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Creation of an inflationary universe out of a black hole

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ABSTRACT

We discuss a two-step mechanism to create a new inflationary domain beyond a wormhole throat which is created by a phase transition around an evaporating black hole. The first step is creation of a false vacuum bubble with a thin-wall boundary by the thermal effects of Hawking radiation. Then this wall induces a quantum tunneling to create a wormhole-like configuration. As the space beyond the wormhole throat can expand exponentially, being filled with false vacuum energy, this may be interpreted as creation of another inflationary universe in the final stage of the black hole evaporation.

Inflation in the early universe provides answers to a number of fundamental questions in cosmology such as why our Universe is big, old, full of structures, and devoid of unwanted relics predicted by particle physics models [1]. Furthermore, despite the great advancements in precision observations of cosmic microwave background (CMB) radiation, there is no observational result that is in contradiction with inflationary cosmology so far [2,3].

Inflationary cosmology has also revolutionized our view of the cosmos, namely, our Universe may not be the one and the only entity but there may be many universes. Indeed already in the context of the old inflation model [4,5], Sato and his collaborators found possible production of child (and grand child...) universes [6–8].

Furthermore, if the observed dark energy consists of a cosmological constant Λ, our Universe will asymptotically approach the de Sitter space which may up-tunnel to another de Sitter universe with a larger vacuum energy density [9–13] to induce inflation again to repeat the entire evolution of another inflationary universe. In such a recycling universe scenario, the Universe we live in may not be of first generation, and we may not need the real beginning of the cosmos from the initial singularity [12].

In this context, so far only a phase transition between two pure de Sitter space has been considered [14]. However, phase transitions which we encounter in daily life or laboratories are usually induced around some impurities which act as catalysts or boiling stones. In cosmological phase transitions, black holes may play such roles. In this manuscript we discuss a cosmological phase transition around an evaporating black hole to show that a wormhole-like configuration with an inflationary domain beyond the throat may be created after the transition.

The study of a phase transition around a black hole was pioneered by Hiscock [15]. More recently, Gregory, Moss and Withers revisited the problem [16]. They have observed that the black hole mass may change in the phase transition and calculated the Euclidean action taking conical deficits into account [16–18]. Moreover, a symmetry restoration activated by Hawking radiation [19,20] near a microscopic black hole has been investigated by Moss [21].

We consider a high energy field theory of a scalar field ϕ whose potential allows a thin-wall bubble solution of a metastable local minimum at $\phi = 0$ with the energy density ϵ^4 surrounded by the true vacuum with a field value ϕ_0 where the mass square is given by m^2. In such a theory Moss [21] argues that the symmetry is restored in the vicinity of the black hole horizon inside a thin wall bubble as the Hawking temperature, $T_H = M_{Pl}^2/(8\pi M_+)$, reaches the mass scale of the theory. Here M_+ and M_{Pl} are the black hole mass and the Planck mass, respectively. In the presence of plausible couplings of the relevant fields, he shows that the medium inside the bubble, where fields coupled to ϕ are massless, is thermalized with a temperature T which is substantially smaller than $m \sim T_H$. Then the free energy of the bubble configuration is given by

$$F(r,T) = \frac{4}{3}\pi r^3 \epsilon^4 + 4\pi r^2 \sigma - \frac{\pi}{18} q\tilde{m}^2 T^2 r^3 \tag{1}$$

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as a function of its radius r and T, where σ is the surface tension of the wall and $\tilde{m}^2$ denotes sum of the mass squared of species which receive a mass from ϕ outside the bubble. For simplicity we assume $\tilde{m}$ is of the same order of m and omit the tilde hereafter. Here q is related to the scattering parameter C defined by Moss [21] as $q \equiv (192\pi^2 C)^{-2/3}$, which can take a value of order of unity or even larger.

The relation between the thermalized temperature T and the bubble radius r_w is obtained by solving the Boltzmann equations for the radiated beam particles and thermalized medium with the boundary condition that only particles with energy larger than m would escape the bubble wall, which reads

$$\frac{1}{216} q^{-3/2} T^3 r^3 + 48 m T r^2 e^{-\beta m} = 1, \tag{2}$$

at $r = r_w$ with $\beta \equiv T^{-1}$.

