

An alternative approach to $b \rightarrow s\gamma$ in the unconstrained MSSM

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The gluino contributions to the $C'_{7,8}$ Wilson coefficients for $b \rightarrow s\gamma$ are calculated within the unconstrained MSSM. New stringent bounds on the δ_{23}^{RI} and δ_{23}^{RI} mass insertion parameters are obtained in the limit in which the SM and SUSY contributions to $C_{7,8}$ approximately cancel. Such a cancellation can plausibly appear within several classes of SUSY breaking models. Assuming this cancellation takes place, we perform an analysis of the $b \rightarrow s\gamma$ decay. We show that, in the uMSSM such an alternative is reasonable and it is possible to saturate the $b \rightarrow s\gamma$ branching ratio and produce a CP asymmetry of up to 20%, from only the gluino contribution to $C'_{7,8}$ coefficients. Using photon polarization a LR asymmetry can be defined that in principle allows the $C_{7,8}$ and $C'_{7,8}$ contributions to the $b \rightarrow s\gamma$ decay to be disentangled.

1 Introduction

The precision measurements of the inclusive radiative decay $B \rightarrow X_s\gamma$ provides an important benchmark for the Standard Model (SM) and New Physics (NP) models at the weak scale, such as low-energy supersymmetric (SUSY) models. In the SM, flavour changing neutral currents (FCNC) are forbidden at tree level. The first SM contribution to the $b \rightarrow s\gamma$ transition appears at one loop level from the CKM flavour changing structure, showing the characteristic Cabibbo suppression. NP contributions to $b \rightarrow s\gamma$ typically also arise at one loop, and in general can be much larger than the SM contributions if no mechanisms for suppressing the new sources of flavour violation exist (see ¹ for a complete set of references).

Experimentally, the inclusive $B \rightarrow X_s\gamma$ Branching Ratio (BR) has been measured by ALEPH, BELLE and CLEO, resulting in the current experimental weighted average $BR(B \rightarrow X_s\gamma)_{exp} = (3.23 \pm 0.41) \times 10^{-4}$, with new results expected shortly from BABAR and BELLE which could further reduce the experimental errors. Squeezing the theoretical uncertainties down to the 10% level has been (and still is) a crucial task. The SM theoretical prediction has been the subject of intensive theoretical investigation in the past several years, leading to the completion of the NLO QCD calculations. The original SM NLO calculation² gives, for

$\sqrt{z} = m_c/m_b = 0.29$, the following result: $BR(B \rightarrow X_s \gamma)_{SM} = (3.28 \pm 0.33) \times 10^{-4}$. The main source of theoretical uncertainty is due to NNLO QCD ambiguities. In³ it was shown that using $\sqrt{z} = 0.22$ (i.e. the running charm mass instead of the pole mass) is more justifiable and causes an enhancement of about 10% of the $b \rightarrow s \gamma$ BR, leading to the current preferred value: $BR(B \rightarrow X_s \gamma)_{SM} = (3.73 \pm 0.30) \times 10^{-4}$. Although these theoretical uncertainties can be addressed only with a complete NNLO calculation, the SM value for the BR is in agreement with the experimental measurement within the $1 - 2\sigma$ level.

The general agreement between the SM theoretical prediction and the experimental results has provided useful guidelines for constraining the parameter space of models with NP present at the electroweak scale, such as the 2HDM and the minimal supersymmetric standard model (MSSM). In SUSY models superpartners and charged Higgs loops contribute to $b \rightarrow s \gamma$, with contributions that typically rival or exceed the SM one in size. For calculational ease only simplified MSSM scenarios (like cMSSM or MSSM with minimal flavour violation (MFV)) have usually been assumed. Nevertheless, as the origin and dynamical mechanism of SUSY breaking are unknown, there is no reason *a priori* to expect that the soft parameters will be flavour-blind (or violate flavour in the same way as the SM). Of course, the kaon system has provided strong FCNC constraints for the mixing of the first and second generations, which severely limit the possibility of flavour violation in that sector. However the constraints for third generation mixings are significantly weaker, with $b \rightarrow s \gamma$ usually providing the most stringent constraints.

A discussion of the $b \rightarrow s \gamma$ process in the general unconstrained MSSM is in principle possible, but it is necessary to deal with two unavoidable problems: (i) a large number of free, essentially unconstrained parameters; (ii) the need to achieve a quite accurate cancellation between the sizeable different contributions (SM, Higgs, chargino, neutralino and gluino). In the following we'll provide a particularly interesting and simple analysis of $b \rightarrow s \gamma$ in the uMSSM.

