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# ATOMIC SHIFTS NEAR ABSORPTIVE MIRRORS*

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The Huttner-Barnett-Loudon theory is adapted to yield the Van der Waals (nonretarded) energy shifts and frictional forces experienced by an ion or molecule near an absorptive mirror.

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For atoms near nonabsorptive mirrors (having no or at most strictly zero-width absorption lines), the most elementary way to deal with proximity effects (i) determines the normal modes of the electromagnetic field; (ii) quantizes them so that the energy of field plus mirror becomes the usual sum of oscillator Hamiltonians; (iii) couples the atom to the field; and (iv) uses leading-order perturbation theory to evaluate the effects of this coupling. But if the mirrors are absorptive, then the familiar normal modes are damped, and it is not immediately obvious how to incorporate the random statistical mechanisms responsible for the damping into a *convenient* Hermitean Hamiltonian.

Here, the topic in the title serves largely as an occasion to adapt the approach due to Huttner, Barnett, Loudon, and others [1], so that an absorptive (complex) dielectric mirror response

$$\varepsilon(\omega) = \frac{\omega_T^2 + \omega_p^2 - \omega^2 - i\omega\Gamma}{\omega_T^2 - \omega^2 - i\omega\Gamma} \tag{1}$$

can be dealt with by essentially the same elementary method as was outlined above for use without absorption. What is crucial is their way of

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introducing a heat bath responsible for the damping (i.e. for $\Gamma \neq 0$). *New here are (i) an extension of their method to media that are both three-dimensional and bounded, and (ii) the determination of the exact normal modes of bath plus mirror through appropriate and highly convenient Lippmann-Schwinger equations*. The model for the mirror is restricted to the nonretarded regime (atom-mirror distance $Z$ far below all absorption wavelengths), where Coulomb forces dominate, and where one can work in the limit $c \to \infty$ from the outset. (Then there are no photons, and nearby atoms can decay only by transferring energy to the mirror. The generalization to Maxwell's equations is straightforward in principle, though laborious in practice.) Our motivation is chiefly methodological, with the atom largely a focus for questions about the mirror (for realistic detail see [2]).

The mirror occupies the half-space $z < 0$, and is modelled as jellium, i.e. by a charged fluid, charge and mass densities $ne$ and $nm$, plus an over-all-neutralizing immobile background (squared plasma frequency $\omega_p^2 = 4\pi ne^2/m$), and harmonic restoring forces that in a neutral fluid would produce frequencies $\omega_T$. The displacement of the fluid from equilibrium and the electric field are $\xi = -\nabla\Psi$ and $E = -\nabla\Phi$, so that $P = -ne\nabla\Psi$ and $\rho_{pol} = ne\nabla^2\Psi$. Regardless of the equations of motion, it follows from Gauss' law alone that $\Phi$ outside stems wholly from surface modes, with charge densities $\rho_{pol}$ proprtional to $\delta(z)$, and with

$$\Phi = -2\pi ne\Psi \qquad \text{(for surface modes, at } z < 0\text{).} \tag{2}$$

Only surface modes will be retained from here on.

In the absence of a heat bath the equation of motion for the normal-mode amplitudes (lower case) reads $(-\omega^2 + \omega_T^2)\phi = e\psi/m$, which jointly with (2) eventually reproduces (1) with $\Gamma = 0$, and entails a sharp surface-mode frequency

$$\omega_S^2 = w_T^2 + w_p^2/2.$$

Define also (with $\Gamma$ not necessarily zero) the image-strength factor

$$\alpha(\omega) \equiv \frac{\varepsilon - 1}{\varepsilon + 1} = \frac{\omega_p^2/2}{\omega_S^2 - \omega^2 - i\omega\Gamma}, \quad \alpha(0) = \omega_p^2/2\omega_S^2.$$

On quantization* via $nm\dot{\Psi} \equiv \Pi$ and (see [3])

$$[\nabla^2\Psi(\mathbf{R}), \Pi(\mathbf{R}')] = i\delta(\mathbf{R} - \mathbf{R}')$$

* We use natural units, such that $\hbar = 1$.

one eventually gets

$$H_{field} = \omega_S \int d^2k(\alpha_{\mathbf{k}}^{+} a_{\mathbf{k}} + 1/2), \tag{3}$$

$$\Phi(z > 0) = -\sqrt{\alpha(0)} \int d^2k \sqrt{\frac{\omega_S}{4\pi k}} \exp(i\mathbf{k}\cdot\mathbf{r} - kz)a_{\mathbf{k}} + Hc, \tag{4}$$

where *Hc* stands for Hermitean conjugate and **k**, **r** are two-component vectors parallel to the surface.