The radius of the wall r_w is obtained by minimizing the free energy (1) under the condition (2). For example, when the inequality

$$m r_w \gg 10^4 q^{2/3} (\beta m)^2 e^{-\beta m} \tag{3}$$

is satisfied and the first term dominates the left hand side of (2), we find $T = 6\sqrt{q}/r_w$, so that the free energy is minimized at

$$r = r_w = \sqrt{\frac{3q}{2} \frac{m}{\epsilon^2}}. \tag{4}$$

For consistency of this solution with (3), ϵ and m must satisfy

$$\frac{m}{T} = \frac{1}{6\sqrt{2}} \frac{m^2}{\epsilon^2} \gtrsim 10, \tag{5}$$

which we assume hereafter. Then the thin wall condition $m r_w = \sqrt{\frac{q}{2} \frac{m^2}{\epsilon^2}} \gg 1$ is naturally satisfied.

Under the condition (5) thermal energy inside the bubble is subdominant compared with ϵ^4, so the geometry inside the bubble can be described by the Schwarzschild de Sitter metric. Furthermore, as the radiation temperature increases in association with the increase of the Hawking temperature, more high energy particles, which escape from the bubble and do not contribute to support the wall, are created to lower the effect of the radiation pressure. Thus, contrary to naive expectation, thermal effects on the created bubble become less important as the temperature increases, which can be also understood from the inequality $dr_w/dT < 0$ derived from (2).

Thus the system can be approximated by a spherically symmetric thin wall with tension σ separating outside Schwarzschild geometry with mass parameter M_+ and inside Schwarzschild de Sitter geometry with vacuum energy density $\epsilon^4 \equiv 3M_{Pl}^2 H^2/(8\pi)$ whose mass parameter we denote by M_-.

We use the equation of motion of the wall obtained by Israel's junction condition to discuss quantum tunneling of the bubble to show that the final state is a wormhole-like configuration. Beyond the throat is a false vacuum state which inflates to create another big universe. Then one may regard that the final fate of an evaporating black hole is actually another universe. We do not take thermal effects to tunneling into account, as they would only enhance the tunneling rate.

We label the inner Schwarzschild de Sitter geometry with a suffix $-$ and outer Schwarzschild geometry with a suffix $+$. Then the outer and inner metrices are given by

$$ds^2 = -f_\pm(r) dt^2 + \frac{dr^2}{f_\pm(r)} + r^2 d\Omega^2, \tag{6}$$

$$f_+(r) \equiv 1 - \frac{2GM_+}{r}, \quad f_-(r) \equiv 1 - \frac{2GM_-}{r} - H^2 r^2.$$

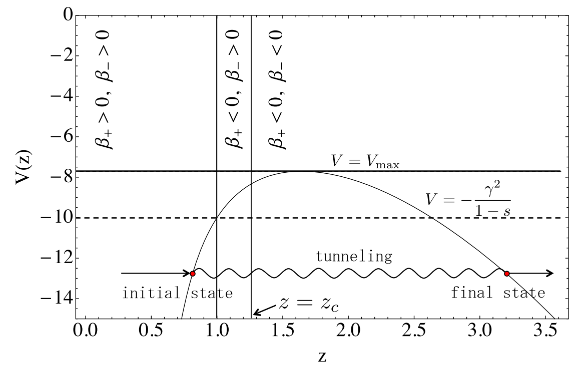

Fig. 1. Shape of the potential $V(z)$ as a function of z with $s = 0.9$. We have taken $\gamma = 1$ for illustrative purpose, although we actually expect $\gamma \ll 1$ for $\phi_0 \ll M_{\rm Pl}$. β_+ changes its sign at $z = 1$, and β_- at $z = (1 - \gamma^2/2)^{-1/2} \equiv z_c$.

We describe the wall trajectory in terms of the local coordinates $(t_\pm(\tau), r_\pm(\tau), \theta, \varphi)$ on each side depending on the proper time τ of an observer on the wall. They satisfy

$$f_\pm(r_\pm)\dot{t}_\pm^2(\tau) - \frac{\dot{r}_\pm^2(\tau)}{f_\pm(r_\pm)} = 1, \tag{7}$$

where a dot denotes derivative with respect to τ. We take the radial coordinates so that the radius of the bubble is given by $R = r_+ = r_-$ in both outer and inner coordinates. The evolution of the bubble wall is described by the following equation [16,22,23] based on Israel's junction condition [24]

$$\beta_- - \beta_+ = 4\pi G\sigma R \equiv \Sigma R, \tag{8}$$

where $\beta_+ \equiv f_+\dot{t}_+ = \pm\sqrt{f_+ + \dot{R}^2}$ and $\beta_- \equiv f_-\dot{t}_- = \pm\sqrt{f_- + \dot{R}^2}$. From (8) we find the wall radius satisfies the following equation similar to an energy conservation equation of a particle in a potential $V(z)$.

