

Scale and electroweak first-order phase transitions

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We consider phase transitions in the standard model (SM) without the Higgs mass term, which is coupled through a Higgs portal term to an SM singlet, classically scale-invariant gauge sector with SM singlet scalar fields. At lower energies the gauge-invariant scalar bilinear in the hidden sector forms a condensate, dynamically creating a robust energy scale, which is transmitted through the Higgs portal term to the SM sector. A scale phase transition is a transition between phases with zero and nonzero condensates. An interplay between the electroweak (EW) and scale phase transitions is therefore expected. We find that in a certain parameter space both the EW and scale phase transitions can be a strong first-order phase transition. The result is obtained by means of an effective theory for the condensation of the scalar bilinear in the mean field approximation.
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Subject Index B50, B53, B57, B59

1. Introduction

Thanks to the recent discovery of the Higgs boson at LHC [1,2] the standard model (SM) describing the dynamics of elementary particles is now complete. However, the SM accommodates neither dark matter (DM) nor neutrinos with a finite mass. Therefore, the SM is incomplete as a theory to explain phenomena in our Universe, and consequently it has to be extended. These unsatisfactory features are the main motivations for probing both theoretically and experimentally new physics around the TeV scale.

Besides the problems mentioned above there are also problems of a more theoretical nature. One of them is the origin of the electroweak (EW) scale. Certainly, the SM cannot explain it, but a hint might exist in the SM: The Higgs mass term is the only term that breaks scale invariance at the classical level. In fact there have recently been many studies on a scale-invariant extension of the SM. There are basically two types of scenario: one [3–47] relies on the Coleman–Weinberg (CW) potential [48], while the other [49–59] is based on non-perturbative effects in non-abelian gauge theory such as dynamical chiral symmetry breaking [60–62] or condensation of the gauge-invariant scalar bilinear [63–65]. The common thinking is that a classically scale-invariant physics around TeV is responsible for the origin of the SM scale.

Along this line of thought we have suggested a new model [59], in which SM singlet scalar fields S are coupled with non-abelian gauge fields in a hidden sector. Below a certain energy scale the scalar fields condensate in the form of the bilinear, i.e. $\langle S^\dagger S \rangle$, by a non-perturbative effect of the hidden sector. Because of the condensate the Higgs portal term turns to a Higgs mass term with a squared mass proportional to $\langle S^\dagger S \rangle$. However, this is too naive, because it is a non-perturbative effect, and there is a back reaction on the condensate from the Higgs through the portal. In [59] we have proposed

an effective theory for the condensation of the scalar bilinear and investigated the vacuum structure in the self-consistent mean field approximation (SCMFA) [66,67]. Furthermore, we have introduced flavors to the scalar fields and shown that realistic DM candidates, which are the excited states above the vacuum, exist in the model. Thus, the DM and EW scales have the same origin.

In this paper we will study phase transitions at finite temperature in our model. There will be EW and scale phase transitions. As is well known, a strong first-order EW phase transition is important for baryon asymmetry in the Universe [68–75]. By the scale phase transition we mean a transition between phases with a zero and nonzero condensates of the scalar bilinear. Note that (to the best of our knowledge) the scale phase transition in a non-abelian gauge theory has not been studied and therefore the nature of the phase transition is not known. Since we have an effective theory for the condensation of the scalar bilinear at hand, we will address this problem by means of the effective theory. The first sections will be used to explain the model as well as the effective theory. We expect that there exists a nontrivial interplay between the EW and scale phase transitions, because the EW scale is created by the condensate in the hidden sector. We will be able to confirm this expectation in Sect. 5. Moreover, it will turn out that the EW and scale phase transitions can be a strong first-order phase transition in a certain parameter space of the model. Section 6 will be devoted to a summary.

2. The model and its effective Lagrangian

Our hidden sector [59] consists of strongly interacting $SU(N_c)$ gauge fields coupled with the scalar fields S_i^a ($a = 1, \dots, N_c, i = 1, \dots, N_f$) in the fundamental representation of $SU(N_c)$. The hidden sector Lagrangian is given by

$$\begin{aligned} \mathcal{L}_H = & -\frac{1}{2} \text{tr} F^2 + ([D^\mu S_i]^\dagger D_\mu S_i) - \hat{\lambda}_S (S_i^\dagger S_i) (S_j^\dagger S_j) \\ & - \hat{\lambda}'_S (S_i^\dagger S_j) (S_j^\dagger S_i) + \hat{\lambda}_{HS} (S_i^\dagger S_i) H^\dagger H, \end{aligned} \quad (1)$$

where $D_\mu S_i = \partial_\mu S_i - i g_H G_\mu S_i$, G_μ is the matrix-valued gauge field, the trace is taken over the color indices, and the SM Higgs doublet field is denoted by H . The total Lagrangian is the sum of $\mathcal{L}_H$ and $\mathcal{L}_{\text{SM}}$, where the scalar potential of the SM part, $\mathcal{L}_{\text{SM}}$, is

$$V_{\text{SM}} = \lambda_H (H^\dagger H)^2. \quad (2)$$

Note that the Higgs mass term is absent. Below a certain energy scale the gauge coupling g_H becomes so large that the $SU(N_c)$ invariant scalar bilinear dynamically forms a $U(N_f)$ invariant condensate [64,65],

$$\langle (S_i^\dagger S_j) \rangle = \left\langle \sum_{a=1}^{N_c} S_i^{a\dagger} S_j^a \right\rangle \propto \delta_{ij}, \quad (3)$$

which breaks classical scale invariance. But the condensate (3) is not an order parameter, because scale invariance is broken by scale anomaly, too [76,77]. This hard breaking by anomaly is only logarithmic, and it implies that the coupling constants depend on the energy scale [76,77]. Therefore, we have assumed in [59] that the non-perturbative breaking is dominant, so that we can ignore the scale anomaly in writing down an effective Lagrangian to the condensation of the scalar bilinear at the tree level. The effective Lagrangian does not contain the $SU(N_c)$ gauge fields, because they are integrated out, while it contains the “constituent” scalar fields S_i^a . Since the effective theory should dynamically describe the condensation of the scalar bilinear, which should be the origin of the

breaking of scale invariance, the effective Lagrangian has to be invariant under scale transformation:

$$\begin{aligned}\mathcal{L}_{\text{eff}} = & \left([\partial^\mu S_i]^\dagger \partial_\mu S_i \right) - \lambda_S \left(S_i^\dagger S_i \right) \left(S_j^\dagger S_j \right) - \lambda'_S \left(S_i^\dagger S_j \right) \left(S_j^\dagger S_i \right) \\ & + \lambda_{HS} \left(S_i^\dagger S_i \right) H^\dagger H - \lambda_H \left(H^\dagger H \right)^2,\end{aligned}\quad (4)$$

where we assume that all λ 's are positive. This is the most general form which is consistent with the $SU(N_c) \times U(N_f)$ symmetry and the classical scale invariance, where the kinetic term for H is included in $\mathcal{L}_{\text{SM}}$.¹ That is, $\mathcal{L}_H - V_{\text{SM}}$ has the same global symmetry as $\mathcal{L}_{\text{eff}}$ even at the quantum level, where $\mathcal{L}_H$ and V_{SM} are given in (1) and (2), respectively. Note that the couplings $\hat{\lambda}_S$, $\hat{\lambda}'_S$, and $\hat{\lambda}_{HS}$ in $\mathcal{L}_H$ are not the same as λ_S , λ'_S , and λ_{HS} in $\mathcal{L}_{\text{eff}}$, because the latter are effective couplings which are dressed by hidden gluon contributions.