2 Alternative solution to $b \rightarrow s \gamma$ branching ratio

The low-energy effective Hamiltonian, at the bottom mass scale μ_b , is defined as $\mathcal{H}_{eff} = -(4G_F/\sqrt{2})V_{tb}V_{ts}^* \sum_i C_i(\mu_b)Q_i(\mu_b)$. The operators relevant to the $b \rightarrow s \gamma$ process are:

$$\begin{aligned}
 Q_2 &= \bar{s}_L \gamma_\mu c_L \bar{c}_L \gamma^\mu b_L, & Q'_2 &= \bar{s}_R \gamma_\mu c_R \bar{c}_R \gamma^\mu b_R, \\
 Q_7 &= \frac{e}{16\pi^2} m_b \bar{s}_L \sigma^{\mu\nu} b_R F_{\mu\nu}, & Q'_7 &= \frac{e}{16\pi^2} m_b \bar{s}_R \sigma^{\mu\nu} b_L F_{\mu\nu}, \\
 Q_8 &= \frac{g_s}{16\pi^2} m_b \bar{s}_L \sigma^{\mu\nu} G_{\mu\nu}^a T_a b_R, & Q'_8 &= \frac{e}{16\pi^2} m_b \bar{s}_R \sigma^{\mu\nu} G_{\mu\nu}^a T_a b_L.
 \end{aligned} \tag{1}$$

Effects of NP generally appear as modifications of the Wilson Coefficients (WC) $C_{7,8}^{(i)}$ associated to the operators $Q_{7,8}$ and their chirality conjugated ones. In the majority of the previous studies of the $b \rightarrow s \gamma$ process, the main focus was to calculate the SM or NP contributions to $C_{7,8}$. The contributions coming from $C_{7,8}^{(i)}$ have usually been neglected on the assumption that they are suppressed with respect to $C_{7,8}$ by the ratio m_s/m_b . While this is always valid in the SM, in the 2HDM or within specific MSSM scenarios (with MFV), this mass suppression can be absent in the uMSSM where the gluino contributions to $C_{7,8}$ and $C_{7,8}^{(i)}$ are naturally of the same order⁴.

Therefore, in the following we present an alternative approach to $b \rightarrow s \gamma$ in the uMSSM. We assume a particular scenario in which the *total* contribution to $C_{7,8}$ is negligible and the main contribution to the $b \rightarrow s \gamma$ BR is given by $C_{7,8}^{(i)}$. This “ $C_7^{(i)}$ -dominated” scenario is realized when the chargino, neutralino, and gluino contributions to $C_{7,8}$ sum up in such a way as to cancel the W and Higgs contributions almost completely. In our opinion this situation does not require substantially more fine tuning than what is required in the usual MFV scenario, where conversely the NP contributions to $C_{7,8}$ essentially cancel between themselves.

In the following we will focus on the gluino contribution to $C_{7,8}$ and $C'_{7,8}$. There is only one gluino diagram that contributes to C_7 and C'_7 , with the external photon line attached to the down-squark line, while two diagrams can contribute to the C_8 and C'_8 coefficients, as the gluon external line can be attached to the squark or the gluino lines. The one-loop gluino contributions to the $C'_{7,8}$ coefficients are given, at first and second order in the MI, respectively by:

$$C'_{7\tilde{g}}(1) = \frac{8g_s^2}{3g^2} \frac{Q_d}{V_{tb}V_{ts}^*} \frac{m_W^2}{\tilde{m}_D^2} \left\{ \delta_{23}^{RR} F_2^{(1)}(x_D^g) - \frac{\tilde{m}_{\tilde{g}}}{m_b} \delta_{23}^{RL} F_4^{(1)}(x_D^g) \right\}, \quad (2)$$

$$C'_{8\tilde{g}}(1) = -\frac{g_s^2}{3g^2} \frac{Q_d}{V_{tb}V_{ts}^*} \frac{m_W^2}{\tilde{m}_D^2} \left\{ \delta_{23}^{RR} F_{21}^{(1)}(x_D^g) - \frac{\tilde{m}_{\tilde{g}}}{m_b} \delta_{23}^{RL} F_{43}^{(1)}(x_D^g) \right\} \quad (3)$$

