PLANCK SCALE EFFECTS IN UNRUH RADIATION

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In this work, we consider the problem of Unruh effect in the framework of Generalized Uncertainty Principle (GUP) of Quantum Gravity. The quantum gravitational effects at the Planck scale physics, as a consequence of GUP, induces the corrections to the Unruh radiation. In this set-up, we find an energy-dependent effective temperature which leads to a non-thermal emission in the Unruh radiation spectrum.

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1. Introduction

The theory of quantum fields in curved space-times includes many attractive and noticeable phenomena. One of them being the radiation that arises from placing a quantum field in a background metric with a horizon. The Hawking radiation is the first significant instance of this effect that predicts the evaporation of black holes [1]. Another instance is Unruh radiation and affirms that an observer encountering a uniform acceleration experiences the Minkowski vacuum as a thermal state [2]. There are profound links between Hawking radiation and the Unruh effect via the principle of equivalence. In fact, a freely falling observer is locally equivalent to an inertial observer in flat space-time and also a stationary observer near a black hole is locally comparable to an accelerated observer in Minkowski space-time. Considering the close connection of black hole radiation and acceleration radiation it is justifiable to presume that some of the hardships concerning the Hawking effect could be reflected on the simpler problem of the Unruh effect.

For both Hawking and Unruh effects, temperature comes into view from information loss allied with true and accelerated-observer horizons, respectively. Recently, the authors in Ref. [3] have utilized the WKB/tunnelling

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formalism of calculating Hawking temperature to acquire a possible resolution to the information loss paradox taking into account both quantum corrections to all orders in $\hbar$ and back reaction. They have shown that the quantum corrections alone cannot solve the information loss paradox. However, by taking both quantum corrections and back reaction into consideration it may be possible under particular conditions to solve the information paradox by possessing the black hole evaporate entirely with the information leaked out by the correlations of the outgoing radiation. The tunnelling is generally applied through two different approaches, the radial null geodesic approach [4] and the Hamilton–Jacobi approach [5]. The primary work on the tunnelling formalism was incapable to achieve the thermal essence of the Hawking radiation. In 2009, Banerjee and Majhi [6] have explained this shortcoming by acquiring the thermal spectrum for Hawking radiation utilizing the tunnelling perspective. The results of Ref. [3] shows that the quantum corrections alone cannot change the thermal nature of the Hawking black body spectrum, *i.e.* to all orders in $\hbar$ the spectrum still remains thermal. Hence, quantum corrections by themselves cannot find a solution to the problem of lost information. Corrections to the Hawking temperature stimulates the interesting question as to whether the Unruh temperature would also have suitable corrections. Quantum corrections to the Unruh temperature, by using a WKB approximation within a Hamilton–Jacobi analysis in the tunnelling perspective, to all orders in $\hbar$ have computed similarly and a connection of the corrected Unruh temperature with the corrected Hawking temperature has established in Ref. [7]. Thereby, it is natural to ask whether these connections can clarify and unify the two Hawking and Unruh effects.

The resulting anticipation of different models of Quantum Gravity, *e.g.* String Theory [8], Non-commutative Quantum Theory [9], Loop Quantum Gravity [10], Black Hole Physics [11], is the impression of a minimal observable distance on the order of the Planck length that cannot be inspected, for example in String Theory there exists a constraint to inspect the distances smaller than the string length. Thereby, Heisenberg Uncertainty Principle (HUP) should be modified to comprise this confined resolution of the space-time structure. The consequence of this modification is the so-called Generalized Uncertainty Principle (GUP) which indeed has the origin on the quantum fluctuations of the space-time at Planck scale and can be formulated in a more practical form as follows [8, 9, 10, 11, 12]

$$\Delta x \geq \frac{\hbar}{\Delta p} + \frac{\alpha l_{\mathrm{P}}^2}{\hbar}\Delta p\,, \tag{1}$$

where α is a dimensionless constant of the order of unity that depends on the details of the Quantum Gravity proposals. In the standard limit, $\Delta x \gg l_{\mathrm{P}}$,

it yields the HUP, $\Delta x \Delta p \geq \hbar$. The second term in r.h.s. of GUP relation plays an important role when the momentum and distance scales are in the vicinity of the Planck scale. In a heuristic way, by using the HUP, the thermodynamic quantities for a spherical black hole can be obtained [13]. Also, the application of the GUP to black hole thermodynamics in the same way, modifies the consequences by inclusion of Quantum Gravity influences at the ultimate phases of evaporation process with a rich phenomenology [14]. Lately, the modifications of the Hawking radiation via the GUP and the tunnelling process has investigated by the authors of Refs. [15,16]. By using the GUP-corrected de Broglie wavelength, the squeezing of the fundamental momentum cell, and accordingly a GUP-corrected energy, they found the non-thermal effects in the black hole radiation spectrum. These features of the Planck-scale corrections are capable to clarify the information problem in black hole evaporation. In fact, information can be recovered as non-thermal GUP correlations between tunnelling probabilities of different modes [15,16].