3. Self-consistent mean field approximation

In the SCMF approximation [66], which has proved to be a successful approximation for the Nambu–Jona-Lasinio theory [61,62], the perturbative vacuum is Bogoliubov–Valatin (BV) transformed to $|0_B\rangle$, such that

$$\langle 0_B | \left(S_i^\dagger S_j \right) | 0_B \rangle = f_{ij} = \langle f_{ij} \rangle + Z_\sigma^{1/2} \delta_{ij} \sigma + Z_\phi^{1/2} t_{ji}^\alpha \phi^\alpha, \quad (5)$$

where the real mean fields σ and ϕ^α ($\alpha = 1, \dots, N_f^2 - 1$) are introduced as the excitations of the condensate $\langle f_{ij} \rangle$. Here, t^α (normalized as $\text{Tr}(t^\alpha t^\beta) = \delta^{\alpha\beta}/2$) are the $SU(N_f)$ generators in the hermitian matrix representation, and Z_σ and Z_ϕ are the wave function renormalization constants of a canonical dimension 2. The unbroken $U(N_f)$ flavor symmetry implies

$$\langle f_{ij} \rangle = \delta_{ij} f \quad \text{and} \quad \langle \phi^\alpha \rangle = 0, \quad (6)$$

where a nonzero $\langle \sigma \rangle$ can be absorbed into f , so that we can always assume $\langle \sigma \rangle = 0$.

In the SCMF approximation, f is determined in a self-consistent way as follows. One first splits up the effective Lagrangian (4) into the sum, i.e., $\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{MFA}} + \mathcal{L}_I$, where $\mathcal{L}_I$ is normal ordered (i.e. $\langle 0_B | \mathcal{L}_I | 0_B \rangle = 0$), and $\mathcal{L}_{\text{MFA}}$ contains at most bilinear terms of S which are not normal ordered. Using the Wick theorem,

$$\left(S_i^\dagger S_j \right) = \left(S_i^\dagger S_j \right) : + f_{ij}, \quad \left(S_i^\dagger S_j \right) \left(S_j^\dagger S_i \right) = \left(S_i^\dagger S_j \right) \left(S_j^\dagger S_i \right) : + 2f_{ij} \left(S_j^\dagger S_i \right) - |f_{ij}|^2, \quad (7)$$

etc., we find

$$\begin{aligned}\mathcal{L}_{\text{MFA}} = & \left(\partial^\mu S_i^\dagger \partial_\mu S_i \right) - M^2 \left(S_i^\dagger S_i \right) + N_f (N_f \lambda_S + \lambda'_S) Z_\sigma \sigma^2 + \frac{\lambda'_S}{2} Z_\phi \phi^\alpha \phi^\alpha \\ & - 2(N_f \lambda_S + \lambda'_S) Z_\sigma^{1/2} \sigma \left(S_i^\dagger S_i \right) - 2\lambda'_S Z_\phi^{1/2} \left(S_i^\dagger t_{ij}^\alpha \phi^\alpha S_j \right) \\ & + \lambda_{HS} \left(S_i^\dagger S_i \right) H^\dagger H - \lambda_H \left(H^\dagger H \right)^2,\end{aligned}\quad (8)$$

where

$$M^2 = 2(N_f \lambda_S + \lambda'_S) f - \lambda_{HS} H^\dagger H, \quad (9)$$

and the linear term in σ is suppressed because it will be cancelled against the corresponding tad pole correction. To the lowest order in the SCMF approximation, the “interacting” part $\mathcal{L}_I$ does not contribute to the amplitudes without external S 's (the mean field vacuum amplitudes). We emphasize that,

¹ Quantum field theory defined by (4) with the kinetic term for H is renormalizable in perturbation theory [78].

in applying the Wick theorem, only the $SU(N_c)$ invariant bilinear product $(S_i^\dagger S_j) = \sum_a^{N_c} S_i^{a\dagger} S_j^a$ has a non-zero (BV transformed) vacuum expectation value.

Given the effective Lagrangian $\mathcal{L}_{\text{MFA}}$, we next compute an effective potential V_{MFA} by integrating out the mean field fluctuations S_i^a , where the fluctuations of the SM fields including H will be taken into account later on when discussing finite temperature effects. We employ the $\overline{\text{MS}}$ scheme, because dimensional regularization does not break scale invariance. To the lowest order the divergences can be removed by renormalization of λ_I ($I = H, S, HS$), i.e. $\lambda_I \rightarrow (\mu^2)^\epsilon (\lambda_I + \delta\lambda_I)$, and also by the shift $f \rightarrow f + \delta f$, where $\epsilon = (4 - D)/2$, and μ is the scale introduced in dimensional regularization. The effective potential for $\mathcal{L}_{\text{MFA}}$ can be straightforwardly computed:

$$V_{\text{MFA}} = M^2 (S_i^\dagger S_i) + \lambda_H (H^\dagger H)^2 - N_f (N_f \lambda_S + \lambda'_S) f^2 + \frac{N_c N_f}{32\pi^2} M^4 \ln \frac{M^2}{\Lambda_H^2}, \quad (10)$$

where $\Lambda_H = \mu \exp(3/4)$ is so chosen that the loop correction vanishes at $M^2 = \Lambda_H^2$. V_{MFA} with a term linear in f included but without the Higgs doublet H has also been discussed in [79–83]. The classical scale invariance forbids the presence of this linear term. To find the minimum of V_{MFA} we look for the solutions of

$$0 = \frac{\partial}{\partial S_i^a} V_{\text{MFA}} = \frac{\partial}{\partial f} V_{\text{MFA}} = \frac{\partial}{\partial H_l} V_{\text{MFA}} \quad (l = 1, 2). \quad (11)$$

