

Experimental Tests of the Electroweak Standard Model at High Energies



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Abstract

In recent years, a new generation of particle physics experiments explore electron-positron and proton-antiproton collisions at centre-of-mass energies never reached before. Large detectors measure accurately all details of the collision processes. This article reviews the wealth of precise, mostly still preliminary measurements which are used to determine masses and coupling constants of heavy fermions and bosons with unprecedented accuracy. The measurements are so precise that they test the Standard Model of particle physics not only at lowest order but also at the level of its higher-order radiative corrections, and constrain its parameters. The results of these tests of the electroweak Standard Model, constraints on its parameters and consequences for new physics are presented.

The main results are: the Standard Model of electroweak interactions describes successfully the complete set of measurements. Based on the analysis of electroweak radiative corrections, the masses of the top quark and the W boson are indirectly determined, and the results agree well with the direct measurements. For the Standard Model Higgs boson, a small mass value is determined, with $M_{\text{H}} < 262 \text{ GeV}$ at 95% confidence level.

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Experimental results becomes even more useful if there is a model or theory for their interpretation and comparison with other experimental results. Over the years I have greatly benefited from many detailed discussions on theoretical aspects of the Standard Model with Dima Bardin and Giampiero Passarino, two theorists who are able to bridge in a meaningful way the gap towards the experimental side of particle physics.

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Chapter 1

Introduction and Overview

1.1 Structure of this Review

In this article experimental tests of the Standard Model of electroweak interactions [1] at high energies are reviewed. Currently, the experiments at the high-energy electron-positron colliders SLC and LEP provide a wealth of precise results on the electroweak and strong interaction, in particular on the neutral and charged heavy gauge bosons, Z and W. The W boson and especially the sixth and heaviest quark, the top quark, are studied by the experiments at the TEVATRON proton-antiproton collider. Neutrino-nucleon scattering experiments also measure precisely the electroweak mixing angle.

This article is organised as follows: In Chapter 2 the theoretical framework of the Standard Model of particle physics is presented, which is applied in Chapter 3 to discuss the specific physics studied at high-energy e^+e^- colliders. The modern e^+e^- colliders, SLC and LEP and their experiments are briefly described in Chapter 4. In Chapter 5 experimental measurements and results, mostly still preliminary, are discussed which are used in the subsequent analyses. Tests of the Standard Model are performed and analysed in Chapter 6. Expectations for the future are summarised in Chapter 7, and conclusions are given in Chapter 8.

The system of units adopted here is that of particle physics, where the reduced Planck constant, $\hbar = h/(2\pi)$, and the speed of light, c , are set to unity, $\hbar = c = 1$. The electromagnetic finestructure constant, α_{em} , is given by $\alpha_{em} = e^2/(4\pi)$ in units of the positron charge e . Energies, momenta and masses are measured in units of electron volts, $1 \text{ eV} = 1.60217733(49) \cdot 10^{-19} \text{ J}$. Cross sections are measured in units of barns, $1 \text{ b} \equiv 10^{-28} \text{ m}^2$. Otherwise, SI units are used.

1.2 Goals of Particle Physics

The aim of particle physics is to describe the elementary constituents of matter and the interactions between them. This field of physics entered its modern phase at the end of the nineteenth century with a series of exciting discoveries: X-rays by W.C. Röntgen in 1895 [2], radioactivity by H. Becquerel in 1896 [3], and the electron as the first particle still considered elementary today by J.J. Thomson in 1897 [4].

Radioactive decays, discovered in uranium, invalidated the common belief in unchangeable chemical elements or atoms. Indeed the phenomenon of radioactivity involves the three interactions still studied today in particle physics: the strong, the electromagnetic and the weak interaction. Radioactive decays of heavy nuclei under emission of α -rays, which are identified as Helium nuclei, show that the atomic nucleus is composed of neutral and charged nucleons, neutrons and protons, bound together by the strong interaction. As a composite object, fission of the atomic nucleus is possible. Furthermore, radioactive decays under emission of γ -rays, identified as photons, are expected, because the atomic nucleus as a composite object of charged and neutral particles may undergo transitions, for example from an excited state to the ground state. Radioactive decays under emission of β -rays, which are

electrons, change neutrons into protons. This is a manifestation of the charged weak interaction where again a change of particle identity appears, now on the more elementary level of nucleons.

Over the last 100 years, experimental results on the weak interaction in particular have shown many surprises, sometimes causing changes to physical concepts deemed to be fundamental. Among these are: postulation of neutrinos by W. Pauli in 1930 [5] to ensure energy-momentum and angular-momentum conservation in β decays, discovery of parity violation in charged weak decays by C.S. Wu *et al.* in 1957 [6], discovery of CP violation by J.W. Cronin, V.L. Fitch *et al.* in 1964 [7], and discovery of neutral weak interactions by F.J. Hasert *et al.* in 1973 [8]. Within the framework of gauge theories, the weak interaction, being of short range and weak at low momentum transfer, requires the exchange of very massive spin-1 gauge bosons, the charged W boson and the neutral Z boson. These intermediate vector bosons were discovered by C. Rubbia *et al.* in 1983 [9, 10].

In order to describe the three phenomenologically vastly different strong, weak and electromagnetic interactions in a common framework, the concepts of special relativity, quantum theory, local gauge symmetry and spontaneous symmetry breaking are used. The Standard Model of particle physics is a renormalisable quantum field theory of the electroweak and strong interactions. This theory is able to explain or at least accommodate all experimental results in particle physics obtained so far.

The electroweak theory, developed by S.L. Glashow, S. Weinberg and A. Salam [1] from 1961 to 1968, provides an integrated description of the weak and electromagnetic interactions. The problem of mass generation in gauge theories was solved in 1964 by the Higgs mechanism named after P.W. Higgs [11, 12]. The strong interaction theory of quantum chromodynamics was developed in 1973 by H. Fritzsch, M. Gell-Mann, H. Leytwyler, D.J. Gross, F. Wilczek [13] and many others. The concept of confinement of quarks and gluons inside hadrons was suggested by S. Weinberg [14]. The asymptotic freedom property was discovered by D.J. Gross, F. Wilczek and H.D. Politzer [15]. Renormalisability of theories like the Standard Model of strong and electroweak interactions was proven by G. 't Hooft [16] in 1971. These developments comprise the theoretical foundation of the Standard Model of particle physics.

1.3 Experiments in Particle Physics

Until the middle of the twentieth century, particle physics experiments were mainly based on the study of radioactive materials and cosmic rays. With the advent of particle accelerators, the experimental emphasis shifted towards fixed-target and colliding-beam experiments, allowing to perform experiments under controlled beam conditions.

The colliding-beam particles must be stable charged particles, leaving electrons, protons and their antiparticles as possibilities.¹ On circular orbits, protons and antiprotons can be accelerated to much higher energy than electrons and positrons, because their masses are higher and the energy loss due to synchrotron radiation is thus much reduced. Therefore, higher centre-of-mass energies in collisions are more easily obtained at proton-antiproton colliders, allowing the production and thus discovery of new particles with high masses.

An example is given by the SPS accelerator at CERN in Geneva, having provided $p\bar{p}$ collisions at centre-of-mass energies of up to 0.6 TeV. In 1983, the SPS experiments UA1 and UA2 discovered the heavy intermediate vector bosons, the charged W boson and the neutral Z boson [9, 10], with masses around 80 GeV and 90 GeV, respectively. By the end of data taking at the SPS in 1990, a few hundred W and Z bosons were accumulated by the SPS experiments.

In contrast, electron-positron colliders offer a much cleaner experimental environment due to the pointlike nature of the colliding beam particles. Particles and interactions are studied and theories are tested with high experimental precision.

In 1989, the e^+e^- colliders SLC at SLAC, Stanford, and LEP at CERN started to operate at centre-of-mass energies close to 91 GeV. This energy corresponds to the mass of the neutral Z boson, which is thus resonantly produced, $e^+e^- \rightarrow Z$.

¹However, since a few years, the design of a $\mu^+\mu^-$ collider is actively studied [17].

Until 1995, more than 16 million Z-boson decays have been recorded by the four LEP experiments ALEPH, DELPHI, L3 and OPAL at LEP-I together, and until 1998, more than half a million Z bosons have been produced with longitudinally polarised electron beams at SLC. This large amount of Z data combined with the clean experimental situation leads to high-precision measurements of the properties of the Z boson, such as its mass, total and partial decay widths, and the neutral-current coupling constants of fermions.

From 1996 until the year 2000 the LEP collider operates in its second phase, called LEP-II, where the centre-of-mass energy is more than doubled to a range from 160 GeV up to 200 GeV. These centre-of-mass energies allow the pair-production of on-shell $W^\pm$ bosons, $e^+e^- \rightarrow W^+W^-$. Nearly 10,000 W-pair events in total are expected per experiment, making it possible to study the W boson precisely and in particular to measure its mass and its gauge couplings.

In parallel to the SLC/LEP program, a new phase of $p\bar{p}$ physics commenced with the TEVATRON collider at Fermilab near Chicago. Its centre-of-mass energy of 1.8 TeV is a factor of three larger than that of the SPS. By the end of run I in 1996, the TEVATRON experiments CDF and DØ have collected more than 100 pb^{-1} of luminosity each, yielding several 10,000 W and Z bosons. The most important discovery at the TEVATRON is that of the sixth and heaviest quark, the top quark, by CDF in 1994 [18] and DØ in 1995 [19]. The top-quark mass of about 175 GeV is so large that it cannot be pair-produced at current e^+e^- colliders. After upgrading the TEVATRON collider and the CDF and DØ detectors, data taking will recommence with run II in the year 2000.

The most recent experiment to study neutrino-nucleon interactions is the NUTEV experiment. It is located at Fermilab and makes use of the detector of the older neutrino-nucleon experiment CCFR. The main advantage with respect to previous neutrino experiments lies in the availability of both a neutrino and an antineutrino beam, leading to measurements with reduced systematic errors. Data taking at NUTEV was completed in the fall of 1997.

For the future two new large accelerators are planned, increasing the centre-of-mass energy by about another order of magnitude. At CERN, the large hadron collider, LHC, a proton-proton collider with a centre-of-mass energy of 14 TeV will be installed in the existing LEP tunnel. Starting in the year 2005, two general-purpose experiments, ATLAS and CMS, will search for the important missing piece of the Standard Model, the Higgs boson, and for new particles predicted by extended theories such as supersymmetry. For e^+e^- physics an international effort is currently under way to design a linear collider and detectors for centre-of-mass energies ranging from 0.3 TeV to 3 TeV, high enough to study the properties of the top quark precisely and to search for the Higgs boson and manifestations of new physics beyond the Standard Model.

Chapter 2

The Standard Model of Particle Physics

The Standard Model (SM) of particle physics [1] culminates decades of experimental and theoretical research aimed at constructing a consistent theory describing elementary particles and their interactions as observed in experiments. Developed during the 1960s and early 1970s the SM provides a mathematical framework to describe particle physics, incorporating the fundamental concepts of special relativity, quantum theory and local gauge symmetry.

This chapter presents an overview on the SM in general and its electroweak interactions in particular. The application to the physics of high-energy e^+e^- collisions is discussed in Chapter 3. Quantitative calculations are performed with the semianalytical programs SMATASY [20], TOPAZ0 [21] and ZFITTER [22], discussed in Section 3.3.2.

2.1 The Minimal Standard Model

The Minimal Standard Model (MSM) is most economic in implementing the key dynamic ingredients of the Standard Model, local gauge symmetry and spontaneous symmetry breaking in form of the Higgs mechanism [11, 12], while describing all experimental measurements. Electromagnetic, weak and strong interactions between elementary particles are incorporated, but not gravity. For the latter, a consistent quantum field theory has yet to be developed. This shortcoming of the SM does not inhibit the investigation of the electroweak and strong interactions in experiments, because the gravitational force between elementary particles is about 40 orders of magnitude smaller than the electromagnetic force. Conceptually however, this fundamental incompleteness of the SM shows that it must be part of an encompassing “theory of everything”.

2.2 Local Gauge Symmetries

Gauge theories are an important concept in field theory as such theories are always renormalisable. Renormalisation of quantum field theories is necessary in order to obtain a unique relationship between calculated and measured quantities.

2.2.1 Gauge Transformations

In quantum field theories, particles, for example, fermions, are described by complex fields $\psi(x)$ depending on the space-time coordinate x . In gauge theories, interactions between fermions are introduced by the requirement of the invariance of the theory under local, *i.e.*, x -dependent transformations $U(x)$:

$$\psi(x) \rightarrow \psi'(x) = U(x)\psi(x), \quad (2.1)$$

The gauge transformations U describe a symmetry of the theory if the equations of motion remain invariant. The transformations may either be simple phase transformations, or, more complicated, mix the components corresponding to the internal degrees of freedom of $\psi(x)$. For each symmetry of the theory, the set of all unitary gauge transformations $U(x)$ form a Lie group, G . The transformations $U(x)$ are normally expressed using the n hermitian generators H^j , $j = 1, \dots, n$, of the Lie group:

$$U(x) = \exp \left[-i \sum_{j=1}^n \theta_j(x) H_j \right], \quad (2.2)$$

where $\theta_j(x)$ are real functions of the space-time coordinate x specifying the local transformation. The n generators H_j obey the Lie algebra:

$$[H_j, H_k] \equiv H_j H_k - H_k H_j = i \sum_{l=1}^n h_{jkl} H_l, \quad (2.3)$$

where h_{jkl} , real numbers, are the structure constants of the Lie algebra of the generators H_j . They are totally antisymmetric and vanish if the group is Abelian.

Equations of motion such as the Klein-Gordon or the Dirac equation contain partial derivatives of the fields $\psi(x)$ with respect to the space-time coordinates x^μ , $\partial_\mu = \partial/\partial x^\mu$. Applying a local gauge transformation to the field $\psi(x)$ entering the equation of motion leads to additional terms when acted upon by the derivative operator ∂_μ . Invariance of the equation of motion is reestablished by adding terms to the partial derivative ∂_μ to form the covariant derivative $\mathcal{D}_\mu$:

$$\partial_\mu \rightarrow \mathcal{D}_\mu = \partial_\mu + ig \sum_{j=1}^n H_j A_\mu^j(x). \quad (2.4)$$

The n spin-1 boson fields $A_\mu^j(x)$, $j = 1, \dots, n$, are called gauge fields. They couple to the fermion fields with the coupling strength g , thus introducing interactions between the formerly free particles. The value of the coupling constant g is not predicted by the theory.

Also the gauge fields $A^j(x)$ transform under a gauge transformation $U(x)$, where the transformation is again determined by the overall invariance requirement of the theory:

$$A_\mu^j(x) \rightarrow A_\mu^{j'}(x) = A_\mu^j(x) - \frac{1}{g} \partial_\mu \theta_j(x) - \sum_{k,l}^n h_{jkl} \theta_k(x) A_\mu^l(x). \quad (2.5)$$

The field-strength tensor $A_{\mu\nu}^j(x)$ of the gauge field $A_\mu^j(x)$ is given by:

$$A_{\mu\nu}^j(x) = \partial_\mu A_\nu^j - \partial_\nu A_\mu^j - g \sum_{k,l}^n h_{jkl} A_\mu^k A_\nu^l. \quad (2.6)$$

The first non-Abelian gauge theory was formulated by O. Klein in 1939 [23]. Theories containing non-Abelian symmetry groups are nowadays called Yang-Mills theories [24].

2.2.2 Local Symmetry Groups of the Standard Model

Quantum electrodynamics, QED, is invariant under local transformations of an Abelian $U(1)$ symmetry group denoted as $U(1)_{EM}$. The associated gauge field is nothing else but the electromagnetic potential, A_μ , describing the photon. The corresponding generator and coupling strength between a fermion and a photon is simply the electromagnetic charge of the fermion, Q_f . Since $U(1)_{EM}$ is an Abelian group photon self-interactions do not exist; the photon does not carry electromagnetic charge.

The strong-interaction theory of quarks, quantum chromodynamics or QCD [13–15], is a Yang-Mills theory based on the non-Abelian symmetry group $SU(3)$ denoted as $SU(3)_C$ where the subscript

C stands for colour. Each quark flavour corresponds to an $SU(3)_C$ quark triplet in a three-dimensional space, the so-called colour space visualised as being spanned by three base colours, red, green and blue. The eight generators of the Lie group $SU(3)_C$ correspondingly require eight massless spin-1 fields G_μ^a , $a = 1, \dots, 8$, called gluons. Since $SU(3)_C$ is a non-Abelian group gluon self-interactions occur; the gluons themselves carry colour charge.

About 30 years ago, the integration of the electromagnetic and weak interaction was proposed. At that time, only charged weak interactions were known. The charged weak interactions, raising or lowering the electromagnetic charge by one unit, involve left-handed fermions and right-handed antifermions only. The left-handed and right-handed parts of a generic fermion field ψ , ψ_L and ψ_R , are given by the normalised orthogonal projections:¹

$$\psi_L \equiv \frac{1 - \gamma^5}{2} \psi \quad \psi_R \equiv \frac{1 + \gamma^5}{2} \psi \quad \psi = \psi_L + \psi_R. \quad (2.7)$$

The smallest Lie group with three generators required for three interactions, two charged weak and a neutral electromagnetic interaction, is $SU(2)$. However, $SU(2)$ alone leads to inconsistencies as in contrast to the weak interactions the electromagnetic interaction does not distinguish between left- and right-handed fermions.

The simplest way out is to enlarge the symmetry group of the electroweak interaction from an $SU(2)_L$ symmetry group of left-handed fields by an additional $U(1)$ symmetry group, called $U(1)_Y$, which, although being mathematically the same $U(1)$ Lie group as that of QED, has a different physical meaning. As a consequence, a neutral weak interaction is predicted. Its discovery in neutrino interactions established that the symmetry group of the electroweak theory, G_{EW} , must be at least as large as the direct product $SU(2)_L \otimes U(1)_Y$ with $U(1)_{EM} \neq U(1)_Y$ being a subgroup of G_{EW} .

The three generators of the group $SU(2)_L$ are called the weak-isospin operators, T_1 , T_2 , and T_3 , in analogy to ordinary spin. The generator of the group $U(1)_Y$ is called the weak-hypercharge operator Y . The corresponding gauge fields consist of a vector-boson triplet under $SU(2)_L$, W_μ^i , $i = 1, 2, 3$, and a vector-boson singlet under $SU(2)_L$, B_μ . Three independent linear combinations of these four gauge-boson fields acquire mass as a result of the Higgs mechanism of spontaneous symmetry breaking [11, 12] while one remains massless. The three massive gauge bosons are the $W^\pm$ bosons mediating the charged weak current, and the Z boson mediating the neutral weak current. The massless boson is identified as the photon of the electromagnetic interaction.

In summary, the symmetry group of the minimal SM of strong and electroweak interactions, G_{MSM} , is given by:

$$G_{MSM} = U(1)_Y \otimes SU(2)_L \otimes SU(3)_C. \quad (2.8)$$

Since G_{MSM} is a direct product of three independent Lie groups, the MSM contains three independent coupling constants, g_1 , g_2 , g_3 , for $U(1)_Y$, $SU(2)_L$, $SU(3)_C$, respectively. The covariant derivative of the MSM reads:

$$\mathcal{D}_\mu = \partial_\mu + ig_1 Y B_\mu + ig_2 \frac{\tau_j}{2} W_\mu^j + ig_3 \frac{\lambda_a}{2} G_\mu^a, \quad (2.9)$$

where the generators $\tau_i/2 \equiv T_i$ and $\lambda_a/2$ are those of the $SU(2)_L$ and $SU(3)_C$ groups, respectively. The matrix representations of the τ_i 's and λ_a 's are known as Pauli and Gell-Mann matrices [25], respectively.

¹The customary language also used here is somewhat confusing. To be more precise, the fields ψ_L and ψ_R defined above and participating in the charged weak interaction are fermion states of definite chirality. They correspond to states of definite helicity, -1 and $+1$, respectively, only in the case of massless fermions.

2.3 Elementary Particles

The particle content of the MSM is reported in Table 2.1. Left-handed fermions are grouped into $SU(2)_L$ weak-isospin doublets, Ψ_L . Right-handed fermions, Ψ_R , are singlets under $SU(2)_L$. Right-handed neutrinos are usually assumed not to exist, implying that neutrinos are massless. However, recent experimental results based on solar, atmospheric and reactor neutrino experiments [26, 27] indicate the possible existence of neutrino oscillations which would require that neutrinos have non-vanishing mass. Confirmation of these results and their consistent interpretation is needed [28].

The fermions appear in generations or families, at least three, with increasing mass but otherwise identical quantum numbers. The assignment of quantum numbers for the non-Abelian $SU(2)_L$ symmetry group, total weak isospin, T , and its third component, T_3 , is fixed by the assignment of the particles to the $SU(2)_L$ multiplets. In contrast, for the Abelian group $U(1)_Y$ of weak hypercharge Y , the group structure alone does not provide any guidelines in the assignment of the weak hypercharge quantum number, *c.f.* electric charge. Gauge invariance of the theory and the requirement of a linear relation between Y , T_3 and the electromagnetic charge, Q , determine Y up to an overall factor [29]:

$$Q = T_3 + a \cdot Y. \quad (2.10)$$

Historically, $a = 1/2$ yielding the Gell-Mann/Nishijima relation first established for the strong-isospin symmetry [30]. In Table 2.1, the convention $a = 1$ is used.

2.4 Standard Model Lagrangian

The Lagrangian of the Minimal Standard Model can be written as a sum of four contributions²:

$$\mathcal{L} = \mathcal{L}_{\text{Fermion}} + \mathcal{L}_{\text{Yang-Mills}} + \mathcal{L}_{\text{Higgs}} + \mathcal{L}_{\text{Yukawa}}. \quad (2.11)$$

The Fermion Lagrangian describes the dynamics of the fermions, *i.e.*, their kinetic energy and interactions with the gauge bosons through the covariant derivative as given above:

$$\mathcal{L}_{\text{Fermion}} = \sum_{\Psi_L} \bar{\Psi}_L i \gamma^\mu \mathcal{D}_\mu \Psi_L + \sum_{\Psi_R} \bar{\Psi}_R i \gamma^\mu \mathcal{D}_\mu \Psi_R. \quad (2.12)$$

The Yang-Mills Lagrangian contains the kinetic-energy and self-interaction terms of the various gauge fields associated with the local symmetry groups. In terms of the field strength tensors of the gauge boson fields, Equation 2.6, it is given by:

$$\mathcal{L}_{\text{Yang-Mills}} = -\frac{1}{4} B^{\mu\nu} B_{\mu\nu} - \frac{1}{4} W_i^{\mu\nu} W_{\mu\nu}^i - \frac{1}{4} G_a^{\mu\nu} G_{\mu\nu}^a, \quad (2.13)$$

predicting trilinear and quadrilinear couplings within the set of gauge bosons of each non-Abelian symmetry group. Because of the antisymmetry of the structure constants, the resulting triple and quadruple gauge boson vertices involve at most two identical gauge bosons.

Problems arise with the inclusion of particle masses in the Lagrangian for both fermions and gauge bosons. For spin-1/2 fermions, mass terms have the form $m \bar{\Psi} \Psi$, or, written in terms of left-handed and right-handed components, $m(\bar{\Psi}_R \Psi_L + \bar{\Psi}_L \Psi_R)$, which is not an $SU(2)_L$ singlet and hence does not yield an $SU(2)_L$ invariant Lagrangian. For spin-1 bosons, mass terms are proportional to $m^2 A^\mu A_\mu$ which is not invariant under a gauge transformation of the field A . Therefore, such mass terms are cannot be part of the Lagrangian.

Both problems are circumvented by the Higgs field, invoking, however, independent mechanisms of mass generation for gauge bosons and for fermions [11, 12]. The Higgs Lagrangian provides mass terms for the gauge bosons introduced by spontaneous symmetry breaking. The Yukawa Lagrangian contains mass terms for fermions introduced by Yukawa couplings of the Higgs to the fermions. The price to pay for the solution of the mass problems is the addition of at least one additional $SU(2)_L$ doublet in the theory, the spin-0 Higgs field.

²Contributions from gauge-fixing terms and ghost fields are not shown.

Fermions — Spin-1/2							
1.	2.	3.	$SU(3)_C$	$SU(2)_L$	T_3	Y	Q
$\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L$	$\begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}_L$	$\begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix}_L$	$\underline{1}$	$\underline{2}$	$+1/2$	$-1/2$	0
ν_{eR}	$\nu_{\mu R}$	$\nu_{\tau R}$	$\underline{1}$	$\underline{1}$	0	0	0
e_R^-	μ_R^-	τ_R^-	$\underline{1}$	$\underline{1}$	0	-1	-1
$\begin{pmatrix} u \\ d' \end{pmatrix}_L$	$\begin{pmatrix} c \\ s' \end{pmatrix}_L$	$\begin{pmatrix} t \\ b' \end{pmatrix}_L$	$\underline{3}$	$\underline{2}$	$+1/2$	$+1/6$	$+2/3$
u_R	c_R	t_R	$\underline{3}$	$\underline{1}$	0	$+2/3$	$+2/3$
d_R	s_R	b_R	$\underline{3}$	$\underline{1}$	0	$-1/3$	$-1/3$
Gauge Bosons — Spin-1							
			$SU(3)_C$	$SU(2)_L$	T_3	Y	Q
B			$\underline{1}$	$\underline{1}$	0	0	0
$\begin{pmatrix} W^+ \\ W^3 \\ W^- \end{pmatrix}$			$\underline{1}$	$\underline{3}$	$+1$	0	$+1$
			$\underline{1}$	$\underline{3}$	0	0	0
			$\underline{1}$	$\underline{3}$	-1	0	-1
G^a			$\underline{8}$	$\underline{1}$	0	0	0
Higgs Bosons — Spin-0							
			$SU(3)_C$	$SU(2)_L$	T_3	Y	Q
$\begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$			$\underline{1}$	$\underline{2}$	$+1/2$	$+1/2$	$+1$
			$\underline{1}$	$\underline{2}$	$-1/2$	$+1/2$	0

Table 2.1: Multiplet assignments and quantum numbers of leptons ν_ℓ , ℓ^- ($\ell = e, \mu, \tau$), quarks u , d' ($u = u, c, \tau$; $d = d, s, b$), gauge bosons (B, W and G), and Higgs boson (ϕ) in the MSM. Right-handed neutrinos are hypothetical, there is no indication for their existence. The prime on the d -type quarks (d') denotes symmetry eigenstates, which arise from the mass eigenstates d by the unitary Cabibbo-Kobayashi-Maskawa quark mixing matrix, see Section 2.7. Indices L and R denote left-handed and right-handed fermions. The electromagnetic charge Q is given by $Q = T_3 + Y$.

2.5 Higgs Mechanism

In the minimal SM the Higgs sector [11, 12] is minimal, containing just one $SU(2)_L$ complex Higgs doublet, Φ . The Higgs Lagrangian has the form:

$$\mathcal{L}_{\text{Higgs}} = (\mathcal{D}^\mu \Phi)^\dagger (\mathcal{D}_\mu \Phi) - V(\Phi^\dagger \Phi), \quad (2.14)$$

where V is the $SU(2)_L$ invariant potential of the Higgs field:

$$V(\Phi^\dagger \Phi) = \mu^2 \Phi^\dagger \Phi + \lambda (\Phi^\dagger \Phi)^2. \quad (2.15)$$

For stability reasons the potential must increase for large $\Phi^\dagger \Phi$ which implies $\lambda > 0$. The mass parameter, μ^2 , however, may still be smaller than zero. In this case the potential has a non-trivial minimum $V_{\min}$ for:

$$\Phi^\dagger \Phi = -\frac{\mu^2}{2\lambda} \equiv \frac{v^2}{2} > 0, \quad (2.16)$$

where $v/\sqrt{2}$ is the vacuum expectation value of the Higgs field. The $SU(2)_L$ symmetry of the Higgs potential leads to a whole family of non-trivial minima of the Higgs potential V . Choosing a specific one as the vacuum ground state and expansion point amounts to breaking this symmetry. Since Φ is a complex $SU(2)_L$ doublet field it is possible to write it as:

$$\Phi(x) = \exp \left[i\theta_j(x) \frac{\tau_j}{2} \right] \begin{pmatrix} 0 \\ [v + \phi(x)]/\sqrt{2} \end{pmatrix}, \quad (2.17)$$

where $\theta_j(x)$, $j = 1, 2, 3$, and $\phi(x)$ correspond to the four real degrees of freedom of Φ . The exponential term is removed by a local $SU(2)_L$ gauge transformation:

$$\Phi(x) \rightarrow \Phi'(x) = \exp \left[-i\theta_j(x) \frac{\tau_j}{2} \right] \Phi(x) = \begin{pmatrix} 0 \\ [v + \phi(x)]/\sqrt{2} \end{pmatrix}, \quad (2.18)$$

leaving only one physical Higgs boson, $\phi(x)$. The other three degrees of freedom disappear due to the local gauge transformation. They reappear as mass terms and thus longitudinal degrees of freedom for three of the four gauge bosons of the electroweak gauge group G_{EW} .

2.5.1 Boson Masses

Using the above representation of the Higgs field Φ , and expanding the sum $\tau_i W_\mu^i$ in a spherical basis, $\tau_i W_\mu^i = \sqrt{2}(\tau^+ W_\mu^+ + \tau^- W_\mu^-) + \tau_3 W_\mu^3$, the Higgs Lagrangian becomes:

$$\begin{aligned} \mathcal{L}_{\text{Higgs}} = & \frac{g_2^2}{4} W_-^\mu W_\mu^+ (v + \phi)^2 + \frac{1}{8} (g_2 W_3^\mu - g_1 B^\mu) (g_2 W_\mu^3 - g_1 B_\mu) (v + \phi)^2 \\ & + \frac{1}{2} (\partial^\mu \phi) (\partial_\mu \phi) - \frac{\mu^2}{2} (v + \phi)^2 - \frac{\lambda}{4} (v + \phi)^4. \end{aligned} \quad (2.19)$$

The terms proportional to v^2 in the first line describe mass terms for spin-1 bosons. For charged spin-1 bosons, mass terms are of the form $m^2 W_-^\mu W_\mu^+$, therefore:

$$M_W = \frac{v}{2} g_2. \quad (2.20)$$

The combination of fields $Z_\mu \propto (g_2 W_\mu^3 - g_1 B_\mu)$ is identified as a neutral spin-1 boson, the Z boson. The mass term for a neutral spin-1 particle has the form $m^2 Z^\mu Z_\mu/2$, therefore:

$$M_Z = \frac{v}{2} \sqrt{g_1^2 + g_2^2}. \quad (2.21)$$

The field Z_μ arises from the original fields W_μ^3 and B_μ by a rotation:

$$\begin{pmatrix} A_\mu \\ Z_\mu \end{pmatrix} = \begin{pmatrix} \cos \theta_W & \sin \theta_W \\ -\sin \theta_W & \cos \theta_W \end{pmatrix} \cdot \begin{pmatrix} B_\mu \\ W_\mu^3 \end{pmatrix}. \quad (2.22)$$

The orthogonal combination A_μ does not appear in the Higgs Lagrangian. It corresponds to a massless spin-1 boson, the photon.

The rotation angle, θ_W , called the electroweak mixing angle, is given in terms of the electroweak couplings g_1 and g_2 :

$$\sin \theta_W \equiv \frac{g_1}{\sqrt{g_1^2 + g_2^2}} \quad \cos \theta_W \equiv \frac{g_2}{\sqrt{g_1^2 + g_2^2}} \quad \tan \theta_W \equiv \frac{g_1}{g_2}, \quad (2.23)$$

also relating the masses of the spin-1 gauge bosons W and Z:

$$\frac{M_W}{M_Z} = \cos \theta_W \quad \sin^2 \theta_W = 1 - \frac{M_W^2}{M_Z^2}. \quad (2.24)$$

The remaining terms of the Higgs Lagrangian govern the dynamics of the surviving spin-0 Higgs boson ϕ , namely its kinetic energy, self-couplings and couplings to the gauge bosons W and Z, predicting trilinear and quadrilinear couplings of the Higgs boson to itself and to the massive gauge bosons. In particular, the mass of the Higgs boson is fixed by the terms containing only ϕ^2 as fields. Since the mass term for a scalar spin-0 boson has the form $m^2\phi^2/2$, the mass of the Higgs boson is:

$$M_H = \sqrt{-2\mu^2} = \sqrt{2\lambda}v. \quad (2.25)$$

The measured masses of the bosons are reported in Table 2.2.

2.5.2 Fermion Masses

Mass terms for fermions are introduced via Yukawa coupling of the left and right-handed fermion fields to the $SU(2)_L$ doublet Higgs field. For left-handed quark and lepton doublets, q_L and ℓ_L , and right-handed singlets, u_R , d_R , ν_R , and ℓ_R , one has for each generation:

$$\begin{aligned} \mathcal{L}_{\text{Yukawa}} = & -g_\nu \bar{\ell}_L \tilde{\Phi} \nu_R - g_\ell \bar{\ell}_L \Phi e_R \\ & -g_u \bar{q}_L \tilde{\Phi} u_R - g_d \bar{q}_L \Phi d_R + \text{Hermitian conjugate}, \end{aligned} \quad (2.26)$$

where $\tilde{\Phi} \equiv i\tau_2 \Phi^*$. If right-handed neutrinos are assumed to be absent, implying massless neutrinos, ν_R does not exist and $g_\nu = 0$. Using the special representation of the Higgs field Φ , this simplifies to:

$$\mathcal{L}_{\text{Yukawa}} = -\frac{v + \phi}{\sqrt{2}} (g_\nu \bar{\nu} \nu + g_\ell \bar{\ell} \ell + g_u \bar{u} u + g_d \bar{d} d), \quad (2.27)$$

which is left-right symmetric. The terms proportional to v have the form of mass terms for fermions, $m_f \bar{\Psi} \Psi$, with masses:

$$m_f = \frac{v}{\sqrt{2}} g_f. \quad (2.28)$$

The terms proportional to ϕ describe fermion-Higgs couplings with a coupling strength m_f/v . This is the reason for stating that the Higgs couples to the mass of a particle. The measured masses of the fermions are reported in Table 2.2. The mass differences between fermions are huge, it is not understood why the Yukawa couplings are so different between the fermions.

Particle	Mass	Particle	Mass	Particle	Mass
Fermions — Spin-1/2					
ν_e	$< 15 \text{ eV}$	ν_μ	$< 0.17 \text{ MeV}$	ν_τ	$< 18.2 \text{ MeV}$
e	0.510999 MeV	μ	105.6584 MeV	τ	1777 GeV
u	$1.5 - 5 \text{ MeV}$	c	$1.1 - 1.4 \text{ GeV}$	t	$173.8 \pm 5.2 \text{ GeV}$
d	$3 - 9 \text{ MeV}$	s	$60 - 170 \text{ MeV}$	b	$4.1 - 4.4 \text{ GeV}$
Gauge Bosons — Spin-1					
γ	$< 2 \cdot 10^{-16} \text{ eV}$	W	$80.41 \pm 0.10 \text{ GeV}$	Z	$91.187 \pm 0.007 \text{ GeV}$
Higgs Boson — Spin-0					
H	$> 77.5 \text{ GeV}$				

Table 2.2: Masses or mass limits of neutrinos, charged leptons, quarks, gauge bosons and Standard Model Higgs boson as of early 1998 [31]. Improved results on M_t , M_Z , M_W and M_H will be given in this review.

2.6 Quantum Chromodynamics

In contrast to the electroweak sector of the MSM, the strong interaction does not participate in the process of symmetry breaking. The Higgs field is a singlet under the symmetry group of quantum chromodynamics, $SU(3)_C$ [13–15]. Thus the eight gluons of QCD remain massless. QCD enters the study of the electroweak interaction through higher-order radiative corrections involving quark-gluon and multi-gluon vertices as discussed in Sections 2.11.2 and 3.1.3.

Unlike QED each quark carries the same strong colour charge. The analogon to the finestructure constant of QED is given by the strong coupling constant, $\alpha_S = g_3^2/(4\pi)$. Since $\alpha_S \gg \alpha_{em}$ and because of gluon-selfinteractions, QCD has a much richer structure than QED. The strong interaction is assumed to confine the coloured quarks and gluons [14], so that only white, colourless bound states are observable. These are systems of qqq or $q\bar{q}$ bound by the strong force, which correspond to baryons and mesons. Because of gluon-selfinteractions, also glueballs may exist, which are colourless bound states solely consisting of gluons. Studies and reviews of experimental results and tests of QCD are given in [32, 33].

2.7 Fermion Mixing

As there exist more than one generation of fermions, the electroweak symmetry eigenstates of the theory may be different from the mass eigenstates. Mixing may occur between the fermions having the same quantum numbers, *i.e.*, within the set of neutrinos ν , charged leptons ℓ , up-type quarks u , and down-type quarks d , respectively. The bases of symmetry eigenstates and mass eigenstates are related by a unitary transformation in flavour space, $S_{L,R}^\alpha$, $\alpha = u, d, \ell, \nu$.

The electromagnetic and neutral weak currents are unaffected, as the transformations $S_{L,R}^\alpha$ and $S_{L,R}^{\alpha\dagger}$ acting on Ψ_α and $\bar{\Psi}_\alpha$ commute with the Dirac γ matrices to yield $S_{L,R}^{\alpha\dagger} S_{L,R}^\alpha = 1$. Thus the absence of flavour-changing neutral currents at tree level is preserved. The charged weak current, however, is affected. Within the leptonic sector, the basis of the three neutrinos can simply be changed by the combined transformation $S_L^{\ell\dagger} S_L^\nu$ without any observable effects as long as all neutrinos have the same mass. Otherwise, this is not possible and the treatment is the same as in the quark sector discussed in the following.

By convention, the S matrices are collected to assign the mixing to the $Q = -1/3$ down-type quarks:

$$d_{L,i} \rightarrow (S_L^{u\dagger} S_L^d)_{ij} d_{L,j}. \quad (2.29)$$

The product of the two S matrices is called the Cabibbo-Kobayashi-Maskawa quark mixing matrix, $V_{CKM} = S_L^{u\dagger} S_L^d$ [34]. For three generations of fermions, the unitary V_{CKM} matrix describes the following transformation between symmetry eigenstates q' and mass eigenstates q :

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \cdot \begin{pmatrix} d \\ s \\ b \end{pmatrix}. \quad (2.30)$$

The matrix elements V_{ij} of V_{CKM} are not predicted by theory but must be inferred from experimental results on the charged weak current. They enter in processes involving the charged intermediate vector boson, $W^\pm$. In case of three or more generations, the general quark mixing matrix contains one or more non-trivial complex phases which lead to CP violating effects in the SM.

2.8 Currents in Electroweak Theory

Electroweak interactions between fermions, mediated by γ , Z , $W^\pm$ or H exchange, are usually written in terms of currents, *i.e.*, terms bilinear in the fermion fields. With the help of Dirac's γ matrices, the

most general term bilinear in fermion fields can be decomposed into a linear combination of 16 basic terms, which are defined by their transformation properties under Lorentz transformations. These 16 terms are: scalar ($\bar{\Psi}1\Psi$, one term), pseudoscalar ($\bar{\Psi}\gamma^5\Psi$, one term), vector ($\bar{\Psi}\gamma^\mu\Psi$, four terms), axial-vector ($\bar{\Psi}\gamma^\mu\gamma^5\Psi$, four terms), and tensor ($\bar{\Psi}\frac{i}{2}[\gamma^\mu, \gamma^\nu]\Psi$, six terms).

Using the following natural definitions for hypercharge and left-handed charged and neutral weak currents:

$$J_\mu^Y \equiv \bar{\Psi}\gamma_\mu Y\Psi \quad (2.31)$$

$$J_\mu^\pm \equiv \bar{\Psi}_L\gamma_\mu\tau^\pm\Psi_L \quad (2.32)$$

$$J_\mu^3 \equiv \bar{\Psi}_L\gamma_\mu\frac{\tau^3}{2}\Psi_L, \quad (2.33)$$

and collecting the pieces of the Lagrangian which lead to couplings of fermions to gauge-bosons:

$$\mathcal{L}_{\text{int}} = -\frac{g_1 g_2}{\sqrt{g_1^2 + g_2^2}} A_\mu J_{EM}^\mu - \sqrt{g_1^2 + g_2^2} Z_\mu J_{NC}^\mu - \frac{g_2}{\sqrt{2}} (W_\mu^+ J_+^\mu + W_\mu^- J_-^\mu), \quad (2.34)$$

one can derive the fermionic currents that couple to the gauge-boson fields A , Z and W :

$$J_{EM}^\mu = J_3^\mu + J_Y^\mu = Q\bar{\Psi}\gamma^\mu\Psi \quad (2.35)$$

$$J_{NC}^\mu = J_3^\mu - \sin^2\theta_W J_{EM}^\mu = \bar{\Psi}\gamma^\mu\frac{g_V - g_A\gamma^5}{2}\Psi \quad (2.36)$$

$$J_\pm^\mu = \bar{\Psi}\gamma^\mu\frac{V - A\gamma^5}{2}\tau^\pm\Psi, \quad (2.37)$$

respectively. In particular, the photon couples with equal strength to left-handed and right-handed charged fermions and does not couple to neutrinos. Denoting with Q the electromagnetic charge of a fermion in units of the positron charge e , $e^2/(4\pi) = \alpha_{em}$ being the finestructure constant, the relation between e and the coupling constants g_1 and g_2 is given by:

$$e = \frac{g_1 g_2}{\sqrt{g_1^2 + g_2^2}} = g_1 \cos\theta_W = g_2 \sin\theta_W. \quad (2.38)$$

The above equations define the customary vector and axial-vector coupling constants of the weak neutral current, which depend on the fermion species f :

$$g_{Af} \equiv T_3^f \quad g_{Vf} \equiv T_3^f - 2q_f \sin^2\theta_W, \quad (2.39)$$

where q_f is the electromagnetic charge of fermion f in units of the positron charge e . The vector and axial-vector coupling constants of the weak charged current are identical for all fermions:

$$A \equiv 1 \quad V \equiv 1. \quad (2.40)$$

The relative sign between the vector and axial-vector part of the charged weak current is the reason for stating that the charged weak interaction has a $V - A$ structure. The corresponding left-handed and right-handed couplings are:

$$g_{Lf} \equiv T_3^f - q_f \sin^2\theta_W \quad g_{Rf} \equiv -q_f \sin^2\theta_W, \quad (2.41)$$

for the neutral weak current, and:

$$L \equiv 1 \quad R \equiv 0, \quad (2.42)$$

for the charged weak current.

2.9 Electroweak Mixing Angle

There are three relations involving the electroweak mixing angle:

1. The relation between the electromagnetic coupling, e , and the weak couplings, g_1 and g_2 .
2. The ratio between the $SU(2)_L$ and $U(1)_{EM}$ component of the neutral weak current.
3. The ratio of the heavy gauge-boson masses.

The latter two are experimentally accessible through the measurement of the neutral weak current at SLC and LEP-I, and the measurement of the W and Z boson masses at the SPS, TEVATRON and LEP. Radiative corrections as discussed in the following modify these relations in a different way, leading to different possibilities in defining the electroweak mixing angle.

2.10 Four-Fermion Theory

E. Fermi made a first attempt to formulate a theory of charged weak interactions in 1934 [35]. As an ansatz for the Lagrangian, he used a product of two currents, i.e., two terms bilinear in the fermion fields. They are evaluated at the same space-time point, therefore describing a vertex with four fermion lines. Initially, Fermi connected the two currents by a vector interaction. Experimental results collected in the following years, in particular the discovery of parity violation in charged weak decays by C.S. Wu in 1957 [6], lead R. Feynman and M. Gell-Mann to suggest in 1958 [36], that the interaction Lagrangian should be the product of two $(V - A)$ currents. The coupling constant connecting both currents in the four-fermion matrix element is nowadays called the Fermi constant, G_F . Within the MSM, the charged weak interaction is still described by two $(V - A)$ currents, now coupled by the propagator of the charged intermediate vector boson $W^\pm$. The two different concepts are compared in Figure 2.1.

The experiments at that time (1930–1960) measured processes, which take place at low energies and low momentum transfers $q^2 \rightarrow 0$ [37]. In this limit, the propagator term $G_V^{\mu\nu}(q^2)$ of a heavy spin-1 vector boson V with four-momentum q^μ becomes:

$$G_V^{\mu\nu}(q^2) = \frac{-g^{\mu\nu} + q^\mu q^\nu / M_V^2}{q^2 - M_V^2} \xrightarrow{q^2 \rightarrow 0} \frac{g^{\mu\nu}}{M_V^2}. \quad (2.43)$$

Thus the low energy limit of the MSM recovers the old four-fermion theory, connecting the Fermi constant G_F to the mass of the intermediate W boson:

$$\frac{G_{CC}(0)}{\sqrt{2}} = \frac{g_2^2}{8M_W^2} = \frac{\pi\alpha_{em}}{2\sin^2\theta_W M_W^2} = \frac{1}{2v^2} = \frac{G_F}{\sqrt{2}}. \quad (2.44)$$

Hence the two formulations compared in Figure 2.1 are equivalent at low energies, but differ strongly at high centre-of-mass energies, $\sqrt{s}$. While the predictions of cross sections based on the four-fermion theory violate the unitarity limit³ since $\sigma \propto G_F^2 s$, those based on the SM lead to well behaved expressions.

The Fermi constant is determined precisely in muon decays. As discussed in Appendix A, QED radiative corrections specific to the muon decay process [38–40] as well as fermion-mass and W-propagator effects are explicitly corrected for. The quantity G_F absorbs universal radiative corrections only and is thus a process independent quantity. The result for G_F is [40]:

$$G_F = 1.16637(1) \cdot 10^{-5} \text{GeV}^{-2}, \quad (2.45)$$

where the error is dominated by the experimental error on the measured lifetime of the muon [31].

³The unitarity limit is a consequence of the conservation of probability in scattering processes, i.e., the intensity of an outgoing partial wave cannot exceed the intensity of the corresponding incoming partial wave. For a process with angular momentum J , this requires $\sigma(s) < 16\pi(2J+1)/s$.

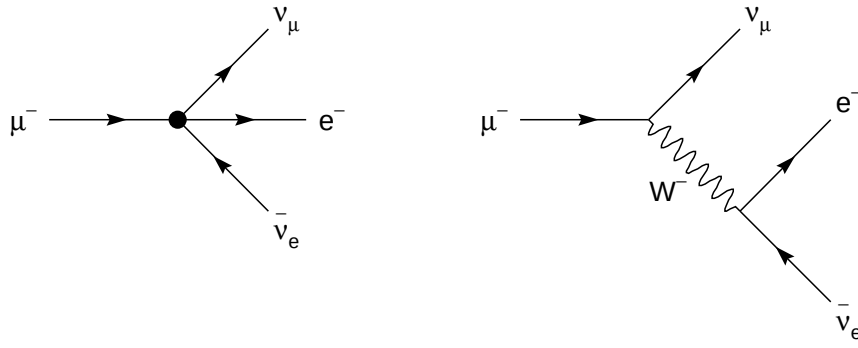


Figure 2.1: The decay of the muon. Left: Four-fermion theory; right: minimal SM. The two charged weak currents are $\mu^- \nu_\mu$ and $\nu_e e^-$.

2.11 Interdependence of Gauge-Boson Masses

2.11.1 Born Term

Taking the limit $q^\lambda \rightarrow 0$ for the neutral weak interaction yields:

$$\frac{G_{\text{NC}}(0)}{\sqrt{2}} = \frac{g_1^2 + g_2^2}{8M_Z^2} = \frac{g_2^2}{8\cos^2\theta_W M_Z^2} = \frac{G_F}{\sqrt{2}}. \quad (2.46)$$

The ratio of the neutral to charged weak current at zero momentum transfer is therefore:

$$\rho \equiv \frac{G_{\text{NC}}(0)}{G_{\text{CC}}(0)} = \frac{g_1^2 + g_2^2}{g_2^2} \frac{M_W^2}{M_Z^2} = \frac{1}{\cos^2\theta_W} \frac{M_W^2}{M_Z^2} = 1, \quad (2.47)$$

to lowest order. The different boson masses appearing in the vector-boson propagators, M_W and M_Z , are compensated by the different charged and neutral weak current couplings, so that the overall coupling strength of the neutral and charged weak current are equal. The precisely known Fermi constant G_F thus establishes an interdependence between M_W and M_Z :

$$M_W^2 \left(1 - \frac{M_W^2}{M_Z^2}\right) = \frac{\pi\alpha_{em}}{\sqrt{2}G_F}, \quad (2.48)$$

to lowest order. In terms of the electroweak mixing angle, Equation 2.24, this relation becomes:

$$\frac{\pi\alpha_{em}}{\sqrt{2}G_F} = M_W^2 \sin^2\theta_W = M_Z^2 \sin^2\theta_W \cos^2\theta_W, \quad (2.49)$$

to lowest order.

