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A POSSIBLE METHOD TO REDUCE SPIN-TUNE SPREAD

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In high-energy electron storage rings with beam energy above a few tens of GeV, a large depolarization occurs in the radiative polarization process because of the large spin-tune spread due to the beam energy spread. A new method is proposed which reduces the spin-tune spread and is expected to provide a high degree of radiative polarization.

§1. INTRODUCTION

Although radiative polarization in electron storage rings predicted by Sokolov and Ternov[1] has been confirmed in many machines, in future higher energy machines depolarization due to resonances occurring every 440 MeV will be inevitable because of the large spin-tune spread due to increased energy spread. Therefore, the so called Siberian snake[2] scheme is being investigated for projects such as LEP and CESR II. This scheme has the advantages that (a) it has no resonance because spin-tune is always one half and that (b) the spin-tune spread is nearly zero even if the beam has a large energy spread. The first point is very useful for polarized proton acceleration but is not essential in considering radiative polarization of electrons. The second point is very important for electron machines. Unfortunately, however, it is very difficult to obtain radiative polarization in the Siberian snake scheme and one has to polarize the electron beam by other methods such as Compton scattering by circularly polarized laser beams. Moreover, the life time of polarization is not long.

In planar machines the spin-tune ν_0 of an equilibrium particle with Lorentz factor γ_0 is $\nu_0 = \gamma_0 a$ and that of a particle with Lorentz factor γ is $\nu = \gamma a$, a being the coefficient of anomalous magnetic moment. They are both one half in the Siberian snake scheme. However one has to make sure that what is essential for radiative polarization is $\nu - \nu_0 = 0$ rather than $\nu_0 = 1/2$.

In this paper a method is investigated by which the condition $\nu - \nu_0 = 0$ is satisfied and still one can obtain radiative polarization in this scheme. Spin-tune of the equilibrium particle is not made to be one half as in the Siberian snake scheme, but chosen to be $\nu_0 = \gamma_0 a$. It may be very difficult to eliminate $\nu - \nu_0$ perfectly but it seems feasible to reduce spintune spread by half amount.

§2. SPIN CHROMATICITY

The title of this section should not be confused with the frequently used terminology that means the coupling coefficient between spin and orbit motions. When we do not take into account synchrotron oscillations, the spin-tune ν is a function of $\epsilon = \Delta E/E$, the relative energy deviation from the equilibrium energy. In the present paper we call $(d\nu/d\epsilon)_{\epsilon=0}$ "spin chromaticity", which we believe to be a more proper terminology. In this section we derive an explicit expression for the spin chromaticity.

Let us start with B.M.T. equation[3] in the absence of electric fields,

$$\frac{d\mathbf{s}}{d\theta} = -R\frac{e}{p}\left(1 + \frac{x}{\rho_x} + \frac{y}{\rho_y}\right)[(\gamma a + 1)(\mathbf{B} - (\mathbf{B}\cdot\mathbf{e}_v)\mathbf{e}_v) + (a + 1)(\mathbf{B}\cdot\mathbf{e}_v)\mathbf{e}_v]. \quad (2.1)$$

Here the notations are as follows:

$\mathbf{s}$ = the spin vector in the rest frame of the particle.
θ = generalized azimuth of the machine.
R = average radius of the machine.
p = momentum of the particle.
γ = energy of the particle divided by the rest mass energy.
$\mathbf{e}_v$ = unit vector along the trajectory of the particle.
$\mathbf{B}$ = magnetic field on the particle trajectory.
a = the coefficient of the anomalous magnetic moment.
ρ_x, ρ_y = horizontal and vertical curvature radius of the design orbit. (positive for bends of $-x$ and $-y$ direction)
x, y = horizontal and vertical deviation from the design orbit.

