SPLITTING OF THE PROTON FORM FACTORS AND DIFFRACTION IN THE PROTON

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(presented by R. Hofstadter)

1. INTRODUCTION

Electron-scattering studies of the proton obtained in the last few years have been summarized recently[1]. The measurements showed that the proton form factors (F_1, F_2) were less then unity, implying a finite structure, and lay in a region in which they were approximately equal to each other at momentum transfers (q) as high as $q^2 = 9.3$ in units of squared inverse fermis. At this value of the momentum transfer the measured ratio was $F_1/F_2 = 1.23 \pm 0.20$[2]. The experiments were confined to angles larger than 60° at the highest energies then obtainable (650 MeV) because of the limitation imposed by the energy-handling ability of the 36″ (500 MeV) spectrometer. It was therefore not possible to solve for F_1 and F_2 separately at values of $q^2 > 9.3$. Several independent experiments [2, 3] indicated that the F_1 values were slightly greater than the F_2 values at the same momentum transfer, but for simplicity and ease of calculation, in the past, the ratio of form factors was usually taken to be unity.

We have now succeeded in splitting apart the two proton form factors. Because of the great interest in the proton form factors, and because our data appear to be internally consistent, we wish to present in this paper some conclusions drawn from the experimental results presented below.

2. APPARATUS AND EXPERIMENTAL DATA

We have recently put into operation a new large double-focusing magnetic spectrometer capable of analyzing electrons or other singly-charged particles up to a momentum value of 1000 MeV/c. This spectrometer is of the 180° type previously used in this laboratory and has a mean radius of curvature of 72″. We have employed this spectrometer in electron-scattering work on the proton between incident electron energies of 650 MeV and 900 MeV, and between scattering angles of 45° and 145°. A typical value of the solid angle employed in detecting electrons is 5.6×10^{-3} steradians. The higher energies (>650 MeV) have been realized by virtue of the recent extension in the length of the Stanford linear accelerator.

The 72″ spectrometer forms part of a two-target system; the second part is the 36″ spectrometer described previously. Fig. 1 is a schematic drawing of the two spectrometers in a position in which they are 120° apart. Both spectrometers are arranged so that they can be rotated independently about a common scattering center. We have taken data simultaneously with both spectrometers, usually employing the 36″ magnet at large angles, e.g. up to 145°. The 36″ spectrometer can handle scattered electrons only up to 500 MeV/c without excessive deterioration of focusing and we have used the 36″ spectrometer in these experiments only in the very safe region below 370 MeV/c. Such a procedure limits, and has limited in the past, the ability of this spectrometer in obtaining scattering data at high energies and small angles, i.e., in those circumstances where the energy of the scattered electron is high.

Čerenkov counters were used as detectors. In Fig. 2*(a)* we show a typical electron-scattering peak