2.11.2 Radiative Corrections

The $M_W - M_Z$ interdependence arising from G_F is modified by higher-order radiative corrections:

$$M_W^2 \left(1 - \frac{M_W^2}{M_Z^2}\right) = \frac{\pi\alpha_{em}}{\sqrt{2}G_F} \cdot \frac{1}{1 - \Delta r}. \quad (2.50)$$

The corrections arise due to propagator corrections as shown in Figure 2.2. The resulting higher-order quantum correction Δr consists of a QED contribution, denoted as $\Delta\alpha_{em}$, and a weak contribution, denoted as Δr_w :

$$\frac{1}{1 - \Delta r} = \frac{1}{1 - \Delta\alpha_{em}(M_Z^2)} \cdot \frac{1}{1 - \Delta r_w}, \quad (2.51)$$

which are discussed in the following [41, 42].

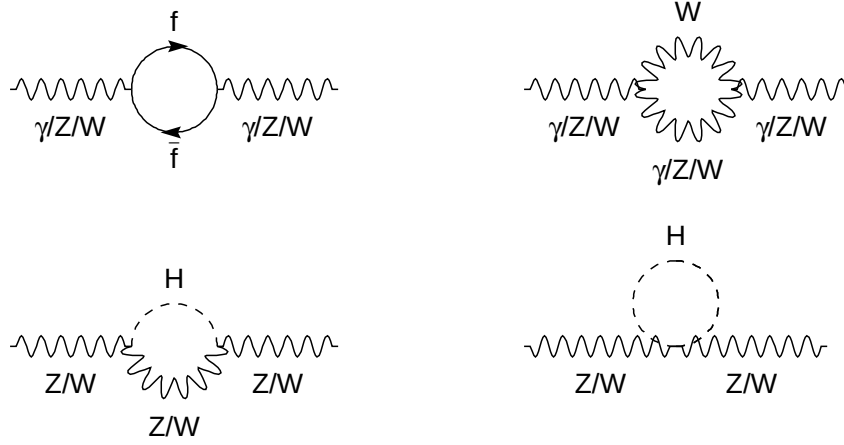


Figure 2.2: Higher-order corrections to the gauge-boson propagators arising due to insertion of fermion and boson loops. Contributions from unphysical degrees of freedom are not shown. The loop insertions may be iterated. At two-loop order, QCD enters through gluon exchange in the quark loops.

2.11.2.1 QED Corrections

The QED contribution arises from the photonic vacuum polarisation, also called photon self energy, consisting of fermion-loop insertions in the propagator of the photon. This effect is expected in any theory containing QED. The correction is usually reinterpreted as the dependence of the electromagnetic coupling strength on the energy of the probing photon, leading to an effective finestructure constant, α_{em} , running with momentum transfer:

$$\alpha_{em} \equiv \alpha_{em}(0) \rightarrow \alpha_{em}(s) = \frac{\alpha_{em}}{1 - \Delta\alpha_{em}(s)}, \quad (2.52)$$

where $1/\alpha_{em}(0) = 137.0359895(61)$ [31].

Each light charged fermion, $4m_f^2 < s$, contributes to $\Delta\alpha_{em}(s)$ by an amount of:

$$\Delta\alpha_{em}^{(f)}(s) = \frac{\alpha_{em}}{3\pi} N_C^f q_f^2 \left[-\frac{5}{3} - \frac{4m_f^2}{s} + \beta_f \left(1 + \frac{2m_f^2}{s} \right) \ln \frac{1 + \beta_f}{1 - \beta_f} \right] \quad (2.53)$$

$$\rightarrow \frac{\alpha_{em}}{3\pi} N_C^f q_f^2 \left[\ln \frac{s}{m_f^2} - \frac{5}{3} \right] \quad \text{for } 4m_f^2 \ll s, \quad (2.54)$$

where N_C^f is the QCD colour factor, $N_C^f = 1$ for leptons and $N_C^f = 3$ for quarks, and $\beta_f = \sqrt{1 - 4m_f^2/s}$ is the fermion velocity. The running of $\alpha_{em}(s)$ is driven by the light charged fermions, while heavy charged fermions, $4m_f^2 > s$, decouple and are not visible:

$$\Delta\alpha_{em}^{(f)}(s) = -\frac{\alpha_{em}}{3\pi} N_C^f q_f^2 \frac{3}{5} \frac{s}{m_f^2} \quad \text{for } 4m_f^2 \gg s. \quad (2.55)$$

For Δr , $\alpha_{em}(s)$ must be evolved from the Thomson limit $s = 0$, where α_{em} is defined, to the scale $s = M_Z^2$ set by the Z mass. The contribution of the three charged leptons is calculated up to three-loop order, $\Delta\alpha_{em}^{(e\mu\tau)}(M_Z^2) = 0.03150$ with negligible uncertainty [43]. The top contribution is small, $\Delta\alpha_{em}^{(t)}(M_Z^2) = -0.00007(1)$, showing numerically the decoupling of the heavy top quark.

For the light quarks $q = d, u, s, c, b$ with $m_q \ll M_Z$, large QCD corrections make the above expression for $\Delta\alpha_{em}^{(q)}$ unreliable. Instead, the contribution of the five light quarks is calculated based

on the measured cross section of e^+e^- annihilations into hadrons at low centre-of-mass energies, $\sqrt{s} \ll M_Z$, as discussed in Appendix B. Since it is derived from data, the hadronic contribution includes all corrections to all orders. The hadronic contribution, $\Delta\alpha_{em}^{(\text{had})}(M_Z^2) = +0.02804(65)$ [44, 45], is nearly as large as the leptonic contribution. Its error, arising from the accuracy of the hadronic cross section measurement at low $\sqrt{s} \ll M_Z$, completely dominates the error on $\Delta\alpha_{em}(M_Z^2)$. This is the reason why the error on $\alpha_{em}(M_Z^2)$ is large despite α_{em} being known very precisely [43–45]:

$$\frac{1}{\alpha_{em}(M_Z^2)} = 128.886 \pm 0.090. \quad (2.56)$$

Recent theoretical developments aiming at reducing the error on $\Delta\alpha_{em}^{(\text{had})}(M_Z^2)$ are discussed in Appendix B.

The correction $\Delta\alpha_{em}$ arises from the well known theory of QED. Since charged particles with masses $4m_f^2 \gg s$ do not contribute to $\Delta\alpha_{em}(s)$, $\alpha_{em}(M_Z^2)$ is insensitive to new physics in form of new particles with large masses. Taking $\alpha_{em}(M_Z^2)$ as an input parameter, the interdependence between the mass of the W and Z boson now reads:

$$M_W^2 \left(1 - \frac{M_W^2}{M_Z^2}\right) = \frac{\pi\alpha_{em}(M_Z^2)}{\sqrt{2}G_F} \cdot \frac{1}{1 - \Delta r_w}. \quad (2.57)$$

The remaining quantum correction Δr_w is discussed in the following.

2.11.2.2 Weak Corrections

Within the MSM, the weak correction Δr_w contains a leading term with $\Delta\rho$, and a remainder, $\Delta r_{\text{remainder}}$:

$$\Delta r_w = -\cot^2 \theta_W \Delta\rho + \Delta r_{\text{remainder}}. \quad (2.58)$$

The ρ parameter, $\rho = 1/(1 - \Delta\rho)$, is defined previously as the ratio of the neutral weak and charged weak current amplitude in the limit of zero momentum transfer. To lowest order, $\Delta\rho = 0$. The corrections to the ρ parameter arise from loops in the propagators of the heavy gauge bosons W and Z involving fermions and the Higgs boson. Each weak isospin doublet of left-handed fermions with masses $m_1 \geq m_2$ contributes to $\Delta\rho$ by an amount of [46]:

$$\Delta\rho^{(f)} = N_C^f \frac{G_F}{8\sqrt{2}\pi^2} \cdot \left(m_1^2 + m_2^2 - \frac{2m_1^2 m_2^2}{m_1^2 - m_2^2} \ln \frac{m_1^2}{m_2^2} \right) \quad (2.59)$$

$$\rightarrow \begin{cases} 0 & \text{for } m_1 = m_2 \\ N_C^f \frac{G_F m_1^2}{8\sqrt{2}\pi^2} & \text{for } m_1 \gg m_2 \end{cases}. \quad (2.60)$$

In contrast to $\Delta\alpha_{em}^{(f)}$, $\Delta\rho^{(f)}$ is negligible for light fermions but large for heavy fermions with light weak-isospin partners such as the top quark. Since for large top-quark masses the corrections are large, two-loop corrections need to be taken into account as well, reducing slightly the M_t dependence of the leading $\Delta\rho^{(t)}$ correction given above. For large Higgs-boson masses, the Higgs contribution to $\Delta\rho$ is given by:

$$\Delta\rho^{(H)} = -3 \frac{G_F M_W^2}{8\sqrt{2}\pi^2} \tan^2 \theta_W \left(\ln \frac{M_H^2}{M_W^2} - \frac{5}{6} \right) \quad \text{for } M_H \gg M_W. \quad (2.61)$$

Top-quark and Higgs-boson contributions to Δr also appear in $\Delta r_{\text{remainder}}$. The combined contributions to Δr arising from the top quark and the Higgs boson are given by:

$$\Delta r^{(t)} = -\frac{G_F M_W^2}{8\sqrt{2}\pi^2} \left[3 \cot^2 \theta_W \frac{M_t^2}{M_W^2} + 2 \left(\cot^2 \theta_W - \frac{1}{3} \right) \ln \frac{M_t^2}{M_W^2} + \right. \quad (2.62)$$

$$\left. \frac{4}{3} \ln \cos^2 \theta_W + \cot^2 \theta_W - \frac{7}{9} \right] \quad (2.63)$$

$$\Delta r^{(H)} = \frac{11}{3} \frac{G_F M_W^2}{8\sqrt{2}\pi^2} \left(\ln \frac{M_H^2}{M_W^2} - \frac{5}{6} \right) \quad \text{for } M_H \gg M_W. \quad (2.64)$$

Electroweak radiative corrections are calculated to one-loop order and leading two-loop order. Recently, subleading two-loop corrections are also calculated [47].

2.11.2.3 Sensitivity to Top Quark and Higgs Boson

The one-loop weak corrections discussed above show a quadratic dependence on the top-quark mass, but only a logarithmic dependence on the Higgs-boson mass, *i.e.*, the effect of a heavy Higgs boson is screened [48]. This so-called Veltman screening is due to the accidental $SU(2)_R$ symmetry of the Higgs sector of the MSM. Terms quadratic in M_H only appear at two-loop order and are numerically small.

In order to show quantitatively the sensitivity of Δr to its various contributions, the variations δ in $\Delta\alpha_{em}$, M_t and M_H are determined which lead to the same uncertainty $\delta\Delta r$ on Δr . Taking only the leading dependence into account, one obtains:

$$\frac{\partial\Delta r}{\partial\Delta\alpha_{em}}\delta\Delta\alpha_{em} = +1\delta\Delta\alpha_{em} \quad (2.65)$$

$$\frac{\partial\Delta r}{\partial M_t}\delta M_t = -\frac{3\cot^2\theta_W G_F}{4\sqrt{2}\pi^2} M_t \delta M_t = -0.0019 \left(\frac{M_t}{175 \text{ GeV}} \right) \left(\frac{\delta M_t}{5 \text{ GeV}} \right) \quad (2.66)$$

$$\frac{\partial\Delta r}{\partial M_H}\delta M_H = +\frac{11G_F M_W^2}{12\sqrt{2}\pi^2} \frac{\delta M_H}{M_H} = +0.0050 \left(\frac{\delta M_H}{M_H} \right). \quad (2.67)$$

While the sensitivity to M_t is large, the sensitivity to M_H is rather weak, for example, a 40% uncertainty on M_H causes an effect as large as a 3% uncertainty on M_t . The error on the hadronic vacuum polarisation of ± 0.00065 [44, 45] is equivalent to an error of 1.7 GeV on M_t and an error of 13% on M_H .

2.12 Standard Model Input Parameters

The *a-priori* unknown parameters of the Standard Model Lagrangian as introduced above are the coupling constants associated to the three gauge groups, g_1 , g_2 and g_3 , the parameters describing the Higgs potential, μ^2 and λ , and the coupling constants of the Yukawa coupling between the Higgs boson and the fermions, g_f . For charged-current processes containing $Wq\bar{q}'$ vertices, the elements of the Cabibbo-Kobayashi-Maskawa quark mixing matrix appear in addition.

Renormalisability of the Standard Model ensures that any observable is calculable in terms of these parameters to any order of perturbation theory. However, it is useful to change the input parameters to more physical parameters, in particular if some of these parameters are also precisely measured. Such a set consists, for example, of the QED and QCD coupling constants, $\alpha_{em}(M_Z^2)$ and $\alpha_S(M_Z^2)$, the boson masses M_W , M_Z , M_H , and the fermion masses m_f . Because the fermion masses with the exception of the top quark are small compared to the centre-of-mass energy of high-energy interactions and known well enough, there are essentially six relevant Standard Model parameters: α_{em} , α_S , M_W , M_Z , M_H and M_t . All of them should be measured accurately.

The precisely-known Fermi constant G_F usually replaces M_W in this set so that M_W is then calculated. In case of the running electromagnetic finestructure constant, the top-quark contribution is removed. Because it is M_t dependent, it is calculated as a function of the Standard Model input parameter M_t and added explicitly to the five-flavour $\alpha_{em}^{(5)}(M_Z^2)$ taken as the Standard Model input parameter.

2.13 Status of the Standard Model

The MSM as a theory describes all experimental results in particle physics very successfully. Nevertheless it is regarded as an incomplete theory, because it does not include gravity. Another drawback is that it contains many parameters which are not predicted or calculable within the MSM but must be inferred from experiment, in particular number of families, masses and quantum numbers of particles, coupling constants and mixing angles.

Physical observables such as cross sections are calculated in terms of these parameters. Measurements of physical observables then lead to a determination of these parameters. Consistency between the values derived from various measurements constitutes a test of the MSM.

2.13.1 Missing Particles

The most recently discovered fundamental particle is the heaviest known elementary particle, the top quark, with a mass of about 175 GeV. It completes the third generation of quark families. Measurements of Z boson decays as discussed in this article show, that there are no additional fermion generations or families, at least not with light neutrinos. The particles reported in Table 2.1 have all been directly observed experimentally with two exceptions, the τ neutrino and the Higgs boson.

2.13.1.1 The τ Neutrino

Measurements of Z boson decays as discussed in this article require the existence of a third light neutrino species besides the electron neutrino and the muon neutrino. The existence of the τ neutrino is also inferred from measurements of τ -lepton decay properties relating to the weak isospin of the τ lepton. These measurements confirm the assignment of the τ^- to a weak-isospin doublet, which implies the existence of its isopartner, by definition the τ neutrino, which is distinct from the electron-neutrino and the muon-neutrino. With this hypothesis measurements of τ decays yield a mass limit of [31]:

$$m_{\nu_\tau} < 0.0182 \text{ GeV} \quad (95\% \text{ CL}). \quad (2.68)$$

Experimentally, the difficulty in observing and identifying the τ neutrino directly arises from the fact that it is produced and interacting only weakly. The Fermilab experiment E872, called DONUT for direct observation of ν_τ [49], aims to measure the charged-current neutrino-nucleon reaction $\nu_\tau \mathcal{N} \rightarrow \tau \mathcal{X}$ through identification of the final-state charged τ lepton via its characteristic decay. For a SM τ neutrino, the total data sample is expected to establish the signal unambiguously.

2.13.1.2 The Higgs Boson

The existence of the Higgs boson is required within the theoretical framework of the MSM. The theory needs a mathematically consistent mechanism for the generation of fermion and gauge-boson masses, for which the Higgs mechanism offers an elegant and simple solution compatible with experimental results. On theoretical grounds the Higgs boson must have a mass of less than about 1 TeV [50]. The analysis of precision measurements of electroweak parameters in the framework of the MSM constraints M_H through Higgs-boson dependent electroweak radiative corrections, as discussed in this article.

The MSM Higgs boson as a free particle has been searched for directly without success. It must be so heavy that it cannot be produced at current colliders at a measurable rate. A mass limit of:

$$M_H > 90 \text{ GeV} \quad (95\% \text{ CL}), \quad (2.69)$$

is obtained from negative direct searches at LEP-II [51].

2.13.2 Extended Higgs Sector

The MSM is minimal in the sense that its Higgs sector is the smallest one which accommodates spontaneous symmetry breaking and mass generation. The Higgs sector might be more complicated, including additional doublets or consisting of larger multiplets, e.g., triplets of Higgs bosons. If there are several Higgs multiplets the gauge boson mass relation is modified as follows:

$$\frac{M_W^2}{M_Z^2} = \rho_0 \cos^2 \theta_W, \quad (2.70)$$

where:

$$\rho_0 = \frac{\sum_i v_i^2 (T^2(i) - T_3^2(i))}{2 \sum_i v_i^2 T_3^2(i)}, \quad (2.71)$$

and where v_i is the vacuum expectation value of the charge-zero component of the Higgs multiplet i with weak isospin $T^2(i) = t(i)(t(i) + 1)$ and third component $T_3(i) = t_3(i)$. Within the MSM, $\rho_0 = 1$, where the subscript 0 denotes the lowest-order calculation. Any deviation from $\rho_0 = 1$ signals an extended Higgs sector, for example $\rho_0 = 1/2$ for a Higgs triplet, and $0.5 < \rho_0 < 1$ for a mixture of doublet and triplet Higgs multiplets. In such a case ρ_0 becomes an additional independent parameter of the theory [52]. On top of such Born-term effects, higher-order electroweak radiative corrections as discussed in Sections 2.11.2 and 3.1.3 modify ρ in the order of a few permille already in the minimal Standard Model.

2.13.3 Conceptual Problems of the Standard Model

Particular fine-tuning problems arise with the values of some parameters, namely the CP-violating phase of QCD [53], the problem of massive scalar particles, often also referred to as naturalness or hierarchy problem [54], and the cosmological constant problem [55].

The theory of QCD has a non-trivial topological structure, which leads to an additional contribution to the QCD Lagrangian of $\Delta\mathcal{L}_{QCD} = \theta \frac{g_3^2}{64\pi^2} \epsilon_{\mu\nu\rho\sigma} G_{\mu\nu}^a G_{\rho\sigma}^a$, which is P-odd and T-odd, thus CP violating. Such a θ term would create a non-vanishing electric dipole moment of the neutron. As this has not been observed, an upper limit of $|\theta| < 10^{-10}$ is derived for a parameter, which a priori is allowed to have any value between 0 and 2π .

Corrections to the Higgs-boson propagator arising from diagrams similar to the ones shown in Figure 2.2 imply a shift of the Higgs mass between bare mass and physical mass which grows quadratically with the upper cut-off in the integral over the loop momentum. For this cut-off in the order of the Planck mass, $M = M_{Planck} = 10^{19}$ GeV, the bare mass must be negative and fine-tuned to more than 30 digits in order to obtain a physical Higgs mass in the order of 0.1 TeV to 1 TeV.

The Higgs potential in the minimum contains a constant term, $V_0 = -\mu^2 v^2/2 = M_H^2 v^2/4$. This energy density contributes as a constant term to the vacuum energy density and thus to the cosmological constant introduced in the general theory of relativity, $\Lambda \propto V_0$, by an amount which is about 50 orders of magnitude larger than the experimental limit on the cosmological constant Λ . Thus there must exist some new aspects associated with gravity which suppresses or cancels this contribution to an accuracy of 1 in 10^{50} .

2.13.4 Extensions of the Standard Model

The problems discussed above lead to fine-tuning requirements that are mathematically possible but regarded as physically unnatural. In order to evade them, new physics beyond the Standard Model is required already at the TeV scale rather than at the Planck scale of 10^{19} GeV where gravity must be included.

Various hypothetical extensions of the Standard Model are constructed, for example, supersymmetry, supergravity, technicolour, grand-unified theories or string theories; see, for example, [56, 57] for recent reviews. Any extended theory is severely constrained by the fact that new physics effects are not allowed to perturb the excellent agreement of the minimal Standard Model with the experimental measurements. No experimental evidence for extended theories has been found so far.

Chapter 3

Physics at Electron-Positron Colliders

This chapter discusses the physics of high-energy e^+e^- collisions. Quantitative calculations are performed with the semianalytical programs SMATASY [20], TOPAZ0 [21] and ZFITTER [22], discussed in Section 3.3.2.

At electron-positron colliders the initial state of an interaction consists of electrons and positrons, which to our knowledge are pointlike particles. Being charged leptons, they interact only electroweakly. The lowest-order tree-level Feynman diagrams for e^+e^- interactions as derived from the MSM Lagrangian discussed above fall into three classes:

1. s -channel scattering through neutral bosons γ , Z , H ;
2. t -channel scattering through bosons γ , Z , H , $W^\pm$; or fermions e , ν_e ;
3. u -channel scattering through fermion e .

The Higgs coupling to fermion pairs is proportional to the ratio of the fermion mass to the Z boson mass, m_f/M_Z . Therefore, Higgs exchange with a direct coupling of the Higgs boson to the initial-state leptons is suppressed by a factor of m_e^2/M_Z^2 and can safely be neglected. At lowest order, the final state may either be a single massive boson decaying to a fermion-antifermion pair, or a pair of photons, or a pair of massive bosons each decaying to a $f\bar{f}$ pair and thus leading to a four-fermion final state.

In the absence of transverse polarisation of the initial state, the non-trivial degrees of freedom in the final state for a given centre-of-mass energy are twofold:

1. The polar scattering angle, θ , of the fermion or boson with respect to the beam electron in the centre-of-mass system, $\cos\theta \equiv \cos\theta_f = -\cos\theta_{\bar{f}}$, as shown in Figure 3.1. The measurement of $\cos\theta$ is rather simple, especially if only the absolute value, *i.e.*, the event axis, is needed.
2. The helicities of the two final-state particles, which are correlated by angular-momentum conservation. The measurement of the final-state helicities is rather complicated and routinely done only for the $\tau^+\tau^-$ and W^+W^- final states.

The cross sections of fermion-pair and boson-pair production as a function of the centre-of-mass energy are compared in Figure 3.2. The Z boson resonance leads to a sharp enhancement of the cross section, up to 30 nb at the pole $\sqrt{s} = M_Z$. The production of W -pairs and Z -pairs occurs with a much smaller cross section, in the order of 20 pb and 1 pb. The energy dependence shows the expected threshold behaviour at $\sqrt{s} = 2M_W$ and $\sqrt{s} = 2M_Z$, respectively. The calculations of these cross sections and their dependence on the parameters of the Standard Model will be discussed in the following.

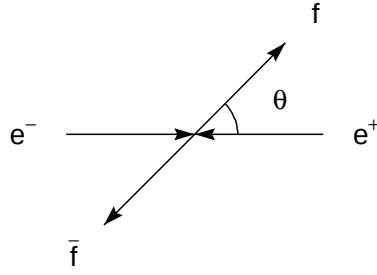


Figure 3.1: The polar scattering angle, θ , is defined as the polar angle between the directions of the incoming electron, $+z$ axis, and the outgoing fermion or W^- boson.

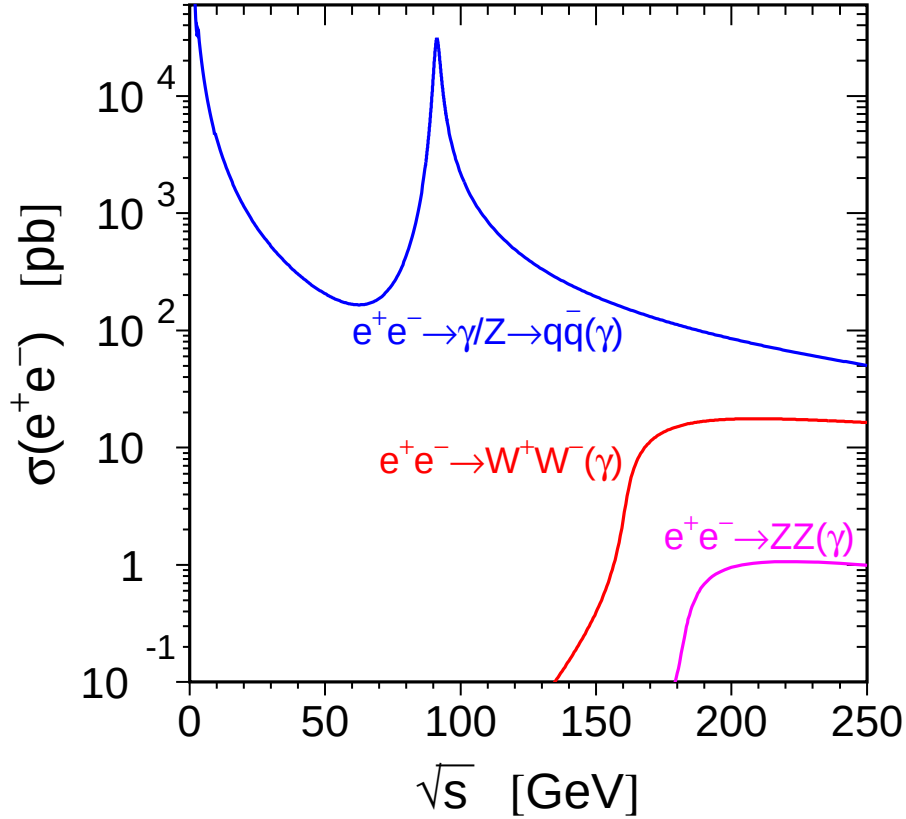


Figure 3.2: Cross sections of $q\bar{q}(\gamma)$, $W^+W^-(\gamma)$ and $ZZ(\gamma)$ production in e^+e^- interactions as a function of the centre-of-mass energy $\sqrt{s}$. The (γ) denotes the possible presence of radiative photons.

3.1 Single-Boson and Fermion-Pair Production

The lowest-order Feynman diagrams for fermion-pair production are shown in Figures 3.3 and 3.4. In general in e^+e^- interactions, fermion pairs are produced in s -channel annihilations via the exchange of a virtual photon or Z boson, Figures 3.3. In the case of t -channel interactions, the initial-state particles and the t -channel exchanged particle specify the final-state particle flavour. The additional Feynman diagrams for e^+e^- and $\nu_e\bar{\nu}_e$ production due to t -channel γ/Z exchange and W exchange, respectively, are shown in Figure 3.4. For e^+e^- final states, *i.e.*, Bhabha scattering, the pure QED t -channel γ exchange dominates the cross section at small polar scattering angles $\theta \rightarrow 0$. All known neutrinos, charged leptons and quarks, with the exception of the heavy top quark, are kinematically

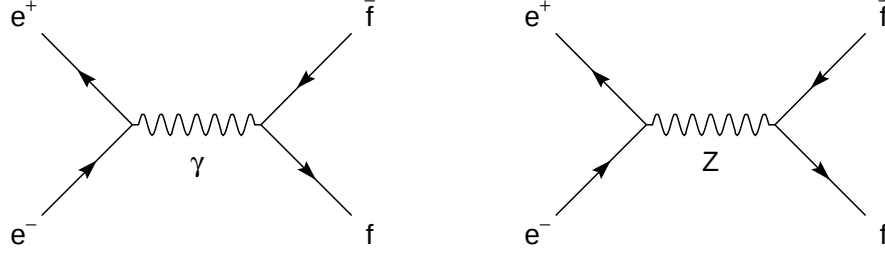


Figure 3.3: Feynman diagrams in s -channel fermion-pair production at e^+e^- colliders: Electromagnetic and neutral weak current.

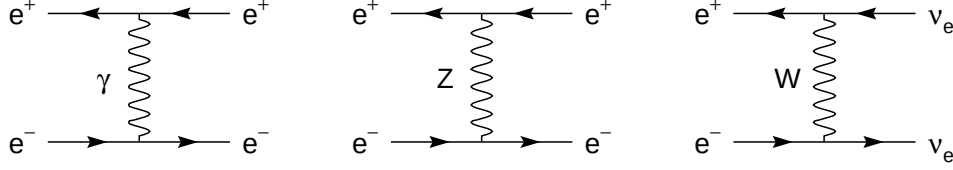


Figure 3.4: Feynman diagrams in t -channel fermion-pair production at e^+e^- colliders: Electromagnetic, neutral weak and charged weak current.

accessible in $f\bar{f}$ production at modern e^+e^- colliders.

3.1.1 Born Level

3.1.1.1 Helicity Amplitudes

The matrix element for s -channel fermion-pair production in e^+e^- interactions, $e^+e^- \rightarrow f\bar{f}$, is a sum of two contributions corresponding to s -channel photon and Z -boson exchange:

$$\mathcal{M} = \mathcal{M}_\gamma + \mathcal{M}_Z \quad (3.1)$$

$$\mathcal{M}_\gamma = i \frac{4\pi\alpha_{em}}{s} [(q_e \gamma_\mu) \otimes (q_f \gamma^\mu)] \quad (3.2)$$

$$\mathcal{M}_Z = i \frac{4\pi\alpha_{em}}{s} \kappa \chi(s) [(\gamma_\mu (g_{Ve} - g_{Ae} \gamma_5)) \otimes (\gamma^\mu (g_{Vf} - g_{Af} \gamma_5))] , \quad (3.3)$$

where:

$$\alpha_{em} = \frac{e^2}{4\pi} \simeq 1/137 \quad (3.4)$$

$$\kappa = \frac{1}{4 \sin^2 \theta_W \cos^2 \theta_W} = \frac{G_F M_Z^2}{2\sqrt{2}\pi\alpha_{em}} \simeq 1.50. \quad (3.5)$$

The term $[J_\mu \otimes J^\mu]$ denotes the product of an initial- and a final-state current:

$$[J_\mu \otimes J^\mu] = (\bar{v}_e J_\mu u_e) (\bar{u}_f J^\mu v_f) , \quad (3.6)$$

where u_f and v_f are spinors describing fermions and antifermions, respectively.

Since the interaction is mediated by spin-1 bosons in the s -channel the angular dependence in θ is determined solely by the d_1^1 functions of angular momentum [58]. The helicities $h = \pm 1$ of the two final-state fermions are not independent of each other due to helicity conservation at vertices

with only vector and axial-vector couplings, yielding $h_{\bar{f}} = -h_f$, which holds for both the initial state electron-positron pair and the final state fermion-antifermion pair. Hence there are four independent helicity amplitudes describing the scattering processes:

$$\begin{aligned} e_L^- e_R^+ &\rightarrow f_L \bar{f}_R \\ e_L^- e_R^+ &\rightarrow f_R \bar{f}_L \\ e_R^- e_L^+ &\rightarrow f_R \bar{f}_L \\ e_R^- e_L^+ &\rightarrow f_L \bar{f}_R, \end{aligned} \quad (3.7)$$

which add incoherently as they do not interfere. They give rise to four independent terms each for the γ exchange, γ/Z interference and Z exchange contribution to the differential cross section.

3.1.1.2 Differential Cross Section

Neglecting fermion masses, the differential cross section for unpolarised e^+e^- beams has the form:

$$\begin{aligned} \frac{d\sigma_{\text{tot}}^0(e^+e^- \rightarrow f\bar{f}; h_f)}{d\cos\theta} &= \frac{\pi\alpha_{em}^2}{4} \times \\ &\left(\begin{array}{c} \frac{s}{(s-M_Z^2)^2 + M_Z^2\Gamma_Z^2} \\ \frac{s-M_Z^2}{(s-M_Z^2)^2 + M_Z^2\Gamma_Z^2} \\ \frac{1}{s} \end{array} \right)^T \cdot \left[\begin{pmatrix} r_f^{\text{tot}} & r_f^{\text{fb}} \\ j_f^{\text{tot}} & j_f^{\text{fb}} \\ g_f^{\text{tot}} & g_f^{\text{fb}} \end{pmatrix} - h_f \begin{pmatrix} r_f^{\text{pol}} & r_f^{\text{fbpol}} \\ j_f^{\text{pol}} & j_f^{\text{fbpol}} \\ g_f^{\text{pol}} & g_f^{\text{fbpol}} \end{pmatrix} \right] \cdot \begin{pmatrix} 1 + \cos^2\theta \\ 2\cos\theta \end{pmatrix}. \end{aligned} \quad (3.8)$$

The parameters r_f^A , j_f^A and g_f^A are real numbers and express the magnitude of the Z exchange, γ/Z interference and γ exchange contributions, respectively. They are called S-Matrix parameters because they are derived within a general S-Matrix formalism describing fermion-pair production [59]. The rationale behind the labels $\mathcal{A} = \text{tot}, \text{fb}, \text{pol}, \text{fbpol}$ will become apparent in the following.

The $\sqrt{s}$ dependence of the three contributions is given by the components of the first matrix. The Z exchange is described by a relativistic Breit-Wigner for a massive spin-1 particle, the Z boson, characterised by its mass, M_Z , and total decay width, Γ_Z . The γ/Z interference contribution vanishes at $\sqrt{s} = M_Z$. The γ exchange shows the typical $1/s$ behaviour of QED.

The dependence on the polar scattering angle, θ , is given by the last matrix. There are two components, a forward-backward symmetric part, proportional to $1 + \cos^2\theta$, and an antisymmetric part, proportional to $2\cos\theta$. The asymmetric part results in a forward-backward asymmetry of the differential cross section but does not contribute to the total cross section within a fiducial volume symmetric in $\cos\theta$.

Within the Standard Model the photon couples with the electromagnetic charge q_f to fermionic vector currents and the Z boson couples with strengths g_{Vf} and g_{Af} to fermionic vector and axial-vector currents, respectively. The S-Matrix parameters for that case are reported in Table 3.1. For the inclusive hadronic final state, a sum over all kinematically accessible quark flavours, $q = d, u, s, c, b$ for $\sqrt{s} < 2M_t$, is performed.

In most cases the helicity of the final-state fermions is not measured so that the final-state helicities $h_f = \pm 1$ are summed over with the result:

$$\begin{aligned} \frac{d\sigma_{\text{tot}}^0(e^+e^- \rightarrow f\bar{f})}{d\cos\theta} &= \frac{\pi\alpha_{em}^2}{2} \times \\ &\left(\begin{array}{c} \frac{s}{(s-M_Z^2)^2 + M_Z^2\Gamma_Z^2} \\ \frac{s-M_Z^2}{(s-M_Z^2)^2 + M_Z^2\Gamma_Z^2} \\ \frac{1}{s} \end{array} \right) \cdot \begin{pmatrix} r_f^{\text{tot}} & r_f^{\text{fb}} \\ j_f^{\text{tot}} & j_f^{\text{fb}} \\ g_f^{\text{tot}} & g_f^{\text{fb}} \end{pmatrix} \cdot \begin{pmatrix} 1 + \cos^2\theta \\ 2\cos\theta \end{pmatrix}. \end{aligned} \quad (3.9)$$

$\mathcal{A}$	r_f^A	j_f^A	g_f^A
tot	$\kappa^2 [g_{Ve}^2 + g_{Ae}^2] [g_{Vf}^2 + g_{Af}^2] N_C^f$	$2\kappa q_e q_f g_{Ve} g_{Vf} N_C^f$	$q_e^2 q_f^2 N_C^f$
fb	$4\kappa^2 g_{Ve} g_{Ae} g_{Vf} g_{Af} N_C^f$	$2\kappa q_e q_f g_{Ae} g_{Af} N_C^f$	0
pol, fb	$2\kappa^2 [g_{Ve}^2 + g_{Ae}^2] g_{Vf} g_{Af} N_C^f$	$2\kappa q_e q_f g_{Ve} g_{Af} N_C^f$	0
fbpol, lr	$2\kappa^2 g_{Ve} g_{Ae} [g_{Vf}^2 + g_{Af}^2] N_C^f$	$2\kappa q_e q_f g_{Ae} g_{Vf} N_C^f$	0

Table 3.1: S-Matrix parameters in the Standard Model.

3.1.1.3 Total Cross Section and Z Lineshape

Integrating over the polar scattering angle θ yields the total production cross section:

$$\sigma_{\text{tot}}^0(e^+e^- \rightarrow f\bar{f}; \sqrt{s}) = \frac{4}{3}\pi\alpha_{em}^2 \left[\frac{g_f^{\text{tot}}}{s} + \frac{sr_f^{\text{tot}} + (s - M_Z^2)j_f^{\text{tot}}}{(s - M_Z^2)^2 + M_Z^2\Gamma_Z^2} \right]. \quad (3.10)$$

The dependence of σ_{tot}^0 on the centre-of-mass energy $\sqrt{s}$ is shown in Figure 3.5. For small energies, $\sqrt{s} \rightarrow 0$, the γ exchange dominates with its $1/s$ pole the production of charged fermions whereas neutrino production vanishes. For large energies, $\sqrt{s} \rightarrow \infty$, all three contributions behave like $1/s$ and contribute proportional to g_f^{tot} , r_f^{tot} , and j_f^{tot} . For $\sqrt{s} \approx M_Z$ the s -channel Z exchange dominates the total cross section with its Breit-Wigner $\sqrt{s}$ dependence:

$$\sigma_Z^0 \equiv \frac{4}{3}\pi\alpha_{em}^2 \cdot r_f^{\text{tot}} \cdot \frac{s}{(s - M_Z^2)^2 + M_Z^2\Gamma_Z^2} \quad (3.11)$$

$$= \frac{12\pi}{M_Z^2} \frac{\Gamma_{ee}\Gamma_{ff}}{\Gamma_Z^2} \frac{s\Gamma_Z^2}{(s - M_Z^2)^2 + M_Z^2\Gamma_Z^2}. \quad (3.12)$$

The partial decay width, Γ_{ff} , of the $Z \rightarrow f\bar{f}$ decay is given by:

$$\Gamma_{ff} = N_C^f \frac{\alpha_{em} M_Z}{12 \sin^2 \theta_W \cos^2 \theta_W} [g_{Vf}^2 + g_{Af}^2] = N_C^f \frac{G_F M_Z^3}{6\sqrt{2}\pi} [g_{Vf}^2 + g_{Af}^2], \quad (3.13)$$

which is proportional to the sum of the squares of the couplings g_{Vf} and g_{Af} . The total decay width of the Z boson, Γ_Z , is the sum of all partial decay widths:

$$\Gamma_Z = \Gamma_{ee} + \Gamma_{\mu\mu} + \Gamma_{\tau\tau} + \Gamma_{\text{inv}} + \Gamma_{\text{had}} \quad (3.14)$$

$$\Gamma_{\text{inv}} = N_\nu \Gamma_{\nu\nu} \quad (3.15)$$

$$\Gamma_{\text{had}} = \sum_{q \neq t} \Gamma_{qq}. \quad (3.16)$$

The decay width of the Z into invisible particles, Γ_{inv} , counts the number of light neutrino species, N_ν , within the Standard Model. Since $2M_t > M_Z$, the decay $Z \rightarrow t\bar{t}$ of on-shell Z bosons is kinematically not allowed. The inclusive hadronic width is thus a sum over the five light quark flavours.

Measurements of the total production cross sections for various final states $f\bar{f}$ determine the basic properties of the Z boson. The mass M_Z , total width Γ_Z and partial decay widths Γ_{ff} are determined by the position, width and height of the Z lineshape, the resonance curve describing the total production cross section as a function of $\sqrt{s}$.

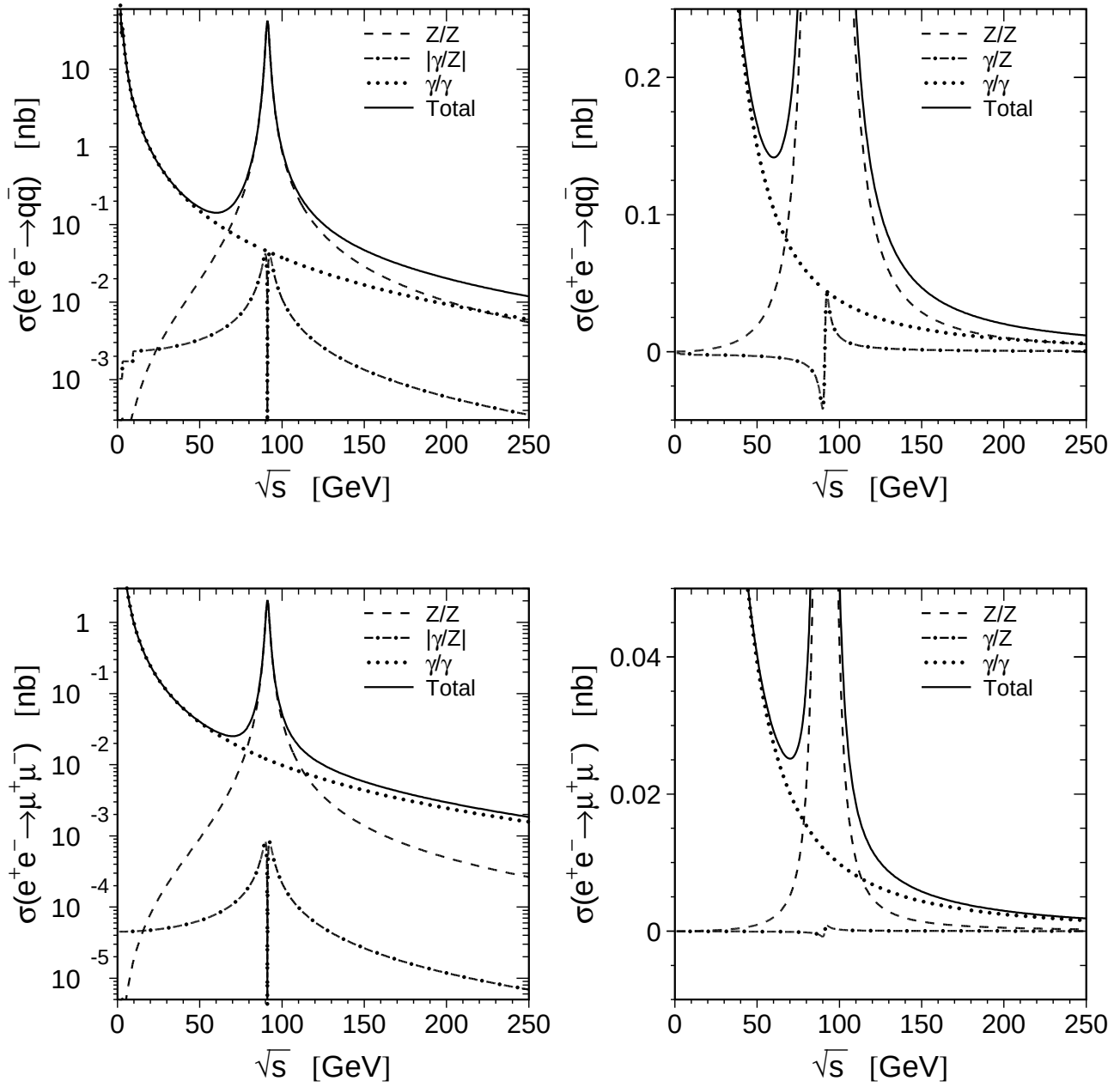


Figure 3.5: Born cross sections as a function of $\sqrt{s}$ for $q\bar{q}$ and $\mu^+\mu^-$ production in e^+e^- annihilations. The total cross section and the contributions arising from Z exchange, γ exchange, and γ/Z interference are shown separately. In case of logarithmic scale, the absolute value of the interference cross section is shown. Because of the larger vector couplings of quarks, the Z exchange and the γ/Z interference are more pronounced in $q\bar{q}$ production than in $\ell^+\ell^-$ production. At energies above the Z resonance, γ and Z exchange have equal importance of $q\bar{q}$ production, with the γ/Z interference term reaching a magnitude of a few percent of the total. In contrast, $\mu^+\mu^-$ production above the Z resonance is dominated by the γ exchange, which is an order of magnitude large than Z exchange and more than two orders of magnitude larger than the γ/Z interference. The reason is again the smallness of the vector coupling of charged leptons.

3.1.1.4 Definition of Z Mass and Width

The $\sqrt{s}$ dependence of the total cross section determines the Z mass and total width. It is fixed by the complex propagator χ_Z for a massive unstable particle. Several possibilities exist for the definition of χ_Z :

1. Breit-Wigner with s -independent width as introduced above:

$$\chi_Z(s) = \frac{s}{s - \bar{M}_Z^2 + i\bar{M}_Z\bar{\Gamma}_Z}, \quad (3.17)$$

2. Breit-Wigner with s -dependent width as suggested by phase space:

$$\chi_Z(s) = \frac{s}{s - \hat{M}_Z^2 + i\sqrt{s}\hat{\Gamma}_Z}, \quad (3.18)$$

3. Breit-Wigner with s -dependent width as suggested by phase-space and Standard Model electroweak radiative corrections:

$$\chi_Z(s) = \frac{s}{s - M_Z^2 + is\Gamma_Z/M_Z}, \quad (3.19)$$

4. Pole in the complex invariant-mass plane:

$$\chi_Z(s) = \frac{s}{s - \left(\tilde{M}_Z - \frac{i}{2}\tilde{\Gamma}_Z\right)^2}. \quad (3.20)$$

The first definition is used in the S-Matrix ansatz. The third definition is motivated by phase-space considerations and Standard Model electroweak radiative corrections associated with the Z-boson propagator, see Section 3.1.3, and is adopted at LEP. These definitions lead to different numerical results for the mass and total width of the Z boson, which are significant compared to the experimental errors. The S-Matrix results are related to the s -dependent width convention adopted at LEP by the relations:

$$M_Z = \bar{M}_Z \sqrt{1 + \bar{\Gamma}_Z^2/\bar{M}_Z^2} \approx \bar{M}_Z + 34.1 \text{ MeV} \quad (3.21)$$

$$\Gamma_Z = \bar{\Gamma}_Z \sqrt{1 + \bar{\Gamma}_Z^2/\bar{M}_Z^2} \approx \bar{\Gamma}_Z + 0.9 \text{ MeV}. \quad (3.22)$$

which are applied when reporting numerical results on M_Z and Γ_Z based on S-Matrix analyses.

3.1.1.5 Asymmetries

The information contained in the fully differential cross section of Equation 3.8, besides the total cross section, is usually expressed in the form of three asymmetries which are ratios of cross sections and which combine differently the forward, backward, left-handed and right-handed production of fermions. Defining $\sigma_{\mathcal{A}}^0$ with $\mathcal{A} = \text{fb}, \text{pol}, \text{fbpol}$, in analogy to σ_{tot}^0 of Equation 3.10, the three asymmetries are:

1. the forward-backward asymmetry, A_{fb} :

$$A_{\text{fb}}(s) = \frac{\sigma^0(\cos\theta > 0) - \sigma^0(\cos\theta < 0)}{\sigma^0(\cos\theta > 0) + \sigma^0(\cos\theta < 0)} = + \frac{3}{4} \frac{\sigma_{\text{fb}}^0}{\sigma_{\text{tot}}^0}, \quad (3.23)$$

2. the polarisation asymmetry, A_{pol} :

$$A_{\text{pol}}(s) = \frac{\sigma^0(h_f = +1) - \sigma^0(h_f = -1)}{\sigma^0(h_f = +1) + \sigma^0(h_f = -1)} = -\frac{\sigma_{\text{pol}}^0}{\sigma_{\text{tot}}^0}, \quad (3.24)$$

3. the forward-backward polarisation asymmetry, A_{fbpol} :

$$A_{\text{fbpol}}(s) = \frac{\sigma^0(h_f \cos \theta > 0) - \sigma^0(h_f \cos \theta < 0)}{\sigma^0(h_f \cos \theta > 0) + \sigma^0(h_f \cos \theta < 0)} = -\frac{3}{4} \frac{\sigma_{\text{fbpol}}^0}{\sigma_{\text{tot}}^0}. \quad (3.25)$$

This is the reason for using the labels $\mathcal{A}$ for the different S-Matrix parameters.

