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Superluminal waves and objects: an overview of the relevant experiments

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1. Introduction.

It is a great privilege and pleasure to be able to present (after a brief theoretical introduction) a panoramic view of the experiments that —revealing the apparent existence of Superluminal motions— seem to confirm the pioneering works published by E.C.George Sudarshan, already in the sixties, about tachyons.

The question of Superluminal ($V^2 > c^2$) objects or waves has a long story, starting perhaps in 50 B.C. with Lucretius' *De Rerum Natura pervolgant* lumina *coelum?"*. Still in pre-relativistic times, one meets various related works, from those by J.J. Thomson to A. Sommerfeld.

With Special Relativity, however, since 1905 the conviction spread over that the speed c of light in vacuum was the *upper* limit of any possible speed.

For instance, R.C.Tolman in 1917 believed to have shown by his "paradox" that the existence of particles endowed with speeds larger than c would have allowed sending information into the past. Such a conviction blocked for more than half a century – aside from an isolated paper (1922) by the Italian mathematician G.Somigliana – any research about Superluminal speeds. Our problem started to be tackled again essentially in the fifties and sixties, in particular after the mentioned, epoch-making papers by George Sudarshan *et al.*[1], which provoked much further work, in particular by E.Recami and coworkers[2], as well as by H.C.Corben and others (to confine ourselves to the *theoretical* researches). The first experiments looking for faster-than-light objects were performed by T.Alväger *et al.*[2].

Superluminal objects were called tachyons, T, by G.Feinberg, from the Greek word $\tau\alpha\chi\acute{\upsilon}\varsigma$, quick (and this induced the present author in 1970 to coin the term bradyon, for ordinary subluminal ($v^2 < c^2$) objects, from the Greek word $\beta\rho\alpha\delta\acute{\upsilon}\varsigma$, slow). Finally, objects travelling exactly at the speed of light are called "luxons".

In recent years, terms as "tachyon" and "superluminal" fell unhappily into the (cunning, rather than crazy) hands of pranotherapists and mere cheats, who started squeezing money out of simple-minded people; for instance by selling plasters (!) that should cure various illnesses by "emitting tachyons"... We are dealing with tachyons here, however, since at least four different experimental sectors of physics seem to indicate the actual existence of Superluminal motions (thus confirming long-standing theoretical predictions [1,3]).

In Sec. 2 (after a non-technical theoretical introduction) we mention the various experimental sectors of physics in which Superluminal motions seem to appear. In particular, a bird's-eye view is presented of the experiments with evanescent waves (and/or tunnelling photons), and with the "localized Superluminal solutions" (SLS) to the wave equations, like the so-called X-shaped

waves; the shortness of this review is compensated by a number of references, sufficient in some cases to provide the interested readers with reasonable bibliographical information.

2. General concepts

As far as classical tachyons are concerned, let us insert Sudarshan's original contributions within the picture provided by Special Relativity (SR), once one *does not restrict* it[2,3] to subluminal motions.

Let us premise that SR, abundantly confirmed by experience, can be built on the two simple, natural Postulates:

1) that the laws (of electromagnetism and mechanics) are valid not only for a particular observer, but for the whole class of the "inertial" observers;

2) that space and time are homogeneous and space is moreover isotropic.

From these Postulates one can theoretically *infer* that one, and only one, *invariant* speed exists: and experience tells us such a speed to be the one, c, of light in vacuum (namely, 299.792458 km/s). Indeed, ordinary light possesses the peculiar feature of presenting always the same speed in vacuum, even when we run towards or away from it. It is just that feature, of being invariant, that makes the speed c quite exceptional: no bradyons, and no tachyons, can enjoy the same property.

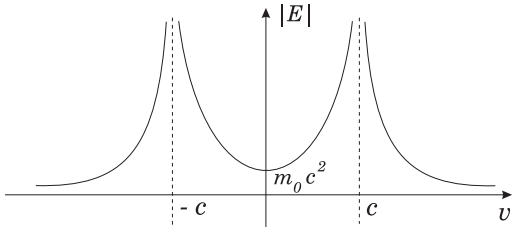

Figure 1. Energy of a free object as a function of its speed.

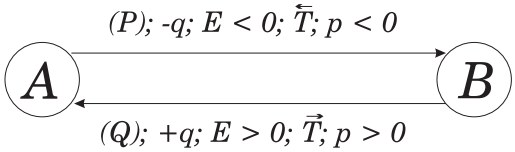

Figure 2. The "switching rule" (or reinterpretation principle) by Stueckelberg-Feynman-Sudarshan.

Another (known) consequence of our Postulates is that the total energy of an ordinary particle increases when its speed v increases, tending to infinity when v tends to c. Therefore, infinite forces would be needed for a bradyon to reach the speed c. This fact generated the popular opinion that speed c can be neither achieved nor overcome.