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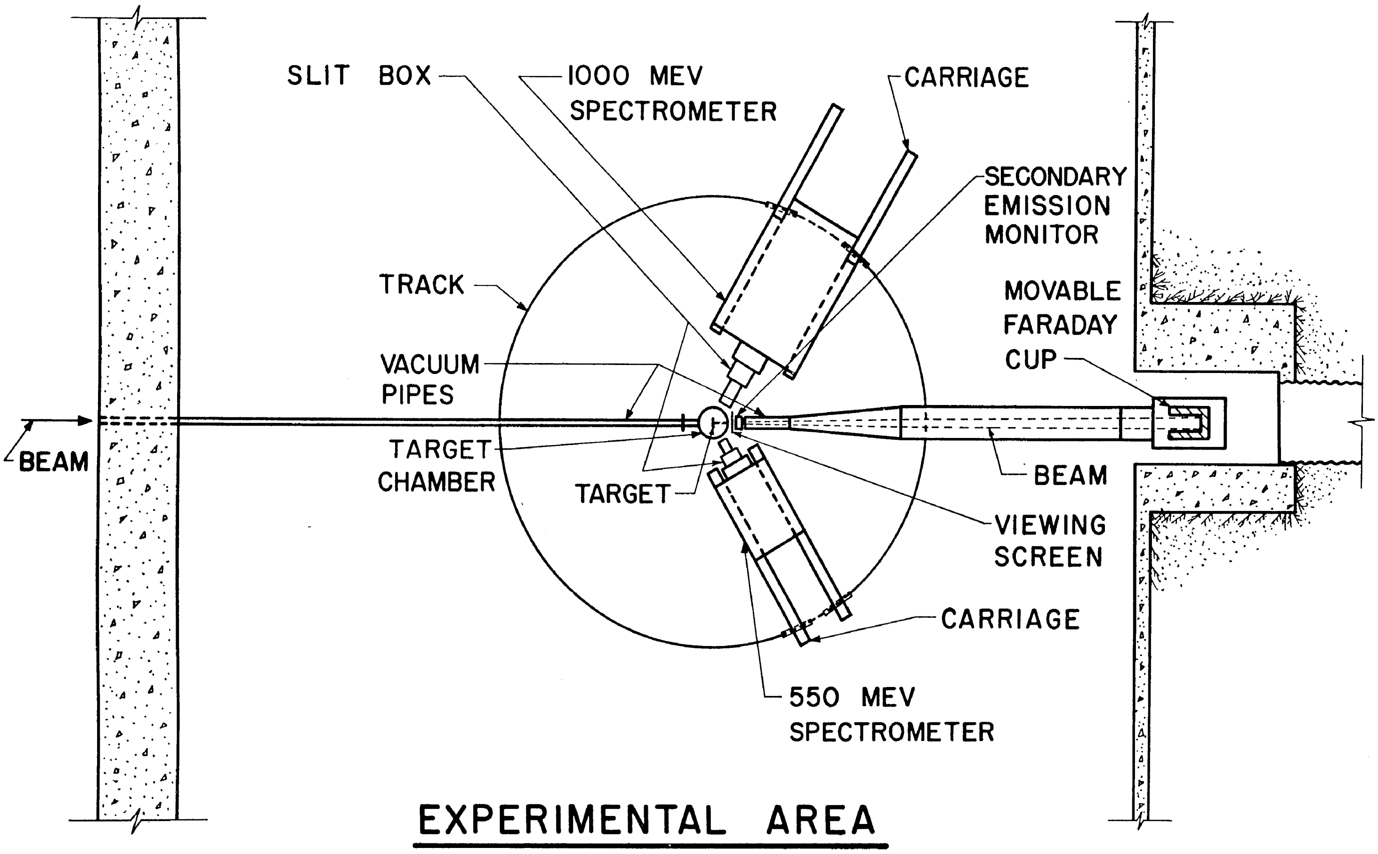
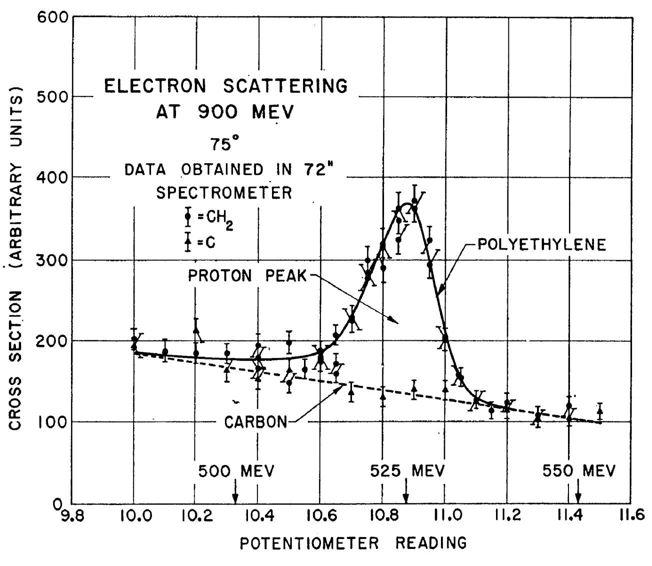


Fig. 1 The double spectrometer system used in these experiments.



