are flat, then this gives us the relation

$$\zeta(t_2; x^i) = \mathcal{N}_A(t_2, t_1; x^i) - N(t_2, t_1) \equiv \delta N. \tag{2.18}$$

Diagrammatically the situation is shown in Fig. 1.

Reducing Everything to Background Dynamics

The next important step is to show that — on choosing an appropriate time coordinate — Einstein's equations and the equations of motion for the scalar fields at full non-linear level take on exactly the same form as those at background level to leading order in ϵ. This then leads us to the conclusion that the perturbed solutions can be derived from the background solutions by simply changing the initial conditions appropriately. In the following we will use the explicit form of energy-momentum tensor associated with the Lagrangian given in (2.10).

Using the ADM decomposition given in (2.13) one can derive an expression for the extrinsic curvature K_{ij}, which can in turn be decomposed into trace and trace-free parts as

$$K_{ij} = a^2 e^{2\mathcal{R}} \left(\frac{\gamma_{ij}}{3} K + A_{ij}\right), \tag{2.19}$$

where the indices of A_{ij} are raised and lowered with γ^{ij} and γ_{ij}. Taking β^i, ∂_i, $\dot{\gamma}_{ij} \sim \mathcal{O}(\epsilon)$ and making a perturbative expansion in ϵ, from the dynamical equation for γ_{ij} we are able to determine that $A_{ij} \sim \mathcal{O}(\epsilon)$. Using this result and taking the flat gauge, i.e. $\mathcal{R} = 0$, one then finds that the Hamiltonian constraint takes the form

$$3H^2 = \frac{1}{M_{Pl}^2}\left[\frac{H^2}{2}h_{ab}\phi_N^a\phi_N^b + V\right] + \mathcal{O}(\epsilon^2). \tag{2.20}$$

where $\phi_N^a = d\phi^a/dN$. We see that the form is exactly the same as the background Hamiltonian constraint. This is thanks to the fact that we have used N as the time coordinate, as this quantity remains unperturbed in the flat gauge $\mathcal{R} = 0$.

Similarly, turning to the equations of motion for the scalar fields, taking the flat gauge we find that they can be written as

$$H\frac{d}{dN}\left(H\phi_N^a\right) + 3H^2\phi_N^a + \Gamma_{bc}^a H^2\phi_N^b\phi_N^c + h^{ab}V_b + \mathcal{O}(\epsilon) = 0, \tag{2.21}$$

which is again of the same form as the background results, but this time only to order ϵ.

We can, however, go a step further. Using the fact that $A_{ij} \sim \mathcal{O}(\epsilon)$ and that the anisotropic stress of the energy-momentum tensor associated with (2.10) is vanishing, the equation of motion for A_{ij} in the flat gauge tells us that $A_{ij} \propto 1/a^3$. As we only have a decaying solution, this allows us to effectively ignore A_{ij} at the order of accuracy to which we are working, so that effectively we have $A_{ij} \sim \mathcal{O}(\epsilon^2)$. Substituting this result into the equation of motion for γ_{ij} leads us to the conclusion that $\dot\gamma_{ij} \sim \mathcal{O}(\epsilon^2)$ as opposed to $\mathcal{O}(\epsilon)$. As such, we find that the equations of motion for the scalar fields are of the same form as the background result up to $\mathcal{O}(\epsilon^2)$ rather than just $\mathcal{O}(\epsilon)$, namely

$$H\frac{d}{dN}\left(H\phi_N^a\right) + 3H^2\phi_N^a + \Gamma_{bc}^a H^2\phi_N^b\phi_N^c + h^{ab}V_b + \mathcal{O}(\epsilon^2) = 0. \tag{2.22}$$

Given the above results, we conclude that on a flat hypersurface, to leading order in ϵ we have

$$\phi^a(N, x^i) = \bar\phi^a(N, \phi_*^b(x^i), \phi_{N*}^c(x^i)). \tag{2.23}$$

That is, we can determine $\phi^a(N, x^i)$ by locally specifying different initial conditions, where $\phi_*^a(x^i) = \bar\phi_*^a + \delta\phi_{\mathcal{R}*}^a$ and similarly for $\phi_{N*}^a(x^i)$. For notational simplicity, let us introduce the vector

$$\phi^a = \begin{pmatrix} \phi^b \\ \phi_N^c \end{pmatrix}, \tag{2.24}$$

where $a = 1....2n$. In this notation we may simply write $\phi^a(N, x^i) = \bar\phi^a(N, \phi_*^a(x^i))$.

Given that we are ultimately interested in the curvature perturbation on hypersurface of constant energy density, let us turn our attention to the energy density. We follow the analysis given in [13]. The energy density on a flat hypersurface is not constant, but is given as $\rho(N, x^i) = \bar\rho(N, \phi_*^a(x^i))$. The energy density on a hypersurface of constant energy density is given as $\hat\rho(\mathcal{N}, x^i) = \bar\rho(\mathcal{N})$. However, due to the fact that the density is a four-scalar, we have the relation $\bar\rho(N, \phi_*^a(x^i)) = \bar\rho(\mathcal{N})$, which can be inverted to give us $\mathcal{N} = \mathcal{N}(N, \phi_*^a(x^i))$. Namely, we have

$$\begin{aligned}\mathcal{N}(t_2, t_*, x^i) &= N(t_2, t_*) + N_a\delta\phi_{\mathcal{R}*}^a \\ &\quad + \frac{1}{2}N_{ab}\delta\phi_{\mathcal{R}*}^a\delta\phi_{\mathcal{R}*}^b + ...\end{aligned} \tag{2.25}$$

where $N_a = \partial N/\partial\phi_*^a$ and similarly for N_{ab}. Inserting this result into (2.18) we finally find the well-known δN formula

$$\zeta(t, x^i) = \delta N = N_a\delta\phi_{\mathcal{R}*}^a + \frac{1}{2}N_{ab}\delta\phi_{\mathcal{R}*}^a\delta\phi_{\mathcal{R}*}^b + ... \tag{2.26}$$

The Correlation Functions of ζ

In comparing theoretical predictions with observations we are interested in the correlation functions of the curvature perturbation. Given the expansion (2.26) for ζ, and on expanding in terms of Fourier modes, we see that in order to be able to determine the correlation functions of ζ we must know the correlation functions of $\delta\phi_{\mathcal{R}*}^a$.

Starting with the power spectrum, and following the procedure outlined in [10], to lowest order in the slow-roll approximation we find[2]

$$\langle\delta\phi^a_{\mathcal{R}*}(\boldsymbol{k})\delta\phi^b_{\mathcal{R}*}(\boldsymbol{k}')\rangle = (2\pi)^3\delta^3(\boldsymbol{k}+\boldsymbol{k}')\frac{H_*^2}{2k^3}h_*^{ab} \equiv (2\pi)^3\delta^3(\boldsymbol{k}+\boldsymbol{k}')P(k)^{ab}, \quad (2.27)$$

where H_* and h_*^{ab} are evaluated at a time shortly after the two scales k and k' have left the horizon and

$$\mathcal{P}(k)^{ab} \equiv \frac{k^3}{2\pi^2}P(k)^{ab} = h_*^{ab}\left(\frac{H_*}{2\pi}\right)^2. \quad (2.28)$$

We therefore find

$$\mathcal{P}_\zeta(k) = N_aN_bh_*^{ab}\left(\frac{H_*}{2\pi}\right)^2. \quad (2.29)$$

In order to determine the spectral tilt, we again follow the procedure outlined in [10] and find

$$n_s - 1 = -2\epsilon - \frac{2}{N_aN^a} + \frac{2N^aN^b}{3H_*^2N_eN^e}\left[\nabla_a\nabla_bV + R_{acbd}\dot\phi^c\dot\phi^d\right]_*, \quad (2.30)$$

where $\epsilon = -\dot H/H^2$, indices are raised and lowered with h^{ab} and h_{ab}, respectively, and R_{abcd} is the Riemann tensor associated with h_{ab}.