The first equation gives $0 = \langle S_i^a \rangle^\dagger \langle M^2 \rangle = \langle S_i^a \rangle^\dagger \langle 2(N_f \lambda_S + \lambda'_S) f - \lambda_{HS} H^\dagger H \rangle$, which has three solutions: (i) $\langle S_i^a \rangle \neq 0$ and $\langle M^2 \rangle = 0$; (ii) $\langle S_i^a \rangle = 0$ and $\langle M^2 \rangle = 0$; and (iii) $\langle S_i^a \rangle = 0$ and $\langle M^2 \rangle \neq 0$. One can easily convince oneself that the solution (i) implies $G = 0$ if the second and third equations in (11) are used, where

$$G = 4N_f \lambda_H \lambda_S - N_f \lambda_{HS}^2 + 4\lambda_H \lambda'_S. \quad (12)$$

Therefore, the solution (i) is inconsistent, unless we use the fine-tuned relation among the coupling constants. Next, we consider the solution (ii) and find that $\langle S_i^a \rangle = \langle f \rangle = \langle H \rangle = 0$ with $\langle V_{\text{MFA}} \rangle = 0$. The third solution (iii) can exist if $G > 0$ is satisfied, and we find

$$|\langle H \rangle|^2 = \frac{v_h^2}{2} = \frac{N_f \lambda_{HS}}{G} \Lambda_H^2 \exp\left(\frac{32\pi^2 \lambda_H}{N_c G} - \frac{1}{2}\right), \quad \langle f \rangle = \frac{2\lambda_H}{N_f \lambda_{HS}} |\langle H \rangle|^2, \quad (13)$$

$$\langle M^2 \rangle = M_0^2 = \frac{G}{N_f \lambda_{HS}} |\langle H \rangle|^2, \quad \langle V_{\text{MFA}} \rangle < 0. \quad (14)$$

Consequently, the solution (iii) presents the true potential minimum if $G > 0$ is satisfied. The Higgs mass at this level of approximation becomes

$$m_{h0}^2 = |\langle H \rangle|^2 \left(\frac{16\lambda_H^2 (N_f \lambda_S + \lambda'_S)}{G} + \frac{N_c N_f \lambda_{HS}^2}{8\pi^2} \right). \quad (15)$$

In the small λ_{HS} limit we obtain $m_{h0}^2 \simeq 4\lambda_H |\langle H \rangle|^2 = 2\lambda_{HS} \langle f \rangle$, where the first equation is the SM expression, and the second one is simply assumed in [53]. There will be a correction ($\sim 7\%$) to (15) coming from the SM part, which will be calculated later on.

We would like to note that the effective potential V_{MFA} in (24) has a flat direction, which corresponds to the end-point contribution of [82,83]: If $M^2 = 2(N_f \lambda_S + \lambda'_S) f - \lambda_{HS} H^\dagger H = 0$ is satisfied, $V_{\text{MFA}} = 0$ for any value of S_i^a , so that (except for $S_i^a = 0$) the $SU(N_c)$ symmetry is spontaneously broken in this direction. The origin that $\langle V_{\text{MFA}} \rangle < 0$ for the solution (iii) is the absence of a

mass term in the effective Lagrangian (4); we have assumed classical scale invariance to begin with. A mass term in (4) would effectively generate in V_{MFA} a term linear in f . This linear term can lift the $\langle V_{\text{MFA}} \rangle$ into a positive direction [80,81], while $V_{\text{MFA}} = 0$ remains in the flat direction [82,83].

Finally, we would like to recall once again that we regard the Lagrangian (4) together with our approximation method as an effective theory for the condensation of the scalar bilinear, which takes place in the $SU(N_c)$ gauge theory described by (1). That is, we discard fundamental problems such as the intrinsic instability inherent in (4) [82,83], because we assume that such problems are absent in the original theory described by (1).

4. Dark matter

We are now in a position to use the effective Lagrangian $\mathcal{L}_{\text{MFA}}$ (8) to discuss DM. First, we replace M^2 and the Higgs doublet H appearing in $\mathcal{L}_{\text{MFA}}$ by M_0^2 and $H^T = (\chi^+, (v_h + h + i\chi^0)/\sqrt{2})$, respectively, where χ^+ and χ^0 are the would-be Nambu–Goldstone fields, and M_0^2 is given in (14). The linear terms in σ and h in $\mathcal{L}_{\text{MFA}}$ should be suppressed, because they will be cancelled against the corresponding tad pole corrections. We integrate out the constituent scalars S^a to obtain effective interactions among σ , ϕ , and the Higgs h , where σ and ϕ are defined in (5). Their inverse propagators should be computed to obtain their masses and the corresponding wave function renormalization constants. Up to and including one-loop order we find:

$$\Gamma_\phi^{\alpha\beta}(p^2) = Z_\phi \delta^{\alpha\beta} \lambda'_S \Gamma_\phi(p^2) = Z_\phi \delta^{\alpha\beta} \lambda'_S \left[1 + 2\lambda'_S N_c \Gamma(p^2) \right], \quad (16)$$

$$\Gamma_\sigma(p^2) = 2Z_\sigma N_f (N_f \lambda_S + \lambda'_S) \left[1 + 2N_c (N_f \lambda_S + \lambda'_S) \Gamma(p^2) \right],$$

$$\Gamma_{h\sigma}(p^2) = -2Z_\sigma^{1/2} v_h \lambda_{HS} (N_f \lambda_S + \lambda'_S) N_f N_c \Gamma(p^2),$$

$$\Gamma_h(p^2) = p^2 - m_{h1}^2 + (v_h \lambda_{HS})^2 N_f N_c (\Gamma(p^2) - \Gamma(0)),$$

with $m_{h1}^2 = m_{h0}^2 + \delta m_h^2$, where m_{h0}^2 is given in (15), δm_h^2 is the SM correction given in (29), and

$$\Gamma(p^2) = \frac{1}{16\pi^2} \left(2 - \ln \left[\frac{M_0^2}{\Lambda_H^2 \exp(-3/2)} \right] - 2(4/x - 1)^{1/2} \arctan(4/x - 1)^{-1/2} \right) \quad (17)$$

with $x = p^2/M_0^2$. Note that we have included the canonical kinetic term for H , but the wave function renormalization constant for h is ignored, which is approximately equal to one within the approximation here. The DM mass is the zero of the inverse propagator, i.e.