The factor containing $\rho_{x,y}$ comes from the difference between the lengths of the design orbit and the actual particle orbit. Up to the first order in x and y, we can expand $\mathbf{B}$ and $\mathbf{e}_v$ as

$$\mathbf{e}_v = \mathbf{e}_z + x'\mathbf{e}_x + y'\mathbf{e}_y.$$

and

$$\mathbf{B} = \mathbf{B}_0 + x\,\partial\mathbf{B}/\partial x + y\,\partial\mathbf{B}/\partial y,$$

where $\mathbf{e}_x$ and $\mathbf{e}_y$ are horizontal and vertical unit vectors, $\vec{\mathbf{e}}_z$ is the unit vector along the design orbit and primes denote derivatives with respect to the orbit length. Since we are interested in the dependence of the spin tune on the energy error ϵ and not in betatron oscillations, we may put $x = \epsilon\eta_x$ and $y = \epsilon\eta_y$, η_x and η_y being the horizontal and vertical dispersion functions. Now expanding the right-hand side of Eq. (2.1) in terms of ϵ up to the first order, we get

$$\frac{d\mathbf{s}}{d\theta} = [\Omega_0(\theta) + \epsilon\omega_\epsilon(\theta)] \times \mathbf{s}, \quad (2.2)$$

with

$$\omega_\epsilon = R\left\{-\left[(\gamma a + 1)G_x\eta_x - \frac{1}{\rho_x}\right]\mathbf{e}_y + \left[(\gamma a + 1)G_y\eta_y - \frac{1}{\rho_y}\right]\mathbf{e}_x\right\}, \quad (2.3)$$

and

$$G_x = \frac{e}{p_0}\frac{\partial B_y}{\partial x} + \frac{1}{\rho_x{}^2}, \qquad G_y = -\frac{e}{p_0}\frac{\partial B_y}{\partial x} + \frac{1}{\rho_y{}^2}, \quad (2.4)$$

where p_0 and Ω_0 are the momentum and spin-precession angular frequency of the equilibrium particle. In the derivation of Eq. (2.2), we have dropped the terms involving selenoid and skew quadrupole fields and neglected the contributions of x' and y' in bending magnets which are small in large machines. Both Ω_0 and ω_ϵ are periodic functions of θ.

Let us assume that the right-handed orthonormal solutions $\mathbf{n}_0(\theta)$, $\mathbf{m}_0(\theta)$ and $\mathbf{l}_0(\theta)$ for the equilibrium particle $\epsilon = 0$ are known. Here $\mathbf{n}_0$ is the periodic solution and $\mathbf{m}_0$ and $\mathbf{l}_0$ have the periodicity

$$\mathbf{k}_0(\theta + 2\pi) = \exp(2\pi i \nu_0)\mathbf{k}_0(\theta), \tag{2.5}$$

where $\mathbf{k}_0$ is defined by $\mathbf{k}_0 = \mathbf{m}_0 + i\mathbf{l}_0$.

Equation (2.2) can be derived from the Hamiltonian

$$H(J_0, \psi_0, \theta) = \nu_0 J_0 - \epsilon\omega_\epsilon(\theta)\cdot\mathbf{s}, \tag{2.6}$$

where J_0, ψ_0 are defined by

$$\mathbf{s} = J_0\mathbf{n}_0 + \sqrt{1 - J_0{}^2}\,\mathrm{Re}\,[\mathbf{k}_0{}^* \exp - i(\psi_0 - \nu_0\theta)]. \tag{2.7}$$

They are the action and angle variables of the unperturbed system. Let J be the new action variable and $S(J, \psi_0, \theta) = J\psi_0 + S_1(J, \psi_0, \theta)$ be Hamilton's principal function.Then we obtain as the new Hamiltonian

$$\begin{aligned} H_{\mathrm{new}}(J) &= \frac{\partial S}{\partial\theta} + H\left(\frac{\partial S}{\partial\psi_0}, \psi_0, \theta\right) \\ &= \frac{\partial S_1}{\partial\theta} + \nu_0\left(J + \frac{\partial S_1}{\partial\psi_0}\right) - \epsilon\omega_\epsilon\cdot[J\mathbf{n}_0 + \sqrt{1 - J^2}\,\mathrm{Re} \\ &\times (\mathbf{k}_0{}^* \exp - i(\psi_0 - \nu_0\theta))] \end{aligned}$$

up to the first order in ϵ. Averaging the right-hand side over θ gives

$$H_{\mathrm{new}}(J) = \left[\nu_0 - \frac{\epsilon}{2\pi}\int_0^{2\pi} \omega_\epsilon\cdot\mathbf{n}_0\,d\theta\right]J. \tag{2.8}$$