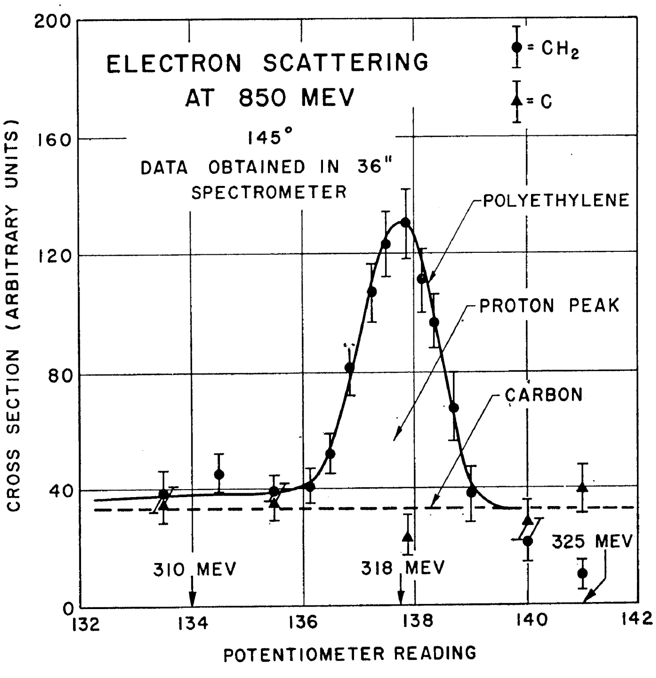


Fig. 2 *(a)* Electron-scattering peak obtained with a 0.237″ polyethylene target at an incident energy of 900 MeV and a scattering angle of 75°, with the 72″ spectrometer. The target was at a 45° with respect to the incident beam. *(b)* Same except parameters are 850 MeV, 145°, 36″ spectrometer, and target at 0°.

Table I. Electron-proton scattering cross sections.

E MeV	θ Degrees	q^2 (inverse fermi)^2	$d\sigma/d\Omega\times10^{33}$ cm^2/ster
597	60°	6.96	67.9
597	90°	11.20	8.76
597	120°	14.06	2.65
650	135°	16.97	1.51
700	60°	9.16	38.0
700	75°	12.01	11.2
700	135°	18.90	0.955
700	145°	19.43	0.778
750	45°	6.86	136.0
750	75°	13.45	10.0
750	90°	16.06	4.08
750	135°	20.86	0.728
750	141.5°	21.24	0.735
750	145°	21.42	0.580
775	135°	21.86	0.644
800	45°	7.70	104.0
800	60°	11.53	23.5
800	75°	14.93	8.19
800	90°	17.75	2.99
800	120°	21.64	0.888
800	135°	22.86	0.605
800	145°	23.44	0.360
850	45°	8.59	80.6
850	60°	12.77	17.8
850	75°	16.46	5.62
850	120°	23.60	0.711
850	135°	24.88	0.509
850	145°	25.50	0.371
875	40°	7.56	127.0
875	45°	9.05	82.0
875	60°	13.42	15.7
875	145°	26.54	0.376
900	45°	9.51	61.5
900	60°	14.06	14.3
900	75°	18.03	5.35
900	90°	21.24	2.09
900	145°	27.58	0.347

obtained with the 72″ spectrometer at an incident energy of 900 MeV and a scattering angle of 75°. The target material was polyethylene and we have subtracted the carbon background in analyzing the data to obtain the proton-scattering peak. The carbon data were obtained in the same run with a separate graphite target. In Fig. 2*(b)* we show a corresponding peak obtained in the 36″ spectrometer at 850 MeV and 145°.

The cross sections obtained in this work are absolute cross sections and are based on the readings of the Faraday cup shown in Fig. 1 which gives the number of electrons in the main beam. In our present experiments the vacuum pipe leading to the Faraday cup was not used.