The asymmetries as a function of the centre-of-mass energy $\sqrt{s}$ are shown in Figure 3.6. In case the final state has the same couplings as the initial state and only vector- and axial-vector couplings are involved, $\frac{3}{4}A_{\text{pol}}(s) = A_{\text{fbpol}}(s)$ holds at Born level independent of $\sqrt{s}$. For small energies, $\sqrt{s} \rightarrow 0$, the asymmetries vanish for charged fermions. For large energies, $\sqrt{s} \rightarrow \infty$, both numerator and denominator behave like $1/s$ so that the asymmetries $A_{\mathcal{A}}$ converge to constants. They change sign at $s = M_Z^2 j_f^A / (r_f^A + j_f^A)$. On the peak of the resonance, $\sqrt{s} = M_Z$, where the interference terms vanish, the asymmetries have very simple expressions when removing the pure photon exchange in the denominator:

$$A_{\text{fb}}(s = M_Z^2) = +\frac{3}{4} \frac{r_f^{\text{fb}}}{r_f^{\text{tot}}} = +\frac{3}{4} A_e A_f \quad (3.26)$$

$$A_{\text{pol}}(s = M_Z^2) = -\frac{r_f^{\text{pol}}}{r_f^{\text{tot}}} = -A_f \quad (3.27)$$

$$A_{\text{fbpol}}(s = M_Z^2) = -\frac{3}{4} \frac{r_f^{\text{pol}}}{r_f^{\text{tot}}} = -\frac{3}{4} A_e. \quad (3.28)$$

The coupling parameter A_f is defined as:

$$A_f \equiv 2 \frac{g_{Vf} \cdot g_{Af}}{g_{Vf}^2 + g_{Af}^2} = 2 \frac{g_{Vf}/g_{Af}}{1 + (g_{Vf}/g_{Af})^2} = \frac{g_{Lf}^2 - g_{Rf}^2}{g_{Lf}^2 + g_{Rf}^2} = \frac{1 - (g_{Rf}/g_{Lf})^2}{1 + (g_{Rf}/g_{Lf})^2}, \quad (3.29)$$

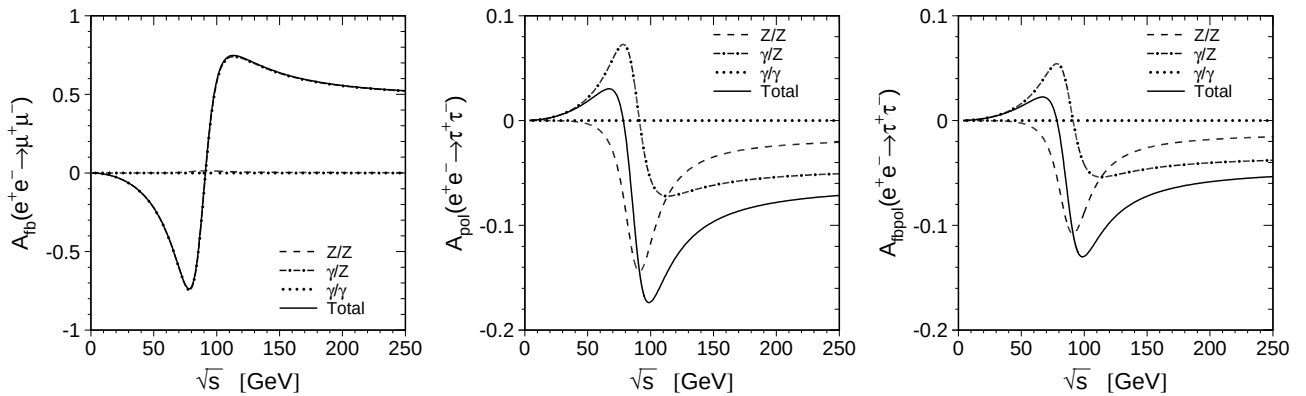


Figure 3.6: Born asymmetries as a function of $\sqrt{s}$ for $\ell^+\ell^-$ production in e^+e^- annihilations. The total asymmetry and the contributions arising from Z exchange, γ exchange and γ/Z interference are shown separately. In all cases, the γ exchange contribution vanishes. The forward-backward asymmetry is by far dominated by the γ/Z interference. The Z contribution is barely visible around $\sqrt{s} = M_Z$. In contrast, both Z exchange and γ/Z interference are equally important for the polarisation asymmetries.

which is a function of the ratio of the neutral-current couplings.

For $\sqrt{s}$ close to M_Z the s -channel Z exchange dominates the total cross section in the denominator. In that region the energy dependence of the asymmetries $A_{\mathcal{A}}$ is quantified by the ratio j_f^A/r_f^{tot} .

The dependence of the average fermion helicity, *i.e.*, the longitudinal polarisation $\mathcal{P}_f$, on the scattering angle $\cos \theta$ simplifies on the peak to:

$$\mathcal{P}_f(\cos \theta) = -\frac{r_f^{\text{pol}}(1 + \cos^2 \theta) + r_f^{\text{fbpol}}2 \cos \theta}{r_f^{\text{tot}}(1 + \cos^2 \theta) + r_f^{\text{fb}}2 \cos \theta} = -\frac{A_f(1 + \cos^2 \theta) + A_e 2 \cos \theta}{(1 + \cos^2 \theta) + A_e A_f 2 \cos \theta}. \quad (3.30)$$

This quantity is measurable in $\tau^+\tau^-$ production, where the parity-violating charged-current decay of the τ lepton serves as a polarisation analyser.

3.1.1.6 Polarised Electron-Positron Beams

The above formulae are derived assuming unpolarised electron and positron beams. It is possible, however, to obtain both transverse as well as longitudinal polarised beams. Transverse polarisation causes a modulation of the differential cross section in azimuthal angle, ϕ . The modulation, proportional to $\sin^2 \theta$, averages out at each value of θ when integrating over the full azimuthal angular range. Transverse polarisation is thus not particularly useful except for the determination of the beam energy, see Section 4.6. In contrast, longitudinal polarisation is interesting from a physics point of view. Denoting with $P_L^\pm$ the degree of longitudinal polarisation of the $e^\pm$ beam, $P_L = -1$ for fully left-handed, $P_L = +1$ for fully right-handed, the differential cross section becomes:

$$\frac{d\sigma^0(e^+e^- \rightarrow f\bar{f}, P_L^\pm)}{d\cos \theta} = (1 - P_L^- P_L^+) \frac{d\sigma_{\text{tot}}^0(e^+e^- \rightarrow f\bar{f})}{d\cos \theta} - (P_L^- - P_L^+) \frac{d\sigma_{\text{lr}}^0(e^+e^- \rightarrow f\bar{f})}{d\cos \theta} \quad (3.31)$$

where $d\sigma_{\text{lr}}^0/d\cos \theta$ is obtained from $d\sigma_{\text{tot}}^0/d\cos \theta$ by replacing the labels $\mathcal{A} = (\text{tot}, \text{fb})$ by $\mathcal{A} = (\text{lr}, \text{fblr})$ in Equation 3.9.

Experimentally, an electron beam polarised to a degree $P_L^- = \pm P$, $P > 0$, is brought into collisions with an unpolarised positron beam, $P_L^+ = 0$. The forward-backward asymmetry is modified:

$$A_{\text{fb}}(s) = \frac{3 \sigma_{\text{fb}}^0 - P_L^- \sigma_{\text{fblr}}^0}{4 \sigma_{\text{tot}}^0 - P_L^- \sigma_{\text{lr}}^0} \quad (3.32)$$

$$\xrightarrow{s \rightarrow M_Z^2} \frac{3 r_f^{\text{fb}} - P_L^- r_f^{\text{fblr}}}{4 r_f^{\text{tot}} - P_L^- r_f^{\text{lr}}} = \frac{3}{4} \frac{A_e - P_L^-}{1 - A_e P_L^-} A_f. \quad (3.33)$$

Furthermore, the measurement of two additional asymmetries is possible:

1. the left-right asymmetry, A_{lr} :

$$A_{\text{lr}}(s) = \frac{1}{P} \cdot \frac{\sigma^0(P_L^- = -P) - \sigma^0(P_L^+ = +P)}{\sigma^0(P_L^- = -P) + \sigma^0(P_L^+ = +P)} = \frac{\sigma_{\text{lr}}^0}{\sigma_{\text{tot}}^0} \quad (3.34)$$

$$\xrightarrow{s \rightarrow M_Z^2} \frac{r_f^{\text{lr}}}{r_f^{\text{tot}}} = A_e, \quad (3.35)$$

2. the forward-backward left-right asymmetry, A_{fblr} :

$$A_{\text{fblr}}(s) = \frac{1}{P} \cdot \frac{\sigma(P_L^- \cos \theta < 0) - \sigma(P_L^- \cos \theta > 0)}{\sigma(P_L^- \cos \theta < 0) + \sigma(P_L^- \cos \theta > 0)} = \frac{3 \sigma_{\text{fblr}}^0}{4 \sigma_{\text{tot}}^0} \quad (3.36)$$

$$\xrightarrow{s \rightarrow M_Z^2} \frac{3 r_f^{\text{fblr}}}{4 r_f^{\text{tot}}} = \frac{3}{4} A_f, \quad (3.37)$$

where the $\sigma_{\mathcal{A}}^0$, $\mathcal{A} = \text{lr}, \text{fblr}$, are defined in analogy to σ_{tot}^0 of Equation 3.10. At Born level with only vector- and axial-vector couplings one finds $\frac{3}{4} A_{\text{lr}}(s) = -A_{\text{fbpol}}(s)$ and $A_{\text{fblr}}(s) = -\frac{3}{4} A_{\text{pol}}(s)$.

3.1.2 Bhabha Scattering

Besides the two s -channel diagrams shown in Figure 3.3, two additional Feynman diagrams contribute to the process $e^+e^- \rightarrow e^+e^-$ in lowest order, t -channel γ and Z boson exchange as shown in Figure 3.4. The differential and total cross section is therefore given by a sum of ten terms [60]:

$$\frac{d\sigma^0(e^+e^- \rightarrow e^+e^-)}{d\cos\theta} = \sum_{i=1}^{10} \frac{d\sigma_i^0(e^+e^- \rightarrow e^+e^-)}{d\cos\theta}. \quad (3.38)$$

For unpolarised e^+e^- beams and denoting $z = \cos\theta$ the ten terms are:

$$\begin{aligned} \frac{d\sigma_1^0}{d\cos\theta} &= \frac{d\sigma_s^0(s_Z s_Z)}{dz} = \frac{\pi\alpha_{em}^2}{2s} |\chi(s)|^2 \kappa^2 [(g_{Ve}^2 + g_{Ae}^2)^2 (1+z^2) + 8g_{Ve}^2 g_{Ae}^2 z] \\ \frac{d\sigma_2^0}{d\cos\theta} &= \frac{d\sigma_s^0(s_\gamma s_Z)}{dz} = \frac{\pi\alpha_{em}^2}{2s} 2\Re(\chi(s))\kappa [g_{Ve}^2 (1+z^2) + 2g_{Ae}^2 z] \\ \frac{d\sigma_3^0}{d\cos\theta} &= \frac{d\sigma_s^0(s_\gamma s_\gamma)}{dz} = \frac{\pi\alpha_{em}^2}{2s} (1+z^2) \\ \frac{d\sigma_4^0}{d\cos\theta} &= \frac{d\sigma_t^0(t_Z t_Z)}{dz} = \frac{\pi\alpha_{em}^2}{2s} \frac{[\tilde{\chi}(t)]^2}{2} \kappa^2 [(g_{Ve}^2 + g_{Ae}^2)^2 + 4g_{Ve}^2 g_{Ae}^2] (1+z)^2 + \\ &\quad 4((g_{Ve}^2 + g_{Ae}^2)^2 - 4g_{Ve}^2 g_{Ae}^2) \\ \frac{d\sigma_5^0}{d\cos\theta} &= \frac{d\sigma_t^0(t_\gamma t_Z)}{dz} = \frac{\pi\alpha_{em}^2}{2s} (-2)\tilde{\chi}(t)\kappa \frac{[(g_{Ve}^2 + g_{Ae}^2)(1+z)^2 + 4(g_{Ve}^2 - g_{Ae}^2)]}{1-z} \\ \frac{d\sigma_6^0}{d\cos\theta} &= \frac{d\sigma_t^0(t_\gamma t_\gamma)}{dz} = \frac{\pi\alpha_{em}^2}{2s} 2 \frac{[(1+z)^2 + 4]}{(1-z)^2} \\ \frac{d\sigma_7^0}{d\cos\theta} &= \frac{d\sigma_i^0(s_Z t_Z)}{dz} = \frac{\pi\alpha_{em}^2}{2s} \Re(\chi(s))\tilde{\chi}(t)\kappa^2 ((g_{Ve}^2 + g_{Ae}^2)^2 + 4g_{Ve}^2 g_{Ae}^2) (1+z)^2 \\ \frac{d\sigma_8^0}{d\cos\theta} &= \frac{d\sigma_i^0(s_Z t_\gamma)}{dz} = \frac{\pi\alpha_{em}^2}{2s} (-2)\Re(\chi(s))\kappa(g_{Ve}^2 + g_{Ae}^2) \frac{(1+z)^2}{1-z} \\ \frac{d\sigma_9^0}{d\cos\theta} &= \frac{d\sigma_i^0(s_\gamma t_Z)}{dz} = \frac{\pi\alpha_{em}^2}{2s} [\tilde{\chi}(t)\kappa(g_{Ve}^2 + g_{Ae}^2)(1+z)^2] \\ \frac{d\sigma_{10}^0}{d\cos\theta} &= \frac{d\sigma_i^0(s_\gamma t_\gamma)}{dz} = \frac{\pi\alpha_{em}^2}{2s} (-2) \frac{(1+z)^2}{1-z}, \end{aligned} \quad (3.39)$$

where the t -channel Z propagator is given by:

$$\tilde{\chi}(t) = \frac{s}{t - M_Z^2}. \quad (3.40)$$

The indices denote the contributions arising from products of s - and t -channel, γ and Z -boson matrix elements.

For small polar scattering angles, $\theta \rightarrow 0$, the cross section is clearly dominated by the pole of the t -channel photon exchange, $d\sigma_i^0(t_\gamma t_\gamma)/d\cos\theta$, a pure QED process. The s -channel contributions to the total cross section are enhanced for large polar scattering angles, for example $|\cos\theta| < \sqrt{2}$. For this region of phase space, the contributions of s -channel, t -channel and s/t -interference to the total cross section and the forward-backward asymmetry are shown in Figure 3.7.

3.1.3 Radiative Corrections

In order to interpret the precise measurements around the Z pole correctly, the theoretical predictions need to take into account higher-order radiative corrections, reviewed in detail in [41, 42, 61] and references therein. Based on the specific interaction, the corrections are loosely classified as QED

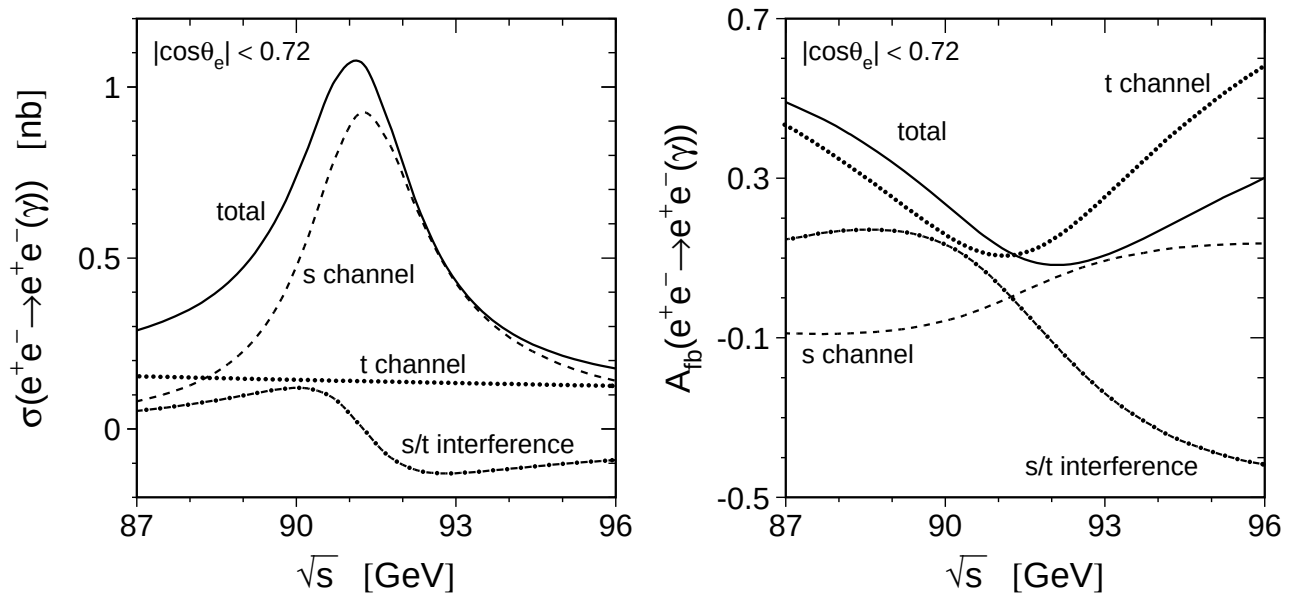


Figure 3.7: Bhabha cross section and forward-backward asymmetry in the fiducial volume defined by $|\cos\theta| < 0.72$ for the polar angle of both final-state charged leptons, including radiative corrections discussed in Section 3.1.3. The contributions arising from s -channel, t -channel and s/t -interference, and the total are shown separately. While the total cross section clearly shows the Z resonance, the forward-backward asymmetry is dominated by t -channel and s/t -interference effects.

corrections, QCD corrections, and weak corrections. Based on the topology of the Feynman diagrams involved, the higher-order corrections are also classified as bremsstrahlung corrections, vertex corrections, propagator corrections and box contributions.

QED corrections arise in any theory containing the electromagnetic gauge group $U(1)_{EM}$. In terms of Feynman diagrams, they consist of those higher-order diagrams where extra photons are attached to the charged particles in Born-level diagrams, either as a bremsstrahlung photon or as a virtual photon connecting two charged particles. QCD corrections are similar to QED corrections, replacing the additional photon in QED corrections by a gluon coupling to quarks. Weak corrections collect the effects of all other higher-order diagrams with internal loops consisting of fermions or additional lines with heavy gauge bosons.

In the electroweak theory the separation of QED and weak corrections is sensible both experimentally and theoretically, the latter at least to one-loop order. Theoretically, QED corrections can be defined in such a way that they form a gauge-invariant subset of the complete set of electroweak corrections. They are independent of the detailed structure of the non-QED part of the theory. Experimentally, QED corrections depend on the details of the measurements, for example the spatial and energy resolution of the detector to resolve radiative photons, and the event selection criteria applied. In contrast, weak corrections are independent of the experimental setup as they are internal to the Feynman diagrams and do not modify the particle content of the observed final state. They depend on the detailed structure of the full theory through corrections arising from internal loops involving all particles of the theory.

QED and QCD bremsstrahlung corrections are shown in Figure 3.8 and QED and QCD vertex corrections are shown in Figures 3.9. As the initial state does not couple to gluons, QCD bremsstrahlung- and vertex corrections appear only as final-state corrections. Propagator corrections, already discussed in Section 2.11.2 are shown in Figure 2.2. Weak vertex corrections and box contributions are shown in Figures 3.10 and 3.11, respectively.

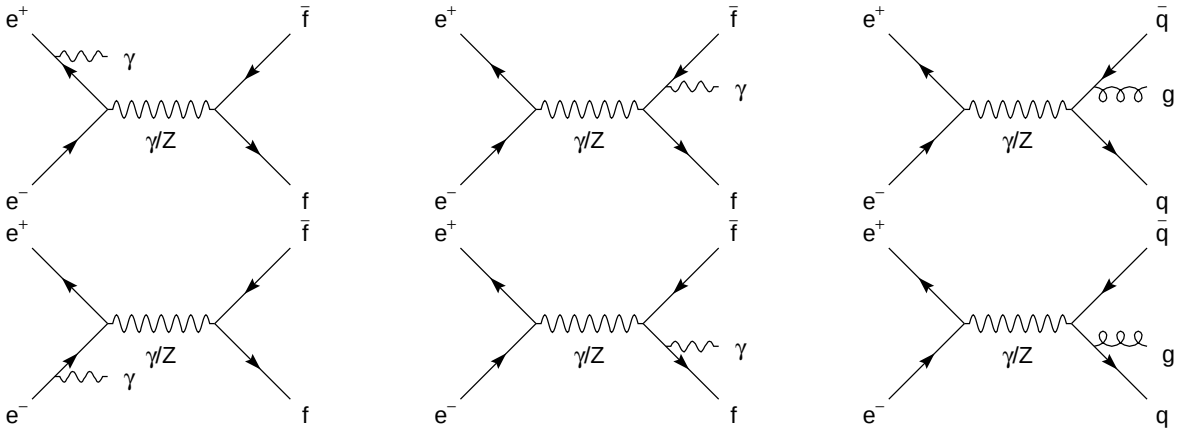


Figure 3.8: QED and QCD bremsstrahlung corrections in s -channel fermion-pair production. Left: Initial-state QED radiation; middle: final-state QED radiation; right: final-state QCD radiation in $q\bar{q}$ production.

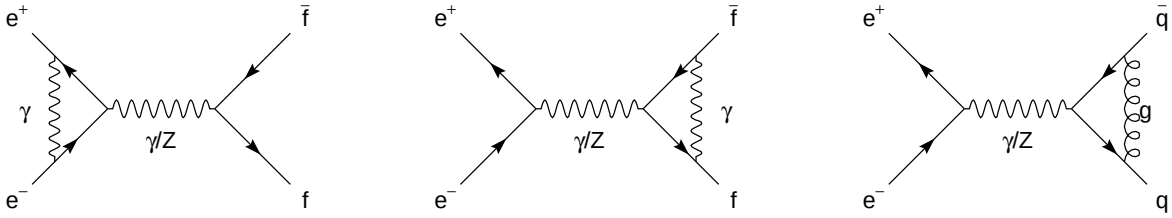


Figure 3.9: QED and QCD vertex corrections in s -channel fermion-pair production. Left: Initial-state QED vertex correction; middle: final-state QED vertex correction; right: final-state QCD vertex correction in $q\bar{q}$ production.

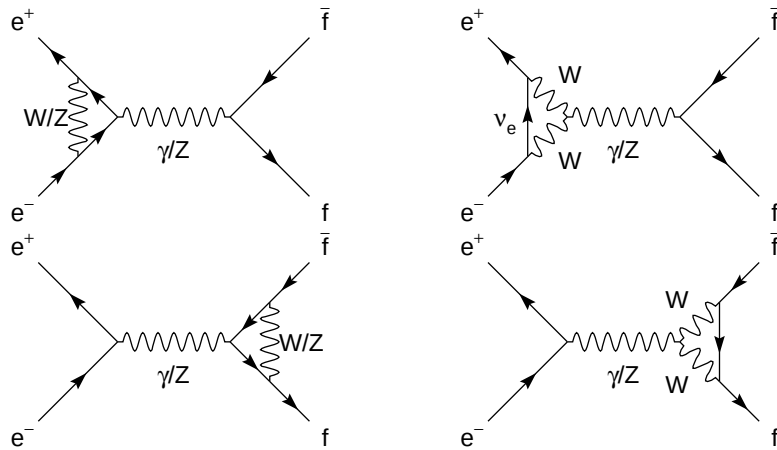


Figure 3.10: Weak vertex corrections in s -channel fermion-pair production.

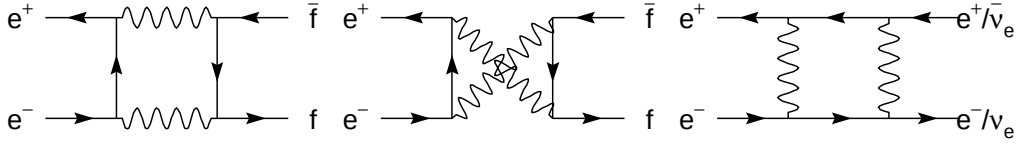


Figure 3.11: Contributions of s -, u - and t -channel box diagrams to fermion-pair production.

Non-vanishing fermion masses introduce a kinematic effect as they restrict the available phase-space, thus modifying the expressions given above for pair production of massless fermions. For quarks, confinement causes quark masses to be an ill-defined concept especially for light quarks, where the confinement energy is comparable to the mass. Because of strong QCD effects quark masses must be treated as running masses, so that QCD and mass corrections, if both need to be considered, cannot be factorised. This also prevents for quarks a simple definition of effective electroweak coupling constants, absorbing only electroweak effects, beyond first order.

Analyses of precision measurements in terms of radiative corrections allow to search for virtual effects of new particles otherwise too heavy to be produced directly. In the case of the MSM, the interest lies in particular in the analysis of the measurements in terms of effects caused by the top quark and the Higgs boson. Mass values inferred for these particles as well as for the W boson are then compared to the direct measurements, testing the Standard Model at the level of its electroweak radiative corrections.

3.1.3.1 QED Bremsstrahlung Corrections

QED corrections to fermion-pair production are rather large. For example, the total cross section on top of the Z pole is reduced by about 25%, and the cross section at energies $\sqrt{s} \gg M_Z$ is increased by several 100%, as shown in Figure 3.12. Thus QED corrections need to be calculated very precisely in order to retain the sensitivity to the weak corrections which are of the order of a few permille.

As a neutral boson is exchanged in s -channel fermion-pair production, a gauge invariant separation of QED corrections into initial-state, final-state and initial-final interference corrections is possible and discussed in the following. QED radiative corrections to Bhabha scattering are discussed in [60, 62] and references therein.

The bulk of the QED corrections is caused by initial-state radiation. QED corrected cross sections are obtained by convoluting the Born cross section, σ_0 , with a radiator function R :

$$\sigma(e^+e^- \rightarrow f\bar{f}(\gamma); s) = \int_{4m_f^2/s}^1 dz R(z, s) \cdot \sigma^0(e^+e^- \rightarrow f\bar{f}; zs), \quad (3.41)$$

where (γ) denotes the possible presence of radiative photons in the final state. The differential cross section is treated in the same way by using a radiator function which depends on the scattering angle.

The radiator functions for symmetric total cross sections, such as σ_{tot} , and antisymmetric forward-backward cross sections, such as σ_{fb} , are different. Therefore, the Born level relations in models with only vector- and axial-vector couplings, $\frac{3}{4}A_{\text{lr}}(s) = -A_{\text{fbpol}}(s)$ and $A_{\text{fbir}}(s) = -\frac{3}{4}A_{\text{pol}}(s)$, do no longer hold after the inclusion of QED radiative corrections.

The idea behind the convolution integral is that initial-state radiative photons, Figure 3.8, carry away four-momentum such that the hard e^+e^- interaction takes place at the reduced centre-of-mass energy, $\sqrt{s'}$, where $s' = zs$. The radiator function $R(z, s)$ expresses the probability for such photon radiation which decreases monotonically with increasing photon energy.

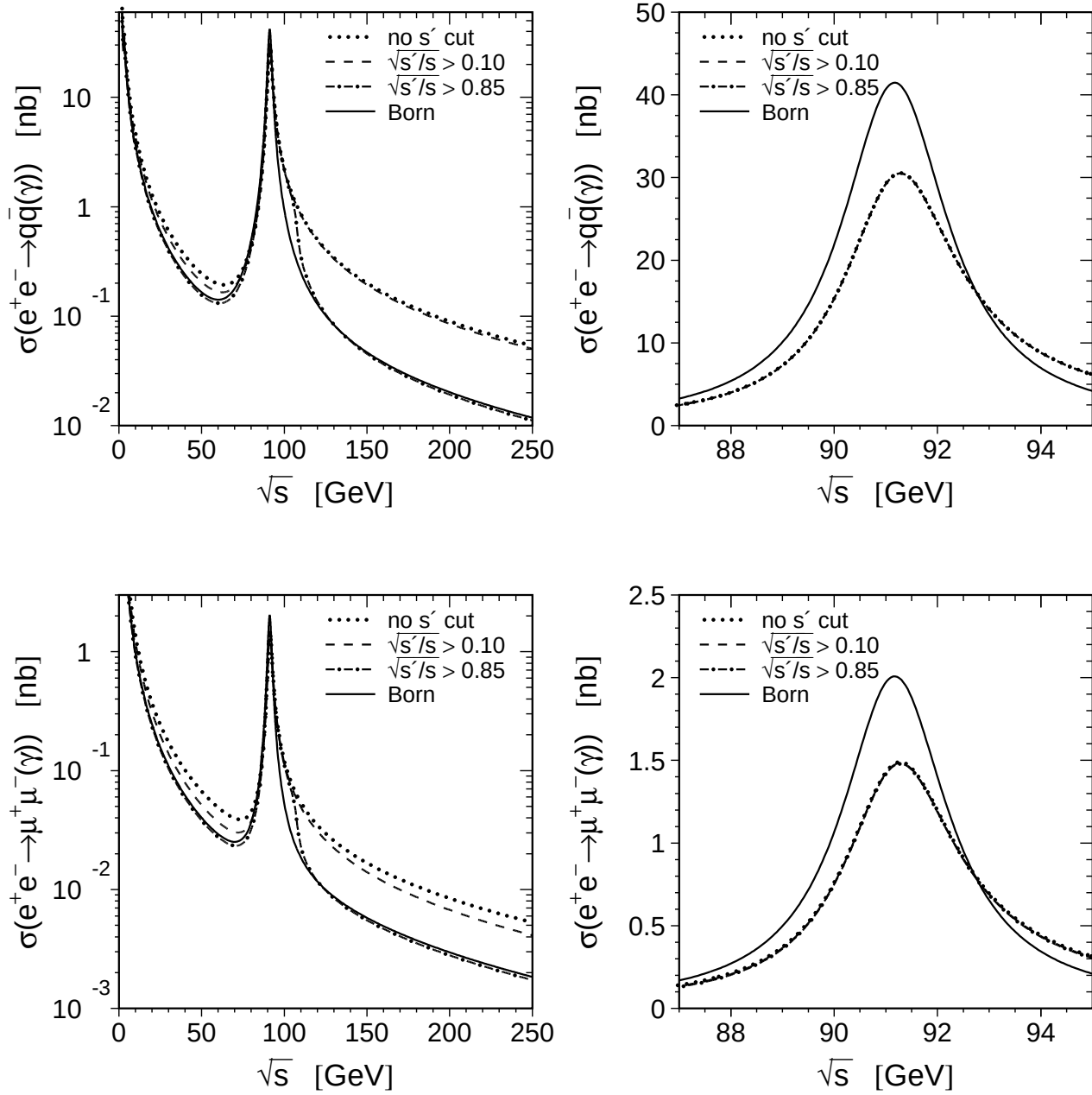


Figure 3.12: Cross sections as a function of $\sqrt{s}$, comparing Born term with radiatively corrected cross sections under various cuts in $\sqrt{s'}$: no $\sqrt{s'}$ cut, $\sqrt{s'}/s > 0.10$, and $\sqrt{s'}/s > 0.85$. Note the step in the latter when the radiative return to the Z is cut out at $\sqrt{s} = M_Z/0.85 = 107$ GeV. At the Z pole, QED radiative correction lower the Z-peak cross section by about 25% and skew the Z lineshape towards higher values.

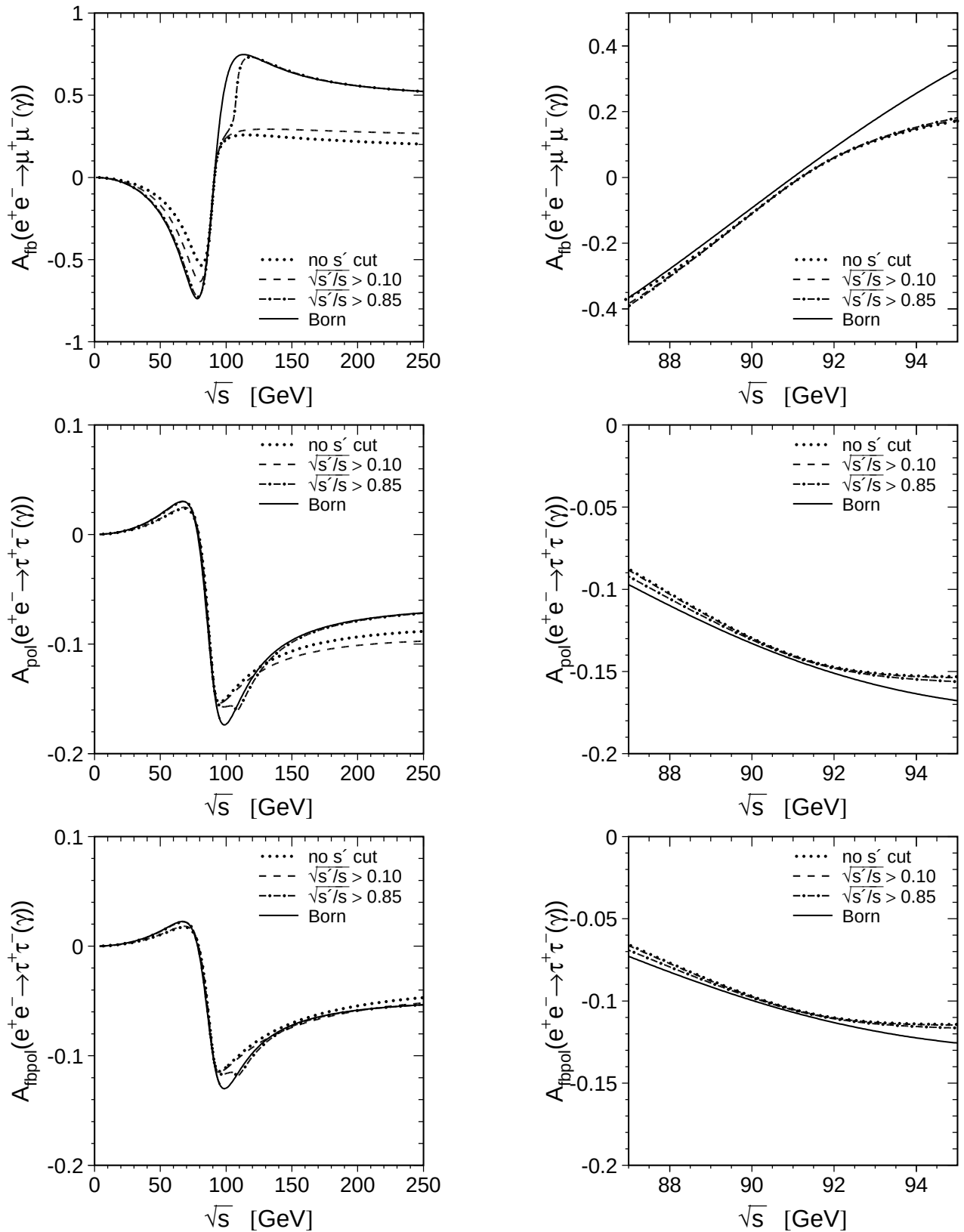


Figure 3.13: Asymmetries as a function of $\sqrt{s}$, comparing Born term with radiatively corrected asymmetries under various cuts in $\sqrt{s'}$: no $\sqrt{s'}$ cut, $\sqrt{s'}/s > 0.10$, and $\sqrt{s'}/s > 0.85$. Note the step in the latter when the radiative return to the Z is cut out at $\sqrt{s} = M_Z/0.85 = 107$ GeV. Especially above the Z pole, the radiative corrections on the forward-backward asymmetry are large, while the polarised asymmetries are less affected.

Technically, the radiator function $R(z, s)$ is split into contributions from virtual-, soft- and hard-photon corrections, δ^V , δ^S , δ^H . Soft-photon corrections are combined with virtual QED corrections at the e^+e^-Z vertex in order to obtain finite results. Summing soft-photon radiation to all orders, the radiator function is given by:

$$R(z, s) = \beta(1-z)^{\beta-1} \left[1 + \sum_{n=1}^{\infty} \left(\frac{\alpha_{em}}{\pi} \right)^n \delta_n^{V+S} \right] + \sum_{n=1}^{\infty} \left[\left(\frac{\alpha_{em}}{\pi} \right)^n \delta_n^H(z) \right] \quad (3.42)$$

$$\beta = \frac{2\alpha_{em}}{\pi}(L-1) \quad (3.43)$$

$$L = \ln \frac{s}{m_e^2}. \quad (3.44)$$

The corrections to first order in α_{em}/π are:

$$\delta_1^{V+S} = \left(\frac{3}{2}L + \frac{\pi^2}{3} - 2 \right) \quad (3.45)$$

$$\delta_1^H(z) = (1-L)(1+z), \quad (3.46)$$

while for the forward-backward cross section the hard-photon correction is more complicated:

$$\delta_1^H(z) = (1-L) \left(\frac{2}{1-z} - \frac{4z}{(1+z)^2} \frac{1+z^2}{1-z} \right) - \frac{4z}{(1+z)^2} \ln \frac{4z}{(1+z)^2}. \quad (3.47)$$

In general the corrections δ_n are n -th order polynomials in the large logarithm L and are calculated order-by-order in perturbation theory. They are known to complete second order and including the summation of soft and virtual photons to all orders. For total cross sections inside a symmetric fiducial volume in $\cos \theta$, the third order correction is known to leading order in the large logarithm L . A collection of radiator functions in the additive and multiplicative form is given in [60, 61, 63–65]. At the Z pole, $\sqrt{s} \approx M_Z$, virtual- and soft-photon corrections dominate as the Z resonance, $\sigma^0(s')$ in the convolution integral, serves as an effective cutoff for hard initial-state photon radiation. In contrast, for $\sqrt{s} > M_Z$, the largest effect arises from hard initial-state radiative corrections. Because of the Z pole in $\sigma^0(s')$, initial-state radiation with the photon lowering $\sqrt{s'}$ to values close to M_Z , called the radiative return to the Z , is preferred. Applying a cut in s' with $s' > s'_{cut} \gg M_Z^2$ restricts the phase-space for photon emission and removes the radiative-return events to keep only the interesting genuine high-energy events.

Final-state QED radiative corrections exhibit a much smaller effect. In the absence of cuts restricting the phase space for final-state radiation, the effect corresponds to an overall normalisation factor on the total cross section:

$$R_{QED}^{(f)} = 1 + \Delta_{QED}^{(f)} \quad (3.48)$$

$$\Delta_{QED}^{(f)} = \frac{3}{4} \frac{\alpha_{em}}{\pi} q_f^2 + \mathcal{O}(\alpha_{em}^2), \quad (3.49)$$

where these factors are also known with cuts on s' and $\cos \theta_f$. The replacement $\alpha_{em} \equiv \alpha_{em}(0) \rightarrow \alpha_{em}(s)$ incorporates the bulk of second- and higher-order corrections such as multiple final-state photon radiation and final-state pair production through radiation of off-shell photons [66].

The radiated photons move preferentially along the direction of the radiating charged particle. Thus, the bulk of initial-state radiative photons is not seen as it is lost inside the beam pipe.

The interference between initial- and final-state radiation is treated as before by extending the above ansatz of convoluting with a radiator function. These corrections need to be combined with QED box contributions, where an additional γ line connects an initial state fermion with a final state fermion. Since the box contributions differ for γ and Z exchange, the cross section to be convoluted must be separated into γ and Z exchange and its interference:

$$\sigma(s) = \int_{4m_f^2/s}^1 dz \left[\sigma_{ZZ}^0(z, s) R_{ZZ}(z, s) + \sigma_{\gamma Z}^0(z, s) R_{\gamma Z}(z, s) + \sigma_{\gamma\gamma}^0(z, s) R_{\gamma\gamma}(z, s) \right] \quad (3.50)$$

The cross sections inside the convolution integral depend on two scales, s for initial-state and $s' = zs$ for final-state radiation:

$$\sigma_{ZZ}^0(z, s) = \frac{4}{3}\pi\alpha_{em}^2 \frac{r_f^{\text{tot}}}{s'} \frac{1}{2} [\chi(s)\chi^*(s') + \chi^*(s)\chi(s')] \quad (3.51)$$

$$\sigma_{\gamma Z}^0(z, s) = \frac{4}{3}\pi\alpha_{em}^2 \frac{j_f^{\text{tot}}}{s'} \frac{1}{4} [\chi(s) + \chi^*(s) + \chi(s') + \chi^*(s')] \quad (3.52)$$

$$\sigma_{\gamma\gamma}^0(z, s) = \frac{4}{3}\pi\alpha_{em}^2 \frac{g_f^{\text{tot}}}{s'} \quad (3.53)$$

$$\chi(s) \equiv \frac{s}{s - M_Z^2 + iM_Z\Gamma_Z}. \quad (3.54)$$

The interference of initial- and final state radiation and the box diagrams make the radiator functions dependent on both s and t , *i.e.*, also on the scattering angle $\cos\theta$. The interference correction is proportional to $\ln(t/u) = \ln \tan(\theta/2)$, thus the interference effect is largest for $|\cos\theta| \rightarrow 1$.

For $\sqrt{s}$ close to M_Z , the corrections arising from the interference between initial- and final-state QED radiation is suppressed due to the finite lifetime of the Z boson [67]. In the absence of strong cuts on the energy of photon radiation, the correction is of order $\mathcal{O}((\alpha_{em}/\pi)(\Gamma_Z/M_Z)^2) = 2 \cdot 10^{-6}$ for cross sections [68], and of order $\mathcal{O}((\alpha_{em}/\pi)(\Gamma_Z/M_Z)) = 6 \cdot 10^{-5}$ for forward-backward asymmetries [69], as no large logarithms $L = \ln \frac{s}{m_e^2}$ appear.

With final-state radiation present, $\sqrt{s'}$ can no longer be identified with the invariant mass of the final-state $f\bar{f}$ system. Care must be taken in the interpretation of results and the comparisons with theoretical calculations, as sometimes the convention is used that $\sqrt{s'}$ is defined as $m_{f\bar{f}}$. Furthermore, the inclusion of initial-final interference leaves only the invariant $f\bar{f}$ mass as an experimentally measurable quantity, but not $\sqrt{s'}$.

3.1.3.2 Pair Production

Photons radiated off the initial- or final-state fermions may also be off-shell, γ^* with $m_{\gamma^*} > 0$, leading to an additional $f\bar{f}$ pair in the final state due to the conversion process $\gamma^* \rightarrow f\bar{f}$. When the additional $f\bar{f}$ system becomes more and more massive, also radiation of virtual Z bosons needs to be included.

The complete final state now consists of four fermions, $e^+e^- \rightarrow f\bar{f}f\bar{f}$. For an additional pair of low invariant mass, pair production is regarded as a radiative correction to fermion-pair production. The production of two pairs of comparable mass, on the other hand, is considered as genuine four-fermion production. Because of the continuous transition region between the two cases, there arises the question of the signal definition for fermion-pair and four-fermion production, *i.e.*, the question which cut to use in order to separate $f\bar{f}f\bar{f}$ production into radiative corrections belonging to fermion-pair production and into genuine four-fermion production.

Experimentally, pair production via low-mass off-shell photons radiated from the initial state are uncritical, as the photon and its decay products are predominantly close to the initial-state particles. Thus the additional soft fermion-antifermion pair is not visible within the detector as it is confined inside the beam pipe. The total correction on the fermion-pair cross section varies with $\sqrt{s}$ and is of the order of $-3 \cdot 10^{-3}$ [70, 71].

In contrast, pair production via off-shell photons radiated from the final state leads to additional fermion-antifermion pairs moving along the direction of the primary final-state fermions. Thus they become visible in the detector. In the absence of cuts restricting the phase space for the radiation of final-state pairs, real and virtual corrections cancel each other [72].

3.1.3.3 QCD Bremsstrahlung Corrections

Final-state QCD radiative corrections are treated similar to final-state QED radiative corrections, replacing the QED coupling constant α_{em} with the strong coupling constant α_S and including QCD

colour factors. Since the QCD coupling is much stronger, $\alpha_S \gg \alpha_{em}$, the correction needs to be known to higher than just first order. For total cross sections, they are known up to third order, thus incorporating multiple gluon radiation and off-shell gluon splitting, $g^* \rightarrow q\bar{q}, gg, ggg$. In the limit of massless quarks, the correction is flavour independent [60, 63]:

$$R_{QCD}^{(f)} = 1 + \Delta_{QCD}^{(f)} \quad (3.55)$$

$$\Delta_{QCD}^{(f)} = \begin{cases} \frac{\alpha_S}{\pi} + 1.41 \left(\frac{\alpha_S}{\pi}\right)^2 - 12.77 \left(\frac{\alpha_S}{\pi}\right)^3 + \mathcal{O}(\alpha_S^4) & \text{for } f = q \\ 0 & \text{for } f = \ell, \nu \end{cases} \quad (3.56)$$

Beyond first order, the vector and axial-vector currents receive different corrections. Simple effective formula for the $b\bar{b}$ and inclusive hadronic final state, which take the leading effects into account, are [60, 63]:

$$\Delta_{QCD}^{(\text{had})} = \frac{\alpha_S}{\pi} + (0.78 \pm 0.04) \left(\frac{\alpha_S}{\pi}\right)^2 - (15.46 \pm 0.06) \left(\frac{\alpha_S}{\pi}\right)^3 + \mathcal{O}(\alpha_S^4) \quad (3.57)$$

$$\Delta_{QCD}^{(b)} = \frac{\alpha_S}{\pi} - (2.46 \pm 0.17) \left(\frac{\alpha_S}{\pi}\right)^2 - (24.7 \pm 0.3) \left(\frac{\alpha_S}{\pi}\right)^3 + \mathcal{O}(\alpha_S^4). \quad (3.58)$$

The corrections Δ_{QCD} are shown in Figure 3.14 as a function of α_S .

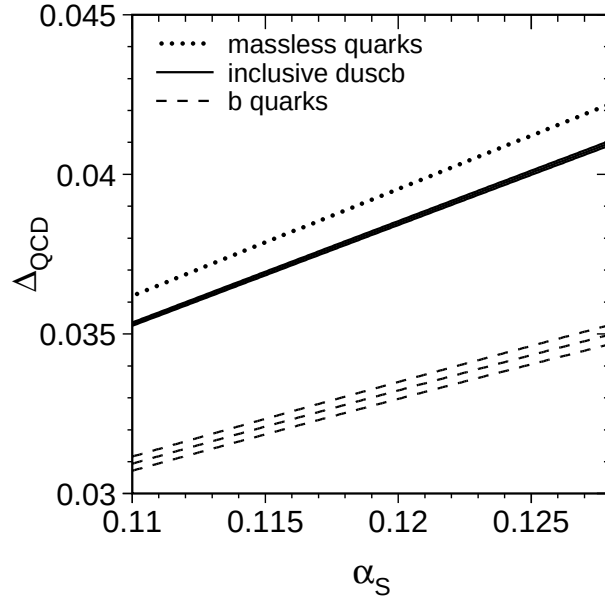


Figure 3.14: QCD correction for partial Z decay widths, Δ_{QCD} , as a function of the strong coupling constant, α_S . Shown are the calculations for massless quarks, the inclusive hadronic width, and the special case of b quarks. For the latter, the triple lines show central value and ± 1 sigma theoretical error.

3.1.3.4 Combination of QED and QCD Bremsstrahlung Corrections

Mixed QED/QCD bremsstrahlung corrections arise due to the competing radiation of both a photon and a gluon off the final-state quarks. The additional normalisation factor is given by [66]:

$$R_{QED/QCD}^{(f)} = 1 + \Delta_{QED/QCD}^{(f)} \quad (3.59)$$

$$\Delta_{QED/QCD}^{(f)} = \begin{cases} -\frac{1}{4} \frac{\alpha_{em}}{\pi} \frac{\alpha_S}{\pi} q_q^2 & \text{for } f = q \\ 0 & \text{for } f = \nu, \ell \end{cases} \quad (3.60)$$

If QED, QCD and mixed QED/QCD corrections are combined, the overall correction is given by:

$$R^{(f)} = 1 + \Delta_{QED}^{(f)} + \Delta_{QCD}^{(f)} + \Delta_{QED/QCD}^{(f)}. \quad (3.61)$$

Note that there is no factorisation of QED and QCD bremsstrahlung corrections, $R^{(f)} \neq R_{QED}^{(f)} R_{QCD}^{(f)}$.

