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PAPER

Ohmura's extended electrodynamics: longitudinal aspects in general relativity

Ole Keller[1] and Lee M Hively[2]

[1] Institute of Physics, Aalborg University, Skjernvej 4, DK-9220 Aalborg Øst, Denmark
[2] Oak Ridge National Lab (retired), 4947 Ardley Drive, Colorado Springs, CO 80922, United States of America

E-mail: okeller@nano.aau.dk and lee.hively314@comcast.net.us

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Abstract

Jiménez and Maroto ((2011) *Phys. Rev.* D **83**, 023514) predicted that free-space, longitudinal electrodynamic waves can propagate in curved space-time, if the Lorenz condition is relaxed. The present work studies this possibility by combining and extending the original theory by Ohmura ((1956) *Prog. Theor. Phys.* **16**, 684) and Woodside's uniqueness theorem ((2009) *Am. J. Phys.* **77**, 438) to general relativity. Our formulation results in a theory that applies to both the field- (**E**, **B**) and potential- (Φ, **A**) domains. We establish a self-consistent, longitudinal wave-propagation theory for the microscopic longitudinal part of the electric field ($\mathbf{E}^L$). We first show that the product of the parameters used previously for the extension of classical electrodynamics can be expressed as a superposition of microscopic displacement modes, which are confined to the energy shell, $|\omega| = cq$. We then show that nonlinear electrodynamic mixing allows creation of longitudinal waves in the near-field region of a source. A propagator approach gives substantial physical insight into the emission process.

1. Introduction

A 1956 paper by Ohmura [1] suggested two independent extensions of the classical microscopic Maxwell-Lorentz equations. One extension used double-potentials to avoid the pathological string concept in Dirac's theory of magnetic monopoles [2, 3], as demonstrated by Cabibbo and Ferrari [4]. Recent work [5] showed that a non-local, space-time transformation of the electric- and magnetic-vector potentials maintains a minimum-coupling, singularity-free, photon-wave description. Ohmura's other extension introduced new scalar (C^{OHMURA}) and pseudo-scalar fields. Only C^{OHMURA} appears in the present paper, because we do not include magnetic monopoles. C^{OHMURA} is a key quantity in the special- and general-relativistic extension of classical electrodynamics, as discussed below.

Prior to Ohmura's work, Fock and Podolsky [6] suggested a new electrodynamic Lagrangian with a scalar field without deriving the resultant field equations. Other notable work under special relativity includes [7–12].

An outline of our paper follows. Section 2 deals with extended electrodynamics (EED) under special relativity. Section 2.1 briefly summarizes the various forms of EED under special and non-extended Lagrangian densities, showing that EED changes only the irrotational (longitudinal) part of the dynamics [13]. Section 2.2 discusses EED in the potential formulation with particular attention to the wave equations for the scalar- and longitudinal-vector-potentials. The Lorenz-gauge-breaking-scaling parameter (λ) connects the potential description to uniqueness theorems in special- [14, 15] and general-relativity [13], involving a scalar field, $C = \nabla \cdot \mathbf{A} + c^{-2}\partial\Phi/\partial t$. **A** and Φ are the classical vector and scalar potentials, respectively; c is the speed of light; t is time. *Only* when λ is explicitly taken into account does a self-consistent EED theory occur in both the electromagnetic-field- (**B**, **E**) and in the potential- (**A**, Φ) formulations. **B** and **E** are the magnetic and electric field vectors, respectively. We obtain $-C^{OHMURA} = \lambda C \equiv C_\lambda$ for $\lambda \neq 1$. For $\lambda = 1$, an apparent extension arises in the (**B**, **E**) equations, while the wave equations for (**A**, Φ) are non-extended, hyperbolic wave equations for all choices of λ [13–15]. The magnetic field is transverse, and thus is unaffected by the extension [13].

Section 3 explains the general-relativistic (GR) EED formulation in 3-vector form [16–21]. Jiménez and Maroto (JM) [22] extended the potential theory of general relativity to account for unexplained cosmological phenomena. The JM theory relaxes the Lorenz gauge condition, and consequently predicts a new propagating, longitudinal electric field. Section 4 elucidates the longitudinal (L) field propagation via a so-called microscopic L-displacement field ($\mathbf{D}^L$). Two remarkable properties arise for free-space propagation of the L-displacement field (sections 4.1, 4.2). First, C_λ can be eliminated in favor of $\mathbf{D}^L$. Second, C_λ is completely determined by a plane-wave, Fourier-integral decomposition ($\mathbf{q}, \omega$) of $\mathbf{D}^L$, in which only values on the energy shell enter ($|\omega| = c\mathbf{q}$). Section 4.3 shows the derivation of a matter- and curvature-driven wave equation for the longitudinal electric field ($\mathbf{E}^L$). No wave propagation occurs for $\mathbf{E}^L$ in classical electrodynamics (CED), although a non-propagating $\mathbf{E}^L$-field exists in the free-space, near-field of a transmitter. $\mathbf{E}^L$ plays a fundamental role in understanding the photon-localization problem [23].

Section 5 discusses the possibility of L-wave excitation in curved space-time via a propagator formalism (section 5.1) and in the wave-vector-frequency domain (section 5.2). We conclude that CED provides no *direct* possibilities for L-wave excitation [22]. Near-field electrodynamics provides *indirect* possibilities, in which case off-energy-shell contributions to the classical current density are needed. Section 6 explains our prediction for launching $\mathbf{E}^L$-waves in free, curved, space-time by non-linear optical mixing.

We briefly summarize the results of this paper, as follows. Under special relativity, Gauss' law ($\nabla \cdot \mathbf{E} = \rho/\varepsilon_o$) in microscopic, classical electrodynamics shows that the electric field in charge-free space ($\rho = 0$) is transverse. Thus, only transversely-polarized electromagnetic fields can propagate in vacuum. Extension of Gauss' law to general relativity replaces the usual derivative (∇) by the covariant derivative ($\nabla_{COV} \equiv \{\nabla_\mu\}$). These operators differ by a so-called gauge term. Gauss' law then has an extra term under general relativity [$\nabla \cdot \mathbf{E} = \varepsilon_o^{-1}(\rho + \rho_{CURV})$]. The new term ($\rho_{CURV}$) is an 'effective' charge density that is associated with the space-time curvature. The electric field now has a longitudinal component ($\mathbf{E}^L$) in charge-free space ($\rho = 0$). We show in sections 3, 4 that the $\mathbf{E}^L$-field leads to longitudinal wave propagation when CED is extended to EED. More specifically, relaxation of the Lorenz-gauge condition in the potential formalism allows $\mathbf{E}^L$-wave propagation under general relativity.

