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Black holes in loop quantum gravity

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Abstract. We summarize recent results concerning the quantization of the complete extension of the Schwarzschild space-time using spherically symmetric loop quantum gravity. We find an exact solution of the polymerized theory, that is expected to capture features of the semi-classical limit. The singularity is eliminated but the space-time still contains a horizon. Although the solution is known partially numerically and therefore a proper global analysis is not possible, a global structure akin to a singularity-free Reissner–Nordström space-time including a Cauchy horizon is suggested.

Loop quantum gravity inspired techniques have been applied to mini-superspace models (loop quantum cosmology) in the last few years. Among the results one generically finds in these models that the big bang singularity is replaced by a bounce (see for instance [1] and references therein). Since the interior of a black hole is classically isometric to a Kantowski–Sachs cosmology (that also sees its singularity eliminated in certain treatments as a loop quantum cosmology), it is natural to expect that the black hole singularity may also disappear in a similar way [2, 3]. A complete treatment of the space-time of a black hole in loop quantum gravity was lacking, even within a midi-superspace type of quantization, and we made progress on this issue recently [4]. What we did was to consider space-times with spherical symmetry and set up their canonical theory. Using a further gauge fixing to avoid the hard problem of having structure functions in the constraint algebra (see [5] for a good discussion) we obtain a model that has only Abelian constraints and a true Hamiltonian for evolution. We proceed to study classically the "polymerized" theory of such a model that can be straightforwardly quantized in the loop representation. It is known that such polymerized theories can capture many effects that one would find in a more systematic quantization followed by a semi-classical approximation. We found that the complete space-time can be covered and a solution can be constructed that replaces the singularities (black and white hole) of the usual Kruskal diagram by regular surfaces. Such surfaces can be smoothly matched so where one expected a "black hole" one tunnels into a "white hole" region of another universe and this can be continued indefinitely. The resulting solution therefore has a Cauchy horizon and can be characterized as the analog in semiclassical loop quantum gravity of an eternal black hole.

We used the Ashtekar new variables to describe the spherically symmetric space-times. Previous work on this subject was done in modern language by Bojowald and Swiderski [6] so we refer the reader to them for details. There is only one non-trivial spatial direction (the radial) which we call x since it is not necessarily parameterized by the usual radial coordinate. We will elaborate more on the range of x later. The canonical variables usual in loop quantum gravity are a set of triads E^a_i and $SO(3)$ connections A^i_a; after the imposition of spherical symmetry one is left with three pairs of canonical variables $(\eta, P^\eta, A_\varphi, E^\varphi, A_x, E^x)$. Instead of using triads in the directions transverse to the radial one, a "polar" set of variables E^φ, η and their canonical momenta is chosen. It is convenient to introduce the gauge invariant variable K_x defined by $2\gamma K_x = A_x + \eta'$ and also K_φ defined as $A_\varphi = 2\gamma K_\varphi$, where γ is the Immirzi parameter of loop quantum gravity. The canonically conjugate pairs are now E^x, K_x and E^φ, K_φ. The relationship to more traditional metric variables is,

$$\begin{aligned} g_{xx} &= \frac{(E^\varphi)^2}{|E^x|}, \qquad g_{\theta\theta} = |E^x|, \\ K_{xx} &= -K_x \text{sign}(E^x)\frac{(E^\varphi)^2}{\sqrt{|E^x|}}, \quad K_{\theta\theta} = -\sqrt{|E^x|}K_\varphi, \end{aligned} \tag{1}$$

and the latter two are the components of the extrinsic curvature. The diffeomorphism and Hamiltonian constraints can be seen in detail in ref. [7]. These constraints have the usual constraint algebra for gravity in $1+1$ dimensions, which includes structure functions. This implies the usual "problem of dynamics" of canonical quantum gravity. Our strategy to treat this model will be to further fix the gauge so we are left with a model with Abelian constraints and a true Hamiltonian. That way it can be treated using the standard Dirac procedure and it can be quantized with loop quantum gravity techniques. If one were to fix the gauge further before quantization, one would be led back to the standard quantization of Kuchař [8], which cannot add insights on the question of singularities. We eliminate the diffeomorphism constraint by choosing a gauge that determines the functional form for $E^x = f(x,t)$. Imposing the constraint strongly determines K_x. This also fixes the corresponding Lagrange multiplier (the shift) $N^r = -\dot{f}(x,t)/f'(x,t)$ and also breaks reparametrization invariance. One is left with a theory with Abelian constraints and with a true Hamiltonian, the dynamical variables are E^φ and K_φ.