$$\Gamma_\phi^{\alpha\beta}(p^2 = m_{\text{DM}}^2) = 0, \quad (18)$$

and Z_ϕ (which has a canonical dimension 2) can be obtained from

$$\begin{aligned} Z_\phi^{-1} &= 2(\lambda'_S)^2 N_c (d\Gamma/dp^2) \Big|_{p^2=m_{\text{DM}}^2} \\ &= \frac{2(\lambda'_S)^2 N_c}{m_{\text{DM}}^2 16\pi^2} \left(4[y(4-y)]^{-1/2} \arctan(4/y - 1)^{-1/2} - 1 \right) \end{aligned} \quad (19)$$

with $y = m_{\text{DM}}^2/M_0^2$. The Higgs and σ masses can be similarly obtained from the eigenvalues of the h – σ mixing matrix

$$\Gamma(p^2) = \begin{pmatrix} \Gamma_h(p^2) & \Gamma_{h\sigma}(p^2) \\ \Gamma_{h\sigma}(p^2) & \Gamma_\sigma(p^2) \end{pmatrix}. \quad (20)$$

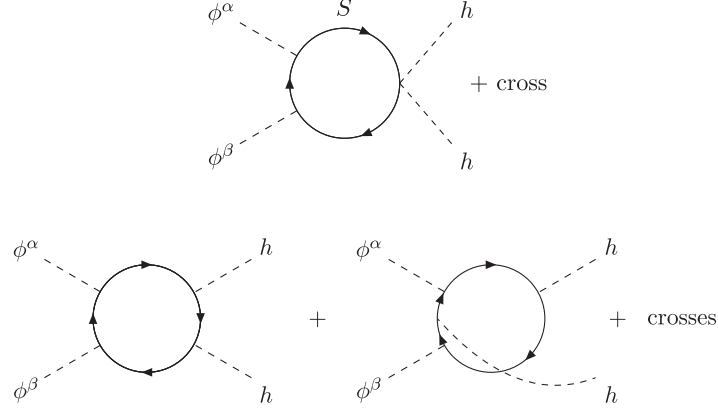


Fig. 1. The interaction between DM and the Higgs h arises at the one-loop level. The lower diagrams are $\sim \lambda_{HS}^2 (v_h/M_0)^2$, so that the upper diagrams are dominant, because $\lambda_{HS}^2 (v_h/M_0)^2 \ll \lambda_{HS}$ in a realistic parameter space.

The squared Higgs and σ masses, m_h^2 and m_σ^2 , are zeros of $\det \Gamma(p^2)$. That is, the SM correction (29) and the correction from the mixing (20) are included in m_h . This mixing has to be taken into account in determining the renormalization constants, which we will ignore in the following discussions, because the effect is very small (as mentioned above). In contrast, the mixing can have a non-negligible effect on the masses. If $m_{\text{DM}}, m_\sigma > 2M_0$, DM or σ would decay into two S 's within the framework of the effective theory, because the effective theory cannot incorporate confinement. Therefore, we will consider only the parameter space with $m_{\text{DM}}, m_\sigma < 2M_0$.

The link of ϕ to the SM model is established through the interaction with the Higgs, which is generated at one-loop as shown in Fig. 1, yielding the effective couplings

$$\kappa_{s(t)} \delta^{\alpha\beta} = \delta^{\alpha\beta} \Gamma_{\phi^2 h^2}(M_0, m_{\text{DM}}, \epsilon = 1(-1)), \quad (21)$$

where²

$$\begin{aligned} \Gamma_{\phi^2 h^2}(M_0, m_{\text{DM}}, \epsilon) &= \frac{Z_\phi N_c (\lambda'_S)^2 \lambda_{HS}}{4\pi^2} \\ &\times \left(\lambda_{HS} \frac{v_h^2}{4M_0^4} - \left\{ \begin{array}{ll} \frac{2}{m_{\text{DM}}^2} \left(\frac{\arctan(4/y - 1)^{-1/2}}{(4/y - 1)^{-1/2}} - \frac{\arctan(1/y - 1)^{-1/2}}{(1/y - 1)^{-1/2}} \right) & \text{for } \epsilon = 1 \\ \frac{2 \arctan(4/y - 1)^{-1/2}}{M_0 m_{\text{DM}} (4 - y)^{1/2}} & \text{for } \epsilon = -1 \end{array} \right\} \right), \end{aligned} \quad (22)$$

$y = m_{\text{DM}}^2/M_0^2$, and $v_h = 246 \text{ GeV}$. We have used the s-channel ($\epsilon = 1$) momenta $p = p' = (m_{\text{DM}}, \mathbf{0})$ for DM annihilation, because we restrict ourselves to the s-wave part of the velocity-averaged annihilation cross section $\langle v\sigma \rangle$. Similarly, we have used the t-channel ($\epsilon = -1$) momenta $p = -p' = (m_{\text{DM}}, \mathbf{0})$ for the spin-independent elastic cross section off the nucleon σ_{SI} .

² Since the contribution of the lower diagrams in Fig. 1 is small, we compute them at $p = 0$, which is the ϵ -independent term in (22).

We obtain

$$\langle v\sigma \rangle = \frac{1}{32\pi m_{\text{DM}}^3} \sum_{I=W,Z,t,h} (m_{\text{DM}}^2 - m_I^2)^{1/2} a_I + \mathcal{O}(v^2),$$

where $m_W = 80.4 \text{ GeV}$, $m_Z = 91.2 \text{ GeV}$, and $m_t = 174 \text{ GeV}$ are the W , Z boson and the top quark masses, respectively, and

$$a_{W(Z)} = 4(2)[\text{Re}(\kappa_s)]^2 \Delta_h^2 m_{W(Z)}^4 \left(3 + 4 \frac{m_{\text{DM}}^4}{m_{W(Z)}^4} - 4 \frac{m_{\text{DM}}^2}{m_{W(Z)}^2} \right),$$

$$a_t = 24[\text{Re}(\kappa_s)]^2 \Delta_h^2 m_t^2 (m_{\text{DM}}^2 - m_t^2), \quad a_h = [\text{Re}(\kappa_s)]^2 \left(1 + 24\lambda_H \Delta_h \frac{m_W^2}{g^2} \right)^2. \quad (23)$$

Here, $g = 0.65$ is the $SU(2)_L$ gauge coupling constant, and $\Delta_h = (4m_{\text{DM}}^2 - m_h^2)^{-1}$ is the Higgs propagator. The DM relic abundance³ is $\Omega \hat{h}^2 = (N_f^2 - 1) \times (Y_\infty s_0 m_{\text{DM}}) / (\rho_c / \hat{h}^2)$, where Y_∞ is the asymptotic value of the ratio Y of the DM number density to entropy, $s_0 = 2890 \text{ cm}^{-3}$ is the entropy density at present, $\rho_c = 1.05 \times 10^{-5} \hat{h}^2 \text{ GeV cm}^{-3}$ is the critical density, and $\hat{h}$ is the dimensionless Hubble parameter. To obtain Y_∞ we solve the Boltzmann equation for Y . The spin-independent elastic cross section off the nucleon σ_{SI} is [84]

$$\sigma_{SI} = \frac{1}{4\pi} \left(\frac{\kappa_t \hat{r} m_N^2}{m_{\text{DM}} m_h^2} \right)^2 \left(\frac{m_{\text{DM}}}{m_N + m_{\text{DM}}} \right)^2,$$

where κ_t is given in (21), m_N is the nucleon mass, and $\hat{r} \sim 0.3$ stems from the nucleonic matrix element [85]. In [59] we have shown that there is a parameter space in the model with various N_f and N_c in which the DM mass is of $O(1) \text{ TeV}$ and σ_{SI} and $\Omega \hat{h}^2$ are, respectively, consistent with the recent experimental measurements in [86] and [87].