Now the spin chromaticity is found to be

$$\begin{aligned} \frac{d\nu}{d\epsilon} &= -\frac{1}{2\pi}\int_0^{2\pi} \omega_\epsilon\cdot\mathbf{n}_0\,d\theta \\ &= \frac{1}{2\pi}\int_0^C \left\{\left[(\gamma a + 1)G_x\eta_x - \frac{1}{\rho_x}\right]n_y - \left[(\gamma a + 1)G_y\eta_y - \frac{1}{\rho_y}\right]n_x\right\}dl, \end{aligned}$$

where l is the length along the equilibrium orbit and C the ring circumference. Using the equations of motion of η_x and η_y,

$$\eta_j'' + G_j\eta_j = 1/\rho_j, \qquad (j = x, y) \tag{2.9}$$

we derive the final result

$$\frac{d\nu}{d\epsilon} = \frac{1}{2\pi}\int_0^C \left\{\left[(\gamma a + 1)\eta_x'' + \frac{\gamma a}{\rho_x}\right] n_y - \left[(\gamma a + 1)\eta_y'' + \frac{\gamma a}{\rho_y}\right] n_x\right\} dl. \qquad (2.10)$$

In planar rings we find $d\nu/d\epsilon = \gamma a$, which is the contribution of the term $\gamma a/\rho_x$ since $n_x = 0$, $n_y = 1$ and the integration of η_x'' vanishes. In the double Siberian snake scheme, the contribution of the term $\gamma a/\rho_x$ is cancelled because $n_y = 1$ in one half of the ring and -1 in the other half.

§3. SPIN-CHROMATICITY CORRECTION

The idea of the present scheme is to reduce $d\nu/d\epsilon$ by making use of the term η_x'' and/or η_y'' in Eq. (2.10), keeping $n_y = 1$ in most of the ring.

Let us consider a ring of superperiodicity N, whose one superperiod is shown in Fig. 1. The dotted line is the equilibrium orbit and the full line is η_x drawn to the same scale as the equilibrium orbit. Suppose $n_y = 0$ in the intervals AA' and BB' and $n_y = 1$ in all the other parts. Then we find that if the full line at point A and that at point B are mutually parallel, the spin chromaticity is almost zero. Indeed, the orbit rotation angle of a particle of energy deviation ϵ between A and B is equal to $2\pi/N - (\eta_B' - \eta_A')\epsilon$ or $(2\pi/N)(1 - \epsilon)$. Since the particle does not precess around $\mathbf{n}$ in the sections AA' and BB', the net precession angle in one revolution is

$$N\cdot\frac{2\pi}{N}(1 - \epsilon)(\gamma a + 1) = 2\pi(\gamma_0 a + 1 - \epsilon),$$

and hence,

$$\nu(\epsilon) = \nu_0 - \epsilon$$

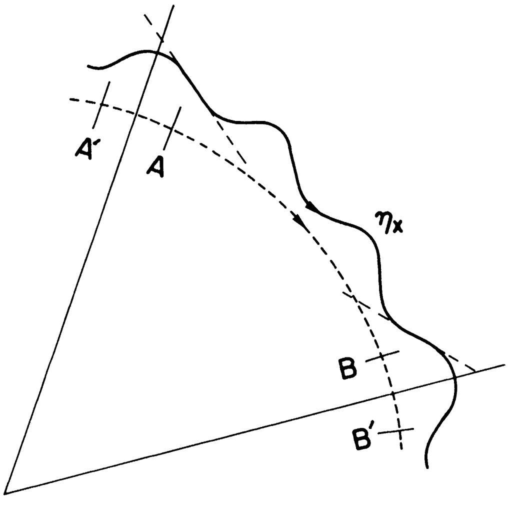

FIGURE 1 Example 1. One superperiod of a ring of superperiodicity N.