However, as speed c photons exist which are born, live, and die always at the speed of light[1] (without any need of accelerating from rest to the light speed), so objects can exist[4] always endowed with speeds V larger than c (see Fig.1). This circumstance has been picturesquely illustrated by Sudarshan (1972) with reference to an imaginary demographer studying the population patterns of the Indian subcontinent:

> Suppose a demographer calmly asserts that there are no people North of the Himalayas, since none could climb over the mountain ranges! That would be an absurd conclusion. People of central Asia are born there and live there: they did not have to be born in India and cross the mountain range. So with faster-than-light particles.

Let us add that, still starting from the above two Postulates (besides a third postulate, even more obvious, the theory of relativity can be generalized[1,3] in such a way as to accommodate also Superluminal objects; such a *non-restricted* version of SR is sometimes called "extended relativity". Also within extended relativity[3] the speed c, besides being invariant, is a limiting velocity: but every limiting value has two sides, and one can a priori approach it both from the left and from the right.

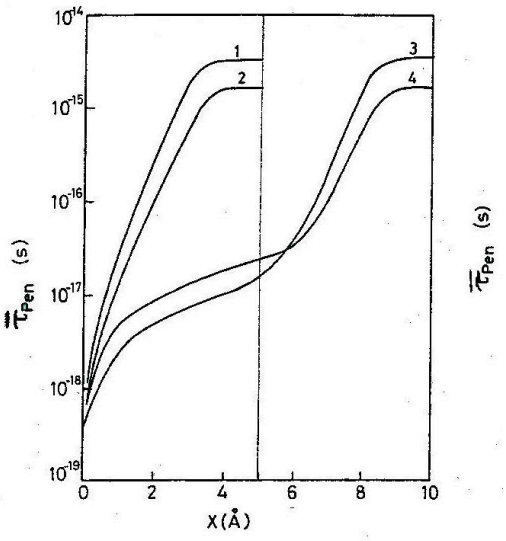

Figure 3. Behaviour of the average "penetration time" spent by a tunnelling wavepacket, as a function of the penetration depth down a potential barrier.

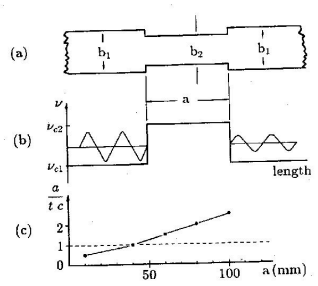

Figure 4. Simulation of quantum tunnelling by experiments with classical evanescent waves.

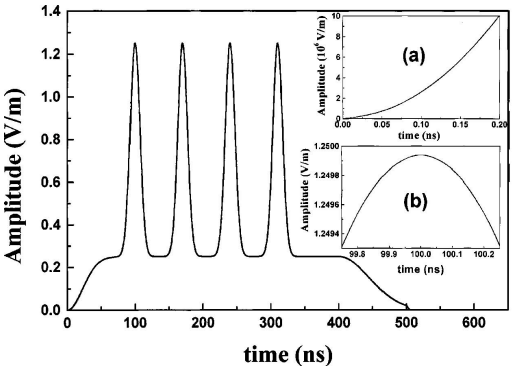

Figure 5. Experiment along a metallic waveguide with two barriers (undersized guide segments), i.e., with two evanescence regions

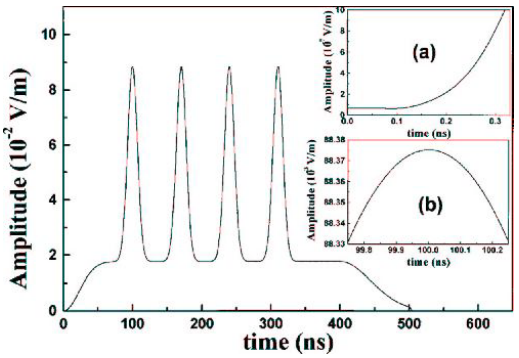

Figure 6. Appearances of a spherical (or pointlike, at the limit) object in vacuum at $v < c$ and $v > c$.

Actually, as we were saying, the ordinary formulation of SR has been restricted *too much*. For instance, even leaving Superluminal speeds aside, it can be easily so widened as to include antimatter[5]. Then, one finds space-time to be a priori populated by normal particles P (which travel forward in time carrying positive energy), *and* by dual particles Q "which travel backwards in time carrying negative energy". The latter shall appear to us as antiparticles, i.e., as particles – regularly travelling forward in time with positive energy, but – with all their "additive" charges (e.g., the electric charge) reversed in sign[5,1]: see Fig.2.