3.1.3.5 Final-State Corrections on Asymmetries

To first order in α_{em} and α_S , final state corrections cancel in the forward-backward cross section, $\sigma_{fb} = \sigma(\cos\theta > 0) - \sigma(\cos\theta < 0)$, of $f\bar{f}$ production. Thus the forward-backward asymmetry, being the ratio of the forward-backward and the total cross section, is affected by the inverse of the correction factor for total cross sections.

3.1.3.6 Weak Corrections and Effective Coupling Constants

Weak corrections to fermion-pair production fall into three classes, propagator or self-energy corrections of the intermediate gauge bosons, vertex corrections at the boson-fermion vertices, and contributions from box diagrams. The effect of γ/Z mixing in the internal gauge-boson propagator, possible due to fermion loop insertions, does no longer allow a clean separation of the matrix element in a γ and Z part. If the box contributions are neglected, it is still possible, however, to retain the structure of the Born-term γ and Z exchange matrix elements summing up to the total matrix element of the $e^+e^- \rightarrow f\bar{f}$ scattering process. The electroweak radiative corrections are incorporated in the two pieces in such a way that a gauge invariant separation is obtained.

Weak radiative corrections modify the photon and Z -boson matrix elements by introducing complex valued $\sqrt{s}$ dependent formfactors, $F_A(s)$, $G_{Vf}(s)$ and $G_{Af}(s)$:

$$\mathcal{M} = \mathcal{M}_\gamma + \mathcal{M}_Z \quad (3.62)$$

$$\mathcal{M}_\gamma = i \frac{4\pi\alpha_{em}}{s} F_A(s) [(q_e\gamma_\mu) \otimes (q_f\gamma^\mu)] \quad (3.63)$$

$$\mathcal{M}_Z = i\sqrt{2}G_F M_Z^2 \chi(s) [(\gamma_\mu(G_{Ve}(s) - G_{Ae}(s)\gamma_5)) \otimes (\gamma^\mu(G_{Vf}(s) - G_{Af}(s)\gamma_5))] . \quad (3.64)$$

Expressions for cross sections and decay widths are rewritten in terms of complex vector- and axial-vector couplings by using the following prescription given in symbolic notation:

- Z boson exchange:

$$\{g_{Vf}^2, g_{Af}^2, g_{Vf}g_{Af}\} \longrightarrow \{|G_{Vf}|^2, |G_{Af}|^2, \Re(G_{Vf}G_{Af}^*)\} \quad (3.65)$$

- γ exchange:

$$q_e^2 q_f^2 \longrightarrow q_e^2 q_f^2 |F_A(s)|^2 \quad (3.66)$$

- γ/Z interference:

$$q_e q_f \{g_{Ve}g_{Vf}, g_{Ae}g_{Af}\} \Re(\chi_Z(s)) \longrightarrow q_e q_f \Re(\{G_{Ve}G_{Vf}, G_{Ae}G_{Af}\} F_A^*(s) \chi_Z(s)) \quad (3.67)$$

$$q_e q_f \{g_{Ve}g_{Af}, g_{Ae}g_{Vf}\} \Re(\chi_Z(s)) \longrightarrow q_e q_f \Re(\{G_{Ve}G_{Af}, G_{Ae}G_{Vf}\} F_A^*(s) \chi_Z(s)) . \quad (3.68)$$

The photonic matrix element $\mathcal{M}_\gamma$ contains the effect of fermion loop insertions in the photon propagator, expressed by the complex photon vacuum polarisation, $F_A(s)$. The real part of $F_A(s)$ is simply the running of the finestructure constant α_{em} as introduced during the discussion of Δr in Section 2.11.2:

$$\Re(F_A(s)) = \frac{1}{1 - \Delta\alpha_{em}(s)}. \quad (3.69)$$

The imaginary part is given by:

$$\Im(F_A(s)) = -\frac{1}{3} \sum_f N_C^f q_f^2 \beta_f \left(1 + \frac{2m_f^2}{s}\right) \theta(s - 4m_f^2) \quad (3.70)$$

$$\Im(F_A(M_Z^2)) \rightarrow -\frac{1}{3} \sum_{f \neq t} q_f^2 N_C^f. \quad (3.71)$$

Within the total matrix element, the imaginary part of $F_A(s)$ picks up the imaginary part of the Z-boson propagator, leading to an additional γ/Z interference contribution to cross sections and asymmetries of the order of $\alpha_{em} \Im(F_A(M_Z^2)) \Gamma_Z/M_Z$. Through γ/Z mixing in the propagator, it also contributes to the resonant Z-exchange terms.

The complex vector and axial-vector couplings are expressed in terms of complex formfactors $\rho_f(s)$ and $\kappa_f(s)$:

$$G_{Vf}(s) = \sqrt{\rho_f(s)} \cdot (T_3^f - 2q_f \kappa_f(s) \sin^2 \theta_W) \quad (3.72)$$

$$G_{Af}(s) = \sqrt{\rho_f(s)} \cdot T_3^f. \quad (3.73)$$

Loop insertions to the Z boson propagator are interpreted as a change in the weak coupling strength. The coupling in the Z-boson matrix element $\mathcal{M}_Z$ is changed:

$$\frac{\pi \alpha_{em}}{\sin^2 \theta_W \cos^2 \theta_W} \rightarrow \rho(s) \sqrt{2} G_F M_Z^2, \quad (3.74)$$

where $\Re(\rho) = 1/(1 - \Delta\rho)$ is, to leading order, nothing else but the ρ parameter denoting the ratio between the neutral weak and charge weak current. In general the ρ parameter is expected when expressing a neutral weak current amplitude in terms of the charged weak current coupling strength G_F . The large logarithms due to fermionic loop insertions as seen in the photon propagator and $\alpha_{em}(s)$ are cancelled by introducing the charged current coupling G_F .

The Z/γ mixing in the propagator with subsequent coupling of the photon to the fermion pair is interpreted as a correction to the charge dependent part of the vector coupling of the Z boson. The factor $\kappa(s)$ in front of the electroweak mixing angle expresses this correction:

$$\sin^2 \theta_W \rightarrow \kappa(s) \sin^2 \theta_W, \quad (3.75)$$

where $\kappa = 1 + \Delta\kappa$.

For Z-pole measurements, the complex s -dependent couplings may be treated as constant by setting $s = M_Z^2$, called the Z-pole approximation. In order to avoid a complex mixing angle and complex vector- and axial-vector couplings, only the real parts are usually retained as effective couplings whereas the imaginary parts are added explicitly:

$$\alpha_{em}(M_Z^2) = \Re(\alpha_{em} \cdot F_A(M_Z^2)) = \frac{\alpha_{em}}{1 - \Delta\alpha_{em}(M_Z^2)} \quad (3.76)$$

$$g_{Af} = \Re(\sqrt{\rho_f}) \cdot T_3^f \quad (3.77)$$

$$g_{Vf} = \Re(\sqrt{\rho_f}) \cdot (T_3^f - 2\Re(\kappa_f) q_f \sin^2 \theta_W) \quad (3.78)$$

$$\sin^2 \bar{\theta}_f = \Re(\kappa_f) \sin^2 \theta_W. \quad (3.79)$$

The approximation where the imaginary parts are neglected is called the improved Born approximation. This approximation is not adequate for the precise results obtained at SLC and LEP-I. For the extraction of the effective couplings from the experimental data, the imaginary parts are therefore fixed to their Standard Model prediction.

In general theories the three weak correction terms Δr_w , $\Delta\rho$, and $\Delta\kappa$ are independent of each other. Within the MSM, the leading terms are the same and given by $\Delta\rho$:

$$\Delta r_w = -\cot^2 \theta_W \Delta\rho \quad (3.80)$$

$$\Delta\kappa = +\cot^2 \theta_W \Delta\rho. \quad (3.81)$$

The corrections discussed so far are of universal nature, *i.e.*, they are independent of the flavour of the external fermions. Fermion-specific vertex corrections introduce dependences on the flavour of the external fermions:

$$\rho_f = \rho + \Delta\rho_f \quad (3.82)$$

$$\kappa_f = \kappa + \Delta\kappa_f. \quad (3.83)$$

For all fermions except the b quark, the flavour dependence of the vertex correction is very small. However, for b quarks, vertex corrections involving Wtb vertices, as shown in Figure 3.15, are large due to the high mass of the isopartner of the b quark, the top quark, and the large Cabibbo-Kobayashi-Maskawa quark mixing matrix element $|V_{tb}| \approx 1$:

$$\Delta\rho_b = -\frac{4}{3}\Delta\rho \quad (3.84)$$

$$\Delta\kappa_b = -\frac{1}{2}\Delta\rho_b = +\frac{2}{3}\Delta\rho. \quad (3.85)$$

The inclusion of the specific vertex corrections inverts the M_t dependence of ρ_b and increases the M_t dependence of κ_b as compared to the other fermions. A comparison of the electroweak corrections for $f = b$ and $f \neq b$ is shown in Figures 3.16.

Based on the master equation:

$$\sin^2 \theta (1 - \sin^2 \theta) = \frac{\pi\alpha_{em}}{\sqrt{2}G_F M_Z^2} \cdot \frac{1}{1 - \Delta}, \quad (3.86)$$

several possibilities exist to define an electroweak mixing angle:

$$\sin^2 \theta = \frac{1}{2} \left[1 - \sqrt{1 - 4 \frac{\pi\alpha_{em}}{\sqrt{2}G_F M_Z^2} \frac{1}{1 - \Delta}} \right], \quad (3.87)$$

which are summarised in Table 3.2. The comparison of the top-quark and Higgs-boson mass dependence of the different definitions of the electroweak mixing angle, on-shell $\sin^2 \theta_W$ and effective $\sin^2 \bar{\theta}_W$, is shown in Figure 3.17.

While the measurement of the W-boson mass at the TEVATRON and LEP-II determines the on-shell quantities $\sin^2 \theta_W$ and Δr , measurement of fermion-pair production at the Z pole performed at SLC and LEP-I determine the effective quantities $\sin^2 \bar{\theta}_f$ and $\Delta\bar{r}_f$. For a heavy top quark and a heavy Higgs boson, the effective Z-pole quantities are given by:

$$\Delta\bar{r}^{(t)} = -\frac{G_F M_W^2}{8\sqrt{2}\pi^2} \left[3 \frac{M_t^2}{M_W^2} - \frac{2}{3 \cos^2 \theta_W} \ln \frac{M_t^2}{M_W^2} \right] \quad (3.88)$$

$$\Delta\bar{r}^{(H)} = \frac{G_F M_W^2}{8\sqrt{2}\pi^2} \frac{1 + 9 \sin^2 \theta_W}{3 \cos^2 \theta_W} \left(\ln \frac{M_H^2}{M_W^2} - \frac{5}{6} \right). \quad (3.89)$$

The sensitivity of the effective Z pole quantities to $\Delta\alpha_{em}$, M_t and M_H , given by:

$$\frac{\partial \Delta\bar{r}}{\partial \Delta\alpha_{em}} \delta \Delta\alpha_{em} = +1 \delta \Delta\alpha_{em} \quad (3.90)$$

$$\frac{\partial \Delta\bar{r}}{\partial M_t} \delta M_t = -\frac{3G_F}{4\sqrt{2}\pi^2} M_t \delta M_t = -0.0005 \left(\frac{M_t}{175 \text{ GeV}} \right) \left(\frac{\delta M_t}{5 \text{ GeV}} \right) \quad (3.91)$$

$$\frac{\partial \Delta\bar{r}}{\partial M_H} \delta M_H = +\frac{G_F M_W^2}{4\sqrt{2}\pi^2} \frac{1 + 9 \sin^2 \theta_W}{3 \cos^2 \theta_W} \left(\frac{\delta M_H}{M_H} \right) = +0.0017 \frac{\delta M_H}{M_H}. \quad (3.92)$$

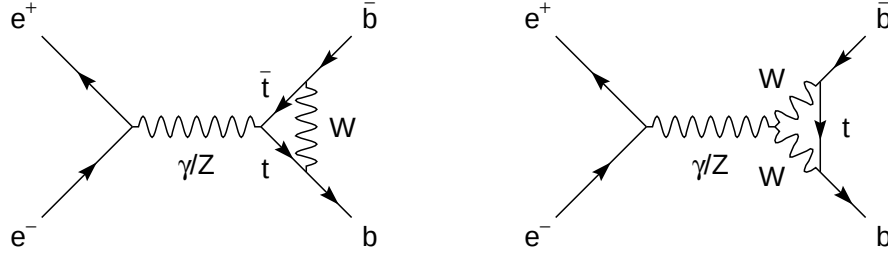


Figure 3.15: Vertex corrections in $b\bar{b}$ production involving the top quark.

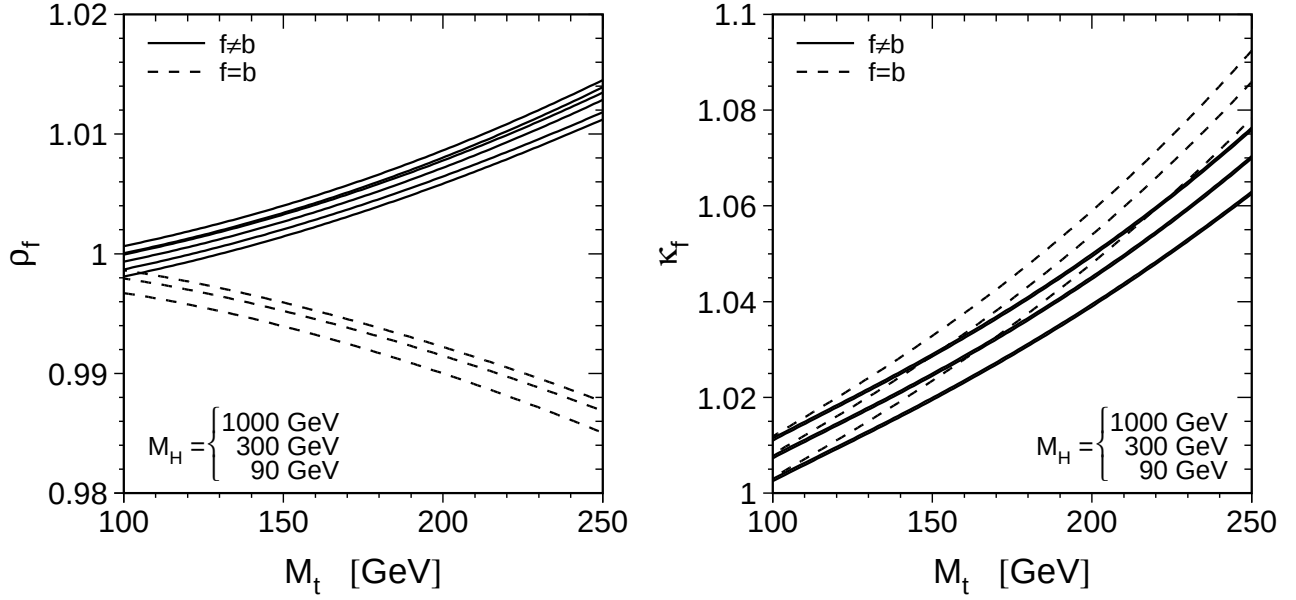


Figure 3.16: Electroweak formfactors ρ_f and κ_f as a function of M_t comparing b and non-b fermions. In each case, three lines are shown corresponding to Higgs-boson masses of 90 GeV, 300 GeV, and 1000 GeV. For fixed top-quark mass, both ρ_f and κ_f increase with M_H .

Scheme	Δ	$\sin^2 \theta$
Born	0	$\sin^2 \theta$
+ QED	$\Delta\alpha_{em}$	$\sin^2 \theta_0$
+ self energies	$\Delta\bar{r} = \Delta\alpha_{em} - \Delta\rho + \Delta\bar{r}_{rem}$	$\sin^2 \bar{\theta}_W = (1 + \Delta\kappa) \sin^2 \theta_W$
+ vertex corrections	$\Delta\bar{r}_f = \Delta\alpha_{em} - \Delta\rho + \Delta\bar{r}_{f,rem}$	$\sin^2 \bar{\theta}_f = \kappa_f \sin^2 \theta_W$
M_W prediction	$\Delta r = \Delta\alpha_{em} - \cot^2 \theta_W \Delta\rho + \Delta r_{rem}$	$\sin^2 \theta_W = 1 - M_W^2/M_Z^2$

Table 3.2: Various definitions of the electroweak mixing angle.

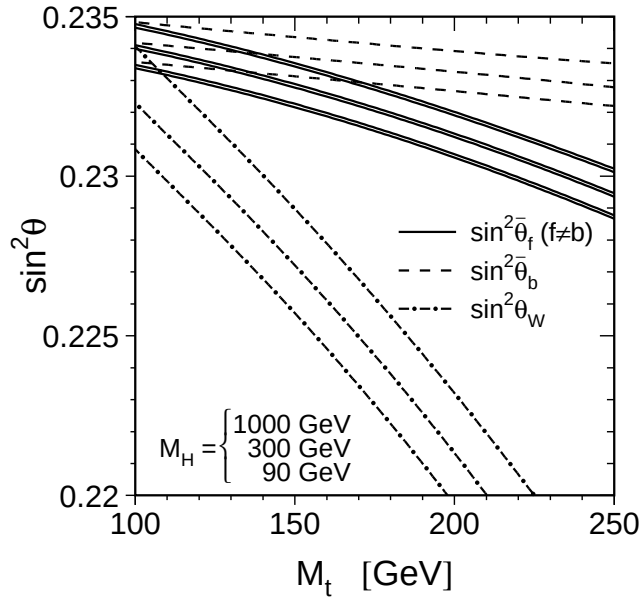


Figure 3.17: The effective and the on-shell electroweak mixing angle as a function of M_t , comparing b and non-b fermions. In each case, three lines are shown corresponding to Higgs-boson masses of 90 GeV, 300 GeV, and 1000 GeV. For fixed top-quark mass, all increase with M_H .

Compared to the on-shell quantities Δr and $\sin^2 \theta_W$, the sensitivity is reduced by a factor of $\cot^2 \theta_W = 3.5$ for M_t and $11 \cos^2 \theta_W / (1 + 9 \sin^2 \theta_W) = 2.8$ for M_H , as visible in Figure 3.17. However, the large statistics of the Z-pole measurements at SLC and LEP-I more than compensates this loss of sensitivity. Nevertheless when interpreting measurements of $\Delta \bar{r}$, the error on the hadronic vacuum polarisation of ± 0.00065 [44, 45] is equivalent to an error of 6 GeV on M_t and an error of 37% on M_H .

The dependences of the electroweak correction terms Δ on top-quark and Higgs-boson mass are of opposite sign. Constraining electroweak corrections by an experimental measurement implies a determination of M_t and M_H with a positive correlation.

3.1.4 The Decay Widths of the Z Boson

The Z boson propagator needs to be considered together with the photon propagator due to γ/Z mixing effects through loop insertions in the propagator. The imaginary part of the Z self energy appears in the total decay width, Γ_Z , entering the Z-boson propagator. Both these higher-order corrections and phase-space effects lead to a linear increase of the total decay width Γ_Z with $\sqrt{s}$. Overall, Γ_Z varies linear with s :

$$\Gamma_Z \longrightarrow \Gamma_Z(s) \equiv \frac{s}{M_Z^2} \Gamma_Z(M_Z), \quad (3.93)$$

so that the Breit-Wigner description of the Z pole naturally changes from an s -independent width to an s -dependent width, see also Section 3.1.1.4. This leads to the above redefinition of the parameter called total width to be $\Gamma_Z = \Gamma_Z(M_Z)$ so that the s -dependence appears explicitly in the Z-boson Breit-Wigner $\chi_Z(s)$.

Owing to factorisation, partial decay widths of the Z boson are corrected by the same terms as cross sections to obtain radiatively corrected decay widths. The individual partial decay widths are written in terms of G_F and effective couplings absorbing weak corrections and are corrected by the same factors R for final state QED and QCD radiation as discussed before. Including fermion-mass

effects, the partial widths of the Z boson are:

$$\Gamma_{ff} = N_C^f \frac{G_F M_Z^3}{6\sqrt{2}\pi} \sqrt{1 - \frac{4m_f^2}{M_Z^2}} \left[|G_{Vf}|^2 \left(1 + \frac{2m_f^2}{M_Z^2}\right) + |G_{Af}|^2 \left(1 - \frac{4m_f^2}{M_Z^2}\right) \right] R^{(f)} \quad (3.94)$$

$$R^{(f)} = 1 + \Delta_{QED}^{(f)} + \Delta_{QCD}^{(f)} + \Delta_{QED/QCD}^{(f)}. \quad (3.95)$$

For the calculation of partial width to one-loop order, the complex formfactors G_V and G_A may be replaced by their real parts g_V and g_A , as there is no other imaginary part to interfere with. If, however, cross sections are calculated, the imaginary part picks up the imaginary part of the boson propagator at Born level, thus cannot be neglected.

In case the Z-exchange cross section is written as a product of the radiatively corrected partial widths into initial- and final-state fermion-pairs, $\sigma \propto \Gamma_{ee} \Gamma_{ff}$, then the QED correction of Γ_{ee} , $\Delta_{QED}^{(e)}$, must be taken out, as initial-state radiation is explicitly accounted for through the convolution integral.

3.1.5 The ϵ Parameters

Within the framework of the Standard Model, most electroweak radiative corrections contain the potentially large quadratic contribution from the top-quark mass. It has been studied how the Higgs dependence and new physics can be disentangled from this leading top quark correction. For this purpose, four new parameters, ϵ_1 , ϵ_2 , ϵ_3 and ϵ_b , are introduced [56, 73]. They are defined such that they absorb genuine weak corrections and vanish in the approximation when only effects due to pure QED and QCD are taken into account.

$$\epsilon_1 = \Delta\rho \quad (3.96)$$

$$\epsilon_2 = \cos^2 \theta_0 \Delta\rho + \frac{\sin^2 \theta_0}{\cos^2 \theta_0 - \sin^2 \theta_0} \Delta r_w - 2 \sin^2 \theta_0 \Delta\kappa' \quad (3.97)$$

$$\epsilon_3 = \cos^2 \theta_0 \Delta\rho + (\cos^2 \theta_0 - \sin^2 \theta_0) \Delta\kappa' \quad (3.98)$$

$$\epsilon_b = \frac{1}{2} \Delta\rho_b, \quad (3.99)$$

where $\sin^2 \theta_0$ is the electroweak mixing angle absorbing only QED effects:

$$\sin^2 \theta_0 \cos^2 \theta_0 = \frac{\pi \alpha_{em} (M_Z^2)}{\sqrt{2} G_F M_Z^2}, \quad (3.100)$$

and $\Delta\kappa'$ transforms this electroweak mixing angle into the effective electroweak mixing angle:

$$\sin^2 \bar{\theta}_f = (1 + \Delta\kappa') \sin^2 \theta_0. \quad (3.101)$$

Within the Standard Model one obtains the following large asymptotic contributions:

$$\epsilon_1 = \frac{3G_F M_t^2}{8\sqrt{2}\pi^2} - \frac{3G_F M_W^2}{4\sqrt{2}\pi} \tan^2 \theta_W \ln \frac{M_H}{M_Z} + \dots \quad (3.102)$$

$$\epsilon_2 = -\frac{G_F M_W^2}{2\sqrt{2}\pi^2} \ln \frac{M_t}{M_Z} + \dots \quad (3.103)$$

$$\epsilon_3 = \frac{G_F M_W^2}{12\sqrt{2}\pi^2} \ln \frac{M_H}{M_Z} - \frac{G_F M_W^2}{6\sqrt{2}\pi^2} \ln \frac{M_t}{M_Z} + \dots \quad (3.104)$$

$$\epsilon_b = -\frac{G_F M_t^2}{4\sqrt{2}\pi^2} + \dots \quad (3.105)$$

Another commonly used description is based on the so-called S, T and U parameters [74].

While the ϵ parameters are merely rearrangements of corrections arising from higher-order diagrams, their merit lies in isolating the large quadratic M_t effects in ϵ_1 and ϵ_b . The quantities ϵ_2 and ϵ_3

contain only logarithmic dependences on M_t . The leading terms for large Higgs masses are contained in ϵ_1 and ϵ_3 .

A second aspect in the construction of the ϵ parameters is the search for new physics effects. In the case of the ϵ parameters, it is assumed that new physics effects enter mainly through vacuum polarisation diagrams. There are three independent vacuum polarisation contributions, in analogy to the three independent terms $\Delta\rho$, $\Delta\kappa$, and Δr_W in theories more general than the Minimal Standard Model. Such an ansatz includes lepton-, neutrino- and light-quark universality, thus ignoring fermion-specific vertex corrections with the exception of those for the b quark.

The Z pole measurements at SLC and LEP-I constrain ϵ_1 , ϵ_3 and ϵ_b through the measurement of the Z mass and the effective couplings. The parameter ϵ_2 is determined through Δr_W which requires the measurement of the W-boson mass, performed at the TEVATRON and at LEP-II.

3.1.6 Summary of Fermion-Pair Production

The measurement of the fermion-pair production cross section as a function of the centre-of-mass energy in the vicinity of the Z pole determines the mass and total and partial decay widths of the Z boson. The partial decay widths in turn determine the sum of the squares of the effective vector- and axial-vector coupling constants, $g_{Vf}^2 + g_{Af}^2$. The various asymmetries determine the ratio of the vector and the axial vector couplings, g_{Vf}/g_{Af} , in form of the couplings parameter, A_f . Together they determine g_{Af} and g_{Vf} up to the ambiguity $g_{Vf} \leftrightarrow g_{Af}$, which is resolved by the energy-dependence of the asymmetries. The measurements of cross sections and asymmetries at centre-of-mass energies away from the Z pole determine the γ/Z interference terms, j_f .

Electroweak radiative corrections are absorbed in effective vector and axial-vector coupling constants. Analyses of the measured effective couplings test the Standard Model at the level of radiative corrections. In particular, since the top quark and the Higgs boson appear as virtual particles in loop corrections, their masses are constrained by precision measurements even if they are too heavy to be produced directly at current e^+e^- colliders.

3.2 Boson-Pair and Four-Fermion Production

In e^+e^- interactions, pairs of neutral bosons γ and Z are produced in neutral-current t - and u -channel interactions as shown in Figure 3.18. The set of these two diagrams is denoted as NC02 [62]. As only the already known $\gamma f\bar{f}$ and $Z f\bar{f}$ vertices and couplings are involved, pair production of neutral gauge bosons and their decay does not reveal new information about the Standard Model. However, an observation of neutral boson pair production involving ZZZ , γZZ or $\gamma\gamma Z$ tree-level vertices, absent in the SM, would signal new physics beyond the SM.

Topologically, the NC02 diagrams with at least one boson decaying into a fermion-antifermion pair are identical to s -channel fermion-antifermion pair production with a radiative photon or additional pair-production attached to the initial state. This shows again the problem of the signal definition, *i.e.*, what is considered as radiative correction to fermion-pair production, and what is considered as genuine four-fermion production, now from the side of four-fermion production.

Pair production of charged bosons, $e^+e^- \rightarrow W^+W^-$, proceeds through both charged-current t -channel and neutral-current s -channel interactions as shown in Figure 3.19. The set of these three diagrams is called CC03 [62]. The s -channel diagrams are particularly interesting because they appear as a consequence of the triple gauge boson vertices γW^+W^- and ZW^+W^- which are expected due to the non-Abelian nature of the electroweak gauge group $SU(2)_L$. Measurement of W -pair production thus tests the Standard Model of electroweak interactions in this very fundamental area.



Figure 3.18: Feynman diagrams in γ/Z -pair production in e^+e^- interactions. The u -channel diagram exists for pair-production of identical bosons.

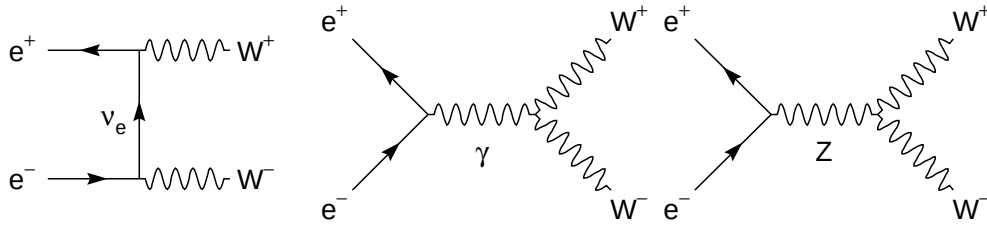


Figure 3.19: Feynman diagrams in W -pair production in e^+e^- interactions.

3.2.1 Born Level

3.2.1.1 Helicity Amplitudes

The matrix element for W -pair production in e^+e^- interactions, $e^+e^- \rightarrow W^+W^-$, is a sum of three contributions corresponding to s -channel γ and Z exchange and t -channel neutrino exchange. The decomposition in terms of helicity amplitudes is given by:

$$\mathcal{M}(\sigma, \bar{\sigma}, \lambda, \bar{\lambda}) = \mathcal{M}_\gamma(\sigma, \bar{\sigma}, \lambda, \bar{\lambda}) + \mathcal{M}_Z(\sigma, \bar{\sigma}, \lambda, \bar{\lambda}) + \mathcal{M}_\nu(\sigma, \bar{\sigma}, \lambda, \bar{\lambda}), \quad (3.106)$$

where $\sigma, \bar{\sigma}$ are the helicities of the initial-state electron and positron, normalised to $\sigma = \pm 1$, and $\lambda, \bar{\lambda}$ are the helicities of the outgoing W^- and W^+ bosons, $\lambda = \pm 1, 0$. The total spin of the initial and the final state is given by $\Delta\sigma = (\sigma - \bar{\sigma})/2$ and $\Delta\lambda = \lambda - \bar{\lambda}$, respectively.

The t -channel neutrino exchange requires left-handed electrons and right-handed positrons, thus contributes to the $\Delta\sigma = -1$ cross section σ_{LR} only. Since due to the exchange of spin-1 bosons only vector- and axial-vector couplings are involved, helicity conservation in the limit of massless electrons implies that amplitudes with $\Delta\sigma = 0$ vanish,¹ $\sigma_{LL} = \sigma_{RR} = 0$. The s -channel diagrams contribute both to σ_{LR} and σ_{RL} . For the final-state W-boson pair, nine helicity states are possible.

The polar angular dependence due to kinematics is given by the d functions of angular momentum [58], $d_{\Delta\sigma\Delta\lambda}^{J_0}$, where $J_0 = 1, 2$ denotes the lowest angular momentum contributing for a given helicity contribution, $J_0 = \max(|\Delta\sigma|, |\Delta\lambda|)$. In order to separate the purely kinematic d functions from the interesting dynamics, one writes $\mathcal{M}$ in terms of reduced matrix elements $\tilde{\mathcal{M}}$:

$$\mathcal{M}(\sigma, \bar{\sigma}, \lambda, \bar{\lambda}, \theta) = 4\pi\alpha_{em}\tilde{\mathcal{M}}(\Delta\sigma, \bar{\sigma}, \lambda, \bar{\lambda})d_{\Delta\sigma\Delta\lambda}^{J_0}(\theta). \quad (3.107)$$

Only the t -channel diagram contributes to the two $J_0 = |\Delta\lambda| = 2$ states:

$$\tilde{\mathcal{M}}(\Delta\sigma, |\Delta\lambda| = 2) = \tilde{\mathcal{M}}_\nu(\Delta\sigma = -1, |\Delta\lambda| = 2) \quad (3.108)$$

$$= \frac{\delta_{\Delta\sigma, -1}}{\sin^2\theta_W} \frac{s}{4t} = -\frac{\delta_{\Delta\sigma, -1}}{\sin^2\theta_W} \frac{1}{1 + \beta_W^2 - 2\beta_W \cos\theta}, \quad (3.109)$$

where the Kronecker δ ensures $\Delta\sigma = -1$, and $\beta_W = \sqrt{1 - 4M_W^2/s}$ is the W-boson velocity. The remaining seven helicity states are accessible via both the s -channel and the t -channel diagrams:

$$\tilde{\mathcal{M}}_\gamma(\Delta\sigma, \lambda, \bar{\lambda}) = -\sqrt{2}\beta_W A_{\Delta\sigma\lambda\bar{\lambda}}^\gamma \delta_{|\Delta\sigma|, +1} \quad (3.110)$$

$$\tilde{\mathcal{M}}_Z(\Delta\sigma, \lambda, \bar{\lambda}) = +\sqrt{2}\beta_W \left[\delta_{|\Delta\sigma|, +1} - \frac{1}{2\sin^2\theta_W} \delta_{\Delta\sigma, -1} \right] \frac{s}{s - M_Z^2} A_{\Delta\sigma\lambda\bar{\lambda}}^Z \quad (3.111)$$

$$\tilde{\mathcal{M}}_\nu(\Delta\sigma, \lambda, \bar{\lambda}) = \frac{1}{\sqrt{2}\beta_W \sin^2\theta_W} \left[B_{\lambda\bar{\lambda}} + \frac{s}{4t} C_{\lambda\bar{\lambda}} \right]. \quad (3.112)$$

Two components contribute to the total matrix element, an electromagnetic part with coupling $e^2 = 4\pi\alpha_{em}$, and a weak part with coupling $e^2/\sin^2\theta_W = g_2^2$. The d functions and contributing Feynman diagrams for the nine helicity states of the W bosons are reported in Table 3.3. The subamplitudes $A_{\Delta\sigma\lambda\bar{\lambda}}^V$ for $V = \gamma, Z$, $B_{\lambda\bar{\lambda}}$ and $C_{\lambda\bar{\lambda}}$ as expected in the Standard Model are given in Table 3.4.

3.2.1.2 Triple Gauge Boson Couplings

In order to test the sector of non-Abelian self couplings among the gauge bosons, triple gauge boson couplings more general than those appearing in the Standard Model are considered. The most general model considers each of the seven helicity states of the W-pair final state, which are accessible via diagrams containing triple gauge boson vertices, as independent, yielding seven coupling constants each for photon and Z boson exchange. Several conventions for these couplings appear in the literature. Here the set is used which corresponds to the effective Lagrangian yielding the most general Lorentz invariant triple gauge boson vertices observable in processes where the gauge bosons couple

¹Since the electron mass is small but finite, the LL - and RR -amplitudes are suppressed by $m_e/\sqrt{s}$ compared to the other amplitudes but non-vanishing. They lead to a constant contribution to the total cross section. Unitarity problems at very high centre-of-mass energies are avoided by including the s -channel Higgs contribution to W-pair production, $e^+e^- \rightarrow H \rightarrow W^+W^-$.

$\Delta\sigma = -1 \quad (\sigma_{LR})$				
$\Delta\lambda$	$(\lambda, \bar{\lambda})$	J_0	$d_{\Delta\sigma\Delta\lambda}^{J_0}$	Diagrams
+2	(+-)	2	$+\frac{1}{2}(1 - \cos \theta) \sin \theta$	ν
-2	(-+)	2	$-\frac{1}{2}(1 + \cos \theta) \sin \theta$	ν
+1	(0-), (+0)	1	$+\frac{1}{2}(1 - \cos \theta)$	γ, Z, ν
0	(--), (00), (++)	1	$+\frac{1}{\sqrt{2}} \sin \theta$	γ, Z, ν
-1	(-0), (0+)	1	$+\frac{1}{2}(1 + \cos \theta)$	γ, Z, ν

$\Delta\sigma = +1 \quad (\sigma_{RL})$				
$\Delta\lambda$	$(\lambda, \bar{\lambda})$	J_0	$d_{\Delta\sigma\Delta\lambda}^{J_0}$	Diagrams
+1	(0-), (+0)	1	$+\frac{1}{2}(1 + \cos \theta)$	γ, Z
0	(--), (00), (++)	1	$-\frac{1}{\sqrt{2}} \sin \theta$	γ, Z
-1	(-0), (0+)	1	$+\frac{1}{2}(1 - \cos \theta)$	γ, Z

Table 3.3: Helicity states in W-pair production: Amplitudes, d functions and contributing Feynman diagrams.

to effectively massless fermions [75]. With $V = \gamma, Z$:

$$\begin{aligned}
i \frac{\mathcal{L}_{VWW}^{\text{eff}}}{g_{VWW}} = & +g_1^V V^\mu (W_{\mu\nu}^- W^{+\nu} - W_{\mu\nu}^+ W^{-\nu}) \\
& +\kappa_V W_\mu^+ W_\nu^- V^{\mu\nu} + \frac{\lambda_V}{M_W^2} V^{\mu\nu} W_\nu^{+\rho} W_{\rho\mu}^- \\
& +ig_4^V W_\mu^- W_\nu^+ (\partial^\mu V^\nu + \partial^\nu V^\mu) \\
& +ig_5^V \epsilon_{\mu\nu\rho\sigma} ((\partial^\rho W^{-\mu}) W^{+\nu} - W^{-\mu} (\partial^\rho W^{+\nu})) V^\sigma \\
& -\frac{\tilde{\kappa}_V}{2} W_\mu^- W_\nu^+ \epsilon^{\mu\nu\rho\sigma} V_{\rho\sigma} - \frac{\tilde{\lambda}_V}{2M_W^2} W_{\rho\mu}^- W_\nu^{+\mu} \epsilon^{\nu\rho\alpha\beta} V_{\alpha\beta},
\end{aligned} \tag{3.113}$$

where here $F_{\mu\nu} = \partial_\mu F_\nu - \partial_\nu F_\mu$ for $F = V, W$. The overall normalisations g_{VWW} are fixed with respect to the electromagnetic charge $e = \sqrt{4\pi\alpha_{em}}$:

$$g_{\gamma WW} = e \tag{3.114}$$

$$g_{ZWW} = e \cot \theta_W. \tag{3.115}$$

The subamplitudes $A_{\Delta\sigma\lambda\bar{\lambda}}^V, V = \gamma, Z$, resulting from the general Lagrangian are also given in Table 3.4. Within the Standard Model, $g_1^V = \kappa_V = 1$ for $V = \gamma, Z$, while all other couplings vanish. The parts of the Lagrangian associated to the various couplings do not necessarily conserve all of the discrete symmetries C, P and CP. The complete list conserved discrete symmetries is given in Table 3.5.

With the above conventions, simple expressions for the electromagnetic properties of the W^- boson, its electric charge, magnetic dipole moment, electric quadrupole moment, electric dipole moment and

$\Delta\lambda$	$(\lambda, \bar{\lambda})$	$\Delta\sigma = \pm 1$		$\Delta\sigma = -1$	
		$A_{\Delta\sigma\lambda\bar{\lambda}}^V$	$A_{\Delta\sigma\lambda\bar{\lambda}}^V$ (SM)	$B_{\lambda\bar{\lambda}}$	$C_{\lambda\bar{\lambda}}$
+1	(+0)	$\pm\gamma(g_1^V + \kappa_V + \lambda_V - ig_4^V + \beta_W g_5^V + \frac{i}{\beta_W}(\tilde{\kappa}_V - \tilde{\lambda}_V))$	$\pm 2\gamma$	$-2\gamma^2$	$-\frac{2(1+\beta_W)}{\gamma}$
+1	(0-)	$\mp\gamma(g_1^V + \kappa_V + \lambda_V + ig_4^V + \beta_W g_5^V - \frac{i}{\beta_W}(\tilde{\kappa}_V - \tilde{\lambda}_V))$	$\mp 2\gamma$	$+2\gamma$	$+\frac{2(1-\beta_W)}{\gamma}$
0	(++)	$\mp(g_1^V + 2\gamma^2\lambda_V + \frac{i}{\beta_W}(\tilde{\kappa}_V + (1-2\gamma^2)\tilde{\lambda}_V))$	∓ 1	1	$\frac{1}{\gamma^2}$
0	(00)	$\pm(g_1^V + 2\gamma^2\kappa_V)$	$\pm(1+2\gamma^2)$	$+2\gamma^2$	$\frac{2}{\gamma^2}$
0	(--)	$\mp(g_1^V + 2\gamma^2\lambda_V - \frac{i}{\beta_W}(\tilde{\kappa}_V + (1-2\gamma^2)\tilde{\lambda}_V))$	∓ 1	1	$\frac{1}{\gamma^2}$
-1	(0+)	$\mp\gamma(g_1^V + \kappa_V + \lambda_V + ig_4^V - \beta_W g_5^V + \frac{i}{\beta_W}(\tilde{\kappa}_V - \tilde{\lambda}_V))$	$\mp 2\gamma$	$+2\gamma$	$+\frac{2(1-\beta_W)}{\gamma}$
-1	(-0)	$\pm\gamma(g_1^V + \kappa_V + \lambda_V - ig_4^V - \beta_W g_5^V - \frac{i}{\beta_W}(\tilde{\kappa}_V - \tilde{\lambda}_V))$	$\pm 2\gamma$	-2γ	$-\frac{2(1-\beta_W)}{\gamma}$

Table 3.4: Subamplitudes in W-pair production: $A_{\Delta\sigma\lambda\bar{\lambda}}^V$ for $V = \gamma, Z$ and $\Delta\sigma = \pm 1$; $B_{\lambda\bar{\lambda}}$ and $C_{\lambda\bar{\lambda}}$ for $\Delta\sigma = -1$. For subamplitude A , the general expression as well as the Standard Model expectations are shown, for the subamplitudes B and C the SM SM formulae expected from the structure of the $Wf\bar{f}'$ vertex.

Coupling	g_1^V	κ_V	λ_V	g_4^V	g_5^V	$\tilde{\kappa}_V$	$\tilde{\lambda}_V$
Invariance	C, P, CP	C, P, CP	C, P, CP	P	CP	C	C

Table 3.5: Discrete symmetries C, P and CP of the general VW^+W^- interaction, $V = \gamma, Z$.

magnetic quadrupole moment are derived:

$$\begin{aligned}
Q_W &= -eg_1^\gamma & \xrightarrow{\text{SM}} & -e \\
\mu_W &= \frac{-e}{2M_W}(g_1^\gamma + \kappa_\gamma + \lambda_\gamma) & \xrightarrow{\text{SM}} & -\frac{e}{M_W} \\
\mathcal{Q}_W &= \frac{-e}{M_W^2}(\lambda_\gamma - \kappa_\gamma) & \xrightarrow{\text{SM}} & \frac{e}{M_W^2} \\
d_W &= \frac{-e}{2M_W}(\tilde{\kappa}_\gamma + \tilde{\lambda}_\gamma) & \xrightarrow{\text{SM}} & 0 \\
\mathcal{M}_W &= \frac{-e}{M_W^2}(\tilde{\lambda}_\gamma - \tilde{\kappa}_\gamma) & \xrightarrow{\text{SM}} & 0,
\end{aligned} \tag{3.116}$$

respectively, where the latter two are CP violating quantities. The number of free coupling constants in the general case, 7 each for photon and Z boson, is reduced by making additional assumptions. First, the couplings are approximated to be real, neglecting small imaginary parts appearing above the W-pair threshold. Electromagnetic gauge invariance requires $g_1^\gamma = 1$ and $g_5^\gamma = 0$. Furthermore, invariance under C and P transformations leaves just five couplings, $g_1^Z, \kappa_\gamma, \kappa_Z, \lambda_\gamma, \lambda_Z$. Gauge symmetry requirements establish relations between γW^+W^- and ZW^+W^- couplings, for example:

$$\Delta\kappa_Z = \Delta g_1^Z - \Delta\kappa_\gamma \tan^2 \theta_W \tag{3.117}$$

$$\lambda_Z = \lambda_\gamma, \tag{3.118}$$

where Δ denotes the difference of the respective coupling to its Standard Model value. These relations reduce the number of independent couplings to three, which are usually chosen to be g_1^Z, κ_γ and λ_γ .

Also the non-observation of anomalous effects in e^+e^- interactions at the Z pole leads to restrictions in the parameter space of anomalous triple-gauge-boson couplings [76, 77].

In order to investigate the general triple gauge boson vertex, both the scattering angle of the W boson as well as the W polarisation must be analysed. Because of the $(V-A)$ structure of the charged weak current the angular distributions of the W decay fermions are ideal W polarisation analysers.

In the case the final-state fermion helicities are not measured, all information is contained in five phase-space angles, the polar scattering angle of the W^- boson, and the polar and azimuthal decay angles of the fermion-antifermion pair in the respective W rest system.

3.2.1.3 Differential Cross Section

The lowest order differential cross section in the Standard Model for the production of two equal-mass W bosons, $M_- = M_+ = M_W$ ($\Gamma_W = 0$), has the form [78, 79]:

$$\frac{d\sigma^0(e^+e^- \rightarrow W^+W^-, \Gamma_W = 0)}{d\cos\theta} = \frac{\pi\alpha_{em}^2\beta_W}{8s} [a_{\gamma\gamma} + a_{ZZ} + a_{\nu\nu} + a_{\gamma Z} + a_{\gamma\nu} + a_{Z\nu}] \quad (3.119)$$

where in the Standard Model:

$$a_{\gamma\gamma} = a_s \quad (3.120)$$

$$a_{ZZ} = \left(1 - \frac{1}{2\sin^2\theta_W} + \frac{1}{8\sin^4\theta_W}\right) \cdot \frac{s^2}{(s - M_Z^2)^2} \cdot a_s \quad (3.121)$$

$$a_{\nu\nu} = \frac{1}{\sin^4\theta_W} \cdot a_t \quad (3.122)$$

$$a_{\gamma Z} = \left(\frac{1}{2\sin^2\theta_W} - 2\right) \cdot \frac{s}{s - M_Z^2} \cdot a_s \quad (3.123)$$

$$a_{\gamma\nu} = -\frac{1}{\sin^2\theta_W} \cdot a_i \quad (3.124)$$

$$a_{Z\nu} = \left(\frac{1}{\sin^2\theta_W} - \frac{1}{2\sin^4\theta_W}\right) \cdot \frac{s}{s - M_Z^2} \cdot a_i. \quad (3.125)$$

The s - and t -channel and the s/t -interference terms are:

$$a_s = \beta_W^2 \left[\frac{16s}{M_W^2} + \sin^2\theta \left(\frac{s^2}{M_W^4} - \frac{4s}{M_W^2} + 12 \right) \right] \quad (3.126)$$

$$a_t = \frac{2s}{M_W^2} + \beta_W^2 \frac{\sin^2\theta}{2} \left[\frac{s^2}{4M_W^2} + \frac{s^2}{t^2} \right] \quad (3.127)$$

$$a_i = 16 \left(1 + \frac{M_W^2}{t} \right) + \beta_W^2 \left[\frac{8s}{M_W^2} + \frac{\sin^2\theta}{2} \left(\frac{s^2}{M_W^4} - \frac{2s}{M_W^2} - \frac{4s}{t} \right) \right]. \quad (3.128)$$

Besides the kinematic d functions, another θ dependence of the differential cross section is introduced due to the dependence of the t -channel matrix elements on the Mandelstamm variable t , denoting the momentum transfer between incoming e^- and outgoing W^- , $t = M_W^2 - s(1 - \beta_W \cos\theta)/2$.

3.2.1.4 Total Cross Section

By integrating over the polar scattering angle, the total cross section is obtained. The integrals $A = \int a d\theta$ are [78, 79]:

$$A_s(s) = \beta_W^2 \left[\frac{16s}{M_W^2} + \frac{2}{3} \left(\frac{s^2}{M_W^4} - \frac{4s}{M_W^2} + 12 \right) \right] \quad (3.129)$$

$$A_t(s) = \frac{2s}{M_W^2} + \beta_W^2 \frac{s^2}{12M_W^4} + 4 \left[\left(1 - \frac{2M_W^2}{s} \right) L - 1 \right] \quad (3.130)$$

$$A_i(s) = 16 - 32 \frac{M_W^2}{s} L + \beta_W^2 \left[\frac{8s}{M_W^2} + \frac{1}{3} \frac{s^2}{M_W^4} - \frac{2}{3} \frac{s}{M_W^2} \right] \quad (3.131)$$

$$+ 4 \left(1 - \frac{2M_W^2}{s} \right) - 16 \frac{M_W^4}{s^2} L, \quad (3.132)$$

with:

$$L = \frac{1}{\beta_W} \ln \frac{1 + \beta_W}{1 - \beta_W}. \quad (3.133)$$

The six contributions to the total cross section are shown in Figure 3.20. At the kinematic threshold, $\sqrt{s} \approx 2M_W$, the cross section is dominated by the t -channel neutrino diagram. At higher energies the s -channel diagrams, which depend on the non-Abelian gauge couplings, become more and more important. In the absence of self-interactions among the gauge bosons, the cross section would be determined by the t -channel neutrino graph only. In that case the cross section would grow proportional to s and eventually violate the unitarity limit. Within the Standard Model, considering all diagrams, good high-energy behaviour of the cross section is restored through interference of all diagrams.

