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ISSUES IN LARGE-ANGLE SCATTERING*

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ABSTRACT

We analyze phenomenologically several models for the high-energy scattering of hadrons through fixed, large angles. The emphasis is on trying to isolate and understand those aspects of hadronic forces which are important at large angles. We review the fixed-angle-lower bounds derived from analyticity and discuss how simple geometrical concepts can be used to guide our extrapolation of cross sections away from the forward and backward peaks into the large angle region. This extrapolation is important in understanding whether or not we need a new, hard component of the hadronic force to interpret the data. We try to isolate the important features of dual models, statistical models and constituent models and to clarify the possibility of experimental distinction between these approaches.

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I. INTRODUCTION AND KINEMATICS

The small-t region of hadronic scattering processes has been extensively investigated experimentally and has been subjected to thorough theoretical analysis. [1] The Regge exchange picture which describes data in this region has been tested, adjusted, and retested. Though it has not remained simple through all these adjustments there seems little doubt that the approach is viable. Those questions which are still open seem quite capable of being resolved within the framework of the basic exchange picture.

In comparison, the description of high energy scattering of hadrons through a fixed large angle is anything but decided. Although experimental results are not new, [2] the topic has not attracted corresponding theoretical interest. The one clear observation is that cross sections at fixed angles fall rapidly. This fact, in turn, implies small experimental counting rates at high energy and rules out the kind of detailed comparison of theory with experiment which is possible in the peripheral peaks. At this point several quite distinct models for the wide-angle hadronic processes are roughly consistent with experiment. Which, if any, of these models will ultimately prove correct is unclear but the new, high-statistics, experiments which may resolve the issue are now being considered. It seems appropriate, therefore, to compare these models in order to provide a framework within which new experimental results can be interpreted.

In this paper we will consider the high energy scattering of spinless hadrons through a fixed, large angle. The neglect of spin effects at large angles seems reasonable and allows us to simplify the formalism. We will refer to the process as ab → cd and label the kinematic variables

$$s = (p_a + p_b)^2$$
$$t = (p_a - p_c)^2 \qquad (1.1)$$
$$u = (p_a - p_d)^2$$

The CM momentum of particles a and b is given by

$$q_{ab}^2 = \frac{1}{4s}\left[s - (m_a + m_b)^2\right]\left[s - (m_a - m_d)^2\right] \sim \frac{s}{4} \qquad (1.2)$$

The scattering angle is given by

$$z = \cos\theta_s = \frac{s^2 + s(2t - (m_a^2 + m_b^2 + m_c^2 + m_d^2)) + (m_a^2 - m_b^2)(m_c^2 - m_d^2)}{4s\, q_{ab}\, q_{cd}} \qquad (1.3)$$

Some other useful approximations at high energy include

$$z \sim 1 + \frac{2t}{s}$$

$$(1 - z)(1 + z) = \sin^2\theta_s \sim \frac{4tu}{s^2} \qquad (1.4)$$

Because we are discussing spinless particles there is a single amplitude, $A(s,t) = A(s,z)$, which describes the process in all regions of the Mandelstam plane and gives the differential cross section

$$\frac{d\sigma}{d\Omega} = \frac{q_{cd}}{q_{ab}} \frac{1}{(8\pi)^2 s} |A(s,z)|^2 \qquad (1.5)$$

and we can use ϕ invariance to write

$$\frac{d\sigma}{dt} = \frac{1}{64\pi s\, q_{ab}^2} |A(s,t)|^2 . \qquad (1.6)$$

The partial wave series for A(s,t),

$$A(s,z) = \sum_{\ell=0}^{\infty} (2\ell + 1) a_\ell(s) P_\ell(z) , \tag{1.7}$$

converges inside the Lehmann ellipse and will prove a valuable tool at large angles, $z \cong 0$. For modest angles the approximation

$$P_\ell(\cos\theta) = J_0(\eta) + \sin^2(\theta/2) \left[\frac{J_1(\eta)}{2\eta} - J_2(\eta) + \frac{\eta}{3} J_3(\eta)\right] + 0(\sin^4\theta/2) \tag{1.8}$$

where $\eta = (2\ell + 1)\sin(\theta/2) \cong (\ell + 1/2) q_T/q$ gives a semiclassical impact parameter representation

$$A(s,t) \cong 2 \int_0^\infty b\,db\, \hat{a}(b,s) J_0(bq_T) \tag{1.9}$$

where

$$b = \frac{(\ell + 1/2)}{q}$$

and

$$\hat{a}(b,s) \cong a_\ell(s)\big|_{\ell \cong (bq - 1/2)} .$$

The expression (1.8) is useful in defining, at a given energy, a distinction between small and large angles. Small angles are those for which $(2\ell + 1)_{max} \sin(\theta/2)$ is small so that an impact parameter description is appropriate.

The plan of this paper is as follows: In Section II we discuss what can be learned from quite general analyticity principles. These results take the form of fixed-angle lower bounds as derived by Cerulus and Martin [3] and extended by Chiu and Tan. [4] Precisely because the experimental fixed angle cross sections fall so rapidly it is interesting to examine the theoretical conditions under which there are limitations on the asymptotic behavior. In Section III we discuss the fixed-angle asymptotic behavior in the framework of a simple semiclassical geometrical picture. This discussion will help clarify how the exact nature of absorption needs to be understood in order to specify the fixed angle "tail" of the peripheral peaks. In Section IV we examine possible resonance contributions to the fixed angle behavior in three different forms. First we discuss peripheral direct channel resonances in the form of the so-called direct channel Regge pole model of Chu and Hendry [5] and Schrempp and Schrempp. [6] This is found to be an explicit representation of many of the geometrical ideas discussed in Section III. We also discuss narrow resonance models and give a simple argument which relates the behavior of the amplitude at fixed angle to the asymptotic behavior of trajectory functions. Finally, we examine the class of statistical models which result from making a random phase assumption for the resonance contribution. In Section V we discuss field theory or constituent models for fixed angle scattering. The emphasis in this discussion is on obtaining some idea of the energy regime in which the scaling laws obtained in the field theory approaches might be valid. We also examine the assumptions which separate the constituent models from the other models without explicit constituents.

Although several comparisons with data are included in Sections II-V, the emphasis is on an exposition of the concepts behind the theoretical models. In Section VI we summarize the results of the models and address directly the question of how well experimental data can discriminate between the different approaches. Experimental evidence which bears indirectly on the models is also discussed, and we attempt to draw some conclusions.

II. ANALYTICITY BOUNDS AND KINOSHITA'S MINIMAL INTERACTION

Historically, one of the first formal discussions of high energy fixed angle amplitudes was in the form of a lower bound developed by Cerulus and Martin. [3] This work was very important in that it first showed how an amplitude at fixed angle is constrained by analyticity postulates.

The bound of Cerulus and Martin occurs if we assume:

1. The amplitude, $A(s,z)$, has the usual Mandelstam analyticity. That is, it is analytic in the z plane cut from $-\infty$ to $-(1+c/q^2)$ and from $(1+c/q^2)$ to $+\infty$ where c is some constant;
2. There is a finite domain in the z-plane in which the amplitude is bounded by s^N.

Through the use of a clever conformal mapping and the application of Hadamard's three circle theorem, [7] Cerulus and Martin showed that these assumptions imply the fixed angle lower bound,

$$|A(s,z)| \geq d \exp\left[-c(z)\, s^{1/2} \ln s\right] \qquad (2.1)$$

where $c(z)$ is some positive function of $z = \cos\theta$.

Aside from being a triumph in the application of complex variable techniques to high energy physics, the bound (2.1) has turned out to have phenomenological impact. Motivated, in part, by an empirical fit to the differential cross section of pp scattering by Orear, [8]

$$\left(\frac{d\sigma}{d\Omega}\right) \cong A \exp(-q_T/q_0) = A \exp(-q \sin\theta/q_0) \qquad (2.2)$$

where $A = 34$ mb/sr and $q_0 = 0.151$ GeV/c, Kinoshita [9] proposed that the bound (2.1) may be saturated by physical amplitudes for angles outside of the peripheral peaks. He formulated the principle of a "minimal interaction" which implies that fixed angle scattering amplitudes should assume the smallest value consistent with the general requirement of analyticity and unitarity. This hypothesis implies the absence of any really "hard" component in hadronic scattering so that instead of observing frequent collisions in which hadrons scatter through large angles we find instead the production of new hadrons at high energy.

In order to evaluate further the significance of Kinoshita's suggestion we must analyze the assumptions of Cerulus and Martin. In fact, the assumptions may be too strong. In particular, assumption (2) would not hold if Regge trajectories rise indefinitely so that for $z \neq 0$ there is a region of s for which the amplitude rises faster than any fixed power. Martin [10] has been able to rederive the bound (2.1) under the weaker assumption that the leading Regge singularity surface, $\alpha(t)$, has the asymptotic behavior

$$\alpha(t) = 0(t^{1/2}) \tag{2.3}$$

Even this assumption may be too strong since dual models have linear trajectory functions. For example, the Veneziano model [11] has the fixed angle asymptotic behavior [11,12]

$$|A_{\text{veneziano}}(s,z)| \sim G(s,z) \exp\left[-c(z)s\right] \tag{2.4}$$

which conflicts with (2.1). The Veneziano model does not have the usual Mandelstam analyticity in that the cuts on the real axis possessed by a physical amplitude are replaced by an infinite series of poles. It can be seen, however, that the important facet of the model which fixes the asymptotic form (2.4) is the existence of linear trajectories. This will be discussed more completely in Section IV. What is important here is the fact that it is possible to have a reasonable amplitude with a more rapid decrease at fixed angle than the original Cerulus-Martin bound (2.1).

Chiu and Tan first extended the methods of Ref. 3 to discuss Regge asymptotics more general than (2.3). [13] They showed that a generalized bound of the form

$$|A(s,z)| \leq \exp\left[-c_\gamma(z) s^\gamma \ln s\right] \tag{2.5}$$

can be written and that $\gamma = 1$ is appropriate for a linear Regge trajectory. By analysis of the phase contour structure of physical amplitudes Eden and Tan [14] showed that there is a value of γ

$$1/2 \leq \gamma \leq 1 \tag{2.6}$$

under which (2.5) is valid. Kaiser [15] has repeated this result under slightly different technical assumptions.

The interpretation of Kinoshita's conjecture is therefore seen to be very dependent on which version of the fixed angle bound (2.5) is appropriate. This is, in turn, related to the question of the asymptotic form of Regge singularity surfaces. In spite of the many other phenomenological successes of dual models, there is no experimental support for a falloff as rapid as than given by (2.4). If Regge trajectories are indeed linear, or approximately so, then the apparent absence of any exponential falloff in s would indicate a violation of the principle of a "minimum interaction." The question of the asymptotic behavior of Regge singularity surfaces is, of course, extremely difficult to pin down and without hypothesizing a breakthrough in experimental resonance spectroscopy techniques it seems unlikely that the dual model assumption of approximately linear trajectory functions will be either confirmed or ruled out in the near future. [16]

The possibility that singularity surfaces obey (2. 3) should therefore be kept in mind. If they rise no faster than a square root or if there is some J_{max} beyond which there are no J-plane singularities then the original version of the Cerulus-Martin bound is appropriate. In this case Kinoshita's conjecture has some chance of being valid. As we shall see, the hypothesis that the bound (2.1) is saturated for some c(z) has a simple geometric interpretation. This form also emerges naturally in several models. We can evaluate more carefully what is meant by a minimal interaction through examining the models.

III. SIMPLE GEOMETRICAL CONSIDERATIONS

The convenience of describing hadronic scattering in an equivalent semi-classical geometrical picture has long been emphasized. [17] Even if we had a complete theoretical picture the translation of this theory into familiar geometrical concepts would provide a useful mnemonic device. In the absence of a definitive theory the abstraction of a geometrical picture from phenomenological analysis can aid in the development of tractable theoretical ideas.

A. The Geometry of Hadronic Scattering as Determined at Small Momentum Transfer

The most reliable source of information about the geometrical structure of hadrons has been Regge pole phenomenology. Because Regge fits are done only over a limited range of transverse momentum the feature in the picture are necessarily crude. As a practical matter we can require of models for large-angle scattering processes that they be roughly consistent with the crude structure deduced from the small-momentum-transfer region. We can then apply the picture at large angles making the assumption that there is no finer structure and the results can be used to normalize and to examine the possibilities for new effects.

The basic picture of a hadron is that of an extended object with diameter approximately 1 fm (5.1 GeV^{-1}). In addition, we know that a hadron is fragile in high energy collisions. The fragility of a hadron is an important dynamical characteristic. It means simply that a hadron is likely to break into pieces in a collision. Collisions of hadrons bear certain similarities to collisions of ordinary macroscopic fragile objects (such as glass ashtrays) except that the pieces of a hadron are themselves hadrons or groups of hadrons. As indicated schematically in Fig. 3.1, the hadrons in a collision usually emerge retaining the direction of the momenta of the incident particles. When there are only two particles in the final state, the assumed fragility tends to mean that the event was either diffractive or peripheral.

The distinction between diffractive and peripheral scattering injects an important nonclassical element into the discussion. We will attempt to understand diffraction as the feedback of other processes back on the elastic channel so that a diffractive component of the elastic amplitude must be present at all impact parameters at which a collision takes place. [18] In order to understand the diffractive component, it is obviously important that we know something about production processes. There are many approaches to diffractive processes which involve making simple approximations for the production amplitudes. [19] It is not our purpose to review here these efforts but it is obvious that the overall geometrical picture we are discussing here depends on the nature of diffraction.

There is also a nondiffractive peripheral component in two-body processes. In Regge language this peripheral piece corresponds to quantum number exchange. It is <u>not</u>, however, the exchange of a simple Regge pole. A single Regge pole contains contributions at small impact parameter while our intuitive notion of fragility suggests that a collision at small impact parameter is unlikely to lead to a final state with only two hadrons. Phenomenological Regge models implement this dynamical constraint by <u>absorbing</u> the low partial waves of the Regge pole. Absorption consists of modifying a single Regge pole exchange with correction terms corresponding to cuts as indicated in Fig. 3.2. There is considerable disagreement among practitioners of Regge fits both about the precise nature of the basic Regge pole exchange and the treatment of the many-body intermediate states in the Regge cut diagram in Fig. 3.2. [20] All of the various approaches agree to some extent with the implications of our semi-classical notion of fragility in that quantum number exchange amplitudes tend to be large only within a band of impact parameter corresponding to the edges of the hadrons. As shown in Fig. 3.3 this feature of nondiffractive scattering also emerges from the more-or-less model independent extraction of amplitudes from data.

Since we are interested here in fixed-angle scattering, it is important to notice that these peripheral processes necessarily involve a smaller range of partial waves than the diffractive ones. The uncertainty principle in the form

$$A(s, \cos\theta) \gtrsim c(s)e^{-\Delta L(s)\Delta\theta} \tag{3.1}$$

can be used to relate the size of the fixed angle amplitude to the "coherence length" ΔL in angular momentum. Diffractive partial waves are roughly coherent from $\ell = 0$ out to $\ell = L_{max} \cong \frac{b_0}{2} s^{1/2}$ where $b_0 = b_0(s)$ is the diameter of a hadron so that using (3.1) we can get

$$\frac{A^{diff.}(s, \cos\theta)}{A^{diff.}(s, 1)} \gtrsim \exp\left\{-\frac{b_0}{2} s^{1/2} \theta\right\} \tag{3.2}$$

If $b_0(s)$ grows logarithmically with s the simple uncertainty principle result coincides in form with the lower bound of Cerulus and Martin. [3] This can be understood as giving a rough geometric interpretation to the analyticity assumptions of Ref. 3. In models with linear Regge trajectories we see we can have some type of coherence out to $\ell = L_{max} \approx \alpha' s$ which gives the weaker bound of Chiu and Tan. [13]

The peripheral or edge scattering should then dominate over diffractive scattering at large angles since it has a smaller coherence, ΔL^{periph}. This range is governed by the amount of overlap that two relativistic hadrons can have such that they exchange quantum numbers but do not fragment. This defines in some sense a "skin depth" of the hadrons. We obviously need a very detailed theory of absorption in order to obtain a quantitative expression for the skin depth. In simple models where Regge cuts are built iteratively from pole exchange such as indicated schematically in Fig. 3.2 the absorptive cuts are damped relative to the poles by logarithmic factors. In this case the exchange amplitude eventually behaves at fixed angle much like the diffractive amplitude. There seems, however, to be no fundamental reason why this iterative approach to absorption must be correct so we must consider a wider range of possibilities for ΔL^{periph}. Among these are growth corresponding to a ring in impact parameter space

$$\Delta L^{periph} \cong \left(\frac{b_1(s) - b_2(s)}{2}\right) s^{1/2} \tag{3.3}$$

where $b_1 - b_2 < b_0$. It is also possible that

$$\Delta L^{periph} \cong a \ln s \tag{3.4}$$

or even that it becomes asymptotically constant. At this point we can do little more than compare the various possibilities with data.

B. Simple Classical Model for the Forward and Backward Peripheral Peaks

It is an amusing exercise to consider a simple classical model for peripheral scattering. This exercise helps us understand under what conditions the peripheral amplitude can be important at large angles and give at the same time a reasonable description of the forward and backward peaks.

Assume that the partial waves of a nondiffractive amplitude are given by

$$a_\ell(s) = h_\ell(s)\, e^{i\eta_\ell(s)} \qquad \ell \in \left(\frac{b_0 s^{1/2}}{2} - c,\ \frac{b_0 s^{1/2}}{2} + c\right)$$

$$a_\ell(s) = 0 \ \text{ otherwise} . \tag{3.5}$$

In (3.5) $h_\ell(s)$ is a smooth function of ℓ and is real and positive while the phase, $\eta_\ell(s) \cong \eta(s)$, does not depend sensitively on ℓ over this range. We then have a nondiffractive amplitude

$$A^{n.d}(s,\cos\theta) \cong e^{i\eta(s)} \sum_{(\frac{1}{2})b_0 s^{1/2} - c}^{(\frac{1}{2})b_0 s^{1/2} + c} (2\ell + 1)\, h_\ell(s)\, P_\ell(\cos\theta) \tag{3.6}$$

The assumptions on the partial waves are quite severe and not necessarily realistic. Presumably a physical absorption mechanism would change both the phase and the modulus of the partial waves. By making $h_\ell(s)$ positive and by not separating even and odd partial waves we are imposing a "classical" definition of the forward direction for the process ab → cd and ignoring parity.