To clarify this point, we can here recall only what follows: We, as macroscopic observers, have to move in time along a single, well-defined direction, to such an extent that we cannot even see a motion backwards in time... and every object like Q, travelling backwards in time (with negative energy), will be *necessarily* reinterpreted by us as an anti-object, with opposite

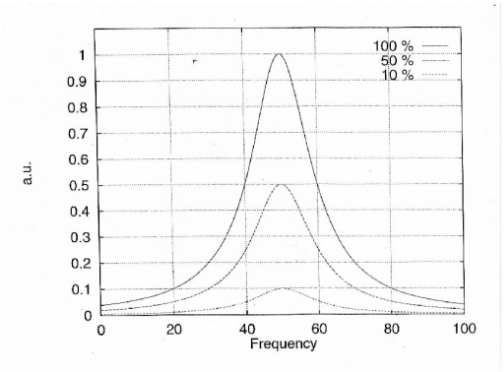

Figure 7. Intersections of an "X-shaped wave" with planes orthogonal to its motion line.

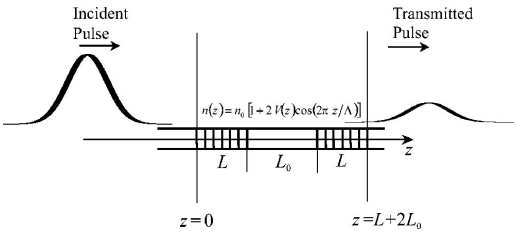

Figure 9. Theoretical prediction of Superluminal localized "X-shaped" waves for the electromagnetic case.

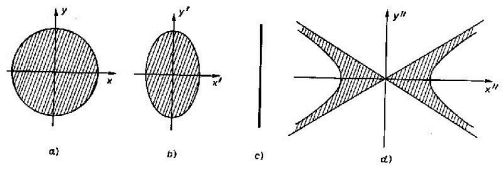

Figure 11. The square magnitude of the "classical", X-shaped Superluminal Localized Solution (SLS).

Figure 8. C

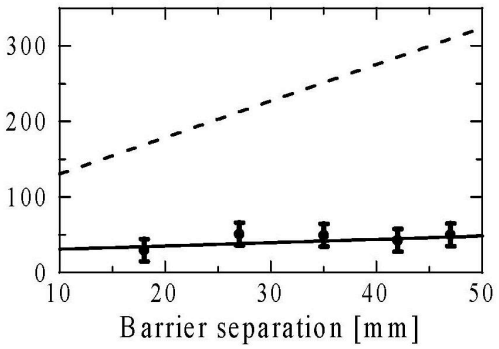

Figure 10. Scheme of the experiment by Saari *et al.*, who announced the production, in optics, of the waves depicted in Fig.8.

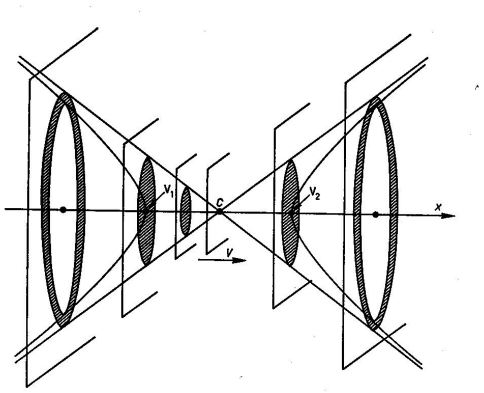

Figure 12. C

charges but travelling forward in time (with positive energy): cf. Fig.2 and refs.[3-5,1].

But let us forget about antimatter and go back to "tachyons". A strong objection against their existence is based on the opinion that by using tachyons it would be possible to send signals

into the past, owing to the fact that a tachyon T which, say, appears to a first observer O as emitted by A and absorbed by B, can appear to a second observer O' as a tachyon T' which travels backwards in time with negative energy[1,3]. However, by applying (as it is obligatory to do) the same "reinterpretation rule" or switching procedure seen above, T' will appear to the new observer O' just as an antitachyon $\overline{\mathrm{T}}$ emitted by B and absorbed by A, and therefore travelling forward in time, even if in the contrary *space* direction. In such a way, every travel towards the past, and every negative energy, disappear[1,3-5]. The mentioned reinterpretation procedure[1,3-5] ought to be called the Sudarshan's principle, or the Stueckelberg-Feynman-Sudarshan principle: indeed, it was Sudarshan[1] who stated it clearly, by taking proper account of the interplay between the signs *both* of the motion direction in time *and* of the energy.

Starting from this observation, it is possible to solve[1,6] the so-called causal paradoxes associated with Superluminal motions: paradoxes which result to be the more instructive and amusing, the more sophisticated they are, but that cannot be re-examined here.

Let us mention here just the following. The reinterpretation principle, according to which, in simple words, signals are carried only by objects which appear to be endowed with positive energy, does eliminate any information transfer backwards in time; but this has a price: that of abandoning the ingrained conviction that the judgement about what is cause and what is effect is independent of the observer[1-6]. In fact, in the case examined above, the first observer O considers the event at A to be the cause of the event at B. By contrast, the second observer O' will consider the event at B as causing the event at A. All the observers will however see the cause to happen *before* its effect[1-6].