In this article, we study the thermal properties of Unruh radiation including the corrections both due to quantum effects to all orders in $\hbar$ and due to gravitational uncertainty. Here, we use a WKB approximation within a Hamilton–Jacobi analysis in the tunnelling formalism to explore the quantum inspection at the level of semiclassical Quantum Gravity.

2. Quantum gravitational corrections

In this section, we briefly investigate the quantum corrections to all orders in $\hbar$ in the tunnelling formalism. In this way, the GUP corrections is included to our problem. Firstly, we incorporate the minimal length scale from Quantum Gravity via the GUP which motivates modification of the standard dispersion relation [17, 18, 19]. Amelino-Camelia *et al.* [17] investigated the black hole evaporation procedure after an analysis of the GUP-induced modification of the black body radiation spectrum. If GUP is fundamental concept to Quantum Gravity, it should emerge in de Broglie relation as follows

$$\lambda \simeq \frac{\hbar}{p}\left[1+\alpha\left(\frac{l_{\mathrm{P}}p}{\hbar}\right)^2\right], \tag{2}$$

or

$$\mathcal{E} \simeq E\left[1+\alpha\left(\frac{l_{\mathrm{P}}E}{\hbar}\right)^2\right], \tag{3}$$

where we have kept only the first GUP-induced term of order $O(\alpha)$. There are other convincing reasons from Non-commutative Geometry and Loop Quantum Gravity that support these relations (see for instance [18,20] and references therein).

We are now ready to commence the Unruh effect by considering the specific form (Schwarzschild-like) of the Rindler metric

$$ds^2 = -(1+2ax)dt^2 + (1+2ax)^{-1}dx^2 + dy^2 + dz^2\,, \tag{4}$$

where a is the constant acceleration in the instantaneous rest frame of the Rindler observer. This kind of the metric is more appropriate because it has the suitable coordinates system for the accelerated observer which cover the entire Minkowski plane in four separate coordinate pieces. In addition, the determinant of the metric is 1 everywhere. These coordinates have been used in Hamilton–Jacobi method to study a thermal spectrum of particles in the traditional Minkowski vacuum state. Consider the massless scalar field ϕ in the Rindler metric, which obey the Klein–Gordon equation

$$-\frac{\hbar^2}{\sqrt{-g}}\partial_\mu\left(g^{\mu\nu}\sqrt{-g}\partial_\nu\right)\phi = 0\,. \tag{5}$$

In the tunnelling formalism we are concerned about the radial path, so that only the $x-t$ sector of the metric (4) is relevant. For a two-dimensional theory, particle's tunnelling from a black hole can be considered as a two-dimensional quantum procedure in the x–t sector. As can be seen from Eq. (5), this equation cannot be solved exactly, therefore we express the standard WKB ansatz for the wave function ϕ as

$$\phi(x,t) = \exp\left[\frac{i}{\hbar}I(x,t)\right]. \tag{6}$$

The single particle action $I(x,t)$ can be expanded in powers of $\hbar$ as

$$I(x,t) = I_0(x,t) + \sum_{j=1}^{\infty}\hbar^j I_j(x,t)\,, \tag{7}$$

where $I_0(x,t)$ is the semiclassical action and the other terms are quantum corrections. According to the method of which has been developed to investigate quantum corrections to all orders in $\hbar$ in the tunnelling approach [21], $I_j(x,t)$ are proportional to $I_0(x,t)$, we have

$$I(x,t) = I_0(x,t)\left(1+\sum_{j=1}^{\infty}\gamma_j\hbar^j\right)\,, \tag{8}$$

where γ_js are proportionality constants and have the dimension of inverse of $\hbar^j$. Since one sets the units as $G = c = k_B = 1$, therefore the Planck

constant $\hbar$ is of the order of square of the Planck length l_{P}. The solution of the semiclassical action is given by

$$I_0(x,t) = \mathcal{E}\left(t \pm \int_0^x \frac{dx}{1+2ax}\right), \tag{9}$$

where $\mathcal{E}$ is the GUP-corrected energy of the particle. The $-(+)$ sign indicates to outgoing (ingoing) paths. Utilizing some basic dimensional analysis it can be exhibited that the coefficients γ_j have dimension x_{H}^{-2j}. This leads to

$$\gamma_j = \frac{\beta_j}{x_{\mathrm{H}}^{2j}}, \tag{10}$$

where β_js are dimensionless constants and $x_{\mathrm{H}} = -\frac{1}{2a}$ is the accelerated-observer horizon. Using Eqs. (8), (9), and (10), we find the wave function as

$$\phi(x,t) = \exp\left[\frac{i}{\hbar}\left(1+\sum_{j=1}^{\infty}\beta_j\frac{\hbar^j}{x_{\mathrm{H}}^{2j}}\right)\mathcal{E}\left(t \pm \int_0^x \frac{dx}{1+2ax}\right)\right]. \tag{11}$$

