BEAM DYNAMICS STUDIES OF THE TRANSVERSE GRADIENT UNDULATOR AND ITS APPLICATION TO SUPPRESSION OF MICROBUNCHING INSTABILITY

Tao Liu,[1,2*] Weilun Qin,[1] Yuantao Ding,[1] Dong Wang,[2] Zhirong Huang[1†]
[1]SLAC National Laboratory, Menlo Park, CA 94025, USA
[2]Shanghai Institute of Applied Physics, Shanghai 201800, China

Abstract

A transverse gradient undulator (TGU) which was initially proposed for high gain free electron lasers (FELs) driven by electron beams with relatively large energy spread, can be extended to the application of beam dynamics, such as phase-merging enhanced harmonic generation FEL and suppression of microbunching instability. In this contribution we present beam dynamics studies of the TGU, analyze the resulting focusing and dispersion, and discuss the effects of an additional corrector on the TGU. As an application to beam dynamics, we show a feasible transport system based on the TGU as a reversible electron beam heater to suppress the microbunching instability of the electron beam.

INTRODUCTION

The performance of a modern free-electron laser (FEL) is significantly limited by the uncorrelated energy spread of the electron beam. Typically, a large energy spread results in a substantial reduction of the FEL gain, which hinders the FEL operation on laser plasma accelerators (LPA) and storage rings. The concept of transverse gradient undulator [1, 2] was proposed to mitigate the problem of the energy spread by properly introducing a transverse dispersion for such beam, which is under test in a LPA facility [3]. Recently, a TGU is also considered for some applications to manipulate the electron beam, e.g. to perform the phase merging effect for improving the frequency up-conversion efficiency of a seeded FEL [4] and to suppress microbunching instability in a linear accelerator [5,6].

In order to gain more insight into the process of beam transport in a TGU, we study on the beam dynamics of a TGU. For an FEL based on TGU, there are two actions concerned here. One is to maintain the TGU based FEL resonant condition and another is the natural focusing, which are different from those of a planar undulator. Due to the dispersion induced by a TGU, it is also possible to apply it into a reversible heating system for suppressing the microbunching instability by destroying the energy and density modulation in a beam, which leads to less bunching generation and accumulation. For the reversible system, an increase of the uncorrelated energy spread and a dilution of the transverse emittance should be avoided.

Based on the magnetic field of a TGU, we derive the motion equation and the transport matrix, analyze the focusing, dispersion and so on. We show the effects of beam dynamics on a TGU based FEL, e.g. the magnetic gradient of the attached corrector. As a typical application of the beam dynamics, we present a reversible heater case to suppress microbunching instability based on TGUs.

THEORY OF MOTION IN A TGU

To begin with, we give the magnetic field of a TGU associated with a correcting coil and analyze the transverse motion. The magnetic field is given in Ref. [7], as shown in the following

$$B_{ux} = B_0 \frac{\alpha}{k_u} \sinh(k_u y) \sin(k_u z)$$
$$B_{uy} = B_0(1 + \alpha x) \cosh(k_u y) \sin(k_u z)$$
$$B_{uz} = B_0(1 + \alpha x) \sinh(k_u y) \cos(k_u z). \quad (1)$$

Here B_0 is the on-axis peak field, $k_u = 2\pi/\lambda_u$ with λ_u the undulator period and α is the transverse field gradient. Here we can introduce the following relations given by

$$K_0 = \frac{eB_0}{mck_u} \quad \text{and} \quad \eta_\alpha = \frac{2 + K_0^2}{\alpha K_0^2}, \quad (2)$$

where the former is the on-axis undulator parameter and the latter is the TGU resonant dispersion based on FEL resonant condition. An additional correcting coil, also called corrector, is used to correct the deflection of a TGU and even keep dispersion constant along the TGU. It is assumed as a magnetic field B_c with a gradient α_c given by

$$B_{cx} = B_c \alpha_c y$$
$$B_{cy} = B_c(1 + \alpha_c x)$$
$$B_{cz} = 0. \quad (3)$$

To take into account the horizontal motion without radiation damping, we have the motion equation governed by Lorentz Force Law

$$\frac{d(\gamma m v_x)}{dt} = \gamma m v_z^2 x'' = -e(v_y B_z - v_z B_y). \quad (4)$$

For simplicity, we focus on the plane of $y = 0$ and have the approximations of $v_z \approx c$ and $v_y = cy'$. The equation of motion can be written as

$$x'' = \frac{e}{\gamma mc}[B_0(1 + \alpha x)\sin(k_u z) + B_c(1 + \alpha_c x)]. \quad (5)$$