Taking new objects or entities into consideration always forces us to a criticism of our prejudices. If we require the phenomena to obey the *law* of (retarded) causality with respect to all the observers, then we cannot demand also the *description* "details" of the phenomena to be invariant: namely, we cannot demand in that case also the invariance of the "cause" and "effect" *labels*[6,2].

To illustrate the nature of our difficulties in accepting that, e.g., the labels of cause and effect depend on the observer, let us cite an analogous situation that does not imply present-day prejudices:

> For ancient Egyptians, who knew only the Nile and its tributaries, which all flow South to North, the meaning of the word "south" coincided with the one of "upstream", and the meaning of the word "north" coincided with the one of "downstream". When Egyptians discovered the Euphrates, which unfortunately happens to flow North to South, they passed through such a crisis that it is mentioned in the stele of Tuthmosis I, which tells us about *that inverted water that goes downstream (i.e. towards the North) in going upstream* [Csonka, 1970].

In the last century, theoretical physics led us in a natural way to suppose the existence of various types of objects: like magnetic monopoles, quarks, strings, tachyons, besides black-holes etcetera: and various sectors of physics could not go on without them, even if the existence of most of them is uncertain (perhaps, also because attention has not yet been paid to some links existing among them: e.g., a Superluminal electric charge is expected to behave as a magnetic monopole; and a black-hole a priori can be the source of tachyonic matter). According to Democritus of Abdera, everything that was thinkable without meeting contradictions had to exist somewhere in the unlimited universe. This point of view – which was given by M.Gell-Mann the name of "totalitarian principle" – was later on expressed (T.H.White) in the humorous form "Anything not forbidden is compulsory"...

3. A glance at the experimental status-of-the-art

Extended Relativity can allow a better understanding of many aspects also of *ordinary* physics; and this remains true independently of the circumstance that tachyons do or do not exist in

our cosmos as asymptotically free objects (their existence as "intermediate states" is, of course, obvious[1,3]). As already said, we are dealing with Superluminal motions, however, since this topic has recently returned in fashion, especially because of the fact that at least three or four different experimental sectors of physics seem to suggest the possible existence of faster-than-light motions. Our first aim is putting forth in the following some information about the experimental results obtained in a couple of those different physics sectors, with a mere mention of the others.

3.1. Neutrinos

A long series of experiments, started in 1971, seems to show that the square ${m_0}^2$ of the mass m_0 of muon-neutrinos, and more recently of electron-neutrinos too, is negative; which, if confirmed, would mean that (when using a naïve language, commonly adopted) such neutrinos possess an "imaginary mass" and are therefore tachyonic, or mainly tachyonic[7,3]. Notice, incidentally, that in extended relativity the dispersion relation for a free Superluminal object becomes

$$\omega^2 - \boldsymbol{k}^2 = -\Omega^2, \text{ or } E^2 - \boldsymbol{p}^2 = -m_{\mathrm{o}}^2,$$

and there is *no* need therefore of imaginary masses! The present author can testify that at least by 1971 —and probably some years before (as well as we ourselves, by the way)— George Sudarshan had got the idea that neutrinos could be tachyons: an idea proposed in print by Cawley[7] later on, in 1972.

3.2. Galactic Micro-quasars

As to the *apparent* Superluminal expansions observed in the core of quasars[8] and, recently, in the so-called galactic microquasars[9], we shall not deal here with that problem, because it is far from the other topics of this paper: not to mention that for those astronomical observations there exist orthodox interpretations, based on ref.[10], that —even if "statistically" weak— are accepted by the majority of the astrophysicists.

Here, let us mention only that simple geometrical considerations in Minkowski space show that a *single* Superluminal light source would appear[11,3]: (i) initially, as a source in the "optical boom" phase (analogous to the acoustic "boom" produced by an airplane travelling with constant supersonic speed): namely, as an intense source which suddenly comes into view; and that (ii) afterwards seems to split into TWO objects receding one from the other with speed $V > 2c$ [both phenomena being similar to those actually observed, according to refs.[9]].

3.3. Evanescent waves and "tunnelling photons"

Within quantum mechanics (and precisely in the *tunnelling* processes), it had been shown that the tunnelling time —firstly evaluated as a simple "phase time" and later on calculated through the analysis of the wavepacket behaviour— does not depend on the barrier width in the case of opaque barriers ("Hartman effect")[12]. This implies Superluminal and arbitrarily large (group) velocities V inside long enough barriers: see Fig.3.