5. Phase transitions at finite temperature

At a certain finite temperature the condensation of the scalar bilinear will be dissolved, and above that temperature the EW symmetry will be restored. The nature of the EW symmetry breaking is crucial for baryon asymmetry in the Universe [68–71]. Here we investigate how the scale and EW symmetry breakings disappear as temperature increases from a low temperature.⁴ To this end, we integrate out the quantum fluctuations at finite temperature within the framework of the effective theory in the mean field approximation. As a result we obtain an effective potential at finite temperature consisting of four components [72–75]:

$$V_{\text{eff}}(f, h, T) = V_{\text{MFA}}(f, h) + V_{\text{CW}}(h) + V_{\text{FT}}(f, h, T) + V_{\text{RING}}(h, T), \quad (24)$$

where $V_{\text{MFA}}(f, h)$ is the effective potential given in (10) with $S_i^a = 0$ and H replaced by $h/\sqrt{2}$, and f (the condensate) is defined in (6). Further, $V_{\text{CW}}(h)$ and $V_{\text{FT}}(f, h, T)$ are the one-loop contributions

³ There are $(N_f^2 - 1)$ DM particles, and the number of the effectively massless degrees of freedom at the freeze-out temperature is $g_* = 106.75 + N_f^2 - 1$.

⁴ EW baryogenesis in a scale-invariant extension of the two-Higgs doublet model has been analyzed in [88–91].

at zero and finite temperature T , respectively, and V_{RING} is the ring contribution. The Coleman–Weinberg potential $V_{\text{CW}}(h)$ is normalized such that

$$V_{\text{CW}}(h = v_h) = 0, \quad \frac{dV_{\text{CW}}(h)}{dh} \Big|_{h=v_h} = 0, \quad (25)$$

where we use $v_h = \langle h \rangle|_{T=0} = 246 \text{ GeV}$. This normalization ensures that the potential $V_{\text{CW}}(h)$ does not change v_h given in (13) obtained from $V_{\text{MFA}}(f, h)$. It can be explicitly written as

$$V_{\text{CW}}(h) = C_0(h^4 - v_h^4) + \frac{1}{64\pi^2} \left[6\tilde{m}_W^4 \ln(\tilde{m}_W^2/m_W^2) + 3\tilde{m}_Z^4 \ln(\tilde{m}_Z^2/m_Z^2) + \tilde{m}_h^4 \ln(\tilde{m}_h^2/m_h^2) - 12\tilde{m}_t^4 \ln(\tilde{m}_t^2/m_t^2) \right], \quad (26)$$

where

$$C_0 \simeq -\frac{1}{64\pi^2 v_h^4} \left(3m_W^4 + (3/2)m_Z^4 + (3/4)m_h^4 - 6m_t^4 \right), \quad (27)$$

$$\tilde{m}_W^2 = (m_W/v_h)^2 h^2, \quad \tilde{m}_Z^2 = (m_Z/v_h)^2 h^2, \quad \tilde{m}_t^2 = (m_t/v_h)^2 h^2,$$

$$\tilde{m}_h^2 = 3\lambda_H h^2 + \frac{\lambda_{HS}}{64\pi^2} \left\{ 7N_c N_f \lambda_{HS} h^2 - 4f N_c N_f (N_f \lambda_S + \lambda'_S) - 2N_c N_f \left[-3\lambda_{HS} h^2 + 4f (N_f \lambda_S + \lambda'_S) \right] \ln \frac{4f (N_f \lambda_S + \lambda'_S) - \lambda_{HS} h^2}{2\Lambda_H^2} \right\}. \quad (28)$$

We work in the Landau gauge, in which the Faddeev–Popov ghost fields are massless even at finite temperature, so that they do not contribute to V_{eff} . The would-be NG bosons are massless only at the potential minimum. But we have neglected their contributions in (26), because they are negligibly small. The tedious expression for $\tilde{m}_h^2$ comes from the fact that the Higgs mass is generated from the condensation of the scalar bilinear: it is the second derivative of V_{MFA} in (10) with respect to h . Note that $V_{\text{CW}}(h)$ contributes to the Higgs mass correction⁵

$$\delta m_h^2 \simeq -16C_0 v_h^2, \quad (29)$$

which is about 7% in m_h . We follow [73] and find

$$V_{\text{FT}}(f, h, T) = \frac{T^4}{2\pi^2} \left(2N_c N_f J_B(\tilde{M}^2(T)/T^2) + J_B(\tilde{m}_h^2(T)/T^2) + 6J_B(\tilde{m}_W^2(T)/T^2) + 3J_B(\tilde{m}_Z^2(T)/T^2) - 12J_F(\tilde{m}_t^2(T)/T^2) \right), \quad (30)$$

where the thermal masses are

$$\tilde{M}^2(T) = M^2 + \frac{T^2}{6} \left((N_c N_f + 1)\lambda_S + (N_f + N_c)\lambda'_S - \lambda_{HS} \right), \quad (31)$$

$$\tilde{m}_h^2(T) = \tilde{m}_h^2 + \frac{T^2}{12} \left(\frac{9}{4}g^2 + \frac{3}{4}g'^2 + 3y_t^2 + 6\lambda_H - N_c N_f \lambda_{HS} \right), \quad (32)$$

the coupling constants $g = 0.65$, $g' = 0.36$, and $y_t = 1.0$ stand for the $SU(2)_L$, $U(1)_Y$ gauge coupling constants and the top Yukawa coupling constant, respectively, and M is defined in (9) with

⁵ The Higgs mass correction and also C_0 in (27) look more complicated if we use the Higgs mass (28). So, the term $\propto m_h^4$ in (27) and (29) is only an approximate expression.