Our results are presented in Table I and include standard radiative corrections. In addition to the cross sections in the Table, we give the values of the square of the momentum transfer. In many cases we have given a single value as an average of several measured cross sections. As a general rule we believe our results are accurate to about $\pm10\%$ because of the aforementioned possible errors. Relatively speaking, errors are probably less than $\pm5\%$.

3. INTERPRETATION OF RESULTS

Our procedure has been to solve for the separate form factors (F_1, F_2) at conditions lying between $7.7\leq q^2\leq25$ by choosing a pair of experimentally measured cross sections at the same value of q^2 but at different correlated values of energy and angle. We have used the method of intersecting ellipses to find the form factors.

Table II shows the values selected and the form factors found by combining the results. In a few

Table II. Proton form factors.

q^2	E_1(MeV)	θ_1	$\left(\frac{d\sigma}{d\Omega}\right)_1 \frac{\text{cm}^2}{\text{ster}}$	E_2(MeV)	θ_2	$\left(\frac{d\sigma}{d\Omega}\right)_2 \frac{\text{cm}^2}{\text{ster}}$	F_1	F_2
7.70	800	45°	1.04×10^{-31}	400	124°	*1.06×10^{-32}	0.520	0.490
9.16	700	60°	3.80×10^{-32}	464	135°	*6.26×10^{-33}	0.500	0.420
11.50	800	60°	2.35×10^{-32}	500	135°	*4.18×10^{-33}	0.451	0.341
14.06	900	60°	1.43×10^{-32}	597	120°	2.65×10^{-33}	0.423	0.214
16.97	866	75°	5.56×10^{-33}	650	135°	1.51×10^{-33}	0.430	0.160
18.03	900	75°	5.35×10^{-33}	675	135°	1.23×10^{-33}	0.451	0.108
21.24	900	90°	2.09×10^{-33}	750	141.5°	7.35×10^{-34}	0.405	0.087

cases, indicated by asterisks, we have used older data and combined the older values with the newly-measured cross section at the same value of q^2. In two cases (866 MeV, 75°; 675 MeV, 135°) we have interpolated between two newly-measured results in order to obtain properly matched pairs of cross sections.

The form factor results now show the behavior plotted in Fig. 3. The dashed line is the form factor corresponding to the exponential model and $F_1 = F_2$. Apparently our new F_2, which is seen to approach zero, indicates qualitatively that the Pauli magnetic moment cloud is a " soft ", spread-out distribution. On the other hand the constancy of F_1 suggests qualitatively that the Dirac electric and magnetic cloud has a small, perhaps point-like, core.

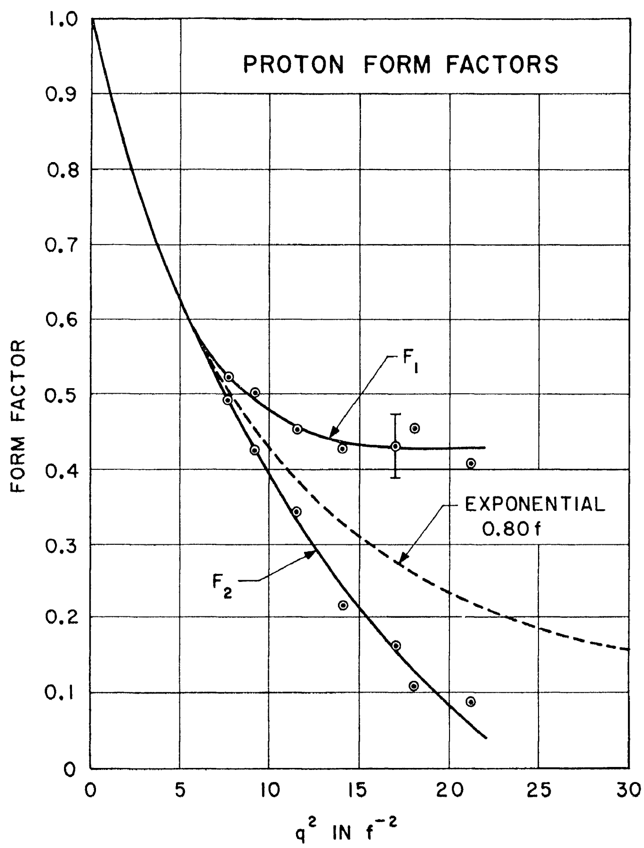


Fig. 3 The values obtained in Table II when plotted against q^2. F_2 may be approaching a diffraction zero. A typical, roughly estimated, error is shown.