To include magnetic-monopole electrodynamics in EED, we start from the double-potential formalism [1, 4, 5]. This approach predicts propagation of a longitudinal magnetic field ($\mathbf{B}^L$). The transverse dynamics (with and without magnetic monopoles) are unaffected by the symmetrized extension of electrodynamics. Consequently, the $\mathbf{E}^L$- and $\mathbf{B}^L$-waves are not accompanied by magnetic- and electric-field components, respectively. Inclusion of magnetic monopoles in the present form of EED makes the analysis of the canonical particle momentum, angular momentum balance, and photon dynamics quite complicated. Therefore, a detailed EED theory with magnetic monopoles is beyond the scope of the present work.

2. Special relativistic EED

We begin with the covariant notation that is used. The contravariant 4-potential is $\{A^\mu\} = (\Phi/c, \mathbf{A})$, for $\mu = 0$–3. Bold symbols denote 3-vectors. Here, Φ and $\mathbf{A}$ are the usual scalar and vector potentials, respectively. The speed of light in vacuum is $c = (\varepsilon_o\mu_o)^{-1/2}$; ε_o and μ_o are the vacuum permittivity and permeability, respectively. The contravariant 4-current is $\{J^\mu\} = (c\rho, \mathbf{J})$, where ρ and $\mathbf{J}$ are the microscopic electric charge and current densities, respectively. The covariant 4-derivative is $\{\partial_\mu\} = (c^{-1}\partial/\partial t, \nabla)$, where t is time. Indices are raised and lowered, using a metric signature of $(-,+,+,+)$. The covariant metric tensor is $\{g_{\mu\nu}\}$ with a determinant of $g < 0$, which is shared by the contravariant metric tensor, $\{g^{\mu\nu}\}$. Summation over repeated lower and upper indices is implicit throughout this work. The wave operator (d'Alembertian) has the form, $\partial_\mu\partial^\mu = \nabla^2 - c^{-2}\partial^2/\partial t^2 \equiv \Box$, as denoted by the box symbol. SI units are used throughout this paper.

2.1. Extended covariant Lagrangian density

The particle component of the Lagrangian density is omitted in the subsequent description, because it is of no importance in this theoretical formulation. The extended Lagrangian density that we use is:

$$\mathcal{L} = \mathcal{L}_\mathrm{F} + \mathcal{L}_\mathrm{I}. \tag{1}$$

The field (F) Lagrangian density with parameter (λ) is:

$$\mathcal{L}_\mathrm{F} = -\frac{\varepsilon_o c^2}{2}[(\partial_\mu A^\nu)(\partial^\mu A_\nu) - (\lambda - 1)(\partial_\mu A^\mu)^2]. \tag{2}$$

The field-matter interaction Lagrangian density is:

$$\mathcal{L}_\mathrm{I} = J_\mu A^\mu. \tag{3}$$

For $\lambda = 1$, $\mathcal{L}_F$ reduces to the well-known (not extended) covariant field Lagrangian density:

$$\mathcal{L}_F{}^{COV} = -\frac{\varepsilon_o c^2}{2}(\partial_\mu A^\nu)(\partial^\mu A_\nu). \tag{4}$$

An alternative covariant electrodynamic formulation uses the Fermi (F) Lagrangian density [24], instead of $\mathcal{L}_F{}^{COV}$:

$$\mathcal{L}_F{}^{F} = -\frac{\varepsilon_o c^2}{2}\left[\frac{F_{\mu\nu}F^{\mu\nu}}{2} + (\partial_\mu A^\mu)^2\right]. \tag{5}$$

Equations (4) and (5) are equivalent, because they differ only by a 4-divergence. In equation (5), $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the $\mu\nu$-th component of the Maxwell field tensor. A form often used in extended electrodynamics is:

$$\mathcal{L}_F = -\frac{\varepsilon_o c^2}{2}\left[\frac{F_{\mu\nu}F^{\mu\nu}}{2} + \lambda(\partial_\mu A^\mu)^2\right]. \tag{6}$$

Equation (6) reduces to the Fermi Lagrangian density for $\lambda = 1$, making equations (2) and (6) equivalent. The generic form of the Euler-Lagrange equation is:

$$\partial_\alpha\left(\frac{\partial\mathcal{L}}{\partial_\alpha(\partial A^\beta)}\right) = \frac{\partial\mathcal{L}}{\partial A^\beta}. \tag{7}$$

Application of equation (7) to equation (1) for $\mathcal{L}_F$ from equation (2) yields:

$$\partial_\mu\partial^\mu A^\nu - (1-\lambda)\partial^\nu(\partial_\mu A^\mu) = -\mu_o J^\nu. \tag{8}$$

However, all covariant gauges have the following form:

$$\partial_\mu A^\mu = K. \tag{9}$$

Here, K is a constant that is independent of space and time. When equation (9) holds, then equation (8) reduces to the classical set ($\nu = 0$–3) of covariant wave equations. The well-known Lorenz gauge uses $K = 0$.

2.2. Generalized uniqueness theorem

The extension to classical theory is the second (4-gradient) term in equation (8). This extension affects only the wave equations for the scalar potential and the irrotational part of the vector potential. The Helmholtz theorem guarantees the unique decomposition of any 3-vector field ($\mathbf{V}$) into longitudinal (L) [rotation or curl free] and transverse (T) [divergence free] components, $\mathbf{V} = \mathbf{V}^L + \mathbf{V}^T$. Consequently, equation (8) yields the following wave equations:

$$\Box\mathbf{A}^T = -\mu_o\mathbf{J}^T; \tag{10}$$

$$\Box\mathbf{A}^L - (1-\lambda)\nabla C = -\mu_o\mathbf{J}^L; \tag{11}$$

$$\Box\Phi + (1-\lambda)\frac{\partial C}{\partial t} = -\frac{\rho}{\varepsilon_o}. \tag{12}$$

The new scalar term ($C \equiv \partial_\mu A^\mu$) in equations (11), (12) is:

$$C \equiv \nabla\cdot\mathbf{A}^L + \frac{1}{c^2}\frac{\partial\Phi}{\partial t}. \tag{13}$$

The extension does not affect the magnetic field, $\mathbf{B} = \nabla\times\mathbf{A}^T$, as is sometimes indicated in the literature. Indeed, equation (10) shows that transverse dynamics satisfies classical electrodynamics (CED). The T-L decomposition is not relativistically invariant. However, T- and L-dynamics is not mixed in a transformation between inertial frames in relative motion. Rather, $\mathbf{E}^T$ and $\mathbf{B}$ are transformed together, while $\mathbf{E}^L$ and $\mathbf{J}^L$ mix separately.