It should be noted that there is a problem here recognized as "factor 2 problem" [22]. Recently, in Ref. [23], a solution to this problem was prepared concerning the overlooked temporal contribution to the tunnelling amplitude. When one comprises this temporal contribution one obtains exact the correct temperature. Therefore, as shown in [23], for the tunnelling of a particle across the horizon the nature of the time coordinate changes. This alteration points out that on crossing the horizon the time coordinate t picks up an imaginary part. Thereby, the ingoing and outgoing probabilities are given by

$$P_{\mathrm{in}} = |\phi_{\mathrm{in}}|^2 = \exp\left[\frac{2}{\hbar}\left(1+\sum_{j=1}^{\infty}\beta_j\frac{\hbar^j}{x_{\mathrm{H}}^{2j}}\right)\mathcal{E}\left(\mathrm{Im}\,t + \mathrm{Im}\int_0^x \frac{dx}{1+2ax}\right)\right], \tag{12}$$

and

$$P_{\mathrm{out}} = |\phi_{\mathrm{out}}|^2 = \exp\left[\frac{2}{\hbar}\left(1+\sum_{j=1}^{\infty}\beta_j\frac{\hbar^j}{x_{\mathrm{H}}^{2j}}\right)\mathcal{E}\left(\mathrm{Im}\,t - \mathrm{Im}\int_0^x \frac{dx}{1+2ax}\right)\right]. \tag{13}$$

In the classical limit, $\hbar \to 0$, the ingoing probability has to be unity which leads to

$$\mathrm{Im}\,t = -\mathrm{Im}\int_0^x \frac{dx}{1+2ax} = -\frac{\pi}{2a}. \tag{14}$$

In accordance with the Ref. [23], the above result is exactly the imaginary part of the transformation $t \to t - i\frac{\pi}{2a}$ when one unites the two regions across the horizon. Hence, the outgoing probability can be written as

$$P_{\text{out}} = \exp\left[-\frac{2\pi}{\hbar a}\left(1 + \sum_{j=1}^{\infty} \beta_j \frac{\hbar^j}{x_{\text{H}}^{2j}}\right)\mathcal{E}\right]. \tag{15}$$

Now, via Detailed Balance Method [5] for ingoing and outgoing probabilities, we get

$$P_{\text{out}} = \exp\left[-\frac{E}{T_{\text{eff}}}\right] P_{\text{in}}\,. \tag{16}$$

These probabilities can be utilized to acquire the effective Unruh temperature as

$$T_{\text{eff}} = T_{\text{U}} \frac{\left(1 - \alpha\frac{E^2}{\hbar} + O\left(\alpha^2 E^4\right)\right)}{\left(1 + \sum\limits_{j=1}^{\infty} \beta_j \frac{\hbar^j}{x_{\text{H}}^{2j}}\right)}\,, \tag{17}$$

where $T_{\text{U}} = \frac{\hbar a}{2\pi}$ is the semiclassical Unruh temperature and other terms are corrections due to the higher order quantum effects and gravitational uncertainty. Note, that we have expanded the relation $\left(1 + \alpha\frac{E^2}{\hbar}\right)^{-1}$ with neglecting second and higher order terms of α. For all $\beta_j = 0$ and $\alpha = 0$ the effective Unruh temperature boils down to the standard form dictated by the Unruh result. As expected, due to GUP-induced term ($\alpha \neq 0$), we find an energy-dependent effective temperature and, consequently, a non-thermal spectrum. Appearance of this term originates from the gravitational uncertainty at the Planck scale, thereby accounting for the GUP effects. It can be concluded easily that once the GUP effects have been included, there is no longer any reason to expect a purely thermal flux from the static observer viewpoint when he sees the vacuum state of the freely falling observer.

3. Summary

In summary, new results for corrections to the Unruh effect are presented. We have utilized a WKB approximation within a Hamilton–Jacobi analysis in the tunnelling perspective to calculate effective Unruh temperature taking into account both quantum corrections to all orders in $\hbar$ and GUP. This effective Unruh temperature is equivalent to the usual static value plus a GUP-corrected term of order $O(\alpha)$ and an infinite power-series expansion in $\hbar$. The GUP-corrected term leads to an energy-dependent temperature and then the spectrum becomes non-thermal.

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