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PARTON MODELS AND DEEP HADRON-HADRON SCATTERING*

Robert Savit
Stanford Linear Accelerator Center
Stanford University, Stanford, Calif. 94305

and

National Accelerator Laboratory†
Batavia, Illinois 60510

ABSTRACT

We review the recent applications of various parton models, to the problem of hadron-hadron scattering at large momentum transfers. Models which allow direct parton-parton interactions as well as those that do not are considered. Predictions for the structure of hadronic final states (whether or not transverse jets exist), associated multiplicities, and the s and t dependencies of inclusive and exclusive cross-sections are described for the two types of models and compared with available data. The relation of the large momentum transfer (deep) region to the small momentum transfer (Regge) region is also discussed.

During the last couple of years, several attempts have been made to extend the intuitions about partons which have been developed in the study of deep electroproduction to purely hadronic interactions at large momentum transfers. In these applications, the hope has been that in this, the deep hadronic region, some sort of impulse approximation is valid so that only the simplest (in some sense) interactions have time to take place. Today I want to discuss some of the major developments which have taken place along these lines. I will describe, in a general way, the theoretical ideas behind various approaches, and then discuss some of the experimental consequences both for inclusive and exclusive (two-body) hadronic interactions. I will also address myself to the question of providing a more precise definition of the deep region. In physical terms this becomes the problem of determining the kinematic region in which coherent (Regge) effects can be ignored.

In parton theories of deep hadron scattering one pictures the incident hadrons as being composed of partonic constituents which mediate the basic interactions between the hadrons. The constituents then recombine to produce the final state hadrons. It is convenient to classify such theories according to whether or not they allow direct parton-parton interactions between partons belonging to different hadrons. A number of authors have described theories which allow direct parton-parton scattering (type I).[1] While these models differ in their details, they all share the common belief that some sort of

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† Present address.

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gluon exchange is important in describing deep hadronic scattering. On the other hand, a theory which does not allow such direct parton-parton scattering (type II) has been developed by Blankenbecler, Brodsky and Gunion,[2] and has been discussed using a different formalism by Landshoff and Polkinghorne.[3]

Type I theories generally describe deep hadron-hadron scattering as shown in Fig. 1. Two partons, one from each hadron scatter off each other with some large momentum transfer, (in the example of Fig. 1 via single gluon exchange), and then, by some as yet unknown mechanism, turn back into hadrons.

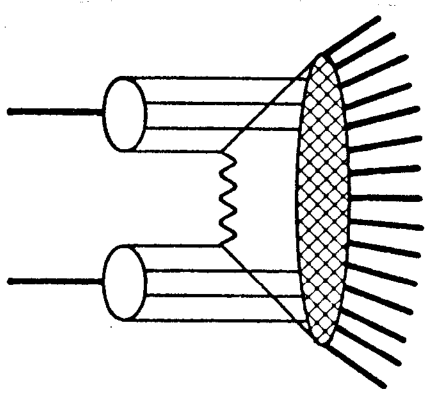

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FIG. 1--Contribution of direct parton-parton scattering (in this case, via gluon exchange) to deep hadron-hadron scattering.

On the basis of some very general arguments, one expects in this picture to see transverse jets of hadrons. Dynamically, these jets arise because the partons are supposed to like to radiate particles (in some sense) at lowish subenergies. Many hadrons will, therefore, be produced as the partons cascade step by step back toward the origin of momentum space. Of course, a high subenergy event is required in the first place in order to produce a high transverse momentum parton. The gross structure of these jets can be inferred by extending Feynman's argument for ordinary hadron scattering to parton scattering. Feynman[4] argues that in ordinary (i.e, no large momentum transfers) hadron scattering at high energies the hadron wave functions become Lorentz contracted, so that the energy density looks like a delta-function along the collision axis in configuration space. Fourier transforming this leads to a constant energy density in momentum space along p_z, and therefore, a uniform population of hadron constituents in rapidity. (Logarithmic multiplicities of hadrons in ordinary hadron-hadron scattering then follows from assuming that the final hadron distribution is similar to the final parton distribution.) This argument can clearly be applied to the two exiting, widely scattered partons in a deep event,[5] and thus leads to logarithmic multiplicities in the transverse jets.