The form factors found in the above manner were then put back into the well-known Rosenbluth equation [reference [1] Eq. (40)] :

$$\frac{d\sigma}{d\Omega} = \sigma_{NS}\{a_{11}F_1^2 + a_{12}F_1F_2 + a_{22}F_2^2\} \qquad (1)$$

where the values of the coefficients a_{11}, a_{12} and a_{22} are taken from the tables [1] at the appropriate energies and angles. When this is done we obtain the results shown by solid lines in Fig. 4. Notice that in Fig. 4 *(c)* the cross section appears to be going through a *diffraction dip*, so characteristic of electron-scattering studies on heavier nuclei. The experimental data indeed show this diffraction dip and we believe that this is the first time diffraction has been observed in the proton.

Within experimental error the new experimental results appear to be in agreement with the split form factor curves. It is very interesting to observe that the new form factors account for an increase of the cross section *above* the exponential case at 120°, and drift *below* the exponential case at the large angles 135° and 145°. This is what the experiments appear to indicate and the result is a rather complicated pattern of cross sections which the form factors must satisfy. The experimental data appear to fit the calculated curves for separate form factors absolutely as well as relatively.

The data are in excellent agreement with the earlier experimental results[1, 2, 3]. The measurements of a proton root-mean-square radius appear to remain undisturbed because those measurements were made at low q values. However we are aware that at higher values of q^2 the conclusions about the neutron's form factors may be influenced slightly[1, 4]. It may be pointed out that the *inelastic* electron scattering studies on the deuteron should perhaps yield new information on the F_1 form factor of the neutron when combined with these results. It is interesting to speculate on whether the proton's F_2 factor rises again after approaching zero at about $q^2 \cong 24$ or whether it becomes negative at that point. In our analyses we assumed $F_2 = 0$ at $q^2 \geq 24$.

By use of these results new information on F_2 of the neutron should result from a study of the deuteron's elastic scattering at large angles.

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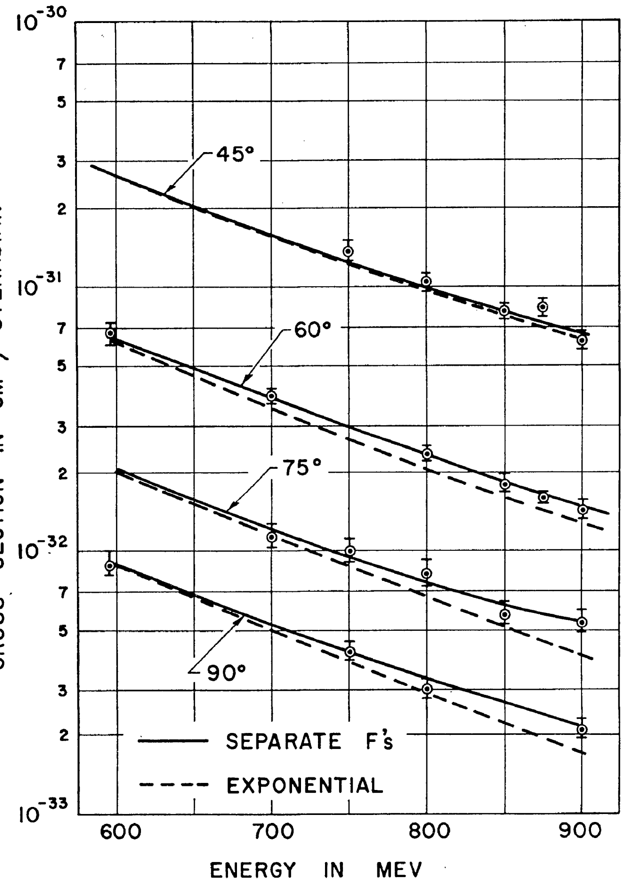