Previous uniqueness theorems in Minkowski and pseudo-Riemann space involve $C(\mathbf{r}, t)$, but not λ [13, 15]. The uniqueness theorems are useful in EED, but the connection to λ has been obscure until now. We clarify the role of λ by rewriting equations (11), (12) as:

$$\Box\mathbf{A}^L = -\mu_o\mathbf{J}_\lambda^L; \tag{14}$$

$$\Box\Phi = -\frac{\rho_\lambda}{\varepsilon_o}. \tag{15}$$

Equations (14), (15) have new terms on the right-hand side (RHS), which are:

$$\mathbf{J}^L_\lambda = \mathbf{J}^L + \left(\frac{\lambda - 1}{\mu_o}\right)\nabla C; \tag{16}$$

$$\rho_\lambda = \rho + \varepsilon_o(1 - \lambda)\frac{\partial C}{\partial t}. \tag{17}$$

The longitudinal part of the electric field ($\mathbf{E}^L$) is:

$$\mathbf{E}^L = -\frac{\partial \mathbf{A}^L}{\partial t} - \nabla\Phi. \tag{18}$$

A longitudinal extension of Ampère's law arises by: use of $c^{-2}\partial/\partial t$ on equation (18); inclusion of $\nabla^2\mathbf{A}^L - \nabla^2\mathbf{A}^L = 0$ via the identity $\nabla^2\mathbf{A}^L = \nabla(\nabla \cdot \mathbf{A}^L)$, knowing that $\nabla \times \mathbf{A}^L = 0$; simplification via the definition of C from equation (13); and use of equation (14) to further simplify the expression. The result is:

$$-\frac{1}{c^2}\frac{\partial \mathbf{E}^L}{\partial t} - \nabla C = \mu_o \mathbf{J}^L_\lambda. \tag{19}$$

A longitudinal extension of Gauss' law can be obtained by taking the divergence of equation (18); inclusion of $-\partial/\partial t(\varepsilon_o\mu_o\partial\Phi/\partial t) + \varepsilon_o\mu_o\partial^2\Phi/\partial t^2 = 0$; use of the definition of C from equation (13); and application of equation (15) to simplify the expression. The result is:

$$\nabla \cdot \mathbf{E}^L + \frac{\partial C}{\partial t} = \frac{\rho_\lambda}{\varepsilon_o}. \tag{20}$$

Equations (19), (20) are analogous to Woodside's uniqueness theorem for extended electrodynamics, which assumes only Minkowski space [15]. The definitions of C in equation (13) and $\mathbf{E}^L$ in equation (18) are uniquely specified via the source terms, $\mathbf{J}^L_\lambda$ and ρ_λ, through the hyperbolic wave equations (14), (15). This result holds even if $\mathbf{J}^L_\lambda$ and ρ_λ depend on C and $\mathbf{E}^L$, from the definitions in equations (16), (17). A dependence on $\mathbf{E}^L$ arises (e.g., in linear response) when $\mathbf{J}^L$ is proportional to $\mathbf{E}^L$. This generalized uniqueness theorem includes the factor, $1-\lambda$, as expected.

Further simplification is possible by introduction of:

$$C_\lambda = \lambda C. \tag{21}$$

Equations (19), (20) can then be rewritten in the form:

$$-\frac{1}{c^2}\frac{\partial \mathbf{E}^L}{\partial t} - \nabla C_\lambda = \mu_o \mathbf{J}^L; \tag{22}$$

$$\nabla \cdot \mathbf{E}^L + \frac{\partial C_\lambda}{\partial t} = \frac{\rho}{\varepsilon_o}. \tag{23}$$

The wave equations for $\mathbf{A}^L$ in equation (14) and Φ in equation (15) can then be expressed in terms of C_λ:

$$\Box\mathbf{A}^L + \left(1 - \frac{1}{\lambda}\right)\nabla C_\lambda = -\mu_o \mathbf{J}^L; \tag{24}$$

$$\Box\Phi - \left(1 - \frac{1}{\lambda}\right)\frac{\partial C_\lambda}{\partial t} = -\frac{\rho}{\varepsilon_o}. \tag{25}$$

The divergence of equation (22) added to $c^{-2}\partial/\partial t$ on equation (23) is:

$$\Box C_\lambda = -\mu_o\left(\nabla \cdot \mathbf{J}^L + \frac{\partial \rho}{\partial t}\right). \tag{26}$$

Classical charge conservation requires the RHS of equation (26) to be zero, leaving the left-hand side (LHS) zero as well. Since C_λ is simply scaled by λ, the C-wave equation is:

$$\Box C = 0. \tag{27}$$

Equation (27) is a source free wave equation for C (and also C_λ). As shown previously, a gauge transformation with a gauge function (Λ) creates the bridge from the old ($C_{OLD} = C$) to the new ($C_{NEW} = \lambda C$) C in equation (21), requiring the condition:

$$\Box\Lambda = C_{NEW} - C_{OLD} = (\lambda - 1)C. \tag{28}$$

The effective charge (ρ_λ) and current ($\mathbf{J}_\lambda^L$) densities also satisfy a continuity equation:

$$\nabla \cdot \mathbf{J}_\lambda^L + \frac{\partial \rho_\lambda}{\partial t} = 0. \quad (29)$$

To obtain equation (29), we apply $\mu_o \nabla \cdot$ to equation (16), and add the result to $(\varepsilon_o c^{-2})\partial/\partial t$ as applied to equation (17). We then employ equation (27) plus classical charge conservation, $\nabla \cdot \mathbf{J}^L + \partial\rho/\partial t = 0$.

3. General relativistic EED

[16–21] showed that Maxwell's equations can be extended to general relativity (GR) in 3-vector notation. The extension in [21] uses so-called generalized microscopic polarization and magnetization fields. The resultant equations are form-identical to the macroscopic Maxwell's equations, but with a very different physical interpretation, as discussed next.

3.1. General relativistic CED in 3-vector form

The manifestly covariant form of the inhomogeneous GR field equation is:

$$\nabla_\mu F^{\mu\nu}(x) = -\mu_o J^\nu(x). \quad (30)$$

The GR covariant derivative is $\{\nabla_\mu\}$, and $F^{\mu\nu} = \nabla^\mu A^\nu - \nabla^\nu A^\mu$. Equation (30) is equivalent to the 3-vector equations:

$$\nabla \times \mathbf{B} = \frac{1}{c^2}\frac{\partial \mathbf{E}}{\partial t} + \mu_o \mathbf{J}_{GR}; \quad (31)$$

$$\nabla \cdot \mathbf{E} = \frac{\rho_{GR}}{\varepsilon_o}. \quad (32)$$

The GR charge (ρ_{GR}) and current ($\mathbf{J}_{GR}$) densities are:

$$\rho_{GR} = \rho + \rho_{CURV}; \quad (33)$$

$$\mathbf{J}_{GR} = \mathbf{J} + \mathbf{J}_{CURV}. \quad (34)$$

Here, ρ_{CURV} and $\mathbf{J}_{CURV}$ are contributions from the space-time curvature (CURV).