At the kinematic threshold, the total cross section rises quickly with $\sqrt{s}$ which is exploited to measure the mass of the W bosons. The finite width of the W boson causes a softening of the sharp rise of the total cross section, also shown in Figure 3.20. The lowest-order total cross section for off-shell W -pair production is written as a double integral over the masses of the two W -bosons, $s_- = M_-^2$ and $s_+ = M_+^2$:

$$\sigma^0(s) = \int_0^s \int_0^s ds_- ds_+ \rho(s_-) \rho(s_+) \sigma_{CC03}^0(s; s_-, s_+), \quad (3.134)$$

where the Breit-Wigner densities are given by:

$$\rho(s_{\pm}) = \frac{1}{\pi} \frac{M_W \Gamma_W}{(s_{\pm} - M_W^2)^2 + M_W^2 \Gamma_W^2}. \quad (3.135)$$

Analytic expressions for $\sigma_{CC03}^0(s; s_-, s_+)$ are given in [80]. They contain the above on-shell results, $\sigma^0(s; s_- = s_+ = M_W, \Gamma_W = 0) = \sigma^0(s; M_W)$.

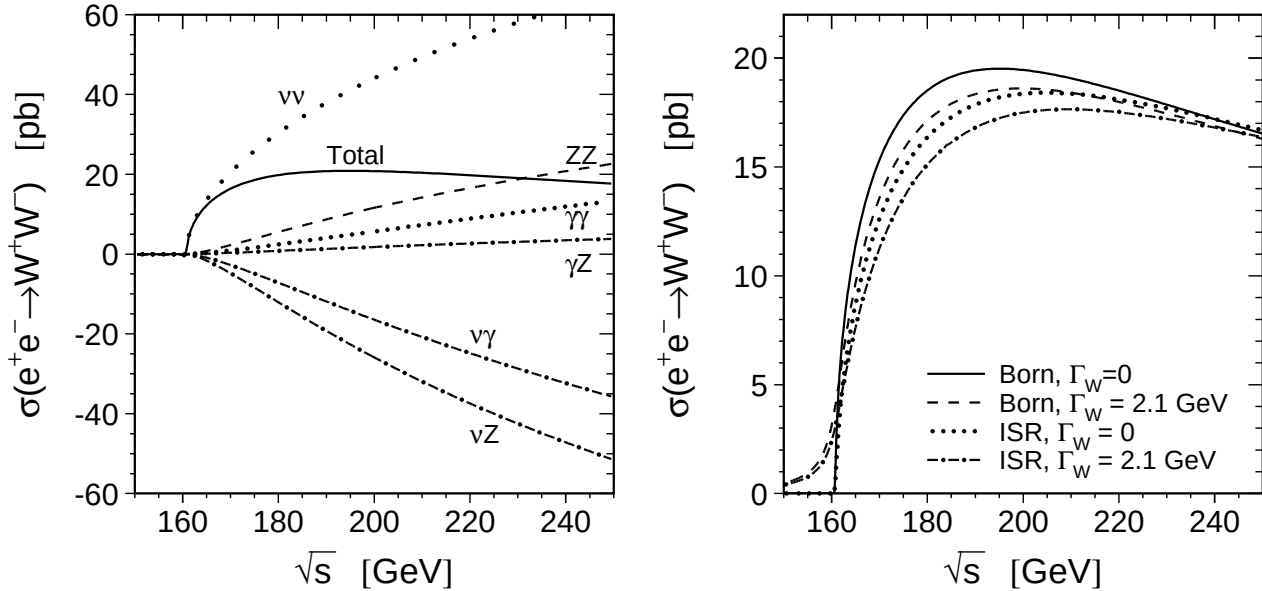


Figure 3.20: Cross section of the reaction $e^+e^- \rightarrow W^+W^-$ as a function of $\sqrt{s}$. Left: Born cross section in the zero-width approximation ($\Gamma_W = 0$). The contributions arising from the three Feynman diagrams and their interferences are shown separately. Right: Total cross section, comparing the effects of finite W width and QED radiative corrections on the total W -pair production cross section.

3.2.2 Radiative Corrections

Radiative corrections in boson pair production are treated rather similar to radiative corrections in fermion pair production. The accuracy required for theoretical predictions is, however, much less due to the lower statistical accuracy of the measurements at LEP-II as compared to LEP-I.

3.2.2.1 QED Bremsstrahlung Corrections

As in fermion-pair production, QED radiative corrections are incorporated by convolution [81, 82], adding an integration over s' :

$$\sigma(s) = \int_0^s ds' \sigma^0(s') = \quad (3.136)$$

$$= \int_0^s ds' \int_0^{s'} \int_0^{s'} ds_- ds_+ R(s, s') \rho(s_-) \rho(s_+) R(s, s') \sigma_{CC03}^0(s'; s_-, s_+). \quad (3.137)$$

To lowest leading order, the radiator function $R(s' = zs, s)$ is identical to the one used in fermion-pair production. For the t -channel and s/t -interference part, additional small non-leading pieces must be added to the radiator function.

The Coulomb attraction between the slowly separating oppositely charged W bosons amounts to a sizeable correction to the total cross section at threshold, because the scale of the correction is set by α_{em}/β_W rather than α_{em} alone:

$$R_{\text{Coulomb}} = 1 + \frac{\alpha_{em}\pi}{2\beta_W} \left[1 - \frac{2}{\pi} \arctan \left(\frac{|\beta_M + \Delta|^2 - \bar{\beta}_W^2}{2\beta_W \Im(\beta_M)} \right) \right], \quad (3.138)$$

where:

$$\bar{\beta}_W = \sqrt{1 - 2 \frac{s_+ + s_-}{s} + \Delta^2} \quad (3.139)$$

$$\Delta = \frac{|s_+ - s_-|}{s} \quad (3.140)$$

$$\beta_M = \sqrt{1 - \frac{4}{s} (M_W^2 - iM_W\Gamma_W - i\epsilon)}. \quad (3.141)$$

Here $\bar{\beta}_W$ is the average velocity of the two W bosons with invariant masses $\sqrt{s_+}$ and $\sqrt{s_-}$. In the approximation of stable W bosons, the Coulomb correction summed to all orders is:

$$R_{\text{Coulomb}} \xrightarrow{\Gamma_W \rightarrow 0} \frac{\alpha_{em}\pi/\beta_W}{1 - \exp(-\alpha_{em}\pi/\beta_W)} \approx 1 + \frac{\alpha_{em}\pi}{2\beta_W}, \quad (3.142)$$

therefore the name Coulomb singularity.

The effect of QED radiative corrections is shown in Figure 3.20.

3.2.2.2 Quadruple Gauge Boson Couplings

Within the Standard Model, also quadruple gauge boson vertices with at most two identical bosons exist, namely $\gamma\gamma W^+W^-$, γZW^+W^- , ZZW^+W^- and $W^+W^-W^+W^-$ vertices. They lead to $W^+W^- \gamma$ and W^+W^-Z final states in e^+e^- interactions as shown in Figure 3.21. The corresponding cross sections are of the same order in α_{em} as first order QED bremsstrahlung correction in W^+W^- production. Thus these processes should be taken into account at the same level as QED radiative corrections. At LEP-I, these processes are strongly suppressed due to kinematics. At LEP-II well above the W-pair threshold, $W^+W^- \gamma$ production becomes visible, similar to QED radiative corrections to W^+W^- production, while W^+W^-Z production is still below the kinematic threshold.

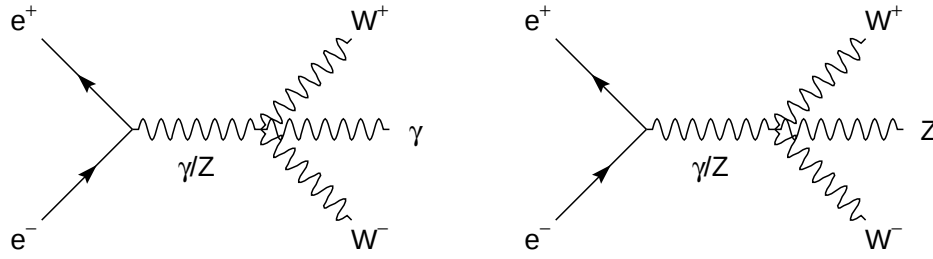


Figure 3.21: Triple gauge boson production involving quadruple gauge boson vertices.

3.2.2.3 Weak Corrections

In the case of fermion-pair production, the Standard Model input parameters chosen for the calculation of electroweak radiative corrections are α_{em} , α_S , M_Z , M_t , M_H , and G_F replacing M_W . This is justified since M_W is not a directly observable quantity in those processes. However, in case of W physics, this is no longer true. The W boson mass appears in the calculations of cross sections in both the matrix element and the phase-space. Thus it is more convenient to use a parameter set which contains M_W .

For W-pair production, the leading weak corrections are incorporated in the matrix elements discussed above by the replacements:

$$\alpha_{em} \longrightarrow \alpha_{em}(s) \quad (3.143)$$

$$\frac{\alpha_{em}}{\sin^2 \theta_W} \longrightarrow \frac{\sqrt{2}}{\pi} G_F M_W^2, \quad (3.144)$$

for the electromagnetic and weak part, respectively.

3.2.3 The Decay Widths of the W Boson

For W bosons, which decay via the left-handed charged weak current, $V = A = 1$, one finds:

$$\Gamma_{W \rightarrow f \bar{f}'} = N_C^f \frac{\alpha_{em} M_W}{12 \sin^2 \theta_W} |V_{f \bar{f}'}|^2 = N_C^f \frac{G_F M_W^3}{6 \sqrt{2} \pi} |V_{f \bar{f}'}|^2, \quad (3.145)$$

where for leptonic decays the mixing matrix is the unit matrix. Writing the partial widths in terms of the Fermi constant G_F incorporates most of the electroweak radiative corrections on W-decay partial widths.

3.2.3.1 QCD and Mass Corrections

Including also fermion-mass effects and QCD corrections arising in hadronic W decays, the partial widths are modified:

$$\Gamma_{W \rightarrow f \bar{f}'} = \sqrt{1 - \frac{(m_f + m_{f'})^2}{M_W^2}} \sqrt{1 - \frac{(m_f - m_{f'})^2}{M_W^2}} \left[1 - \frac{m_f^2 + m_{f'}^2}{2 M_W^2} - \frac{(m_f^2 - m_{f'}^2)^2}{2 M_W^4} \right] \times \\ N_C^f \frac{G_F M_W^3}{6 \sqrt{2} \pi} |V_{f \bar{f}'}|^2 R_{QCD}^{(f)}, \quad (3.146)$$

with $R_{QCD}^{(f)} = 1 + \Delta_{QCD}^{(f)}$ as given before. The inclusive hadronic decay width is given by the sum over all $W \rightarrow q \bar{q}'$ decay widths:

$$\Gamma_{W \rightarrow \text{had}} = \sum_{q \neq t} \Gamma_{W \rightarrow q \bar{q}'} = \frac{G_F M_W^3}{\sqrt{2} \pi} \left(1 + \Delta_{QCD}^{(\text{had})} \right), \quad (3.147)$$

where for the latter equality fermion-mass effects are neglected. Since $M_t > M_W$, the decay $W \rightarrow tb$ of on-shell W bosons is kinematically not allowed. The total decay width is the sum of all partial decay widths:

$$\Gamma_W = \Gamma_{e\nu} + \Gamma_{\mu\nu} + \Gamma_{\tau\nu} + \Gamma_{W \rightarrow \text{had}} = \frac{3G_F M_W^3}{2\sqrt{2}\pi} \left(1 + \frac{2}{3} \Delta_{QCD}^{(\text{had})} \right). \quad (3.148)$$

Like in the case of Z boson production, fermion-mass corrections and QCD corrections for hadronic W decays also correct the total W -pair production cross section due to its dependence on Γ_W arising from the W -boson propagator. For cross sections of specific four-fermion final-states in W -pair production, the change in W -decay branching fractions must also be taken into account.

3.2.4 ZZ Production

The two Feynman diagrams contributing to ZZ production are shown in Figure 3.18. The differential cross section for Z -pair production is given in [83, 84]. The total cross section is given by [85]:

$$\begin{aligned} \sigma^0(e^+e^- \rightarrow ZZ, \Gamma_Z = 0) &= \beta_Z \frac{G_F^2 M_Z^4}{8\pi s} [(g_{Ve} + g_{Ae})^4 + (g_{Ve} - g_{Ae})^4] \\ &\quad \left(\frac{s^2 + 4M_Z^4}{s - 2M_Z^2} \frac{1}{\beta_Z s} \ln \frac{s(1 + \beta_Z) - 2M_Z^2}{s(1 - \beta_Z) - 2M_Z^2} - 2 \right), \end{aligned} \quad (3.149)$$

where $\beta_Z = \sqrt{1 - 4M_Z^2/s}$ is the Z -boson velocity. This Born cross section is modified by the inclusion of the finite width of the Z boson [86]. QED radiative corrections are incorporated as before by convolution with a radiator function [82, 87]. The cross sections on Born level, corresponding to the above Equation, and including QED radiative corrections and width effects are shown in Figure 3.22. Within the SM, the Z -pair cross section is about a factor 15 smaller than the W -pair cross section.

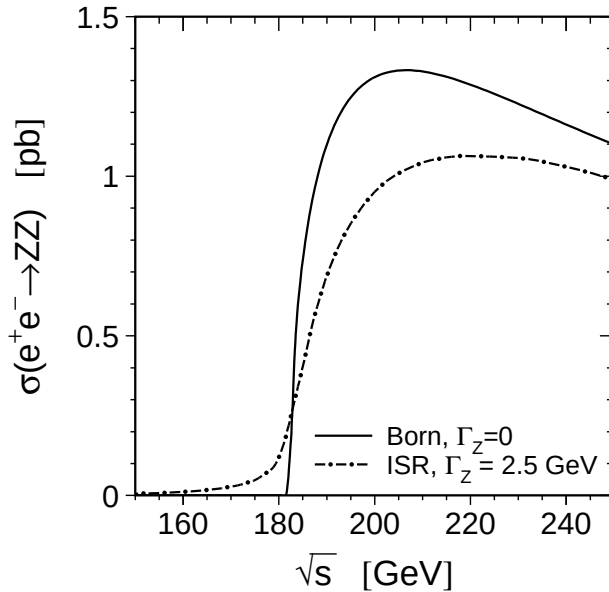


Figure 3.22: Cross section for the reaction $e^+e^- \rightarrow ZZ$ as a function of $\sqrt{s}$, comparing Born term in the zero-width approximation to the calculation including QED radiative correction and the finite Z width.

3.2.5 Four Fermion Production

The diagrams for Z- and W-pair production, NC02 and CC03, as shown in Figures 3.18 and 3.19 lead to the production of two heavy gauge bosons both of which may be on-shell, called double resonant production. Each heavy boson decays into a pair of light fermions so that the final state observed in the detector consists of four fermions. In case of boson-pair production, products of branching fractions of the heavy bosons determine the four-fermion event sample composition:

$$\frac{\sigma(e^+e^- \rightarrow VV \rightarrow (f\bar{f})_1(f\bar{f})_2)}{\sigma(e^+e^- \rightarrow VV)} = B(V \rightarrow (f\bar{f})_1) \cdot B(V \rightarrow (f\bar{f})_2) \quad (3.150)$$

$$= \frac{\Gamma(V \rightarrow (f\bar{f})_1)}{\Gamma_V} \cdot \frac{\Gamma(V \rightarrow (f\bar{f})_2)}{\Gamma_V}, \quad (3.151)$$

where $V = Z, W$.

For a given four-fermion final state, however, there exist many more Feynman diagrams contributing to it than just the NC02 and CC03 graphs with two resonant bosons. A classification of all four-fermion processes is given in [62]. For four-fermion final states consisting of two charged-current type fermion-antifermion pairs (CC), the number of contributing Feynman diagrams ranges from 9 to 56 depending on the final state. For four-fermion final states consisting of neutral-current type fermion-antifermion pairs (NC), between 6 and 144 diagrams contribute. The various possibilities are reported in Tables 3.6 and 3.7. In particular, certain final states belong to both classes as they are produced by both CC and NC type Feynman diagrams. Because of the large number of diagrams involved, no complete calculation of electroweak radiative correction exists for four-fermion production.

The analysis of W-pair production needs to take into account that there are other diagrams in four-fermion production besides CC03, such as single resonant boson production as shown in Figure 3.23 and non-resonant diagrams. As an example, the CC20 set of 20 diagrams contributing to the process $e^+e^- \rightarrow q\bar{q}e\nu$ at lowest order is shown in Figure 3.24. The additional diagrams modify the results for total and differential cross sections as calculated on the basis of the CC03 set of diagrams. The effects are usually small except if electrons appear in the final state. In that case t -channel diagrams with the electron line going from the initial to the final state contribute, leading to additional poles for small scattering angles, $\theta \rightarrow 0$, of the t -channel electron.

In neutral-current four fermion production, the typical two-photon type of diagrams are part of the diagrams contributing to the processes $e^+e^- \rightarrow e^+e^-f\bar{f}$. In the case of $f\bar{f} = q\bar{q}$, the diagrammatic calculation on fermion level is inadequate if the $f\bar{f}$ system is of low invariant mass, *i.e.*, in the typical two-photon physics region. In that region of phase space the cross section is dominated by the multiperipheral two-photon diagrams topologically identical to graphs 1 to 4 of Figure 3.24, where all internal gauge-boson propagators consist of photons. Effectively, the internal photons may become off-shell and fragment into a low mass hadronic resonance with the same quantum numbers. Thus, hadron-hadron collisions occur which cannot be described at the parton level.

CC	$\bar{d}u$	$\bar{s}c$	$e^+\nu_e$	$\mu^+\nu_\mu$	$\tau^+\nu_\tau$
$d\bar{u}$	43	11	20	10	10
$e^-\bar{\nu}_e$	20	20	56	18	18
$\mu^-\bar{\nu}_\mu$	10	10	18	19	9

Table 3.6: Number of lowest-order Feynman diagrams contributing to the four-fermion final state composed of two charged-current type fermion-antifermion pairs. Combinations not reported above are given by family generation symmetry and particle-antiparticle interchange.

NC	$\bar{d}d$	$\bar{u}u$	e^+e^-	$\mu^+\mu^-$	$\bar{\nu}_e\nu_e$	$\bar{\nu}_\mu\nu_\mu$
$\bar{d}d$	64	43	48	24	21	10
$\bar{s}s$	32	43	48	24	21	10
$\bar{u}u$	43	64	48	24	21	10
e^+e^-	48	48	144	48	56	20
$\mu^+\mu^-$	24	24	48	64	19	19
$\tau^+\tau^-$	24	24	48	24	19	10
$\bar{\nu}_e\nu_e$	21	21	56	19	36	12
$\bar{\nu}_\mu\nu_\mu$	10	10	20	19	12	12
$\bar{\nu}_\tau\nu_\tau$	10	10	20	10	12	6

Table 3.7: Number of lowest-order Feynman diagrams contributing to the four-fermion final states composed of two neutral-current type fermion-antifermion pairs. Combinations not reported above are given by family generation symmetry.

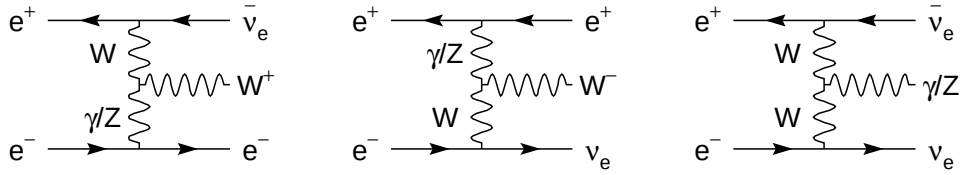


Figure 3.23: Feynman diagrams with single resonant W/Z bosons in four-fermion production which contain triple gauge boson vertices. In case of single W production the diagrams with an internal propagator of a massless photon dominate over those with a massive Z boson, making single W production sensitive to the electromagnetic gauge couplings of the W boson only. Single Z production with a ZW^+W^- vertex is strongly suppressed as all three boson propagators are massive. The single Z boson may be replaced by a massless photon, with the consequence that single-photon production, $e^+e^- \rightarrow \nu\bar{\nu}\gamma$, is sensitive to the electromagnetic gauge couplings of the W boson.

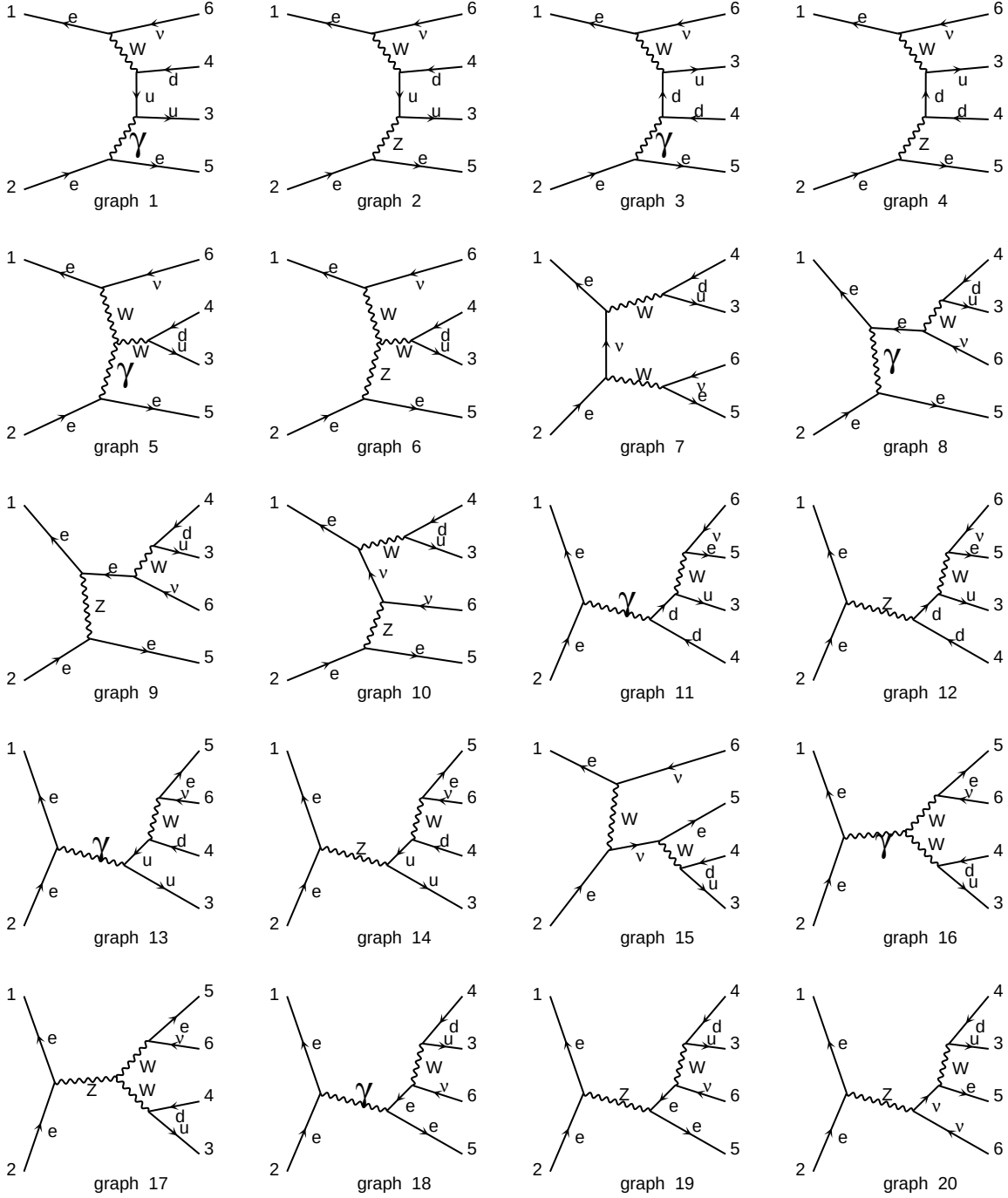


Figure 3.24: All Feynman diagrams contributing to the CC20 process $e^+e^- \rightarrow u\bar{d}\bar{\nu}_e$ at lowest order. Graphs 7, 16 and 17 correspond to the CC03 double-resonant diagrams. Graphs 1, 2, 3 and 4 are the non-resonant diagrams. Topologically, they are identical to the multiperipheral diagrams in two-photon collision processes. All other graphs are singly-resonant diagrams. Triple gauge boson couplings appear in graphs 5, 6, 16 and 17. The CC10 set is given by the CC03 graphs plus the graphs 11, 12, 13, 14, 18, 19, and 20, replacing $e^-\bar{\nu}_e$ by $\mu^-\bar{\nu}_\mu$. The CC09 set is obtained from the CC10 family by replacing $u\bar{d}$ by $\nu_\tau\tau^+$ and removing graph 13, since the photon does not couple to neutrinos. The CC11 set is obtained from the CC10 set by replacing $\mu^-\bar{\nu}_\mu$ by $s\bar{c}$ and adding graph 20 again with the Z replaced by a photon, as the photon does couple to quarks.

3.2.5.1 Single W Boson Production

An interesting example is the process of single-W production, $e^+e^- \rightarrow We\nu$, which is part of the CC20 process $e^+e^- \rightarrow q\bar{q}e\nu$ shown in Figure 3.24. The differential cross section in four-fermion $q\bar{q}e\nu$ production is shown in Figure 3.25. In addition to double-resonant W-pair production with one W decaying to $e\nu$, t -channel diagrams of the type shown in Figure 3.23 contribute in particular for small values of the polar scattering angle of the t -channel electron [88, 89]. As then the electron is lost in the beam pipe, only the decay products of a single W boson are registered by the detector, either two hadronic jets from the decay $W \rightarrow q\bar{q}'$, or a single charged lepton from the decay $W \rightarrow \ell\nu$.

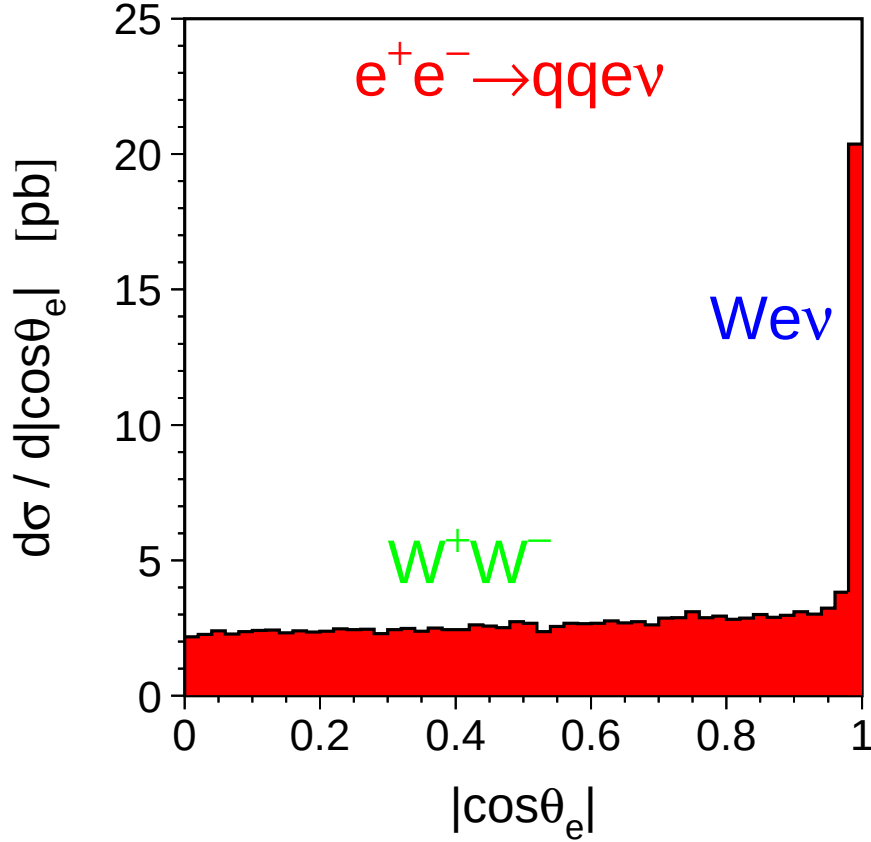


Figure 3.25: Distribution of electron polar angle in $e^+e^- \rightarrow q\bar{q}e\nu$ production. The flat part of the distribution is mainly caused by the CC03 diagrams. The peak at $\cos \theta_e \rightarrow 1$ is mainly caused by single W production, $e^+e^- \rightarrow We\nu$.

3.2.5.2 Top Quark Production

The process $e^+e^- \rightarrow t\bar{t}$ requires $\sqrt{s} > 2M_t$ for the production of top-quark pairs. In contrast, the production of single top quarks in four-fermion production, $e^+e^- \rightarrow tb f\bar{f}'$, is kinematically possible already for $\sqrt{s}$ larger than M_t plus a few GeV to account for the other three fermions and to gain some phase-space. Single top production proceeds via the CC10, CC11, and CC20 set of diagrams, where one quark pair contains the top quark and the other $f\bar{f}'$ pair consists of leptons or light quarks. However, for top-quark masses of 175 GeV, the single-top cross section is below 10^{-3} fb at $\sqrt{s} = 200$ GeV [90]. Single top production is thus completely out of the reach of LEP-II.

The single-top cross section increases with centre-of-mass energy and reaches a few fb just at the kinematic threshold of $t\bar{t}$ pair production [91]. Above the $t\bar{t}$ threshold, top-pair production, with a

cross section of about 0.5 pb [91], is by far the dominant production mechanism for top quarks in e^+e^- interactions.

Besides the important question of the top-quark mass, the top decay width will test both the electroweak theory and QCD [92]. For $M_t > M_W + m_b$, the top quark decays into a b quark and an on-shell W boson, $t \rightarrow Wb$, leading to a Standard Model decay width of:

$$\Gamma_t = \frac{G_F M_t^3}{8\sqrt{2}\pi} \left(1 - \frac{M_W^2}{M_t^2}\right)^2 \left(1 + 2\frac{M_W^2}{M_t^2}\right) \left[1 - \frac{2}{3} \frac{\alpha_S}{\pi} \left(\frac{2\pi^2}{3} - \frac{5}{2}\right)\right]. \quad (3.152)$$

The total decay width of the top quark ranges from 0.8 GeV for $M_t = 150$ GeV to 2.2 GeV for $M_t = 200$ GeV, and is 1.4 GeV for $M_t = 175$ GeV.

3.2.5.3 Higgs Boson Production

The trilinear couplings of the Higgs boson imply the existence of HW^+W^- , HZZ and HHH vertices. They lead to Higgs-boson production in e^+e^- interactions with larger cross section than through direct coupling of the Higgs to the initial e^+e^- pair. The Feynman diagrams of Higgs production in e^+e^- interactions are shown in Figure 3.26.

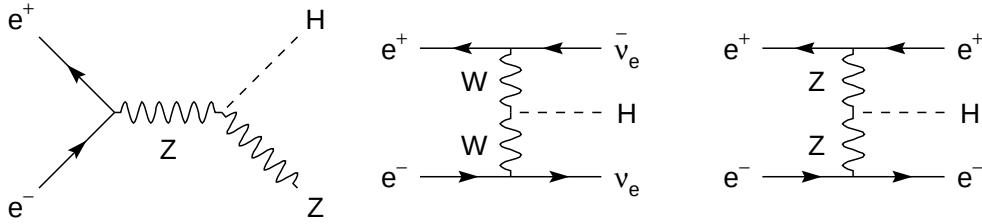


Figure 3.26: Main Feynman diagrams contributing to Higgs-boson production in e^+e^- collisions.

For M_H up to $\sqrt{s} - M_Z$ Higgs radiation off the s -channel Z boson is dominant, $e^+e^- \rightarrow Z^* \rightarrow ZH$, with a cross section given by [63]:

$$\sigma^0(e^+e^- \rightarrow Z^* \rightarrow ZH) = \frac{G_F^2 M_Z^4}{24\pi s^2} (g_{Ve}^2 + g_{Ae}^2) \sqrt{\lambda} \frac{\lambda + 12sM_Z^2}{(s - M_Z^2)^2} \quad (3.153)$$

$$\lambda = (s - M_H^2 - M_Z^2)^2 - 4M_H^2 M_Z^2, \quad (3.154)$$

which stays above 0.1 pb for Higgs-boson masses up to the nominal kinematic limit of $\sqrt{s} = M_Z + M_H$ due to the finite width of the Z boson neglected in the expression given above. For Higgs masses up to $\sqrt{s}$ Higgs production is only possible through W^+W^- or ZZ fusion, $e^+e^- \rightarrow \nu_e \bar{\nu}_e H$ or $e^+e^- \rightarrow e^+e^- H$, respectively. However, the cross section is much smaller, a few fb only [63].

The quadrilinear couplings of the Higgs boson imply the existence of HHW^+W^- , $HHZZ$ and $HHHH$ vertices. As in the case of quadrilinear gauge couplings, they are suppressed by a factor α_{em} relative to trilinear Higgs couplings.

For the analysis of e^+e^- interactions, direct Higgs boson production must be evaluated. Dedicated searches for direct Higgs boson production are performed, so far with negative results. Thus the Higgs boson must be so heavy that it cannot be produced at a measurable rate at current colliders. The highest lower limit on the mass of the Higgs boson is derived from the negative search for ZH production in e^+e^- annihilations, and is given by $M_H > \sqrt{s} - M_Z$ as indicated above. The current limit of $M_H > 90$ GeV at 95% CL [51] is so high that any remaining contribution of direct Higgs boson production to the e^+e^- processes of interest here is negligible. However, virtual effects arising through trilinear and quadrilinear Higgs couplings to the massive gauge bosons lead to gauge-boson propagator corrections and thus to the M_H dependence of the effective couplings of the neutral weak current as discussed in Section 2.11.2.

3.2.6 Final State Interactions

In case of four-fermion production, final state interactions are more complicated than expected from a simple linear superposition of two two-fermion systems, as cross talk between the two systems may occur. In particular, there are two types of strong final state interactions which potentially have severe consequences for the experimental determination of the W-boson mass in the channel $e^+e^- \rightarrow W^+W^- \rightarrow q\bar{q}q\bar{q}$ [63].

First, there are colour-reconnection (CR) effects arising from QCD interactions [62, 63]. Each W boson decays to a colour-singlet $q\bar{q}$ pair which then undergoes parton-shower evolution and hadronisation. Because of the very short W lifetime, their separation is just 0.1 fm compared to the typical hadronic scale of 1 fm. Thus the two hadronic systems evolve on top of each other. Cross talk between the two systems may lead to a reconnection of the two colour strings between the $q\bar{q}$ pair from each W boson to two colour strings connecting quark-antiquarks from different W bosons. Such a colour reconnection may occur both during the perturbative shower evolution through gluon exchange, as shown in Figure 3.27, and during the non-perturbative hadronisation process for which only phenomenological models are available. Visible effects include modifications of event shapes and particle multiplicities.

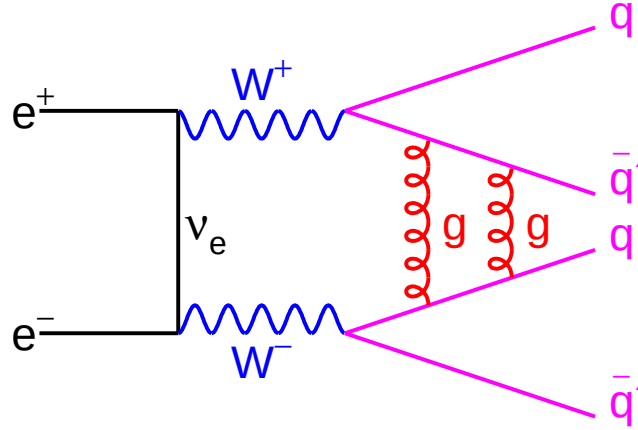


Figure 3.27: Perturbative colour reconnection in $q\bar{q}q\bar{q}$ production through two-gluon exchange.

Second, there are effects due to Bose-Einstein correlations [62, 63]. Such correlations are of quantum-mechanical origin as the overall wavefunction of the hadronic final state must be symmetric under the exchange of identical bosons, for example pions. Effectively, this causes particle correlations in phase space, since identical bosons in the hadronic final state tend to be closer in phase space than in a world without Bose-Einstein correlations.

Effects of colour reconnection and Bose-Einstein correlations within the hadronic system of a hadronically decaying W boson are uncritical for the experimental determination of the W-boson mass, as the total four-momentum of the system is unchanged. However, cross talk between the two hadronic systems due to colour reconnection or Bose-Einstein correlations may introduce a non-vanishing four-momentum transfer between the two systems, as schematically shown in Figure 3.28. In that case the concept of individually conserved invariant masses of the two W bosons, assumed in the experimental determination of the W mass, is no longer valid.

The problem is that these phenomena in W-pair production are studied based on phenomenological models only. Except for the case of colour reconnection during the perturbative phase, they cannot be calculated from first principles. Depending on the particular Monte Carlo modelling, mass shifts on M_W of up to 100 MeV in the $e^+e^- \rightarrow W^+W^- \rightarrow q\bar{q}q\bar{q}$ are possible, which is large compared to the statistical accuracy on the W boson mass expected at the end of the LEP-II program.

The size of both effects depends on how close two jets from different W bosons are in phase space.

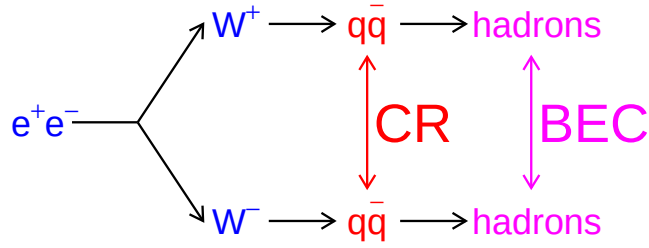


Figure 3.28: Schematic view of colour reconnection and Bose-Einstein correlations in W-pair production.

Thus, the experimental selection procedure must be taken into account, as it intrinsically has lower efficiency for overlapping jets reducing the number of visible jets. In addition, the bias in M_W depends on how M_W is extracted from the distribution of invariant masses, for example, the simple mean of the mass distribution is much more affected by tails than the pole mass of a Breit-Wigner fit to the data.

For a detailed analysis, different models depending on several parameters should first be tuned using data by analysing other distributions sensitive to such effects. Examples are Bose-Einstein correlation functions as observed at the Z peak, or particle multiplicities and spectra comparing semileptonic and hadronic W pair events. Such a tuning of the Monte Carlo models to describe the data will lead to refined models with restricted variation of model parameters. Applying these models to the experimental procedure to reconstruct the W-boson mass allows to study the effect on M_W , to tune the mass determination to be less sensitive to such effects, and to assess remaining systematic biases.

3.2.7 Summary of Boson-Pair Production

Boson-pair production is especially interesting in the case of W-pair production because it allows to measure the mass and the gauge couplings of the W boson.

The measurement of the total W-pair production at the kinematic threshold determines the W-boson mass. Above the kinematic threshold, the W-boson mass is best determined from the invariant mass of the W decay products. Care must be taken in the analysis of invariant masses in W-pair events where both W bosons decay hadronically, as strong final-state interaction effects between the two overlapping hadronic W decay systems may lead to a bias.

The measurement of the total and differential cross sections in W-pair mediated four-fermion production also allows the measurement of gauge boson couplings. These are expected due to the non-Abelian structure of the electroweak theory and their measurement constitutes a crucial test of the MSM. In the literature this is often referred to as a search for anomalous gauge boson couplings. The reason is that the LEP-II data statistics is only sufficient to search for relatively large deviations from the Born-term values rather than to measure electroweak radiative corrections to gauge boson couplings.

3.3 Monte Carlo Generators and Semianalytical Programs

3.3.1 Monte Carlo Generators

Monte Carlo generators simulate e^+e^- interactions for various final states, usually distributed in phase-space according to the Standard Model expectation, or, to be more precise, according to the Standard Model calculation as implemented in the event generator. The most widely used Monte Carlo generators are: BHLUMI [93] for $e^+e^- \rightarrow e^+e^-(\gamma)$ for luminosity applications; BABAMC [94], BHAGENE3 [95], BHWIDE [96], UNIBAB [97] and TEEGG [98] for $e^+e^- \rightarrow e^+e^-(\gamma)$; KORALZ [99] for $e^+e^- \rightarrow \mu^+\mu^-(\gamma)$, $\tau^+\tau^-(\gamma)$; ARIADNE [100] and JETSET [101] for $e^+e^- \rightarrow q\bar{q}(\gamma)$; DIAG36 [102] and LEP4F [103] for two-photon process $e^+e^- \rightarrow e^+e^-\ell^+\ell^-$; PHOJET [104] for two-photon process $e^+e^- \rightarrow e^+e^- \text{had}$; KORALW [105], EXCALIBUR [106] and GRC4F [107] for $e^+e^- \rightarrow f\bar{f}f\bar{f}(\gamma)$; PYTHIA [101] and HERWIG [108] for $e^+e^- \rightarrow q\bar{q}(\gamma)$, $W^+W^-(\gamma)$, $ZZ(\gamma)$.

The generated final-state particles are propagated through the detector simulation. Most experiments model the response of the detector with the detector simulation program GEANT [109], which includes the effects of energy loss, multiple scattering and showering in the detector materials and in the beam pipe. Hadronic showers are usually simulated with the GHEISHA [110] program.

The detector simulation makes direct comparisons with the recorded data possible. Thus efficiencies of selections for specific final states are evaluated as well as background contributions arising from other processes.

Through the detailed simulation of detector effects, one is able to correct the number of selected data events for background and selection efficiency, and thus arrive at a measurement for a specific process free of detector effects. These measurements are then analysed by comparing them to predictions calculated based on various physics assumptions. These predictions are usually calculated by the semianalytical programs discussed in the following.

3.3.2 Semianalytical Programs

Semianalytical programs calculate cross sections integrating out most of the kinematic variables specifying the final state, such as azimuthal or polar angles or energies of the final state fermions and radiative photons. These integrations are performed analytically or numerically. Only a few simple cuts on the phase-space variables describing the final state are taken into account, such as polar angles, acollinearity or amount of photon radiation.

On one side there are programs coding Standard Model calculations. As a function of the chosen Standard Model input parameter set, these programs calculate all other quantities, such as effective couplings, partial decay widths, cross sections and forward-backward asymmetries, both with or without including QED radiative effects. The most widely used programs in this respect are ALIBABA [111], TOPAZ0 [21] and ZFITTER [22] for fermion-pair production, and GENTLE [112] and WTO [113] for W-pair and four-fermion production.

In addition, there are theory programs which calculate cross sections and asymmetries in so-called model-independent approaches. Such calculations of cross sections and asymmetries are based on masses, partial widths or effective coupling constants. They allow the determination of these parameters from the measurements without relying on the full framework of the Standard Model, thus also allowing to perform tests of the Standard Model. The programs used in this area are MIZA [114] and ZFITTER, implementing the methods of partial widths and effective couplings, and SMATASY [20], implementing the S-Matrix ansatz within the ZFITTER framework. In this review, the theoretical calculations are performed with the semianalytical programs SMATASY, TOPAZ0 and ZFITTER.

3.3.3 Theoretical Uncertainties

The Standard Model calculations necessarily contain uncertainties due to uncalculated and thus missing higher order radiative corrections. The precision of the calculations and their theoretical uncertainties are assessed in [60, 62, 63, 115–117]. A detailed comparison of the various computer codes, in particular of the the semianalytical programs and electroweak libraries for the calculation of electroweak observables, have been made by the working group on precision calculations for the Z resonance [116] in 1994. Since then, additional higher-order corrections have been calculated and the new results are incorporated in the semianalytical programs TOPAZ0 and ZFITTER. In order of increasing importance, the newly available corrections are:

- Four-loop QCD corrections [118].
The effect of these corrections is less than 10^{-5} relative and thus negligible compared to the experimental errors.
- Nonfactorisable mixed $\mathcal{O}(\alpha_{em}\alpha_S)$ QCD-electroweak corrections [119].
These corrections reduce the hadronic Z width by about 0.6 MeV and thus increase the value of α_S derived from the measurement by 0.001.
- Next-to-leading two-loop electroweak corrections [47].
Depending on the mass of the top quark and of the Higgs boson, these corrections decrease $\Delta\rho$ by up to $4 \cdot 10^{-4}$, increase $\sin^2 \bar{\theta}_W$ by up to $2 \cdot 10^{-4}$, and decrease M_W by up to 15 MeV.

The combined effect of all new corrections on the electroweak parameters as calculated by the two programs TOPAZ0 and ZFITTER is summarised in [120]. The inclusion of the new subleading two-loop electroweak corrections reduces the theoretical uncertainty on the SM calculations of $\sin^2 \bar{\theta}_W$ and M_W to less than 10^{-4} and 10 MeV, respectively [121]. Detailed comparisons between the calculations of TOPAZ0 and ZFITTER and studies of the remaining theoretical uncertainties are compiled in [122].

Chapter 4

Electron-Positron Colliders and Detectors

4.1 The Stanford Linear Collider SLC

The SLC collider at SLAC, Stanford, is based on the old linear accelerator initially built during the 1960s for fixed-target lepton-nucleon scattering experiments. During the 1980s it was modified to accelerate both electrons and positrons which are brought into collisions at a single interaction point by guiding them along opposing arcs as shown in Figure 4.1. In June 1989 the first e^+e^- collisions at 91 GeV centre-of-mass energy were recorded by the upgraded MARK II detector [123], which had already taken data in e^+e^- collisions at lower centre-of-mass energies. By August 1989, the first measurement of Z resonance parameters in e^+e^- collisions was performed. Since 1992, the new SLD detector [124], in design similar to the LEP detectors, takes data at the SLC.

The SLC provides the unique opportunity of longitudinally polarised electron beams. The longitudinal polarisation of the electron beam, P_e , is defined as the average electron helicity, $h_e = -1$ for left-handed electrons and $h_e = +1$ for right-handed electrons, averaged over the electrons in the beam. Nowadays, SLC routinely achieves beam polarisations in excess of $|P_e| = P = 70\%$. Because of the small SLC beam pipe radius of only 4.5 cm at the interaction vertex, the silicon micro vertex detector of SLD is positioned very close to the interaction point. This makes precise reconstruction of vertices, impact parameters and decay lengths possible.

Until June 1998, the SLD experiment has collected $5.6 \cdot 10^5$ Z events at 91.28 GeV where the Z pole e^+e^- annihilation cross section has its maximum. The amount of polarisation obtained for the different data sets is shown in Table 4.1. In view of the excellent performance in 1997 and 1998, the SLD collaboration and SLAC propose a run extension with the goal of collecting more than $7 \cdot 10^5$ additional Z decays.

4.2 The Large Electron Positron Collider LEP

The LEP collider has been built at CERN close to Geneva during the 1980s. As shown in Figure 4.2, it is a circular accelerator with a circumference of 27 km, which makes it the largest machine in the world. Existing colliders at CERN, such as the PS and SPS, are used as pre-accelerators to ramp the electrons and positrons to the LEP injection energy between 20 GeV and 22 GeV as shown in Figure 4.3. Inside the LEP ring, the e^+e^- beams are further accelerated and brought into collisions at four interaction points, which are equipped with the detectors ALEPH [126, 127], DELPHI [128, 129], L3 [130] and OPAL [131]. The LEP machine started its operation with a pilot run in August 1989, yielding a handful of Z events per experiment. The first energy scan of the Z resonance was performed in the fall of 1989.