Similar as the planar undulator, the horizontal motion of the electron can be decomposed into a fast wiggle oscillatory motion and a slow betatron motion i.e. $x = x_u + \bar{x}$. Generally, we have to solve the wiggle motion before the betatron motion, so we can obtain the wiggle motion as

$$x_u'' \approx \frac{k_u K_0}{\gamma}(1 + \alpha \bar{x}) \sin(k_u z), \qquad (6)$$

by which x_u' and x_u can be given as

$$x_u' = -\frac{K_0}{\gamma}(1 + \alpha \bar{x}) \cos(k_u z) \qquad (7)$$

and

$$x_u = -\frac{K_0}{\gamma k_u}(1 + \alpha \bar{x}) \sin(k_u z). \qquad (8)$$

Relying on the wiggle motions x_u and x_u' given, we can average Eq. (5) over one undulator period to solve the betatron motion $\bar{x}$. Introducing an energy deviation δ with $\gamma = \gamma_0(1 + \delta)$ and expanding up to the second order, we can remove the TGU deflection term by choosing $B_c = \alpha K_0 B_0/2\gamma_0 k_u$ and define $\eta_c = 1/(\alpha - \alpha_c)$ to obtain the equation

$$\bar{x}'' = -\Delta_\delta^2 k_x^2 \left[\bar{x} + \frac{\eta_c(\delta - 2\delta^2)}{1 - (1 + \alpha\eta_c)\delta} \right] \qquad (9)$$

with the abbreviations

$$\Delta_\delta = \sqrt{1 - (1 + \alpha\eta_c)\delta} \quad \text{and} \quad k_x = \sqrt{\alpha K_0^2/2\gamma_0^2\eta_c}.$$

Consequently, the expressions of $\bar{x}$ and $\bar{x}'$ can be calculated. Further we rewrite them in the form of the matrix elements given by

$$\begin{aligned}
\bar{x} =\ & \bar{x}_0 \cos(k_x z) + \bar{x}_0' \sin(k_x z)/k_x + \delta\eta_c[1 - \cos(k_x z)] \\
& + \bar{x}_0 \delta(1 + \alpha\eta_c)k_x z \sin(k_x z)/2 \\
& + \bar{x}_0' \delta(1 + \alpha\eta_c)[\sin(k_x z)/k_x - z \cos(k_x z)]/2 \\
& + \delta^2[\alpha_c\eta_c^2(1 - \cos(k_x z)) - (1 + \alpha\eta_c)\eta_c k_x z \sin(k_x z)/2]
\end{aligned} \qquad (10)$$

$$\begin{aligned}
\bar{x}' =\ & -\bar{x}_0 k_x \sin(k_x z) + \bar{x}_0' \cos(k_x z) + \delta\eta_c k_x \sin(k_x z) \\
& + \bar{x}_0 \delta(1 + \alpha\eta_c)k_x[\sin(k_x z) + k_x z \cos(k_x z)]/2 \\
& + \bar{x}_0' \delta(1 + \alpha\eta_c)k_x z \sin(k_x z)/2 \\
& + \delta^2[(\alpha\eta_c - 3)\sin(k_x z) - k_x z(1 + \alpha\eta_c)\cos(k_x z)]\eta_c k_x/2.
\end{aligned} \qquad (11)$$

With the analogous method, we give the equations of the betatron motion in the vertical direction

$$\begin{aligned}
\bar{y} =\ & \bar{y}_0 \cos(k_y z) + \bar{y}_0' \sin(k_y z)/k_y + \bar{y}_0 \delta k_y z \sin(k_y z)AB/2 \\
& + \bar{y}_0' \delta[\sin(k_y z)/k_y - z \cos(k_y z)]AB/2
\end{aligned} \qquad (12)$$

$$\begin{aligned}
\bar{y}' =\ & -\bar{y}_0 k_y \sin(k_y z) + \bar{y}_0' \cos(k_y z) \\
& + \bar{y}_0 \delta k_y[\sin(k_y z) + k_y z \cos(k_y z)]AB/2 \\
& + \bar{y}_0' \delta k_y z \sin(k_y z)AB/2
\end{aligned} \qquad (13)$$

with the abbreviations $k_y^2 = K_0^2/2A\gamma_0^2$, $1/A = k_u^2 + \alpha^2 + \alpha\alpha_c$ and $B = 2k_u^2 + 2\alpha^2 + \alpha\alpha_c$.

