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Scattering in ICOOL

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We review the scattering models available in ICOOL. Two new options have recently been added using the scattering models of U. Fano and of A. Tollestrup. We find excellent agreement between the Monte Carlo distributions from ICOOL using these models and the experimental results from the MuScat experiment. We also find good agreement of the hydrogen data with the ELMS model. The new scattering models lead to ~5% improvement in accepted muons for the Study 2a neutrino factory and ~65-150% improvement in the merit factor for the ideal RFOFO cooling ring.

1. Introduction

Multiple Coulomb scattering plays a crucial role in the effectiveness of ionization cooling. It is therefore important that Monte Carlo simulations of ionization cooling include accurate models of multiple scattering. This is commonly done using the Moliere theory in the form given by Bethe [1]. However, there are two problems with the Bethe model that have potentially important consequences for ionization cooling [2]. First the contributions from scattering from atomic electrons is not handled correctly for a heavy incident particle like a muon. Second the model used for atomic screening is not correct for the low-Z materials used as absorbers in ionization cooling. These issues were first addressed by Fano [3] who modified some of the parameters used in the Moliere theory. The subject was reexamined later by Tollestrup [4] who proposed a different set of changes to the parameters in the Moliere theory. Monte Carlo versions of the Fano and Tollestrup models have now been included in ICOOL.[1] In this report we will compare predictions of the following scattering models:

- Bethe
- Fano
- Tollestrup
- ELMS

[1] Available in version 2.93 or later of ICOOL.

ELMS is a combined model of energy loss and multiple scattering based on photo absorption cross sections [5]. A version of this model for hydrogen already exists in ICOOL.[2] These model predictions will be compared with recent scattering data from the MuScat experiment [6].

2. New scattering models in ICOOL

Two new scattering models have been incorporated into ICOOL.

Fano model

For incident muons this model uses Z^2 instead of $Z(Z+1)$ for calculating the Moliere characteristic angle χ_C^2. The Moliere parameter *b* (which is related to the effective number of scatters) is increased by a quantity u_{in} that depends on an integral over the inelastic form factor. The values of u_{in} used in the Monte Carlo model are shown in Table 1.

Table 1. Values of the Fano u_{in} parameter.

H	He	Li	Be	LiH	all others
3.6	4.2	4.6	4.3	4.1	5

The values for He and Be were determined by numerical integration using experimental values of the inelastic form factor [7].

Tollestrup model

This model takes explicit account of the fact that the maximum scattering angle for muons from atomic electrons is $\theta_o \sim 4.8$ mrad. The model has two branches depending on whether the rms scattering angle θ_{rms} is smaller or larger than θ_o. For the comparisons in this report we always take small step sizes, so that $\theta_{rms} < \theta_o$. In this case it is appropriate to use $Z(Z+1)$ in computing χ_C^2. The large angle tail is forced to obey the Z^2 dependence of Rutherford scattering. Whenever a Monte Carlo angle θ is generated that is larger than θ_o, an additional random number *r* is generated. If $r > Z/(Z+1)$ the angle θ is discarded and another scattering angle is generated. This continues until we have a case where $r < Z/(Z+1)$. Experimental form factor data is used to compute required elastic and inelastic integrals for $Z < 5$. Standard Moliere theory is used to compute the screening angle for larger Z.

[2] Based on code provided by Simon Holmes, November 2004.

3. Comparison with the MuScat data

Results from the MuScat scattering experiment at TRIUMF are now available. This experiment examined many of the low-Z elements used for ionization cooling. We can use this data to compare the predictions of the different Monte Carlo scattering models. All the models are used with small step sizes. Figure 1 shows a comparison for liquid hydrogen.