The quantization of the Abelian constraints is straightforward and can be carried out in the same Hilbert space that was considered in the exterior case [7]. In brief, one discretizes the radial direction and the Hilbert space is a tensor product of Hilbert spaces of loop quantum cosmology, one per spatial point. In such a space the constraint of the model is not well defined, but one can work with an expression where K_φ is replaced by $\sin(\mu K_\varphi)/\mu$. The latter is immediately expressible in terms of holonomies and therefore naturally exists in the loop representation. The resulting theory agrees with general relativity in the limit $\mu \to 0$. In loop quantum gravity it is natural to consider a finite value of μ, usually associated with the elementary quantum of area [1].

Instead of quantizing the theory and then studying the semiclassical limit, we will follow a procedure that is known [3] to capture some of the semiclassical behaviors, in particular the elimination of the singularity, at least in simple examples with a constant value of μ as the one we are considering. We analyze the resulting classical "polymerized" theory with finite μ. One is then considering a classical theory of gravitation, different from general relativity that contains some of the ingredients of the quantum theory, akin to when one works out in an effective theory.

We wish to choose the function $f(t,x)$ in such a way that in the limit $\mu \to 0$ one recovers the standard Schwarzschild metric in Kruskal-like coordinates. That is, a metric with a singularity at $x^2 - t^2 = -1$. On the other hand, in the case of finite μ we will make a gauge choice such that no singularities are present on the surface $x^2 - t^2 = -1$ (one could choose gauges with coordinate singularities there). To be more specific we will choose $E^x = f(u,t,\delta)$, where $u = x^2 - t^2 + 1$ and $\delta(\mu)$ a positive parameter such that when $\mu \to 0$, $\delta \to 0$ and we recover the standard Kruskal form of the Schwarzschild space-time. To completely fix the gauge and obtain an explicit solution we set $K_\varphi = g(u,t,\delta)$ after polymerization. In the quantum theory such a gauge fixing would be equivalent to the study of an evolving constant [9, 10] E^φ in terms of c-number variable K_φ.

We require the following conditions on the gauge fixing. We choose the u in the range $[0,\infty]$ and is such that the radial variable has a logarithmic dependence on u for $u \to \infty$ just like in ordinary Kruskal coordinates. Moreover, we want asymptotically $E^\varphi \sim r + M$ in ordinary Schwarzschild coordinates, which translates conditions on the falloff in u. The conjugate variables are exponentially small in the radial coordinate $K_x \sim K_\varphi \sim 1/\sqrt{u}$. These boundary conditions are very similar to those in Kruskal coordinates [11]. At $u = 0$ we will require that all variables be t-independent and we will choose their derivatives to vanish. This ensures that one can easily continue the manifold without shells of matter present at $u = 0$. There might be other possibilities for this boundary condition but we have not explored them. Finally we would like that in the limit $\delta \to 0$ we get a gauge choice that covers the entire extension of the Schwarzschild space-time. Although the choice of coordinates we are making is not unique, it is computationally laborious to actually find a coordinate system that satisfies all the conditions we listed and that involves variables that do not turn complex in certain regions and that has the variable K_φ taking correct values in the Bohr compactification.

The specific choice we make for E^x is,

$$\begin{aligned} E^x \quad = \quad & \left\{ \frac{\left[\delta\,(1+u) + \left(10u^2 + u^{7/2}\right)\left(\delta\left(t^2-1\right)+1\right)\right]}{u^{7/2} + \left(t^2-1\right)\left(\delta u^{7/2} + \delta^2\right) + \frac{1}{2}\delta^2 u} \right. \\ & \left. \times \left[\ln(1+u)\right]^2 + \delta^8 \right\} M^2. \end{aligned} \tag{2}$$

This choice has the property that for $u \to 0$ $E^x = M^2\delta^8$ independent of t and in the limit $\delta \to 0$ we have that $E^x = M^2(10u^{3/2} + u^3)\ln^2(1+u)/u^3$ tends to 0 when $u = 0$, as in the Kruskal coordinates, giving rise to the singularity. It can be checked that the first derivative with respect to x of E^x vanishes for $u = 0$ for any finite value of δ. This choice for E^x is not unique, in the sense that other choices may satisfy the above conditions. It might be possible to find simpler choices.