In the forward direction the nondiffractive amplitude (3.6) can be written

$$A^{n.d.}(s,0) \cong e^{i\eta(s)}\, 2c(s)\, b_0 s^{1/2} \left. \langle h_\ell(s) \rangle \right|_{\ell \cong \frac{b_0 s^{1/2}}{2}} \tag{3.7}$$

where the bracket denotes mean value. We now require the forward amplitude to have, within logarithmic factors, Regge asymptotic behavior,

$$A^{n.d.}(s,0) \sim \beta(0)\, s^{\alpha_M(0)} . \tag{3.8}$$

Let's assume that the trajectory intercept $\alpha_M(0)$ is the same as that empirically observed in hadron scattering, $\alpha_M(0) \cong 1/2$. We can combine (3.7) and (3.8) in the form

$$c(s) < h_\ell(s) > \sim \frac{\beta(0)\, e^{-i\eta(s)}}{2\, b_0} \; s^{\alpha_M(0) - 1/2} \tag{3.9}$$

Under our assumptions, the r.h.s. of (3.9) is a slowly varying function of s. From (3.5) we have

$$c(s) < \frac{b_0}{2}\, s^{1/2} \tag{3.10}$$

and from generalized partial wave unitarity,

$$h_\ell(s) \leq 1 \,. \tag{3.11}$$

Within the context of our simple model there are several ways in which we can get a slowly varying function of s on the left hand side of (3.9). The extremes are that

$$h_\ell(s)\Big|_{\ell \cong \frac{b_0 s^{1/2}}{2}} \cong \text{const.}$$

$$c(s) \cong \text{const.} \tag{3.12}$$

which "saturates" the unitarity bound with a fixed or slowly varying number of partial waves, or that

$$h_\ell(s)\Big|_{\ell \cong \frac{b_0 s^{1/2}}{2}} \cong d s^{-1/2}$$

$$c(s) \cong \frac{b_2 - b_1}{2} s^{1/2} \qquad (3.13)$$

where a ring of nearly fixed width in impact parameter is filled with partial wave amplitudes which vanish asymptotically.

The second possibility, (3.13), is closer to the situation which emerges from simple absorptive corrections to Regge exchange but since we cannot pretend to understand the total effect of all the absorptive corrections we cannot yet rule out the possibility (3.12) or some form of intermediate behavior. Using (3.1) we see that the slower the growth of c(s) the larger the asymptotic cross section at fixed angles.

In the backward peak,

$$A^{n.d.}(s,\pi) \cong e^{i\eta(s)} \sum_{(\frac{1}{2})b_0 s^{1/2} - c}^{(\frac{1}{2})b_0 s^{1/2} + c} (2\ell + 1)\, h_\ell(s)\, (-1)^\ell . \qquad (3.14)$$

If we write

$$g_n(s) = (4n + 1)\, h_{2n}(s) - (4n + 3)\, h_{2n+1}(s) \qquad (3.15)$$

then the assumed smoothness of $h_\ell(s)$ makes this well behaved and we can write

$$A^{n.d.}(s,\pi) \cong e^{i\eta(s)} \sum_{(\frac{1}{4})\, b_0 s^{1/2} - c/2}^{(\frac{1}{4})\, b_0 s^{1/2} + c/2} g_n(s) \tag{3.16}$$

$$\cong e^{i\eta(s)} \; c(s) \; \langle g_n(s) \rangle \Big|_{n \cong \frac{b_0 s^{1/2}}{4}} \tag{3.17}$$

Since g_n represents the difference between two partial waves of different parity we do not have a constraint from partial wave unitarity. Our classical picture would expect it also to be bounded by a constant but, for example, if the positive parity partial waves were systematically larger than the negative parity ones $g_n(s)$ could grow with n.

If we wanted a semiclassical picture of, for example πN scattering where the backward peak has power behavior characteristic of baryon exchange $\alpha_B(0) \cong 0$ we might expect

$$\frac{\langle h_\ell(s) \rangle}{\langle g_{\ell/2}(s) \rangle} \cong \text{const.} \tag{3.18}$$

so that

$$\frac{A^{n.d.}(s,0)}{A^{n.d.}(s,\pi)} \propto 2\, b_0(s)\, s^{1/2} \tag{3.19}$$

Although we have not dealt here with spin constraints we have seen that, at least for mnemonic purposes, it is possible to think of both forward and backward nondiffractive scattering occurring within the same peripheral band of partial waves. It therefore makes sense to consider the extrapolation of this simple picture to the intermediate angles between the two peaks and this extrapolation is important to the question of whether we see anything "new" at large angles.

C. Geometry as a Constraint on Models for Fixed Angle Scattering

Straightforward application of the uncertainty relation (3.1) can then give bounds or constraints on scattering amplitudes at large angles. Let's, for example, in the spirit of Kinoshita's conjecture, [9] assume that there is an absence of fine structure so that the uncertainty relation can be interpreted as an approximate equality for some $\Delta L(s)$.

$$\frac{A(s,\theta)}{A(s,0)} \cong \exp\left\{-\Delta L(s)\,\theta\right\} \qquad \theta \leq \pi/2 \tag{3.20}$$

For pp scattering we can ask whether (3.20) can be valid with $\Delta L(s)$ determined by the diffractive channel alone,

$$\Delta L(s) \cong \frac{b_0}{2}\, s^{1/2} \tag{3.21}$$

with $b_0 \cong 1$ fm (5.1 GeV^{-1}). We take this size to be the approximate width of a gaussian b-space amplitude which describes pp scattering and, to first approximation, neglect shrinkage effects.

$$\begin{aligned} s^2 \frac{d\sigma}{dt}(s,\ \theta = \pi/2) &\propto \exp\left\{-5.1\,\frac{\pi}{2}\,\sqrt{s}\right\} \\ &\propto \exp\left\{-8.0\, s^{1/2}\right\} \end{aligned} \tag{3.22}$$

which is much more rapid than the experimental falloff as seen in Fig. 3.4. We conclude that diffraction is negligible at 90° and that a geometrical model for pp scattering must contain a peripheral component as well as a diffractive one.

The best fit of the data in Fig. 3.4 to a form

$$s^2 \frac{d\sigma}{dt} (s, \theta = \pi/2) \propto \exp \left\{ -(\Delta b) \frac{\pi}{2} s^{1/2} \right\}, \tag{3.23}$$

where Δb can be interpreted as the width of the peripheral band in impact parameter gives

$$\Delta b \cong 0.48 \text{ fm} . \tag{3.24}$$

in rough agreement with the width of the band of partial waves inferred from $K^{\pm}p$ scattering in Fig. 3.3. We could get different numbers from fitting different portions of the curve, however, since the data does not follow a simple exponential.

More direct evidence for a peripheral component in pp scattering is found in a geometrical interpretation of structure in the polarization data. This is discussed in some detail by Hendry and Abshire [22] to whom we refer the interested reader. This is important since a straightforward application of duality or exchange degeneracy ideas might imply that pp is entirely diffractive.

Detailed fits to the shape and energy dependence of pp scattering which embody these simple geometric ideas have been presented by Chu and Hendry. [23] These fits are excellent at all angles and reproduce some quite complicated structure.

A comment on the applicability of geometrical concepts is in order. In Section V we will discuss field theory models for large angle processes. In these models the large-angle scattering is due to the pointlike constituents of the hadrons. The fixed angle differential cross sections fall off with a large power of s such as s^{-8} or s^{-10} instead of exponentially like (3.23). We could reproduce the fixed-angle behavior of these models if we allowed the band of important peripheral partial waves to grow logarithmically with s as in (3.6) instead of as $\sqrt{s}$. We cannot be sure if this type of interpretation of these constituent models is correct since, because they do not introduce a distance scale, there is no way of calculating in these models what the diameter of a hadron is and what a peripheral collision is. However, an extremely interesting development in the interpretation of constituent models for hadrons has been the formulation of so-called "bag models." [24] These models allow both for the quasi-free behavior of the constituents and for the comparatively sharp boundaries of the composite hadrons in a natural way. It would not be surprising to find that a version of the bag model could incorporate the concept of fragility we have discussed here. If this can be done it would represent a major step in the interpretation of collision processes.

Without a complete understanding of the binding mechanism it is difficult to know whether a hard scattering between two point constituents represents a "new" component of the hadronic cross section. If the constituents or partons are located randomly in the interior of the hadron this component would add incoherently to the tail of the small angle contributions we have considered here. One test for the region of validity of these models, therefore, is that fixed angle contribution of peripheral quantum number exchange be negligible. To a certain extent this can be guaranteed only after detailed amplitude analyses at different energies clarify the situation. At this point it is not clear that we need such a new component.

IV. RESONANCE CONTRIBUTIONS AT LARGE ANGLES

Resonance formation in hadron collisions provides a strong clue to the strong interaction force and resonance spectroscopy is therefore an active subfield of high energy physics. Large angle may be a particularly fruitful place to investigate the properties of direct channel resonances.

A. The "Direct Channel" Regge Pole Model

An interesting approach to the inclusion of direct channel resonances is due originally to Chu and Hendry. [23] This is the suggestion that direct channel resonances can be approximately accounted for by a "Regge Pole" in the direct channel. That is, we presume that the partial wave amplitude for spinless particles contains, in those channels with nonexotic quantum numbers, a contribution

$$a_\ell^{pole}(s) \cong \frac{f(s)}{(\ell - \alpha(s))(\ell + 1 + \alpha(s))} \tag{4.1}$$

where $\alpha(s)$ is a complex trajectory function. This corresponds to an amplitude of the form

$$A^{pole}(s,z) = \frac{f(s)\,P_{\alpha(s)}(-z)}{\sin \pi\alpha(s)}$$

$$\theta \neq 0 \,. \tag{4.2}$$

where $P_\alpha(z)$ is a Legendre function. In the strict forward direction, $\theta = 0$, the partial wave series (1.7) with (4.1) diverges but this can be handled by introducing some extra convergence factor in the sum over ℓ without altering significantly the validity of the approximation (4.2) away from the forward direction.

Chu and Hendry [5, 23] originally parameterized separately the s-channel helicity flip and nonflip amplitudes in πN scattering in terms of a central component given by a gaussian and a peripheral resonance component given by pole term analogous to (4.1). With a fair amount of freedom allowed by fitting separately the parameters at different energies they achieved a good fit to the differential cross section and polarization data at all angles. Typical fits to $\pi^- p \rightarrow \pi^- p$, $\pi^+ p \rightarrow \pi^+ p$ and $\pi^- p \rightarrow \pi^o n$ are shown in Fig. 4.1.

The fits of Chu and Hendry should be considered an explicit parameterization of the simple geometrical ideas discussed in Section III. They verify the connection of the central component with diffraction and the peripheral component with quantum number exchange. The fits offer a convenient way of summarizing a great deal of data.

B. Schrempp and F. Schrempp [6,25] have formulated a quite similar s-channel Regge pole model which they call the dual peripheral model. This model is motivated to some extent by the dual absorption model of Harari. [26] They take from the dual model the decomposition of the amplitude

$$A(s,t) = V(s,t) + V(s,u) + V(u,t) \tag{4.3}$$

and then use an amplitude very similar to $A^{pole}(s,z)$ in (4.2) for the terms with s-channel poles. A major improvement over the approach of Chu and Hendry is that Schrempp and Schrempp use a specific complex trajectory function

$$\alpha(s) = -\frac{1}{2} + \frac{i2R}{\pi} \; q \, \ell n \left(e \, \frac{-i\pi}{2} \; \frac{q}{q_0} \right) \tag{4.4}$$

with R = 1 fm in order to enforce the basic geometrical structure of a hadronic scattering event. With (4.3) they are also able to impose crossing relations. They show how their peripheral dual model explains a great deal of the systematics of $0^- \frac{1}{2}^+ \to 0^- \frac{1}{2}^+$ reactions. A good example of the predictions are those for polarization in $\pi p \to K\Lambda(\Sigma)$ and crossed reactions illustrated in Fig. 4.2.

Still another fit involving a direct channel Regge pole has been done by Kondo, Shimizu and Sugawara. [27]

One thing that can be done with the explicit form of the scattering amplitude (4.2) is to check the asymptotic fixed-angle behavior and compare it with our expectations based on the uncertainty relation (3.1). Inserting the asymptotic form for $P_\alpha(-\cos\theta)$ valid when $\mathrm{Re}\,\alpha \to \infty$ into (4.2) we get

$$|A^{\mathrm{pole}}(s,\cos\theta)| \sim \left[\frac{f(s)}{\sin(\pi\,\mathrm{Re}\,\alpha)\cosh(\pi\,\mathrm{Im}\,\alpha) + \cos(\pi\,\mathrm{Re}\,\alpha)\sinh(\pi\,\mathrm{Im}\,\alpha)}\right]$$
$$\times \left(\frac{2}{\pi|\alpha|\sin\theta}\right)^{1/2}\left[\sin^2\left((\mathrm{Re}\alpha+\tfrac{1}{2})(\pi-\theta)+\tfrac{\pi}{4}\right)+\sinh^2(\mathrm{Im}\alpha(\pi-\theta))\right]^{1/2}. \tag{4.5}$$

If $\mathrm{Im}\,\alpha$ is also large this simplifies to

$$|A^{\mathrm{pole}}(s,\cos\theta)| \sim \frac{2f(s)}{\sin(\pi\,\mathrm{Re}\,\alpha)+\cos(\pi\,\mathrm{Re}\,\alpha)}\left(\frac{2}{\pi|\alpha|\sin\theta}\right)^{1/2}\exp\left\{-\mathrm{Im}\,\alpha\;\theta\right\} \tag{4.6}$$

Since $\mathrm{Im}\,\alpha$ in the Breit-Wigner form of (4.1) determines the width, ΔL, of the band of partial waves which are important this is in agreement with the estimate of the amplitude based on (3.1).

If the direct channel Regge pole model is valid we can use the relation (4.6) which gives the shrinkage at fixed angle to compare with the value of $\mathrm{Im}\,\alpha(s)$ which determines the total widths of the peripheral resonances through

$$M_N\Gamma_N \cong \left.\frac{\mathrm{Im}\,\alpha(M_N^2)}{\frac{d}{ds}\,\mathrm{Re}\,\alpha(M_N^2)}\right|_{\mathrm{Re}\,\alpha\,=\,N} \tag{4.7}$$

One important qualification concerning the use of the direct channel Regge pole model exists. Taken at face value, it implies that only those channels with nonexotic s-channel quantum numbers and, hence, resonances should have a peripheral component in the amplitude. This agrees with the basic ideas of 2-component duality and exchange degeneracy. However, we have already noted that the interpretation of the pp elastic polarization data given by Hendry and Abshire [22] requires a peripheral band of partial waves in at least one of the helicity amplitudes in this process.

B. Fixed Angle Behavior of Meromorphic Amplitudes

The asymptotic behavior at fixed angle of the Veneziano model [28] was discussed in Section I as an example of the violation of the original form of the Cerulus-Martin bound. [3] We would like to present here a simple way to estimate the fixed angle asymptotic behavior of a more general class of meromorphic functions (functions whose only finite singularities are poles). This exercise is instructive and, if we assume Veneziano's interpretation [29] of dual models as being a realistic approximation of physical amplitudes, will give us some insight into the possible behavior of the data. The convenient feature of meromorphic amplitudes is the absence of any normal threshold singularities. Because of this we can use simple analyticity arguments to discuss asymptotic behavior in terms of the spacing of poles and zeros.

Suppose we have a crossing-symmetric meromorphic amplitude which can be decomposed

$$A(s,t) = M(s,t) + M(s,u) + M(t,u) \tag{4.8}$$

in the usual way. The function $M(s,t)$ has poles located on the real axis at $\alpha(s_N) = N$ and at $\alpha(t_M) = M$. We want to get an estimate of the asymptotic behavior of $A(s,t)$ as $s \to \infty$ with $t/s = -\xi$ fixed. At high energy using (1.3)

$$\xi \cong \frac{1-z}{2} \tag{4.9}$$

In this limit the Beta function has the behavior [11,12]

$$\left.\frac{\Gamma(-s)\Gamma(-t)}{\Gamma(-s-t)}\right|_{t/s=-\xi} \sim \frac{\Gamma(\frac{\xi}{1-\xi}\,\mathrm{Re}\,s)}{\Gamma(1+\frac{\xi}{1-\xi}\,\mathrm{Re}\,s)}\left(\frac{2\pi\xi\,\mathrm{Re}\,s}{1-\xi}\right)^{1/2} \exp\left\{i\left[\xi\pi\,\mathrm{Re}\,s + \mathrm{Im}\,s\,\log(1-\xi)\right]\right\}$$

$$\times \exp\left\{\mathrm{Re}\,s\left[\xi\log\xi + (1-\xi)\log(1-\xi)\right] - \frac{\xi}{1-\xi}\frac{(\mathrm{Im}\,s)^2}{\mathrm{Re}\,s} + O\left(\frac{\mathrm{Im}\,s}{\mathrm{Re}\,s}\right)\right\} \tag{4.10}$$

which simplifies to (2.4). [30]

We can get a fairly good estimate of the fixed angle asymptotic behavior without knowing the specific form for the amplitude $M(s,t)$ in terms of Beta functions. Consider the argument principle [31] applied to the function $F_\xi(s) = M(s, -\xi s)$,

$$n_z(\gamma, F) - n_p(\gamma, F) = \frac{1}{2\pi i}\int_\gamma ds\,\frac{F_\xi'(s)}{F_\xi(s)} \tag{4.11}$$

Here γ is a closed contour which does not intersect any poles or zeros, $n_z(\gamma, F)$ is the number of zeros of F, and $n_p(\gamma, F)$ is the number of poles of F contained within γ. For simplicity we can choose γ to be the closed circle

$$s = Re^{i\theta} \qquad \theta \in (0, 2\pi) \tag{4.12}$$

with the understanding that, if necessary, we treat separately the regions near $\theta = 0$ and $\theta = \pi$ where we might be close to poles or zeros.

By our definitions, the number of poles enclosed in this contour is given by

$$n_p(\gamma, F) = [\alpha(R)] + [\alpha(\xi R)] + 2 \tag{4.13}$$

where the brackets denote the greatest integer function. We will assume that the trajectory function grows indefinitely and that asymptotically it has the simple form

$$\alpha(s) \sim b s^{\beta} \tag{4.14}$$

Then the l.h.s. of (4.11) is large and negative. The fastest growth occurs in the situation where the number of zeros is small compared to the number of poles.

Let $h_\xi(s) = -\ln F(s)$, as $R \to \infty$ suppose we have an asymptotic approximation to $h(s)$ and $h'(s)$, that is, there is some $h^{(0)}(s)$ such that

$$\begin{aligned} h_\xi(s) &\sim h_\xi^{(0)}(s) \\ h_\xi'(s) &\sim h_\xi^{(0)'}(s) \end{aligned} \tag{4.15}$$

for s on γ.