Experiments that may verify this prediction by, say, electrons are difficult: And, in fact, only preliminary results for tunnelling neutrons exist[12,26]. Luckily enough, however, the Schroedinger equation in the presence of a potential barrier is mathematically identical to the Helmholtz equation for an electromagnetic wave propagating, for instance, down a metallic waveguide along the x-axis (as recalled, e.g., by R.Chiao *et al.*[13]); and a barrier height U greater than the electron energy E corresponds (for a given wave frequency) to a waveguide of transverse size lower than a cut-off value. A segment of "undersized" guide —to go on with our example— does therefore behave as a barrier for the wave (photonic barrier)[16,13], as well as any other photonic band-gap filters. The wave assumes therein —like a particle inside

a quantum barrier— an imaginary momentum or wave-number and gets, as a consequence, exponentially damped along x. In other words, it becomes an *evanescent* wave (going back to normal propagation, even if with reduced amplitude, when the narrowing ends and the guide returns to its initial transverse size). Thus, a tunnelling experiment can be simulated[13,16] by having recourse to evanescent waves (for which the concept of group velocity can be properly extended[14]).

The fact that evanescent waves travel with Superluminal speeds (cf., e.g., Fig.4) has been actually *verified* in a series of famous experiments.

Namely, various experiments, performed since 1992 onwards by G.Nimtz *et al.* at Cologne[15], by R.Chiao, P.G.Kwiat and A.Steinberg's group at Berkeley[16], by A.Ranfagni and colleagues at Florence[17], and by others at Vienna, Orsay, Rennes[17] etc., verified that "tunnelling photons" travel with Superluminal group velocities. Let us add that also extended relativity had predicted[19] evanescent waves to be endowed with faster-than-c speeds; the whole matter appears to be therefore theoretically self-consistent. The debate in the current literature does not refer to the experimental results: which can be correctly reproduced even by numerical elaborations[20,21] based on Maxwell equations only (cf., e.g., the illuminating figures 11 and 13 in ref.[21], here reproduced as Figs.5* and 6* of this paper), but rather to the question whether they allow, or do not allow, sending signals or information with Superluminal speed[22,21,14]. Actually, a peaked wavepacket suffers a strong amplitude attenuation while travelling inside a quantum or classical barrier; its width, however, remains unaffected (cf. Fig.7*): Something that might have some relevance when thinking of attempting transmissions by Morse's alphabet). Moreover, many authors have emphasized that —at least in the case of quantum barriers— the tunnelling of particles is a statistical process, in the sense that one cannot know a priori which particle, or photon, will pass through the barrier. This is true, of course; but the weight of such a consideration becomes lower when the number of the particles at our disposal for attempting a "signal" transmission does increase. To remain within the Morse alphabet example, one can send out *dots* and *dashes* by emitting pulses of, say, one thousand and ten thousand particles each, respectively; the dot and dashes will then be recognized also after the tunnelling!... This becomes even more meaningful when one approaches the classical limit. The claim that superluminal tunnelling cannot be used to transmit any information is in need, therefore, of further discussion, especially at the light of what will follow below: More details can be found, anyway, in refs.[50,51].

Let us repeat that all the phenomena mentioned in this, as well as in the following, subsections can be accomodated into the standard frameworks of quantum physics or of classical relativistic physics: It is therefore obvious that such phenomena can receive explication or interpretation in terms of standard physics (e.g., sometimes but not always, in terms of suitable "reshapings"): but this does not eliminate the fact that Superluminal motions take place.

As we already said, in the above-mentioned experiments one meets a substantial attenuation of the considered pulses during tunnelling (or during propagation in an absorbing medium). However, by having recourse to suitable devices, as a "gain doublet", it has been recently reported the observation of undistorted pulses propagating with Superluminal group-velocity with a *small* change in amplitude[23].

Let us underline that some of the most interesting experiments of this series seem to be the ones with TWO "barriers" (e.g., with two gratings in an optical fiber, or with two segments of undersized waveguide separated by a piece of normal-sized waveguide: Fig.5=8*). For suitable frequency bands —i.e., for "tunnelling" far from resonances—, it was found that the total crossing time does *not* depend on the length of the intermediate (normal) guide: namely, that the wavepacket speed along it is infinite[24,25]... This agrees with what predicted by Quantum Mechanics for the non-resonant tunnelling through two successive opaque barriers (namely, the tunnelling phase time, which depends on the entering energy, has been shown by

us to be *independent* of the distance between the two barriers[26]); something that has been theoretically confirmed, and generalized, by Y.Aharonov *et al.*[26]. Such a prediction has been experimentally verified a second time, with a cleaner experiment, taking advantage of the circumstance that quite interesting evanescence regions can be constructed in the most varied manners, like by means of different photonic band-gap materials or gratings (it being possible to use from multilayer dielectric mirrors, or semiconductors, to photonic crystals...). Indeed, a very recent confirmation came from an experiment having recourse to two gratings in an optical fiber[25]. On this respect, rather interesting are the figures 1 and 5 of ref.[25], here reported as Figs.9* and 10* of this paper (see especially the experimental results depicted in Fig.10*).