3.2. Extended longitudinal electrodynamics

From equations (31), (32), one obtains the CED longitudinal field equations:

$$-\frac{1}{c^2}\frac{\partial \mathbf{E}^L}{\partial t} = \mu_o \mathbf{J}_{GR}^L; \quad (35)$$

$$\nabla \cdot \mathbf{E}^L = \frac{\rho_{GR}}{\varepsilon_o}. \quad (36)$$

Comparison of equations (35), (36) to equations (22), (23) shows that GR EED satisfies:

$$-\frac{1}{c^2}\frac{\partial \mathbf{E}^L}{\partial t} - \nabla C_\lambda = \mu_o \mathbf{J}_{GR}^L; \quad (37)$$

$$\nabla \cdot \mathbf{E}^L + \frac{\partial C_\lambda}{\partial t} = \frac{\rho_{GR}}{\varepsilon_o}. \quad (38)$$

A similar comparison to equations (24), (25) shows that the GR potential wave equations with $\mathbf{E}^L$ from equation (18) are:

$$\Box \mathbf{A}^L + \left(1 - \frac{1}{\lambda}\right)\nabla C_\lambda = -\mu_o \mathbf{J}_{GR}^L; \quad (39)$$

$$\Box \Phi - \left(1 - \frac{1}{\lambda}\right)\frac{\partial C_\lambda}{\partial t} = -\frac{\rho_{GR}}{\varepsilon_o}. \quad (40)$$

The potential formalism in the extended GR Lagrangian density (*c.f.*, section 2.1 and [19]) has the parameter, λ:

$$\mathcal{L} = \sqrt{-g}\left\{-\frac{\varepsilon_o c^2}{2}\left[\frac{F_{\mu\nu}F^{\mu\nu}}{2} + \lambda(\nabla_\mu A^\mu)^2\right] + J_\mu A^\mu\right\}. \quad (41)$$

Application of the GR Euler-Lagrange equation [equation (7) with ∂_α replaced by ∇_α] to equation (41) with neglect of the explicit curvature couplings allows the derivation of equations (39), (40). This approach is unneeded in the following.

4. GR longitudinal waves

We next introduce a so-called longitudinal displacement field by the definition:

$$\mathbf{D}^L \equiv \varepsilon_o \mathbf{E}^L + \mathbf{P}^L_{GR}. \tag{42}$$

The longitudinal component of the GR-polarization density ($\mathbf{P}^L$) has the following properties:

$$\mathbf{J}^L_{GR} = \frac{\partial \mathbf{P}^L_{GR}}{\partial t}; \tag{43}$$

$$\rho_{GR} = -\nabla \cdot \mathbf{P}^L_{GR}. \tag{44}$$

The curvature part of the polarization density is:

$$\mathbf{P}^L_{CURV} = \mathbf{P}^L_{GR} - \mathbf{P}^L, \tag{45}$$

provided that the GR metric tensor is known. The curvature polarization density is discussed further in section 5.5.

4.1. Longitudinal displacement field equations

Insertion of equations (43)–(45) into equations (37), (38) yields:

$$\frac{\partial \mathbf{D}^L}{\partial t} + \frac{\nabla C_\lambda}{\mu_o} = 0; \tag{46}$$

$$\nabla \cdot \mathbf{D}^L + \varepsilon_o \frac{\partial C_\lambda}{\partial t} = 0. \tag{47}$$

Subtraction of the partial-time derivative of equation (46) from the gradient of equation (47) shows that $\Box \mathbf{D}^L = \mathbf{0}$, which is a well-known result in CED.

4.2. Elimination of C_λ in favor of $\mathbf{D}^L$

The vector fields in this work (generically denoted as **F**) all can be represented as Fourier integrals:

$$\mathbf{F}(\mathbf{r}, t) = \int_{-\infty}^{\infty} \mathbf{F}(\mathbf{q}, \omega) e^{i(\mathbf{q}\cdot\mathbf{r} - \omega t)} \frac{d^3 q d\omega}{(2\pi)^4}, \tag{48}$$

with an inverse transformation:

$$\mathbf{F}(\mathbf{q}, \omega) = \int_{-\infty}^{\infty} \mathbf{F}(\mathbf{r}, t) e^{-i(\mathbf{q}\cdot\mathbf{r} - \omega t)} d^3 r dt. \tag{49}$$

The position vector is $\mathbf{r}$; ω is the angular frequency; and $\boldsymbol{q}$ is the wave vector with an amplitude, q. The gradient (∇) and partial-time derivative ($\partial/\partial t$) operators transform into multiplication by $i\mathbf{q}$ and $-i\omega$ in the (ω, $\mathbf{q}$)-domain. Hence, equations (46), (47) become:

$$-\omega \mathbf{D}^L(\mathbf{q}, \omega) + \frac{\mathbf{q} C_\lambda(\mathbf{q}, \omega)}{\mu_o} = 0; \tag{50}$$

$$\mathbf{q} \cdot \mathbf{D}^L(\mathbf{q}, \omega) - \varepsilon_o \omega C_\lambda(\mathbf{q}, \omega) = 0. \tag{51}$$

Both $\mathbf{D}^L$ and $\mathbf{q}$ are in the longitudinal direction, $\hat{\mathbf{q}} = \mathbf{q}/q$. $\mathbf{D}^L$ can then be rewritten as $D^L = \hat{\mathbf{q}} \cdot \mathbf{D}^L$ resulting in:

$$-\omega[\mu_o D^L(\mathbf{q}, \omega)] + q C_\lambda(\mathbf{q}, \omega) = 0; \tag{52}$$

$$q[\mu_o D^L(\mathbf{q}, \omega)] - \frac{\omega C_\lambda(\mathbf{q}, \omega)}{c^2} = 0. \tag{53}$$

A non-zero solution for C_λ and $\mu_o D^L$ exists only if the determinant of equations (52), (53) is zero, yielding:

$$\omega = \pm cq. \tag{54}$$

Equation (54) has two branches of the vacuum photon dispersion [$\omega = cq\,(\omega > 0)$] and anti-photons [$\omega = -cq\,(\omega < 0)$], leading to:

$$C_\lambda(\mathbf{q}, \omega) = \begin{cases} \pm c\mu_o D^L(\mathbf{q}, \omega), \text{ for} \omega = \pm cq; \\ 0, \text{ for} \omega \neq \pm cq. \end{cases} \tag{55}$$