Going beyond the power spectrum, if we consider the expansion (2.26) up to second order, then to leading order we find the three-point correlation function to be given as

$$\begin{aligned}&\langle\zeta(\boldsymbol{k}_1)\zeta(\boldsymbol{k}_2)\zeta(\boldsymbol{k}_3)\rangle\\ &= N_aN_bN_c\langle\delta\phi^a_{\mathcal{R}*}(\boldsymbol{k}_1)\delta\phi^b_{\mathcal{R}*}(\boldsymbol{k}_2)\delta\phi^c_{\mathcal{R}*}(\boldsymbol{k}_3)\rangle\\ &\quad+(2\pi)^3\delta^{(3)}(\boldsymbol{k}_1+\boldsymbol{k}_2+\boldsymbol{k}_3)\frac{6}{5}f_{NL}\left[N_aN^a\right]^2\\ &\quad\frac{H^4}{4}\frac{k_1^3+k_2^3+k_3^3}{k_1^3k_2^3k_3^3},\end{aligned} \quad (2.31)$$

where

$$f_{NL} = \frac{5}{6}\frac{N_aN_b\nabla_c\nabla_dNh^{ac}h^{bd}}{[N_eN^e]^2}. \quad (2.32)$$

[2]Note that here we are making the slow-roll approximation and neglecting the dependence of N on ϕ^a_N. This means here we only need the power spectra of $\delta\phi^a_{\mathcal{R}*}$ and not of $\delta\phi^a_{N\mathcal{R}*}$.

The first term in (2.31) corresponds to the intrinsic non-Gaussianity in the perturbations $\delta\phi^a_{\mathcal{R}*}$, resulting from non-linear interactions before horizon crossing. The second term corresponds to the non-Gaussianity generated due to the non-linear dependence of the evolution of super-horizon patches on the initial conditions of each of the fields. If we assume the initial perturbations $\delta\phi^a_{\mathcal{R}*}$ to be Gaussian, the first term vanishes and consequently f_{NL} parameterises the magnitude of the non-Gaussianity. In the case of single-field inflation it is known that f_{NL} is slow-roll suppressed, but this is no longer the case in multi-field inflation.

Before moving on, we close this section by commenting on the conservation of the non-linear curvature perturbation. In [6] it was shown that in the single-field case, provided that the system is in the attractor regime, where $\partial_\tau\phi = F(\phi)$ ($d\tau = \alpha dt$), then the uniform ϕ slicing coincides with the comoving slicing and the non-linear comoving curvature perturbation is found to be conserved.

3. Non-Minimal Coupling

So far, whilst some of our results relating to the curvature perturbation have been independent of the choice of gravity theory, many have explicitly assumed GR. In this section we review how the analysis presented in the previous section is modified if we consider models that contain a non-minimal coupling of one or more scalar fields to the Ricci scalar.

3.1 *Motivation, Formulation and Application to Inflation*

The general action under consideration takes the form

$$S = \int d^4x\sqrt{-g}\quad\left\{f(\vec\phi)R - \frac{1}{2}h_{ab}(\vec\phi)g^{\mu\nu}\partial_\mu\phi^a\partial_\nu\phi^b - V(\vec\phi) + \mathcal{L}^{(m)}(g_{\mu\nu},\psi)\right\}, \quad (3.1)$$

where the first term represents a non-minimal coupling of some function of one or more fields to the Ricci scalar and we have also allowed for additional, minimally coupled matter, represented by the field ψ. Such a form of action is theoretically well motivated:

- In the context of field theory in curved spacetime [22, 23], on dimensional grounds we should allow for terms of the form $\xi R\phi^2$ in our action, where ξ is a dimensionless constant. Even if such terms are not present in the original action, we would expect them to appear when we consider higher-order loop corrections.

- Mach's principle and Dirac's large-numbers hypothesis both led to the idea that the gravitational constant may be time dependent, which in turn led to the development of scalar-tensor theories of gravity [24–30].

- It is well known that $f(R)$ gravity — a natural extension of GR in which the Ricci scalar in the Einstein-Hilbert action is replaced by some general function of the Ricci scalar [31] — can be re-expressed as a scalar-tensor theory, where the scalar field captures the additional scalar degree of freedom associated with $f'(R)$ [32].

- Finally, in the context of higher-dimensional unifying theories, non-minimal coupling appears naturally when we derive the 4-dimensional effective actions via compactification, see e.g. [33].

Jordan Frame Analysis

The action in its original form (3.1) is said to be in the Jordan frame. Despite the non-minimal coupling, we are still able to express Einstein's equations in the form $G_{\mu\nu} = T^e_{\mu\nu}/M^2_{\rm Pl}$, but where $T^e_{\mu\nu}$ is now some effective energy-momentum tensor given as

$$T^e_{\mu\nu} = \frac{M^2_{pl}}{2f}\left[T^{(\phi)}_{\mu\nu} + T^{(m)}_{\mu\nu} + 2\nabla_\mu\nabla_\nu f - 2g_{\mu\nu}\Box f\right], \tag{3.2}$$

where $T^{(\phi)}_{\mu\nu}$ and $T^{(m)}_{\mu\nu}$ are the standard energy-momentum tensors associated with the middle two terms of (3.1) and $\mathcal{L}^{(m)}$, respectively. Using the equations of motion it is possible to show that, despite the non-minimal coupling, the Biancci identity does indeed give us $\nabla^\mu T^{(m)}_{\mu\nu} = 0$, meaning that test particles still follow geodesics of $g_{\mu\nu}$ [29].

Einstein Frame Analysis

Mixing between the scalar and gravitational sectors that results from the non-minimal coupling in (3.1) makes analysing this form of action somewhat complicated. A common trick, however, is to perform a conformal transformation that allows us to remove this mixing. If we consider our original metric to be given in terms of some new metric $\tilde{g}_{\mu\nu}$ as

$$g_{\mu\nu} = \frac{M^2_{pl}}{2f}\tilde{g}_{\mu\nu}, \tag{3.3}$$

we obtain an action whose gravitational part is of the canonical Einstein-Hilbert form, namely

$$S = \int d^4x\sqrt{-\tilde{g}} \quad \left\{\frac{M^2_{Pl}\tilde{R}}{2} - \frac{1}{2}S_{ab}\tilde{g}^{\mu\nu}\tilde{\nabla}_\mu\phi^a\tilde{\nabla}_\nu\phi^b - \tilde{V} + \tilde{\mathcal{L}}^{(m)}\left(\frac{M^2_{Pl}}{2f}\tilde{g}_{\mu\nu}, \psi\right)\right\}, \tag{3.4}$$

where $\tilde{V} = M^4_{Pl}V/(2f)^2$ and S_{ab} defines a new field-space metric that is given explicitly as[3]

$$S_{ab} = \frac{M^2_{Pl}}{2f}\left[h_{ab} + 3\frac{f_a f_b}{f}\right]. \tag{3.5}$$

Einstein's equations then take the standard form $G_{\mu\nu} = (\tilde{T}^{(\phi)}_{\mu\nu} + \tilde{T}^{(m)}_{\mu\nu})/M^2_{\rm Pl}$. Using the relation between $g_{\mu\nu}$ and $\tilde{g}_{\mu\nu}$ we are able to determine that $T^{(m)}_{\mu\nu} = 2f(\vec{\phi})\tilde{T}^{(m)}_{\mu\nu}$, from which we

[3]Note that we require the eigenvalues of the matrix S_{ab} to be positive in order to avoid the appearance of ghosts in our model.

can in turn deduce

$$\tilde{\nabla}^{\mu}\tilde{T}^{(m)}_{\mu\nu} = -\tilde{T}^{(m)}\frac{f_{\nu}}{2f}, \tag{3.6}$$

where $\tilde{T}^{(m)}$ is the trace of the matter energy-momentum tensor. Thus, we see that even if $T^{(m)}_{\mu\nu}$ is covariantly conserved, in general $\tilde{T}^{(m)}_{\mu\nu}$ is not, meaning that test particles will not follow geodesics of the metric $\tilde{g}_{\mu\nu}$. It will, however, be conserved if $\tilde{T}^{(m)} = 0$, which is the case for radiation-like matter.

Inflation with Non-Minimal Coupling

Inflationary models with non-minimal coupling are not something new. In the late 80's and early 90's so-called *Induced Gravity* inflation models were considered [34]. These were based on Zee's idea of induced gravity whereby the current value of the gravitational constant is determined by the vacuum expectation value of some field ϕ after spontaneous symmetry breaking [35]. Chaotic inflationary models with non-minimal coupling were also considered around that time [36, 37], and again received attention around the turn of the century [38, 39]. Recent observational constraints on the scalar tilt, n_s, and scalar-to-tensor ratio, r, coming from WMAP and *Planck* have once again led to a renewed interest in inflationary models with non-minimal coupling [40–42]. In particular, Higgs inflation [43] and a whole class of inflation models introduced by Kallosh and Linde in the context of the superconformal approach to supergravity [44–48] all give the same predictions for n_s and r as R^2 inflation, namely

$$n_s - 1 = -\frac{2}{N} \qquad \text{and} \qquad r = \frac{12}{N^2}. \tag{3.7}$$

For $N \sim 55$, these lie right at the sweet spot of constraints from WMAP and *Planck*. In all of these examples, there are common effects of the non-minimal coupling that lead to ideal conditions for inflation. When one re-writes the theories in their Einstein frame representation and defines a canonically normalised field (which is always possible in the single-field case) one finds that

1. The effective potential in the Einstein frame is suppressed by a factor $1/f^2$ with respect to that in the Jordan frame. For the specific models considered here, at large field values this suppression factor causes the effective potential to approach a constant, equivalent to a cosmological constant.