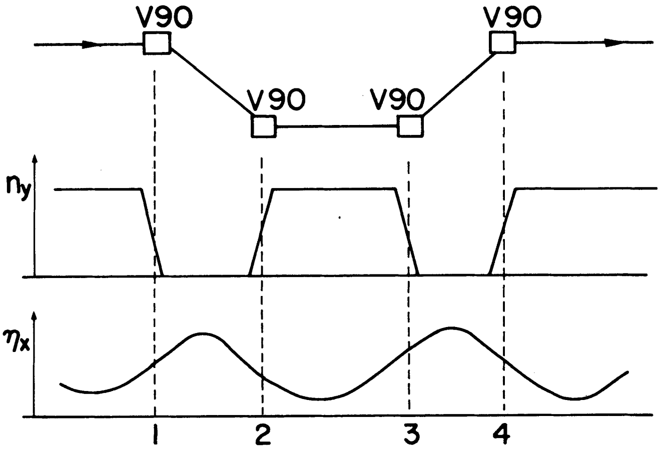

FIGURE 2 Example 2. Side view of a section, vertical component of vector **n** and eta function.

which means that the spin chromaticity is almost zero. One easily finds from Eq. (2.10) or a geometrical consideration that the spin chromaticity can be reduced by making n_y from unity to zero at the point $\eta_x' > 0$ and by making n_y from zero to unity where $\eta_x' < 0$.

A more realistic example is shown in Fig. 2. The upper drawing is the side view of a section which should be inserted where $n_y = 1$. "V90" is a vertical bend which rotates the spin by 90 degrees. Since n_x is zero throughout this section, the contribution of η_y need not be considered. If we make η_x as in the lower drawing, we have a reduction of the spin chromaticity by the amount of about $\gamma_0 a \times 4\eta_0'/2\pi$, assuming, for simplicity, $\eta_1' = -\eta_2' = \eta_3' = -\eta_4' \equiv \eta_0'$, where η_1' is the value of η_x' at location 1, etc.

From the practical viewpoint, these devices had better be imbedded in spin rotators, if any, which rotate the spin to the longitudinal direction. Such an example is shown in Fig. 3, which is based on the HERA design[4], with the orders of some magnets changed

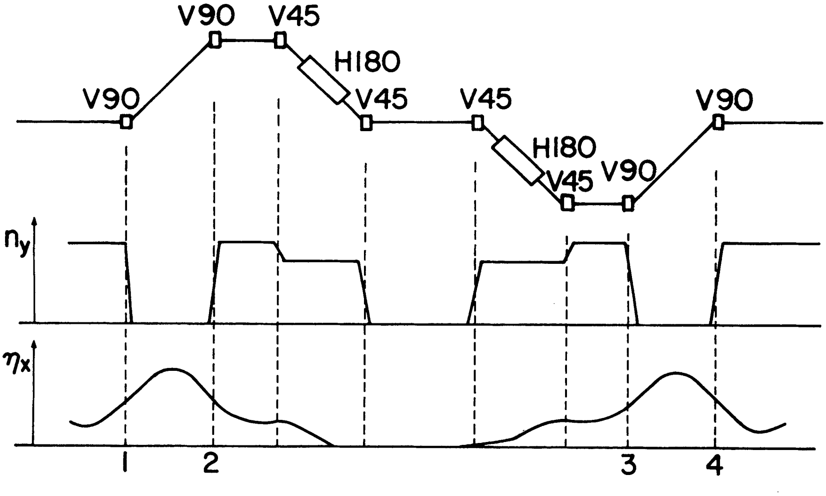

FIGURE 3 Example 3. Spin chromaticity corrector imbedded in a HERA type rotator.

for our purpose. In this figure, "H_x" and "V_x" are vertical and horizontal bends which rotate the spin by x degrees. Assuming again for simplicity, $\eta_1' = -\eta_2' = \eta_3' = -\eta_4' \equiv \eta_0'$ and neglecting the variation of n_y near "V45", we get a reduction of spin chromaticity by $\gamma_0 a \times 4\eta_0'/2\pi$. Unfortunately, if the insertion is completely antisymmetric, this method cannot be used.

In practice, we find various difficulties. The first problem is the required magnitude of η'. If, for example, there are four insertions around the ring, as in the case of Fig. 3, $\eta_0' = \pi/8 \approx 0.4$ is required for complete elimination of the spin chromaticity. However, for our problem, i.e., depolarization due to energy spread, reduction of one half of the spin chromaticity may be sufficient. The second problem is that the spin-transparency conditions[5] may become more complicated or even conflict with spin-chromaticity correction. So we may have to ignore some conditions by making the fields of some magnets in the insertions very weak. At any rate, these points should further be investigated for each ring design.

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