In the space-time ($\mathbf{r}$, t) domain, the solution for $C(\mathbf{r}, t)$ is:

$$C_\lambda(\mathbf{r}, t) = \pi\mu_o c\int_{-\infty}^{\infty}\frac{d^3q d\omega}{(2\pi)^4}D^L(\mathbf{q}, \omega)e^{i(\mathbf{q}\cdot\mathbf{r}-\omega t)}[\delta(\omega - cq) - \delta(\omega + cq)]$$
$$= \frac{\mu_o c}{2}\int_{-\infty}^{\infty}\frac{d^3q d\omega}{(2\pi)^3}D^L(\mathbf{q}, \omega)e^{i(\mathbf{q}\cdot\mathbf{r}-\omega t)}[\delta(\omega - cq) - \delta(\omega + cq)]. \quad (56)$$

Here, $\delta(\dots)$ is the Dirac delta function. A factor of π appears in the first line of equation (56), arising from our choice of $\mathbf{D}^L$ scaling. Then, the factor of ½ in the second line of equation (56) is convenient, when adding a term and its complex conjugate. Integration of equation (56) over ω then yields:

$$C_\lambda(\mathbf{r}, t) = \frac{\mu_o c}{2}\int_{-\infty}^{\infty}\frac{d^3q}{(2\pi)^3}[D^L(\mathbf{q}, cq)e^{i(\mathbf{q}\cdot\mathbf{r}-cqt)} - D^L(\mathbf{q}, -cq)e^{i(\mathbf{q}\cdot\mathbf{r}+cqt)}]. \quad (57)$$

Substitution of $\mathbf{q} \to -\mathbf{q}$ in the last term of equation (57) gives:

$$\int_{-\infty}^{\infty}\frac{d^3q}{(2\pi)^3}D^L(\mathbf{q}, -cq)e^{i(\mathbf{q}\cdot\mathbf{r}+cqt)}$$
$$= \int_{-\infty}^{\infty}\frac{d^3q}{(2\pi)^3}D^L(-\mathbf{q}, -cq)e^{i(-\mathbf{q}\cdot\mathbf{r}+cqt)}$$
$$= -\int_{-\infty}^{\infty}\frac{d^3q}{(2\pi)^3}[D^L(\mathbf{q}, cq)]^* e^{-i(\mathbf{q}\cdot\mathbf{r}-cqt)} \quad (58)$$

The last step in equation (58) relies on $\mathrm{D}^L(\mathrm{r}, t)$ being real, so that $\mathrm{D}^L(-\mathrm{q}, -\mathrm{cq}) = [\mathrm{D}^L(\mathrm{q}, \mathrm{cq})]^*$. This vectorial relation implies:

$$D^L(-\mathbf{q}, -cq) = -\hat{\mathbf{q}}\cdot\mathbf{D}^L(-\mathbf{q}, -cq) = -\hat{\mathbf{q}}\cdot[\mathbf{D}^L(\mathbf{q}, cq)]^*$$
$$= -[\hat{\mathbf{q}}\cdot\mathbf{D}^L(\mathbf{q}, cq)]^* = -[D^L(\mathbf{q}, cq)]^*. \quad (59)$$

Equation (59) allows equation (57) to be written as:

$$C_\lambda(\mathbf{r}, t) = \frac{\mu_o c}{2}\int_{-\infty}^{\infty}\frac{d^3q}{(2\pi)^3}\hat{\mathbf{q}}\cdot\mathbf{D}(\mathbf{q}, cq)e^{i(\mathbf{q}\cdot\mathbf{r}-cqt)} + c.c. \quad (60)$$

Here, '*c.c.*' denotes the complex conjugate. Equation (60) shows that $C_\lambda(\mathbf{r}, t)$ is completely determined by the value of $\mathbf{D}^L(\mathbf{q}, \omega)$ on the energy shell, $|\omega| = cq$. $C_\lambda(\mathbf{r}, t)$ is then composed of plane waves with a wave vector ($\mathbf{q}$), propagating at the vacuum speed of light. Equation (54) also implies that C_λ satisfies equation (26) with the RHS equal to zero for each plane-wave mode.

4.3. Longitudinal wave equations

A wave equation for $\mathbf{D}^L$ arises by application of $c^{-2}\partial/\partial t$ to equation (46) and ∇ to equation (47). We then obtain equation (61) by subtracting the results:

$$\Box\mathbf{D}^L = 0, \quad (61)$$

since $\nabla^2\mathbf{D}^L = \nabla(\nabla\cdot\mathbf{D}^L)$. Equation (61) for the longitudinal displacement field shows that $\mathbf{D}^L$ is composed of energy-shell contributions alone:

$$\mathbf{D}^L(\mathbf{r}, t) = \frac{\mu_o c}{2}\int_{-\infty}^{\infty}\frac{d^3q}{(2\pi)^3}\mathbf{D}^L(\mathbf{q}, cq)e^{i(\mathbf{q}\cdot\mathbf{r}-cqt)} + c.c. \quad (62)$$

This result shows consistency in equations (46), (47), and (60).

A wave equation for $\mathbf{E}^L$ can be obtained by insertion of the definition in equation (42) into equation (61) and utilizing the identity, $\nabla^2\mathbf{P}_{GR}^L = \nabla(\nabla\cdot\mathbf{P}_{GR}^L)$:

$$\Box\mathbf{E}^L = \frac{\Box\mathbf{P}_{GR}^L}{\varepsilon_o} = -\frac{1}{\varepsilon_o}\left[\nabla(\nabla\cdot\mathbf{P}_{GR}^L) - \frac{1}{c^2}\frac{\partial}{\partial t}\left(\frac{\partial\mathbf{P}_{GR}^L}{\partial t}\right)\right]. \quad (63)$$

Equation (63) then can be rewritten as an inhomogeneous wave equation, using equations (43), (44):

$$\Box\mathbf{E}^L = \mu_o\frac{\partial\mathbf{J}_{GR}^L}{\partial t} + \frac{\nabla\rho_{GR}}{\varepsilon_o}. \quad (64)$$

Equation (64) is a genuine EED longitudinal wave equation. In CED however, the gradient of Gauss' law has the form:

$$\nabla(\nabla \cdot \mathbf{E}^L) = \nabla^2\mathbf{E}^L = \frac{\nabla\rho}{\varepsilon_o}. \tag{65}$$

Substitution of equation (65) into the CED $\mathbf{E}^L$-wave equation yields the form is analogous to equation (35), namely:

$$\varepsilon_o\frac{\partial\mathbf{E}^L}{\partial t} + \mathbf{J}^L = 0. \tag{66}$$

As discussed elsewhere [23], the first-order differential equation, equation (66), plays a significant role in understanding the fundamental limitations for spatial photon localization. The presence of the additional non-zero term, $\partial C_\lambda/\partial$t, in the EED form of Gauss' law, equation (38), prevents the reduction of equation (64) to first-order form.