In this case the statement of the argument principle (4.11) assuming (4.14) and (4.15) is

$$\frac{R}{2\pi} \int_0^{2\pi} d\theta \, e^{i\theta} \, h_\xi^{(0)'}(Re^{i\theta}) \sim c(\xi) R^{\beta} \tag{4.16}$$

In the absence of systematic cancellations in the integrand of (4.16) we can conclude

$$h_\xi'(s) \big|_{s \in \gamma} = 0(s^{\beta-1}) \tag{4.17}$$

where the power β is the same as that which gives the asymptotic behavior of the trajectory function. If we assume that, except for exceptional points, the logarithm is suitably smooth we get

$$M(s,\ \xi s)\ \sim \exp\left\{-g(\xi)s^{\beta}\right\} \tag{4.18}$$

The agreement with (3.1) can be understood by noting that $\Delta\ell_{max} \cong \alpha(s)$ in this amplitude.

It is an important question whether this strong connection between fixed-angle behavior and the trajectory functions is more general than meromorphic amplitudes. The derivation of fixed angle bounds suggests that this might be the case. However, we might expect that in a unitary, physical amplitude that Regge poles do not determine the fixed-angle behavior but that Regge cuts dominate. Because of this expectation, the work of Ellis and Freund [31] is interesting. They claim that in dual models with loop corrections where the dominant singularity remains linear, $\alpha(s) \sim \alpha' s$, that the asymptotic expression

$$A(s,z)\ \sim \exp\left\{-\alpha' s\ f(z)\right\} \tag{4.19}$$

remains valid. The expression (4.19) is actually verified only at the one-loop level but Ellis and Freund conjecture that it is a general feature of unitarized dual models.

This conjecture conflicts with the approach to dual models which takes seriously the resonance spectrum of the dual born terms but assumes that the primary effect of the unitarization procedure is to break the tremendous degeneracy of the mass spectrum. This latter approach emphasizes the connection of dual models to the statistical models which we will discuss more fully in

Section IV. C. Since there is no evidence for the rapid falloff (4.19) a resolution of this difference in interpretation is important to the continued viability of dual models.

C. Statistical Models

One line of thought which gives an alternative to constituent models for describing large transverse momentum phenomena can be grouped roughly under the general heading of "statistical models." These approaches have a venerable history [32] in the time scale of the development of ideas concerning hadronic processes. Much of the present thinking can be traced to the work of Hagedorn [33] and Frautschi. [34] There are many ways of motivating the "statistical" approach but perhaps the most direct and instructive is to form an analogy between hadronic physics and nuclear physics. We will briefly review this analogy [35] here in order to place in perspective the application of statistical models to large angle exclusive processes.

The diagram in Fig. 4.3 gives a rough indication of the energy levels in nuclear physics and hadronic physics.

In nuclear physics there are roughly three energy regimes. At low energies there are a few well defined resonances or energy levels which can be calculated in shell models. There is an intermediate energy range in which the number of levels is large and the levels can be treated statistically. There is even a small overlap region where the number of levels is large but they have not begun to overlap so that we can "verify" the statistical approximations to some extent. The situation in hadronic physics is similar even if not so clear cut. At low energies there are well defined levels which are described nicely by the quark model. At higher energies there appear to be more resonances but it is not completely clear where statistical approximations are relevant. Of crucial importance is the absence of an upper bound on the energy range of the statistical region. Most treatments assume that such a bound does not exist or, if it does, is located at super-asymptotic energies where free quarks can be created and that this energy is high enough to be ignored. In view of the fact that resonances overlap already at low energies, it seems impossible by traditional methods, such as phase shift analysis, to establish whether or not very high mass resonances exist and we have to follow our physical intuition. [36]

In nuclear physics it is acknowledged that amplitudes in the resonance region are saturated by the direct-channel resonance contributions. In hadronic physics we also have to consider contributions corresponding to resonances in the crossed channel. It is also not completely clear the role that diffractive processes play in hadronic physics. One assumption that is usually made is then the Freund-Harari hypothesis: [37-38] a finite fraction of nondiffractive amplitudes is given by the direct channel resonances. Once the existence of high-mass resonances in hadronic scattering amplitudes is assumed there are two ways of going about the derivation of the level density. The first is a self-consistent or "bootstrap" approach advocated by Hagedorn and Frautschi. Assuming a fraction, f, of the s-channel nondiffractive amplitude is saturated by resonances we have the "unitarity" constraint

$$\rho(m)\, \Gamma(m) = \frac{f}{2\pi}\, N(m) \tag{4.20}$$

where $\rho(m)$ is the level density. $\Gamma(m)$ the average total width and $N(m)$ is the number of open channels. We are already bringing in some statistical assumptions to say that (4.20) is valid on the average. The number of open channels, $N(m)$, in turn depends in a nonlinear way on the level density, $\rho(m)$, and by solving the "bootstrap" equation with assumptions about $\Gamma(m)$, we can obtain a form for $\rho(m)$. The level densities of Hagedorn and Frautschi can be parameterized asymptotically

$$\rho(m) \sim b m^{a} e^{m/kT_0} \tag{4.21}$$

There are some technical differences in the approaches which give different values of a. The value of kT_0 is derived by Frautschi and Hamer [35] and determined independently by Hagedorn [33] to be approximately

$$kT_0 \cong \Gamma \cong m_\pi \tag{4.22}$$

Notice that the expression (4.22) for the density of states is not valid in exotic channels such as pp or K^+p. One failing of the statistical approach is that selection rules have to be put in by hand. [39]

Instead of the bootstrap approach, one can analyze the level density of specific models. Krzywicki [40] has argued that the level density (4.21) is implied by the basic assumption of duality. Examination of specific dual models [41] has confirmed the validity of this level density under these assumptions. A different asymptotic level density can be found, for example, in the quark model of Feynman, Kislinger and Ravndal [42] but both agree on the number of intermediate mass states. The difference between the level densities is not likely to be resolved directly.

In the statistical model the behavior of amplitudes at large enough angles to be away from the forward diffractive peak and other "coherent" effects is presumed to be given by the incoherent sum of the resonances. It is not clear that the incoherent effect will necessarily dominate the coherent "tail" of peripheral exchange effects considered in the naive geometrical model of Section III. It may be true that the resonance contribution should be important at an intermediate range of energies. We are making a random phase approximation for the large number of resonances.

If we consider the process ab $\rightarrow$ cd, neglecting spin and assume that in the region of interest $|A|^2$ can be approximated by the incoherent sum of resonances

$$|A(s,z)|^2_{\theta \cong 90^0} \cong \sum_{i,\ell} \frac{(2\ell+1)^2 P_\ell^2(\cos\theta)(\gamma_{ab}^{i\ell})^2(\gamma_{cd}^{i\ell})^2}{(\sqrt{s} - m_{i,\ell})^2 + 1/4\, \Gamma_{i,\ell}^2} \tag{4.23}$$

The sum in ℓ extends over the range $\ell \epsilon$ (0, qR), where R is a typical hadronic radius. If we neglect the dependence of the residues over this range we can factor out the θ dependence

$$\xi(s,\theta) = \frac{\sum_0^{qR} (2\ell+1)^2 P_\ell^2(\cos\theta)}{\sum_0^{qR} (2\ell+1)} \tag{4.24}$$

and deal with the appropriate average quantities in the form

$$|A(s,z)|^2 \cong \frac{(2\pi)\,(\gamma_{ab}(\sqrt{s}))^2\,(\gamma_{cd}(\sqrt{s}))^2\,\rho(\sqrt{s})\,\xi(s,\theta)}{\Gamma(\sqrt{s})} \tag{4.25}$$

Now the further statistical assumption of equal partition of probability among channels

$$\frac{(\gamma_{ab}(\sqrt{s}))^2}{\Gamma(\sqrt{s})} \cong \frac{(\gamma_{cd}(\sqrt{s}))^2}{\Gamma(\sqrt{s})} \cong \frac{1}{\rho(\sqrt{s})} \tag{4.26}$$

allows us to simplify further

$$\left.\frac{d\sigma}{d\Omega}\right|_{\theta \cong 90^0} \cong \frac{1}{64\pi^2 s} \frac{\Gamma(\sqrt{s})}{\rho(\sqrt{s})}\, \xi(s,\theta) \tag{4.27}$$

By simple space-time arguments we know that a resonance cannot decay before a signal can pass across a typical hadronic radius and that

$$\Gamma(\sqrt{s}) = 0(\sqrt{s}) \tag{4.28}$$

The statistical bootstrap model gives, of course, a specific prediction for both $\Gamma(\sqrt{s})$, (4.22) and $\rho(\sqrt{s})$ (4.21) and has been compared to data by Eilam, Gell, Margolis, and Meggs. [33] There is only an overall normalization factor corresponding to b in (4.21), the expression for the density of states and a small ambiguity concerning the value of kT_0. Fits to $\pi^{\pm}p$ and $\bar{p}p$ elastic scattering at 90° are shown in Fig. 4.4 and compared to the power behavior of constituent models. The agreement is good.

A major flaw in the approach is that no predictions are made for the behavior of fixed angle cross sections in exotic channels. One possible way around this is to extend the observed spectrum to exotic channels and just state that the density of states in exotic channels is small. This would then imply, for example

$$\left.\frac{d\sigma/d\Omega\ pp \to pp}{d\sigma/d\Omega\ \bar{p}p \to \bar{p}p}\right|_{\theta \cong 90^{\circ}} >> 1 \tag{4.29}$$

$$\left.\frac{d\sigma/d\Omega\ K^{+}p \to K^{+}p}{d\sigma/d\Omega\ K^{-}p \to K^{-}p}\right|_{\theta \cong 90^{\circ}} >> 1 \tag{4.30}$$

which is in agreement with present observations. It would also seem that the naive approach would imply large fluctuations about the mean in such exotic channels. This is not observed as we will discuss in the next section. Presumably, the correct answer for exotic channels involves getting the spectrum in nonexotic channels right and then implementing crossing. This can be investigated in the context of dual models but is somewhat outside the range of the straight statistical models.

D. Ericson Fluctuations in Hadronic Physics

One way of testing for existence of overlapping direct channel resonances is to look for Ericson fluctuations. [44] These are well known in nuclear physics where, in the region of overlapping resonances, peaks and dips are attributed not to individual resonances but to fluctuations in the number and couplings of the overlapping resonances. The differential cross section in $p + {}^{56}F_e \rightarrow p + {}^{56}F_e$ [45] pictured in Fig. 4.5 is a particularly clean example of this effect.

If high mass direct channel resonances are a feature of hadronic scattering we would expect to see these oscillations in hadronic cross sections as well. The equations (4.26) and (4.27) should be expected to have corrections of order $(1/N)^{1/2}$

$$(\gamma_{ab}(\sqrt{s}))^2 = \frac{\Gamma(\sqrt{s})}{\rho(\sqrt{s})} \left(1 + 0(\Gamma/\rho)^{1/2}\right) \tag{4.31}$$

The period of these oscillations in energy is expected to be on the order of $\Gamma(\sqrt{s})$. The average resonance width can therefore be determined if fluctuations are observed and there period measured. In principle, the magnitude of the oscillations can be used as an indirect measure of the density. In hadronic scattering this is not too reliable since, for example, the $\rho(\sqrt{s})$ in the statistical bootstrap model (4.21) varies over the period of one oscillation (4.22). It is also not clear whether oscillations should be attributed to the fluctuations in $(\gamma)^2$ or the interference of fluctuations in γ with a coherent background.

Experimental searches for Ericson fluctuations in pp scattering have been made. Allaby et. al. [46] looked at 16.9 GeV/c over a range of angles and

Akerlof et.al. [47] looked at $\theta_{cm} = 90^{o}$ over a range of energies. Both results were negative. This would argue against trying to take the same statistical approach in pp scattering as in $\bar{p}p$ except for allowing the density of states in exotic channels to be small since this would imply large fluctuations. Frautschi [35] has interpreted structure at 180^{o} in $\pi^{-}p \rightarrow \pi^{-}p$ and $\pi^{+}p \rightarrow \pi^{+}p$ in terms of interference of statistical fluctuations with a coherent background. This is shown in Fig. 4.6.

F. Schmidt et.al. [48] have looked at $\pi^{\pm}p \rightarrow \pi^{\pm}p$ at large angles and at two nearby energies near 5 GeV/c. The structure they see, Fig. 4.7, is indicative of Ericson fluctuations although it would be more clear cut if more energy bins could be examined.

The search for Ericson fluctuations provides an important experimental tool for deciding on the existence of overlapping high mass resonances. If fluctuations are found in a given energy range it would tend to support the interpretations of the energy dependence of fixed angle cross sections in terms of direct channel effects. Simple constituent model explanations should probably not be applied at energies where there are such direct channel effects.

V. CONSTITUENT MODELS AND SCALING LAWS

The models we have discussed so far have all contained an explicit distance scale which governs the asymptotic behavior of fixed angle cross sections. In the dual resonance model [29-30] the scale is determined by $(\alpha')^{1/2}$ where α' is the slope of the dominant Regge singularity. In the other approaches a scale is determined by the size of the hadrons, [9] by their peripheral "skin depth" [5,6] or by the level density of excited states. [33,34] We now want to examine the possibility that the large-angle scattering of hadrons results from the pointlike interaction of elementary constituents. This approach is largely motivated by the fact that simple quark models explain crucial elements of hadron spectroscopy [42,49] while quark-parton models provide a good description of structure functions in deep-inelastic electron scattering. [50] Although composite hadrons are not necessarily neat surgical probes of their own structure, there is, in the context of specific models, a strong connection between electromagnetic form factors and structure functions and high-energy fixed-angle hadronic scattering.

A. Constituent Counting Scaling Laws

Brodsky and Farrar [51] have studied the scattering of composite objects in renormalizable field theories. They begin by studying diagrams such as those shown in Fig. 5-1. Neglecting the binding energy between the quarks in these diagrams it is necessary that all constituents of the same hadrons have equal momenta. In the kinematic region where t and u are large and proportional to s (the fixed-angle region) the CM energy of each constituent is then proportional to $\sqrt{s}$. Dimensional arguments indicate that the invariant amplitude has dimension

$$d = (\text{length})^{n-4} , \qquad (5.1)$$

where n is the number of external lines. If $\sqrt{s}$ is large compared to any masses in the problem the fixed-angle amplitude should be proportional to

$$A^B(s,z) \propto (\sqrt{s})^{-n+4} , \qquad (5.2)$$

From this point, the basic assumption is that there is a scale-invariant interaction between the quarks so that the binding of the quarks in the hadron does not modify this simple result. Brodsky and Farrar have therefore examined the assumption that this free-quark Born diagram has the same behavior as the physical amplitude in specific field theories. They find, for example, that diagrams like 5.1d can contribute a finite number of logarithmic factors but the result (5.2) is approximately valid for the physical amplitude unless some set of these diagrams sums to build up a new power. Diagrams such as 5.1e do not change the behavior (5.2) provided the bound-state wave functions are finite everywhere.

One exception to the Brodsky-Farrar rules has been reported by Landshoff. [52] He has found that the diagram shown in Fig. 5.2 dominates over the Brodsky-Farrar terms when there is a scale-invariant quark-quark scattering. There are reasons, within the context of specific models, [53] why diagrams such as that shown in Fig. 5.2 may not be important but there is still a great deal yet to be understood on this point.

Whatever the diagrammatic justification for the Brodsky-Farrar rules, they must be considered an outstanding empirical success. The formula for the fixed-angle differential cross section,

$$(d\sigma/dt)_{ab \to cd} \sim f(t/s)\, s^{-(n_a+n_b+n_c+n_d)+2} , \qquad (5.3)$$

combined with usual quark-model assignments for the particles has shown remarkable success for correlating the systematics of the different processes. The comparison between the prediction, (5.3), and the result of fitting the data at 90° to a simple power law is displayed in Table 5.1. [54-56]

The empirical success of (5.3) is all the more remarkable in view of the fact that the original assumptions, including the neglect of binding energies between the constituents, would seem to rule out the application of (5.3) in energy regumes where resonant effects are important. However, there is substantial evidence for πN resonances of mass $M_N \gtrsim 3$ GeV. This is in the middle of the range in which (5.3) is compared with data in Table 5.1. It may be that there is a new principle resembling the original form of Dolen-Horn-Schmid duality [58] which allows us to apply this type of asymptotic formula with no corrections even at very low energies. One of the problems in understanding the fact that there is no experimental evidence for unbound quarks arises from the ability of free-quark formulas such as (5.3) to successfully describe data. The very existence of high mass resonances is an interesting question in the context of field theory models because of the bearing such states have on the nature of the constituent binding mechanism.

One explicit dynamical model for the role of constituents in fixed-angle scattering is that of Blankenbecler, Brodsky and Gunion. [53] In this approach it is assumed that any direct interaction between quarks from different hadrons is absent or suppressed. The fixed-angle scattering is assumed dominated by the interchange of constituents. The identification is made between the hadronic constituents and the carriers of the electromagnetic current in order to relate the fixed-angle hadronic cross sections to form factors.

The expression for the fixed-angle invariant cross section in this model is of the form

$$A(s,z) \propto s\, F_a(s)\, F_c(t)\, F_d(u)\, I(z) \qquad (5.4)$$

where the F's are form factors and I(z) is some smooth function. There is some question whether this expression is appropriate for a 3-quark proton. With a dipole proton form factor (5.4) predicts an s^{-12} behavior for the pp elastic cross section which conflicts with the Brodsky-Farrar value in Table 5.1. In other reactions the predictions of (5.4) agree with the constituent counting rules.

In the constituent interchange model the connection between fixed-angle behavior and fixed-t behavior of amplitudes can be studied. The fixed-angle power behavior is found to join on smoothly with the Regge regions providing that Regge trajectories approach negative constants at large momentum transfers. For meson channels, the prediction is

$$\lim_{t \to -\infty} \alpha_m(t) = -1 \qquad (5.5)$$

where $\alpha_m(t)$ is the trajectory which, for example, determines large-t πp elastic scattering. This prediction is connected with the question, mentioned earlier, of the existence of high-spin, high-mass resonances. Experience with potential models [59] where (5.5) is valid combined with the usual analyticity properties of trajectory functions makes it difficult to reconcile (5.5) with an indefinitely rising resonance spectrum at positive t. Studies of effective Regge trajectories may be able to decide whether (5.5) is a better representation of the data than a linearly falling trajectory as we will discuss later. It is important to note that the prediction (5.5) is to be valid in exotic channels as well as those with nonexotic quantum number exchange. That is, it should describe the fixed-t behavior of $\pi^- p \rightarrow K^+ \Sigma^-$ as well as $\pi^- p \rightarrow \pi^o n$.