In these theories, one calculates inclusive cross sections in which the detected particle has a large $p_\perp$ by folding the differential cross section for parton-parton scattering into an integral with the distribution of partons in the initial hadrons and the distribution of hadrons in the struck parton. The detailed behavior of these cross sections is clearly dependent on the distribution functions for partons in hadrons and hadrons in partons.[6] However, for a wide class of theories, the energy dependence for the invariant cross sections does not depend on these details.[7] It is

$$E \frac{d^3\sigma}{dp^3} = s^{-2} F(x_1, x_2) \tag{1}$$

when $s \to \infty$, and $x_1 = -t/s$ and $x_2 = -u/s$ are fixed. This is an important, simple prediction of type I theories. Although present data do not seem to support Eq. (1), its theoretical range of validity is ambiguous, so it may not be fair to apply it to the present experimental situation. We shall have more to say about this later.

What do type I parton theories say about two body reactions at large $|t|$? The natural specialization of Fig. 1 to a $2 \to 2$ hadronic amplitude is shown in Fig. 2. Of course, one could replace the single gluon exchange with a more general parton-parton scattering amplitude, but in view of the popularity of single gluon exchange, let us consider just the present example. If the gluon is a vector, the contribution of this diagram to the differential cross section is

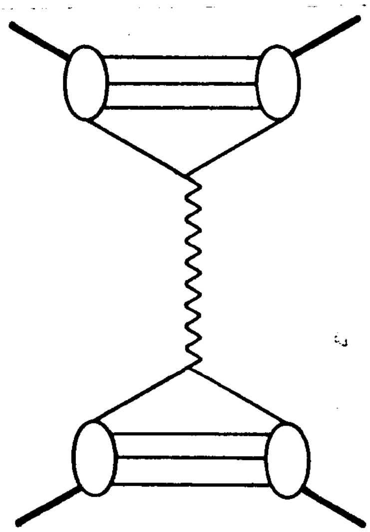

FIG. 2--Direct parton-parton scattering contribution to a deep hadron-hadron scattering event with a two-body final state.

$$\frac{d\sigma}{dt} \propto G^4(t) \left(\frac{d\sigma}{dt}\right)_{\text{parton}}$$

$$\propto \frac{G^4(t)}{t^2} \qquad (\text{large } |t|) \quad (2)$$

where $G(t)$ is the electromagnetic form factor of the hadrons involved in the scattering. This is reminiscent of the Wu-Yang formula.[8] In fact, if we make the gluon infinitely heavy, we recover their expression. Unfortunately, the data for proton-proton scattering is not in agreement with this expression. Equation (2) has no s-dependence, and predicts a fall-off with large $|t|$ which is greater than what is observed. Furthermore, from Fig. 2 one would predict equal cross sections for $pp \to pp$ and $\bar{p}p \to \bar{p}p$ at large s and t. This does not seem to be substantiated by experiment.[9]

Once again, however, we must caution those who would crucify type I theories on the cross of experiment. The spirit of direct parton-parton scattering could be resurrected in a number of ways. First, it is possible that a single gluon exchange does not asymptotically dominate parton-parton scattering. Second, regardless of the correct asymptotic form of the direct constituent scattering, present experiments may not be probing the asymptotic region in the context of these models. We'll say more about this later.

Let us turn now to a brief description of type II parton theories — those in which direct parton-parton scattering is not allowed. In these models, the deep scattering region is assumed to be dominated by interchange of the parton constituents. Typical graphs for two-body and inclusive processes are shown in Figs. 3 and 4, respectively. The internal lines are partons (straight) and "cores" (wiggly). These latter represent the collective effects of the (hadron minus parton) system. The blobs are vertex functions whose

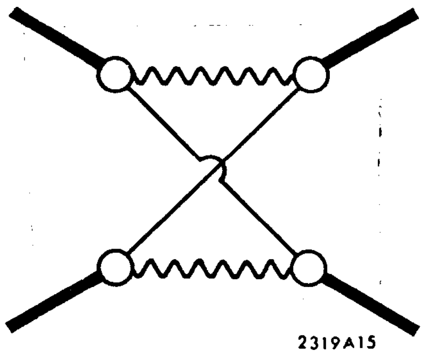

FIG. 3--Interchange contribution (tu graph) to deep hadron-hadron scattering for two body reactions.