5. Longitudinal wave emission

We introduce a longitudinal curvature (*CURV*) displacement field to investigate the electrodynamic excitation of longitudinal waves in curved space-time:

$$\mathbf{D}^L_{CURV} \equiv \varepsilon_o\mathbf{E}^L + \mathbf{P}^L_{CURV}. \tag{67}$$

By a combination of equations (42), (45), and (67), we obtain:

$$\mathbf{D}^L \equiv \mathbf{D}^L_{CURV} + \mathbf{P}^L. \tag{68}$$

5.1. Propagator formalism in the (r, *t*)-domain

A $\mathbf{D}^L_{CURV}$-wave equation can be derived by use of the wave operator on equation (68) and use of $\square\mathbf{D}^L = 0$ from equation (61):

$$\square\mathbf{D}^L = 0 = \square\ \mathbf{D}^L_{CURV} + \square\ \mathbf{P}^L. \tag{69}$$

Expansion of equation (69) yields the following:

$$\square\mathbf{D}^L_{CURV} = -\ \square\ \mathbf{P}^L = \frac{1}{c^2}\frac{\partial}{\partial t}\left(\frac{\partial\mathbf{P}^L}{\partial t}\right) - \nabla(\nabla \cdot \mathbf{P}^L), \tag{70}$$

since $\nabla^2\mathbf{P}^L = \nabla(\nabla \cdot \mathbf{P}^L)$. The longitudinal polarization density is related to the microscopic charge and L-current densities via $\rho = -\nabla \cdot \mathbf{P}^L$ and $\mathbf{J}^L = \partial\mathbf{P}^L/\partial t$. These relations can be substituted into equation (70) to obtain the form:

$$\square\mathbf{D}^L_{CURV} = \nabla\rho + \frac{1}{c^2}\frac{\partial\mathbf{J}^L}{\partial t}. \tag{71}$$

We take the current density distribution (**J**) to be localized to a finite, space-time volume, V. **J** and $\mathbf{J}^L$ are related in a spatially, non-local manner [23]. More specifically, the space-time volume that is associated with the $\mathbf{J}^L$ is V^L, which is different from (larger than) V with both volumes evaluated at the same time. The general solution to equation (71) is then given by:

$$\mathbf{D}^L_{CURV}(\mathbf{r},\ t) = \mathbf{D}^{L,O}_{CURV}(\mathbf{r},\ t) - \int_{V^L} d^3r'dt'g(R,\ \tau)\left[\nabla'\rho(\mathbf{r}',\ t') + \frac{1}{c^2}\frac{\partial\mathbf{J}^L(\mathbf{r}',\ t')}{\partial t'}\right]. \tag{72}$$

The scalar (Huygens) propagator is given by:

$$g(R,\ \tau) = \frac{\delta\left(\frac{R}{c} - \tau\right)}{4\pi R}. \tag{73}$$

The Dirac delta function is $\delta(R/c - \tau)$ with $\mathbf{R} = \mathbf{r} - \mathbf{r}'$; $R = |\mathbf{R}|$; and $\tau = t - t'$. This form guarantees an Einsteinian retarded connection between the source location, $(\mathbf{r}', t')$, and the observation point, $(\mathbf{r}, t)$. The homogeneous part of the solution, $\mathbf{D}^{L,O}_{CURV}(\mathbf{r},\ t)$, originates in electrodynamic sources that are located outside V^L. We assume that the dynamics of the external sources are unaffected by the radiation form the sources inside V^L. Neither V nor V^L has a sharp boundary. Usually, the microscopic current density exhibits an exponential fall-off in the surface region due to the quantum-mechanical, wave-function spill-off, whereas the longitudinal part has a slower algebraic falloff in general. In either case, the integration limits may extend to $\pm\infty$.

We next focus on the inhomogeneous part of equation (72):

$$\mathbf{F}^L(\mathbf{r}, t) = -\int_{-\infty}^{\infty} d^3r'dt'g(R, \tau)\mathbf{S}^L(\mathbf{r}', t'), \quad (74)$$

with the definition:

$$\mathbf{S}^L(\mathbf{r}, t) = \nabla\rho(\mathbf{r}, t) + \frac{1}{c^2}\frac{\partial \mathbf{J}^L(\mathbf{r}, t)}{\partial t}. \quad (75)$$

5.2. Propagator formalism in the (q, ω)-domain

Section 4.2 showed that the CED extension only affects the electrodynamics on the light-energy shells, $\omega = \pm cq$. In this view, equation (50) can be rewritten as:

$$-\omega[\mathbf{D}^L_{CURV}(\mathbf{q}, \omega) + \mathbf{P}^L(\mathbf{q}, \omega)] + \frac{\pi\mathbf{q}}{\mu_o}[\delta(\omega - cq)C_\lambda(\mathbf{q}, cq) + \delta(\omega + cq)C_\lambda(\mathbf{q}, -cq)] = 0. \quad (76)$$

If $\omega \neq \pm cq$, then the second line of equation (76) is zero, yielding $\mathbf{P}^L(\mathbf{q}, \omega) = -\mathbf{D}^L_{CURV}(\mathbf{q}, \omega)$. In the $(\mathbf{q}, \omega)$-domain, we obtain $\mathbf{P}^L(\mathbf{q}, \omega) = \mathbf{J}^L(\mathbf{q}, \omega)/(-i\omega)$, since $\mathbf{J}^L = \partial\mathbf{P}^L/\partial t$. This last equation can be substituted into equation (76) to obtain an enlightening form:

$$i\omega\mathbf{D}^L_{CURV}(\mathbf{q}, \omega) = \mathbf{J}^L(\mathbf{q}, \omega) + \mathbf{J}^L_\lambda(\mathbf{q}, \omega), \quad (77)$$

where the last term in equation (77) is:

$$\mathbf{J}^L_\lambda(\mathbf{q}, \omega) = \frac{i\pi\mathbf{q}}{\mu_o}[\delta(\omega - cq)C_\lambda(\mathbf{q}, cq) + \delta(\omega + cq)C_\lambda(\mathbf{q}, -cq)]. \quad (78)$$

The extension of CED appears in equation (77) as an additional contribution to the longitudinal current density. Such an interpretation is reasonable, because $C_\lambda = \lambda C$ relates to the 4-potential in equations (13) and (21). This relation in turn leads to the longitudinal component of electric field in equation (18). Since $\mathbf{E}^L(\mathbf{q}, \omega) = \mathbf{J}^L(\mathbf{q}, \omega)/(i\varepsilon_o\omega)$ in CED, then EED would naturally have an extension in the propagator description.