$$\left(\frac{dz}{d\tau'}\right)^2 + V(z) = E, \;\; V(z) \equiv -\frac{1}{1-s}\frac{\gamma^2}{z} - \left(\frac{1-z^3}{z^2}\right)^2, \tag{9}$$

$$E \equiv -\frac{\gamma^2}{[2GM_+\chi(1-s)]^{\frac{2}{3}}}, \;\; \chi \equiv (H^2 + \Sigma^2)^{\frac{1}{2}}, \gamma \equiv \frac{2\Sigma}{\chi}. \tag{10}$$

Here dimensionless coordinate variables are defined by

$$\tau' \equiv \frac{\chi^2 \tau}{2\Sigma}, \;\; z^3 \equiv \frac{\chi^2 R^3}{2GM_+(1-s)}, \;\; \text{with } s \equiv \frac{M_-}{M_+}. \tag{11}$$

As is seen in Fig. 1, the potential $V(z)$ has a concave shape with the maximum $V(z_m) \equiv V_{\max}$ given by

$$V_{\max} = -3\frac{z_m^6 - 1}{z_m^4}, \tag{12}$$

with

$$z_m^3 = \left[2 + \left(\frac{1}{2} - \frac{\gamma^2}{4(1-s)}\right)^2\right]^{\frac{1}{2}} - \left(\frac{1}{2} - \frac{\gamma^2}{4(1-s)}\right), \tag{13}$$

for $s < 1$. From (8) we also find

$$M_+ = M_- + \frac{4\pi}{3}R^3\epsilon^4 + 4\pi R^2 \sigma \frac{\beta_+ + \beta_-}{2}. \tag{14}$$

We may consider the evolution of the system taking the initial condition that the bubble is at rest at $R = r_w$ as the Hawking temperature has increased to above m so that thermal support on the

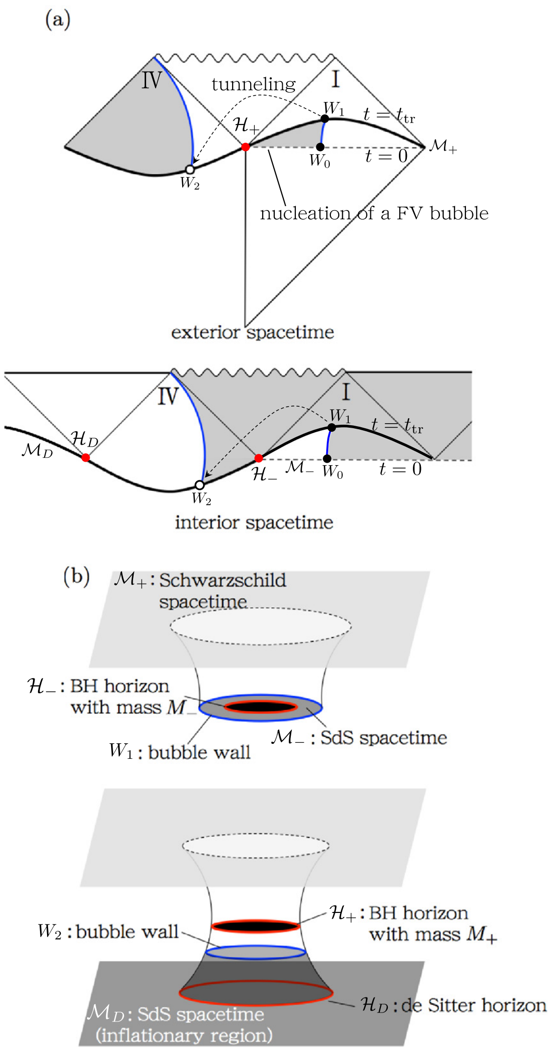

Fig. 2. The trajectories of a bubble wall (blue line) on Penrose diagrams and a schematic figure of a wormhole-like configuration accommodating an inflationary region induced by a phase transition. The upper (lower) diagram in Figure (a) shows the spacetime outside (inside) the wall and a shaded region is to be replaced by the interior (exterior) spacetime. These diagrams depict the case a bubble wall is produced at the point W_0 at $t = t_w < 0$ through thermal effects of Hawking radiation, and the wall tunnels from W_1 to W_2 at $t = 0$ to create a wormhole-like configuration. Figures (b) depict the initial and final configurations schematically. (For interpretation of the colors in the figure(s), the reader is referred to the web version of this article.)

wall has become less important as discussed above. For a thin wall bubble, (14) reads $M_+ \cong M_- + 4\pi r_w^3 \epsilon^4/3$, and one can show that an inequality $E = V(z_w) < -\gamma^2/(1-s) = V(z=1)$ holds where $z_w \equiv \chi^{2/3} r_w [H^2 r_w^3 + \Sigma r_w^2(\beta_+ + \beta_-)]^{-1/3}$ is the value of z corresponding to $R = r_w$. Therefore $\beta_\pm$ are both positive initially as is shown in Fig. 1.