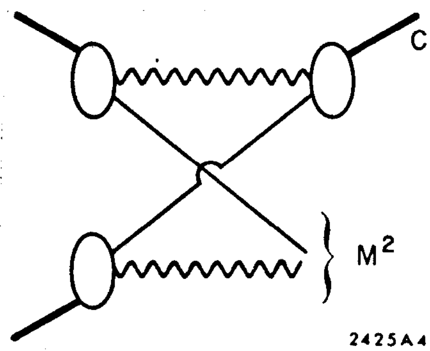

FIG. 4--Interchange contribution to inclusive cross sections in the deep region.

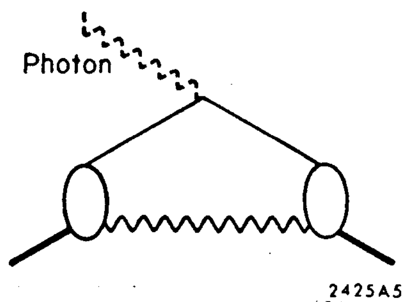

FIG. 5--Asymptotic form factor in terms of the hadron's asymptotic wave function.

asymptotic behavior is determined by calculating the form factor (Fig. 5), and choosing a vertex function which correctly reproduces the experimentally determined form factor. (In practice, meson form factors are assumed to behavior like $(q^2)^{-1}$ and baryon form factors like $(q^2)^{-2}$ for large $|q^2|$.)

The topologies of the interchange graphs are determined by the quantum numbers of the constituents. If the partons are assumed to have quark quantum numbers then the topologies of the interchange graphs are the same as those found in the case of the Harari-Rosner duality diagrams.[10] For instance, Fig. 3 is a two body t-u diagram, and is the only one allowed for, say k^+p elastic scattering. (The relationship between duality diagrams and parton-interchange diagrams is rather interesting. See Refs. (11) and (12) for a discussion.) Notice that only the simplest components of the hadrons' wave functions are assumed to be important in these calculations: that is, the sea of parton-antiparton pairs is assumed to be an asymptotically unimportant component of the hadron.

What are the general features of deep scattering predicted by this theory? First, one should probably not expect two transverse jets of hadrons in a typical deep event. This can be seen by referring to Fig. 4. A jet of hadrons associated with particle C, which has a large transverse momentum is most likely to be produced by bremsstrahlung from that particle. Since C will lose some of its energy if it bremsstrahlungs a jet of hadrons, the interchange interaction which originally produced C will have to take place at a higher energy and with a larger momentum transfer than if a transverse jet were not produced. But since the interchange interaction falls (like inverse powers) with increasing s and t, such events will be suppressed. This argument is, of course, only heuristic. A more careful analysis of this problem within the spirit of the interchange model is clearly needed.

A number of single particle inclusive reactions have been calculated in this model. In the deep region the invariant cross section for detecting a particle with a large transverse momentum has the form

$$E\frac{d^3\sigma}{dp^3} = s^{-n}\,g(x_1, x_2) \tag{3}$$

Unlike type I theories, the power fall-off of s varies from reaction to reaction since it is dependent on the form factors of the particles involved in the scattering. For instance, for $pp \to pX$, $n = 8$, for $\pi p \to \pi X$, $n = 4$, and for $\pi p \to pX$, $n = 6$.