In the $(\mathbf{q}, \omega)$-domain, equation (74) takes the algebraic form:

$$\mathbf{F}^L(\mathbf{q}, \omega) = -g(q, \omega)\mathbf{S}^L(\mathbf{q}, \omega), \quad (79)$$

because equation (74) is a folding integral. The scalar propagator is then given by:

$$g(q, \omega) = \frac{1}{q^2 - q_o^2}, \quad (80)$$

where $q = |\omega|/c$, as the vacuum wave-number of light. The singularities of the propagator at $\omega = \pm cq$ correspond to the singularity at $R = 0$ in the $(\mathbf{r}, t)$-domain [equation (73)]. Contour integration over a complex ω-plane is often used in CED. The poles are located on the real axis at $\omega = \pm cq$. These poles are encircled in the upper or lower half-plane, depending on the specific application.

5.3. Near-field, off-energy-shell, longitudinal dynamics

The propagator formalism in the $(\mathbf{q}, \omega)$-domain reveals a 'hidden contribution' in equation (76) that is non-vanishing only for $\omega = \pm cq$ in the extension of CED. When $\omega \neq \pm cq$, the agreement with CED is complete. Consequently, $\mathbf{S}^L(\mathbf{q}, \omega)$ in the extended theory (EXT) must be replaced by:

$$\mathbf{S}^L_{EXT}(\mathbf{q}, \omega) = \mathbf{S}^L(\mathbf{q}, \omega) - \frac{i\omega\mathbf{J}^L_\lambda(\mathbf{q}, \omega)}{c^2}. \quad (81)$$

The structure of $\mathbf{S}^L(\mathbf{q}, \omega)$ is given by rewriting equation (74):

$$\mathbf{S}^L(\mathbf{q}, \omega) = i\mathbf{q}\rho(\mathbf{q}, \omega) - \frac{i\omega\mathbf{J}^L(\mathbf{q}, \omega)}{c^2}; \\ = \frac{-i\omega}{c^2}\left[\mathbf{J}^L(\mathbf{q}, \omega) - \frac{c^2\mathbf{q}}{\omega}\rho(\mathbf{q}, \omega)\right]. \quad (82)$$

Equation (82) can be rewritten using charge continuity in the $(\mathbf{q}, \omega)$-domain:

$$\mathbf{q}\cdot\mathbf{J}^L(\mathbf{q}, \omega) = \omega\rho(\mathbf{q}, \omega), \quad (83)$$

together with the identity:

$$\hat{\mathbf{q}}\hat{\mathbf{q}} \cdot \mathbf{J}^L(\mathbf{q}, \omega) = \mathbf{J}^L(\mathbf{q}, \omega). \tag{84}$$

Substitution of equations (83), (84) into equation (82) yields:

$$\mathbf{S}^L(\mathbf{q}, \omega) = \frac{\mathbf{J}^L(\mathbf{q}, \omega)}{i\omega}\left(\frac{\omega^2}{c^2} - q^2\right) = \frac{\mathbf{J}^L(\mathbf{q}, \omega)}{i\omega}(q_o^2 - q^2). \tag{85}$$

Equation (85) uses the definition, $q_o^2 = (\omega/c)^2$. Three remarks about equation (85) are important. First, $\mathbf{S}^L(\mathbf{q}, \omega) = \mathbf{0}$ occurs on the light energy shell ($|\omega| = cq$), where EED contributes a non-vanishing component via equation (81) that is proportional to $\mathbf{J}_\lambda^L(\mathbf{q}, \omega)$. Second, $\mathbf{S}^L(\mathbf{q}, \omega) = \mathbf{0}$ on the light energy shell implies that the curvature displacement field cannot be excited by a CED source, $\mathbf{S}^L(\mathbf{r}, t)$. Third, equation (80) shows the CED propagator singularities in $g(q, \omega)$ that are cancelled by the factor of $(q_o^2 - q^2)$ in equation (85). Thus, we obtain:

$$\begin{aligned}\mathbf{F}^L(\mathbf{q}, \omega) &= -g(q, \omega)\mathbf{S}^L(\mathbf{q}, \omega) = \left(\frac{-1}{q^2 - q_o^2}\right)\frac{\mathbf{J}^L(\mathbf{q}, \omega)}{i\omega}(q_o^2 - q^2);\\ &= -\mathbf{P}^L(\mathbf{q}, \omega),\end{aligned} \tag{86}$$

since $\mathbf{J}^L(\mathbf{q}, \omega) = -i\omega\mathbf{P}^L(\mathbf{q}, \omega)$. The inhomogeneous solution to equation (71) is then obtained by transformation to the $(\mathbf{r}, t)$-domain and use of equation (66):

$$\varepsilon_o\mathbf{E}^L(\mathbf{r}, t) + \mathbf{P}_{CURV}^L(\mathbf{r}, t) = -\mathbf{P}_\lambda^L(\mathbf{r}, t). \tag{87}$$

The partial-time derivative of equation (87) gives equation (35).

CED provides no *direct* excitation of longitudinal waves in curved free-space-time. However, CED provides indirect possibilities. Namely, a longitudinally polarized electric field does exist in the near-field (rim) zone of the source [21]. The rim zone extends over the spatial region of $\mathbf{J}^L(\mathbf{r}, t)$, or equivalently $\mathbf{P}^L(\mathbf{r}, t)$. Section 6 shows an example of how a near-field $\mathbf{E}^L$ can be used as a source for L-wave in GR.

5.4. $\mathbf{E}^L$ on the light-energy shell

Equation (77) provides an EED expression for the longitudinal current density, $\mathbf{J}_\lambda^L(\mathbf{q}, \omega)$ in the $(\mathbf{q}, \omega)$-domain. $\mathbf{J}_\lambda^L(\mathbf{r}, t)$ can be obtained by using by using the Fourier integral transformation in equation (48). The delta functions in equation (77) enable immediate evaluation of the ω-integration:

$$\mathbf{J}_\lambda^L(\mathbf{r}, t) = \frac{i}{2\mu_o}\int_{-\infty}^{\infty}\frac{\mathbf{q}d^3q}{(2\pi)^3}\begin{bmatrix}C_\lambda(\mathbf{q}, cq)e^{i(\mathbf{q}\cdot\mathbf{r}-cqt)}\\ +C_\lambda(\mathbf{q}, -cq)e^{i(\mathbf{q}\cdot\mathbf{r}+cqt)}\end{bmatrix}. \tag{88}$$

The second term on the RHS of equation (88) can be re-written:

$$\begin{aligned}&\frac{i}{2\mu_o}\int_{-\infty}^{\infty}\frac{\mathbf{q}d^3q}{(2\pi)^3}C_\lambda(\mathbf{q}, -cq)e^{i(\mathbf{q}\cdot\mathbf{r}+cqt)}\\ &= \frac{i}{2\mu_o}\int_{-\infty}^{\infty}\frac{(-\mathbf{q})d^3q}{(2\pi)^3}C_\lambda(-\mathbf{q}, -cq)e^{-i(\mathbf{q}\cdot\mathbf{r}-cqt)}\\ &= \frac{-i}{2\mu_o}\int_{-\infty}^{\infty}\frac{\mathbf{q}d^3q}{(2\pi)^3}C_\lambda^*(\mathbf{q}, cq)e^{-i(\mathbf{q}\cdot\mathbf{r}-cqt)}\end{aligned} \tag{89}$$