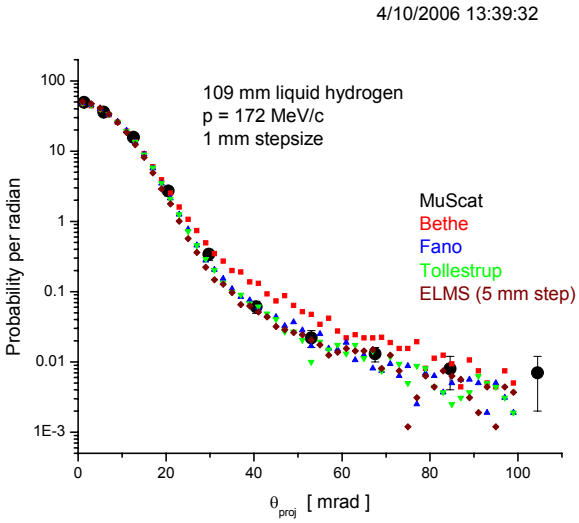

Figure 1. Comparison of models and data for liquid hydrogen.

Outside the central Gaussian region the Bethe model overestimates the amount of large angle scattering. At the largest scattering angles the data flattens out, either from a statistical fluctuation or from some background that was not properly removed. The other three models follow the data up to 80 mrad very well. The models in Figure 1 were computed using the standard liquid hydrogen density of 0.0708 g/cm^3. Figure 2 shows a comparison using the measured density of 0.0755 g/cm^3 from the MuScat experiment.

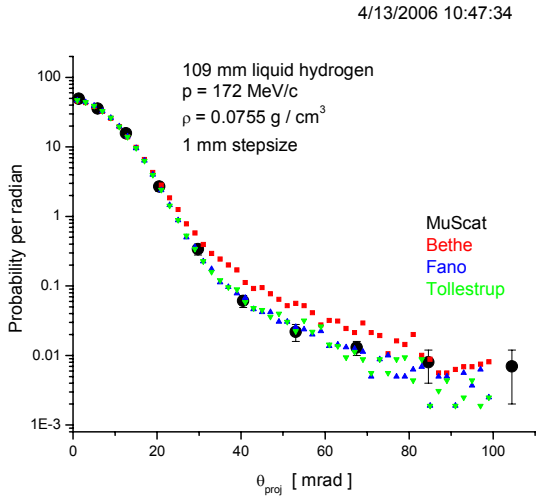

Figure 2. Comparison of models and data for liquid hydrogen with measured density.

The Fano and Tollestrup models do a good job of reproducing the data up to the region where the data background become important[3]. Figure 3 shows a comparison for lithium.

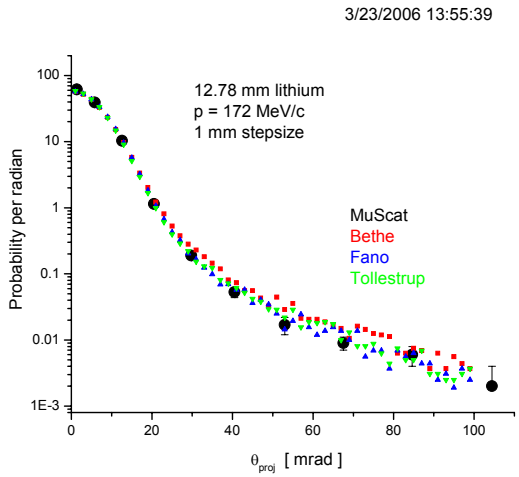

Figure 3. Comparison of models and data for lithium.

The Bethe model overestimates the large angle data. The Fano and Tollestrup models agree well with the data. Figure 4 shows a comparison for beryllium.

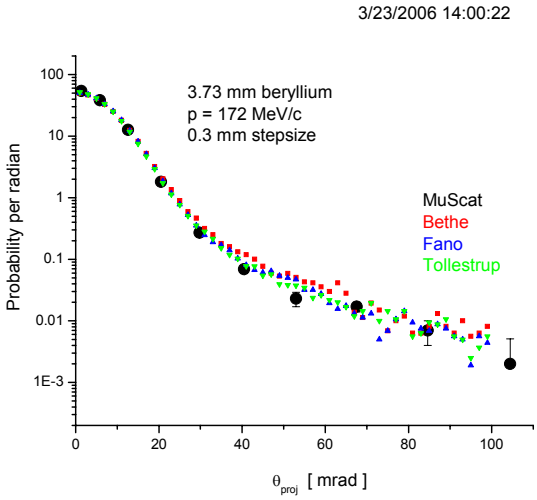

Figure 4. Comparison of models and data for beryllium.