and

$$C'_{7\tilde{g}}(2) = \frac{4g_s^2}{3g^2} \frac{Q_d}{V_{tb}V_{ts}^*} \frac{m_W^2}{\tilde{m}_D^2} \frac{m_b(A_b - \mu \tan \beta)}{\tilde{m}_D^2} \left\{ \delta_{23}^{RL} F_2^{(2)}(x_D^g) - \frac{\tilde{m}_{\tilde{g}}}{m_b} \delta_{23}^{RR} F_4^{(2)}(x_D^g) \right\}, \quad (4)$$

$$C'_{8\tilde{g}}(2) = -\frac{g_s^2}{6g^2} \frac{Q_d}{V_{tb}V_{ts}^*} \frac{m_W^2}{\tilde{m}_D^2} \frac{m_b(A_b - \mu \tan \beta)}{\tilde{m}_D^2} \left\{ \delta_{23}^{RL} F_{21}^{(2)}(x_D^g) - \frac{\tilde{m}_{\tilde{g}}}{m_b} \delta_{23}^{RR} F_{43}^{(2)}(x_D^g) \right\}. \quad (5)$$

The gluino contribution to $C_{7,8}$ can be obtained exchanging $L \leftrightarrow R$ in eqs. (2-5). In deriving eqs.(2-5) to the second order in the MI parameters, we have kept only the dominant term proportional to $\tan \beta$ (the A_b term is retained in the above expression for defining our convention for the μ term; see later) and neglected all of the other off-diagonal MIs⁴. Clearly the dominant terms in eqs. (2-5) are those proportional to the gluino chirality flip, so that the gluino contribution to C_7 (C'_7) depends, at first order, only on the MI term δ_{23}^{LR} (δ_{23}^{RL}). However, for large $\tan \beta$ and $\mu \approx \tilde{m}_A$, the second order MI terms in eqs. (4,5) can become comparable in size with the first order mass insertions. Thus, two different MI parameters are relevant in the L/R sectors: $(\delta_{23}^{LR}, \delta_{23}^{LL})$ and $(\delta_{23}^{RL}, \delta_{23}^{RR})$, contrary to common wisdom. To which extent the LL and RR MIs are relevant depends of course on the values chosen for μ and $\tan \beta$, but in a large part of the allowed SUSY parameter space they cannot in general be neglected. Moreover, the fact that the gluino WCs depend on two different MI parameters will have important consequences in the study of the $b \rightarrow s\gamma$ CP asymmetry.

2.1 Single and multiple MI-dominance analysis

From eqs.(2-5), one can read (in MI language) the off-diagonal entries that are relevant to the gluino contribution to the $C_{7,8}$ and $C'_{7,8}$ WCs. Note that limits on $\delta_{23}^{LR} \approx O(10^{-2})$ have previously been obtained in⁵. No stringent bound has been derived there for δ_{23}^{LL} , as this term does not come, at lowest order, with the $\tilde{m}_{\tilde{g}}/m_b$ enhancement (see eqs.(2)). No limits were showed on δ_{23}^{RL} and δ_{23}^{RR} because the MI formula are symmetric in the $L \leftrightarrow R$ exchange and in the scenarios generally adopted in the literature the ‘‘opposite chirality’’ MIs are suppressed by a factor m_s/m_b and so negligible. An analysis of the δ_{23}^{RL} dependence has been performed in⁴, in which the W contribution to $C_{7,8}$ was not set to zero (sometimes also Higgs and MFV chargino contributions to $C_{7,8}$ were included). Consequently their bounds on the down-squark off-diagonal MIs contributing to $C'_{7,8}$ are more stringent than the bounds we derive in our scenario, for which the total contribution to $C_{7,8}$ is assumed to be negligible. It is clearly only in the scenario we study that an *absolute* constraint on these MIs can be derived. Moreover no analysis on δ_{23}^{LL} and δ_{23}^{RR} was performed in⁴ as these contributions are not relevant in the small $\tan \beta$ region, as can be seen from eqs.(4,5). In fig. 1 we show the dependence of the $b \rightarrow s\gamma$ BR on the MI terms δ_{23}^{RL} and δ_{23}^{RR} for different values of $x_D^g = \tilde{m}_{\tilde{g}}^2/\tilde{m}_D^2$ and for $\tan \beta = 20$ and $\mu = 350$ GeV. All the other off-diagonal entries in the down-squark mass matrix are assumed to vanish. ‘‘Individual’’

⁴In¹ a complete derivation of the general results and conventions used in eqs. (2-5) is presented.