By means of the transverse motion we have the longitudinal component of the relativistic velocity given by $v_s/c = \sqrt{1 - 1/\gamma - (x_u' + \bar{x}')^2 - (y_u' + \bar{y}')^2}$. Compared to the transverse betatron motion, the velocity variation is dominated by the transverse wiggle motion. As a consequence, by inserting the transverse wiggle motion into the longitudinal velocity equation, averaging over one undulator period and retaining the linear terms in x and y, we obtain

$$\frac{\bar{v}_s}{c} \approx 1 - \frac{1 + K_0^2/2}{2\gamma_0^2} - \frac{\alpha K_0^2}{2\gamma_0^2}\bar{x} + \frac{1 + K_0^2/2}{\gamma_0^2}\delta. \qquad (14)$$

Substitute the transverse betatron motion into the velocity above and we can obtain the equation of the relative longitudinal displacement

$$\begin{aligned}
\Delta\bar{s} =\ & -\bar{x}_0 k_x \eta_c \sin(k_x z) - \bar{x}_0' \eta_c[1 - \cos(k_x z)] + \\
& \delta[k_x^2 \eta_c \eta_\alpha z - \eta_x^2 k_x(k_x z - \sin(k_x z))] - \\
& \bar{x}_0 \delta(1 + \alpha\eta_c)\eta_c k_x[\sin(k_x z) - k_x z \cos(k_x z)]/2 - \\
& \bar{x}_0' \delta(1 + \alpha\eta_c)\eta_c[2 - 2\cos(k_x z) - k_x z \sin(k_x z)]/2 - \\
& \delta^2 k_x \eta_c^2[2\alpha_c \eta_c k_x z + (1 + \alpha\eta_c)k_x z \cos(k_x z) + (1 - 3\alpha\eta_c)\sin(k_x z)].
\end{aligned} \qquad (15)$$

As a conclusion above, we have listed the dominant terms of the beam dynamics in a TGU including the first order terms and the fractional second order terms. The first order transport matrix can be given as

$$R_T = \begin{pmatrix}
\cos(k_x z) & \frac{1}{k_x}\sin(k_x z) & 0 \\
-k_x \sin(k_x z) & \cos(k_x z) & 0 \\
0 & 0 & \cos(k_y z) \\
f0 & 0 & -k_y \sin(k_y z) \\
-\eta_c k_x \sin(k_x z) & -\eta_c(1 - \cos(k_x z)) & 0 \\
0 & 0 & 0
\end{pmatrix}$$

$$\begin{pmatrix}
0 & 0 & \eta_c(1 - \cos(k_x z)) \\
0 & 0 & \eta_c k_x \sin(k_x z) \\
\frac{1}{k_y}\sin(k_y z) & 0 & 0 \\
\cos(k_y z) & 0 & 0 \\
0 & 1 & k_x^2 \eta_x \eta_\alpha z - \eta_x^2 k_x(k_x - \sin(k_x z)) \\
0 & 0 & 1
\end{pmatrix}. \qquad (16)$$

TRANSPORT FOR A TGU BASED FEL

According to the beam dynamical analysis above we consider the first order matrix to study the beam transport in a TGU for the realization of FEL.

As seen from Eq. (16), both the horizontal and vertical natural focusing exist and the vertical one is relatively larger. In the horizontal direction it also includes a dispersion term and we can obtain two singular points about transport. One of these is the focusing point that when the focusing strength $k_x = 0$ under the condition of $\alpha_c = \alpha$, the horizontal focusing disappears and it can be treated as a drift space, while meantime the dispersion term is still related to z that will be changed along the undulator. Another one is the dispersion point that is on the condition of $\alpha_c = \alpha - 1/\eta$, i.e.

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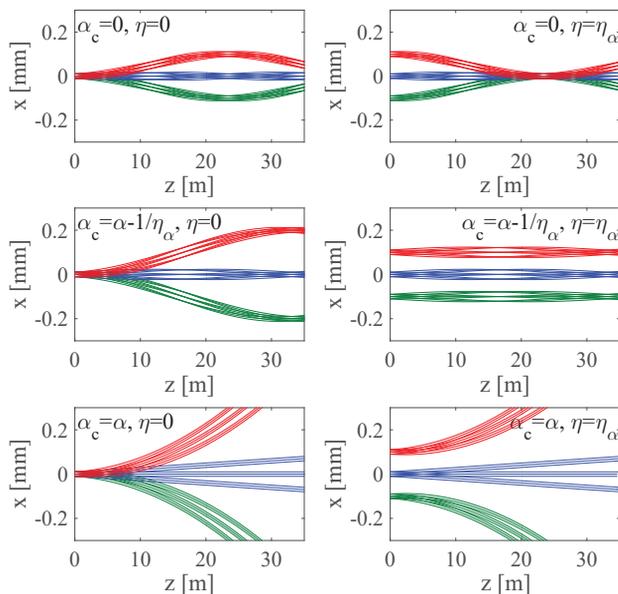
Figure 1: Tracking of the particle transport in a TGU based on correcter gradient and beam dispersion. The color (green, blue, red) corresponds to energy ($-\delta$, 0, δ).