The Bethe model overestimates the large angle data. The Fano and Tollestrup models agree well with the data with the possible exception of the data point near 50 mrad. It is not clear whether the discrepancy there is due to the experiment or to the models or is just a statistical fluctuation.

[3] The ELMS database uses the standard value for the hydrogen density in the current version of ICOOL.

Figure 5 shows a comparison for carbon.

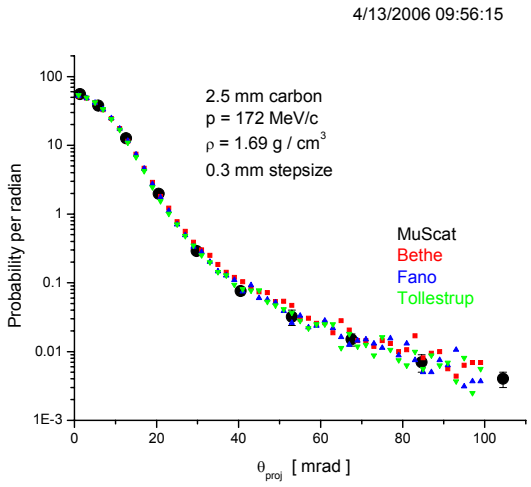

Figure 5. Comparison of models and data for carbon.

The Bethe model slightly overestimates the large angle data. Both the Fano and Tollestrup models agree very well with the data. Figure 6 shows a comparison for aluminum.

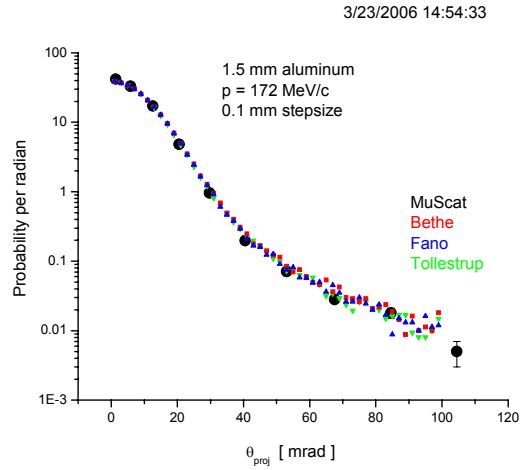

Figure 6. Comparison of models and data for aluminum.

The Bethe model slightly overestimates the large angle data. The Fano and Tollestrup models agree well with the data.

The previous figures showed comparisons for elements. Figure 7 shows a comparison for a compound material, polyethylene.

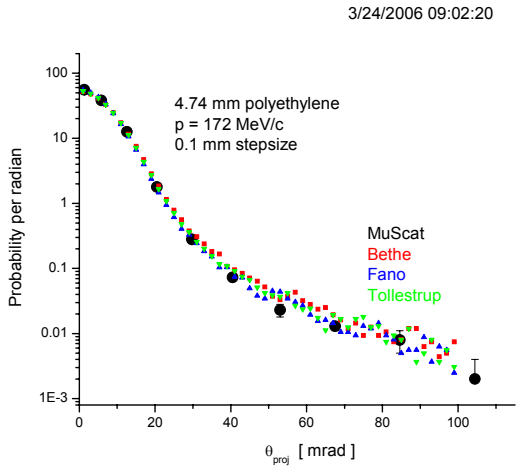

Figure 7. Comparison of models and data for polyethylene.

The Bethe model overestimates the large angle data. The Fano and Tollestrup models agree well with the data with the possible exception of the data point near 50 mrad.

There are small quantitative differences between all the Monte Carlo models and the experimental data[4]. Table 2 shows the value of the scattering probability for the first bin near 0 degrees.

Table 2. Scattering probability per radian near 0 degrees.