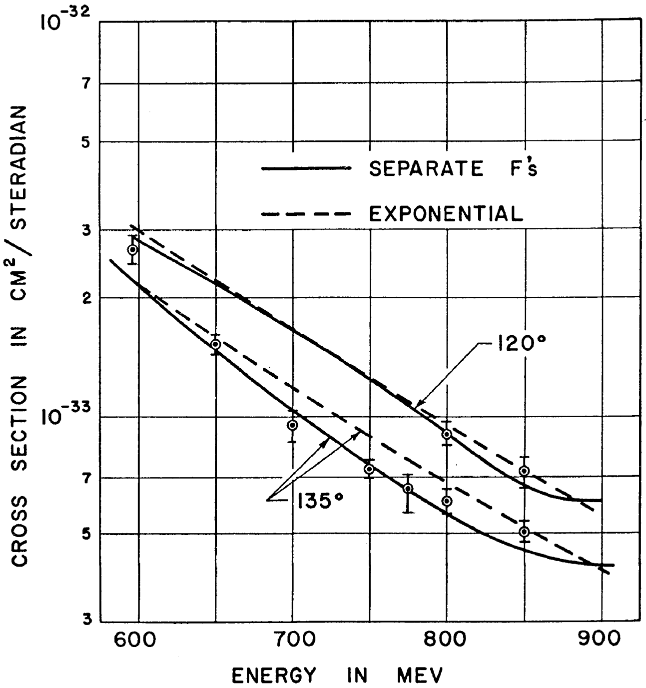

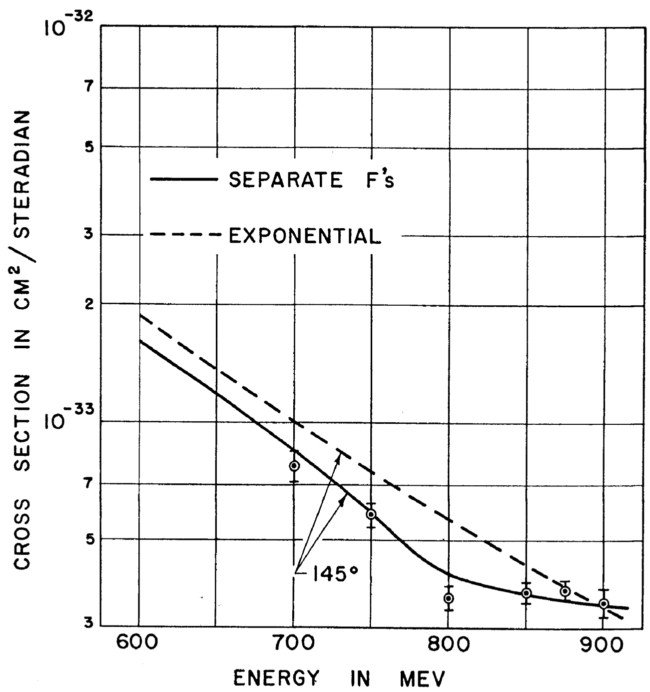

Fig. 4 Electron proton scattering cross sections between 600 and 900 MeV. *(a)* 45° to 90°, *(b)* 120° and 135°, *(c)* 145°. The experimental points are shown by hollow circles. The dashed line refers to the case $F_1 = F_2$ and corresponds to the form factors deduced from the old exponential model[1]. The solid line is obtained from Eq. 1 and the newly-obtained form factors of Table II and Fig. 3.

LIST OF REFERENCES AND NOTES

1. Herman, R. and Hofstadter, R. " High Energy Electron Scattering Tables " (Stanford University Press, Stanford, California, 1960).
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