An important result of some of the work which has been done on the interchange model is a fairly clear idea of the range of validity of the predictions. In particular, the deep region in inclusive experiments may be defined as the region where x_1, x_2 and M^2/s all remain finite (non-zero) as s grows. (M^2 is the missing mass squared.) (As $M^2/s \to 0$, with x_1, x_2 fixed, we are still in a deep region, but we are approaching the exclusive edge of phase space.) Note that since, in this region $p_\perp^2 \propto s$ Eq.(3) can be rewritten as a power of $p_\perp$ times as function of x_1 and x_2. As we move away from this kinematic domain, towards smaller $p_\perp$, other sorts of effects become important and change the character of the inclusive cross section. The reason is the following: as we mentioned before, the interchange process falls with increasing energy. Therefore, if we wish to detect a particle at a given, large $p_\perp$, the most important diagrams will be those that allow the interchange interaction to occur at the lowest possible energy consistent with the observation of a large $p_\perp$ secondary. Hence, for $p_\perp$ large, but significantly away from the edge of phase space, diagrams such as those of Fig. 6 become important. By allowing bremsstrahlung of hadrons from the incident particles, the effective energy of the interchange process is lowered. Furthermore, since the colliding particles can bremsstrahlung baryons, an incident proton can turn into a pion which then takes part in the interchange interaction. This is advantageous because the pion's form factor falls less rapidly for large q^2 than the proton's, thus leading to a less rapidly falling energy dependence in the near deep, or transition region. These effects become especially important near $x_F = 2p_\parallel/\sqrt{s} \sim 0$ (pionization region) when $s \sim M^2$, and $|t|$, $|u| \sim \sqrt{s}$ for large s. For example, the invariant cross section for $pp \to pX$ in the transition region can still be written in the form (3), only now we have $n = 6$ rather than $n = 8$ as in the deep region. Furthermore, the invariant

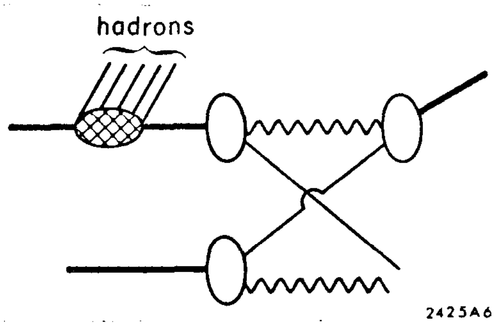

FIG. 6--Bremsstrahlung diagrams which give rise to the transition region in type II theories of inclusive reactions.

cross section generally falls less rapidly with $p_\perp$ for fixed s and $x_F \sim 0$ in this region than in the deep region.

For orientation, we have plotted in Fig. 7 the expected qualitative behavior of $E(d^3\sigma/dp^3)$ as a function of $p_\perp$ for $x_F \sim 0$, and a number of different values of s. Notice that there are three fairly distinct regions. For small $p_\perp$ the inclusive cross section is independent of s as required by Feynman scaling. In the transition region the fall off with $p_\perp$ is slower due to the occurrence of hadronic bremsstrahlung. As we move into the deep region, there is not enough energy for bremsstrahlung to occur and so the fall off with $p_\perp$ becomes sharper. Finally, at the very edge of phase space, the inclusive cross section matches smoothly on to exclusive one. The other regions of phase space can be discussed in a similar way, and are treated in detail in the third article in Ref. (2).

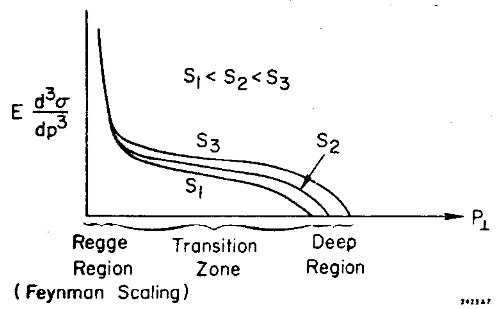

FIG. 7--Schematic representation of the behavior of inclusive cross sections predicted by type II theories for $x_F \sim 0$.

Let us now turn to the predictions of the interchange theory for two body reactions. In the high energy fixed angle region, the differential cross section can be written in the form

$$\frac{d\sigma}{dt} = s^{-p}\, h(t/s) \tag{4}$$

p varies from reaction to reaction, depending on the number of baryons and mesons involved in the reaction. For $BB \to BB$ (or $B\bar{B} \to B\bar{B}$) reactions, $p=12$, for $MB \to MB$ or $B\bar{B} \to MM$, $p=8$, and for $MM \to MM$, $p=6$.

It seems reasonable to suppose that some sort of bremsstrahlung process also Reggeizes these two body reactions as we move in in t from the fixed angle region. This is, indeed, the case.[11,12] However, since we require a specific final state, the brem'ed hadrons must be caught by the exiting particles on their way out. This leads us to consider diagrams such as those shown in Fig. 8. Each blob is some Born term which we iterate in the t-channel to obtain graphs which become increasingly important at smaller and smaller $|t|$. By assuming a Born term which accurately describes fixed angle scattering, the first order corrections can be calculated by t-channel iteration.