It should be noted that the constituent interchange model makes a large number of predictions for inclusive and semi-inclusive processes as well as fixed-angle scattering and that its success in correlating a large amount of data has been surprising. A more thorough discussion of the model is outside the scope of this paper and we refer the reader to the review of Blankenbecler. [60]

In a different interpretation of constituent models Fishbane and Quigg [61] have discussed the ratios of cross sections at 90^o under the assumption that they are proportional to the number of ways the constituents of a and b can be recombined to form c and d. This assumes the complete dissociation of the hadrons in a hard collision and the absence of any interference effects. For example, the ratios of the cross sections for $pp \rightarrow pp$ and $np \rightarrow np$ are obtained by making the usual quark model assignments of u (up) and d (down) quarks

$$\begin{aligned} p &= (uud) \\ n &= (udd) \end{aligned} \tag{5.6}$$

In a pp collision the number of ways we can reform to protons out of a collection (uuuudd) is

$$\binom{4}{2} \binom{2}{1} = 12 \ , \tag{5.7}$$

while in an np collision we have to reform a proton and a neutron out of (uuuddd) with probability.

$$\binom{3}{2} \binom{3}{1} = 9 \ . \tag{5.8}$$

The prediction of Fishbane and Quigg is then

$$\left.\frac{d\sigma/dt(pp \to pp)}{d\sigma/dt(pn \to pn)}\right|_{90^o} = \frac{12}{9} = \frac{4}{3} , \qquad (5.9)$$

a value which is in rough agreement with data. These combinatorics provide several other interesting ratios which should be compared with data. [61] It is important to notice that Fishbane and Quigg assume the "standard" quark model instead of colored quarks. It is interesting, within the assumptions associated with complete dissociation, to investigate how sensitive these ratios are to different constituent schemes.

B. Other Field Theory Models

The picture of Blankenbecler et.al. [53] discussed above is not the only approach to fixed angle scattering based on field theory which has some phenomenological backing. Fried, Kirby and Gaisser [62] advocate a picture where the scattering of hadrons through large angles occurs from a single hard exchange modified by the effect of exchanges of soft, virtual neutral vector mesons between external hadronic legs. They achieve an approximate wide angle formula

$$\frac{d\sigma}{dt}(s,z) = (d\sigma/dt)_H \; S(s,z) \qquad (5.10)$$

where $(d\sigma/dt)_H$ is the Born approximation for single hard meson exchange and $S(s,z)$ is the effect of all the soft external leg insertions. From this point, they fit some parameter and then use the relative strength of physical ω-couplings to make some nontrivial predictions. In this model

$$
\begin{aligned}
d\sigma/dt(pp \rightarrow pp)\Big|_{90^o} &\sim a_1\, s^{-11.2} \\
d\sigma/dt(\pi p \rightarrow \pi p)\Big|_{90^o} &\sim a_2\, s^{-6.6} \qquad (5.11) \\
d\sigma/dt(\gamma p \rightarrow \pi p)\Big|_{90^o} &\sim a_3\, s^{-6.6}
\end{aligned}
$$

In this model pp and kp elastic scattering have the same s-dependence in view of the strong ω-coupling to $K\overline{K}$. The appearance of fractional powers in (5.11) is not attractive and the systematics of Table (5.1) favor the simpler Brodsky-Farrar powers but this work is important in that it shows that, within the context of field theories, it is hard to justify the neglect of external leg insertions since, with physical coupling constants, they can significantly modify the power behavior. The assumption in the Brodsky-Farrar approach that these are unimportant needs further examination.

Preparata [63] has conducted a thorough investigation of large angle scattering within the framework of a massive quark model. The basic postulates of this model are:

(i) Quarks are the fundamental constituents.

(ii) The mass of a quark can be considered very large.

(iii) Hadrons are bound states of quarks with zero triality.

(iv) Green's functions display Regge behavior at high energies and decrease rapidly in the masses of external quark legs.

(v) Physical amplitudes can be constructed by an iterative procedure in the number of intermediate quark legs.

In this approach the interchange diagrams of Ref. 53 are suppressed compared to the diagram in Fig. 5.3. The results of the calculation yield

$$
\begin{aligned}
d\sigma/dt^{MB}\Big|_{90^o} &\sim \frac{\log^2(s/\mu^2)}{s^8} \quad f_{MB}(z=0) \\
d\sigma/dt^{BB}\Big|_{90^o} &\sim \frac{\log^2(s/\mu^2)}{s^{10}} \quad f_{BB}(z=0)
\end{aligned}
\tag{5.12}
$$

which agree with the Brodsky-Farrar rules. The fact that this model makes a definite prediction for the number of logarithmic factors is probably not significant for experiment. Rather it should be emphasized that the results are quite similar to those of Blankenbecler, Brodsky and Gunion in spite of the fact that the starting point is quite different.

VI. SUMMARY AND CONCLUSIONS

Although we have not attempted a complete review of the theoretical approaches to large-angle scattering, we have discussed a large number of models with a wide variety of predictions for the asymptotic cross sections. The models and their predictions are summarized in Table VI. 1. The first order of business for a phenomenologist would seem to be the comparison of the predictions in this table with experiment in order to decide which model is "correct". However, the problems involved in making a direct comparison of all these models with experiment are considerable. In spite of the fact that the asymptotic forms in Table VI. 1 show large differences, the differences in the predictions of the models at energies where data is currently available are actually quite small.

Only three different models can be ruled out by a straightforward comparison with present data. The dual model with linear trajectories makes the prediction (4.10). This fast falloff of the fixed angle cross section with s is simply not indicated by the data. The question of whether the bad, fixed-angle behavior should in any way "discredit" dual models is intriguing. The flaw should probably be considered minor in view of their other successes. As indicated in Section IV the behavior follows rather directly from the linear trajectory function and the degeneracy of states with the same mass and different angular momentum. This degeneracy may not survive a "unitarization" of the model and so the problem of the dual model's disagreement with fixed-angle scattering data is not considered serious by many proponents of the dual approach.

A second viewpoint which can be ruled out by the data is the strictest geometrical interpretation of Kinoshita's minimal interaction where the scale is determined by the size of hadrons. This is essentially the diffraction dominated geometrical model discussed in Section III (see Fig. 3.4). It should be noted, however, that the original formula by Orear, [8] Eq. (2.2), remains in substantial agreement with large angle scattering. If we relax the geometrical interpretation of Kinoshita's conjecture then his approach is not particularly meaningful but it cannot be ruled out.

The final approach to fixed angle scattering which can be ruled out is the behavior of pointlike hadrons interacting through a scale invariant force. This would predict a behavior for elastic scattering similar to that in QED

$$\frac{d\sigma}{dt} \sim f(z)\, s^{-2} \tag{6.1}$$

The most that can be said about this type of behavior is that it is evidently not significant at current energies. We will not say anything more about (6.1) here.

The remainder of the models discussed in this paper can still be considered active contestants in the field. The differences in their predictions for the fixed angle behavior do not work out to be large at present energies. In order to distinguish these models directly and convincingly it would seem as though we need two or three more decades falloff in the experimental data. The situation of having many theoretical models which cannot be distinguished at present energies but which diverge slowly as energy increases seems a fairly typical situation in particle physics. However frustrating it may be for experimentalists wanting a fast straightforward experimental test, it looks like unraveling fixed-angle models will take a lot of time. An important possibility is that new experiments will discredit all the remaining models simultaneously. It would be very convenient if theorists received a clear signal about which approach is correct. As it is, those who emphasize the connection of large-angle scattering with weak and electromagnetic phenomena will be drawn to the constituent approach while those who view wide-angle scattering as a continuation of small-angle hadronic effects are provided with several convenient mass scales.

There are several important subsidiary questions in the discussion of large-angle scattering problems. The first is the problem of high-mass, high-spin resonances. A few years ago, in the heyday of dual models, it seemed heretical to doubt that such resonances exist. There have since occurred several developments which have tended to shake this belief. The first, and most important, result was experimental — the failure of the Northeastern-Stony Brook [64] group to confirm the S, T, U enhancements reported by the CERN Missing Mass Spectrometer. [65] These bumps fitted nicely on a linear ρ-A_2 trajectory complex and gave comfort to those who held the dual model viewpoint.

Theoretical developments such as models for scaling, asymptotic freedom, etc. have tended to support constituent binding schemes in which it is awkward for trajectories to rise linearly. The essence of the parton model and the constituent interchange model discussed earlier is an idea of free or quasi-free constituents. We do not need high mass resonances in these constituent models and the assumption that the fixed angle behavior is given by a simple power becomes clear only when we are "above" the resonance region. We do not understand these models well enough to know whether the results might also be good at lower energies, but this is an area where considerable theoretical progress might be made.

Resonance spectroscopy therefore can provide a very important set of input into the area of knowledge concerning large-angle scattering. As we get to the masses where the phase shifting techniques are unable to make progress then it is important to have other evidence concerning resonant effects. The

Ericson-Frautschi fluctuations discussed in Section IV are an excellent place to start. Further experimental evidence concerning these fluctuations would be very important. The predictions made by Frautschi [35] in the framework of the statistical bootstrap models rely specifically on the assumption that average resonance widths are approximately (m_π) and that the exponential mass spectrum of the statistical model is valid. Some kind of fluctuations should appear, however, whenever resonance contributions are important at fixed angle and their presence should be considered strong evidence for high mass resonances. Whether or not the resonances have the properties predicted by the statistical bootstrap (or some other) model is a harder question which is not easily susceptible to test by the fluctuations.

Relevant to the question of whether or not we are in an energy regime where the simple constituent counting laws of Brodsky and Farrar [51] are apt to be valid is the paper of Hendry. [66] He points out that there is a great deal of fixed-t structure in the cross sections which intersects the fixed-angle behavior at current energies. This structure is easy to understand in the geometrical model of Chu and Hendry [23] discussed in Section III where it corresponds to structure in the Legendre functions which approximate the amplitudes. It is also possible to understand the structure in terms of Odorico zeros [67] in a resonance approach but it is not a feature of any simple class of field theory diagrams.

The graphs in Fig. 6.1 illustrate Hendry's point. It should be noted, however, that straight lines which interpolate the dip structure in these diagrams have a slope which is very close to the predictions of Brodsky and Farrar. This again may be a case where, for some unexplained reason, an asymptotic form extrapolates quite well into regions where it is not strictly applicable.

It is quite important to consider how predictions for fixed-angle connect up with the fixed-t behavior. One of the attractive features of the Chu-Hendry parameterization was that it provided a simple expression which covered the entire angular range. This feature has subsequently been supported by the work of Schrempp and Schrempp using the geometrical model at large, fixed t.

While not providing a description of the complete physical region there has been a considerable effort to extend the Constituent Interchange Model from the fixed-angle region into the fixed-t region. [53] This effort has resulted in predictions for the large-t behavior of Regge trajectories. Let us consider here the status of the predictions

$$\lim_{t \to \infty} \alpha_{\pi p}(t) = -1$$

$$\lim_{t \to \infty} \alpha_{pp}(t) = -2 \tag{6.2}$$

This prediction can be tested by measuring the effective trajectory in $\pi^- p \to \pi^o n$ and pp elastic scattering.

Barger, Halzen and Luthe [68] have calculated an effective trajectory in pp scattering using

$$\ell n \frac{d\sigma}{dt} (pp \to pp) = (2\alpha_{eff}(t) - 2) \, \ell n \, s + \ell n \, \beta(t) \tag{6.3}$$

This is shown in Fig. (6.2). Blankenbecler et. al. [69] have calculated the effective trajectory in Fig. (6.3) using

$$\ell n \frac{d\sigma}{dt} (pp \to pp) = (2 \, \alpha_{eff}(t) - 2) \, \ell n(-u) + \ell n \, \beta(t) \tag{6.4}$$

which they claim is more consistent with the duality properties of the pp elastic amplitude. The differences in the t-dependences of these two trajectory functions is primarily due to

$$\ell n(-u) = \ell n(s + t - 4m^2) = \ell n(s) + \ell n\left(1 + \frac{t-4m^2}{s}\right) \qquad (6.5)$$

and so the fact that the trajectory of Barger et.al. falls below -2 should therefore not necessarily rule out (6.2).

Possibly more significant evidence concerning the large-t behavior of Regge trajectories is the study using Finite Energy Sum Rules of the amplitude structure of $\pi^- p \rightarrow \pi^o n$. Elvekjar, Inanie and Rungland [70] report that the large t region the amplitudes are very similar to what is expected from a simple ρ-Regge pole with a linear trajectory. In particular they point out the right-signature zero at t = -1.6 and a second wrong-signature zero at t $\cong$ -2.4-2.5 consistent with the places where a linear trajectory would pass through -1 and -2 respectively. Their results are shown in Fig. 6.4 . There is some feedback through the "optimized convergence" finite energy sum rule of the form assumed for the trajectory function and the structure in the amplitude. However, if this structure is confirmed independently, for example, by amplitude analysis at large t then it would argue quite strongly for the existence of indefinitely falling trajectories.

A final word of caution is appropriate about the dangers of concentrating on one aspect of hadronic interactions, such as fixed-angle scattering, and drawing sweeping conclusions about models. Clearly, what we want is a theory which explains all the data: inclusive and exclusive. What we have tried to do here is to try to decide which aspects of such a theory might be particularly sensitive to experimental knowledge on fixed-angle scattering.

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FIGURE CAPTIONS

3.1. This diagram schematically suggests one of the basic features of collision processes of fragile hadrons — absorption of low partial waves in the $2 \rightarrow 2$ channels. A central collision is more likely to fragment one of the incident hadrons and contribute to a many-body final state as indicated in Fig. (a). The low partial waves in a 2-2 amplitude are suppressed leaving a peripheral component such as shown in (b).

3.2 Typical approach to absorbing the low partial waves of a Regge pole.

3.3 This figure taken from Ref. 21 demonstrates the geometrical difference between diffractive and nondiffractive scattering. Diagram (a) gives the Legendre coefficients for the "amplitudes" in K^+p and K^-p elastic scattering showing the presence of a central "diffractive" component. Diagram (b) shows the same thing for the difference of K^-p and K^+p and represents peripheral quantum number exchange. For more details see Schmid; Ref. 21.

3.4 $s^2\, d\sigma/dt$ for proton-proton scattering at 90°. If we ask for a peripheral component then the data are roughly consistent with (3.26) with a $\Delta b \cong .48$ fm. The falloff of the data is too rapid to be due to diffraction with $b \cong 1$ fm.

4.1 The differential cross section at 5 GeV/c for $\pi^+p \rightarrow \pi^+p$ (diagram A), $\pi^-p \rightarrow \pi^-p$ (diagram B), $\pi^-p \rightarrow \pi^0n$ (diagram C). The curves are the fits of Chu and Hendry explained in more detail in Ref. 23.

4.2 Figure taken from Ref. 25 where more details of the model can be found. The curves compare predictions of the dual peripheral model of Schrempp and Schrempp for the polarization.

4.3 Sketch of Energy Levels in Nuclear Physics and Hadron Physics.

4.4 (a) $\pi^- p$ elastic scattering data are compared with the theoretical predictions of the statistical model of Ref. 43 with $kT_0 = 140$ MeV (Solid line). The dashed line represents a form s^{-8} which corresponds to the asymptotic behavior of constituent models.

(b) Same for $\pi^+ p$ elastic scattering.

(c) Data on $\bar{p}p$ scattering are compared with the theoretical predictions of the statistical model of Ref. 43 with $kT_0 = 140$ MeV (Solid Line). The dashed line represents a form s^{-10} appropriate for the asymptotic behavior of statistical models.

4.5 $d\sigma/d\Omega$ for $p + {}^{56}Fe$ $p + {}^{56}Fe$ at energies around 9.4 MeV. Taken from Ref. 45.

4.6 Data on $d\sigma/dt$ for $\pi^+ p \rightarrow \pi^+ p$ at 0^o and 180^o. The structure in the backward data is interpreted by Frautschi as evidence for Ericson Fluctuations. For more details see Ref. 35.

4.7 Data from Ref. 48 showing fluctuations with energy of $\pi^\pm p$ elastic cross sections near 5 GeV/c. The quantity A is defined as

$$A = \frac{d\sigma(E + \Delta E) - d\sigma(E)}{d\sigma(E + \Delta E) + d\sigma(E)}$$

where $\Delta E = 36$ MeV.

5.1 Typical Diagrams considered by Brodsky and Farrar in Ref. 51.

5.2 A diagram investigated by Landshoff [52] which does not obey the Brodsky-Farrar constituent counting power law (5.2).

5.3 Diagrams for large-angle scattering in the massive quark model of Ref. 63.

6.1 (a) Structure in $d\sigma/dt$ for $\pi^+ p$ scattering pointed out by Hendry. [66]

(b) Same for pp.