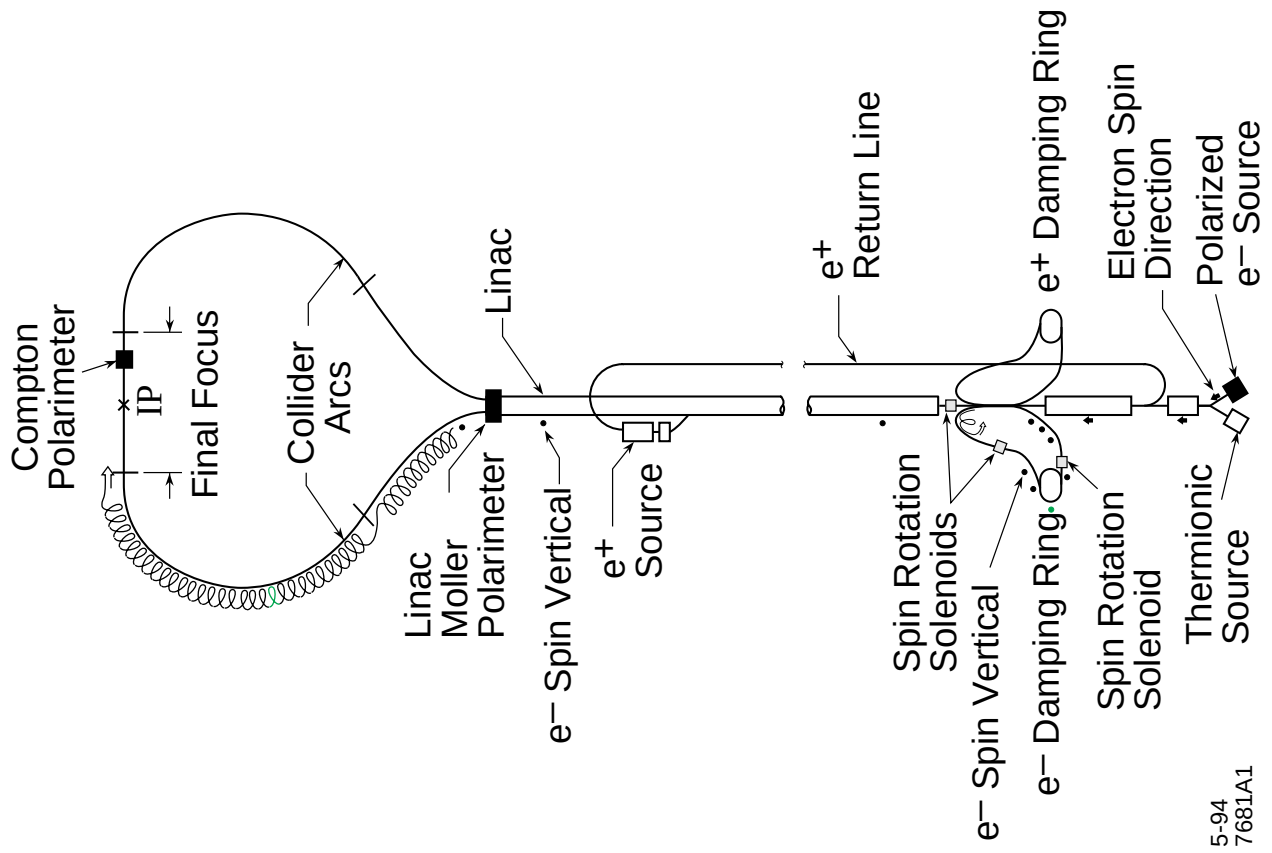


Figure 4.1: The SLC collider at SLAC, Stanford.

Data Taking Period	Z Events [10^5]	Polarisation P [%]
1992	0.1	22.4 ± 0.6
1993	0.5	62.6 ± 1.2
1994+1995	1.0	77.2 ± 0.5
1996	0.5	76.5 ± 0.5
1997	1.0	73.3 ± 0.8
1998	2.5	73.1 ± 0.8

Table 4.1: Number of Z events selected by SLD and degree of beam polarisation at SLC [125]. The error on the polarisation is dominated by systematic effects. The polarisation results for the 1996-1998 data are preliminary.

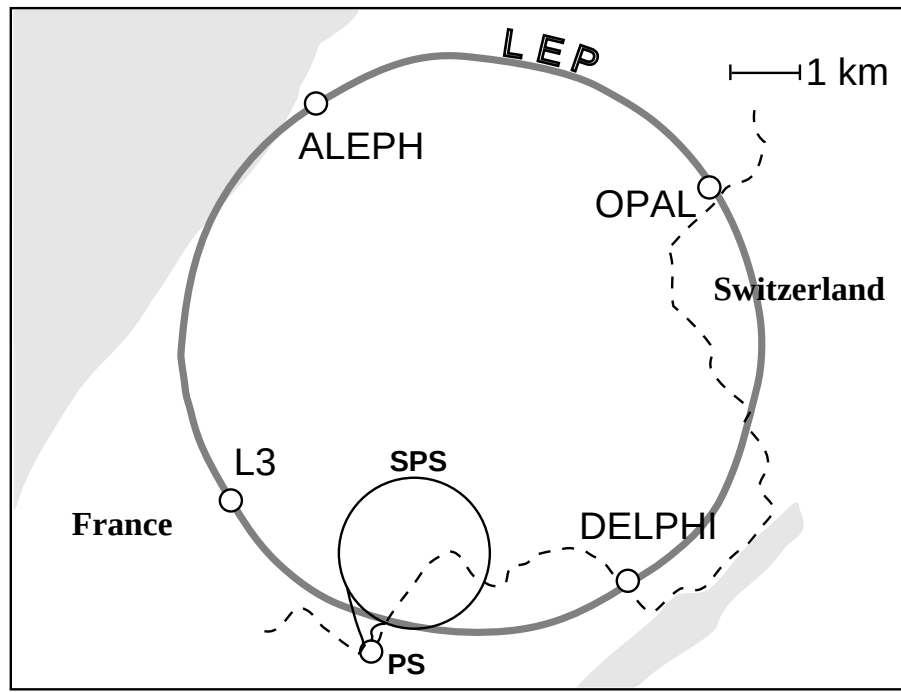


Figure 4.2: The LEP collider and the four LEP experiments ALEPH, DELPHI, L3 and OPAL at CERN, Geneva.

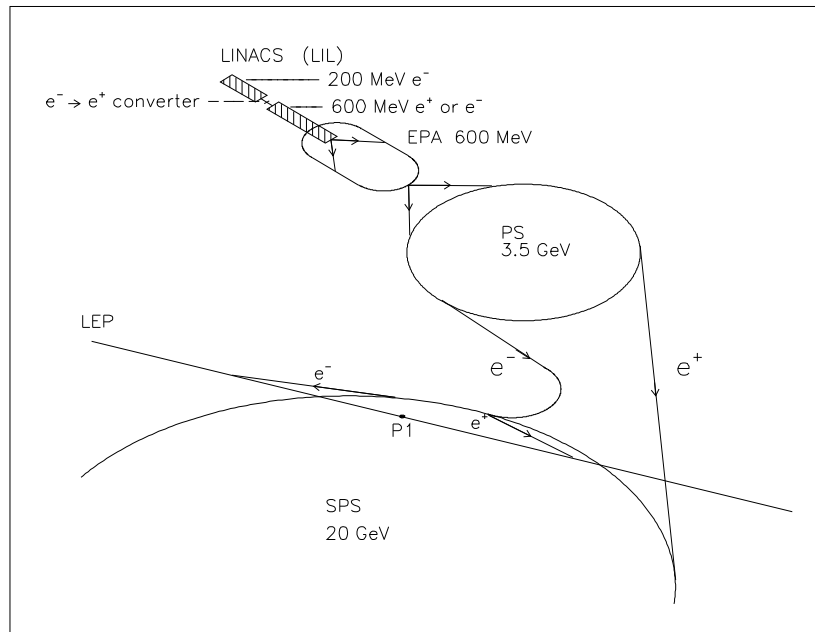


Figure 4.3: The injection scheme of the LEP collider at CERN.

Since then, LEP delivered an increasing amount of integrated luminosity each year as shown in Table 4.2. Peak luminosities reached $10^{31}\text{cm}^{-2}\text{s}^{-1}$. The LEP accelerator is considered as a Z factory, delivering luminosity at a rate nearly two orders of magnitude higher than SLC. Until 1995, LEP was operated at various centre-of-mass energies close to the Z mass, $|\sqrt{s} - M_Z| < 3 \text{ GeV}$, in order to scan the Z resonance. During this phase, called LEP-I, each experiment collected about 160 pb^{-1} of integrated luminosity and selected on average $4 \cdot 10^6$ hadronic and $4 \cdot 10^5$ leptonic events.

The Z phase of LEP ended in the final weeks of the 1995 data taking period, when the centre-of-mass energy was increased by a factor of 1.5 to a range of 130 GeV to 140 GeV. Since then the LEP accelerator provides by far the highest centre-of-mass energies ever achieved in e^+e^- collisions. During the LEP-II program from 1996 until the year 2000, the centre-of-mass energy gradually increases from the threshold of W-pair production, 161 GeV, to a maximum of about 200 GeV. The increase in centre-of-mass energy with respect to LEP-I is achieved by adding 288 superconducting cavities over the years 1995 to 1999. At LEP-II, peak luminosities of $10^{32}\text{cm}^{-2}\text{s}^{-1}$ are reached and a total integrated luminosity approaching 500 pb^{-1} per experiment is expected until the end of the year 2000, when data taking at LEP must stop due to preparations for the LHC project.

In contrast to SLC, LEP operates with unpolarised electron and positron beams during physics data taking and the measurements are interpreted under this assumption. Only under special running conditions, transverse beam polarisation is obtained, which is used for the calibration of the LEP beam energy as discussed in Section 4.6. Transverse beam polarisation leads to an azimuthal modulation of differential cross sections which, however, integrates out since the detectors are azimuthally symmetric. An unknown net longitudinal polarisation at the interaction points would be dangerous. Studies show that the magnitude of the electron and positron polarisations are smaller than 0.5%, and that the polarisation vectors of electrons and positrons are opposite for identical orbits [132]. Thus modifications of observables linear in the polarisation vanish, while quadratic effects are limited to be less than $2.5 \cdot 10^{-5}$ which is negligible.

Data Taking Period	Energy Points	Luminosity pb^{-1}	Centre-of-mass Energies $\sqrt{s}$
1989	10	1	$ \sqrt{s} - M_Z < 4 \text{ GeV}$
1990	7	6	$ \sqrt{s} - M_Z < 3 \text{ GeV}$
1991	7	14	$ \sqrt{s} - M_Z < 3 \text{ GeV}$
1992	1	25	$\sqrt{s} \simeq M_Z$
1993	3	35	$ \sqrt{s} - M_Z < 3 \text{ GeV}$
1994	1	50	$\sqrt{s} \simeq M_Z$
1995	3	30	$ \sqrt{s} - M_Z < 3 \text{ GeV}$
1995	3	5	130 GeV – 140 GeV
1996	1	10	161 GeV
1996	2	10	170 GeV – 172 GeV
1997	2	7	130 GeV – 136 GeV
1997	3	55	182 GeV – 184 GeV
1998	1	180	189 GeV
1999+2000	3	200	192 GeV – 200 GeV

Table 4.2: Number of centre-of-mass energy points and average recorded integrated luminosity per experiment. For 1999-2000, expectations are shown.

4.3 The Experiments at SLC and LEP

The conceptual design of the SLC detector, SLD [124], and the four LEP detectors, ALEPH [126, 127], DELPHI [128, 129], L3 [130] and OPAL [131], is rather similar. They are shown in Figures 4.4, 4.5 and 4.6. All detectors aim as much as possible for hermetic coverage of the full solid angle. Various subdetector systems are layered as cylinders centred along the beam line and on the interaction point. Part or all of the detector is embedded in a magnetic field with field lines parallel to the beam axis.

The inner tracking system consists of multilayered silicon micro-strip vertex detectors surrounded by gas drift chambers. This system measures track parameters such as momentum and angles of charged particles. Extrapolation back to the interaction vertex allows the determination of the event vertex, impact parameters, secondary vertices and decay lengths.

Calorimeters surround the inner tracking system. They are usually divided into two parts, first an electromagnetic calorimeter and then a hadronic calorimeter. All non-minimal-ionising particles are absorbed and their energy is measured. The electromagnetic calorimeter measures precisely the energy and position of electromagnetic showers arising from electrons and photons. The hadronic calorimeter measures the energyflow of both charged and neutral hadronic particles.

An outer tracking system for penetrating minimal-ionising muons completes the detectors. Such a system improves on the accuracy of the measurements of the inner tracking system for muons due to its larger lever arm.

Particle identification is also derived from dE/dx measurements in the central tracking chamber, and for some experiments by ring-imaging Cerenkov counters. In addition, the detectors contain special purpose subdetectors to measure the luminosity in their interaction region, described in Section 4.5.

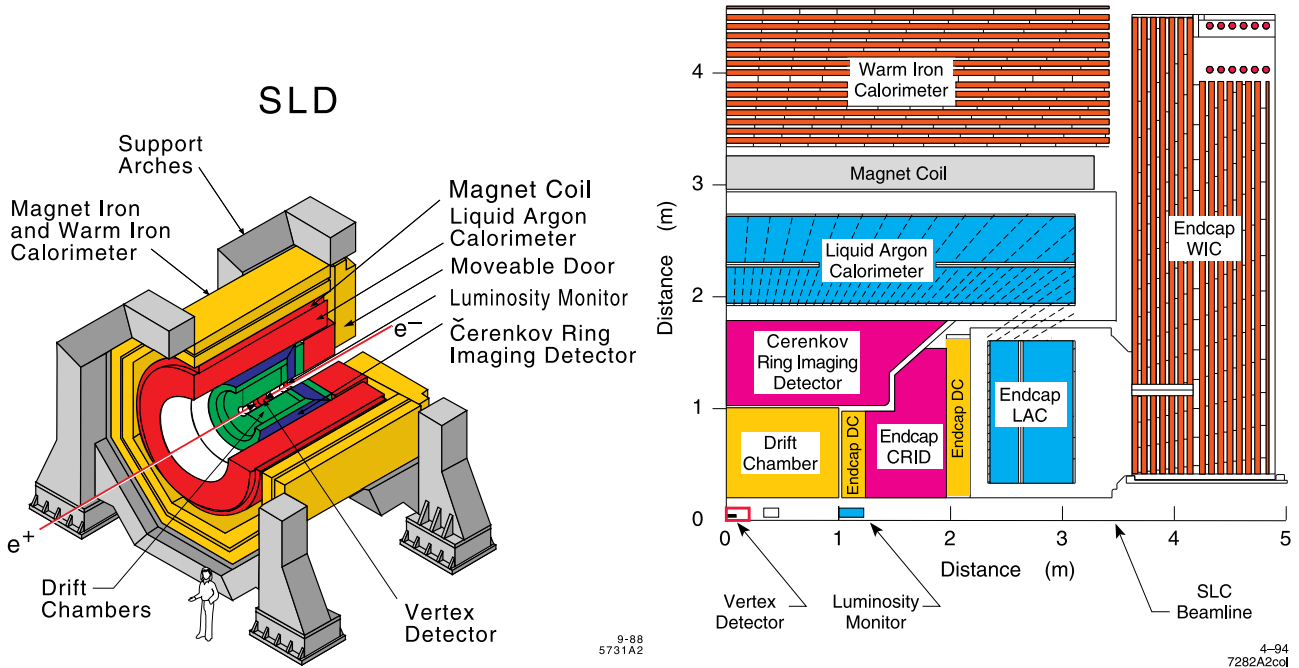


Figure 4.4: The SLD detector at SLC.

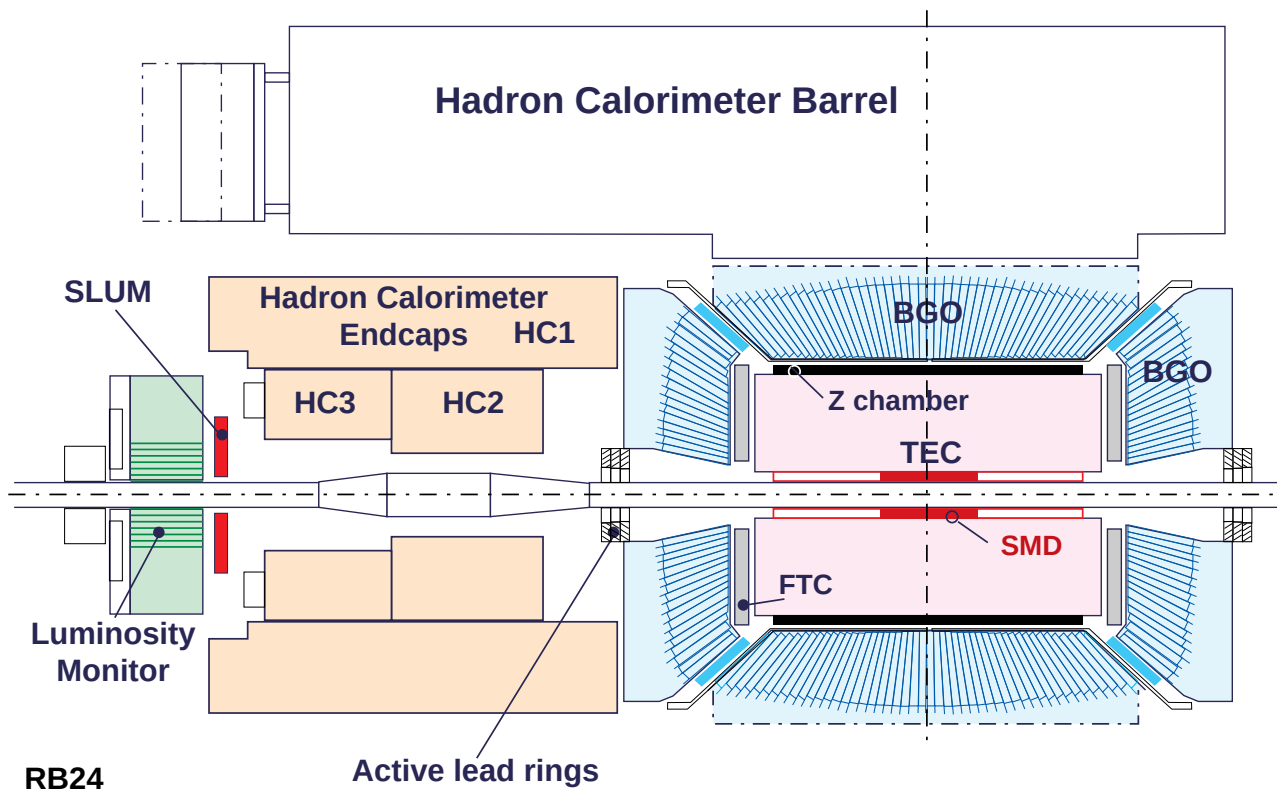
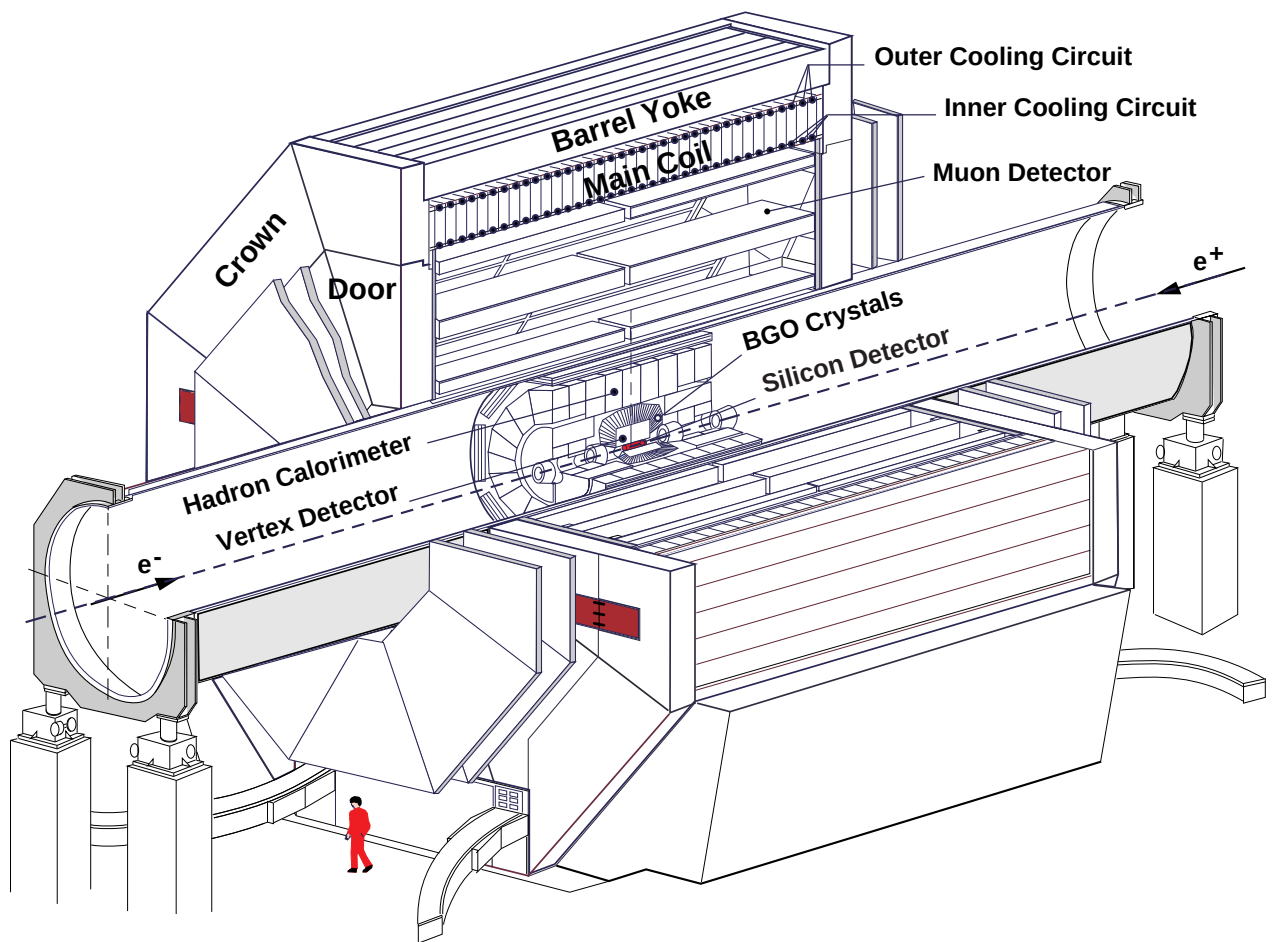


Figure 4.5: The L3 detector at LEP.

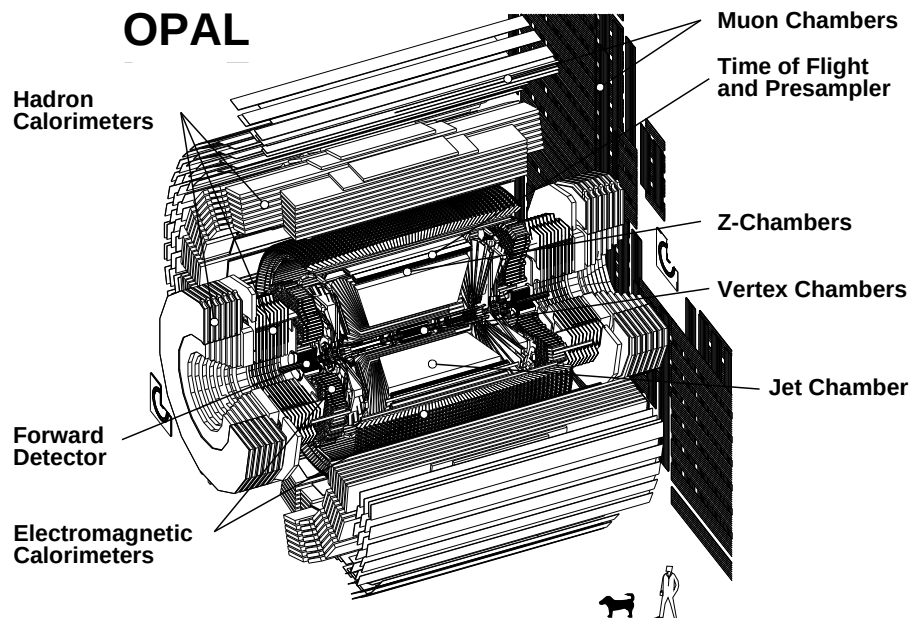
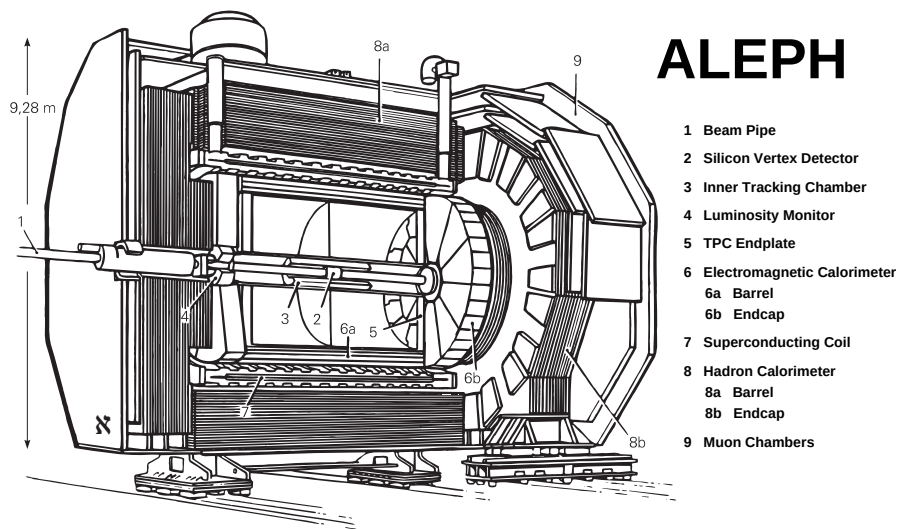
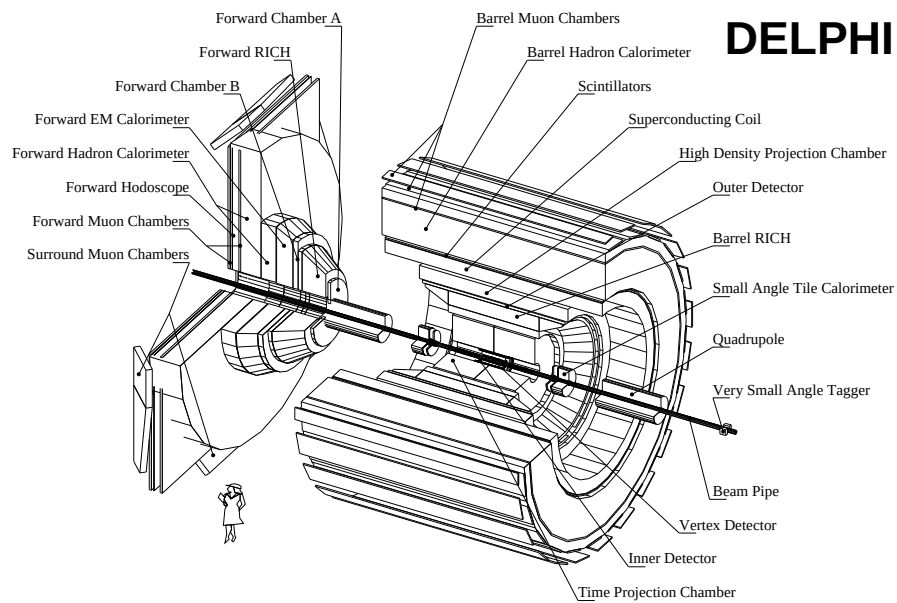


Figure 4.6: The detectors DELPHI, ALEPH and OPAL at LEP.

4.4 Beam Polarisation

4.4.1 Production of Polarised Electrons

Polarised electrons at SLC are photoproduced by shining circularly polarised laser light on a strained GaAs cathode as shown in Figure 4.7. The sign of the laser polarisation, and thus the sign of the electron polarisation, is chosen at random with equal probability. The strain affecting the GaAs lattice is obtained by growing a $0.1\ \mu\text{m}$ thick layer of GaAs on a basis of GaAsP. It lifts the degeneracy of the $J = 3/2$ energy levels in the valence band by about $0.05\ \text{eV}$ as shown in Figure 4.8. Incident photons with positive helicity and an energy between $1.43\ \text{eV}$ and $1.48\ \text{eV}$ excite only the transition from the $m_j = -3/2$ state in the valence band to the $m_j = -1/2$ state in the conduction band, thus producing electrons of one helicity state only. As a consequence the polarisation of the extracted electrons is increased above the limit of 50% accessible in the degenerate case.

4.4.2 Transport of Polarised Electrons

Before entering the damping ring, the longitudinal polarisation of the electron bunch is rotated to become transverse. This transverse polarisation is kept during acceleration up to the nominal beam energy of $46.6\ \text{GeV}$ along the linear accelerator, as shown in Figure 4.1. The electron spin orientation is manipulated in the SLC arc by a pair of vertical betatron oscillations to achieve longitudinal polarisation at the interaction point.

4.4.3 Polarisation Measurement

The polarisation of the beam electrons is measured twice. First it is measured with a Moller polarimeter at the end of the linear part of the accelerator, and then with a Compton polarimeter after the electron beam has crossed the SLD collision point, as shown in Figure 4.9.

The experimental setup of the more precise Compton polarimeter is shown in Figure 4.10. Circularly polarised laser light interacts through Compton scattering with the longitudinally polarised beam electrons. If in the electron-photon centre-of-mass system the electron is scattered backwards, the energy loss of the electrons in the laboratory system becomes quite sizeable. The Compton cross section is measured for parallel, $J = 3/2$, and antiparallel, $J = 1/2$ combinations of the photon and electron beam helicities, and their asymmetry is formed:

$$A(E) = \frac{\frac{d\sigma(E, J = 3/2)}{dE} - \frac{d\sigma(E, J = 1/2)}{dE}}{\frac{d\sigma(E, J = 3/2)}{dE} + \frac{d\sigma(E, J = 1/2)}{dE}} = \kappa P_e P_\gamma A_C(E), \quad (4.1)$$

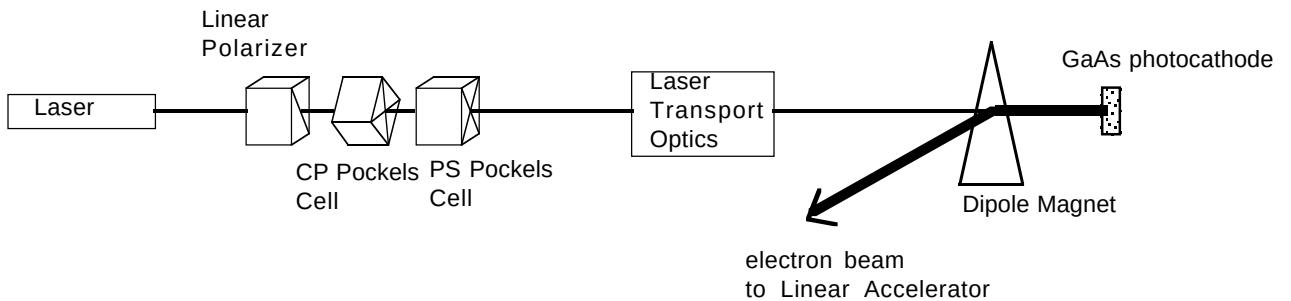


Figure 4.7: Source of polarised beam electrons at SLC [133]. The laser light is polarised with a linear polariser and two Pockels cells allowing to generate arbitrary elliptical polarisation in order to compensate for phase shifts in the laser beam transport optics.

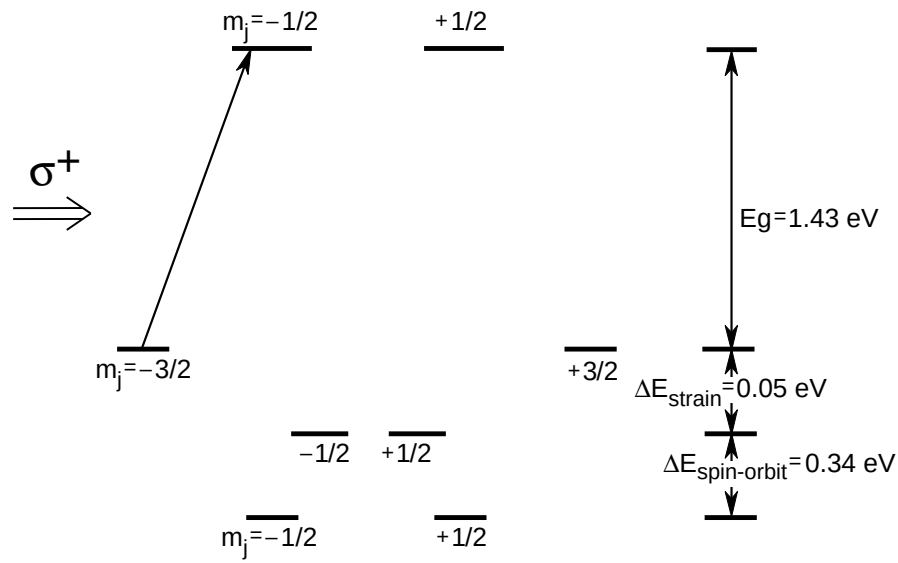


Figure 4.8: Energy levels of the strained GaAs photocathode used to produce polarised electrons for SLC [133]. Shown is the energy gap between the conduction and the valence band and the splits of the energy levels of the valence band caused by the applied strain and the spin-orbit coupling. The stimulated transition is indicated by the arrow pointing from the valence band to the conduction band. The helicity of the extracted electrons is the same as that of the incident photons since the extracted electrons have opposite direction to the incident photons.

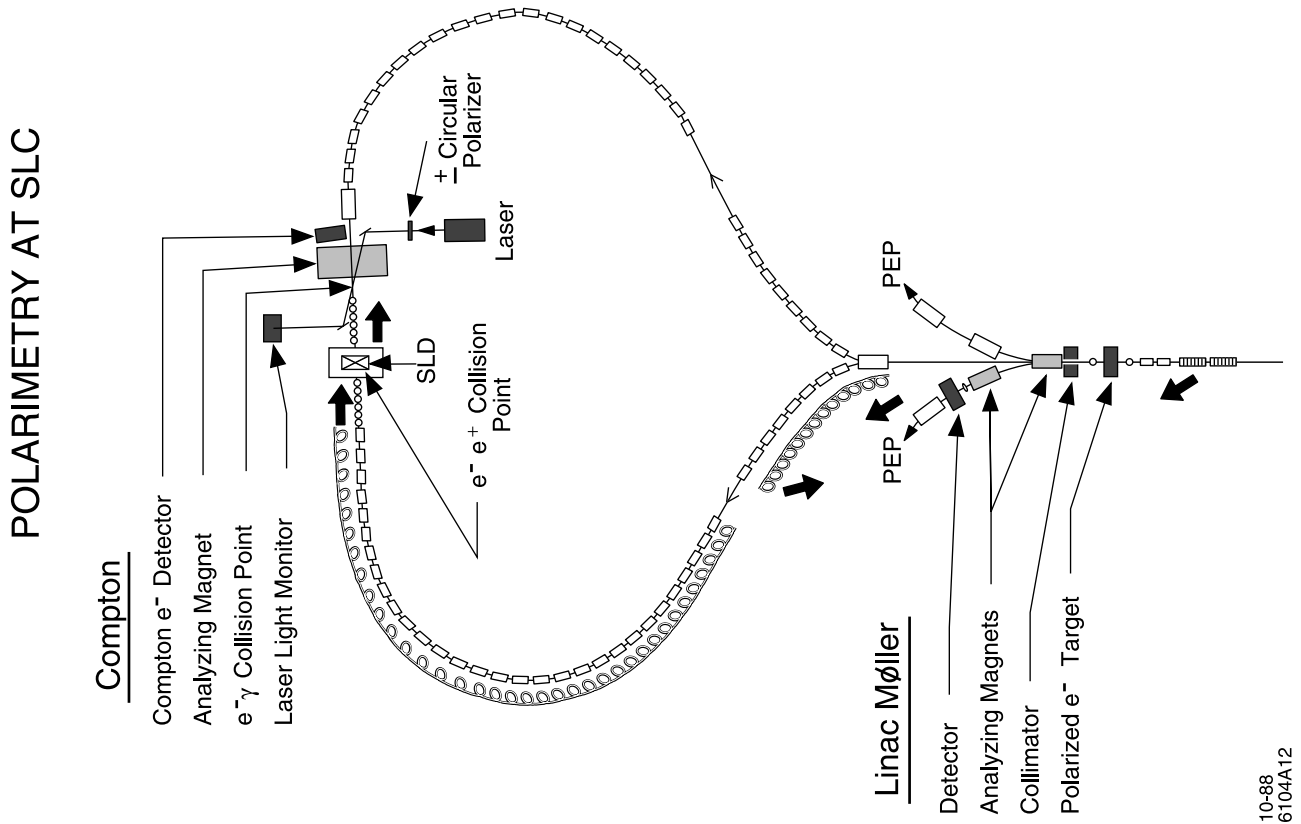


Figure 4.9: Measurements of electron beam polarisation at SLC. Shown are the locations of the Compton and Moller polarimeters along the SLC beam line. The thick arrows indicate the beam polarisation vector along the electron beam line.

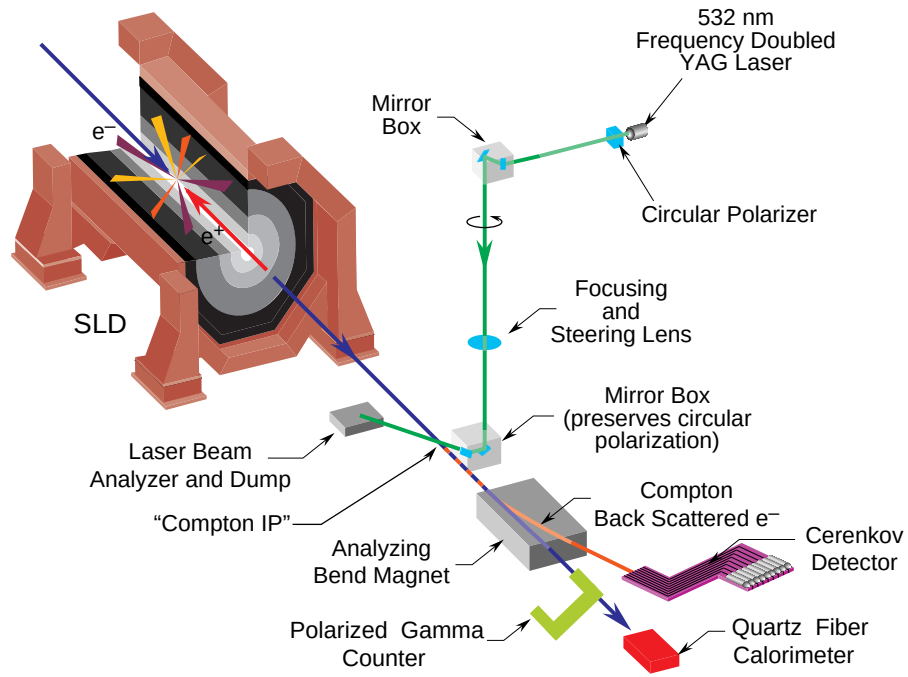


Figure 4.10: Schematic view of Compton polarimeter setup at SLC.

where $A_C(E)$ is the known Compton asymmetry function [134], and κ is the analysing power of the polarimeter accounting for detector effects, typically $\kappa = 0.7$. The asymmetry $A(E)$ is measured as a function of the electron energy E . This is achieved by spatially separating the scattered beam electrons of different energy E by means of a magnetic spectrometer deflecting the electron beam from its nominal path, the so called neutral beam line. As shown in Figure 4.11, the scattered electrons are measured by a nine-channel Cerenkov detector extending from 11 cm to 21 cm in transverse distance to the neutral beam line, corresponding to electron energies in the range from 17 GeV to 30 GeV.

For each energy E , the measurement of $A(E)$ determines the product of electron beam and laser light polarisations, $P_e P_\gamma$. The laser is circularly polarised by means of a linear polariser and two Pockels cells, as shown in Figure 4.12. The polarisation of the laser beam, typically $P_\gamma = 99.6 \pm 0.2\%$, is itself also continuously monitored both before and after passing through the Compton interaction point. In each case, the amount of left-polarised light and right-polarised light making up the total is measured by photodiodes.

Based on this setup, the longitudinal electron beam polarisation is measured downstream of the electron beam every three minutes with a statistical uncertainty δP better than 1%. Machine tuning of the accelerator causes changes to the polarisation in the range of $\Delta P = 0.5\%$ over the time scale of hours. The time dependent drift of the polarisation is well tracked. The measurement of the average polarisation is systematics limited, dominated by the detector linearity and interchannel consistency. In recent years the relative error $\delta P/P$ is reduced to 0.7% as the final error, as reported in Table 4.1. The error on the polarisation directly propagates as a source of systematic error to the measurements of polarised asymmetries, $\delta P/P = \delta A_{lr}/A_{lr} = \delta A_{fblr}/A_{fblr}$.

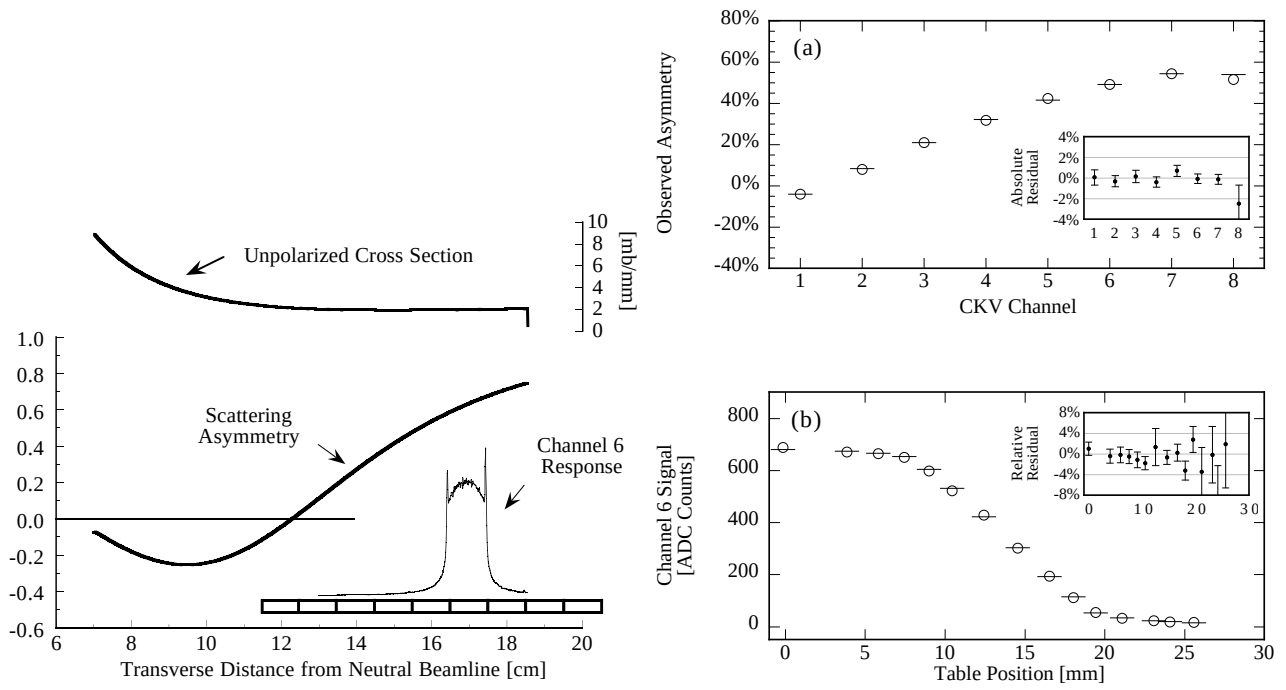


Figure 4.11: Left: Calculated unpolarised cross section and Compton asymmetry [135]. Right: Measurements shown as circles compared to fit result shown as horizontal bar. The fit procedure determines the beam polarisation and the location of the Compton edge, *i.e.*, the position of the detector with respect to the beam line. Part a) shows the the observed asymmetry for each detector channel. Part b) shows the response of channel 6 as a function of the table position or transverse distance to the neutral beamline.

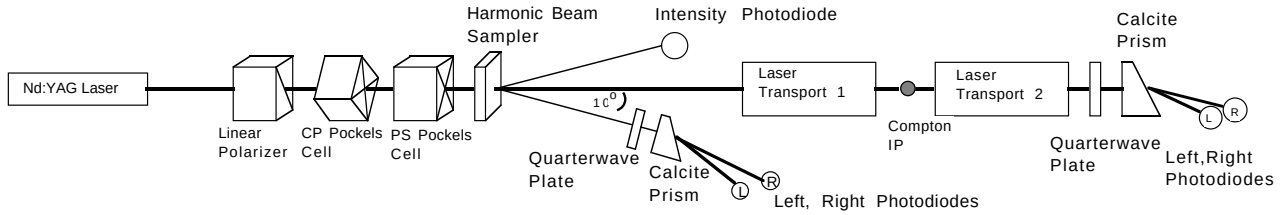


Figure 4.12: Compton laser system and monitoring of laser polarisation at SLC [135].

4.5 Luminosity Measurement

The accurate measurement of the total integrated luminosity, $\mathcal{L}$, at each centre-of-mass energy point is crucial for precise measurements of cross sections, σ , as it enters all cross-section measurements:

$$\sigma = \frac{N_{sel} - N_{bg}}{\epsilon \cdot \mathcal{L}}, \quad (4.2)$$

where the number of events selected in the data, N_{sel} , corrected for the expected number of events selected from background sources, N_{bg} , is divided by the selection efficiency, ϵ , and the total integrated luminosity of the data sample, $\mathcal{L}$.

The parameters of the SLC and LEP machines and fills are not well enough known to calculate the luminosity at the individual experimental interaction points with permille precision. A much more reliable determination is obtained by using the detector itself to measure the luminosity of its interaction region, thereby also taking detector deadtimes and downtimes correctly into account. Given

a specific luminosity reaction e , the luminosity at the centre-of-mass energy $\sqrt{s}$ is simply determined by the inverse of the equation above, written for each $\sqrt{s}$ point:

$$\mathcal{L}(s) = \frac{N_{sel}^e(s) - N_{bg}^e(s)}{\epsilon_e(s) \cdot \sigma_e(s, a)}, \quad (4.3)$$

where $\sigma_e(s, a)$ is the calculated cross section of reaction e within the fiducial volume a . Effectively, the cross section of any new process, $\sigma(s)$, is simply determined relative to the known cross section $\sigma_e(s, a)$.

It is obviously advantageous to use a well known interaction e with a high cross section $\sigma_e(s, a)$ to keep both the theoretical error on the calculation of $\sigma_e(s, a)$ and the statistical error of the luminosity measurement as small as possible. At e^+e^- colliders, the reaction typically used is Bhabha scattering, $e^+e^- \rightarrow e^+e^-(\gamma)$, measured at small polar scattering angles θ . In this region of phase-space, the Bhabha cross section is dominated by the t -channel exchange of a photon leading to a sharp rise of the differential cross section with $\theta \rightarrow 0$, see Equation 3.39. The lowest-order result is:

$$\frac{d\sigma_e(s)}{d\theta} = \frac{d\sigma_t(\gamma_t\gamma_t)}{d\theta} = \frac{32\pi\alpha_{em}^2}{s} \cdot \frac{1}{\theta^3} \quad \text{for } 0 < \theta \ll 1 \text{ rad}. \quad (4.4)$$

At small scattering angles, $t = -s(1 - \cos\theta)/2$ is small and in that region of t QED is experimentally well tested. Furthermore, since t -channel photon exchange is a pure QED process, the luminosity is also nearly completely independent of the Z boson, whose properties are to be determined.

Integrating the differential cross section, the total cross section in the fiducial volume a given by the polar angular range $0 < \theta_{min} < \theta < \theta_{max}$ is obtained:

$$\sigma_e(\theta_{min} < \theta < \theta_{max}) = \frac{16\pi\alpha_{em}^2}{s} \left(\frac{1}{\theta_{min}^2} - \frac{1}{\theta_{max}^2} \right) \quad \text{for } \theta_{max} \ll 1 \text{ rad} \quad (4.5)$$

$$= 0.125 \frac{M_Z^2}{s} \left(\frac{1}{\theta_{min}^2} - \frac{1}{\theta_{max}^2} \right) \text{ nb}. \quad (4.6)$$

An error on the fiducial volume, $\delta\theta$, translates into an error on the Bhabha cross section and thus luminosity of $\delta\mathcal{L}/\mathcal{L} = \delta\sigma_e/\sigma_e = 2\delta\theta/\theta$. In particular the inner edge of the fiducial volume, θ_{min} , needs to be known precisely.