We can discuss quantum tunneling of the bubble wall from $R = r_w$ to a larger R, from which it expands in real time, by manipulating the Euclidean action. As one can see in Fig. 1, physically relevant expanding bubble nucleation is possible only for $\beta_+ < 0$ and $\beta_- < 0$. It has been shown in [22] that in this case the trajectory of the bubble wall after the transition exists in region IV on the Penrose diagram (Fig. 2(a)), that is, a wormhole-like configuration is created [25,26] and the false vacuum exists on the other side of the throat (Fig. 2(b)).

Note that although β_+ and β_- change their signs at different radii, namely $z = 1$ and z_c, their physical separation

$$\Delta R_{c1} = \left[\frac{2GM_+(1-s)}{\chi^2}\right]^{1/3} \left(z_c^{1/3} - 1\right) \cong \frac{\gamma^2}{4} r_w \tag{15}$$

is actually smaller than the width of the wall $1/m$ for realistic values of parameters, so that we can regard that they change sign at the same radius in the thin wall approximation.

Let us now calculate the transition rate to the wormhole-like configuration Γ by solving Euclidean equation of motion starting from the bubble radius $R = r_w$ at rest. Following Gregory, Moss, and Withers [16–18], the transition rate Γ has the form

$$\Gamma = m e^{-I_{\mathcal{M}-\mathcal{B}} - I_{\mathcal{B}}} = m e^{-B_{\rm tunnel} + \Delta S}, \tag{16}$$

where the prefactor m has been introduced on dimensional grounds. Here $I_{\mathcal{M}-\mathcal{B}}$ represents the action over the regular bulk Euclidean spacetime and $I_{\mathcal{B}}$ stands for the contribution of conical singularities. They are given by

$$\begin{aligned} I_{\mathcal{M}-\mathcal{B}} = \int d\tau_E \,\big[&(2R - 6GM_+)\dot{t}_{E+} \\ &- (2R - 6GM_-)\dot{t}_{E-}\big] \equiv B_{\rm tunnel}, \\ I_{\mathcal{B}} = \frac{A_f}{4G} - &\frac{A_i}{4G} \equiv \Delta S, \end{aligned} \tag{17}$$

respectively, where the suffix E indicates the Euclidean time and A_i (A_f) denotes the total horizon area in the initial (final) state. Obviously terms arising from conical singularities are identical to the difference of horizon (Bekenstein) entropies, ΔS, between initial and final states. These terms have been derived using another method of calculation, too [27–30].

It is well known that the Bekenstein entropy of horizon may be related to its number of microscopic states W although so far we do not know what the microscopic degrees of freedom are. In our case, the initial state before tunneling has a Schwarzschild de Sitter black hole horizon with its mass parameter M_-, whose area is denoted by A_-, and the final state has two gravitational horizons, namely, the black hole horizon with mass M_+ and the de Sitter horizon (Fig. 2(b)), whose horizon areas are denoted by A_+ and A_D, respectively. Therefore, we have $A_i = A_-$ and $A_f = A_+ + A_D$ and the numbers of the initial and final microscopic degrees of freedom are given by $W_i = e^{A_i/4G}$ and $W_f = e^{A_f/4G}$, respectively [31].

From $e^{\Delta S} = W_f/W_i$ we can interpret the transition rate we have calculated, (16), as a transition from one microscopic initial state with a statistical weight $1/W_i$ to a final state with W_f microscopic degrees of freedom, and the transition rate from one microscopic state of the initial black hole to another microscopic state of the final wormhole configuration is given by

$$\Gamma_{\rm micro} = m e^{-B_{\rm tunnel}}, \tag{18}$$

up to the uncertainty of the prefactor.

Let us evaluate the transition rate by calculating $B_{\rm tunnel}$ and ΔS which are functions of q, the energy scales m and ϵ, and the tension of bubble wall σ. Here we can evaluate the tension as $\sigma \simeq \xi^4/m$, where ξ^4 is the potential energy density at the top of the potential barrier separating the false vacuum and true vacuum. We take M_+ at a reference value $M_+ = M_{Pl}^2/(8\pi m) \gg M_{\rm Pl}$ corresponding to $T_H = m$. Taking $m^2 = 120\sqrt{2}\epsilon^2$, $q = 1$, and $\xi^4/\epsilon^4 = 25$, as an example, one can satisfy the thin wall condition, $m r_w = 120 \gg 1/m$. $m\Delta R = m\gamma^2 r_w/4 \ll 1$ is also satisfied for $\epsilon \ll 10^{16}$ GeV.