TABLE 5.1

Process	Constituent Power	Experimental Power		Range $\sqrt{s}$
$\gamma N \rightarrow \pi^+ N$	7	7.3 ± 0.3	[54]	2.8-3.8
$K_L^o p \rightarrow K_L^o p$	8	8.5 ± 1.4	[56]	2.2-3.4
$\overline{K}_0 p \rightarrow \pi^+ \Lambda$	8	7.4 ± 1.4	[56]	2.0-4.0
$\overline{K}_0 p \rightarrow \pi^+ \Sigma^o$	8	8.1 ± 1.4	[56]	2.3-3.4
$K^+ p \rightarrow K^+ p$	8	7 ± 1	[55]	2.0-3.6
$\pi^- p \rightarrow \pi^- p$	8	8 ± 1	[57]	2.0-4.1
$\pi^+ p \rightarrow \pi^+ p$	8	7 ± 1	[55]	2.0-3.5
$pp \rightarrow pp$	10	9.7 ± 0.5		(2.5-6.1)
$\overline{p}p \rightarrow \overline{p}p$	10			

TABLE VI. 1

Comparison of Models for Fixed-Angle Hadronic Scattering Processes

A. Models with an Explicit Mass Scale

Model	Mass Scale	Fixed Angle Behavior	Section
Dual Models [11,12] with linear trajectory $\alpha(t) = \alpha(0) + \alpha' t$	$(\alpha')^{-1/2}$	$\exp\{-\alpha' s\, f(z)\}$	II, IV
Minimal [9] Interaction	$\approx 1\ (\mathrm{fm})^{-1}$ $(1/b_0)$ diameter of hadrons	$\exp\{-b_0 \sqrt{s}\ f(z)\}$	II, III
Statistical [33-35] Bootstrap	m_π	$f(z) \exp\{-\sqrt{s}/m_\pi\}$	IV
Fragile Hadrons [23-25] s-channel Regge pole	inverse of "skin depth" $\Delta d \approx 1/M_p$	$\exp\{-(\Delta d) \sqrt{s}\ \theta\}$	III, IV

B. Models with Pointlike Interactions

Model	Mass Scale	Fixed Angle Behavior	Section
Constituent Counting [51] Power Laws		$f(z)\, s^{2-n}$	V
Constituent [53] Interchange Model		$f(z)\, s^{-12}$ for pp if proton quark + "core"	V
Heavy Quark [63] Model		$f(z)\, (\ell n s)^2 s^{-8}$ – πN $f(z)\, (\ell n s)^2 s^{-10}$ – NN (no generalization)	V
Soft Virtual [62] Neutral Vector Mesons		$s^{-11.2}$ pp, Kp $s^{-6.6}$ πp	V
Quark-Gluon [50] "QED"		s^{-2}	

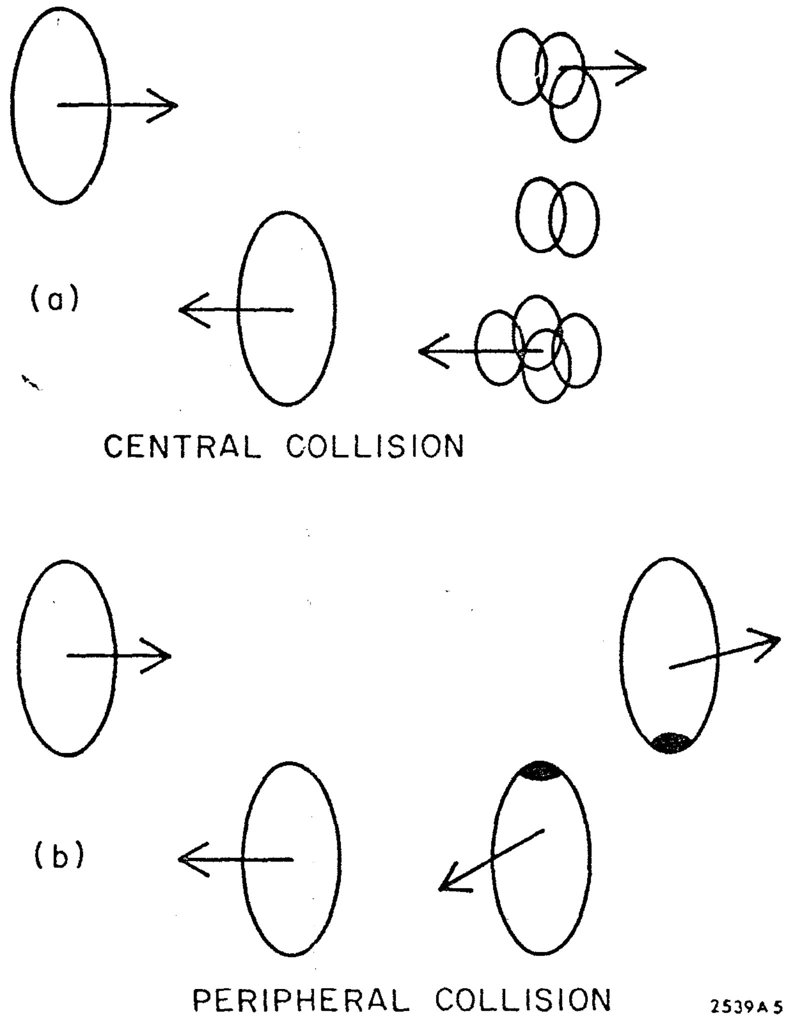


FIG. 3.1

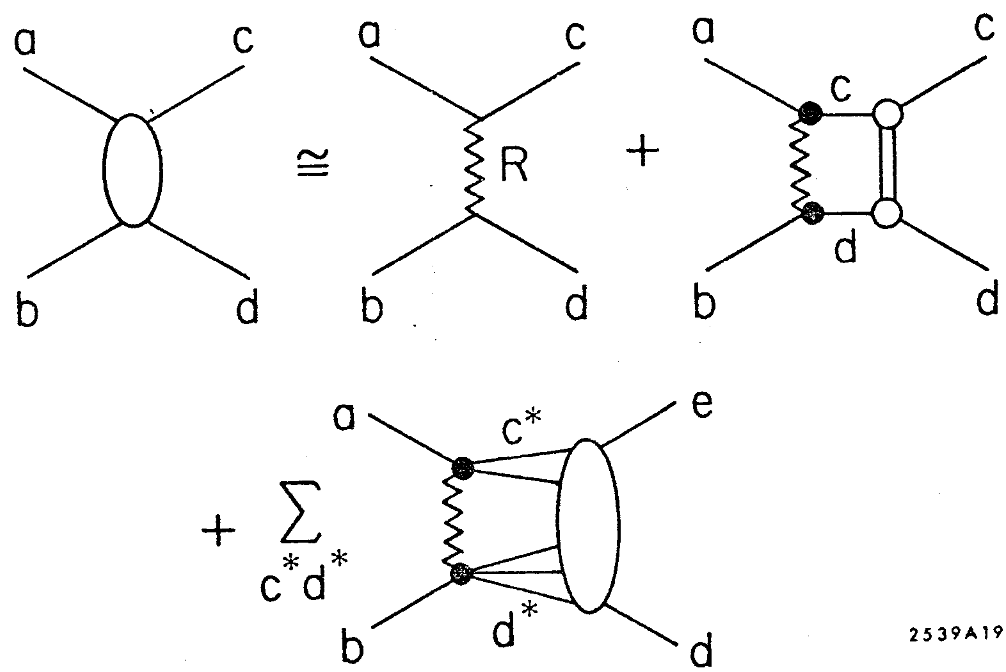


FIG. 3.2

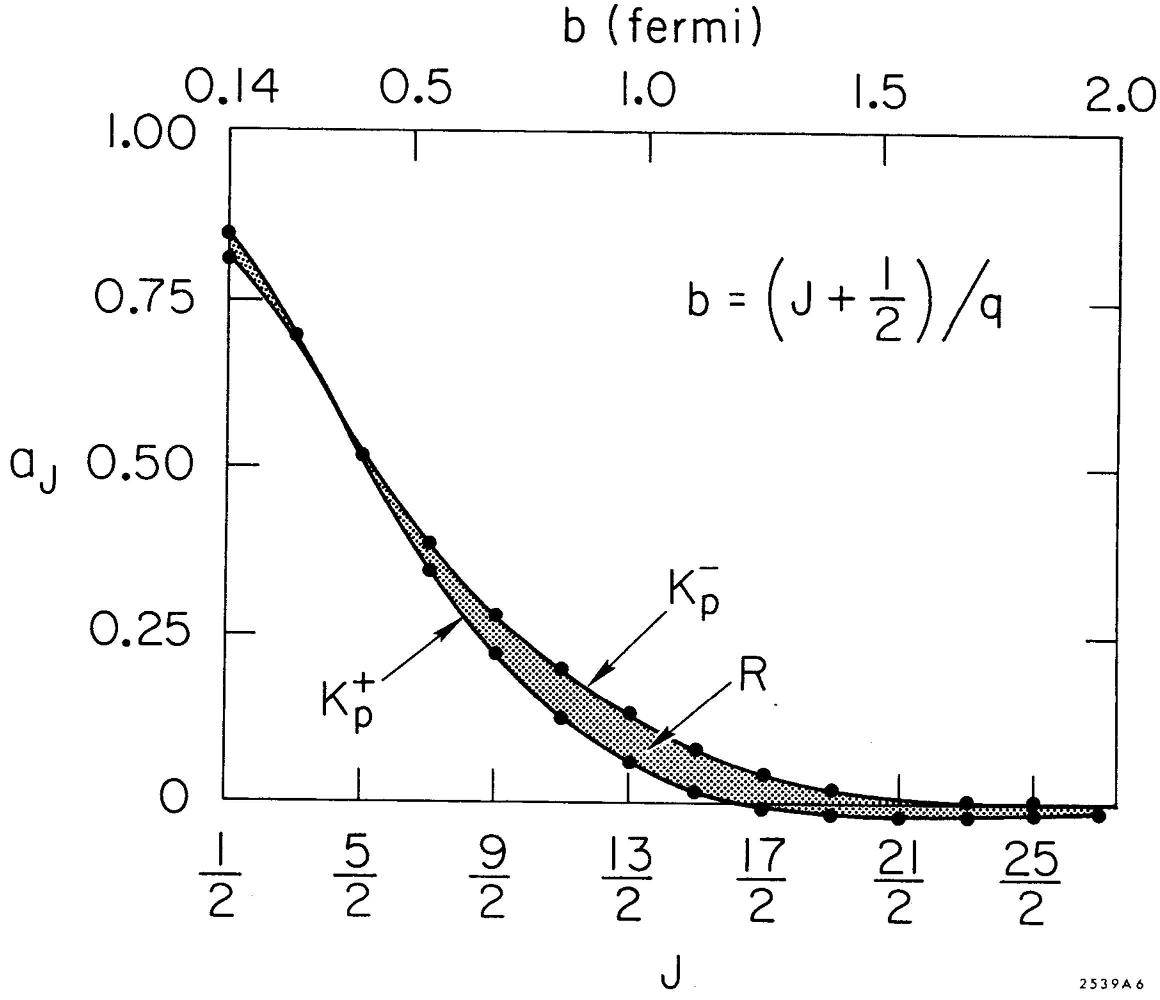

b (fermi)
0.14
0.5
1.0
1.5
2.0
1.00
0.75
a_J 0.50
0.25
0
$b = \left(J + \frac{1}{2}\right)/q$
K_p^-
K_p^+
R
$\frac{1}{2}$
$\frac{5}{2}$
$\frac{9}{2}$
$\frac{13}{2}$
$\frac{17}{2}$
$\frac{21}{2}$
$\frac{25}{2}$
J
2539A6

FIG. 3.3a

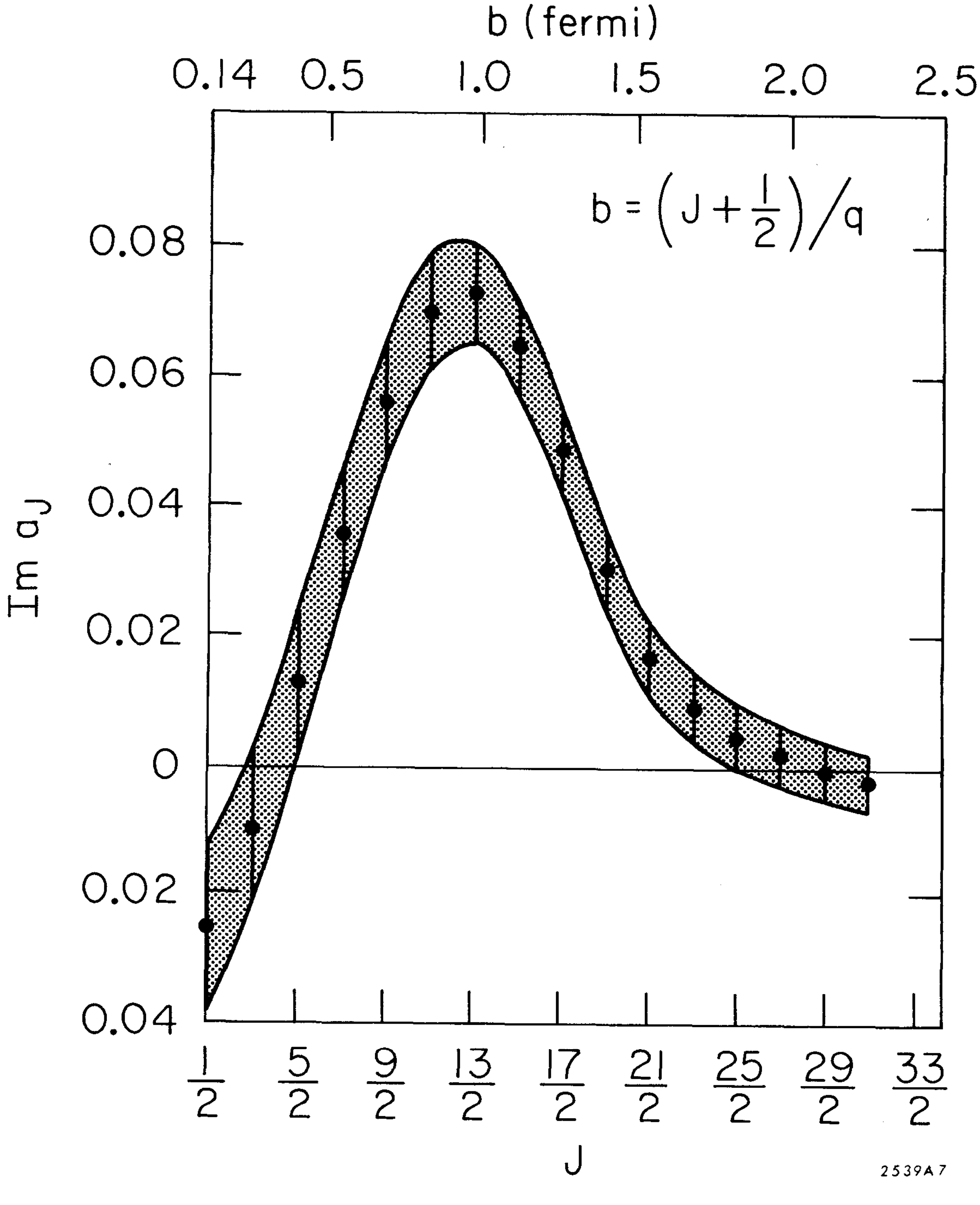

b (fermi)
0.14
0.5
1.0
1.5
2.0
2.5
$b = \left(J+\frac{1}{2}\right)/q$
0.08
0.06
0.04
0.02
0
0.02
0.04
Im a_J
$\frac{1}{2}$
$\frac{5}{2}$
$\frac{9}{2}$
$\frac{13}{2}$
$\frac{17}{2}$
$\frac{21}{2}$
$\frac{25}{2}$
$\frac{29}{2}$
$\frac{33}{2}$
J
2539A7

FIG. 3.3b

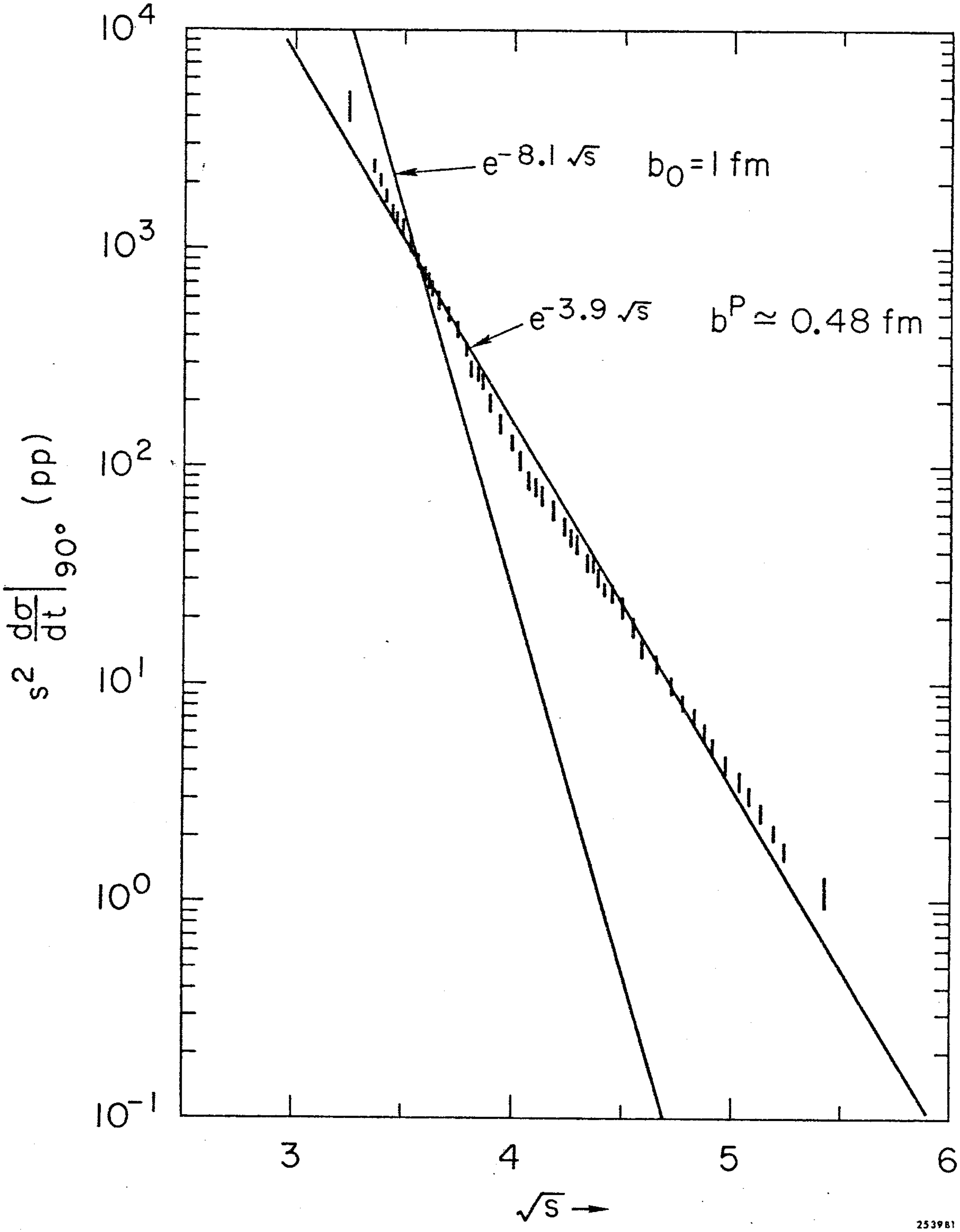
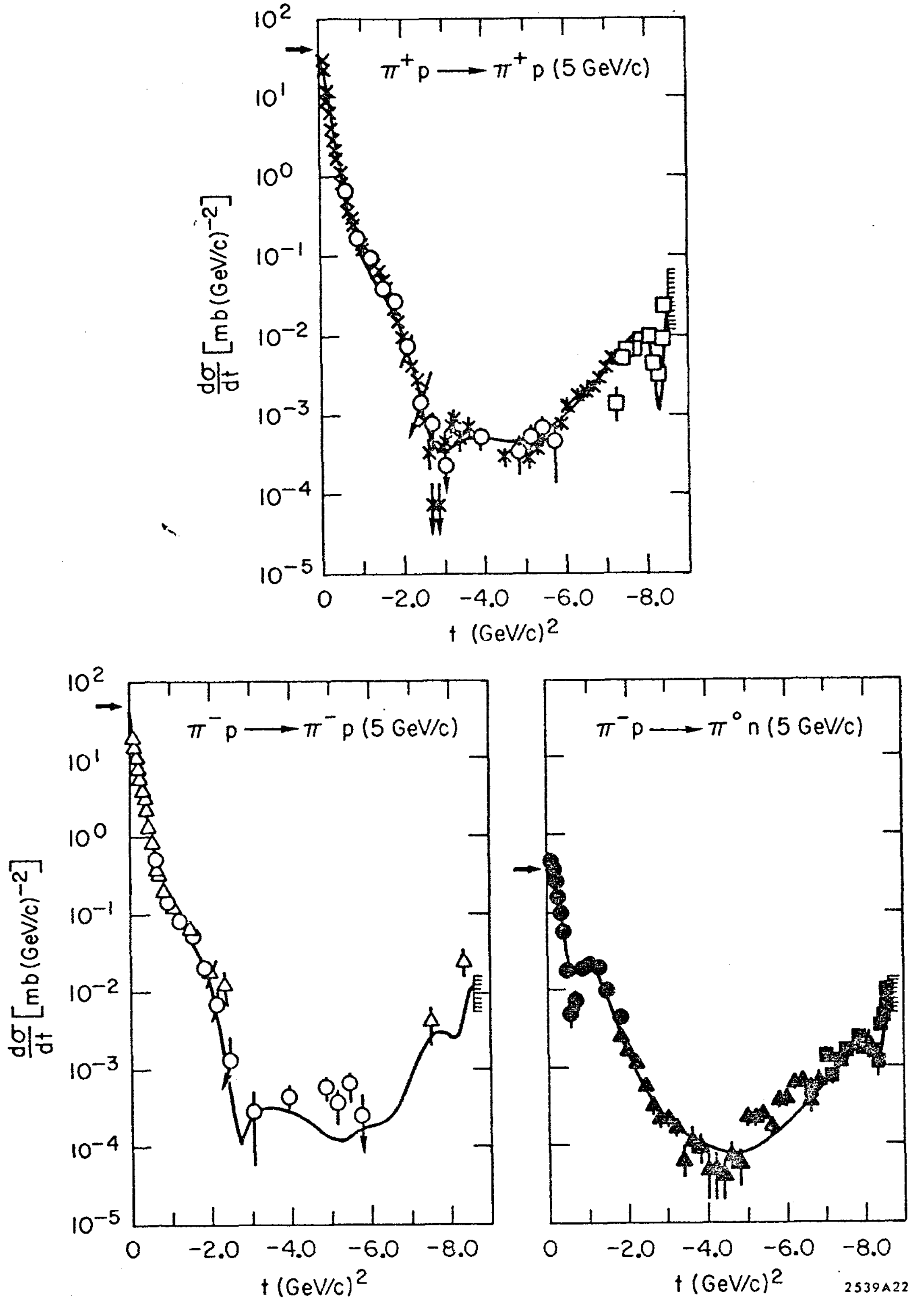
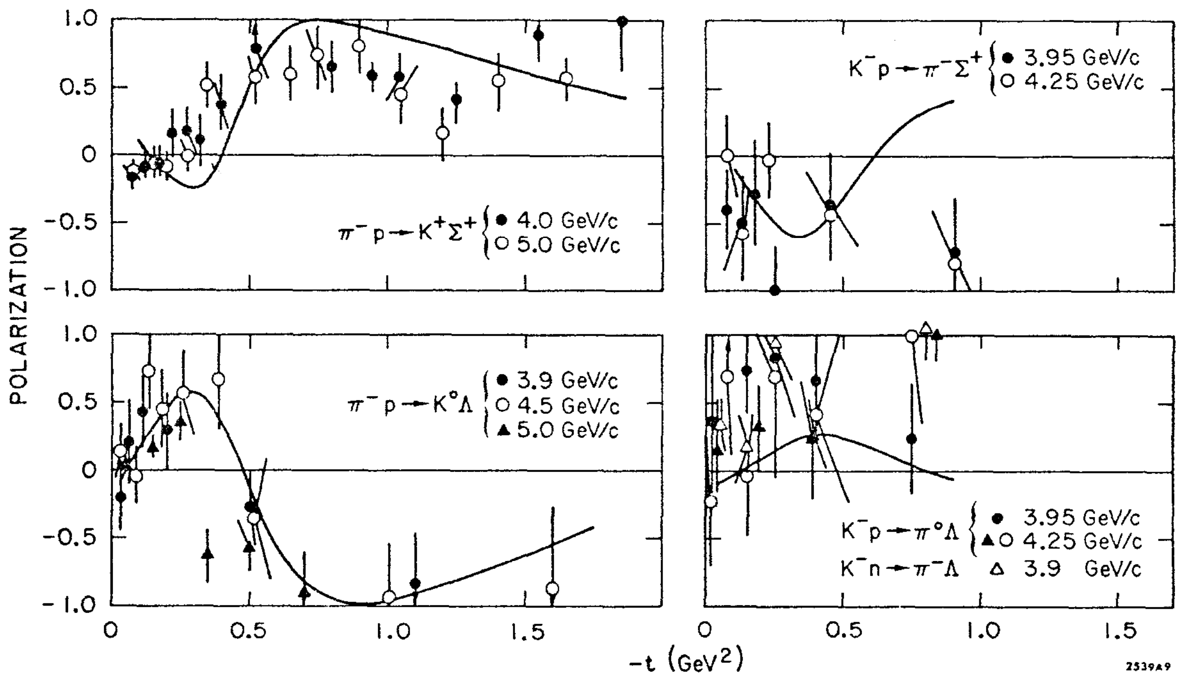