$H^\dagger H = h^2/2$. The thermal functions J_B and J_F are defined as

$$J_B(r^2) = \int_0^\infty dx x^2 \ln \left(1 - e^{-\sqrt{x^2+r^2}} \right) \\ \simeq -\frac{\pi^4}{45} + \frac{\pi^2}{12} r^2 - \frac{\pi}{6} r^3 - \frac{r^4}{32} \left[\ln(r^2/16\pi^2) + 2\gamma_E - \frac{3}{2} \right] \quad \text{for } r^2 \lesssim 2, \quad (33)$$

$$J_F(r^2) = \int_0^\infty dx x^2 \ln \left(1 + e^{-\sqrt{x^2+r^2}} \right) \\ \simeq \frac{7\pi^4}{360} - \frac{\pi^2}{24} r^2 - \frac{r^4}{32} \left[\ln(r^2/\pi^2) + 2\gamma_E - \frac{3}{2} \right] \quad \text{for } r^2 \lesssim 2. \quad (34)$$

In the actual calculations we employ the idea [92] for approximating the thermal functions as

$$J_{B(F)}(r^2) \simeq \exp(-r) \sum_{n=0}^{40} c_n^{B(F)} r^n. \quad (35)$$

Finally, the ring contribution from the gauge bosons is [73]:

$$V_{\text{RING}} = -\frac{T}{12\pi} \left(2a_g^{3/2} + \frac{1}{2\sqrt{2}} \left(a_g + c_g - \left[(a_g - c_g)^2 + 4b_g^2 \right]^{1/2} \right)^{3/2} \right. \\ \left. + \frac{1}{2\sqrt{2}} \left(a_g + c_g + \left[(a_g - c_g)^2 + 4b_g^2 \right]^{1/2} \right)^{3/2} - \frac{1}{4} [g^2 h^2]^{3/2} - \frac{1}{8} [(g^2 + g'^2) h^2]^{3/2} \right), \quad (36)$$

where

$$a_g = \frac{1}{4} g^2 h^2 + \frac{11}{6} g^2 T^2, \quad b_g = -\frac{1}{4} g g' h^2, \quad c_g = \frac{1}{4} g'^2 h^2 + \frac{11}{6} g'^2 T^2. \quad (37)$$

The critical temperatures of the scale phase and EW phase transitions (which we denote by T_S and T_{EW} , respectively) can be different. If T_S and T_{EW} are distant from each other, two phase transitions cannot influence each other much. In the case that they are close or equal, i.e. $T_C \equiv T_S = T_{\text{EW}}$, two phase transitions can influence each other. In fact, depending on the choice of the parameter values, these different cases can be realized in our model. Below we consider some representative examples.

(i) Scale phase transition with $N_f = 1$, $N_c = 6$

First we consider the case with $\lambda_{HS} = 0$, i.e., no connection between the hidden sector and the SM sector. We choose:

$$N_f = 1, \quad N_c = 6, \quad \lambda_S + \lambda'_S = 2.083, \quad (38)$$

where we will use the same N_f and N_c as well as the same parameter values for λ_S and λ'_S when discussing case (ii) with the SM connected. (If $N_f = 1$, only the linear combination $\lambda_S + \lambda'_S$ is an independent coupling.) In Fig. 2 (left) we show $\langle f \rangle^{1/2}/T$ against T/Λ_H . We see from the figure that the scale phase transition is first order with $T_S/\Lambda_H \simeq 7.0$. The right panel shows the form of the potential for $T/\Lambda_H = 7.1$ (red-dashed), T_S/Λ_H (black), and 6.9 (green dash-dotted). As we will see below, the strong first-order scale phase transition in the hidden sector can infect the EW phase transition.

The existence of the first-order phase transition observed here was predicted in [82,83]. In our analysis we have assumed (and will throughout assume) that $\langle S_i^a \rangle = 0$. However, within the framework of the effective theory (even if we assume classical scale invariance), there is no reason to prefer $\langle f \rangle = \langle S_i^a \rangle = 0$ to the flat direction with $\langle S_i^a \rangle \neq 0$ [82,83] (mentioned at the end of Sect. 3) at

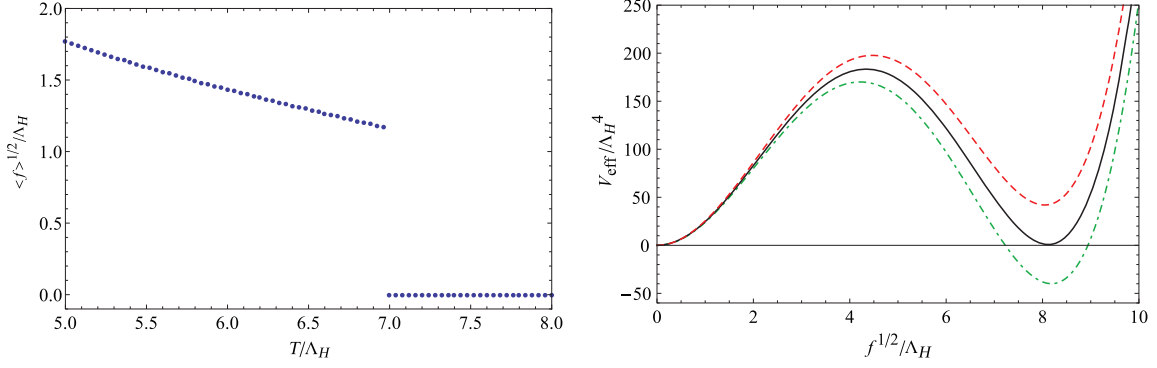


Fig. 2. Left: The scale phase transition for case (i), in which the hidden sector is disconnected from the SM. The (dimensionless) critical temperature is $T_S/\Lambda_H \simeq 7.0$. Right: The (dimensionless) potential $V_{\text{eff}}/\Lambda_H^4$ against $f^{1/2}/\Lambda_H$ for $T/\Lambda_H = 7.1$ (red dashed), T_S/Λ_H (black), and 6.9 (green dash-dotted). The potential energy density at the origin is subtracted from V_{eff} so that the form of the potential for different temperatures can be compared.

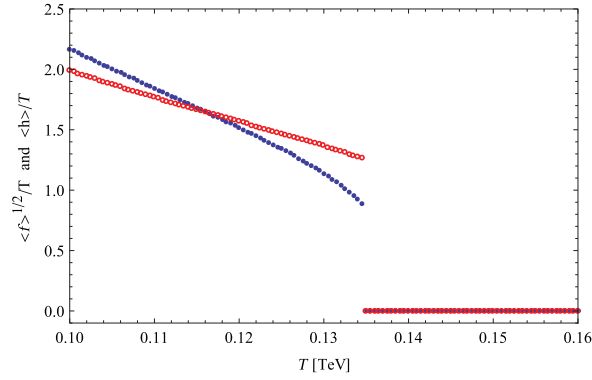


Fig. 3. The scale and EW phase transitions for case (ii) with the critical temperature $T_C \equiv T_S = T_{\text{EW}} \simeq 0.135$ TeV. The phase transitions are both of a strong first order. The red circles stand for $\langle f \rangle^{1/2}/T$ and the blue points are for $\langle h \rangle/T$.