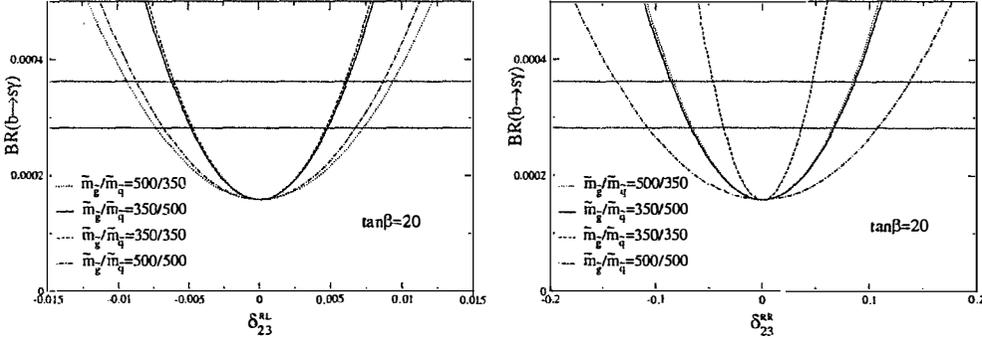


Figure 1: The dependence of $b \rightarrow s\gamma$ branching ratio on δ_{23}^{RL} and δ_{23}^{RR} for different values of $\tilde{m}_{\tilde{g}}/\tilde{m}_D$, for $\tan\beta = 20$ and $\mu = 350$ GeV. All of the other off-diagonal entries, except the one displayed on the axes, are assumed to vanish. $C_{7,8}(\mu_W) = 0$ is assumed. The horizontal lines represent the 1σ experimentally allowed region.

limits $\delta_{23}^{RL} < 10^{-2}$ and $\delta_{23}^{RR} < 1.5 \times 10^{-1}$ can be obtained respectively from the left and right side plot of fig. 1. Of course, the required cancellation of the total $C_{7,8}$ contribution may in general need non-vanishing off-diagonal entries of the up and down squark mass matrices. However, the specific values of these entries do not significantly affect the absolute limits on the δ_{23}^{RL} and δ_{23}^{RR} MIs shown in fig. 1. As expected from eqs.(4,5), the bounds obtained for δ_{23}^{RR} are strongly dependent on the product $\mu \tan\beta$. More stringent bounds on δ_{23}^{RR} can be obtained for larger $\tan\beta$. For $\tan\beta > 35$ the bounds on δ_{23}^{RR} can become as stringent as the δ_{23}^{RL} bounds.

A general analysis of the gluino contribution to $C_{7,8}^I$ depends simultaneously on both the δ_{23}^{RL} and δ_{23}^{RR} MIs. For a complete specification of our scenario the only other free parameters that need be fixed are the ratio between the gluino mass and the common down-squark mass, $\tilde{m}_{\tilde{g}}/\tilde{m}_D$, the product $\mu \tan\beta$, and the relative phase between δ_{23}^{RL} and δ_{23}^{RR} . The influence of all the other down-sector squark matrix off-diagonal entries and MSSM parameters in the $C_{7,8}^I$ sector can safely be neglected. In fig. 2 (left) we show the 1σ experimentally allowed region in the $(\delta_{23}^{RL}, \delta_{23}^{RR})$ parameter space for a specific choice of $\tilde{m}_{\tilde{g}}/\tilde{m}_D = 350/500$, $\mu = 350$ GeV, and for three different values of $\tan\beta = 3, 20$ and 35 . For δ_{23}^{RL} or δ_{23}^{RR} vanishing, one obtains the regions depicted in fig. 1. Larger regions in the $(\delta_{23}^{RL}, \delta_{23}^{RR})$ parameter space are obtained when both the MIs take non-vanishing values. It is clear that no absolute limit can be derived for the two MIs simultaneously. The values $(\delta_{23}^{RL}, \delta_{23}^{RR}) \approx (1, 0.1)$ are, for example, possible for $\tan\beta = 35$. In fact, as can be seen in fig. 2 (left), there is always a “flat direction” where large values of δ_{23}^{RL} and δ_{23}^{RR} can be tuned in such a way that the gluino contribution to $C_{7,8}^I$ is consistent with the experimental bound. This flat direction clearly depends on the chosen values for $\tilde{m}_{\tilde{g}}/\tilde{m}_D$ and $\mu \tan\beta$. The presence of this particular direction is explained by the fact that we are allowing complex off-diagonal entries. Hence the relative phase between δ_{23}^{RL} and δ_{23}^{RR} can be fixed in such a way that the needed amount of cancellation can be obtained between the first and second order MI contribution. In the notation used in eqs.(2–5) the line of maximal cancellation is obtained for $\varphi = \arg[\delta_{23}^{RL} \delta_{23}^{RR}] = \pm\pi$.