2. As a result of the logarithmic relation between the original field and the canonically normalised field, one also finds a stretching of the potential for large field values when written in terms of the canonically normalised field.

It must be noted, however, that these effects are not common to all inflation models with non-minimal coupling.

Finally, we must mention the relevance of the recent findings of the BICEP2 team, who claim to have detected a tensor-to-scalar ratio $r \sim \mathcal{O}(0.1)$ [49]. If this finding is confirmed, then the aforementioned models become decidedly less attractive, as they all predict $r \sim \mathcal{O}(10^{-3})$. However, this by no means spells the end for models with non-minimal coupling. Indeed, classes of non-minimally coupled models that interpolate between models predicting small r and those predicting large r are known [50].

3.2 *Linear Curvature Perturbation*

Having introduced inflationary models with non-minimal coupling, we are now interested in discussing the curvature perturbation on hypersurfaces of constant energy density. In their Einstein frame representation, models with non-minimal coupling are no different from the models discussed in Sec. 2. However, if matter is minimally coupled to the Jordan frame metric rather than that in the Einstein frame, we would like to relate the curvature perturbation associated with $\tilde{g}_{\mu\nu}$ to that associated with $g_{\mu\nu}$.

Using the fact that $d\tilde{s}^2 = (2f/M_{\rm Pl}^2)ds^2$, and decomposing both the Jordan and Einstein metrics in the form (2.1), but with tildes attached to Einstein frame quantities, at background level we find the relations

$$\tilde{a} = \frac{\sqrt{2f}}{M_{pl}}a, \quad d\tilde{t} = \frac{\sqrt{2f}}{M_{pl}}dt,$$
$$d\tilde{x}^i = dx^i \quad \text{and} \quad \tilde{H} = \frac{M_{Pl}}{\sqrt{2f}}\Big(H + \frac{\dot{f}}{2f}\Big), \tag{3.8}$$

and for the perturbations we find

$$\tilde{A} = A + \frac{\delta f}{2f}, \quad \tilde{\mathcal{R}} = \mathcal{R} + \frac{\delta f}{2f},$$
$$\tilde{B} = B \quad \text{and} \quad \tilde{H}_T = H_T. \tag{3.9}$$

At background level we see that there are apparently non-equivalences between the two frames. For example, the notion of an accelerating expansion is different in the two frames [51, 52], $\dot{H}/H^2 \neq \tilde{H}'/\tilde{H}^2$ (where a prime denotes the derivative with respect to $\tilde{t}$), so that the notion of slow-roll is not equivalent and $aH \neq \tilde{a}\tilde{H}$, meaning that the naive definition of super-horizon scales is also not equivalent. At the level of perturbations, we see that they are equivalent if we are able to choose a gauge in which $\delta f = 0$.

In the single-field case $\delta f \propto \delta\phi$, so that $\delta f = 0$ corresponds to the comoving gauges in both frames, i.e. $\delta\phi = 0 \leftrightarrow \delta T^{e0}{}_i = 0 = \delta\tilde{T}^0{}_i$. As such, the comoving curvature perturbation is equivalent in the two frames [53, 54]. Using the fact that on super-horizon scales the comoving and constant-energy-density curvature perturbations coincide, we can conclude that the latter is also equivalent in the two frames.

In the more general case, however, we may not be able to choose a gauge in which $\delta f = 0$, and even when this is possible there is no guarantee that this choice of gauge will coincide with the comoving or constant-energy-density gauges. In comparing ζ and $\tilde{\zeta}$ in the more general case, we in fact choose to work with the curvature perturbations on comoving hypersurfaces, $\mathcal{R}_c$ and $\tilde{\mathcal{R}}_c$. This is because on super horizon scales we have $\mathcal{R}_c \approx \zeta$ and $\tilde{\mathcal{R}}_c \approx \tilde{\zeta}$, and the expressions for $\mathcal{R}_c$ and $\tilde{\mathcal{R}}_c$ turn out to be simpler than those for ζ and $\tilde{\zeta}$. We thus have

$$\zeta - \tilde{\zeta} \simeq \mathcal{R}_c - \tilde{\mathcal{R}}_c = -\frac{\delta f}{2f} + \frac{H\delta q}{\rho + p} - \frac{\tilde{H}\tilde{\delta q}}{\tilde{\rho} + \tilde{p}}. \tag{3.10}$$

where δq and $\delta\tilde{q}$ are as defined in (2.3) for the Jordan and Einstein frames respectively. Note that quantities such as δq in the Jordan frame are those associated with the effective energy-momentum tensor defined in (3.2) (see [55] for explicit expressions). To simplify the calculation further we take the longitudinal gauge. This is possible thanks to the fact that taking the longitudinal gauge in one frame is equivalent to taking it in the other, as can be seen from (3.9).[4] After some manipulation (see [55] for details), we obtain

$$\zeta - \tilde{\zeta} \approx \mathcal{A}_{ab}\mathcal{K}^{ab} + \mathcal{B}_{ab}\dot{\mathcal{K}}^{ab}, \tag{3.11}$$

where $\mathcal{A}_{ab}$ and $\mathcal{B}_{ab}$ are dependent only on background quantities – see [55] for explicit expressions. With this difference written wholly in terms of $\mathcal{K}^{ab}$ and its derivative, it is explicitly clear that it is the isocurvature modes that are responsible for any discrepancy between ζ as defined in the two frames. In particular, we see that the difference vanishes in the single-field case and in any scenario where an effectively single-field adiabatic limit is reached, where $\delta\phi^a \propto \dot{\phi}^a$.

Next let us comment on the conservation of the curvature perturbation. As already mentioned, the analysis in the Einstein frame is entirely equivalent to that presented in Sec. 2, meaning that the expression for $\delta\tilde{p}_{\rm nad}$ is the same as (2.11) but with $h_{ab} \to S_{ab}$, $d/dt \to d/d\tilde{t}$, $\mathcal{K}^{ab} \to \tilde{\mathcal{K}}^{ab} = M_{Pl}\mathcal{K}^{ab}/\sqrt{2f}$ and $D/dt \to \tilde{D}/d\tilde{t}$, where the last object corresponds to the covariant derivative with respect to the field-space metric S_{ab}. With both $\delta\tilde{p}_{\rm nad}$ and $\zeta - \tilde{\zeta}$ being given in terms of $\mathcal{K}^{ab}$ and $\dot{\mathcal{K}}^{ab}$, we can deduce that $\delta p_{\rm nad}$ in the Jordan frame must also be given in terms of $\mathcal{K}^{ab}$ and its derivatives. As

[4] See [56] for a discussion on the relation between gauge choices made in the Jordan and Einstein frames.

such, we can confirm that it is the isocurvature perturbations that source the evolution of ζ, even in the presence of non-minimal coupling. In the absence of isocurvature modes, i.e. when $\mathcal{K}^{ab} = 0$, we thus recover $\dot{\tilde{\zeta}} = \dot{\zeta} = 0$ and $\tilde{\zeta} = \zeta$. The equivalence of the curvature perturbations and their statistical properties in the absence of isocurvature modes is discussed in [57].

More generally, the fact that $\zeta \neq \tilde{\zeta}$ suggests that the evolution of the two curvature perturbations will also be different. In particular, the conservation of one of the two quantities does not necessarily imply the conservation of the other, i.e. $\delta\tilde{p}_{\rm nad} = 0 \not\Leftrightarrow \delta p_{\rm nad} = 0$. It is important to stress, however, that this does not equate to a frame-dependence of observables; any observable predictions should remain independent of the frame. In the case that an adiabatic limit is reached before the end of inflation, i.e. $\mathcal{K}^{ab} \to 0$, the equivalence is manifest, as we eventually recover $\zeta = \tilde{\zeta}$. In the case that isocurvature modes persist, the equivalence may not be so manifest, but should still hold so long as we are careful to keep track of how the non-minimal coupling affects matter, rulers and clocks in the two frames, see e.g. [58, 59]. The physical interpretation in the two frames, however, may be very different, see e.g. [60].