Damping ($\Gamma > 0$) is ascribed to a heat bath consisting of a continuum of otherwise unspecified localized oscillators labelled by their frequecies $\nu$, with displacements $\xi_\nu = -\nabla\Psi_\nu$, canonical conjugate $\Pi_\nu \equiv nm\dot{\Psi}_\nu$, and commutators* $[\nabla^2\Psi_\nu(R), \Pi_{\nu'}(R')] = i\delta(\nu - \nu')\delta(R - R')$. One starts with an as yet unrenormalized Hamiltonian density

$$\mathcal{H}_u = \frac{nm}{2}\left\{\dot{\xi}^2 + \omega_u^2\xi^2 + \frac{1}{nm}\rho_{pol}\Phi + \int_0^{\nu(\max)} d\nu(\dot{\xi}_\nu^2 + \nu^2\xi_\nu^2) - 2\xi\cdot\int_0^{\nu(\max)} d\nu g_\nu \xi_\nu\right\}.$$

The precise form of the mirror-bath interaction (the last term) is largely irrelevant; the coupling function $g_\nu$ will be chosen presently. Remarkably, no further generality would be gained by augmenting the first integrand by another adjustable factor $dn_\nu/d\nu$ , say.

The equations of motion now read

$$(-\omega^2 + \omega_u^2)\xi = (e/m)\mathbf{E} + \int_0^{\nu(\max)} d\nu g_v \xi_\nu, \quad (-\omega^2 + \nu^2)\xi_\nu = g_\nu \xi.$$

They reproduce the response $\varepsilon$ in (1) if

$$\omega_u^2 - f_u(\omega) = \omega_T^2 - i\omega\Gamma, \quad f_u(\omega) \equiv \int_0^{\nu(\max)} d\nu \frac{g_\nu^2}{\nu^2 - \omega^2 - i0}. \tag{5}$$

This is achieved by choosing

$$g_\nu^2 = 2\nu^2\Gamma/\pi, \quad \omega_u^2 = \omega_T^2 + 2\Gamma\nu(\max)/\pi,$$

* The conjugate momenta $\pi_\nu$ and the commutators as given in eqs (6.1) and (6.2) in [2] are oversimplified and wrong, as shown by comparison with section 2.4 in [3]. Fortunately the wrong forms do not enter the actual calculations, and the subsequent Hamiltonians and equations of motion are correct as they stand.

followed by $\nu(\max) \to \infty$. Thereafter we proceed from the renormalized Hamiltonian density

$$\mathcal{H} = \frac{nm}{2}\left\{\dot{\xi}^2 + \omega_T^2\xi^2 + \frac{1}{nm}\rho_{pol}\Phi + \int_0^\infty d\nu(\dot{\xi}_\nu^2 + \nu^2\xi_\nu^2) - 2\xi\cdot\int_0^\infty d\nu g_\nu \xi_\nu\right\}, \tag{6}$$

with the prescription to ignore any further corrections to the restoring forces, and on every encounter to make the replacement

$$f_u(\omega) \to f(\omega) \equiv i\omega\Gamma. \tag{7}$$

Because the Hamiltonian contains no gradients of the bath variables $\xi_\nu$, their variation in configuration as in Fourier space is wholly enslaved by the variation of $\xi$. And, crucially, because the bath has a continuous spectrum, so have the exact normal modes for any given wave-vector **k** parallel to the surface.

For the normal-mode amplitudes, defined by

$$(\Phi, \Psi, \Psi_\nu) = \int d^2k \int_0^\infty d\omega \exp(i\mathbf{k}\cdot\mathbf{r} - k|z|)(\phi, \psi, \psi_\nu)(\omega, \mathbf{k})a_{\mathbf{k}\omega} + Hc, \tag{8}$$

the new equations of motion plus Gauss' law (2) eventually lead for each **k** (we suppress this label) to the coupled Lippmann-Schwinger equations

$$(-\omega^2 + \omega_S^2)\psi(\omega) = \int_0^\infty d\nu g_\nu \psi_\nu(\omega), \quad (-\omega^2 + \nu^2)\psi_\nu(\omega) = g_\nu\psi(\omega). \tag{9}$$

These are solved by

$$\psi(\omega) = \frac{N_\omega g_\omega}{\omega_S^2 - \omega^2 - i\omega\Gamma}, \quad \psi_\nu(\omega) = N_\omega\delta(\nu - \omega) + \frac{g_\nu}{\nu^2 - \omega^2 - i0}\psi(\omega), \tag{10}$$

where (7) has been used. The norming constant $N_\omega$ is chosen so that on substitution from (8) the Hamiltonian $H = \int d^2r \int dz\mathcal{H}$ reduces to

$$H = \int d^2k \int_0^\infty d\omega\omega(a_{\mathbf{k}\omega}^+ a_{\mathbf{k}\omega} + 1/2); \tag{11}$$

in practice this is by far the most tedious part of the work. At the same time one finds

$$\Phi(z>0)= \tag{12}$$
$$-\sqrt{\alpha(0)}\int d^2k\int_0^\infty d\omega\left[\frac{g_\omega}{\omega_S^2-\omega^2-i\omega\Gamma}\right]\sqrt{\frac{\omega_S^2}{4\pi k\omega}}\exp(i\mathbf{k}\cdot\mathbf{r}-kz)a_{\mathbf{k}\omega}+Hc.$$

Equations (11) and (12) are our central results. They show how the undamped structures (3), (4) are as it were dissolved in the bath, where for small $\Gamma/\omega_S$ their influence nevertheless persists through the resonance factor [...] in (12).