The principle of the luminosity measurement is shown in Figure 4.13. The experiments at SLC and LEP are equipped with dedicated luminosity detectors situated at low polar angles, typically covering the angular range of $25 \text{ mrad} < \theta, \pi - \theta < 60 \text{ mrad}$. The accepted cross section of small-angle Bhabha scattering for the luminosity measurement ranges from 50 nb to 100 nb. This is a factor of 2 to 3 larger than the largest Z resonance cross section, which is about 30 nb for the hadronic final state at the Z pole $\sqrt{s} = M_Z$. The accepted cross sections for lepton-pairs are smaller by at least an order of magnitude. Thus the statistical error on the luminosity measurement needs to be taken into account for the measurement of the hadronic cross section while it is negligible for all other cross section measurements. It is always negligible for cross section measurements at centre-of-mass energies far away from the Z resonance, *i.e.*, for the LEP-II measurements.

The systematic error of the luminosity measurement constitutes a correlated error between all cross section measurements of that experiment. When averaging cross section measurements of different experiments, the theoretical error on the calculation of the small-angle Bhabha cross-section must be taken as fully correlated.

4.5.1 Experimental Errors

The experimental systematic errors are dominated by aspects concerning the definition of the fiducial volume in polar angle, such as the reconstruction of the polar coordinate of the electrons, the detector geometry and the alignment of the detector with respect to the interaction vertex. In order to reduce

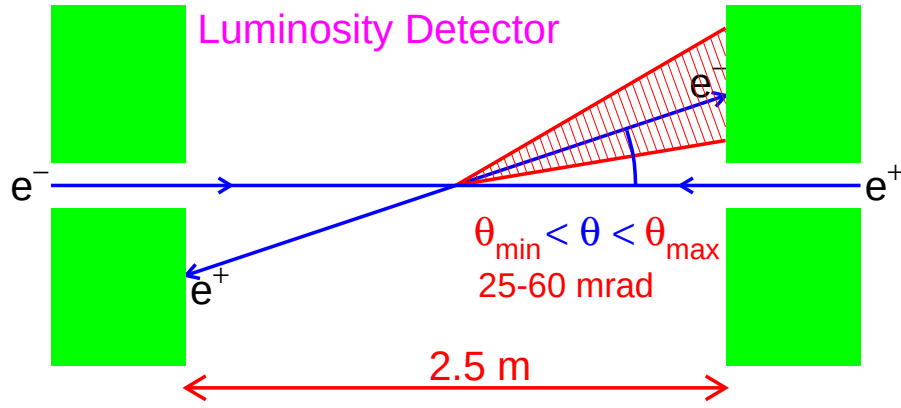


Figure 4.13: Principle of luminosity measurement at e^+e^- collider. The distance of the luminosity detectors to the interaction point is about 2.5 m. The radial coverage typically extends from 6 cm to 16 cm.

the uncertainty on the luminosity due to such geometrical effects, asymmetric fiducial volumes are defined for the two luminosity detectors, a wide region on one side and a narrow region on the other side. The luminosity is measured twice with the wide side and the narrow side interchanged. The average of the two has a reduced sensitivity to effects caused by misalignment.

The first generation of luminosity detectors in the LEP experiments consisted of electromagnetic calorimeters, yielding experimental systematic errors in the order of 0.5%. In order to exploit the higher luminosities and high precision energy scans of the Z resonance performed in the years 1993 to 1995, the LEP experiments have installed a second generation of luminosity detectors, combining calorimetry with silicon strip technology in order to improve the reconstruction of impact coordinates and the definition of the fiducial volume. Experimental systematic errors of 0.03% to 0.1% are now achieved [127, 129, 136–138]. Distributions of typical selection variables are shown in Figure 4.14.

4.5.2 Theoretical Errors

The theoretical calculation of the cross section for small-angle Bhabha scattering at $\sqrt{s} \approx M_Z$ has been improving in parallel with the data taking at LEP-I. Prompted by always decreasing experimental errors, a large effort was undertaken in order to decrease the theoretical uncertainties as well, and to provide adequate Monte Carlo programs for acceptance calculations. The theoretical error on the luminosity measurement for Z pole centre-of-mass energies decreased by an order of magnitude, from about 1% before the start of data taking at SLC and LEP to 0.11% in 1995. This progress is achieved by including higher-order QED radiative corrections and assessment of the remaining uncertainties [60, 62, 116, 140]. Currently, the complete $\mathcal{O}(\alpha_{em})$ corrections and the leading-log corrections to all orders, $\mathcal{O}(\alpha_{em}^n L^n)$ with $L = \ln |t/m_e^2| \approx 15$ are included in both calculations and event generators which allow to study the effects for realistic experimental event selections. The currently remaining uncertainties are summarised in Table 4.3 [62, 140].

At SLC/LEP-I centre-of-mass energies, the total theoretical uncertainty is large compared to the statistical precision of 0.025% given by $16 \cdot 10^6$ hadronic Z decays selected by the four LEP experiments combined. At LEP-II centre-of-mass energies, lower precision is required due to the smaller number of selected $f\bar{f}$ events, yielding statistical accuracies in the order of 0.25% on the number of hadronic events combining the data of the four experiments at the end of the LEP-II program.

The total theoretical error is dominated by uncertainties due to missing second-order subleading QED corrections, $\mathcal{O}(\alpha_{em}^2 L)$. These uncertainties have been evaluated to be 0.10% at $\sqrt{s} = M_Z$ and 0.20% at $\sqrt{s} = 2M_Z$. A recent reevaluation of these uncertainties reduces the corresponding

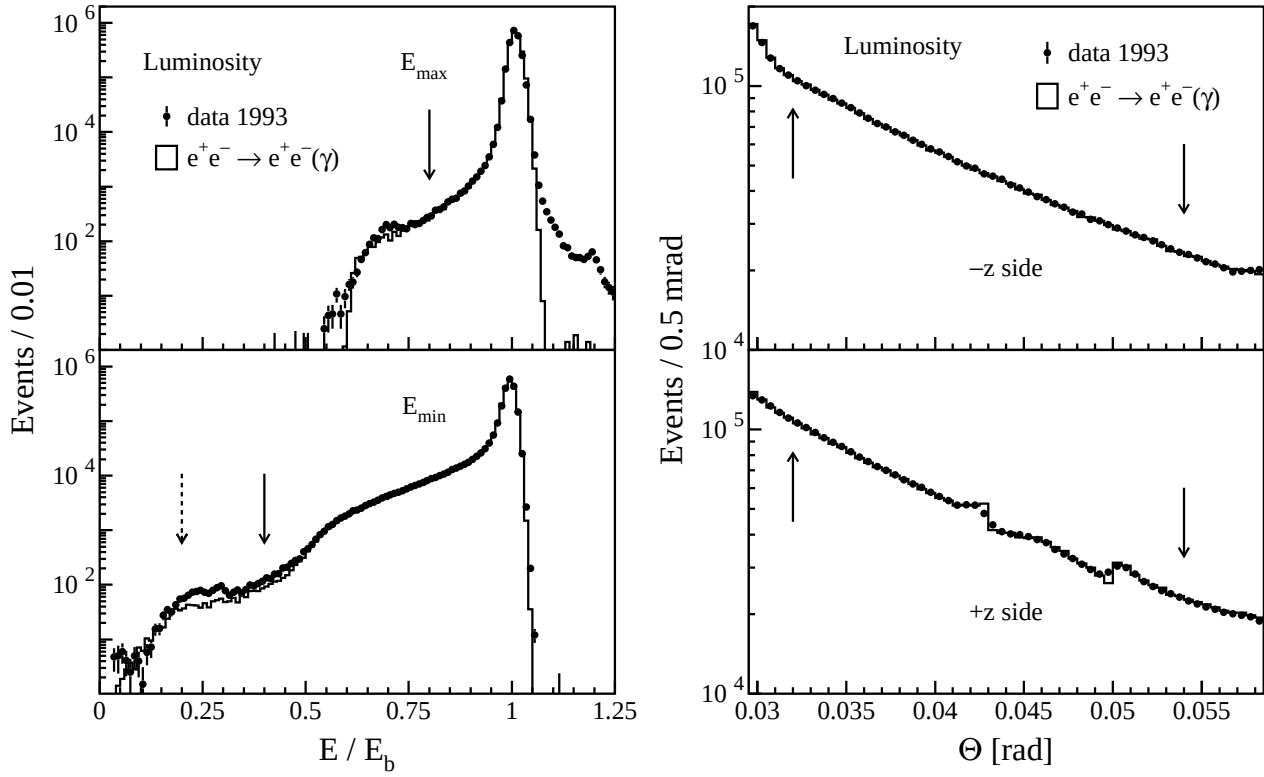


Figure 4.14: Distributions used in the luminosity analysis of L3 at $\sqrt{s} = M_Z$ [137, 139]. Left: Maximal and minimal calorimetric energy of the electron and positron candidate normalised to the beam energy. Right: Polar angles of the electron or positron candidates observed in the luminosity detector on the $+z$ and on the $-z$ side. The structure seen in the central part of the distribution at the $+z$ side is caused by a flare in the beam pipe on that side.

Source of Systematic Uncertainty	Contribution to $\delta\mathcal{L}/\mathcal{L}$ [%]	
	$\sqrt{s} = M_Z$	$\sqrt{s} = 2M_Z$
Missing $\mathcal{O}(\alpha_{em}^2 L)$	0.10	0.20
Vacuum Polarisation	0.04	0.10
Light Pairs	0.03	0.05
Missing $\mathcal{O}(\alpha_{em}^3 L^3)$	0.015	0.03
Z-boson exchange	0.015	—
Total	0.11	0.25

Table 4.3: Contributions to the total theoretical error on the calculation of the cross section of the small-angle Bhabha scattering process [62, 140], which is used in the luminosity analysis. For typical polar angles, the large logarithm L is given by $L = \ln |t/m_e^2| \approx 15$.

errors down to 0.027% and 0.04%, respectively [141]. Thus the total theoretical luminosity error will be decreased by a factor of two, down to 0.06% and 0.12%, respectively. With this improvement, the total theoretical error on the luminosity is then dominated by the uncertainty in the hadronic vacuum polarisation.

4.6 Centre-of-Mass Energy Measurement

In order to determine the resonance parameters of the Z boson such as its mass and total decay width precisely, total cross sections must be measured at several precisely known LEP-I centre-of-mass energies around the mass of the Z boson. Likewise, the determination of the mass of the W boson requires precise knowledge on the centre-of-mass energies where data is taken at LEP-II. The calibration of the LEP centre-of-mass energy is discussed in this section.

At LEP-I, energy scans of the Z resonance have been performed in the years 1989, 1990, 1991, 1993 and 1995. In 1992 and 1994, luminosity was collected at a centre-of-mass energy where the Z cross section is maximal. The various centre-of-mass energies are reported in Table 4.2, together with typical luminosities per experiment. By now, the 1989 data is ignored due to its negligible statistical significance as compared to the data taken in later years and due to significant improvements in the accuracy of the LEP energy calibration. At LEP-II, data has been taken at the threshold of W-pair production, 161 GeV, and above, at 172 GeV, in the year 1996; at 183 GeV in 1997; and at 189 GeV in 1998. In the year 1999 LEP will approach a centre-of-mass energy of 200 GeV to be kept until the end of data taking at LEP in the year 2000.

The SLD detector at SLC has taken data at the maximum of the Z-pole annihilation cross section. The luminosity weighted mean centre-of-mass energy is measured with precision energy spectrometers [142] to be 91.280 ± 0.025 GeV [143]. This measurement is now being verified by also analysing cross sections for Z boson production at 0.9 GeV below and above the peak. From a fit to the three cross sections and knowing the Z mass from LEP-I, a preliminary correction in the centre-of-mass energy of $\Delta\sqrt{s} = -49 \pm 40$ MeV is derived [125].

4.6.1 Calibration of the LEP Energy

4.6.1.1 Beam Energy Measurement

The determination of the LEP beam energy for the data accumulated by the experiments relies on two key points:

1. The precise measurement of the LEP beam energy by the method of resonant depolarisation [144, 145], usually performed just before a LEP machine fill is terminated.
2. The model to describe the time dependent and interaction point specific variation of the LEP beam energy during data taking at LEP-I [146–150] and LEP-II [151–153].

4.6.1.2 Method of Resonant Depolarisation

In an ideal circular orbit, the circulating electrons and positrons become polarised transverse to the plane containing the orbit. The polarisation is built up naturally due to synchrotron radiation, called Sokolov-Ternov effect [154]. Over time, the transverse polarisation, $P(t)$, approaches an asymptotic value:

$$P(t) = P_{\infty} \cdot \left[1 - \exp\left(-\frac{t}{\tau}\right) \right] \quad (4.7)$$

$$P_{\infty} = \frac{8}{5\sqrt{3}} \cdot \frac{\tau_d}{\tau_p + \tau_d} = 0.92376 \cdot \frac{\tau_d}{\tau_p + \tau_d} \quad (4.8)$$

$$\tau = \frac{\tau_p \tau_d}{\tau_p + \tau_d}, \quad (4.9)$$

where τ_p and τ_d are the time constants of self-polarising and depolarising effects competing against each other. Usually, $\tau_d \ll \tau_p$ due to machine imperfections, *i.e.*, depolarisation occurs much faster than self polarisation, so that during normal data taking the transverse polarisation of the LEP beams vanishes.

Under special running conditions, however, τ_d is much reduced and transverse polarisation is built up within an hour or less, as shown in Figure 4.15. It is then possible to measure the spin tune, *i.e.*, the number of spin precessions per complete particle revolution, which is related to the particle energy via the Lorentz γ factor:

$$\nu_s = a_e \gamma_e = a_e \frac{E_e}{m_e} = n_s + \delta_s, \quad (4.10)$$

where $a_e = (g_e - 2)/2$ is the anomalous magnetic moment of the electron. The quantities n_s is the integer part and δ_s is the fractional part of the spin tune ν_s . Standard magnetic measurements are precise enough to determine n_s without ambiguity. At the Z pole, $n_s = 103$ for a beam energy of 45.6 GeV.

A radial field of an RF magnet oscillating with frequency f_{dep} is used to rotate the spin by 0.14 mrad. If the perturbation from the RF magnet is in resonance with the spin precession, these small spin rotations add up coherently. After about 10^4 turns or 1 sec at a particle revolution frequency of $f_e = 11.25$ kHz, the polarisation vector is moved into the horizontal plane, and the vertical polarisation is destroyed. A sweep of f_{dep} around the estimated value f_{dep}^{res} is performed while continuously measuring the beam polarisation. Eventually, depolarisation is observed at a certain setting,

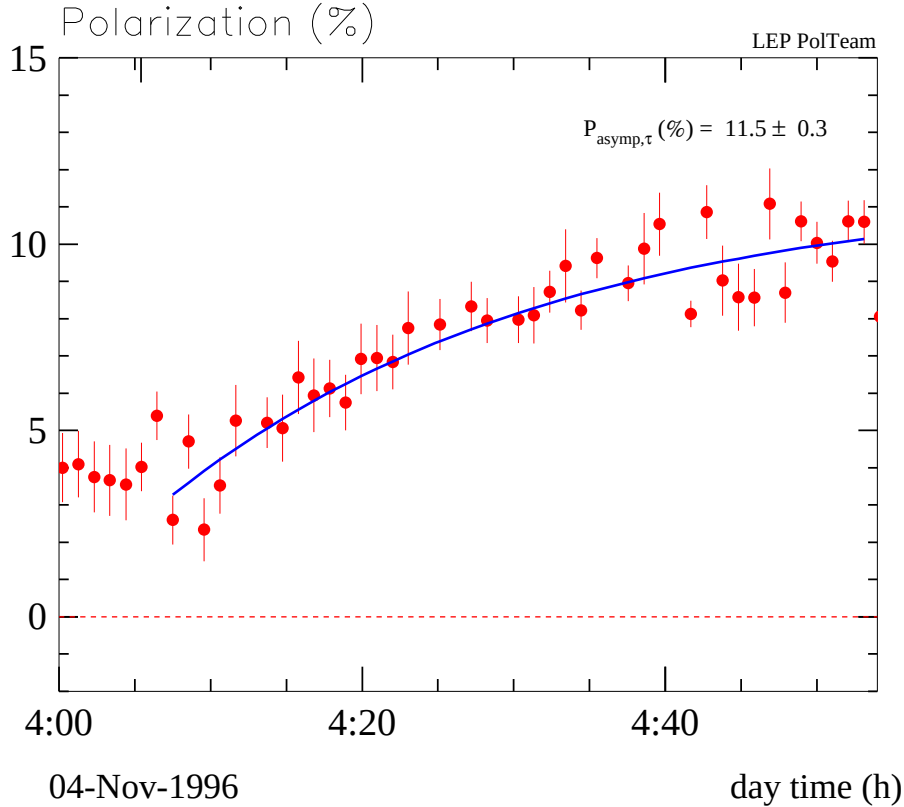


Figure 4.15: Build-up of transverse beam polarisation at LEP-I achieved under special running conditions. The asymptotic value of the polarisation is determined to be $P_\infty = 11.5 \pm 0.3\%$ for this measurement.

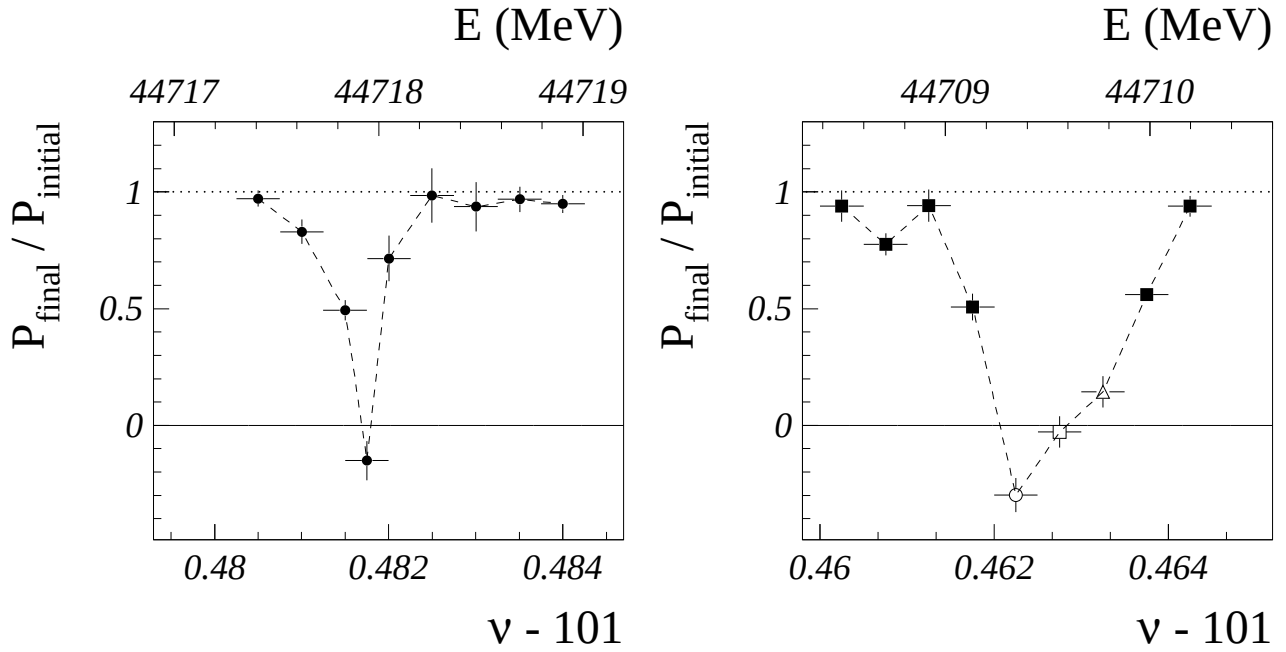


Figure 4.16: Resonant depolarisation of transverse beam polarisation at LEP. The change in transverse polarisation at a beam energy of 44.7 GeV, corresponding to $n_s = 101$, is shown when sweeping through the depolarising resonance [145]. The calculated beam energies corresponding to ν_s are also shown.

yielding f_{dep}^{res} . The condition for depolarisation to occur reads:

$$f_{dep}^{res} = (k \pm \delta_s) f_e, \quad (4.11)$$

where k is an arbitrary integer. The frequencies f_{dep}^{res} used at LEP correspond to the cases $k = 0$ with the frequency $f_{dep}^{res} = \delta_s f_e$, or $k = +1$ with the mirror frequency $f_{dep}^{res} = (1 - \delta_s) f_e$. A measurement of the depolarising frequency at the resonance, f_{dep}^{res} , yields the fractional part of the spin tune, thus determining the average beam energy around the LEP ring:

$$E_e = \frac{m_e}{a_e} \nu_s = 0.4406486(1) \cdot \left(n_s + \frac{f_{dep}^{res}}{f_e} \right) \text{ GeV}. \quad (4.12)$$

A measurement of δ_s to an accuracy of $0.5 \cdot 10^{-3}$ determines E_e to an accuracy of $0.5 \cdot 10^{-5}$ or 0.22 MeV. The width of the depolarisation resonance in spin tune and beam energy is shown in Figure 4.16 for two depolarisation measurements. Intrinsic accuracies of 0.2 MeV and less are achieved.

The beam polarisation P_e itself is measured [155] by a polarimeter [156] based on spin-dependent Compton scattering of circularly polarised photons off the polarised electron beam [157]. The angular distribution of the backscattered photons is measured far downstream, where the beams following their circular orbit are bent away, in terms of the transverse distance Y_γ from the extrapolated electron beam line. The mean transverse position depends on the product of laser polarisation, P_γ , and beam polarisation, P_e . The shift Δ in the mean position $\langle Y_\gamma \rangle$ is determined when reversing the laser polarisation:

$$\Delta \langle Y_\gamma \rangle = \kappa P_\gamma P_e, \quad (4.13)$$

where $\kappa = (500 \pm 30) \mu\text{m}$ is the analysing power of the polarimeter [155]. The idea is similar to the polarisation measurement at SLC discussed before except that the scattered photons are detected, and not the scattered electrons. However, the requirement in accuracy is much less stringent since no absolute polarisation measurement is needed for the purpose of the energy calibration.

4.6.1.3 Energy Model for 1990 to 1992

For the energy scans in 1990 and 1991 [146, 147, 149] and the data taken in 1992 [148], the average LEP beam energy for each energy point i and fill f is derived from the magnetic field measured in a reference dipole which is powered in series with the LEP dipoles. The field measurement determines the so-called field-display energy, $E_{FD}(i, f)$, to which corrections are applied in order to determine the centre-of-mass energy at the interaction points:

$$E_{CM}(i, f) = 2E_{FD}(i, f) \left[1 + a_{abs} + \alpha \frac{2E_{FD}(i, f) - 93 \text{ GeV}}{2E_{FD}(i, f)} + C_{temp}(T(i, f) - \bar{T}) + \Delta E_{RF}(IP, i, f) \right]. \quad (4.14)$$

The individual corrections are described in the following:

1. The absolute energy scale is determined by calibrating the field-display energy against the result from the method of resonant depolarisation, which is performed at the $P + 2$ energy point at $\sqrt{s} = 93 \text{ GeV}$. The resulting relative offset is $a_{abs} = (-73.0 \pm 5.7) \cdot 10^{-5}$.
2. The correction of the local energy scale is determined from flux-loop measurements. It is linearised and normalised to the $P + 2$ calibration point at 93 GeV. The linear correction is $\alpha = (-2.0 \pm 1.5) \cdot 10^{-3}$.
3. A linear temperature correction is applied on a fill-by-fill basis. The temperature coefficient C_{temp} is obtained from flux-loop measurements and dedicated laboratory experiments, $C_{temp} = (1.00 \pm 0.25) \cdot 10^{-4} K^{-1}$.
4. Interaction-point specific corrections, $\Delta E_{RF}(IP, i, f)$, need to be taken into account. They arise from the asymmetric distribution of the RF cavities around the LEP ring and alignment errors of these RF cavities. In addition, these corrections are time dependent as they vary according to the high-voltage and power status of the accelerating RF cavities.

Typical accuracies on $\sqrt{s}$ of 25 MeV in 1990 and of 5 MeV in 1991 are obtained.

4.6.1.4 Energy Model for 1993 to 1995

For the precise energy scans of 1993 and 1995 and the data taken in 1994, measurements of resonant depolarisation are performed at all three energy points of each scan. The energy model is much refined to take additional effects such as earth tides and parasitic currents on the beam pipe created by electrical trains running nearby into account [150]. The latter effect, discovered for the 1995 data taking period, invalidated the LEP energy calibration for 1993 already published before [158]. The 1993 and 1994 energies are retroactively corrected for this effect [150].

The LEP beam energy averaged over the interaction points is calculated every 15 minutes according to the formula:

$$E_e(t) = E_{norm}(f) \cdot [1 + C_{rise}(t)] \cdot [1 + C_{temp}(t)] \cdot [1 + C_{tide}(t)] \cdot [1 + C_{orbit}(f)] \cdot [1 + C_{corr}(t)] \cdot [1 + C_{QFQD}(t)], \quad (4.15)$$

where the energy used for normalisation, E_{norm} , is either given by the resonant depolarisation result if the fill f was calibrated, or by the mean of the calibrations performed for the fills of this energy point if the fill f was not calibrated. The individual corrections are described in the following:

1. C_{rise} accounts for the change in the bending field due to parasitic currents flowing along the beam pipe;

2. C_{temp} accounts for the change in the field of the dipole magnets due to temperature changes;
3. C_{tide} accounts for the effect of earth tides changing the circumference of the LEP ring;
4. C_{orbit} accounts for the deviation of the horizontal orbit position from a central orbit with no quadrupole bending component;
5. C_{corr} accounts for different settings of the horizontal corrector dipoles between physics mode and energy calibration mode, which affects the orbit and thus the field integral seen by the beams;
6. C_{QFQD} accounts for the magnetic field induced by the current imbalance of the focusing quadrupoles, QF , and defocusing quadrupoles, QD , which is due to the different phase advance in the vertical and horizontal plane.

4.6.1.5 Centre-of-Mass Energy

The LEP centre-of-mass energy for an individual experiment is then calculated by applying corrections specific to the interaction point:

$$E_{CM}(t) = 2E_e(t) + \Delta E_{RF}(t) + \Delta E_{disp}(t) + \Delta E_{e+}. \quad (4.16)$$

The correction $\Delta E_{RF}(t)$ describes interaction point specific corrections due to the status of the RF system. The bulk of the effect is given by the asymmetric distribution of RF power along the LEP ring as shown in Figure 4.17. The correction ΔE_{disp} is the interaction point dependent correction in 1995 due to the combined effect of opposite-sign vertical dispersion and vertical beam offsets. The correction ΔE_{e+} accounts for the possible difference in the average energies of positrons and electrons.

The time-dependent centre-of-mass energies are used by the experiments to calculate the luminosity-weighted mean centre-of-mass energies for each energy point. Seven energy points are considered for each LEP experiment, three each for the scans in 1993 and 1995, and the single peak energy of 1994. The errors on the mean centre-of-mass energies are given in terms of a covariance matrix, taking correlations due to the calibration procedure into account. The full covariance matrix has a size of 28 by 28 and takes interaction point specific errors and correlations into account. Assuming equal weight of the measurements of the four experiments, the 28×28 error matrix is simplified to the 7×7 matrix reported in Table 4.4.

Period	93 $P - 2$	93 P	93 $P + 2$	94	95 $P - 2$	95 P	95 $P + 2$
93 $P - 2$	11.71	7.60	6.73	5.06	1.66	1.41	1.45
93 P	7.60	44.81	6.96	5.67	1.30	1.43	1.33
93 $P + 2$	6.73	6.96	8.69	4.67	1.51	1.57	1.76
94 P	5.06	5.67	4.67	13.14	1.51	1.68	1.53
95 $P - 2$	1.66	1.30	1.51	1.51	3.17	1.53	1.49
95 P	1.41	1.43	1.57	1.68	1.53	29.08	1.79
95 $P + 2$	1.45	1.33	1.76	1.53	1.49	1.79	2.83

Table 4.4: Error matrix for the seven LEP-I centre-of-mass energies of 1993 to 1995 in units of MeV^2 [150].

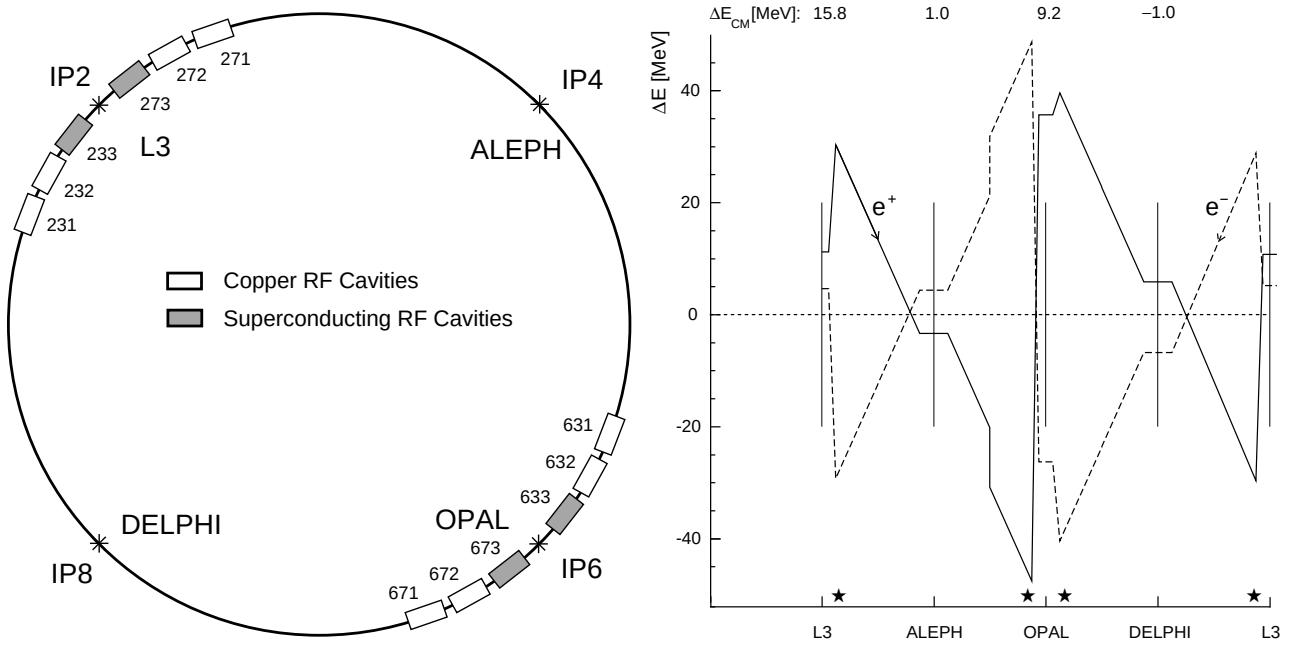


Figure 4.17: Left: Distribution of RF power along the LEP-I ring [150]. Right: Deviation of electron and positron beam energy along the LEP-I ring from the average [150]. The energy sawtooth is caused by the energy loss due to synchrotron radiation, $\Delta E_e \propto E_e^4$, and the asymmetric distribution of the RF power along the LEP-I ring.

4.6.2 Energy Spread

The time-dependence of the centre-of-mass energy causes a spread ω_t of typically 10 MeV to 15 MeV associated to the mean centre-of-mass energy of each energy point. In addition, the intrinsic energy distribution of particles in the LEP beams at any given time is not a δ function but has a finite width. For the 1993 to 1995 data, the spread ω_i typically ranges from 54 MeV to 57 MeV and is known to an accuracy ranging from 1.1 MeV to 1.3 MeV [150]. Therefore, the measured cross sections and asymmetries are the result of a convolution in centre-of-mass energy. The effect of the convolution on the measured quantities is calculated assuming a Gaussian centre-of-mass energy distribution with a width of $\omega_E = \sqrt{\omega_t^2 + \omega_i^2}$.

4.6.2.1 Effect on Cross Sections

For cross sections, the convolution is given by:

$$\sigma_{\text{measured}}(E_{CM}) = \frac{1}{\sqrt{2\pi}\omega_E} \int_{-\infty}^{+\infty} dE \exp \left[-\frac{1}{2} \left(\frac{E - E_{CM}}{\omega_E} \right)^2 \right] \sigma(E) \quad (4.17)$$

A Taylor expansion of the cross section, $\sigma(E)$, around the mean centre-of-mass energy, E_{CM} , yields the effective shift in the cross section, $\Delta\sigma$, due to the energy spread ω_E of:

$$\Delta\sigma \equiv \sigma(E_{CM}) - \sigma_{\text{measured}}(E_{CM}) = -\frac{d^2\sigma}{dE^2} \frac{\omega_E^2}{2}, \quad (4.18)$$

which depends on the second derivative of the cross section with respect to the centre-of-mass energy. The effect is large at the pole where the curvature of the cross section is largest. While the observed pole cross section is reduced, the observed cross sections at the wings of the Z resonance are increased. Thus the energy spread affects in particular the measurement of the total width of the Z boson.

4.6.2.2 Effect on Asymmetries

The effect on asymmetries is calculated by performing the above convolution on the forward and the backward cross section separately, and dividing the convoluted difference by the convoluted total cross section, yielding the result:

$$\Delta A_{\text{fb}} \equiv A_{\text{fb}}(E_{CM}) - A_{\text{fb}}^{\text{measured}}(E_{CM}) = - \left(\frac{d^2 A_{\text{fb}}}{dE^2} + \frac{2}{\sigma} \frac{d\sigma}{dE} \frac{dA_{\text{fb}}}{dE} \right) \frac{\omega_E^2}{2}. \quad (4.19)$$

4.6.2.3 Corrections

An order of magnitude estimate shows the typical size of the corrections:

$$\mathcal{O} \left(\frac{\Delta\sigma}{\sigma} \right) = \mathcal{O}(\Delta A_{\text{fb}}) = \frac{\omega_E^2}{\Gamma_Z^2} \approx 10^{-3}, \quad (4.20)$$

which is smaller than the statistical errors for leptonic final states but larger than the statistical error in the hadronic final state.

The experiments quote their cross sections and forward-backward asymmetries having applied the correction for the energy spread. The corrections are calculated by performing the above convolutions based on the theoretical calculation of cross sections and forward-backward asymmetries. The calculations depend on parameters of the model, such as the mass and the total width of the Z boson, which one wants to measure. However, the uncertainty on the energy spread corrections arising from the variation of the input parameters used in the calculations is negligible. Typical corrections on cross sections and forward-backward asymmetries are reported in Table 4.5. The effect on cross sections is as large as the luminosity theory error, while the effect on asymmetries is negligible.

Quantity	Energy Point		
	$P - 2$	P	$P + 2$
$\frac{1}{\sigma} \frac{d\sigma}{d\sqrt{s}}$ [GeV ⁻¹]	+0.73	+0.06	-0.52
$\frac{1}{\sigma} \frac{d^2\sigma}{d\sqrt{s}^2}$ [GeV ⁻²]	+0.69	-1.09	+0.39
$\frac{dA_{\text{fb}}^\ell}{d\sqrt{s}}$ [GeV ⁻¹]	+0.09	+0.08	+0.04
$\frac{d^2 A_{\text{fb}}^\ell}{d\sqrt{s}^2}$ [GeV ⁻²]	+0.00	-0.02	-0.01
$\frac{\Delta\sigma}{\sigma}$ [10 ⁻³]	-1.1	+1.7	-0.6
$\Delta A_{\text{fb}}^\ell$ [10 ⁻³]	-0.22	+0.02	+0.08

Table 4.5: First and second derivatives of s -channel total cross sections and leptonic forward-backward asymmetries; and fractional changes in cross sections and shifts in leptonic forward-backward asymmetries to be applied to the measurements in order to correct for the effect of the spread in the LEP-I centre-of-mass energy. Numerical values are calculated for the three LEP-I centre-of-mass energies of 1993 to 1995 and a spread of 56 MeV in $\sqrt{s}$. Since the magnitude of the spread correction is quadratic in the spread, the relative error on the correction is twice the relative error on the spread.

4.6.3 Treatment of Energy Errors

The error on the mean centre-of-mass energy and the error on the energy spread induce additional uncertainties on both the theoretical calculation of cross section and asymmetry predictions and the

experimentally measured cross sections. The theoretical cross sections and asymmetries are calculated as a function of the centre-of-mass energy, and are convolutions as discussed above. The measured cross section also depends on the measured luminosity. The measured luminosity in turn depends on the theoretical cross section for small-angle Bhabha scattering, σ_e , which is calculated as a function of the centre-of-mass energy. In leading order, the following uncertainties on cross sections and asymmetries are obtained:

$$\delta\sigma_{theo} = \frac{d\sigma_{theo}}{dE}\delta E \oplus \frac{d^2\sigma_{theo}}{dE^2}\frac{\delta\omega_E^2}{2} \quad (4.21)$$

$$\delta\sigma_{exp} = \frac{d\sigma_{exp}}{d\mathcal{L}}\left(\frac{d\mathcal{L}}{dE}\delta E \oplus \frac{d^2\mathcal{L}}{dE^2}\frac{\delta\omega_E^2}{2}\right) = \frac{\sigma_{exp}}{\sigma_e}\left(\frac{d\sigma_e}{dE}\delta E \oplus \frac{d^2\sigma_e}{dE^2}\frac{\delta\omega_E^2}{2}\right) \quad (4.22)$$

$$\delta A_{fb}^{theo} = \frac{dA_{fb}^{theo}}{dE}\delta E \oplus \left(\frac{d^2A_{fb}^{theo}}{dE^2} + \frac{2}{\sigma_{theo}}\frac{d\sigma_{theo}}{dE}\frac{dA_{fb}^{theo}}{dE}\right)\frac{\delta\omega_E^2}{2}, \quad (4.23)$$

where $\oplus$ stands for addition in quadrature. Since the uncertainties on the centre-of-mass energy and the uncertainties on the energy spread are uncorrelated, their contributions to the error on the cross section are added in quadrature. The combined uncertainty on the difference of theoretical and measured cross section and asymmetry, which enters the χ^2 of the fitting procedure, is given by:

$$\delta(\sigma_{theo} - \sigma_{exp}) = \left[\frac{d\sigma_{theo}}{dE} - \frac{\sigma_{exp}}{\sigma_e}\frac{d\sigma_e}{dE}\right]\delta E \oplus \left[\frac{d^2\sigma_{theo}}{dE^2} - \frac{\sigma_{exp}}{\sigma_e}\frac{d^2\sigma_e}{dE^2}\right]\frac{\delta\omega_E^2}{2} \quad (4.24)$$

$$\delta(A_{fb}^{theo} - A_{fb}^{exp}) = \delta A_{fb}^{theo}. \quad (4.25)$$

Assuming a power-law behaviour of the small-angle Bhabha cross section as a function of E , $\sigma_e \propto E^{-k}$, $k \approx 2$, the expression for the cross section difference simplifies to:

$$\delta(\sigma_{theo} - \sigma_{exp}) = \left[\frac{d\sigma_{theo}}{dE} + k\frac{\sigma_{exp}}{E}\right]\delta E \oplus \left[\frac{d^2\sigma_{theo}}{dE^2} - k(k+1)\frac{\sigma_{exp}}{E^2}\right]\frac{\delta\omega_E^2}{2}. \quad (4.26)$$

An order of magnitude estimate shows the relative importance of the δE induced errors:

$$\mathcal{O}\left(\frac{1}{\sigma}\frac{d\sigma}{dE}\delta E\right) = \mathcal{O}\left(\frac{dA_{fb}}{dE}\delta E\right) = \frac{\delta E}{\Gamma_Z} \approx 10^{-3} \quad (4.27)$$

$$\mathcal{O}\left(\frac{k}{E}\delta E\right) = \frac{\delta E}{M_Z} \approx 10^{-5}, \quad (4.28)$$

and $\delta\omega_E^2$ induced errors:

$$\mathcal{O}\left(\frac{1}{\sigma}\frac{d^2\sigma}{dE^2}\frac{\delta\omega_E^2}{2}\right) = \mathcal{O}\left(\left(\frac{d^2A_{fb}}{dE^2} + \frac{2}{\sigma}\frac{d\sigma}{dE}\frac{dA_{fb}}{dE}\right)\frac{\delta\omega_E^2}{2}\right) = \frac{\delta\omega_E^2}{\Gamma_Z^2} \approx 10^{-5} \quad (4.29)$$

$$\mathcal{O}\left(\frac{k(k+1)}{E^2}\frac{\delta\omega_E^2}{2}\right) = \frac{\delta\omega_E^2}{M_Z^2} \approx 10^{-8}. \quad (4.30)$$

In both cases, the uncertainties entering through the luminosity measurement are negligible. Note that the errors induced by δE and the errors induced by $\delta\omega_E$ are fully correlated between all cross section and asymmetry measurements, all energy points, and all four LEP experiments.

4.6.4 Uncertainties on Mass and Width of the Z Boson

When extracting Z-boson parameters from measured cross sections and asymmetries, the energy errors are propagated and thus taken into account. A simplified analysis [150] allows to estimate the size of the errors on mass and total width induced by the uncertainties on the LEP energy calibration. These two parameters are determined essentially by the high-precision scans of the Z lineshape in 1993 and 1995, where luminosity was recorded at the peak, P , and approximately 1.8 GeV below and above the

peak, denoted as $P \pm 2$. Equal weight of the four LEP experiments and of the two scans in 1993 and 1995 are assumed.

The mass of the M_Z boson is determined by the position of the Z resonance curve. This position is fixed by the average of the two off-peak centre-of-mass energies. The error on the Z mass arising from the LEP energy calibration is therefore determined by the error on the average energy of the two off-peak energy points.

$$\delta M_Z = \frac{1}{2} \delta (E_{P+2} + E_{P-2}) \quad (4.31)$$

$$= 1.9 \text{ MeV} . \quad (4.32)$$

The total decay width of the Z boson is given by the width of the Z resonance curve relative to its height. The measurement of the width is fixed by the difference in the off-peak centre-of-mass energies. The relative error on the total width arising from the LEP energy calibration is therefore determined by the relative error on the energy difference between the two off-peak energy points:

$$\delta \Gamma_Z = \Gamma_Z \cdot \frac{\delta (E_{P+2} - E_{P-2})}{E_{P+2} - E_{P-2}} \quad (4.33)$$

$$= 1.2 \text{ MeV} . \quad (4.34)$$

The 1 MeV uncertainty on the centre-of-mass energy spread contributes a negligible 0.2 MeV uncertainty on Γ_Z .

4.6.5 Energy Calibration above the Z pole

For LEP-II running above the Z pole, additional superconducting RF cavities, in total 288 by the year 1999, are installed around each LEP experiment. The energy sawtooth thus halves its period, as shown in Figure 4.18. Nevertheless the amplitude is much larger, as the energy loss due to synchrotron radiation increases with the fourth power of the beam energy, $\Delta E_e \propto E_e^4$.

Even though the rate of radiative selfpolarisation, $1/\tau_p$, increases with E_e^5 , the rate of depolarisation, $1/\tau_d$, due to resonances driven by machine imperfections increase even faster. A simple model predicts $\tau_p/\tau_d \propto E_e^2$ [63]. Since the beam energy spread is increased by a factor of four at LEP-II energies, $\omega_E \propto E_e^2$, depolarising resonances nearby in phase space cannot be avoided. Levels of transverse polarisation useful for beam energy measurements are limited to a maximal beam energy about 60 GeV, even under special running conditions.

For this reason, extrapolation based on the magnetic field strength of the LEP dipoles is required to move from calibration energy up to the beam energy of physics collisions in the range of 80 GeV to 100 GeV. In order to increase the lever arm on which the extrapolation is based, and to test the linearity of the magnetic measurements, the energy is calibrated by the method of resonant depolarisation at several beam energy points between 40 GeV and 55 GeV [151–153]. Nevertheless, the error on the LEP-II beam energies, reported in Table 4.6, is by far dominated by the extrapolation error. The accuracy at 183 GeV is improved since several polarisation measurements have been made during data taking. For the 189 GeV data taken in 1998, resonant depolarisations were also achieved at a beam energy of 61 GeV.

The expected statistical accuracy on the determination of the W-boson mass at LEP-II is 25 MeV combining all four experiments. Since the error on the beam energy propagates directly to the mass, $\delta M_W/M_W = \delta E_e/E_e$, the beam energy needs to be measured with an accuracy of 10 MeV to 15 MeV for the bulk of the luminosity collected at LEP-II.

In order to achieve this goal, a new precise magnetic spectrometer will be installed in time for data taking in 1999 and 2000 [152], with the intention to propagate improvements in the beam energy determination back to previous years. The angular deflection of the LEP beams in a dipole magnet with precisely mapped field will be measured. The resonant depolarisation technique will be used at low energies to calibrate the angular deflection, and at intermediate energies to test this calibration.

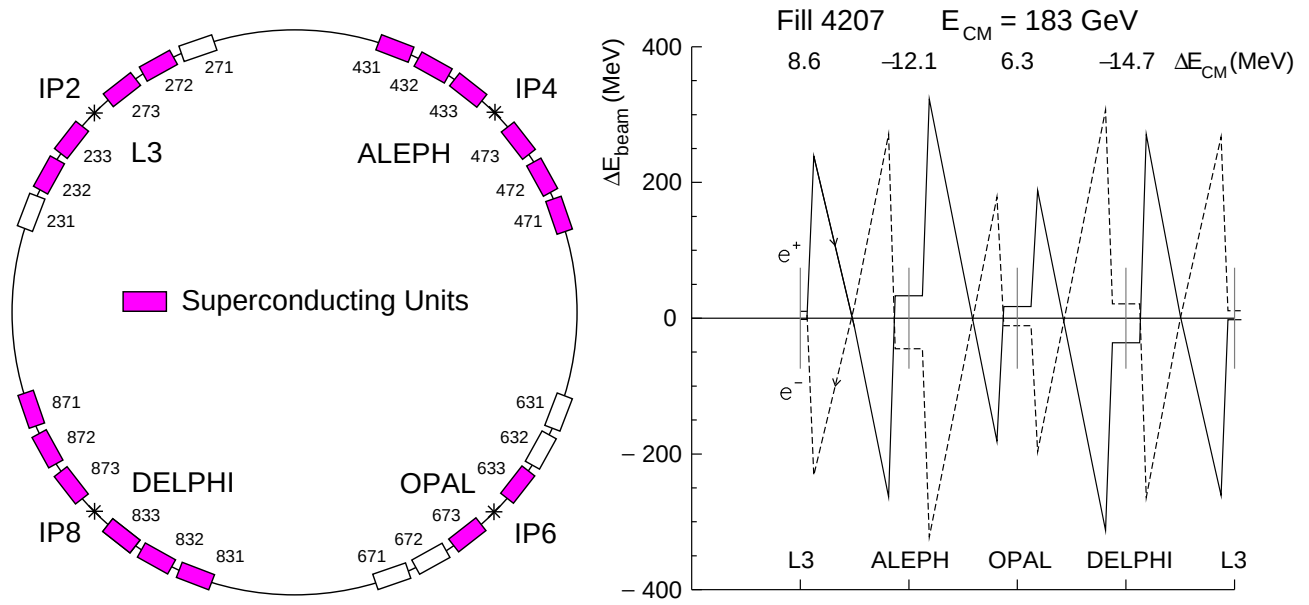


Figure 4.18: Left: Distribution of RF power along the LEP-II ring [152]. Right: Deviation of electron and positron beam energy along the LEP-II ring from the average [151–153]. The situation is shown for 1997 and $\sqrt{s} = 183$ GeV. The energy sawtooth is caused by the energy loss due to synchrotron radiation, $\Delta E_e \propto E_e^4$, and the distribution of the RF power along the LEP-II ring.

The goal is to measure the change in angular deflection at different energies to an anticipated accuracy of 1 in 10^4 , corresponding to an uncertainty of 10 MeV on the final LEP-II beam energy of 100 GeV.

In contrast to the method of resonant depolarisation, this method determines the beam energy at one point along the LEP ring rather than the average energy. An extrapolation to the interaction points is required across the large amplitudes of the energy sawtooth shown in Figure 4.18.