3.3 *Non-Linear Considerations*

In moving beyond linear perturbations, we once again turn to the δN formalism. Let us begin by verifying the validity of the δN formalism in models with non-minimal coupling. We will then point out some subtleties relating to the calculation of correlation functions of ζ in the presence of non-minimal coupling. Finally, based on our findings we will compare the non-linear curvature perturbations associated with the Jordan and Einstein frames.

The Validity of the δN Formalism

Recall that there were two key steps in proving the validity of the δN formalism in subsection 2.3. The first was showing that $\delta N = \zeta$, but in proving this relation we only had to specify the form of metric, and made no assumptions about the gravity theory. As such, no additional proof is required regarding this point. The second step was showing that, under the separate universe approximation, Einstein's equations and the equations of motion for the fields take on exactly the same form as those for the background quantities to order ϵ^2 in the gradient expansion. Extending the proof of this statement to the case of non-minimal coupling follows through almost exactly as in subsection 2.3, replacing $T_{\mu\nu}$ in the original equations with $T^e_{\mu\nu}$. Turning first to the Hamiltonian constraint, we find that the $\Box f$ term appearing in T^e_{00} contains a $\dot{\gamma}_{ij}$, meaning that the result is of the same form as the background equation, but only up to $\mathcal{O}(\epsilon)$. Turning to the equations of motion for the fields, we find that the $\Box\phi^a$ and R terms both contain $\dot{\gamma}_{ij}$ terms, meaning that the form of the equations of motion also only matches that of the background equations to $\mathcal{O}(\epsilon)$.

In the case of Einstein gravity we were able to show that as A_{ij} only has a decaying solution, and was originally $\mathcal{O}(\epsilon)$, we are able to take $A_{ij} \sim \mathcal{O}(\epsilon^2)$, which in turn gives us $\dot{\gamma}_{ij} \sim \mathcal{O}(\epsilon^2)$. In the non-minimally coupled case, the anisotropic stress associated with T^e_{ij} is not vanishing, meaning that we may not be able to draw the same conclusion. However, on closer inspection, we find that the combination $T^e_{ij} - \gamma_{ij}\gamma^{kl}T^e_{kl}/3 \propto \dot{f}A_{ij} + \mathcal{O}(\epsilon^2)$, which means that the dynamical equation for A_{ij} reduces to

$$\dot{A}_{ij} + \left(3\bar{H} + \frac{\dot{f}}{f}\right)A_{ij} + \mathcal{O}(\epsilon^2) = 0, \tag{3.12}$$

giving us

$$A_{ij} \propto \frac{1}{fa^3}. \tag{3.13}$$

As such, as long as f is not decaying faster than $1/a^3$, we still have a decaying solution for A_{ij}, thus allowing us to take A_{ij} and consequently

$\dot{\gamma}_{ij}$ to be $\mathcal{O}(\epsilon^2)$, which means that we recover

$$3H^2 = \frac{1}{2f}\left[\frac{H^2}{2}h_{ab}\phi^a_N\phi^b_N + V - 6H^2 f_N\right] + \mathcal{O}(\epsilon^2), \tag{3.14}$$

$$H\frac{d}{dN}\left(H\phi^a_N\right) + 3H^2\phi^a_N + \Gamma^a_{bc}H^2\phi^b_N\phi^c_N + h^{ab}\left(V_b - f_b\left(12H^2 + 6HH_N\right)\right) + \mathcal{O}(\epsilon^2) = 0. \tag{3.15}$$

We have therefore verified the validity of the δN formalism in the presence of non-minimal coupling, under the condition that f is not decaying faster than $1/a^3$.

Correlation Functions of ζ

In the minimally coupled case, the expansion of δN in terms of the perturbations on the initial flat hypersurface — Eq. (2.26) — allowed us to use the known correlation functions of $\delta\phi^a_{\mathcal{R}*}$ — given in Eq. (2.27) — to determine correlation functions of ζ. In the non-minimally coupled case, however, we cannot use the standard result (2.27) for the correlation functions of $\delta\phi^a_{\mathcal{R}*}$, as the equations of motion for $\delta\phi^a_{\mathcal{R}}$ take on the non-standard form

$$M_2{}^a{}_b\frac{D^2\delta\phi^b_{\mathcal{R}}}{dt^2} + M_1{}^a{}_b\frac{D\delta\phi^b_{\mathcal{R}}}{dt} + M_0{}^a{}_b\delta\phi^b_{\mathcal{R}} = 0, \tag{3.16}$$

i.e. there is mixing not only in the mass term $M_0{}^a{}_b\delta\phi^b$, but also in the derivative terms (see [55] for the explicit forms of $M_i{}^a{}_b$).

In principle we can of course use these equations of motion to determine the correlation functions of $\delta\phi^a_{\mathcal{R}}$, but in fact we find that this is unnecessary once we realise the relation between the $\delta\phi^a_{\mathcal{R}}$ defined on flat hypersurfaces in the Jordan frame and the $\delta\phi^a_{\tilde{\mathcal{R}}}$ defined on flat hypersurfaces in the Einstein frame. As noted previously, the analysis in the Einstein frame follows almost exactly that outlined in Sec. 2, and the correlation functions of $\delta\phi^a_{\tilde{\mathcal{R}}*}$ are given as in (2.27) with $h^{ab}_* \to S^{ab}_*$ and $H_* \to \tilde{H}_*$. In order to relate $\delta\phi^a_{\mathcal{R}}$ to $\delta\phi^a_{\tilde{\mathcal{R}}}$, we note that the Sasaki-Mukhanov variables at first and second order are given as [61–63]

$$\begin{aligned}
\delta\phi^a_{\mathcal{R}(1)} &= \delta\phi^a_{(1)} - \frac{\phi'^a}{\mathcal{H}}\mathcal{R}_{(1)}, \\
\delta\phi^a_{\mathcal{R}(2)} &= \delta\phi^a_{(2)} - \frac{\phi'^a}{\mathcal{H}}\mathcal{R}_{(2)} \\
&\quad + \left(\frac{\mathcal{R}_{(1)}}{\mathcal{H}}\right)^2\left[2\mathcal{H}\phi'^a + \phi''^a - \frac{\mathcal{H}'}{\mathcal{H}}\phi'^a\right] \\
&\quad + 2\frac{\phi'^a}{\mathcal{H}^2}\mathcal{R}'_{(1)}\mathcal{R}_{(1)} - \frac{2}{\mathcal{H}}\mathcal{R}_{(1)}\delta\phi'^a_{(1)},
\end{aligned} \tag{3.17}$$

where a prime denotes the derivative with respect to conformal time η ($ad\eta = dt$) and $\mathcal{H} = a'/a$. In extending the metric decomposition (2.1) to second order in perturbation theory, we have taken e.g. $\mathcal{R} = \mathcal{R}_{(1)} + \mathcal{R}_{(2)}/2$, with $R_{(i)}$ representing a quantity that is of i'th order in the perturbative expansion.

The corresponding quantities in the Einstein frame, $\delta\phi^a_{\tilde{\mathcal{R}}(1)}$ and $\delta\phi^a_{\tilde{\mathcal{R}}(2)}$, are then found by replacing all $\mathcal{R}$ and $\mathcal{H}$ in (3.17) with $\tilde{\mathcal{R}}$ and $\tilde{\mathcal{H}} = \tilde{a}'/\tilde{a}$ respectively. In comparing $\delta\phi^a_{\mathcal{R}(i)}$ and $\delta\phi^a_{\tilde{\mathcal{R}}(i)}$ it is therefore necessary to determine the relation between $\mathcal{R}$ and $\tilde{\mathcal{R}}$ to second order. The first order result is already given in (3.9), and extending the analysis to second order we find

$$\begin{aligned}
\tilde{\mathcal{R}}_{(2)} &= \mathcal{R}_{(2)} + \frac{f_a\delta\phi^a_{(2)}}{2f} + \frac{f_{ab}\delta\phi^a_{(1)}\delta\phi^b_{(1)}}{2f} \\
&\quad - \frac{f_a f_b\delta\phi^a_{(1)}\delta\phi^b_{(1)}}{f^2} + \frac{2f_a\delta\phi^a_{(1)}\tilde{\mathcal{R}}_{(1)}}{f}.
\end{aligned} \tag{3.18}$$