We cannot skip a further topic —which, being delicate, should not appear in a brief overview as this one— since some experimental contributions to it (like the one performed at Princeton by J.Wang *et al.*[23] and published in *Nature* on July 20, 2000) arose a general interest.

Even if in extended relativity all the ordinary causal paradoxes seem to be solvable[6,3,1] on the basis of the above-seen Stueckelberg-Feynman-Sudarshan reinterpretation rule, *nevertheless* —let us repeat— one has to remember that (whenever it has to be considered an object, $\mathcal{O}$, travelling with Superluminal speed) one can meet negative contributions to the *tunnelling times*[27,12]: and this should not be regarded as unphysical. In fact, whenever an "object" (electromagnetic pulse, particle,...) $\mathcal{O}$ *overcomes* the infinite speed[3,6] with respect to a certain observer, it will afterwards appear to the same observer as the "*anti*-object" $\overline{\mathcal{O}}$ travelling in the opposite *space* direction[1,3,6].

More precisely, when going on from the lab to a frame $\mathcal{F}$ moving in the *same* direction as the particles or waves entering the barrier region, the object $\mathcal{O}$ penetrating and travelling through the final part of the barrier (with almost infinite speed[12,21,26,27], like in Figs.3) will appear in the frame $\mathcal{F}$ as an anti-object $\overline{\mathcal{O}}$ crossing that portion of the barrier *in the opposite space-direction*[6,3,1]. In the new frame $\mathcal{F}$, therefore, such anti-object $\overline{\mathcal{O}}$ would yield a *negative* contribution to the tunnelling time: which could even result, in total, to be negative. For clarifications, see refs.[28]. What we want to stress here is that the appearance of such negative times is once more predicted by relativity itself, on the basis of the ordinary postulates[3,6,21,28]. (In the case of a non-polarized wave, the wave anti-packet coincides with the initial wave packet; if a photon is however endowed with helicity $\lambda = +1$, the anti-photon will bear the opposite helicity $\lambda = -1$).

From the theoretical point of view, besides refs.[3,6,12,21,27,28], see refs.[29]. On the (quite interesting!) experimental side, see papers [30].

Let us *add* here that, via quantum interference effects it is possible to obtain dielectrics with refraction indices very rapidly varying as a function of frequency, also in three-level atomic systems, with almost complete absence of light absorption (i.e., with quantum induced transparency)[31]. The group-velocity of a light pulse propagating in such a medium can decrease to very low values, either positive or negatives, with *no* pulse distortion. It is known that experiments have been performed both in atomic samples at room temperature, and in Bose-Einstein condensates, which showed the possibility of reducing the speed of light to a few meters per second. Similar, but negative group velocities, implying a propagation with Superluminal speeds thousands of times higher than the previously mentioned ones, have been recently predicted also in the presence of such an "electromagnetically induced transparency", for light moving in a rubidium condensate[32], while the corresponding experiments are being performed (for instance at the "LENS" laboratory in Florence).

Finally, let us recall that faster-than-c propagation of light pulses can be (and was, in same cases) observed also by taking advantage of anomalous dispersion near an absorbing line, or nonlinear and linear gain lines —as already seen—, or nondispersive dielectric media, or inverted two-level media, as well as of some parametric processes in nonlinear optics (cf., e.g., G.Kurizki *et al.*'s works).

3.4. Superluminal Localized Solutions (SLS) to the wave equations. The "X-shaped waves"
The fourth sector (to leave aside the others) is not less important. It came into fashion again, when some groups of scholars in engineering (for sociological reasons, most physicists had abandoned the field) rediscovered by a series of works that any wave equation —to fix the ideas, let us think of the electromagnetic case— admit also solutions as much sub-luminal as Super-luminal (besides the ordinary —plane, spherical,...— waves endowed with speed c/n).

Let us recall that, starting with the pioneering work by H.Bateman, it had slowly become known that all wave equations (in a general sense: scalar, electromagnetic, spinorial,...) admit wavelet-type solutions with sub-luminal group velocities[33]; namely, soliton-like solutions, even if they are linear equations. Subsequently, also Superluminal solutions started to be written down. A quite important feature of some new solutions of these (which attracted much attention for possible applications) is that they propagate as *localized*, non-diffracting pulses: namely, according to the Courant and Hilbert's terminology[33], as "undistorted progressing waves"; which possess the further property of "self-reconstructing" themselves after obstacles smaller than their *aperture* size (that is, smaller than the width of the antenna generating them, enourmously larger, in general, than their wavelenth). It is easy to realize the practical importance, for instance, of a radio transmission carried out by localized waves, independently of their being sub- or Super-luminal. But non-diffractive wave packets can be of use even in theoretical physics for a reasonable representation of elementary particles[37]; and so on.