2.2 CP asymmetry and LR asymmetry

In addition to the $b \rightarrow s\gamma$ BR, the experimental collaborations will provide in the coming years more precise measurements of the $b \rightarrow s\gamma$ CP asymmetry. The present experimental value gives, at 90% CL, the range $-0.27 < \mathcal{A}_{CP}(b \rightarrow s\gamma) < 0.10$ which is still too imprecise to provide useful

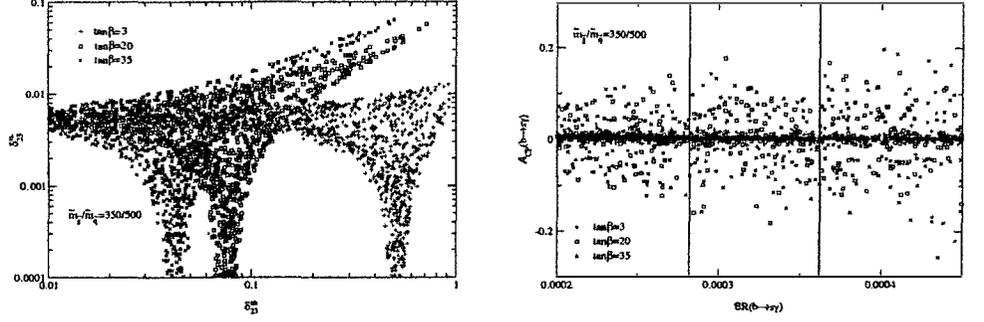


Figure 2: 1σ -allowed region in the $(\delta_{23}^{RL}, \delta_{23}^{RL})$ parameter space (left) and Asymmetry vs Branching Ratio (right) for three different values of $\tan\beta$, with the other parameters fixed to $\tilde{m}_{\tilde{g}}/\tilde{m}_{\tilde{q}} = 350/500$ and $\mu = 350$ GeV. All the other off-diagonal entries, except the one displayed on the axes, are assumed to vanish.

tests for NP, although the measurement is expected to be upgraded soon.

The only flavour-violating and CP-violating source in the SM (and MFV scenarios) is given by the CKM matrix, which results in a very small prediction for the CP asymmetry. In the SM an asymmetry of approximately 0.5%. If other sources of CP violation are present, a much larger CP asymmetry could be produced. In our $C'_{7,8}$ -dominated scenario, one can derive the following approximate relation for the CP asymmetry, in terms of the δ_{23}^{RL} and δ_{23}^{RR} MIs:

$$A_{CP}(b \rightarrow s\gamma) = -\frac{4}{9}\alpha_s(\mu_b)\frac{\text{Im}[C'_7 C'^*_8]}{|C'_7|^2} \approx k(x_D^g)\left(\frac{m_b\mu \tan\beta}{\tilde{m}_D^2}\right)|\delta_{23}^{RL} \delta_{23}^{RR}|\sin\varphi, \quad (6)$$

where φ is the relative phase between δ_{23}^{RL} and δ_{23}^{RR} as previously defined. One can immediately note that if only one MI is considered, the CP asymmetry is automatically zero. In addition a non-vanishing phase in the off-diagonal down-squark mass matrix is necessary. No sensitive bounds on this phase can be extracted from EDMs in a general flavour-violating scenario.

In fig. 2 (right), we show the results obtained for the BR and CP asymmetry in which δ_{23}^{RL} , δ_{23}^{RR} and the relative phase φ are varied arbitrarily for fixed value of $\tilde{m}_{\tilde{g}}/\tilde{m}_{\tilde{q}} = 350/500$ and $\tan\beta = 35$. The full vertical lines represent the 1σ region experimentally allowed by the $b \rightarrow s\gamma$ BR measurements. It is possible, using $C'_{7,8}$ alone, to saturate the $b \rightarrow s\gamma$ measured BR and at the same time have a CP asymmetry even larger than $\pm 10\%$, the sign of the asymmetry being determined by the sign of $\sin\varphi$. As shown in fig. 2 (right), in the relevant BR range the CP asymmetry range is constant. No strong dependence from $\tan\beta$, in the large $\tan\beta$ region, is present. The points with large asymmetry ($> 5\%$) lie in the “flat direction” observed in fig. 2 and they have almost $\varphi \approx \pm\pi$ (obviously for $\varphi = \pm\pi$ the CP asymmetry vanishes). The explanation of this fact is the following. The numerator is proportional to $\sin\varphi$ and so goes to 0 as φ approaches $\pm\pi$. However, at the same time it is enhanced for large MI values. This happens when the flat direction condition is (almost) satisfied. Here, in fact, a cancellation between the two (large) MI terms takes place, providing the enhancement of the CP asymmetry as the denominator remains practically constant, fixed by the allowed experimental measurement on the BR. Note also that for parameter values outside the flat direction condition a CP asymmetry of a few per cent can still be observed, about ten times bigger than the SM prediction. In our scenario even smaller values of the CP asymmetry can be obtained, e.g. if one of the two off-diagonal entries is negligible, or the two MIs are “aligned”.