Figure 2: Layout of the proposed reversible beam heater based on TGUs. The first TGU (T1) is located between the first and the second RF cavities (L1, L2) and the second one (T2) is located downstream of the bunch compressor (BC). Several quadupoles (Q1 - Q4), a harmonic cavity (LX) and a sextupole (S1) are included.

$\eta_c = \eta$, in which η is the initial dispersion of the beam at the entrance of the TGU. Although a horizontal focusing retained, the dispersion η is kept constant in the TGU. As we know, to satisfy the FEL resonant condition of a TGU, the dispersion of the beam should equal the one of the TGU, i.e. $\eta = \eta_\alpha$, therefore under these conditions a consistent resonant dispersion of the beam can be maintained to realize the FEL using the beam with a large energy spread. As shown in Fig. 1, we change the TGU corrector gradient α_c and the beam dispersion η to track the particle trajectory in the TGU. With $\alpha_c = 0$ the beam is focused and has an active oscillation depending on the energy deviation, where the constant dispersion point does not satisfy the resonant dispersion of the beam resulting in a dispersion variation. When $\alpha_c = \alpha - 1/\eta$ still with focusing and active oscillation, the constant dispersion right equals the beam resonant dispersion to maintain the resonant condition for the TGU based FEL. While the corrector gradient equals the TGU gradient, that means defocusing and dispersion is always increased.

APPLICATION TO SUPPRESSION OF MBI

In the previous sections, we have studied the beam dynamics in a TGU and the requirement for a TGU based FEL. Now we consider its applications to manipulate the beam phase space, such as suppression of microbunching instability.

In Ref. [8], we proposed a reversible beam heater based on the dispersion property of TGUs without a rotation of the beam line as shown in Fig. 2. Due to the first energy chirp in L1 and the following dispersion in T1, the beam is heated with an additional uncorrelated energy spread and transverse-to-longitudinal coupling, lenfing to a suppression of the microbunching instability. Finally the second TGU
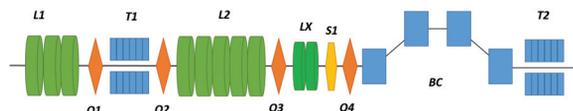
reverses the heated energy spread and the diluted transverse emittance, but also leaks out a coupling to suppress the initial modulation. Therefore both the initial and internal collective effects driven microbunching instabilities are suppressed. With regard to the TGU, the horizontal focusing can be ignored by the choice of $\alpha_c = \alpha$, while the vertical focusing is relatively large due to the requirement of an appropriate dispersion. In addition, the high order effects of the TGU should be taken into account in the case of the reversible system where the nonlinear effect plays an important role in the increased energy spread and transverse emittance. In Eq. (10, 11, 15), the main second order terms have been derived and the deviation from the first order are also shown in Fig. 3. For comparison, the much higher transport is also presented. Consequently, the whole scheme design including several quadrupoles, a harmonic cavity and a sextupole is for the optimization of the beam optics up to the second order. As a result, we can reverse the increased energy spread with a scale of compression factor, and the core transverse emittance growth is less than 6 percent, which is almost completely removed at the exit from the reversible heater.

SUMMARY

In this paper, we studied the beam dynamics of the TGU, gave a relatively exact transport matrix with the first order and part of the second order terms. From the transport matrix, TGU has the properties of focusing and dispersion similar as a dipole but without rotating the beam line. The additional corrector of the TGU effects on its focusing and dispersion that requires us adjust the angle of the corrector and the beam dispersion to maintain the resonant relation of a TGU based FEL. As an application to beam dynamics, it
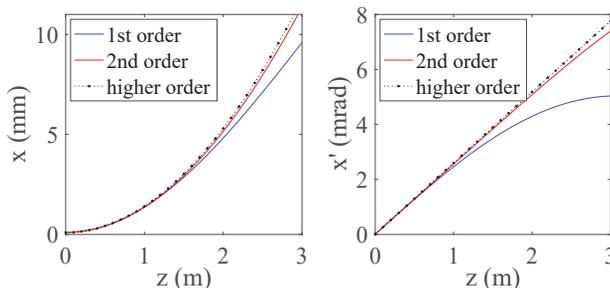
Figure 3: Effects of high order on the horizontal betatron motion in a TGU.

can be used to suppress the microbunching instability in a reversible heating system with an advantage of keeping the beam line straight.

ACKNOWLEDGMENTS

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