$T > T_S$. We discard this problem here, because we assume that the local $SU(N_c)$ gauge symmetry of (1) remains unbroken even at $T > T_S$.

(ii) Scale and EW phase transitions at $T_C \equiv T_S = T_{\text{EW}}$

Now we couple the hidden sector with the SM sector. We use the same parameter values as those given in (38) along with

$$\lambda_{HS} = 0.296, \quad \lambda_H = 0.208. \quad (39)$$

The input parameters (38) with (39) yield $M = 0.410$ TeV, $m_\sigma = 0.796$ TeV, $\Lambda_H = 0.019$ TeV, and $m_h = 0.125$ TeV.⁶ In Fig. 3 we show $\langle f \rangle^{1/2}/T$ (red) and $\langle h \rangle/T$ (blue) against T , and we can see that the scale and EW phase transitions occur at the same critical temperature $T_C \equiv T_S = T_{\text{EW}} \simeq 0.135$ TeV, where the dimensionless critical temperature $T_C/\Lambda_H \simeq 7.0$ is basically the same as that of case (i) with the SM decoupled. This shows that the strong first-order scale phase transition in the hidden sector can indeed infect the EW phase transition.

⁶ Due to a relatively large λ_{HS} there is a relatively large mixing between σ and the Higgs h with a mixing angle of ~ 0.2 , which is still consistent with the LHC constraint at 95% CL [97]. This mixing has a negative effect on m_h , leading to a large λ_H .

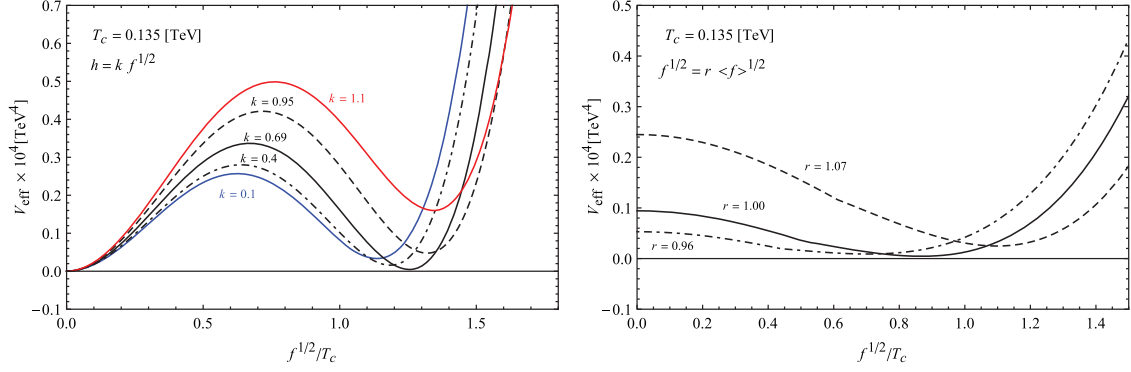


Fig. 4. The form of the potential at $T = T_C$ for case (ii), where the potential energy density at the origin is subtracted from V_{eff} . Left: The potential as a function of $f^{1/2}/T_C$ on the line $h = k f^{1/2}$ in the $f^{1/2}$ – h plane with $k = 1.1$ (red), $k = 0.95$ (black dashed), $k = 0.69$ (black), $k = 0.4$ (black dash-dotted), and $k = 0.1$ (blue). Right: The potential as a function of h/T_C for $f^{1/2}$ fixed at $1.07\langle f \rangle^{1/2} \simeq 1.34T_C$ (dashed), $1.00\langle f \rangle^{1/2} \simeq 1.25T_C$ (black), and $0.96\langle f \rangle^{1/2} \simeq 1.20T_C$ (dash-dotted). The curve with $k = 0.69$ (left) and that with $r = 1.0$ (right) on the potential surface go through the nontrivial potential minimum.

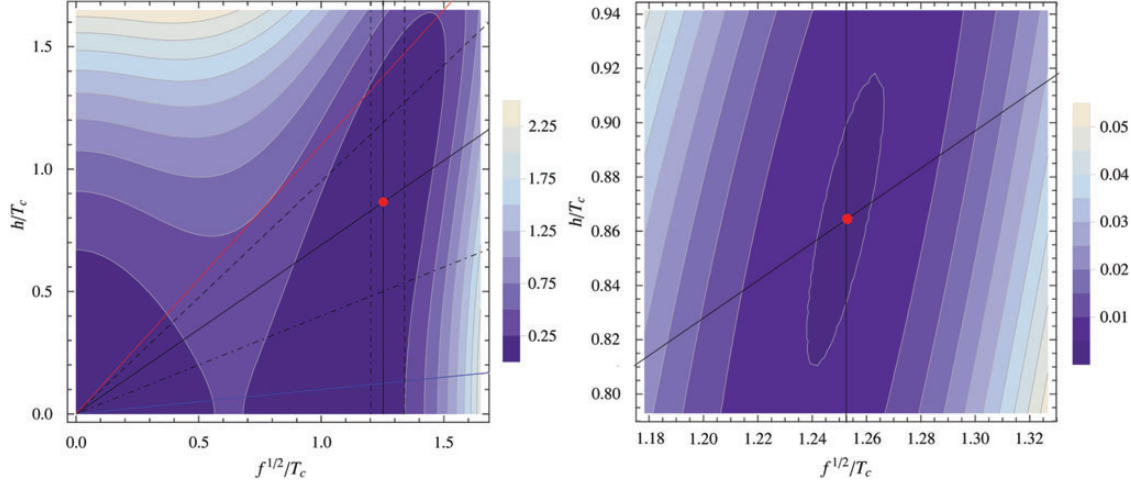


Fig. 5. The lines in the $f^{1/2}/T_C$ – h/T_C plane on which the potential values are computed and plotted in Fig. 4. Two black lines go through the nontrivial potential minimum as one can see from Fig. 4. The intersection of these two black solid lines in Fig. 5 is the location of the nontrivial potential minimum at $T = T_C$, which is marked with a red point. The darker the color, the deeper the depth of the potential.