Suppose, for instance, that we have a large angle scattering Born term of the form

$$M_0 = s^{-m} f(t)$$

The leading behavior of the first iterated graph (Fig. 8) is then

$$M_1 = s^{-m} \frac{f^2(t)}{|t|} \ln s \;.$$

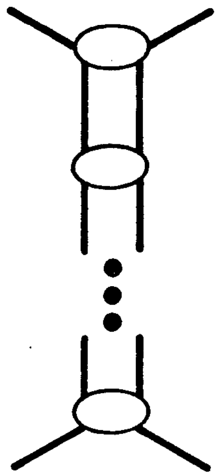

FIG. 8--Generalized ladder graphs which Reggeize two body deep scattering in a wide class of theories (including type II parton theories).

The sum can be written, to this order in the couplings as

$$M = M_0 + M_1 = \beta(t)\ s^{\alpha(t)}$$

with $\beta(t) = f(t)$ and $\alpha(t) = -m + \beta(t)/|t|$. We see then, how we can build up moving Regge trajectories by summing generalized t-channel ladder graphs. This Reggeization procedure also provides us with a definition of the deep region in the case of $2 \rightarrow 2$ amplitudes. The deep region is that region where coherent Regge effects are unimportant, i.e., where $M_1 << M_0$. If, as predicted by type II theories, $f(t) \propto t^{-\ell}$, then for large s the deep region sets in when

$$|t| >> C\,[\ln s]^{\frac{1}{\ell+1}}$$

where C is a constant. Hence, the deep region is neither a fixed angle nor a fixed t region, but sets in someplace inbetween.[13]

There are two lessons to be learned from this. Phenomenologically, we learn that given a description of the deep scattering region, we can, by t-channel iteration learn how this behavior is extended in first order into the smaller t region. In a realistic analysis, it is necessary to deal with a coupled channel problem since, in M_1 the hadrons in the 2 particle intermediate t-channel state may be different from the external hadrons. This leads naturally to sets of Regge trajectories which become degenerate as $t \rightarrow -\infty$. Indeed, such degeneracies are expected and occur in a wide range of physical systems.[14] Second, we learn that since deep scattering is properly described by the large $|t|$ limit of the usual Regge expressions for hadronic amplitudes, it is in principle wrong to add parton dynamics to Regge exchange — they are two manifestations of the same thing, and one commits double counting errors by adding them together.

Before reviewing the experimental situation with regard to the two types of theories, I want to mention some recent work by Brodsky and Farrar.[7] They assume that hadron scattering is describable by a renormalizable field theory. Baryons are composed of three elementary (quark) fields, and mesons are composed of two. Two body reactions are considered in the kinematic domain $s \rightarrow \infty$, with t/s and u/s fixed. Writing the differential cross section in the form (4) these authors determine the power, p, by drawing the simplest connected Born diagrams and counting the propagators which must have large invariant mass ("hot propagators" in Brodsky's parlance). Using this rule, and assuming that higher order terms don't change the result (mod logs), Brodsky and Farrar find the same energy dependence as that found in the interchange theory, with one exception. Their result is conveniently summarized as

$$p = n - 2$$

where n is the number of elementary fields in the initial state plus the number of elementary fields in the final state. (This result can also be extended to photon induced processes.[7]) The one point of disagreement with the interchange model is in the case of baryon-baryon elastic scattering. The interchange theory predicts p = 12 while Brodsky and Farrar find p = 10. However, the difference is most likely traceable to the different models assumed for the baryon wave function.

This approach can also be used to calculate inclusive cross sections under the assumptions appropriate to type I theories (direct parton-parton interactions between partons belonging to different hadrons allowed), and also under the assumptions appropriate to type II theories (no such direct parton-parton interactions allowed). The results of these calculations are consistent with the predictions of the two types of theories as we have so far discussed.