$e^{-8.1\sqrt{s}}$
$b_0 = 1$ fm
$e^{-3.9\sqrt{s}}$
$b^P \simeq 0.48$ fm
10^4
10^3
10^2
10^1
10^0
10^{-1}
$s^2 \left.\frac{d\sigma}{dt}\right|_{90°}$ (pp)
3
4
5
6
$\sqrt{s} \rightarrow$
253981

FIG. 3.4



$\pi^+ p \rightarrow \pi^+ p$ (5 GeV/c)
$\frac{d\sigma}{dt}$ [mb (GeV/c)$^{-2}$]
t (GeV/c)^2
$\pi^- p \rightarrow \pi^- p$ (5 GeV/c)
$\pi^- p \rightarrow \pi^\circ n$ (5 GeV/c)
2539A22

FIG. 4.1



POLARIZATION
1.0
0.5
0
-0.5
-1.0
$\pi^- p \to K^+\Sigma^+$ • 4.0 GeV/c ○ 5.0 GeV/c
$K^- p \to \pi^-\Sigma^+$ • 3.95 GeV/c ○ 4.25 GeV/c
$\pi^- p \to K^0\Lambda$ • 3.9 GeV/c ○ 4.5 GeV/c ▲ 5.0 GeV/c
$K^- p \to \pi^0\Lambda$ • 3.95 GeV/c ▲○ 4.25 GeV/c
$K^- n \to \pi^-\Lambda$ △ 3.9 GeV/c
0
0.5
1.0
1.5
-t (GeV²)
2539A9

FIG. 4.2

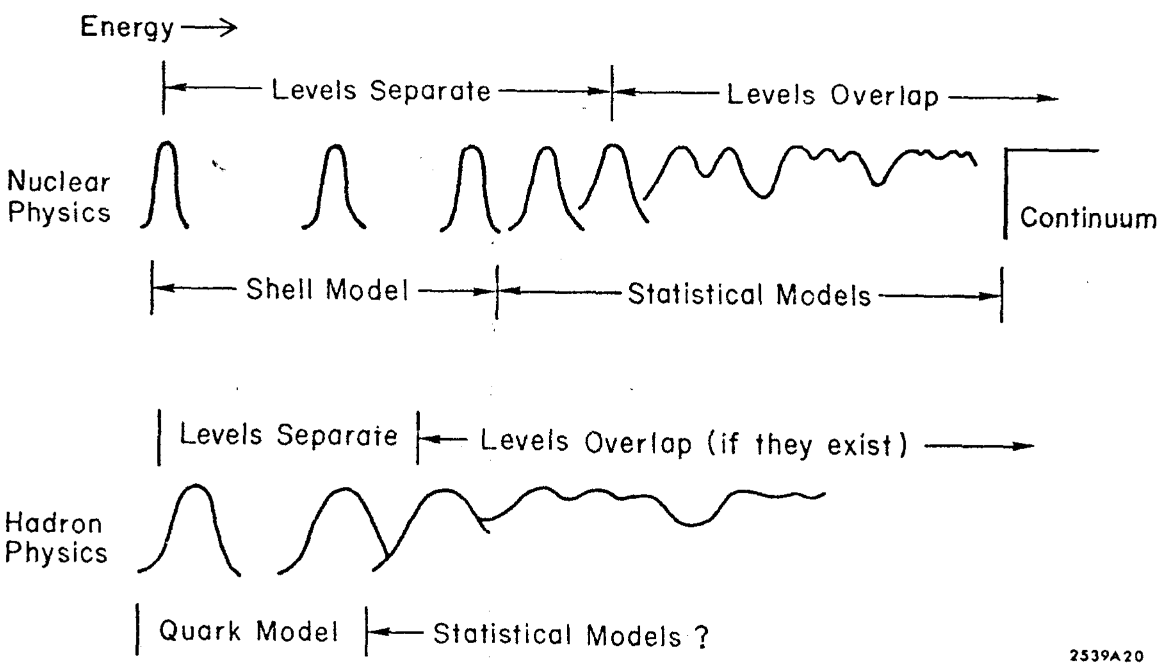
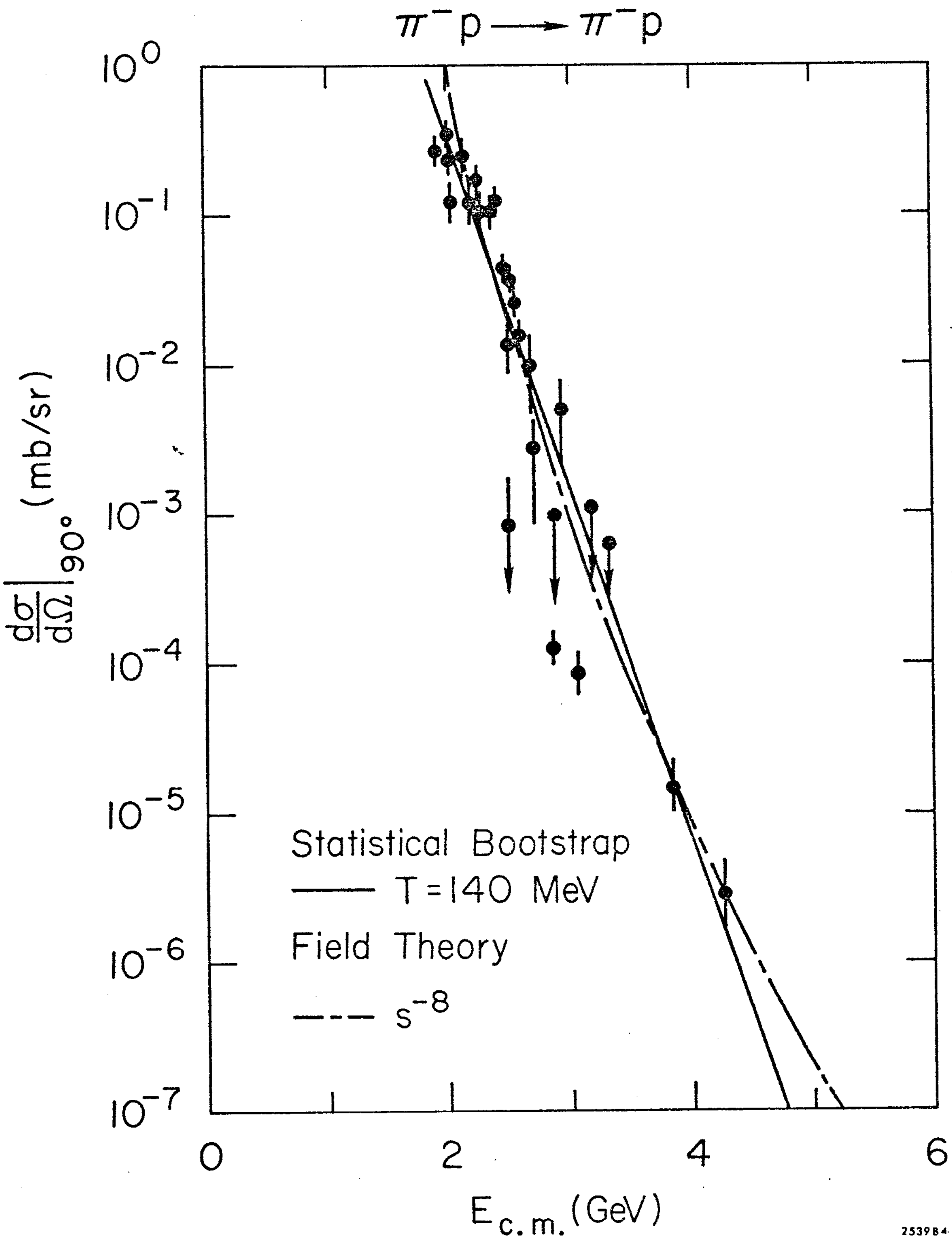

Energy ⟶
Levels Separate
Levels Overlap
Nuclear Physics
Continuum
Shell Model
Statistical Models
Levels Separate
Levels Overlap (if they exist)
Hadron Physics
Quark Model
Statistical Models ?
2539A20

FIG. 4.3



$\pi^- p \longrightarrow \pi^- p$
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
$\left.\frac{d\sigma}{d\Omega}\right|_{90°}$ (mb/sr)
Statistical Bootstrap
T=140 MeV
Field Theory
s^{-8}
0
2
4
6
$E_{c.m.}$ (GeV)
2539B4

FIG. 4.4a

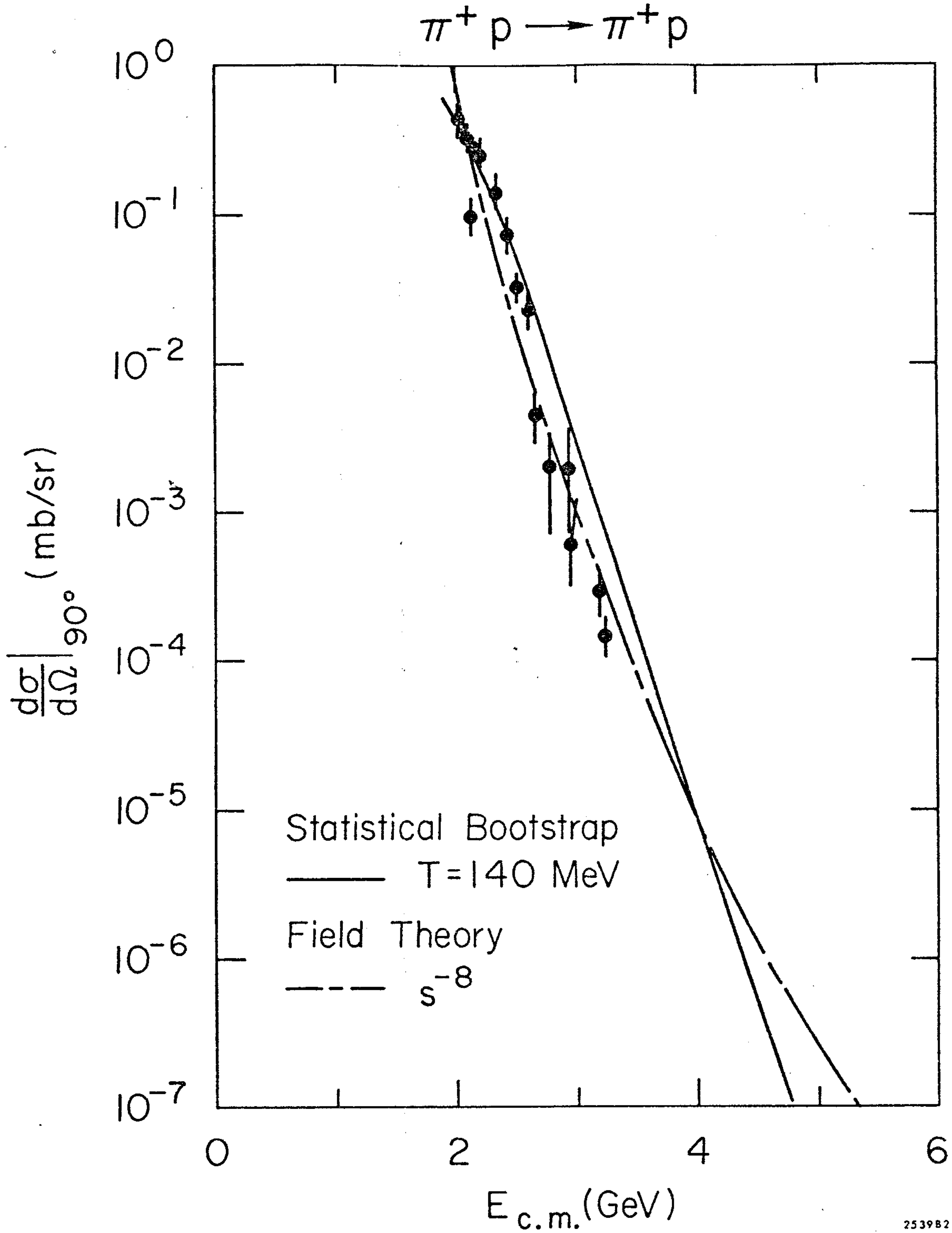

$\pi^+ p \longrightarrow \pi^+ p$
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
$\frac{d\sigma}{d\Omega}\Big|_{90°}$ (mb/sr)
Statistical Bootstrap
T=140 MeV
Field Theory
s^{-8}
0
2
4
6
$E_{c.m.}$(GeV)
253982

FIG. 4.4b

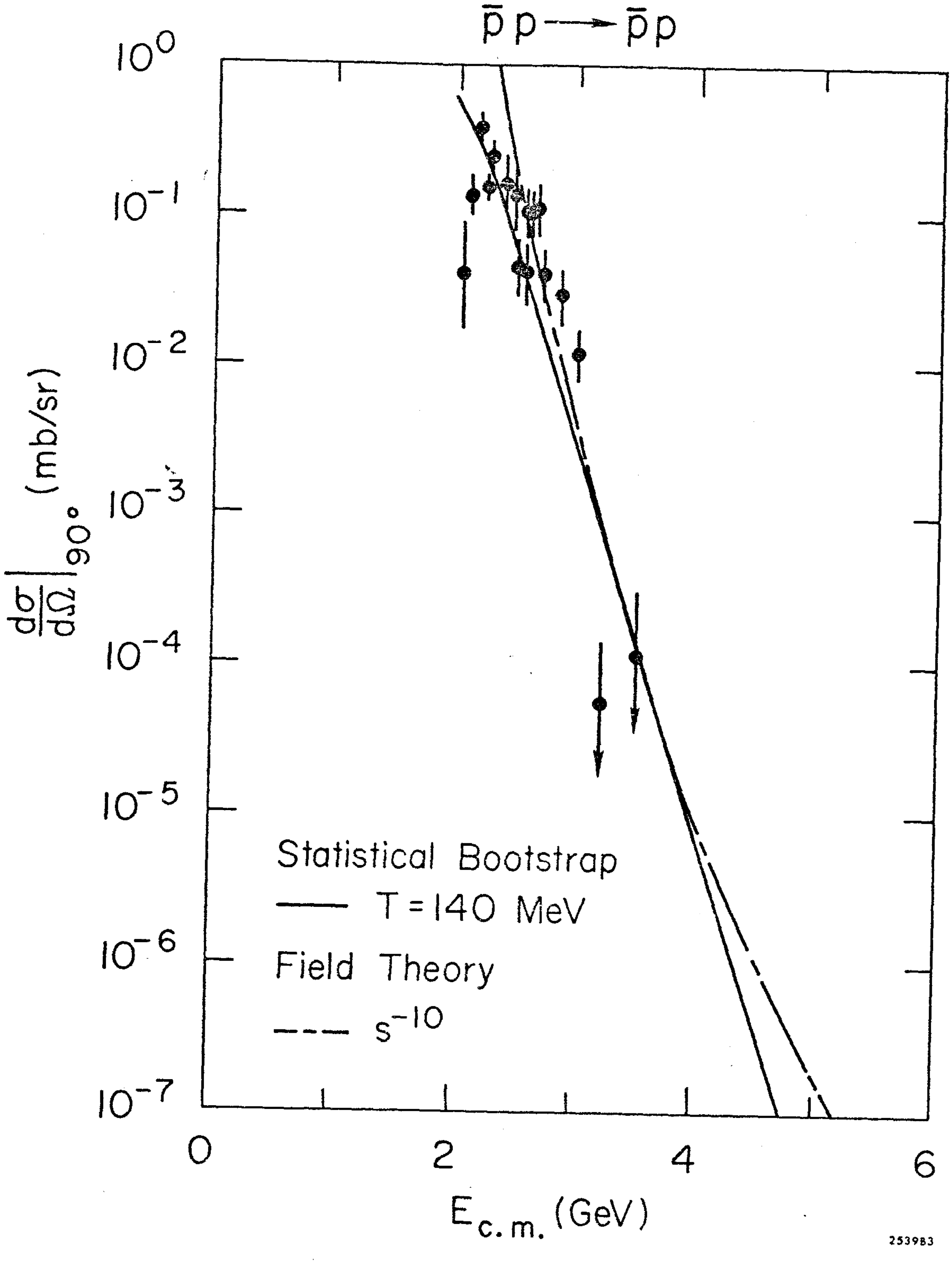

$\bar{p}p \longrightarrow \bar{p}p$
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
$\frac{d\sigma}{d\Omega}\Big|_{90°}$ (mb/sr)
Statistical Bootstrap
T = 140 MeV
Field Theory
s^{-10}
0
2
4
6
$E_{c.m.}$ (GeV)
253983