Using this result, along with (3.9) and the background result $\tilde{\mathcal{H}} = \mathcal{H} + \frac{f'}{2f}$, we are able to express $\delta\phi^a_{\mathcal{R}(i)}$ in terms of $\delta\phi^a_{\tilde{\mathcal{R}}(i)}$, and substituting these relations into the expansion for δN in the Jordan frame we find[5]

$$\begin{aligned}
\zeta &= \delta N \\
&= N_a\delta\phi^a_{\tilde{\mathcal{R}}} + \frac{1}{2}N_{ab}\delta\phi^a_{\tilde{\mathcal{R}}(1)}\delta\phi^b_{\tilde{\mathcal{R}}(1)} - \frac{f_a\delta\phi^a_{\tilde{\mathcal{R}}}}{2f} \\
&\quad - \frac{1}{2}\frac{f_{cb}\delta\phi^c_{\tilde{\mathcal{R}}(1)}\delta\phi^b_{\tilde{\mathcal{R}}(1)}}{2f} + \left(\frac{f_b\delta\phi^b_{\tilde{\mathcal{R}}(1)}}{2f}\right)^2,
\end{aligned} \tag{3.19}$$

[5] See [64] for details.

where we have omitted asterisks attached to f, its derivatives and the $\delta\phi^a_{\tilde{\mathcal{R}}}$ to avoid clutter. Note that, in its current form, the expression for ζ is not explicitly covariant with respect to field-space quantities. In order to make cavariance explicit, we need to replace the $\delta\phi^a_{\tilde{\mathcal{R}}}$ with their corresponding perturbation quantities that transform as vectors of the tangent space defined by the field-space metric S_{ab}. The relation to second order is [65]

$$\delta\phi^a_{\tilde{\mathcal{R}}} = Q^a_{\tilde{\mathcal{R}}} - \frac{1}{2}{}^{(S)}\Gamma^a_{bc} Q^b_{\tilde{\mathcal{R}}(1)} Q^c_{\tilde{\mathcal{R}}(1)}, \tag{3.20}$$

where ${}^{(S)}\Gamma^a_{bc}$ is the Christoffel symbol associated with S_{ab}, and substituting this relation into (3.19) we find

$$\zeta = \mathcal{N}_a Q^a_{\tilde{\mathcal{R}}} + \frac{1}{2}\tilde{\nabla}_a\tilde{\nabla}_b\mathcal{N} Q^a_{\tilde{\mathcal{R}}(1)} Q^b_{\tilde{\mathcal{R}}(1)}, \tag{3.21}$$

where $\mathcal{N} = N - \ln(f)/2$.

Using the result [9, 10, 66]

$$\langle Q^a_{\tilde{\mathcal{R}}(1)}(\boldsymbol{k}_1) Q^b_{\tilde{\mathcal{R}}(1)}(\boldsymbol{k}_2)\rangle = (2\pi)^3\delta^3(\boldsymbol{k}_1+\boldsymbol{k}_2)\frac{\tilde{H}^2}{2k^3}S^{ab}, \tag{3.22}$$

we find the power spectrum and f_{NL} parameter in the Jordan frame

$$\mathcal{P}_\zeta(k) = \mathcal{N}_a\mathcal{N}_b S^{ab}\left(\frac{\tilde{H}}{2\pi}\right)^2 \quad \text{and}$$
$$f_{NL} = \frac{5}{6}\frac{\mathcal{N}_a\mathcal{N}_b\tilde{\nabla}_c\tilde{\nabla}_d\mathcal{N} S^{ac}S^{bd}}{[\mathcal{N}_e\mathcal{N}_f S^{ef}]^2}, \tag{3.23}$$

where, at leading order, the three point correlation function for ζ is given as

$$\begin{aligned}&\langle\zeta(\boldsymbol{k}_1)\zeta(\boldsymbol{k}_2)\zeta(\boldsymbol{k}_3)\rangle\\ &= \mathcal{N}_a\mathcal{N}_b\mathcal{N}_c\langle Q^a_{\tilde{\mathcal{R}}}(\boldsymbol{k}_1) Q^b_{\tilde{\mathcal{R}}}(\boldsymbol{k}_2) Q^c_{\tilde{\mathcal{R}}}(\boldsymbol{k}_3)\rangle\\ &\quad +(2\pi)^3\delta^{(3)}(\boldsymbol{k}_1+\boldsymbol{k}_2+\boldsymbol{k}_3)\frac{6}{5}f_{NL}\left[\mathcal{N}_a\mathcal{N}_b S^{ab}\right]^2\\ &\quad \frac{\tilde{H}^4}{4}\frac{k_1^3+k_2^3+k_3^3}{k_1^3k_2^3k_3^3}.\end{aligned} \tag{3.24}$$

The intrinsic non-Gaussianity of the field perturbations at horizon crossing, which is required to determine the first term in the above equation, can be calculated following the formalism of [66, 67]. Recall that all appearances of $Q^a_{\tilde{\mathcal{R}}}$, S^{ab}, $\tilde{H}$ and $\ln(f)$ in the above equations should have asterisks attached, indicating that they are to be evaluated at the time η_* that is shortly after the modes in question have left the horizon. We remark here that under the slow-roll approximation the horizon crossing times in the two frames coincide, i.e. $k = \mathcal{H} \approx \tilde{\mathcal{H}}$ when $|f'/f| \ll |\mathcal{H}|$.

Using (3.23), and following very closely the calculation outlined in [10], to leading order in the slow-roll approximation we find the spectral index as

$$\begin{aligned} n_s - 1 :&= \frac{d\ln\mathcal{P}_\zeta}{d\ln k}\\ &= 2\frac{d\tilde{H}/d\tilde{t}}{\tilde{H}^2} - \frac{2}{\mathcal{N}_a\mathcal{N}^a}\\ &\quad + \frac{2\mathcal{N}^a\mathcal{N}^b}{3\tilde{H}^2\mathcal{N}_e\mathcal{N}^e}\left[\tilde{\nabla}_a\tilde{\nabla}_b\tilde{V} + \tilde{R}_{acbd}\frac{d\phi^c}{d\tilde{t}}\frac{d\phi^d}{d\tilde{t}}\right].\end{aligned} \tag{3.25}$$

Comparing the Frames

Following the arguments of [53], let us write our conformal transformation in the form

$$\Omega^2 = \Omega_0^2 e^{2\Delta\Omega}, \tag{3.26}$$

where $\Omega^2 = 2f/M^2_{\rm Pl}$ and Ω_0 and $\Delta\Omega$ correspond to the background part of Ω and its non-linear perturbation, respectively. Then, considering only the spatial part of our line elements in the Jordan and Einstein frames, we find

$$\begin{aligned}\tilde{a}^2 e^{2\tilde{\mathcal{R}}}\tilde{\gamma}_{ij}dx^i dx^j &= \Omega_0^2 e^{2\Delta\Omega} a^2 e^{2\mathcal{R}}\gamma_{ij}dx^i dx^j\\ &= \tilde{a}^2 e^{2\mathcal{R}+2\Delta\Omega}\gamma_{ij},\end{aligned} \tag{3.27}$$

from which we determine

$$\tilde{\mathcal{R}} = \mathcal{R} + \Delta\Omega. \tag{3.28}$$

As such, we see that in a similar way as with the linear case, the two curvature perturbations will be equal in a gauge where $\Delta\Omega = 0$. In the single-field case, as Ω is only a function of ϕ, $\Delta\Omega = 0$ corresponds to $\delta\phi = 0$ at the non-linear level. Even at non-linear level, taking $\delta\phi = 0$ corresponds to the comoving gauge in both the Jordan and Einstein frames, and so we conclude

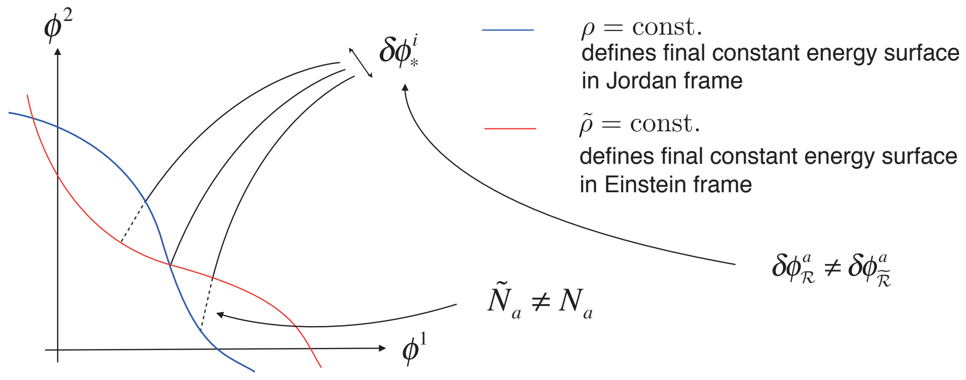

Fig. 2. Figure demonstrating the two key differences between the δN formalism as applied in the Jordan and Einstein frames: the difference in initial conditions compared to some fiducial background trajectory, i.e. $\delta\phi^a_{\mathcal{R}} \neq \delta\phi^a_{\tilde{\mathcal{R}}}$, and the difference in the final surfaces of constant energy density up to which we integrate N in the Jordan frame (blue line) and $\tilde{N}$ in the Einstein frame (red line).

that the two comoving curvature perturbations are the same. In the presence of isocurvature modes, however, this argument no longer holds.