What does experiment tell us about the merits of the two types of theories? As far as the general structure of deep reactions — especially transverse jet formation — is concerned, it is difficult to draw any definite conclusions. There is some slight indication from the Cern-Columbia-Rockefeller ISR collaboration that transverse jets may be formed in large $p_\perp$ events,[15] but neither the transverse momenta nor associated multiplicities are large enough to draw any firm conclusions. The ratios of various particles produced at large $p_\perp$ are certainly important quantities to study, but predictions of these ratios depend on fairly detailed aspects of the theories, and so are probably not appropriate as a first test of the general ideas.

Much more germain to the present discussion is the energy and, to some extent, the angular dependence of exclusive and inclusive processes. As far as the inclusive cross sections are concerned, we emphasize that the only high energy, large $p_\perp$ measurements which have been performed so far have been carried out in a range of $p_\perp$ which, at least in the context of type II theories, corresponds to the transition region. The results of these measurements[16] are consistent with the transition zone predictions of type II theories. However, it is difficult to compare type I theories with these measurements since, as we have stated before, there is no clear delineation of a transition zone, and so it is not known over what range of $p_\perp$ the results are supposed to be valid. For this reason it is extremely important to measure inclusive cross sections at higher values of $p_\perp / \sqrt{s}$ either by increasing $p_\perp$ or decreasing s.

The situation for two body reactions is somewhat clearer. Type II theories appear to predict very well the energy and angular dependence around 90^o in a variety of reactions. For instance, $d\sigma/dt\,|_{90^o}$ as a function of s for meson-baryon reactions is predicted to go like s^{-8}, and measurements of all the following reactions are consistent with this result:[17] $\bar{k}^o p \to \pi \Sigma^o$, $k^o_L p \to k^o_s p$, $\bar{k}^o_p \to \pi^+ \Lambda^o$, $\pi^\pm p \to \pi^\pm p$, $k^\pm p \to k^\pm p$, $p\bar{p} \to \pi^+\pi^-$, and $p\bar{p} \to k^+ k^-$. It should furthermore be noted that this behavior persists in many of these reactions down to s of about 5 GeV^2. For pp and $\bar{p}p$ elastic scattering in the asymptotic fixed angle region, the interchange model implies a differential cross section which behaves as s^{-12}. Brodsky and

Farrar find s^{-10} for the same quantity. At the present level of accuracy the data is consistent with both.[18] The simplest type I theories, on the other hand, such as the single gluon exchange pictured in Fig. (2) predict energy independent differential cross sections and, at least for the case of pp scattering at fixed s, a $(d\sigma/dt)$ which falls too rapidly with t. Of course, these predictions could be modified by using a more complicated parton-parton scattering amplitude, but this does not really seem to be in the spirit of type I models.

Let's conclude with a brief summary of what we've learned. Although the experimental situation is incomplete, type II theories seem to be in better shape in their description of the gross properties of both inclusive and exclusive hadronic reactions. For inclusive reactions, it is difficult to clearly compare type I theories to present data, since we do not really know their range of validity. On the other hand, there is a well-defined and necessary extension of type II theories which gives rise to a transition region, and which agrees well with the present data. Exclusive data near 90^{0} is well described by type II theories, but not by type I theories. Again, however, the deep region predictions of type I theories may not be applicable to the present data — we do not know. Finally, the important question of the existence or nonexistence of transverse jets in the two types of theories has not really been settled theoretically (although there are some arguments which may be brought to bear) and has barely been asked experimentally.

ACKNOWLEDGMENTS

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12. R. Savit, "Multiplicities and Regge Poles in Deep Hadron-Hadron Scattering", Stanford Linear Accelerator Center report; Ph.D. thesis, Stanford University (1973).
13. Many of the results obtained by considering generalized ladder graphs actually follow from much more general assumptions. R. Savit and R. Blankenbecler, to be published.
14. Nearly degenerate energy levels usually occur in physical systems governed by a Hamiltonian with a large piece and a small (perturbative) piece. In the present case, (large $|t|$), the short range part of the Hamiltonian is large, and the long-range, coherent part is small. Near $t = 0$, on the other hand, the situation is reversed. This is similar to what happens in the case of an atom in a weak magnetic field (Zeeman effect) versus a strong magnetic field (Paschen-Back effect). For a further explanation, see Refs. 11 and 12.
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