For K_φ we choose,

$$\begin{aligned} K_\varphi = & \frac{1}{2}\frac{\delta^{5/2}\pi\left(1+\ln\left(1+u^2\right)\right)}{\mu\left(\delta^{5/2}+\ln\left(1+u\right)^2\right)} + \frac{|t|\ln\left(1+u^3\right)}{u^{3/8}} \\ & \times \frac{\left(-1 + \frac{u}{(10+u\ln(1+u))} + \frac{9u}{\left(100+u\ln(1+u)^2\right)}\right)}{\left(\delta^2 t + \ln\left(1+u^3\right)\right)\left(1+u^{1/8}\right)}. \end{aligned} \tag{3}$$

This choice has the property that for $u \to 0$ $K_\varphi = \pi/(2\mu)$ independent of t, so the term that appears in the Hamiltonian goes as $\sin(\mu K_\varphi) \sim 1$. This means that the departure of the polymerized theory from classical general relativity is maximum at the point where the singularity would have occurred in the continuum theory. Therefore loop quantum gravity could remove the classical Schwarzschild singularity. In the limit $\delta \to 0$ we have that K_φ blows up when $u = 0$, as in the Kruskal coordinates, also compatible with the presence of the singularity in the continuum theory. It can be checked that the first derivative with respect to x of $\sin(\mu K_\varphi)$ vanishes for $u = 0$. As in the case of E^x, the choice is not unique. It should also be noted that the choice is only valid in $|t| > 1$. We have extended the solution beyond that domain. The extension is symmetric under $t \to -t, x \to -x$, but it makes the expressions too lengthy, so for reasons of space here we concentrate in the region $|t| > 1$ since it includes the singularity.

We then solve the diffeomorphism constraint for K_x and the remaining constraint for E^φ. The preservation of the gauge conditions in time determine the lapse and shift. The consistency of the system, that is, the preservation of the constraints upon the Hamiltonian evolution guarantees that the evolution equations for the canonical variables are automatically satisfied. All these calculations can be carried out explicitly, except for the determination of the lapse that requires a difficult integral that can be handled numerically. Studying the components of the metric one notices that they behave very closely to the usual Schwarzschild solution except that when one approaches the singularity all relevant quantities remain finite. The singularity is avoided and a picture is suggested in which the spacetime of a (highly idealized) eternal black hole is continued into another region containing a Cauchy horizon, similar to a Reissner–Nordström space-time but without the singularity. In spite of the lack of singularity, there still is a horizon and a causal behavior far away from the singularity similar to that of the usual Schwarzschild solution.

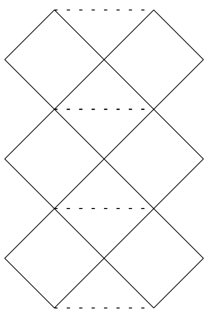

Figure 1. The conjectured global structure of the solution. The singularity is replaced by a regular region indicated with a dashed line. The space-time is continued through into another copy of the same solution. The solution would have a Cauchy horizon similar to that in a Reissner–Nordstöm solution, presumably unstable.

Is the solution unique? At this point we cannot say. There clearly are parameters that can be changed, and choices that were made, but it is not clear if they just correspond to diffeomorphisms. In particular we do not know if all possible choices will lead to non-singular solutions. During our work towards constructing the solution we display here we encountered solutions with singularities, but they ended up being coordinate singularities. Although the treatment of the exterior carried out previously [7] yields a single solution up to diffeomorphisms, it is known that in the treatments of the interior the "polymerization" breaks Birkhoff's theorem [2, 3] suggesting it may not hold in the complete case either. In the interior treatment there appears an additional parameter in the solution which, for instance, controls if the "bounce" is symmetric or not and the extent of the region where the polymerized theory departs from general relativity. Our solution appears to have several free parameters, even though we have imposed by hand that the bounce be symmetric. Clarifying the uniqueness point may shed light on the degrees of freedom that are remnant of the elimination of the singularity in loop quantum gravity and may yield a picture with elements in common with the "fuzzballs" [12] of string theory, although our solutions do not exhibit significant departures from general relativity at the position of the horizon.

Since we completed our manuscript other approaches to the Schwarzschild space-time have appeared. Modesto [13] considered the interior solution to the polymerized theory and analytically continued by exchanging t and r to produce an "exterior" solution. This has the advantage that the solution is completely known analytically. The disadvantage is that the solution solves the analytically continued polymerized equations of the interior. It is not clear if such equations would result from a plausible polymerization of the exterior. Peltola and Kunstatter [14] have considered polymerizing in the traditional variables and noted that perhaps only some of the variables should be polymerized. The result is a spacetime without a Cauchy horizon. This highlights that choices in polymerization may alter importantly the picture of the space-time one obtains. This clearly requires further investigation. Another point of interest is that when one finds solutions in ordinary general relativity one usually works in a given gauge noting that the solution can be later recast in any gauge one wants. In the polymerized theory there are also gauge transformations, but they are somewhat more limited than the full diffeomorphism invariance of ordinary general relativity, as we discussed in [15]. This is another issue that will require further investigation.

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