In the more general case, where isocurvature perturbations are present, let us use try to use the δN formalism applied in both the Jordan and Einstein frames to establish the difference between ζ and $\tilde{\zeta}$. As pointed out already, the analysis in the Einstein frame follows almost exactly that in Sec. 2. The expressions for $\tilde{\zeta}$, $\mathcal{P}_{\tilde{\zeta}}(k)$, $\tilde{f}_{NL}$, $\langle\tilde{\zeta}(\boldsymbol{k}_1)\tilde{\zeta}(\boldsymbol{k}_2)\tilde{\zeta}(\boldsymbol{k}_3)\rangle$ and $\tilde{n}_s - 1$ are found to be of exactly the same form as given in expressions (3.21)– (3.25), but with $\mathcal{N} \to \tilde{N}$, where $\tilde{N}$ is the number of e-foldings in the Einstein frame. As such, we are interested in determining the difference between the two quantities $\tilde{N}$ and $\mathcal{N}$, which are given explicitly as

$$\mathcal{N} = \int_*^{\rho=\text{const.}} \mathcal{H} d\eta - \frac{\ln(f_*)}{2}, \tag{3.29}$$

$$\begin{aligned}\tilde{N} &= \int_*^{\tilde{\rho}=\text{const.}} \tilde{\mathcal{H}} d\eta \\ &= \int_*^{\tilde{\rho}=\text{const.}} \mathcal{H} d\eta + \frac{1}{2}\ln\left(\frac{f_{\tilde{\rho}=\text{const.}}}{f_*}\right),\end{aligned} \tag{3.30}$$

where $\rho = $ const. and $\tilde{\rho} = $ const. denote the final surfaces of constant energy density up to which we integrate in the Jordan and Einstein frames respectively. Taking their difference, we have

$$\tilde{N} - \mathcal{N} = \int_{\rho=\text{const.}}^{\tilde{\rho}=\text{const.}} \mathcal{H} d\eta + \frac{1}{2}\ln(f_{\tilde{\rho}=\text{const.}}). \tag{3.31}$$

Note that this difference is not affected by the difference in definition of the initial flat hypersurface. This is because the difference resulting from $\delta\phi^a_{\mathcal{R}} \leftrightarrow \delta\phi^a_{\tilde{\mathcal{R}}}$, which manifested itself in the $\ln(f_*)/2$ term in the expression for $\mathcal{N}$, exactly cancels with the $\ln(f_*)/2$ term in the expression for $\tilde{N}$, which results from the difference between $\tilde{\mathcal{H}}$ and $\mathcal{H}$. As such, it is only the difference in definition of the final surface of constant energy density which is important. The two key differences between the δN formalism as applied in the Jordan and Einstein frames are depicted schematically in Fig. 2.

If an adiabatic limit is reached we have two simplifications. Firstly, the definition of the final surface of constant energy density becomes unique, meaning that the first term on the right-hand side of (3.31) vanishes. Secondly, the final values of all the fields become independent of the initial conditions, meaning that $\partial f_{\tilde{\rho}=\text{const.}}/\partial\phi^a_* = 0$.

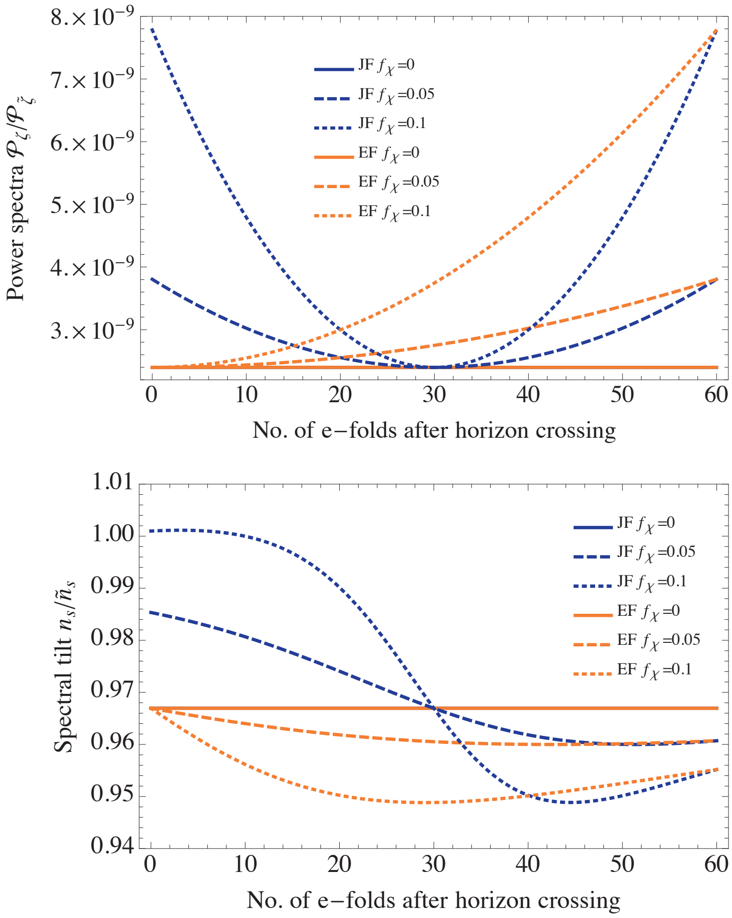

Fig. 3. Taken from [64]: Evolution of the power spectrum (upper plot) and spectral tilt (lower plot) in the Jordan (JF) and Einstein (EF) frames for a range of f_χ. Here we have set $2f = 1$ (which corresponds to taking $M_{Pl} = 1$), $V^{(\chi)} = 1$, $f_{\chi\chi} = W_{\chi\chi} = 0$, $N_* = 60$, $p = 1$ and $m^2 = 1.94 \times 10^{-11}$.

As such, given that we are actually only interested in differences between the derivatives of $\tilde{N}$ and $\mathcal{N}$, such as $\tilde{N}_a - \mathcal{N}_a$, which will depend on derivatives of $f_{\tilde{\rho}=\text{const.}}$, we see that the second term in (3.31) also becomes irrelevant. Consequently, we recover the known result $\zeta = \tilde{\zeta}$ even at the non-linear level [53, 68].

3.4 *An Example: Non-Minimally Coupled "Spectator"*

As an example, let us summarise the results obtained in [64] for what was dubbed the non-minimally coupled "spectator" field model. This model consists of two fields ϕ and χ, corresponding to the inflation and "spectator" field respectively. We then choose the non-minimal coupling function to only depend on χ. The reason we call χ the "spectator" field is that we assume $\tilde{V}_\chi = 0$, meaning that under the slow-roll approximation we have $\chi' = 0$, i.e. the field is not dynamical during inflation. However, it is not a spectator field in the usual sense, as the presence of non-minimal coupling results in the perturbations of χ contributing to the curvature perturbation. Hence we write spectator in quotation marks.