Within extended relativity since 1980 it had been found that —whilst the simplest subluminal object conceivable is a small sphere, or a point as its limit— the simplest Superluminal objects turns out to be instead an "X-shaped" wave (see refs.[38], and Figs.6=11* and 7=12* of this paper), or a double cone as its limit, which moreover travels without deforming —i.e., rigidly— in a homogeneous medium[3]. Analogously, the equipotential surfaces of the electrostatic field, generated by a tiny charged sphere at rest, will assume[3,38] the shape represented in Fig.13* when the source is Superluminal. For clarifying the connection existing between what predicted by SR and the localized X-waves (mathematically and experimentally constructed in recent times, as we are going to see) let us refer ourselves to a paper recently appeared in Physical Review E, i.e., to the last one of refs.[38], where the issue of the (X-shaped) field created by a Superluminal electric charge has been tackled. Localized waves do exist, of course, with any group velocicty (the subluminal ones being ball-like, as expected); but it is not without meaning that the most interesting localized solutions happened to be just the Superluminal ones, and with a shape of that kind. Even more, since from Maxwell equations under simple hypotheses one goes on to the usual *scalar* wave equation for each electric or magnetic field component, one could expect the same solutions to exist also in the field of acoustic waves, and of seismic waves (and of gravitational waves too).

In other words, from the present point of view, it is rather interesting to note that, during the last fifteen years, "X-shaped" waves have been *actually* found as solutions to the Maxwell and to the wave equations [the form of any wave equations is intrinsically relativistic, by the way]. Actually, such waves (as suitable superpositions of Bessel beams[39], that is, of simple solutions to the wave equation already discovered[34] in 1941) were mathematically constructed for the first time by Lu *et al.*[40], *in acoustics*: and later on by Recami *et al.*[41] for electromagnetism; and were then called "X-waves" or rather X-shaped waves. In an elementary Appendix we briefly show how X-shaped solutions to the wave equation (in particular, the "classic" X-wave) can be constructed.

It is more important for us that the X-shaped waves have been indeed *produced in experiments* both with acoustic and with electromagnetic waves; indeed, X-waves were produced that, in their medium, travel undistorted with a speed larger than sound, in the first case, and than light, in the second case. In acoustics, the first experiment was performed by Lu *et al.* themselves[42] in 1992, at the Mayo Clinic (and their papers received the first IEEE 1992 award). In the

electromagnetic case, certainly more intriguing, Superluminal localized X-shaped solutions were first mathematically constructed (cf., e.g., Fig.8=14*) in refs.[41], and later on experimentally produced by Saari *et al.*[43] in 1997 at Tartu by visible light (Fig.9=15*), as announced in their Physical Review Letters article, and more recently, as we already mentioned, by Mugnai, Ranfagni and Ruggeri at Florence by microwaves[44] (paper appeared in Physical Review letters too, three years later, in 2000). Further experimental activity is in progress; while in the theoretical sector the activity has been growing so intensely, that it is not possible to quote here the relevant recent literature; we might recall, e.g., the papers devoted to building up new analogous solutions with finite total energy or more suitable for high frequencies, on one hand, and localized solutions Superluminally propagating even along a normal waveguide, on the other hand[45,46]; or the attempts at focusing X-shaped waves, at a certain instant, in a small region[47]. But we cannot avoid mentioning that suitable superpositions of Bessel beams (which can originate also subluminal pulses) can produce even *stationary* intense wave-field: confined within a tiny region (a static envelope); while the field intensity outside that region is everywhere negligible[48]; such "frozen waves" can have (a patent is pending) very many important applications, for instance as a new kind of tweezers, and especially —of course— in medicine.

Before going on, let us eventually touch the problem of producing an X-shaped Superluminal wave like the one in Fig.7=12*, but truncated – of course – in space and in time (by the use of a finite, dynamic antenna, radiating for a finite time): in such a situation, the wave will keep its localization and Superluminality only along a certain "depth of field", decaying abruptly afterwards[39,41].

We can become convinced about the possibility of realizing it, by imagining the simple ideal case of a negligibly sized Superluminal source S endowed with speed $V > c$ in vacuum and emitting electromagnetic waves W (each one travelling with the invariant speed c). The electromagnetic waves will result to be internally tangent to an enveloping cone C having S as its vertex, and as its axis the propagation line x of the source[3].

This is analogous, as we know, to what happens for an airplane that moves in the air with constant supersonic speed. The waves W interfere mostly negatively inside the cone C, and constructively only on its surface. We can place a plane detector orthogonally to x, and record magnitude and direction of the W waves that hit on it, as (cylindrically symmetric) functions of position and of time. It will be enough, then, to replace the plane detector with a plane antenna which *emits* —instead of recording— exactly the same (axially symmetric) space-time pattern of waves W, for constructing a cone-shaped electromagnetic wave C that will propagate with the Superluminal speed V (of course, without a source any longer at its vertex): even if each wave W travels with the invariant speed c.