FIG. 4.4c

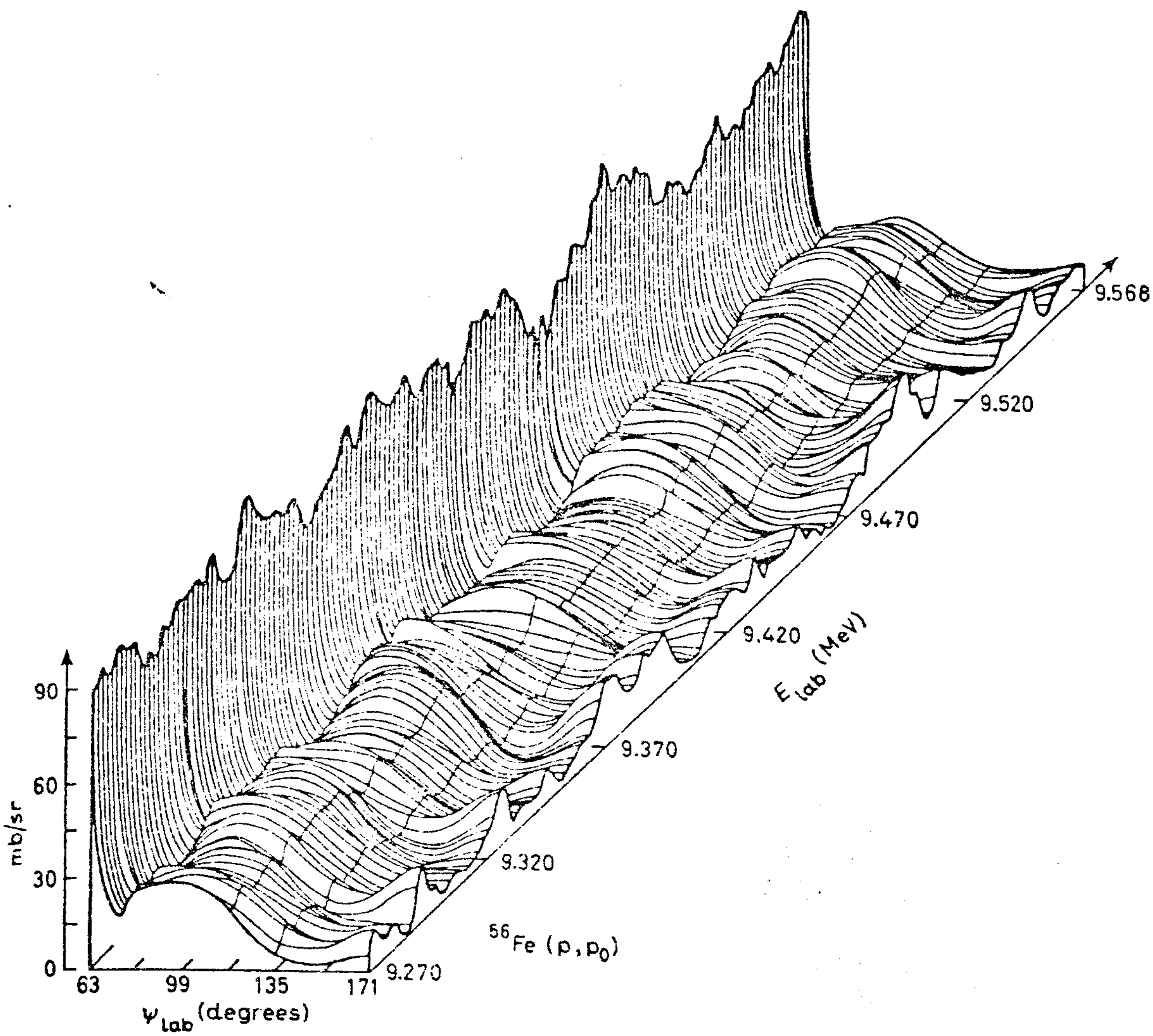

90
60
30
0
mb/sr
63
99
135
171
ψ_{lab} (degrees)
9.270
9.320
9.370
9.420
9.470
9.520
9.568
E_{lab} (MeV)
^{56}Fe (p, p$_0$)

FIG. 4.5

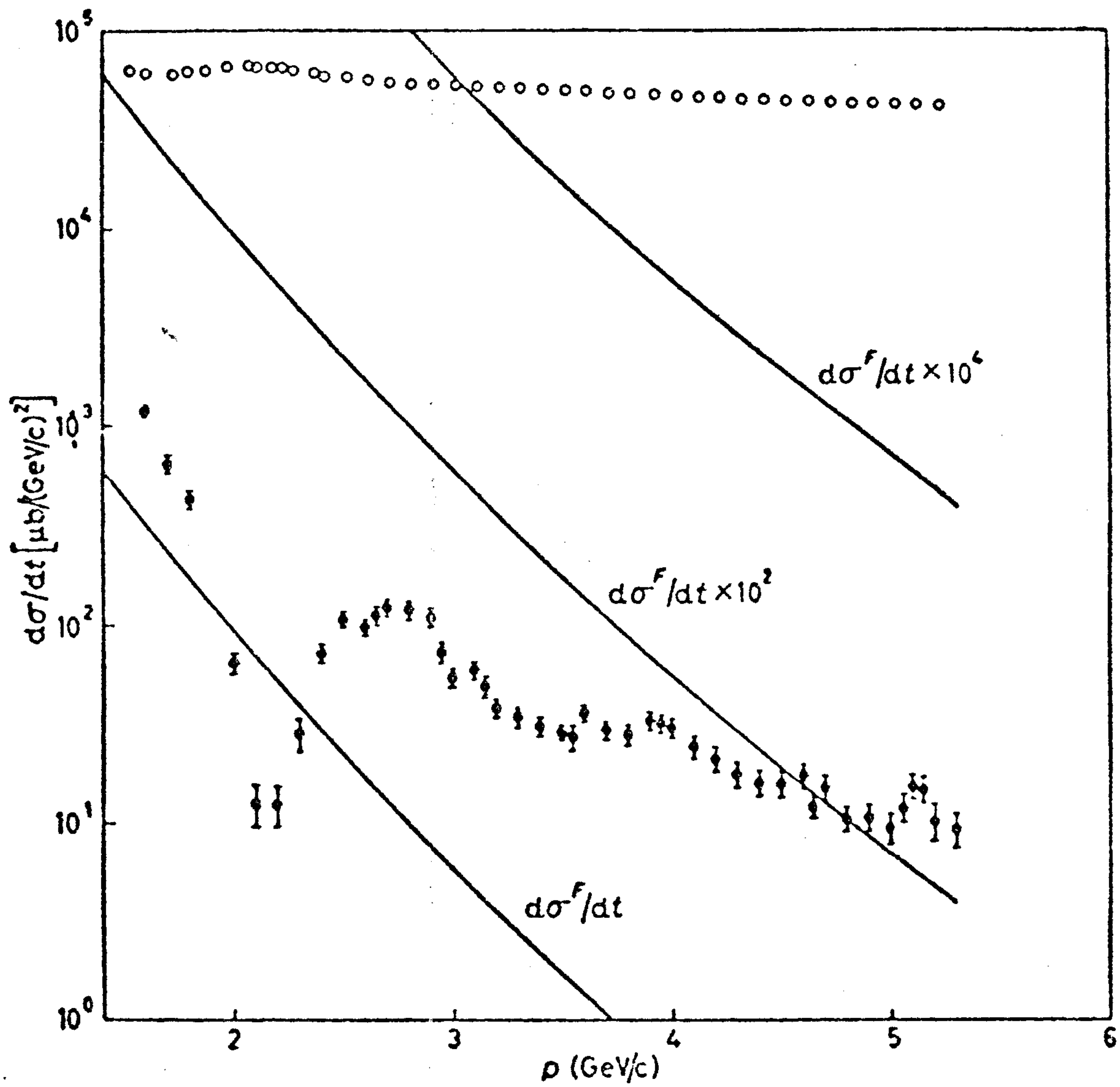
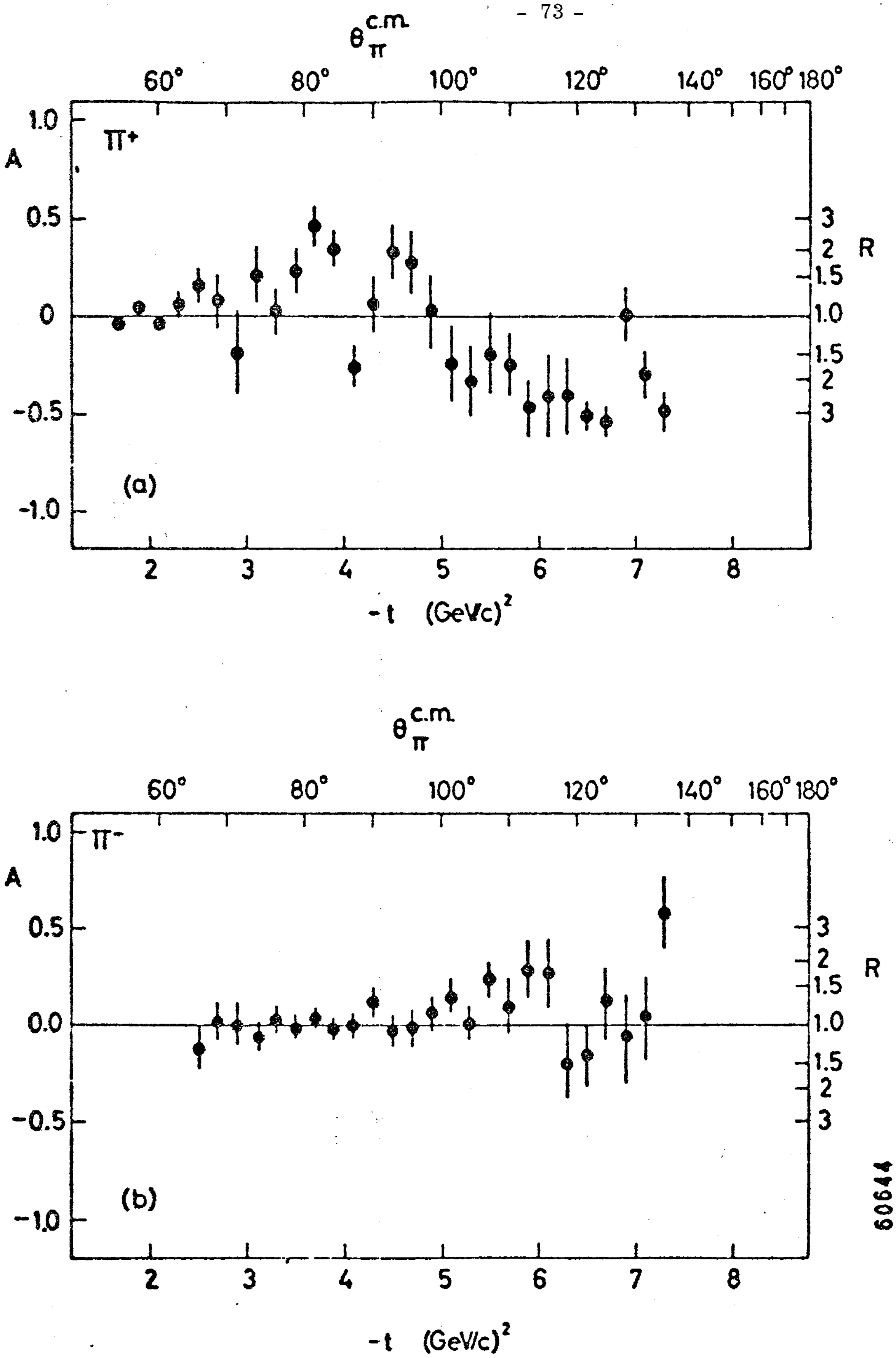

10^5
10^4
10^3
10^2
10^1
10^0
$d\sigma/dt\ [\mu b/(GeV/c)^2]$
2
3
4
5
6
p (GeV/c)
$d\sigma^F/dt \times 10^4$
$d\sigma^F/dt \times 10^2$
$d\sigma^F/dt$

FIG. 4.6



$\theta_\pi^{c.m.}$
60° 80° 100° 120° 140° 160° 180°
A
1.0 0.5 0 -0.5 -1.0
π+
(a)
R
3 2 1.5 1.0 1.5 2 3
2 3 4 5 6 7 8
-t (GeV/c)²
$\theta_\pi^{c.m.}$
60° 80° 100° 120° 140° 160° 180°
A
1.0 0.5 0.0 -0.5 -1.0
π-
(b)
R
3 2 1.5 1.0 1.5 2 3
2 3 4 5 6 7 8
-t (GeV/c)²
60644

FIG. 4.7

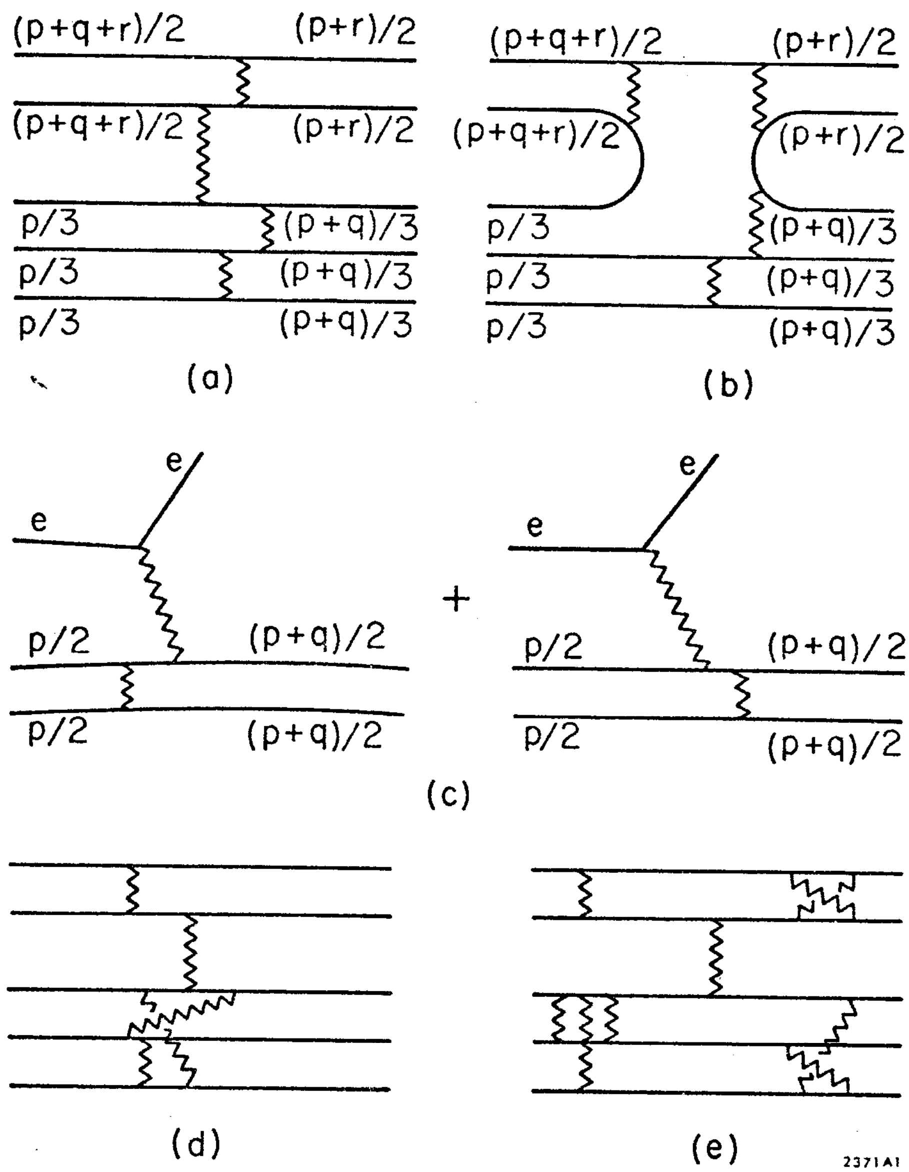

(p+q+r)/2
(p+r)/2
(p+q+r)/2
(p+r)/2
p/3
(p+q)/3
p/3
(p+q)/3
p/3
(p+q)/3
(a)
(p+q+r)/2
(p+r)/2
(p+q+r)/2
(p+r)/2
p/3
(p+q)/3
p/3
(p+q)/3
p/3
(p+q)/3
(b)
e
e
p/2
(p+q)/2
p/2
(p+q)/2
+
e
e
p/2
(p+q)/2
p/2
(p+q)/2
(c)
(d)
(e)
2371A1