We next show the form of the potential at $T = T_C$. The curves in Fig. 4 (left) are the intersections of the potential V_{eff} with the surfaces defined by

$$0 = h - k f^{1/2} \quad (40)$$

for $k = 1.1$ (red), $k = 0.95$ (black dashed), $k = 0.69$ (black), $k = 0.4$ (black dash-dotted), and $k = 0.1$ (blue), where their intersections with the $f^{1/2}/T_C$ – h/T_C plane are shown in Fig. 5. That is, Fig. 4 (left) shows the potential values on the inclined lines in Fig. 5 as a function of $f^{1/2}/T_C$. The potential minimum for $T = T_C$ is located at the origin and at $f^{1/2}/T_C \simeq 1.25$ with $k \simeq 0.69$. Since $T_C \simeq 0.135$ TeV we obtain $\langle f \rangle^{1/2} \simeq 0.169$ TeV and $\langle h \rangle \simeq 0.117$ TeV. Figure 4 (right) shows the potential as a function of h/T_C for $f^{1/2}$ fixed at $1.07\langle f \rangle^{1/2} \simeq 1.34T_C$ (dashed), $1.00\langle f \rangle^{1/2} \simeq 1.25T_C$ (black), and $0.96\langle f \rangle^{1/2} \simeq 1.20T_C$ (dash-dotted), where these fixed values define the vertical lines shown in Fig. 5. The intersection of the two black solid lines in Fig. 5 is

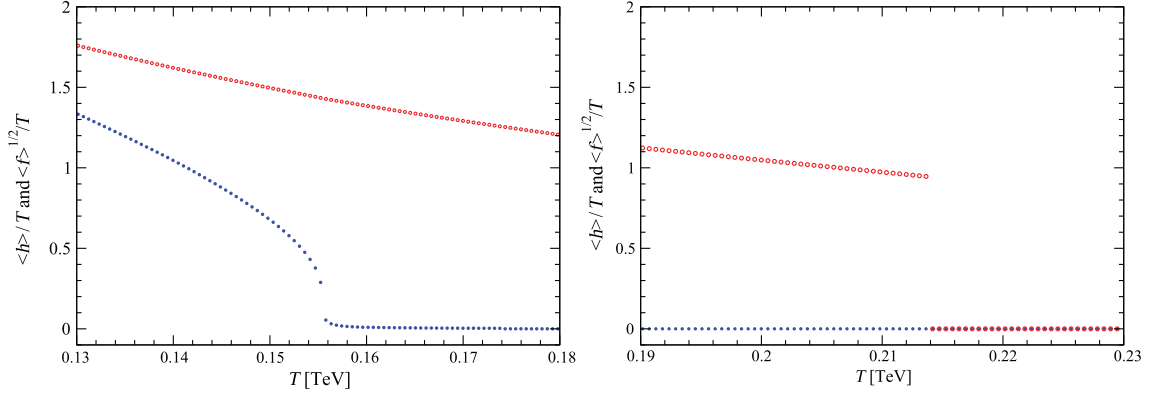


Fig. 6. The scale and EW phase transitions for case (iii), in which $T_S > T_{EW}$ is realized. The red circles stand for $\langle f \rangle^{1/2}/T$, while the blue points are for $\langle h \rangle/T$. The difference in the two figures is the temperature interval. The critical temperatures are, respectively, $T_{EW} \simeq 0.155$ TeV and $T_S \simeq 0.214$ TeV.

the location of the potential minimum (other than the origin) at $T = T_C$, which is marked with a red point. We have computed the potential not only on the lines shown in Fig. 5, but also for the range $0 < f^{1/2}/T_C < 15$, $0 < h/T_C < 15$, and found that there is no other point for a minimum in this range.

(iii) Scale and EW phase transitions with $T_S \gtrsim T_{EW}$

The third example is $N_f = 2$ and $N_c = 6$ along with

$$\lambda_S = 0.165, \quad \lambda'_S = 2.295, \quad \lambda_{HS} = 0.086, \quad \lambda_H = 0.155. \quad (41)$$

These input parameters yield $M = 0.533$ TeV, $m_{DM} = 0.676$ TeV, $m_\sigma = 0.989$ TeV, $\Lambda_H = 0.055$ TeV, $\Omega \hat{h}^2 = 0.119$, and $\sigma_{SI} = 5.76 \times 10^{-45} \text{ cm}^2$. In Fig. 6 we show $\langle f \rangle^{1/2}/T$ (red circles) and $\langle h \rangle/T$ (blue) against T . For the left figure the temperature T varies between 0.13 TeV and 0.18 TeV, while $0.19 \text{ TeV} \lesssim T \lesssim 0.23 \text{ TeV}$ for the right figure. We see from these figures that the critical temperatures are, respectively, $T_{EW} \simeq 0.155$ TeV and $T_S \simeq 0.214$ TeV, and that the nature of the two phase transitions are different: the scale phase transition is clearly first order, while the nature of the EW phase transition is indefinite.

We would like to emphasize that our results are based on the effective theory approach. A more accurate calculation based on lattice simulation could alter the result. If our observation here turns out to be correct, the EW scalegenesis from the condensation of the scalar bilinear in a hidden sector may be an alternative way to realize a strong first-order EW phase transition.

6. Summary

We have considered the SM without the Higgs mass term, which is coupled through a Higgs portal term, the last term of (1), with a classically scale invariant hidden sector. The hidden sector is an SM-singlet and described by an $SU(N_c)$ gauge theory with N_f scalar fields. At lower energies the hidden sector becomes strongly interacting, and consequently the gauge-invariant scalar bilinear forms a condensate (3), thereby violating scale invariance and dynamically creating a robust energy scale. This scale is transmitted through the Higgs portal term to the SM sector, realizing EW scalegenesis. Moreover, the excitation of the condensate can be identified with the DM degrees of freedom, which are consistent with the present experimental observations [59].

The nature of the scale phase transition in a non-abelian gauge theory is not yet known. By the scale phase transition we mean a transition between phases with a zero and nonzero condensates of the

scalar bilinear. We have addressed this problem by means of an effective theory for the condensation of the scalar bilinear. Since the EW scale is (indirectly) created in the hidden sector, it is expected that there exists a nontrivial interplay between the EW and scale phase transitions. We have indeed confirmed this expectation and found that there exists a parameter space in our model in which both the EW and scale phase transitions can be a strong first-order phase transition. This is not the final conclusion, because our result is based on the mean field approximation in the effective theory. A more accurate calculation could change this result. It is well known that a strong first-order phase transition in the early Universe can produce gravitational wave background [93,94], which could be observed by future experiments such as the Evolved Laser Interferometer Space Antenna (eLISA) experiment [95,96]. In our scenario there can exist two strong first-order phase transitions, whose critical temperatures lie close to each other.

The nature of the EW symmetry breaking is crucial for baryon asymmetry in the Universe [68–71]. For a successful EW baryogenesis, there have to exist CP phases other than that of the SM. Unfortunately, there is no such phase in our model as it stands. We will come to an extension of the model so as to realize a successful EW baryogenesis elsewhere.

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