Here let us only remark that such localized Superluminal waves appear to keep their good properties only as long as they are fed by the waves arriving (with speed c) from the antenna: Taking account of the time needed for fostering such Superluminal pulses (i.e., for the arrival of the feeding speed-c waves coming from the aperture). one concludes that these localized Superluminal waves are probably unable to transmit *information* faster than c. However, they don't seem to have anything to do with the illusory "scissors effect", even if the energy feeding them appears to travel with the speed of light. In fact, the spot —endowed, as we know, with Superluminal group-velocity— is able to get, for instance, two (tiny) detectors at a distance L to click after a time *smaller* than L/c). A lot of discussion is still going on about the possible differences among group-velocity, signal-velocity and information speed.

As we mentioned above, the existence of all these X-shaped Superluminal (or "Super-sonic") waves seem to constitute at the moment, together, e.g., with the Superluminality of evanescent waves, some valuable confirmations of refs.[1], as well as of extended relativity: a theory[3], let us recall, based on the ordinary postulates of SR and that consequently does not appear to violate

any of its fundamental principles. It is curious, moreover, that one of the first applications of such X-waves (that takes advantage of their propagation without deformation) is in advanced progress in the field of medicine, and precisely of ultrasound scanners[49].

Before ending, let us remark that a series of new SLSs to the Maxwell equations, suitable for arbitrary frequencies and arbitrary bandwidths have been recently constructed by us: many of them being extremely well localized in the surroundings of their vertex, and some of them being endowed with *finite* total energy. Among the others, we have set forth an infinite family of generalizations of the classical X-shaped wave; and shown how to deal with the case of a *dispersive* medium. Results of this kind may find application in other fields in which an essential role is played by a wave-equation.

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APPENDIX

In this Appendix we want to show how localized superluminal solutions (SLS) to the wave equations, and in particular the X-shaped ones, can be mathematically constructed. Here we shall consider, for simplicity, only the case of a dispersionless medium like vacuum, and of free space (without boundaries).

It is known since more than a century that a particular axially symmetric solution to the wave equation in vacuum ($n = n_0$) is, in cylindrical coordinates, the function $\psi(\rho, z, t) = J_0(k_\rho \rho)\, \mathrm{e}^{+ik_z z}\mathrm{e}^{-i\omega t}$ with $k_\rho^2 = n_0^2 \frac{\omega^2}{c^2} - k_z^2$; $k_\rho^2 \geq 0$, where J_0 is the zeroth-order ordinary *Bessel function*, k_z and k_ρ are the axial and the transverse wavenumber respectively, ω is the angular frequency and c the velocity of light. Using the transformation

$$\begin{cases} k_\rho = \dfrac{\omega}{c}\, n_0\, \sin\,\theta \\ \\ k_z = \dfrac{\omega}{c}\, n_0\, \cos\,\theta \end{cases} \tag{1}$$

such particular solution $\psi(\rho, z, t)$ can be rewritten in the the well-known *Bessel beam* form:

$$\psi(\rho, \zeta) \;=\; J_0(n_0 \frac{\omega}{c} \rho \sin\theta)\; \mathrm{e}^{+in_0 \frac{\omega}{c} \zeta \cos\theta} \tag{2}$$

where $\zeta \equiv z - Vt$ while $V = c/(n_0 \cos\,\theta)$ is the phase velocity, quantity θ $(0 < \theta < \pi/2)$ being the cone angle of the Bessel beam.

More in general, SLSs (with axial symmetry) to the wave equation will be the following ones[40,50]:

$$\psi(\rho, \zeta) \;=\; \int_0^\infty S(\omega) J_0 \left(\frac{\omega}{V} \rho \sqrt{n_0^2\, \frac{V^2}{c^2} - 1} \right)\; \mathrm{e}^{+i\frac{\omega}{V}\zeta} \mathrm{d}\omega, \tag{3}$$

where $S(\omega)$ is the adopted frequency spectrum.

Indeed, such solutions result to be pulses propagating in free space without distortion and with the Superluminal velocity $V = c/(n_0 \cos\theta)$. The most popular spectrum $S(\omega)$ is that one given by $S(\omega) = \mathrm{e}^{-a\omega}$, which provides the ordinary ("classic") X-shaped wave

$$X \equiv \psi(\rho, \zeta) \; = \; \frac{V}{\sqrt{(aV - i\zeta)^2 + \rho^2(n_0^2 \frac{V^2}{c^2} - 1)}}. \tag{4}$$

Because of its non-diffractive properties and its low frequency spectrum, the X-wave is being particularly applied in fields like acoustics[42]. The "classic" X-wave is represented in Fig.10=16*. As we already said in the text, infinite series of SLSs can be constructed, more and more concentrated in the vicinity of their vertex, and corresponding to any desired frequency and bandwidth.

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