A possible method for disentangling the relative contributions to the $b \rightarrow s\gamma$ BR from the Q_7 and Q'_7 operators utilizes an analysis of the photon polarization. For simplicity, let us define the following “theoretical” LR asymmetry at LO:

$$\mathcal{A}_{LR}(b \rightarrow s\gamma) = \frac{BR(b \rightarrow s\gamma_L) - BR(b \rightarrow s\gamma_R)}{BR(b \rightarrow s\gamma_L) + BR(b \rightarrow s\gamma_R)} = \frac{|C_7(\mu_b)|^2 - |C'_7(\mu_b)|^2}{|C_7(\mu_b)|^2 + |C'_7(\mu_b)|^2}, \quad (7)$$

which could in principle distinguish between C_7 and C'_7 dominated scenarios. Here L,R is the polarization of the external photon. This quantity is related to the quark chiralities of the Q_7, Q'_7 operators. Such a measurement is not yet available, as only the average quantity $BR(b \rightarrow s\gamma_L) + BR(b \rightarrow s\gamma_R)$ is reported experimentally. In the SM case, and in general in all the MFV and mSUGRA scenarios, only the C_7 coefficient gives a non-negligible contribution to the $b \rightarrow s\gamma$ BR, in such a way that $\mathcal{A}_{LR}(b \rightarrow s\gamma) = 1$. In our scenario, where the total contribution to C_7 is negligible, one obtains $\mathcal{A}_{LR}(b \rightarrow s\gamma) = -1$. In any other uMSSM scenario, any LR asymmetry between 1 and -1 is allowed. Consequently, a measurement of $\mathcal{A}_{LR}(b \rightarrow s\gamma)$ different from one will be a clear indication of physics beyond the SM with a non-minimal flavour structure.

3 Conclusions

We have discussed an alternative explanation of the $b \rightarrow s\gamma$ BR in the unconstrained MSSM. We analyzed in particular the gluino contribution to the WC C'_7 associated with the chirality operator Q'_7 . We show that this coefficient arises mainly from two off-diagonal entries: δ_{23}^{RL} and δ_{23}^{RR} . For scenarios where the $C_{7,8}$ contributions to $b \rightarrow s\gamma$ are small (i.e. for regions in the MSSM parameter space where Ws, Higgs, chargino and gluino contributions to $C_{7,8}$ tend to cancel each other), $C'_{7,8}$ provides the dominant effect. We derived absolute bounds separately on each of these coefficients. We then described the allowed region of $(\delta_{23}^{RL}, \delta_{23}^{RR})$ parameter space, as a function of $\tan\beta$. We observed that (for a fixed ratio $\tilde{m}_{\tilde{g}}/\tilde{m}_{\tilde{q}}$ and for each chosen value of $\mu \tan\beta$), there exists a “flat direction” where large (even $O(1)$) off-diagonal entries are allowed. For the majority of parameter space the CP asymmetry is less than 5%, even if asymmetries as large as 20% can be obtained along these “flat directions”. Finally, we suggested that the measure of the LR asymmetry could help to disentangle the C_7 from the C'_7 contribution to the $b \rightarrow s\gamma$ BR. Any $\mathcal{A}_{LR}(b \rightarrow s\gamma) \neq 1$ would be an irrefutable proof, not only of physics beyond the SM, but also it would indicate the existence of a non-minimal flavour violation structure of the down-squark mass matrix. It would be very interesting if such a quantity could be measured. One implication of our analysis is that previous results on MSSM parameters, including constraints on the “sign of μ^{th} ” which are more model-dependent than has generally been assumed.

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⁶The relative sign between the parameters μ and A_t . See⁶ for a discussion about the $g-2$ “sign of μ ”.