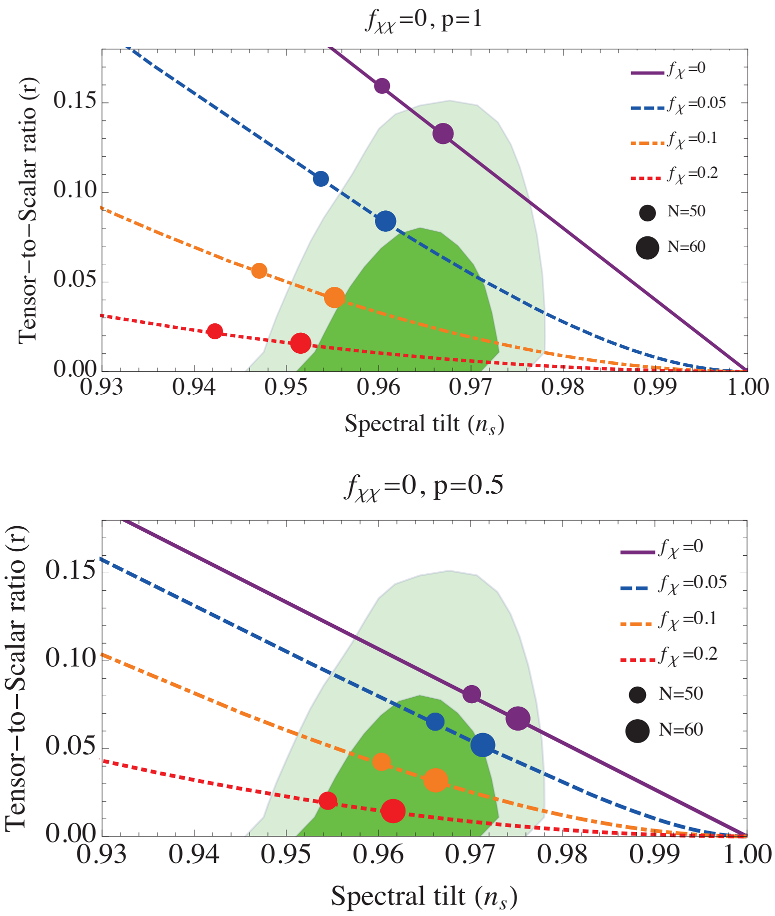

Fig. 4. Taken from [64]: Predictions in the n_s–r plane for models with $p = 1$ (upper plot) and $p = 1/2$ (lower plot). A range of f_χ are considered (see legends for exact values) and the remaining parameters are fixed as $V^{(\chi)} = 2f = 1$, $f_{\chi\chi} = \tilde{V}_{\chi\chi} = 0$ and m^2 is normalised for each curve such that $\mathcal{P}_\zeta = 2.4 \times 10^{-9}$. The small and large discs on each curve correspond to modes leaving the horizon 50 and 60 e-foldings before the end of inflation, respectively. As the Jordan and Einstein frames agree at leading order at the end of inflation, here we only plot the Jordan frame quantities. The 68% and 95% confidence contours from the recent *Planck* results have also been plotted.

In order to allow for an analytic analysis we consider the case where $\tilde{V}$ is product separable and $h_{ab} = \delta_{ab}$. Being a little more specific, we take $V = V^{(\chi)}(\chi)V^{(\phi)}(\phi)$, with $V^{(\phi)}(\phi) = m^2\phi^{2p}$. In the context of our analytic calculations, we find that we require $f_\chi/\sqrt{f} \sim \mathcal{O}(\epsilon^{1/2})$ and $f_{\chi\chi} \lesssim \mathcal{O}(\epsilon)$. To leading order in the slow-roll approximation we then find

$$\zeta - \tilde{\zeta} \simeq \frac{f_\chi}{2f\epsilon}\delta\chi_{\tilde{\mathcal{R}}*}. \tag{3.32}$$

Given the $1/\epsilon$ dependence of this difference, we therefore expect that it might be initially large, when $\epsilon \ll 1$, but that towards the end of inflation, when $\epsilon \sim 1$, it is small.

In Fig. 3 we have plotted the Jordan and Einstein frame power spectra and tilts as functions of N for a range of values of f_χ. The end of inflation is taken to be defined by $\epsilon = 1$, and we consider modes that leave the horizon 60 e-folds before the end of inflation. For definiteness, the remaining parameters are taken as follows: $2f = 1$ (so that the effective Planck mass is in agreement with the current value and we are now taking $M_{Pl} = 1$), $f_{\chi\chi} = \tilde{V}_{\chi\chi} = 0$, $V^{(\chi)}(\chi) = 1$ and $p = 1$. Lastly, we take $m^2 = 1.94 \times 10^{-11}$ in order that for $f_\chi = 0$ the power spectrum is normalised to the observed 2.4×10^{-9}.

We see that the general behaviour of the differences between the power spectra and tilts associated with the Jordan and Einstein frames is as expected from expression (3.32). Namely, whilst initially there is a non-trivial difference between the Jordan and Einstein frame quantities, they become equivalent towards the end of inflation.

A similar behaviour is found for the quantities f_{NL} and $\tilde{f}_{NL}$, but the requirement that $f_{\chi\chi} \lesssim \mathcal{O}(\epsilon)$ deems the non-Gaussianity unobservable, so we do not plot the results here. This example illustrates how, despite the fact that final predictions for ζ and $\tilde{\zeta}$ may coincide, their preceding evolution may be very different, leading to different interpretations of the generation of primordial perturbations in the two frames.

Having looked at the evolution of the power spectra and spectral tilts, let us now turn to their final values and the dependence of these values on the non-minimal coupling. Still imposing $f_{\chi\chi} = \tilde{V}_{\chi\chi} = 0$, in Fig. 4 we have plotted predictions in the n_s–r plane as a function of f_χ for the cases $p = 1$ and $p = 1/2$. As we expect the final Jordan and Einstein frame results to agree at leading order, we choose to only plot the Jordan frame parameters. In each case we choose m such that $\mathcal{P}_\zeta$ is normalised to 2.4×10^{-9}, and the remaining parameters are set as $2f = V^{(\chi)} = 1$. We see that as f_χ is increased, the tensor-to-scalar ratio becomes suppressed and the spectrum becomes more red-tilted. As such, for $p = 1/2$, a combination of these two effects brings predictions within the 68% confidence contours of the recent *Planck* results.[6] See [64] for a more detailed analysis of this example model.

[6]We would like to thank Laila Alabidi for providing us with the data for these confidence contours.

4. Conclusions

In this article we have given a brief review of aspects relating to the curvature perturbation generated during inflation, focusing in particular on the time dependence of ζ at linear order and the δN formalism as a tool to determine ζ to non-linear order.

After reviewing the standard results in the context of GR, in Sec. 3 we considered how the situation changes on the introduction of non-minimal coupling. Even in the presence of non-minimal coupling, it is possible to bring Einstein's equations into the form $G_{\mu\nu} = T^e_{\mu\nu}/M^2_{\rm Pl}$, where $T^e_{\mu\nu}$ is the effective energy-momentum tensor defined in (3.2). As such, the analysis with regard to the time dependence of ζ at linear order follows through exactly as in the minimally coupled case, but with quantities such as $\delta\rho$, δp and $\delta p_{\rm nad}$ now being those associated with $T^e_{\mu\nu}$. In the case of multi-field inflation, we saw that — as in the standard case — $\delta p_{\rm nad}$ is given in terms of isocurvature perturbations that do not lie along the background trajectory in field space. Unlike the standard case, however, ζ is not necessarily conserved when the background trajectory is a geodesic of the field-space.

Whilst the above results pertain to the original Jordan frame, we also discussed the quantity $\tilde{\zeta}$ associated with the conformally related Einstein frame and its relation to ζ in the Jordan frame. The Einstein frame action is of canonical form, meaning that all standard results regarding the time dependence of $\tilde{\zeta}$ and its non-Gaussianity apply. We saw that the difference between ζ and $\tilde{\zeta}$ is given in terms of the isocurvature perturbations associated with multi-field inflation, which agrees with the known result $\zeta = \tilde{\zeta}$ in the single-field case. Under the assumption that an adiabatic limit is reached before the radiation dominated phase — such that eventually $\zeta = \tilde{\zeta}$ — the potentially different evolutions of ζ and $\tilde{\zeta}$ in the preceding non-adiabatic phase gives rise to the possibility that the interpretation of the generation of the primordial curvature perturbation is very different in the two frames. Indeed, this was the case in the non-minimally coupled "spectator" example presented in the final subsection. In the case that isocurvature modes persist, perhaps even until the present day, one has to be very careful to determine exactly how ζ and $\tilde{\zeta}$ are related to the observed temperature fluctuations of the CMB. So long as we are careful to keep track of how the non-minimal coupling affects matter, rulers and clocks in the two frames, then, at least classically, predictions for observables should be independent of the frame in which they are calculated. The physical interpretation in the two frames, however, may be very different.

Moving beyond linear order perturbations we considered the validity of the δN formalism in the presence of non-minimal coupling. Provided that the non-minimal coupling function f does not decay faster than $1/a^3$, we saw that the key results of the δN formalism remain unaffected. In calculating the correlation functions of $\delta N = \zeta$, however, we saw that one has to be careful to take into account the fact that the correlators for $\delta\phi^a_{\mathcal{R}}$ as defined in the Jordan frame do not take on the standard form. By relating $\delta\phi^a_{\mathcal{R}}$ to $\delta\phi^a_{\tilde{\mathcal{R}}}$ as defined in the Einstein frame, however, one is able to determine the correlation functions of ζ. Using our results we were able to compare ζ and $\tilde{\zeta}$ to non-linear order, and were able to confirm that in the adiabatic limit we recover $\zeta = \tilde{\zeta}$.