The second line of equation (89) uses $\mathbf{q} \to -\mathbf{q}$; the third line of equation (89) relies on $C(\mathbf{r}, t)$ being real so that:

$$C_\lambda(-\mathbf{q}, -cq) = C_\lambda^*(\mathbf{q}, cq). \tag{90}$$

Substitution of the third line of equation (89) into equation (88) gives:

$$\mathbf{J}_\lambda^L(\mathbf{r}, t) = \frac{1}{2\mu_o}\int_{-\infty}^{\infty}\frac{\mathbf{q}d^3q}{(2\pi)^3}[iC_\lambda(\mathbf{q}, cq)e^{i(\mathbf{q}\cdot\mathbf{r}-cqt)} + c.c.] \tag{91}$$

Here, 'c.c.' is an abbreviation for complex conjugate. Thus, generation of a current density, $\mathbf{J}_\lambda^L(\mathbf{r}, t)$, launches a longitudinal displacement field. When no *external* field exists, the scalar-propagator description yields:

$$\begin{aligned}\mathbf{D}_{CURV}^L(\mathbf{r}, t) &= \varepsilon_o\mathbf{E}^L(\mathbf{r}, t) + \mathbf{P}_{CURV}^L(\mathbf{r}, t)\\ &= \frac{1}{c^2}\int_{-\infty}^{\infty} d^3r'dt'g(r', \tau)\frac{\partial\mathbf{J}_\lambda^L(\mathbf{r}, t)}{\partial t'}.\end{aligned} \tag{92}$$

The first line of equation (92) comes from equation (66). The second line of equation (92) arises from the Green's-function solution to equation (70) for no gradient in the free-space charge density. Equation (92) implies that a rapidly varying longitudinal current density produces a strong displacement field.

5.5. $\mathbf{E}^L$ propagation in curved space-time

For the following analysis, we assume that $\partial \mathbf{J}_\lambda^L(\mathbf{r}, t)/\partial t$ is non-zero over a finite space-time interval. After $\partial \mathbf{J}_\lambda^L(\mathbf{r}, t)/\partial t$ is zero, equation (92) then implies:

$$\varepsilon_o \mathbf{E}^L(\mathbf{r}, t) + \mathbf{P}_{CURV}^L(\mathbf{r}, t) = \mathbf{0}. \tag{93}$$

In the absence of gyromagnetic effects [21], we have:

$$\mathbf{P}_{CURV} = \varepsilon_o(\overleftrightarrow{\varepsilon}_r - \overleftrightarrow{u}) \cdot \mathbf{E}. \tag{94}$$

The relative dielectric tensor is $\overleftrightarrow{\varepsilon}_r$ with Cartesian elements in terms of the contravariant metric tensor, $g^{\mu\nu}$:

$$\varepsilon^{ij} = \sqrt{-g}(g^{io}g^{jo} - g^{oo}g^{ij}). \tag{95}$$

Equations (93), (94) yield the free-space wave equation for $\mathbf{E}^L$:

$$\Box[\overleftrightarrow{\varepsilon}(\mathbf{r}, t) \cdot \mathbf{E}(\mathbf{r}, t)]^L = 0, \tag{96}$$

with $[\overleftrightarrow{\varepsilon} \cdot \mathbf{E}]^L$ as the longitudinal component of $\overleftrightarrow{\varepsilon} \cdot \mathbf{E}$. equation (96) reduces to equation (97) for a self-consistent propagation of the $\mathbf{E}^L$-field in a spatially flat, expanding universe [13]:

$$\Box[a(t)\mathbf{E}^L(\mathbf{r}, t)] = \mathbf{0}, \tag{97}$$

where $a(t)$ is the Robertson-Walker scale factor [25, 26].

6. $\mathbf{J}_\lambda^L$ by nonlinear optical mixing

Strong electromagnetic fields are present in many astrophysical processes. For example, longitudinal electric-field oscillations (e.g., plasma oscillations) may dominate, which have a frequency that is a function of the plasma density, and the mass and charge of the oscillating ions and electrons. Moreover, sum- and difference-frequency modes are generated in nonlinear (NL) optics at lowest (second) order. Such modes generate a NL current density ($\mathbf{J}^{NL}$) at the sum- and difference-frequencies for a non-vanishing susceptibility tensor, $\overleftrightarrow{\chi}(\mathbf{q}_1, \omega_1; \mathbf{q}_2, \omega_2)$, via the sum:

$$\mathbf{J}^{NL}(\mathbf{q}, \omega) = \int_{-\infty}^{\infty} \frac{d^3q' d\omega'}{(2\pi)^4} \overleftrightarrow{\chi}(\mathbf{q}, \omega; \mathbf{q}', \omega') \colon \mathbf{E}^L(\mathbf{q}', \omega')\mathbf{E}^L(\mathbf{q} - \mathbf{q}', \omega - \omega'). \tag{98}$$

The nonlinear, longitudinal current density for $\omega = cq$ is:

$$\mathbf{J}_\lambda^L(\mathbf{q}, cq) = \mathbf{J}^{NL}(\mathbf{q}, cq). \tag{99}$$

Hence, $\mathbf{E}^L$ in the source's near-field via the generated $\mathbf{J}^{NL}$ can drive L-waves in free, curved space-time. A closely related source is current density fluctuations.

7. Conclusions

The present work provides insight into extended electrodynamics (EED) that was pioneered by Ohmura [1] and studied recently by others [8–15]. In the Lagrangian formulation, EED is described in terms of the well-known λ-parameter (λ not necessarily one). Previous uniqueness theorems [13–15] formulated EED with a non-zero, scalar field, $C = \partial_\mu \mathbf{A}^\mu$. In general relativity, EED allows propagation of longitudinal (L) waves in charge- and current-free, space-time regions. L-waves cannot exist in free-space CED. We show that EED: (i) only affects the longitudinal components of the fields and vector potential, and (ii) needs both C and λ to obtain consistency between the longitudinal electric field ($\mathbf{E}^L$) and the potential descriptions (Φ, $\mathbf{A}^L$) descriptions. We prove that the parameter product, λC, is completely determined by a superposition of longitudinal displacement-field modes, all of which are confined to the energy shell, $|\omega| = cq$. This formulation predicts that L-waves can be emitted into free-space from the near-field of a source by non-linear, electrodynamic mixing. A propagator formalism offers substantial physical insight into the L-wave emission process.

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ORCID iDs

Lee M Hively https://orcid.org/0000-0002-0449-9107

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