FIG. 5.1

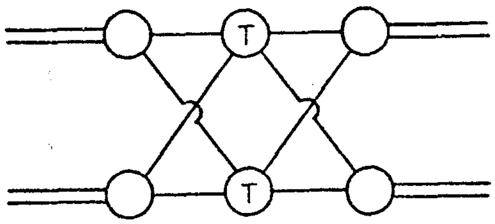


FIG. 5.2

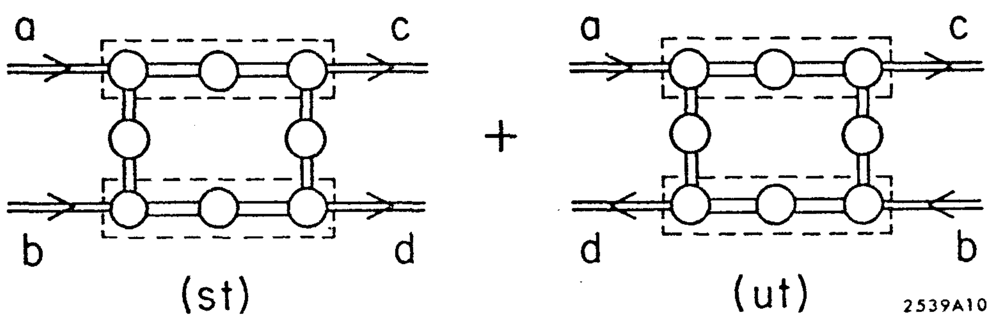


FIG. 5.3

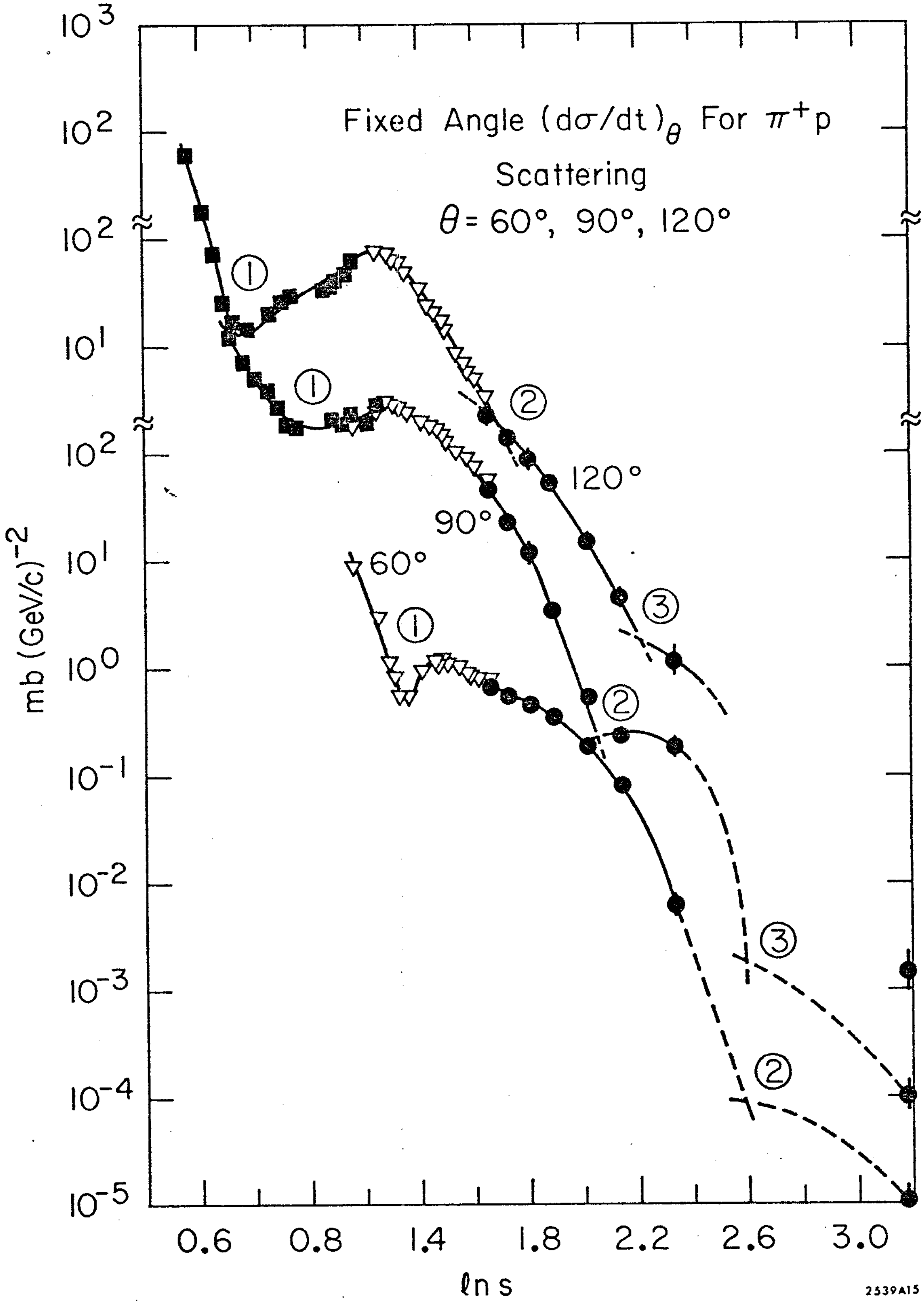

Fixed Angle $(d\sigma/dt)_\theta$ For π^+p
Scattering
θ= 60°, 90°, 120°
120°
90°
60°
mb $(GeV/c)^{-2}$
$\ell n\, s$
2539A15

FIG. 6.1a

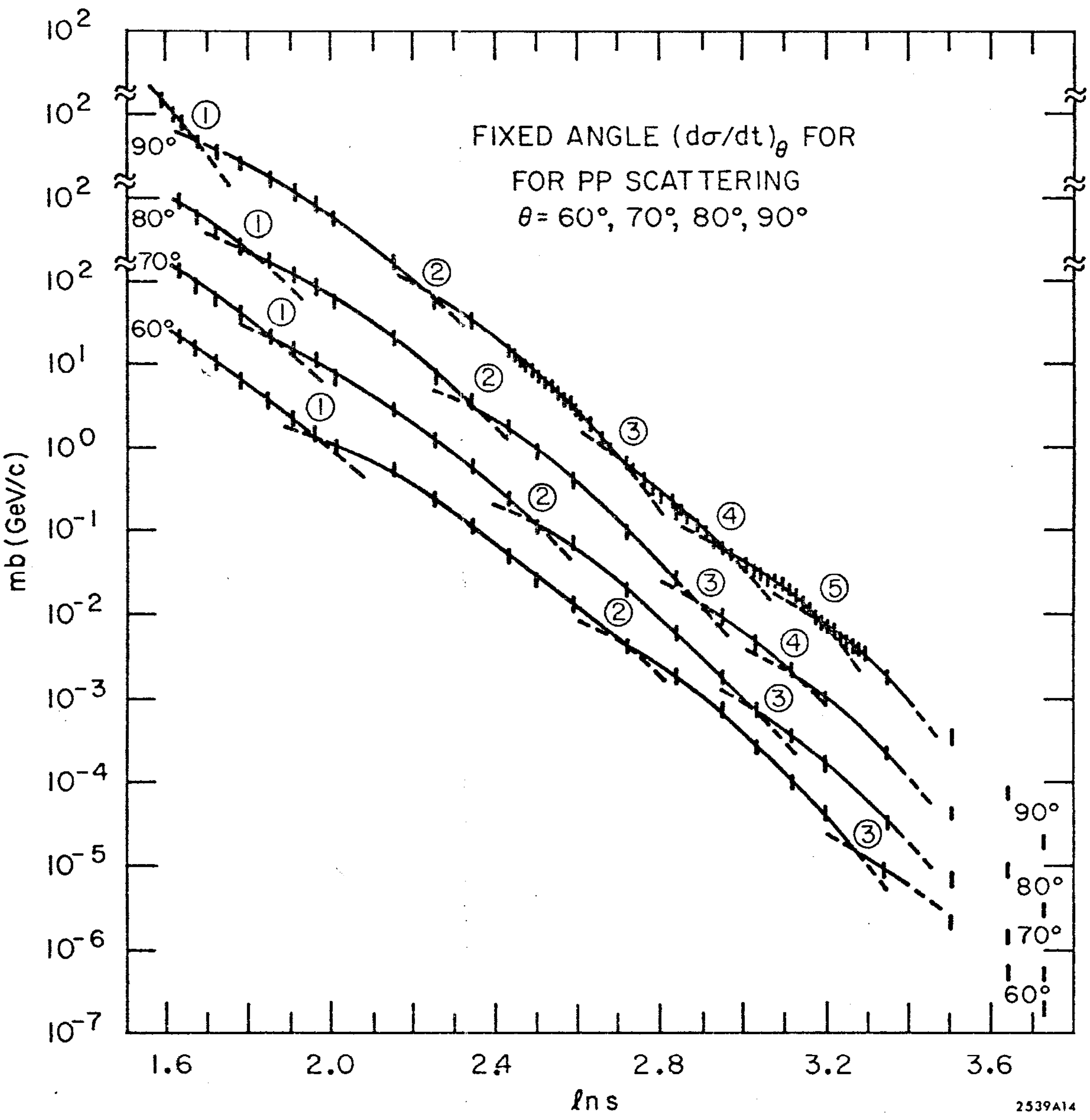

FIXED ANGLE $(d\sigma/dt)_\theta$ FOR
FOR PP SCATTERING
θ = 60°, 70°, 80°, 90°
mb (GeV/c)
ℓn s
90°
80°
70°
60°
2539A14

FIG. 6. 1b

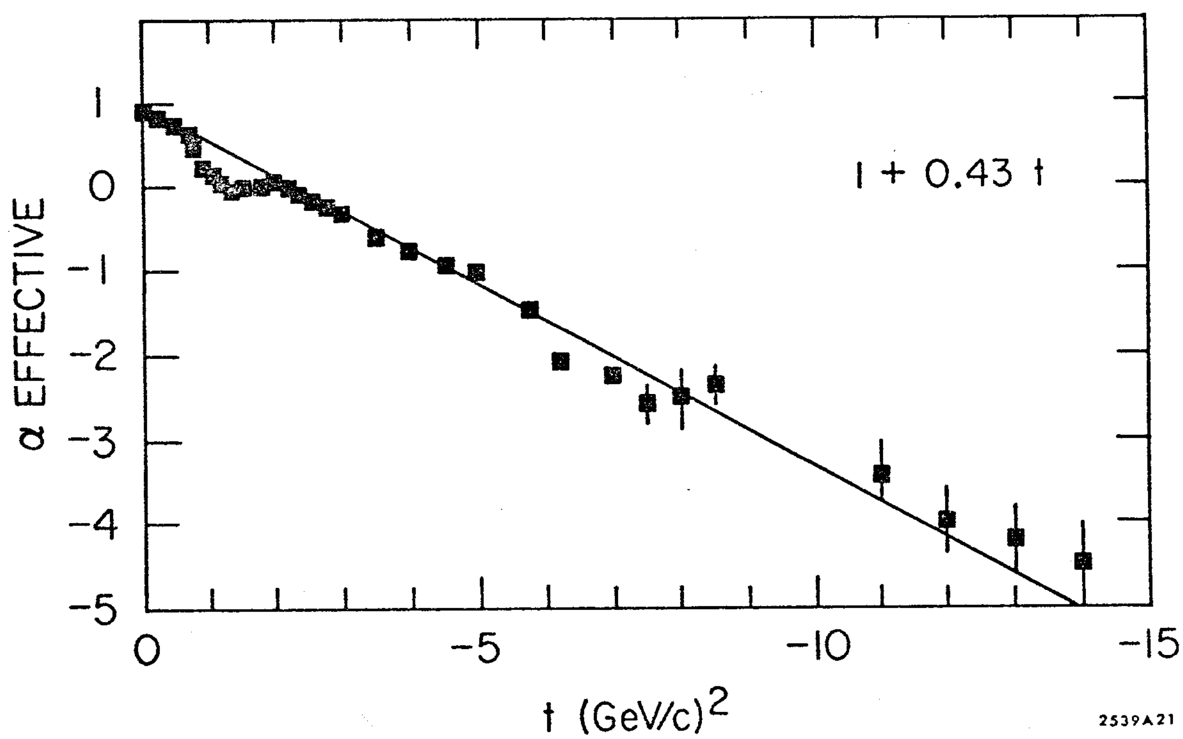


FIG. 6.2

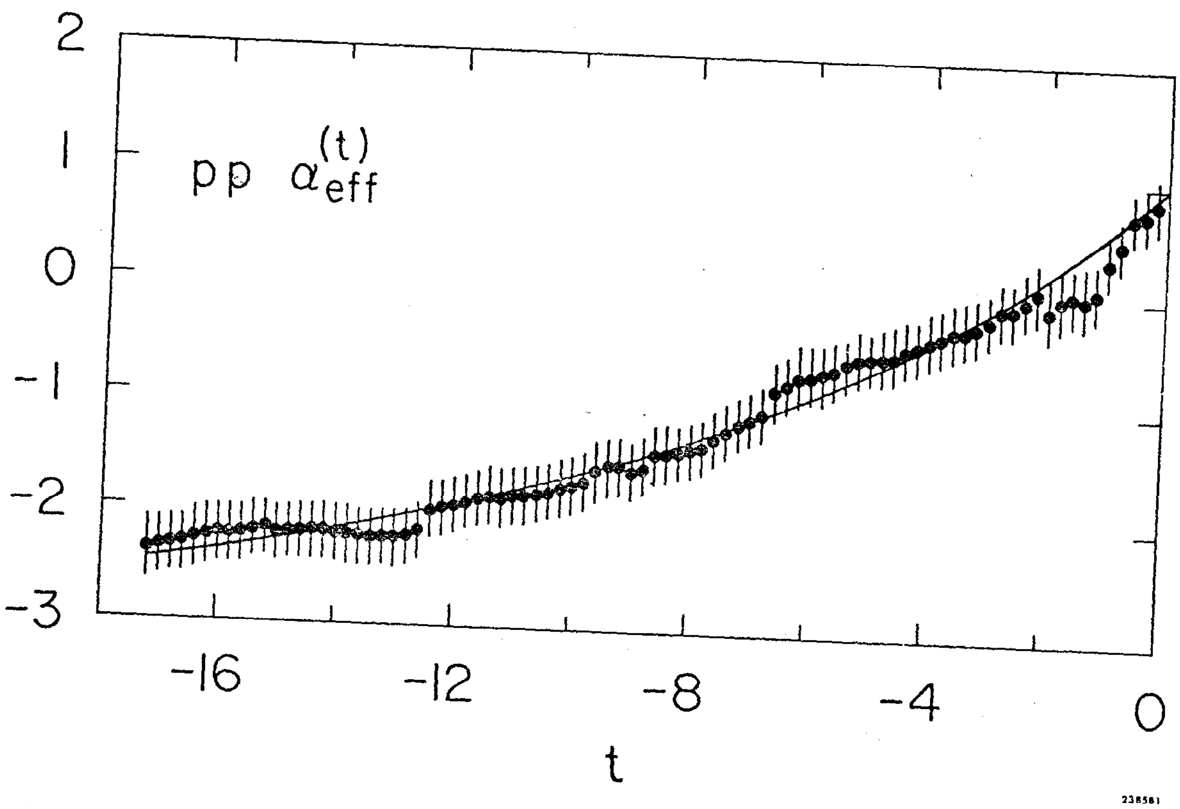


FIG. 6.3

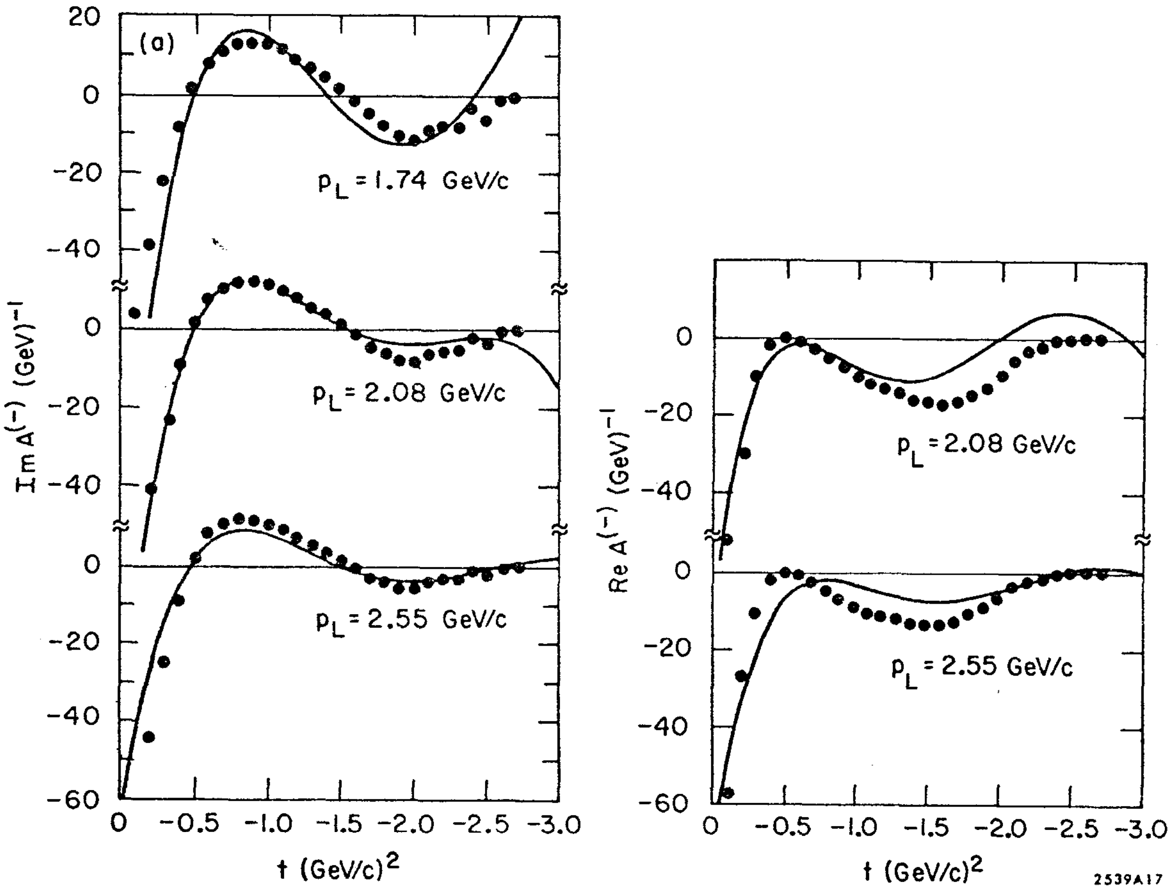

(a)
p_L = 1.74 GeV/c
p_L = 2.08 GeV/c
p_L = 2.55 GeV/c
Im $A^{(-)}$ (GeV)$^{-1}$
Re $A^{(-)}$ (GeV)$^{-1}$
t (GeV/c)^2
2539A17

FIG. 6.4