	ρ [g / cm^3]	Exp.	Bethe	Fano	Tollestrup
H_2	0.0755	49.5 ± 2.7	47.4	48.2	48.5
Li	0.53	61.7 ± 3.4	57.3	57.0	58.8
Be	1.85	54.2 ± 5.4	50.2	50.7	51.9
C	1.69	55.7 ± 3.0	52.0	52.2	53.6
Al	2.70	41.7 ± 2.4	38.4	38.6	39.0
CH_2	0.93	55.4 ± 3.1	52.2	54.5	53.1

It can be seen that all the Monte Carlo data is systematically ~1 sigma below the experimental data. It is unlikely that this small deviation can lead to significant errors in the performance of ionization cooling channels.

[4] This result was pointed out by Sergei Striganov.

4. Comparison of models for LiH

Lithium hydride is an important compound material for ionization cooling. Unfortunately no scattering measurements were made of this material in the MuScat experiment. We show a comparison of the model predictions for 1 cm of LiH. This is the same thickness as the absorber used in Study 2a.

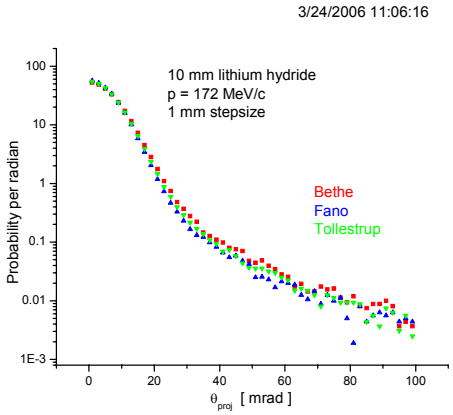

Figure 8. Comparison of models for lithium hydride.

The Fano and Tollestrup models are close together over most of the angular range, while the Bethe model is higher as usual. From the previous discussions we feel save in using either the Fano or the Tollestrup model to predict the scattering.

5. Effect on performance

A comparison of the effects of the scattering models on the Study 2a performance is shown in Table 3. The Bethe model has been the standard (default) scattering model in ICOOL up until this time.

Table 3. Effect of scattering model on Study 2a performance

model	μ_A / p	ε_{TN} [mm]
Bethe	0.178	7.1
Fano	0.188	6.3
Tollestrup	0.186	6.8

Study 2a uses LiH absorbers and Be windows. The usual performance measure for a neutrino factory is the number of muons contained in the accelerator acceptance. We use A_T = 30 mm and A_L = 150 mm for Study 2a. We see that using the Fano or Tollestrup models increases the number of accepted muons by ~5%. We also show the effect on the transverse emittance in the last column of Table 3.

We have seen in section 3 that the biggest deviations of the Bethe model from the MuScat measurements occurred for liquid hydrogen absorbers. Thus let us look at the

effects of the new scattering models on the ideal RFOFO cooling ring, which uses liquid hydrogen absorbers (but no windows or empty cells). The results are shown in Table 4.

Table 4. Effect of scattering model on ideal RFOFO performance

model	M	μ_A / p
Bethe	99	0.461
Fano	168	0.516
Tollestrup	160	0.514
ELMS	250	0.540

The usual performance measure for a cooling ring is the M-factor

$$M(s) = \frac{\varepsilon_{6N}(0)}{\varepsilon_{6N}(s)} \frac{N(s)}{N(0)}$$

where ε_{6N} is the 6-dimensional normalized emittance, N is the number of surviving muons, and s corresponds to 15 turns around the ring. ELMS gives a very large ~150% increase in the M-factor. For the Fano and Tollestrup models the improvement of ~66% is still very large. We also show the number of muons in an acceptance volume in the last column of Table 4. For this case we used A_T = 15 mm and A_L = 35 mm.

6. Conclusions

The Fano and Tollestrup multiple scattering models have been added to ICOOL. Both models agree very well with the measurements from the MuScat experiment. In addition the ELMS model agrees very well with hydrogen data. Previous ICOOL modeling using the Bethe model overestimated the amount of multiple scattering. Using a more correct model only has a small effect on the performance of the Study 2a neutrino factory. However, the corrected scattering models lead to much better performance of the RFOFO cooling ring.

Acknowledgements

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References

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