Looking beyond the results summarised in this article, the frame dependence of quantum corrections is another issue that has recently attracted much attention, see e.g. [69, 70]. Such considerations are of particular importance in the context of Higgs inflation, where predictions are very sensitive to quantum corrections and the running of Standard Model couplings. It is our hope that some of the results discussed in this article will prove useful in resolving these quantum issues.

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References

[1] A. A. Starobinskii, *Spectrum of relict gravitational radiation and the early state of the universe*, JETP Letters (1979).

[2] K. Sato, Mon. Not. Roy. Astron. Soc. **195**, 467 (1981).

[3] A. H. Guth, Physical Review **D23**, No. 2, 347 (1981).

[4] A. D. Linde, Phys. Lett. **B108**, No. 6, 389 (1982).

[5] A. Albrecht and P. J. Steinhardt, Phys. Rev. Lett. **48**, No. 17, 1220 (1982).

[6] A. Naruko and M. Sasaki, Classical and Quantum Gravity **28**, 072001(2011).

[7] J. Maldacena, Journal of High Energy Physics **05**, 013 (2003).

[8] A. A. Starobinsky, JETP Lett. **42**, 152 (1985).

[9] M. Sasaki and E. D. Stewart, Progress of Theoretical Physics **95**, 71 (1996).

[10] T. T. Nakamura and E. D. Stewart, Phys. Lett. **B381**, 413 (1996).

[11] M. Sasaki and T. Tanaka, Progress of Theoretical Physics **99**, 763 (1998).

[12] D. H. Lyth, K. A. Malik, and M. Sasaki, JCAP **0505**, 004 (2005).

[13] N. S. Sugiyama, E. Komatsu, and T. Futamase, Phys. Rev. **D87**, 023530 (2013), arXiv:1208.1073.

[14] H. Kodama and M. Sasaki, Prog. Theor. Phys. Suppl. **78**, 1 (1984).

[15] B. A. Bassett, S. Tsujikawa, and D. Wands, Reviews of Modern Physics **78**, 537 (2006).

[16] S. Groot Nibbelink and B. J. W. van Tent, Classical and Quantum Gravity **19**, 613 (2002).

[17] D. Langlois and S. Renaux-Petel, JCAP **0804**, 017 (2008).

[18] C. M. Peterson and M. Tegmark, Physical Review **D87**, No. 10, 103507 (2013).

[19] A. Mazumdar and L. Wang, JCAP **1209**, 005 (2012).

[20] C. Gordon, D. Wands, B. A. Bassett, and R. Maartens, Phys. Rev. **D63**, No. 2, 023506 (2001).

[21] M. Sasaki, arXiv:1210.7880 (2012).
[22] N. Birrell and P. C. W. Davies, "*Quantum Fields in Curved Space*" (Cambridge University Press, 1984).
[23] L. Parker and D. Toms, "*Quantum Field Theory in Curved Spacetime: Quantized Fields and Gravity*" (Cambridge University Press, 2009).
[24] P. A. M. Dirac, Physical and Engineering Sciences **165**, 199 (1938).
[25] D. W. Sciama, *Monthly Notices of the Royal Astronomical Society* **113**, 34(1953).
[26] P. Jordan, "*Schwerkraft und Weltall*" (Vieweg, Braunschweig, 1955).
[27] R. H. Dicke, American Scientist **47**, No. 1, 25 (1959).
[28] C. Brans and R. H. Dicke, Phys. Rev. **124**, No. 3, 925 (1961).
[29] Y. Fujii and K.-i. Maeda, "*The Scalar-Tensor Theory of Gravitation*" (Cambridge University Press, 2003).
[30] V. Faraoni, "*Cosmology in Scalar Tensor Gravity*" (Springer, 2004).
[31] A. De Felice and S. Tsujikawa, Living Rev. Rel. **13:3** (2010).
[32] K.-i. Maeda, Phys.Rev. **D39**, 3159 (1989).
[33] E. J. Copeland, R. Easther, and D. Wands, Physical Review **D56**, 874 (1997).
[34] R. Fakir and W. G. Unruh, Phys. Rev. **D41**, No. 6, 1792 (1990).
[35] A. Zee, Phys. Rev. Lett. **42**, 417 (1979).
[36] T. Futamase and K.-i. Maeda, Phys. Rev. **D39**, No. 2, 399 (1989).
[37] A. A. Starobinsky and J. Yokoyama, Proc. Fourth Workshop on General Relativity and Gravitation eds. K. Nakao *et al.* (Kyoto University,1995), **381**, arXiv:gr-qc/9502002.
[38] E. Komatsu and T. Futamase, Phys. Rev. **D58**, 023004 (1998), astro-ph/9711340.
[39] E. Komatsu and T. Futamase, Phys. Rev. **D59**, 064029 (1999), astro-ph/9901127.
[40] G. Hinshaw *et al.*, The Astrophysical Journal Supplement **208**, 19 (2013).
[41] C. L. Bennett *et al.*, The Astrophysical Journal Supplement **208**, 20 (2013).
[42] P. A. R. Planck Collaboration, Ade *et al.*, *Planck 2013 results. XXII. Constraints on inflation*, arXiv:1303.5082.
[43] F. Bezrukov and M. Shaposhnikov, Phys. Lett. **B659**, 703 (2008).
[44] R. Kallosh and A. Linde, JCAP **1306**, 027 (2013), arXiv:1306.3211.
[45] R. Kallosh and A. Linde, JCAP **1306**, 028 (2013), arXiv:1306.3214.
[46] R. Kallosh and A. Linde, JCAP **1307**, 002 (2013), arXiv:1306.5220.
[47] R. Kallosh and A. Linde, JCAP **1310**, 033 (2013), arXiv:1307.7938.
[48] R. Kallosh and A. Linde, arXiv:1309.2015.
[49] BICEP2 Collaboration Collaboration, P. Ade *et al.*, Phys. Rev. Lett. **112**, 241101 (2014), arXiv:1403.3985.
[50] R. Kallosh, A. Linde, and D. Roest, arXiv:1310.3950.
[51] Y. S. Piao, arXiv:1112.3737 [hep-th].
[52] T. Qiu, JCAP **1206**, 041 (2012).
[53] J.-O. Gong, J.-c. Hwang, W.-I. Park, M. Sasaki, and Y.-S. Song, JCAP **1109**, 023 (2011).
[54] T. Kubota, N. Misumi, W. Naylor, and N. Okuda, JCAP **1202**, 034 (2012).
[55] J. White, M. Minamitsuji, and M. Sasaki, JCAP **1207**, 039 (2012).
[56] I. A. Brown and A. Hammami, JCAP **1204**, 002 (2012).
[57] T. Chiba and M. Yamaguchi, JCAP **0901**, 019 (2009).
[58] R. Catena, M. Pietroni, and L. Scarabello, Phys. Rev. **D76**, 84039 (2007).
[59] T. Chiba and M. Yamaguchi, JCAP **1310**, 040 (2013), arXiv:1308.1142.
[60] N. Deruelle and M. Sasaki, Cosmology **1**, Chapter 23, 247-260 (2011).
[61] M. Sasaki, Progress of Theoretical Physics **76**, 1036-1046 (1986).
[62] V. F. Mukhanov, Sov. Phys. JETP **67**, 1297-1302 (1988).
[63] K. A. Malik, JCAP **0511**, 005 (2005).
[64] J. White, M. Minamitsuji, and M. Sasaki, JCAP **1309**, 015 (2013).
[65] J. O. Gong and T. Tanaka, JCAP **1103**, 015 (2011).
[66] J. Elliston, D. Seery, and R. Tavakol, JCAP **1211**, 060 (2012).
[67] D. Langlois, S. Renaux-Petel, D. A. Steer, and T. Tanaka, Phys. Rev. **D78**, 63523 (2008).
[68] T. Chiba and M. Yamaguchi, JCAP **0810**, 021 (2008).
[69] C. F. Steinwachs and A. Y. Kamenshchik, AIP Conf. Proc. **1514**, 161 (2012), arXiv:1301.5543.
[70] D. P. George, S. Mooij, and M. Postma, arXiv:1310.2157.