As promised, applications now follow through routine perturbation theory.

(i) *Neutral atom in state* $|j>$ *fixed at a distance Z from the mirror.* The interaction reads $\mathbf{D}.\nabla\Phi$, with $\mathbf{D}$ the dipole-moment operator of the atom. The energy shift is

$$\Delta(j)=-\frac{\alpha(0)}{8Z^3}\sum_i\left\{|D_{z,ij}|^2+|\mathbf{D}_{\|,ij}|^2/2\right\}$$
$$\times\int_0^\infty d\omega\omega\left(\frac{\mathcal{P}}{E_i-E_j+\omega}\right)\left[\frac{2\Gamma\omega_S^2/\pi}{(\omega_S^2-\omega^2)^2+\omega^2\Gamma^2}\right], \tag{13}$$

where the $E_i$ are the atomic energy levels, and $\mathcal{P}$ prescribes the Cauchy principal value. This remains finite even when $E_i-E_j+\omega_S=0$, a case where $\Gamma=0$ entails a divergence. If $E_j>E_i$, the decay rate is given by replacing $\mathcal{P}/(\ldots)\to\pi\delta(\ldots)$. In fact, though different in form, (13) agrees with the formula derived from the Lifshitz theory [4].

(ii) *Point charge q moving parallel to the surface with velocity* $\mathbf{u}$. Here the interaction is $H_{int}(t)=q\Phi(Z,\mathbf{r}=\mathbf{u}t)$; the problem is to determine the drag force $\mathbf{F}$, given by the Feynmann-Hellmann theorem through $\mathbf{F}.\mathbf{u} =<\partial H_{int}/\partial t>$, which must be evaluated with the first-order-perturbed states of the field.

For simplicity we consider only an Ohm's-law mirror with conductivity* $\sigma$, so that

$$\varepsilon=1+4\pi i\sigma/\omega\Rightarrow\omega_T=0,\quad\Gamma=\omega_p^2/4\pi\sigma,\quad\alpha(0)=1. \tag{14}$$

* The conductivity $\sigma$ here must not be confused with the quite different quantity denoted by the same symbol in [2].

The calculation is straightforward:

$$\langle \partial H_{int}/\partial t\rangle =$$
$$2q^2\omega_S^2 \int d^2k e^{-2kZ} \int_0^\infty d\omega \left[\frac{2\omega^2\Gamma/\pi}{(\omega_S^2-\omega^2)^2+\omega^2\Gamma^2}\right] \frac{(\mathbf{k}\cdot\mathbf{u})\delta(\omega-\mathbf{k}\cdot\mathbf{u})}{4k\omega},$$

which for $u/\omega_S Z \ll 1$ leads to

$$\mathbf{F} \simeq -\mathbf{u}q^2/16\pi\sigma Z^3, \tag{15}$$

in agreement with earlier results[5] found differently [5].*

(iii) *Neutral ground-state atom moving parallel to the surface with velocity* $\mathbf{u}$. Here $H_{int} = \mathrm{D}.\nabla\Phi(Z, \mathbf{r} = \mathbf{u}t)$, but the second-order perturbative expression for F is exponentially small, and the true leading term must be found either very laboriously by working to fourth order in $H_{int}$, or else to second order in an effective two-plasmon interaction $H_{eff} = -(\alpha_{atom}/2)(\nabla\Phi)^2$, where $\alpha_{atom}$ is the electrostatic polarizability of the atom. The problem is instructive because $\mathbf{F}$, though certainly very small, depends sensitively on the spectrum of the mirror at low frequencies. In the Ohmic model sketched above, one finds

$$F \sim \alpha_{atom}^2 u^3/\sigma^2 Z^{10} \quad \text{(Ohmic mirror)}.$$

By contrast, a metallic mirror modelled so that its low-frequency excitations are electron-hole pairs leads to the quite different estimate [6]

$$F \sim \alpha_{atom}^2 u e^4/\omega_S^2 Z^{10} \quad \text{(electron gas)}.$$

*That these applications are technically trivial is precisely the point*: the deep physical problems attending irreversibility ($\Gamma \neq 0$) have been pre-empted by the determination of the exact normal modes by means of the Huttner-Barnett method, which yields a simple Hamiltonian ready for use without any further explicit references to statistical mechanics.

*Possible further applications* include (iv) a straightforward calculation of the Van der Waals force between two mirrors described by (1), likely to agree with the Lifshitz result; (v) extension to mirrors in uniform relative motion, for comparison with the recent work of Pendry [7]; (vi) generalization from Coulomb forces to the full Maxwell equations, so as to explore the effects of absorption on mirror-induced radiation, i.e. on the photon pairs radiated when a single mirror moves nonuniformly [8].

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