Fig. 3 depicts ΔS and $B_{\rm tunnel}$ as functions of ϵ. ΔS is proportional to ϵ^{-4} since the de Sitter horizon area, which is proportional to $H^{-2} \propto \epsilon^{-4}$, becomes dominant compared to the black

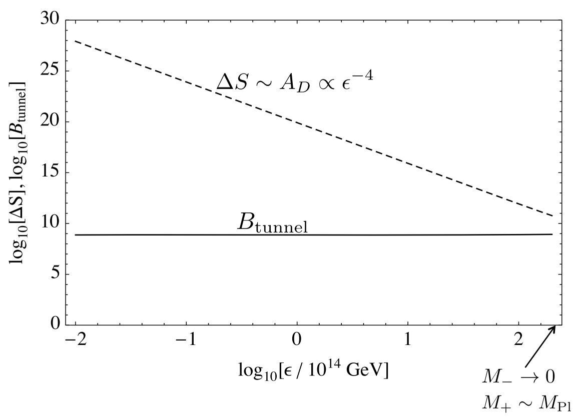

Fig. 3. ΔS (dashed line) and $B_{\rm tunnel}$ (solid lines) as functions of ϵ for $m^2 = 120\sqrt{2}\epsilon^2$, $q = 1$ and $\xi^4/\epsilon^4 = 25$. The inner black hole mass, M_-, approaches to zero and M_+ becomes comparable to the Planck mass at $\epsilon \simeq 2 \times 10^{16}$ GeV, where $M_+ = 4 \times 10^2 M_{Pl}(\epsilon/10^{14}\text{ GeV})^{-1}$.

hole horizon areas for $\epsilon \ll 10^{16}$ GeV. As is seen here, we always find $\Delta S \gg B_{\rm tunnel} \gg 1$. This means that even though the tunneling rate from one microscopic state to another is exponentially suppressed so that the semiclassical approximation is valid, due to the largeness of the number of microscopic degrees of freedom after the transition, the tunneling as a whole is unsuppressed and wormhole creation may take place with the relevant time scale $t \sim 1/T_H \sim 1/m$, once a bubble is thermally excited around an evaporating black hole with the proper conditions we discussed above. Similar enhancement of transition rate due to the large entropy in the final state has been observed by Mathur in a different problem [32].

Then we can sketch the following scenario of cosmic evolution. Typical astrophysical black holes with mass $\sim 10M_\odot$ will evaporate in $\sim 10^{67}$ years from now. As its mass falls below a critical value so that the Hawking temperature become high enough for a false vacuum bubble to spontaneously nucleate around the black hole according to the process described by Moss [21]. Then the bubble wall will experience quantum tunneling rather efficiently to create a wormhole-like configuration with a de Sitter horizon, beyond which the false vacuum region is extended to infinity. Thus the space on the other side of the throat will inflate which is causally disconnected from our patch of the universe. If inflation is appropriately terminated followed by reheating, another big bang universe will result there. For this purpose the old inflation model [4,5] with thin wall bubble nucleation does not work, but we may make use of the results of open inflation models there [33–35] which can also realize an effectively flat universe.

Throughout these processes, the outer geometry remain Schwarzschild space with the mass parameter M_+, so those who live there do not realize a black hole in their universe has created a child universe. To this end alone, our model is similar to the scenario proposed by Frolov, Markov, and Mukhanov [36,37]. However, there are two striking differences between our model and their scenario. One is that theirs is entirely dependent on the limiting curvature hypothesis and the assumption that in the regime of large curvature, the gravitational field equation would take the form in vacuum with a positive cosmological constant. They thereby find a Schwarzschild solution is continued to a deflating de Sitter space inside the black hole horizon which bounces to an inflating de Sitter universe. Our model, on the other hand, does not need such a speculative hypothesis near the singularity but creation of another inflationary universe is achieved by symmetry restoration due to the high Hawking temperature around an evaporating black hole which also induces a phase transition to produce a wormhole-like configuration in quantum field theory. Thus the entire processes can be described by known physics with appropriate values of the model parameters. Another difference lies in the causal structures as described in Fig. 2 of our paper and Figs. 3 and 6 of [36], that is, in our model the inflating domain is causally disconnected from the original universe unlike theirs.

In conclusion, our result may also suggest that our Universe may have been created from a black hole in the previous generation in the cosmos.

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