At LEP-II, the centre-of-mass energy spread, ω_E , is about a factor of four larger than at LEP-I, since it grows quadratically with the energy, $\omega_E \propto s$. The actual values, also depending on the machine optics, are reported in Table 4.6. The effect of the energy spread on cross sections is negligible, since in the LEP-II energy range the cross sections do not show sharp structures with high curvatures and are measured with larger statistical errors. In case the mass of the W boson is reconstructed based on beam energy constraints or four-momentum conservation, it is smeared by an amount of $\delta M_W/M_W = \omega_E/\sqrt{s}$. The effect on the mass and width of the W boson extracted from the invariant mass distribution is negligible, since the additional smearing enters in quadrature to the effects of detector resolution and width itself.

year	$\sqrt{s}$ [GeV]	δE_e [MeV]	ω_E [MeV]
1996	161	27	144 ± 7
1996	172	30	165 ± 8
1997	183	25	219 ± 11

Table 4.6: Uncertainties in the determination of the LEP-II beam energy E_e [151, 152], and spread in centre-of-mass energy $\sqrt{s}$ [153]. The uncertainty of 25 MeV on the beam energy for the 1997 data is common to the 1996 data. The spread in beam energy is a factor of $\sqrt{2}$ smaller than the spread in $\sqrt{s}$ given above.

Chapter 5

Measurements and Results

The experimental measurements used for the determination of electroweak parameters and to test the Standard Model are summarised in this chapter. Nearly all results are preliminary, and sometimes not yet based on the complete data sample collected.

Many results are derived from measurements of fermion-pair production in e^+e^- interactions at the Z pole. These measurements are performed by experiments at the e^+e^- colliders SLC and LEP.

Because of the much larger luminosity available at LEP, the SLD experiment concentrates on exploiting the uniqueness of longitudinally polarised beams at SLC in order to measure polarised asymmetries. In addition, due to the small SLC beam pipe and powerful SLD micro vertex pixel detector, identification of the heavy b quarks and c quarks is performed with higher tagging efficiency at SLD, so that the resulting higher sensitivity compensates for the lower luminosity. The measurements of SLD are performed at a fixed centre-of-mass energy of 91.26 GeV, corresponding to the maximum of the Z pole annihilation cross section.

The dependence of e^+e^- cross sections and asymmetries on the centre-of-mass energy is investigated at LEP. During the LEP-I program from 1989 until 1995, e^+e^- collisions were recorded at centre-of-mass energies in a window of about ± 4 GeV around M_Z . These measurements determine the mass and total decay width of the Z boson. For the LEP-II program lasting from 1996 until the year 2000, the centre-of-mass energy is increased from 161 GeV up to 200 GeV, allowing the pair production of W^+W^- bosons. At LEP-II, the properties of W bosons are measured, in particular its mass, total decay width, decay branching fractions and gauge couplings.

In the past, properties of the W boson were measured at hadron colliders only, first by the SPS experiments UA1 and UA2 at CERN, where the W and Z bosons were discovered in 1983, and nowadays at the TEVATRON collider at Fermilab, Chicago. In 1994 and 1995, the TEVATRON experiments CDF and DØ have discovered the top quark. Until the end of run I in 1996, both experiments have subsequently measured its production cross section, mass and other properties. Data taking at the TEVATRON will recommence with run II in the year 2000.

The analysis of neutrino-nucleon interactions yields a determination of the on-shell electroweak mixing angle, *i.e.*, the mass ratio of W and Z boson, since radiative corrections are small. Especially the new result from the CCFR successor experiment NUTEV, which completed data taking at Fermilab in the fall of 1997, is very precise due to the advantage of taking data with both a neutrino and an anti-neutrino beam.

The measurements are compared to the expectations within the Minimal Standard Model, calculated for $1/\alpha_{em}^{(5)}(M_Z^2) = 128.878 \pm 0.090$, $\alpha_S(M_Z^2) = 0.119 \pm 0.002$, $M_Z = 91186.7 \pm 2.1$ MeV, $M_t = 173.8 \pm 5.0$ GeV and $M_H = 300_{-210}^{+700}$ GeV, where these parametric uncertainties are propagated to the calculated observables. The Standard Model calculations are performed with the semianalytical programs TOPAZ0 and ZFITTER described in Section 3.3.

5.1 Z Lineshape and Leptonic Forward-Backward Asymmetries

The reaction $e^+e^- \rightarrow f\bar{f}(\gamma)$, where the (γ) denotes the possible presence of radiative photons, is measured for all charged fermion-pair final states. For each final state, selection criteria are optimised to obtain the signal with high efficiency and low background. The leptonic final states, $e^+e^-(\gamma)$, $\mu^+\mu^-(\gamma)$ and $\tau^+\tau^-(\gamma)$, are easily distinguishable from another and from the inclusive hadronic events, $e^+e^- \rightarrow \text{hadrons}(\gamma)$, for which the five quark flavours $q = d, u, s, c, b$ produced at LEP are summed over. The sum is considered, because quarks manifest themselves as jets, which are collimated groups consisting of many neutral and charged hadrons, so that the flavour of the primary quark is not easily accessible.

Total cross sections and forward-backward asymmetries are measured at each centre-of-mass energy. Each experiment selects about $4 \cdot 10^6$ hadronic and $4 \cdot 10^5$ lepton-pair events at LEP-I in total, leading to small statistical errors of 0.1% and less. Therefore, systematic errors and their correlations must carefully be investigated in order to match the statistical precision. At LEP-II, the data statistics is three orders of magnitude smaller due to the lower cross sections at the higher LEP-II centre-of-mass energies. The event samples separate into two classes: events where the effective centre-of-mass energy of the hard interaction is close to the nominal centre-of-mass energy, $s' \rightarrow s$, and events where an initial-state radiative photon lowers the effective centre-of-mass energy close to that of the Z mass, $s' \rightarrow M_Z^2$, denoted as radiative return to the Z.

5.1.1 Selection and Total Cross Sections

Total cross sections of a given signal process are measured by counting selected signal events, N_{sel} , subtracting the expected number of background events arising from other processes, N_{bg} , and dividing this difference by the selection efficiency to select the signal, ϵ , and the integrated luminosity, $\mathcal{L}$:

$$\sigma(s) = \frac{N_{sel}(s) - N_{bg}(s)}{\epsilon(s) \cdot \mathcal{L}(s)}. \quad (5.1)$$

Monte-Carlo simulations of signal and background processes are used to determine the signal efficiency, ϵ , and the number of expected background events, N_{bg} . The latter is a sum over all contributing background processes $i = 1, n$:

$$N_{bg} = \sum_{i=1}^n \epsilon_i \cdot \sigma(i) \cdot \mathcal{L}, \quad (5.2)$$

with cross section $\sigma(i)$. The efficiency to select background process i is denoted by ϵ_i . Efficiencies are determined by forming the ratio:

$$\epsilon = \frac{N_{sel}^{MC}}{N_{gen}^{MC}}, \quad (5.3)$$

between the number of selected and the number of generated Monte Carlo events. Besides the signal efficiency, the selection is characterised by the purity, defined as the ratio between accepted signal cross section and total accepted cross section:

$$p = \frac{\epsilon \cdot \sigma}{\epsilon \cdot \sigma + \sum_{i=1}^n \epsilon_i \cdot \sigma(i)}. \quad (5.4)$$

Both efficiency and purity change with changing selection criteria. The product of efficiency and purity must be maximised in order to obtain the cross section measurement with the smallest statistical error.

The selection efficiency ϵ is determined relative to the region of phase-space for which the sample of Monte Carlo events is generated and the Monte Carlo cross section $\sigma(i)$ is quoted. For most $e^+e^- \rightarrow f\bar{f}(\gamma)$ reactions, the generated phase-space covers nearly the full phase space except for some

very loose requirements on the minimal fermion-antifermion energies or invariant masses, typically 0.1 GeV to 1 GeV, or $0.1\sqrt{s}$, the former in order to ensure numerical stability and the latter in order to avoid the region of $q\bar{q}$ resonances. The only exception is given by the Bhabha scattering process. Because of the divergent differential cross section for $\theta \rightarrow 0$, a cut on the polar angular range of the final state e^+e^- particles must be imposed in the event generation.

It must be ensured that the selection criteria do not accept events which lie outside the generated phase-space region, as otherwise efficiencies and accepted cross sections are determined incorrectly. The phase-space cuts for which the measured cross section should be quoted may be tighter than those used in the generation of Monte Carlo events. In that case, the denominator in the efficiency ratio is replaced by the number of events generated in the restricted region, and the cross section prediction is scaled by the ratio of the number of events inside the restricted region and the total number of events generated. As required, this leaves the predicted accepted cross section invariant. Migration due to resolution effects may cause events being selected which lie outside that region. This is automatically corrected for by the modified efficiency calculation. As for any background, it is desirable that the fraction of such events is small.

5.1.1.1 Hadron Production

Experimentally, hadronic events, $e^+e^- \rightarrow q\bar{q}(\gamma) \rightarrow \text{hadrons}(\gamma)$, are selected with the highest efficiency and acceptance, between 95% up to more than 99%, owing to the large particle multiplicity in the final state and full centre-of-mass energy observable in the detector. Backgrounds are very small, typically in the order of 0.1%. Examples of hadronic events recorded at LEP are shown in Figure 5.1, and distributions of typical selection variables are shown in Figure 5.2.

The measurement of hadron production constitutes the most precise cross section measurement due to the large number of selected events accompanied by very small systematic errors. The correlated systematic scale uncertainty on the hadronic cross section arises mainly from the systematic error on the luminosity measurement. The remaining systematic error ranges typically from 0.04% to 0.1%. The correlated systematic absolute uncertainty is given by the subtraction of non-resonant background processes such as hadronic two-photon collision processes but also cosmic-ray showers and machine-related beam-gas and beam-wall interactions. Depending on the analysis, non-resonant background contributions ranging from 10 pb to 50 pb with an associated uncertainty between 3 pb and 7 pb are subtracted, to be compared to a maximum signal cross section of 30 nb.

Because of its statistical power, the hadronic channel largely determines the mass and total width of the Z boson. A scale error on the cross sections correlated between energy points causes a negligible error on these two parameters. However, a correlated absolute error on the cross sections directly propagates to the total width, as Γ_Z is measured as the width of the resonance curve relative to its height. For example, a correlated uncertainty of 3 pb causes an error of 0.4 MeV on the total width.

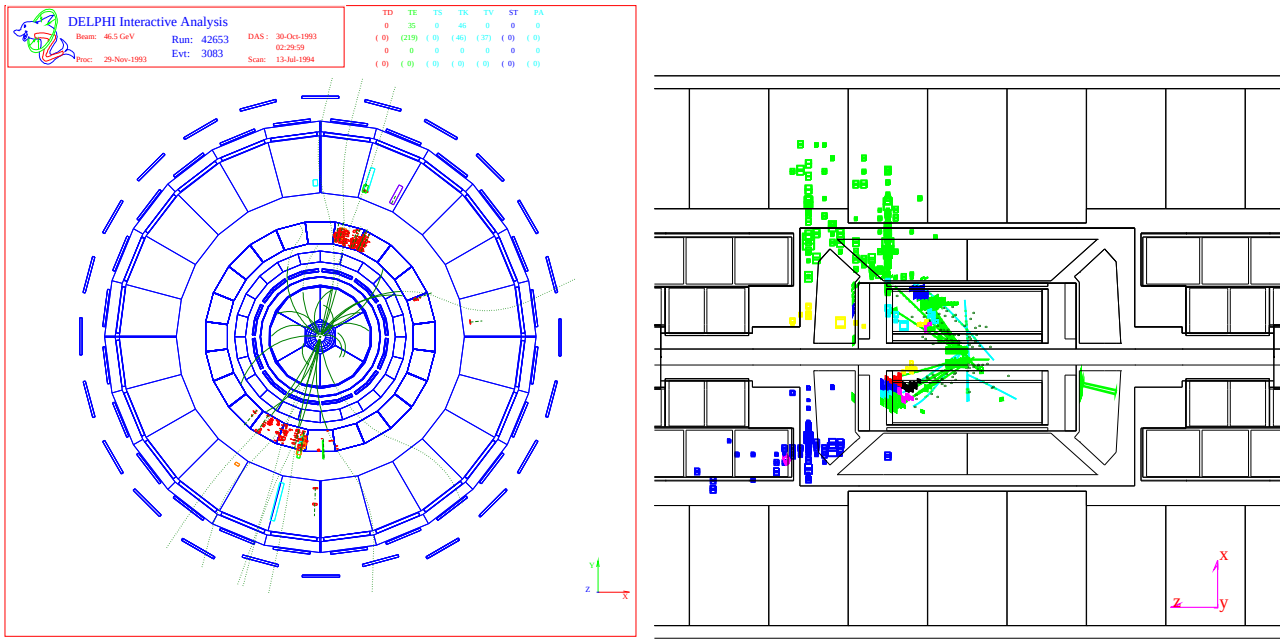


Figure 5.1: Hadronic events recorded at LEP, each showing two jets of tracks in the central tracking chamber and energy depositions in the calorimeters. Left: a LEP-I event recorded by the DELPHI detector. Right: a LEP-II event showing a radiative return to the Z recorded by the L3 detector. The radiative photon is visible in the lower right endcap of the electromagnetic calorimeter.

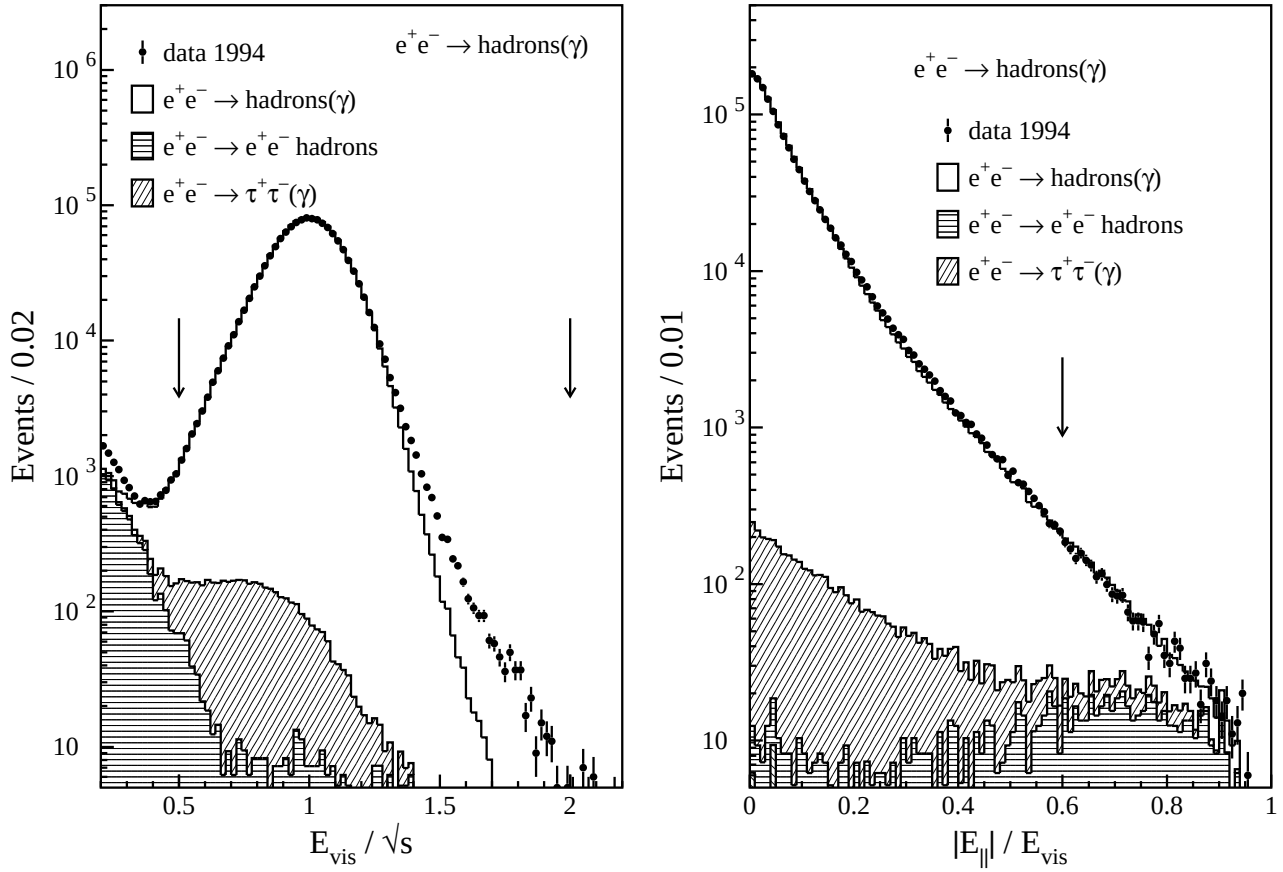


Figure 5.2: Distributions used in the hadron analysis of L3 at $\sqrt{s} = M_Z$ [139]. Left: Visible energy. Right: Longitudinal energy imbalance. The arrows indicate the position of the selection cuts.

5.1.1.2 Lepton Production

The reactions $e^+e^- \rightarrow \mu^+\mu^-(\gamma)$ and $e^+e^- \rightarrow \tau^+\tau^-(\gamma)$ are studied in a polar angular range of typically $|\cos\theta| < 0.7 - 0.9$. The restriction in polar angular range is required by deteriorating particle identification for $|\cos\theta| \rightarrow 1$ and losses in the beam pipe at very low polar angles.

Dimuon events, $e^+e^- \rightarrow \mu^+\mu^-(\gamma)$, recorded at LEP are shown in Figure 5.3. Muons are identified in the outer muon chambers as well as minimum ionising particles traversing the detector from the inner central tracking system through the calorimeters. They are of high energy, up to half the centre-of-mass energy. Distributions of typical selection variables are shown in Figure 5.4. Systematic errors between 0.1% and 0.5% are obtained.

A τ -pair event, $e^+e^- \rightarrow \tau^+\tau^-(\gamma)$, recorded at LEP-I is shown in Figure 5.5. Since τ leptons have such a short lifetime they decay already inside the beam pipe, either into an electron, a muon, or a few light hadrons, accompanied by neutrinos. Since there is no unique event signature for $\tau^+\tau^-(\gamma)$ events, the selection efficiencies and purities are somewhat lower compared to the other charged lepton species. Distributions of typical selection variables are shown in Figure 5.6. Systematic errors between 0.2% and 0.8% are obtained.

The reaction $e^+e^- \rightarrow e^+e^-(\gamma)$ is measured in a reduced fiducial volume, restricting the polar scattering angle to large values, typically $|\cos\theta| < 0.7$. In this phase-space region, the contribution of the interesting s -channel Z exchange diagram to the total cross section is enhanced, as opposed to the small-angle region, $\theta \rightarrow 0$, where the t -channel photon exchange dominates, *c.f.* luminosity measurement discussed in Section 4.5. Bhabha events recorded at LEP are shown in Figure 5.7. Events are characterised by two large energy depositions in the electromagnetic calorimeter, with energies up to half the centre-of-mass energy and electromagnetic shower shape, matched to high-momentum tracks reconstructed in the central tracking system. Distributions of typical selection variables are shown in Figure 5.8. Systematic errors between 0.2% and 0.5% are obtained.

5.1.1.3 Cross Sections at and above the Z pole

The distributions of the reconstructed effective centre-of-mass energy, $\sqrt{s'}$, for data taken well above the Z pole are shown in Figures 5.9 and 5.10. Different methods of reconstructing the effective centre-of-mass energy lead to different $\sqrt{s'}$ distributions, in particular visible for hadronic events.

Two classes of events are clearly visible, the so-called genuine high-energy events at $s' \rightarrow s$, and the events showing a radiative return to the Z at $s' \rightarrow M_Z = 91$ GeV. For Bhabha events, the genuine high-energy events dominate due to t -channel and s/t -interference contributions. A cut on the reconstructed $\sqrt{s'}$, typically $\sqrt{s'} > 0.85\sqrt{s}$, separates the true high-energy events from the radiative return to the Z.

The hadronic and leptonic cross sections measured as a function of the centre-of-mass energy are shown in Figure 5.11 for LEP-I energies close to M_Z . Cross sections at higher centre-of-mass energies, including and excluding the radiative return to the Z, are shown in Figure 5.12.

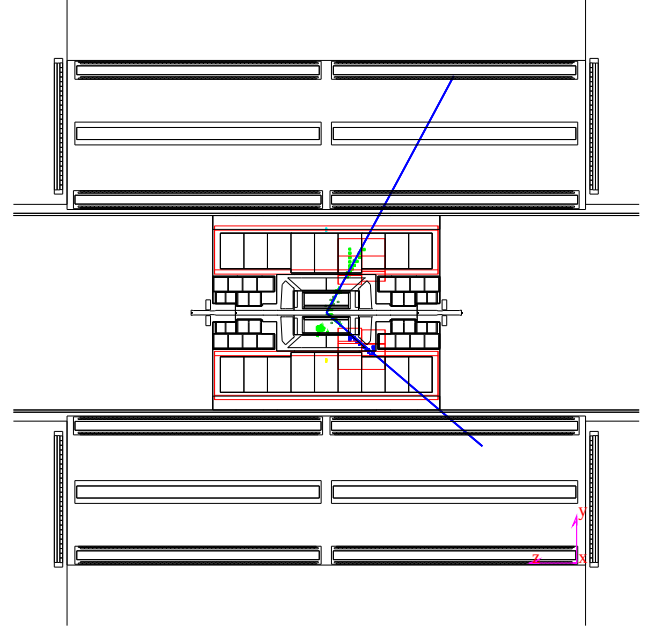
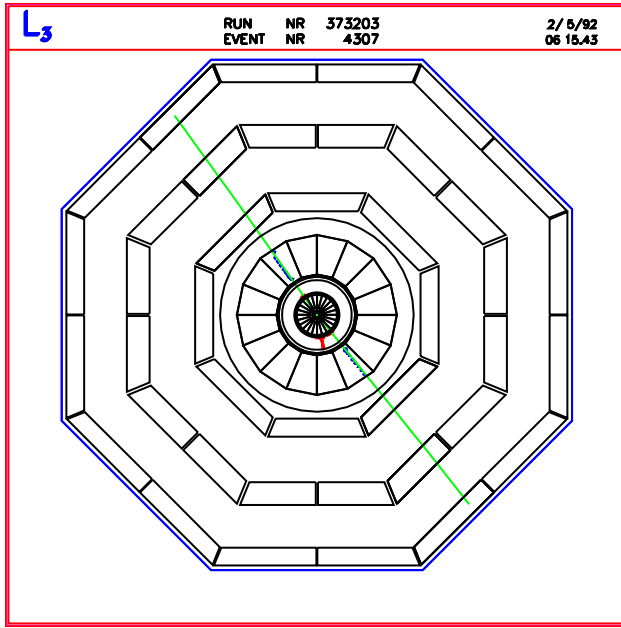


Figure 5.3: Dimuon events recorded at LEP, showing the two muons as minimal ionising tracks traversing the detector. Left: a LEP-I event recorded by the L3 detector. Right: a LEP-II event showing a radiative return to the Z recorded by the L3 detector.

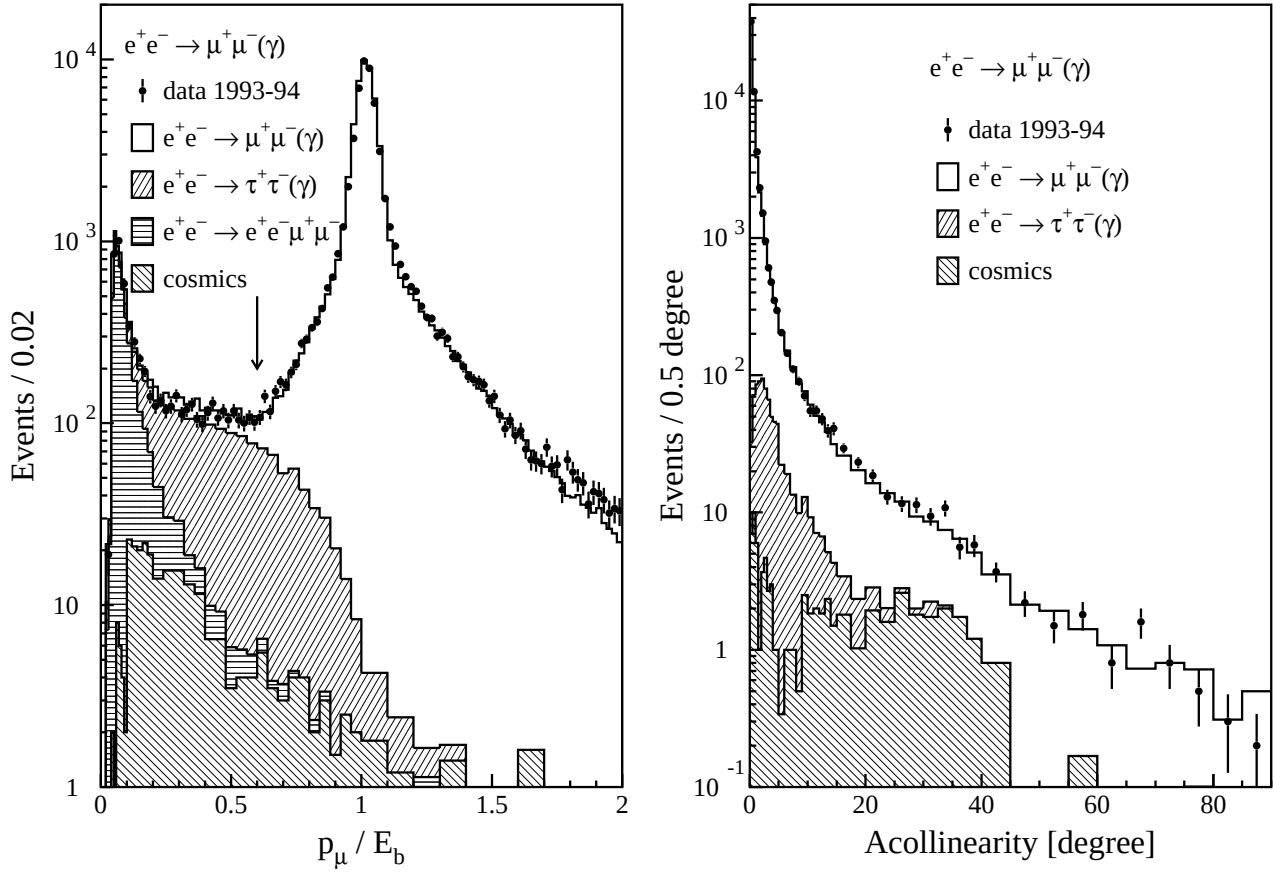


Figure 5.4: Distributions used in the dimuon analysis of L3 at $\sqrt{s} = M_Z$ [139]. Left: Maximal muon momentum. Right: Acollinearity of the two muons.

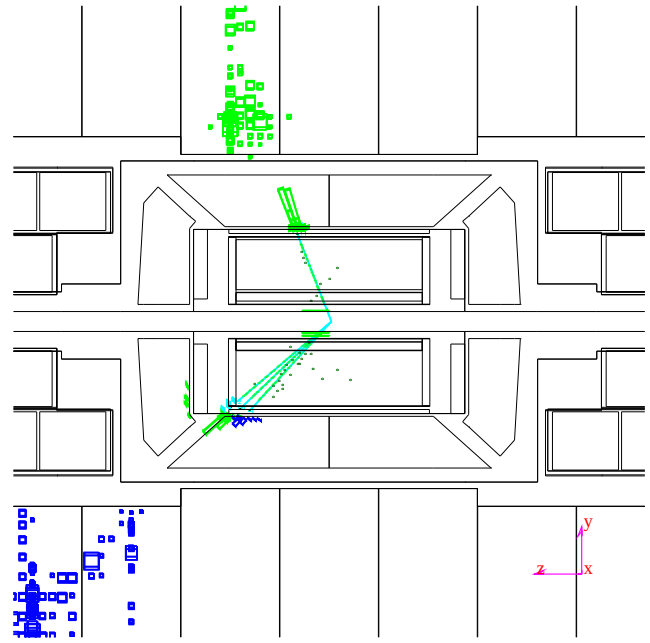
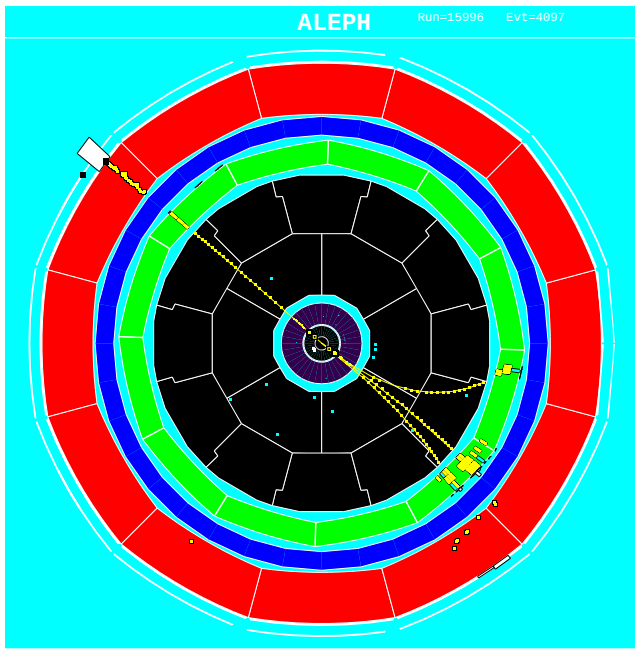


Figure 5.5: Tau-pair events recorded at LEP, each containing a one-prong and a three-prong τ decay. Left: a LEP-I event recorded by the ALEPH detector. Right: a LEP-II event showing a radiative return to the Z recorded by the L3 detector.

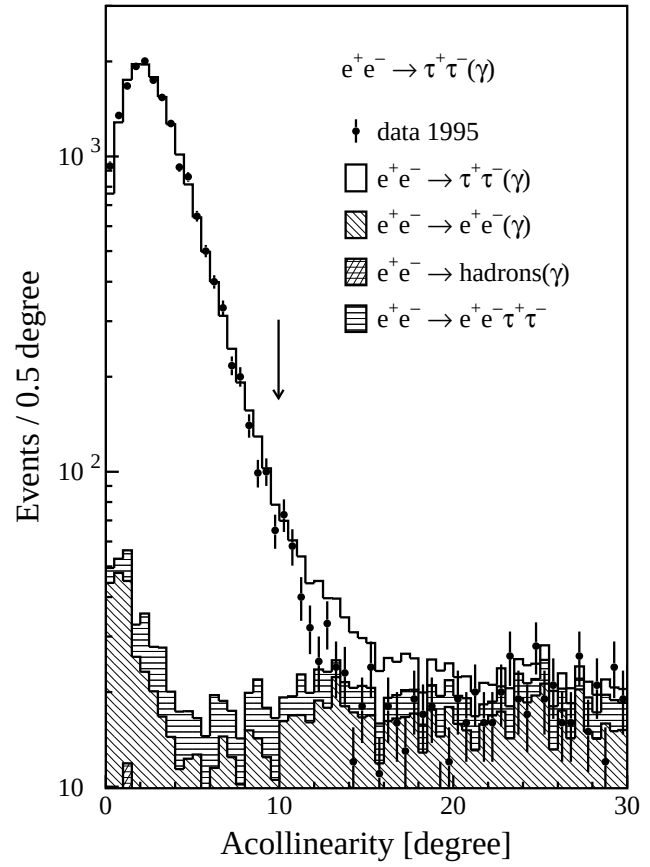
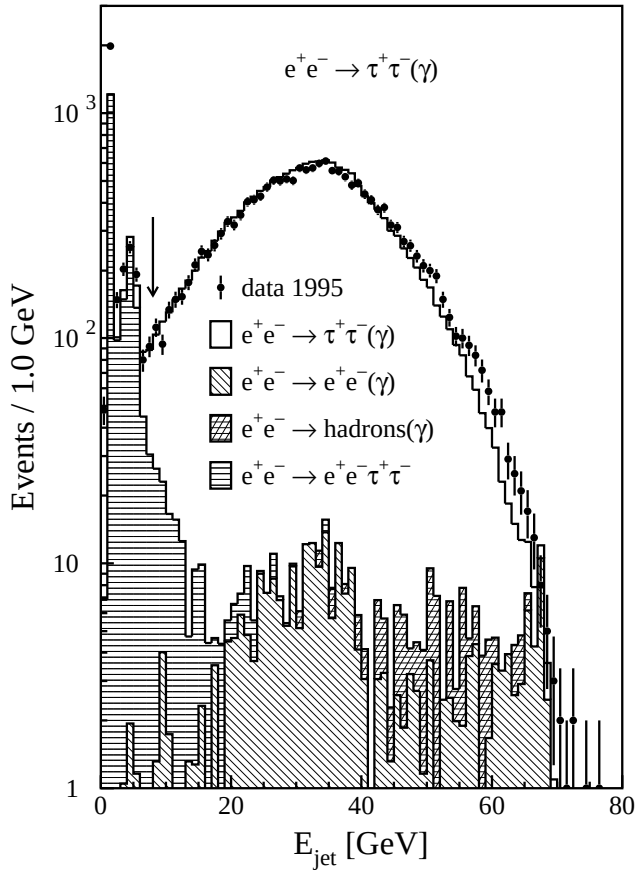


Figure 5.6: Distributions used in the tau-pair analysis of L3 at $\sqrt{s} = M_Z$ [139]. Left: Visible energy of tau jet. Right: Acollinearity of the visible tau jets.

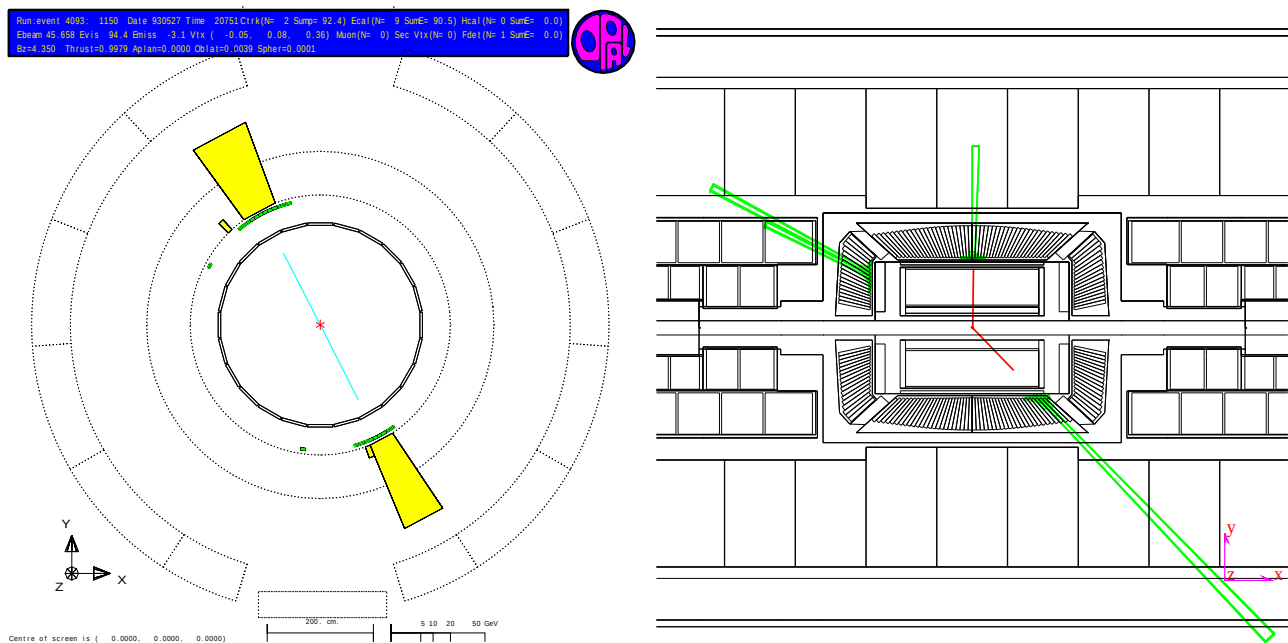


Figure 5.7: Large-angle Bhabha events recorded at LEP, showing two tracks in the central tracking chamber matched with energy depositions in the electromagnetic calorimeter. Left: a LEP-I event recorded by the OPAL detector. Right: a LEP-II event showing a radiative return to the Z recorded by the L3 detector. The radiative photon is visible in the endcap of the electromagnetic calorimeter.

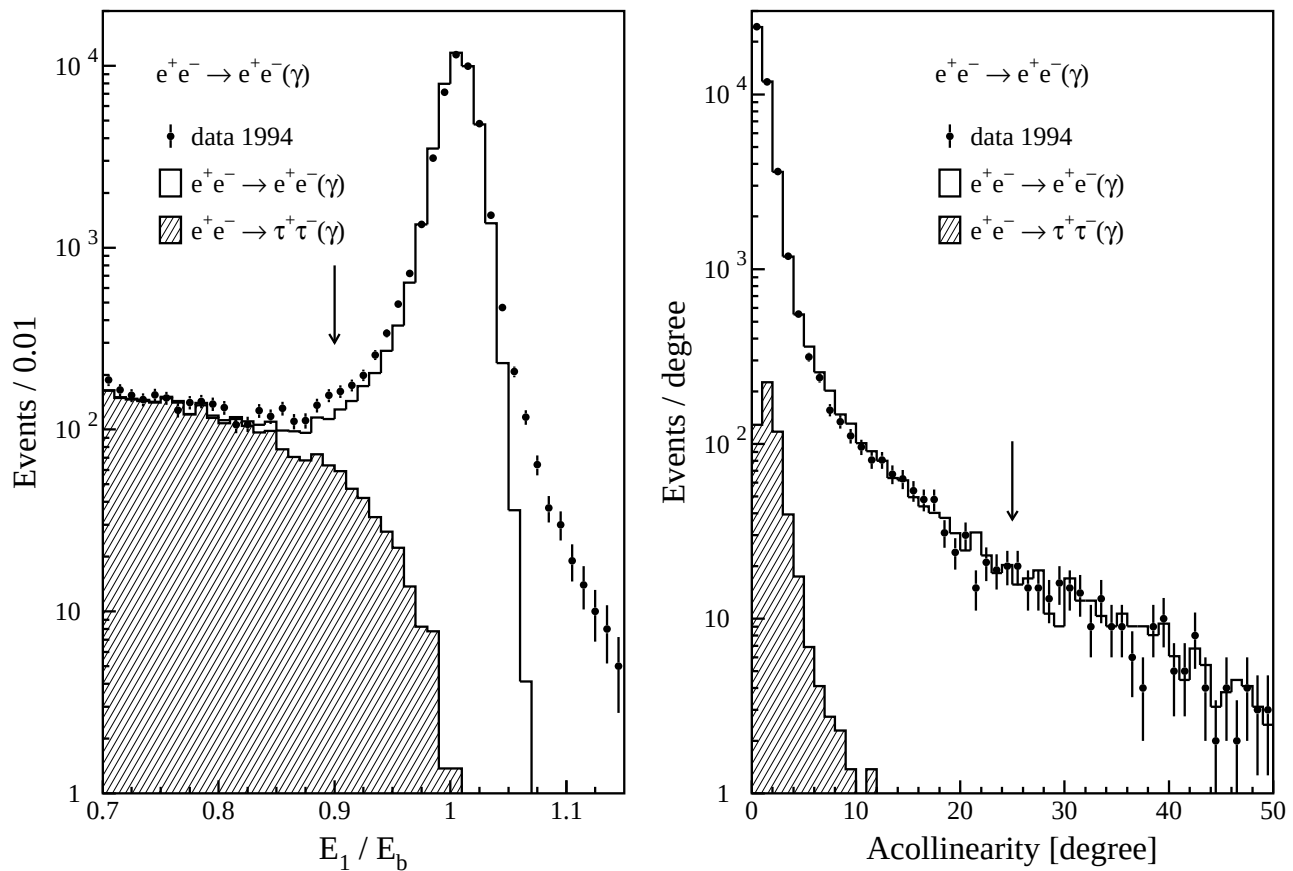


Figure 5.8: Distributions used in the Bhabha analysis of L3 at $\sqrt{s} = M_Z$ [139]. Left: Maximal cluster energy normalised to beam energy. Right: Acollinearity of the two electrons.

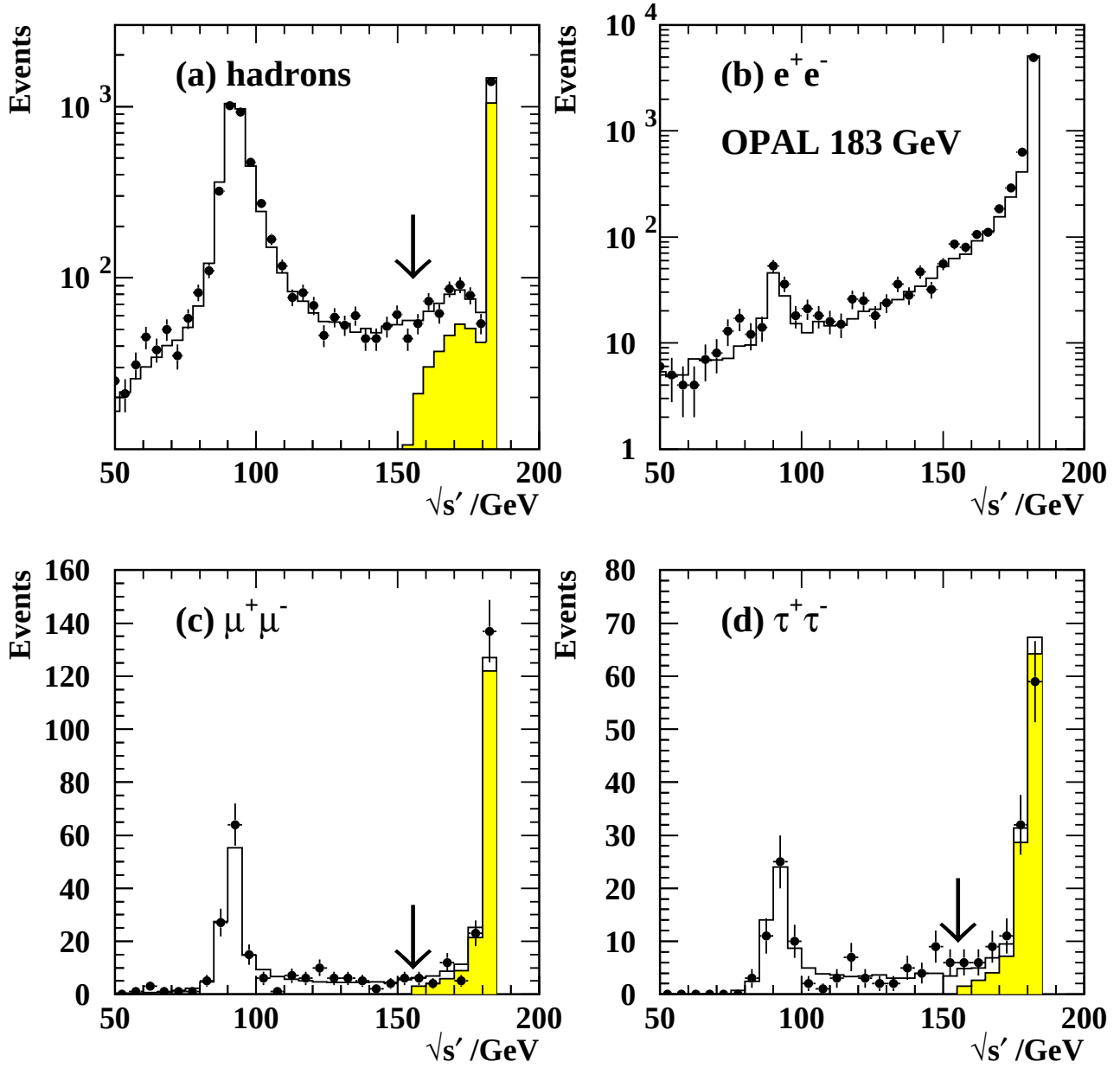


Figure 5.9: Distribution of the reconstructed effective centre-of-mass energy for (a) hadrons, (b) electrons, (c) muons, and (d) taus as measured by OPAL at $\sqrt{s} = 183$ GeV [159]. The shaded part in (a), (c) and (d) shows the Monte Carlo events with generated $\sqrt{s'} > 0.85\sqrt{s} = 155$ GeV.

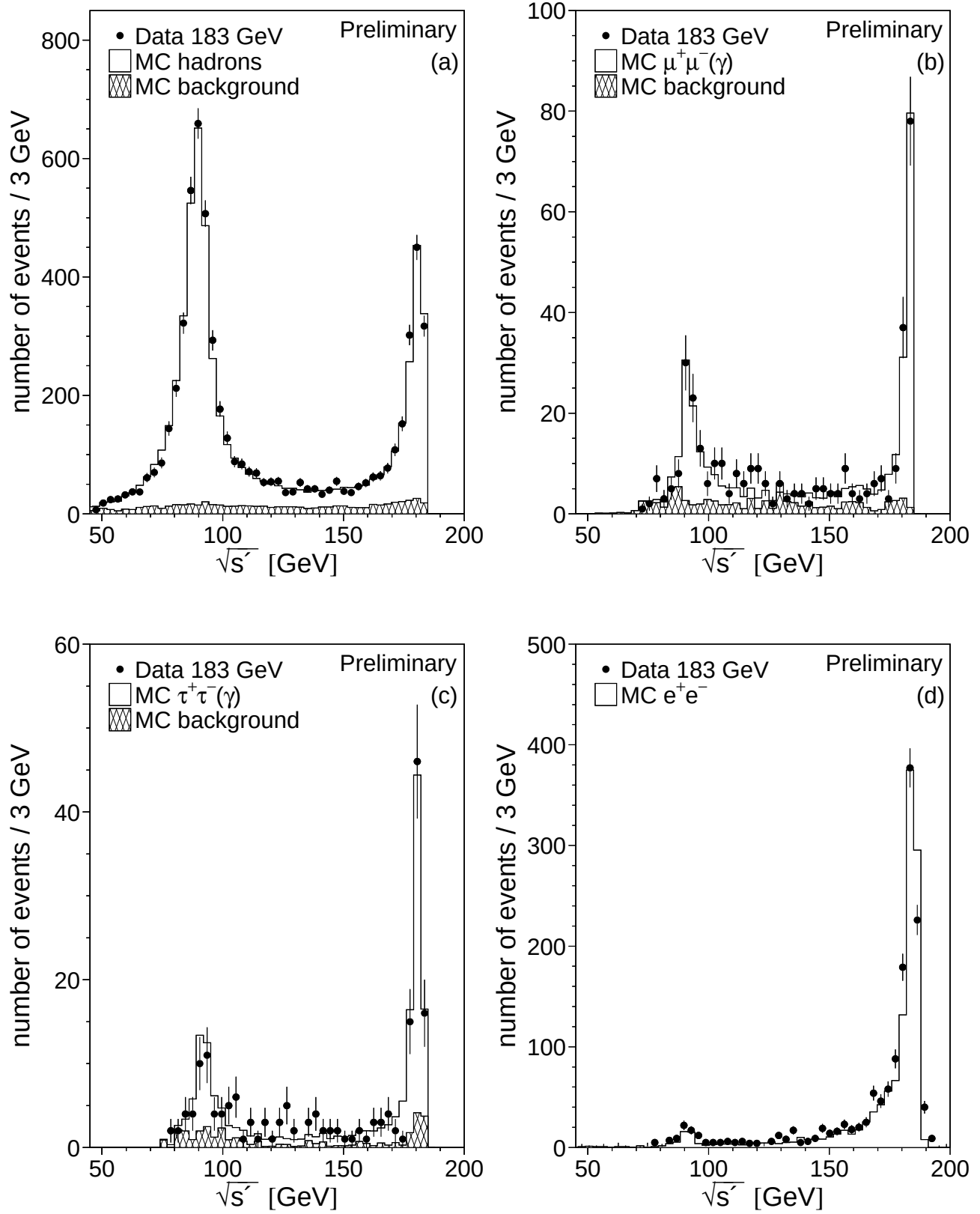


Figure 5.10: Distribution of the reconstructed effective centre-of-mass energy for (a) hadrons, (b) muons, (c) taus, and (d) electrons as measured by L3 at $\sqrt{s} = 183$ GeV [160].

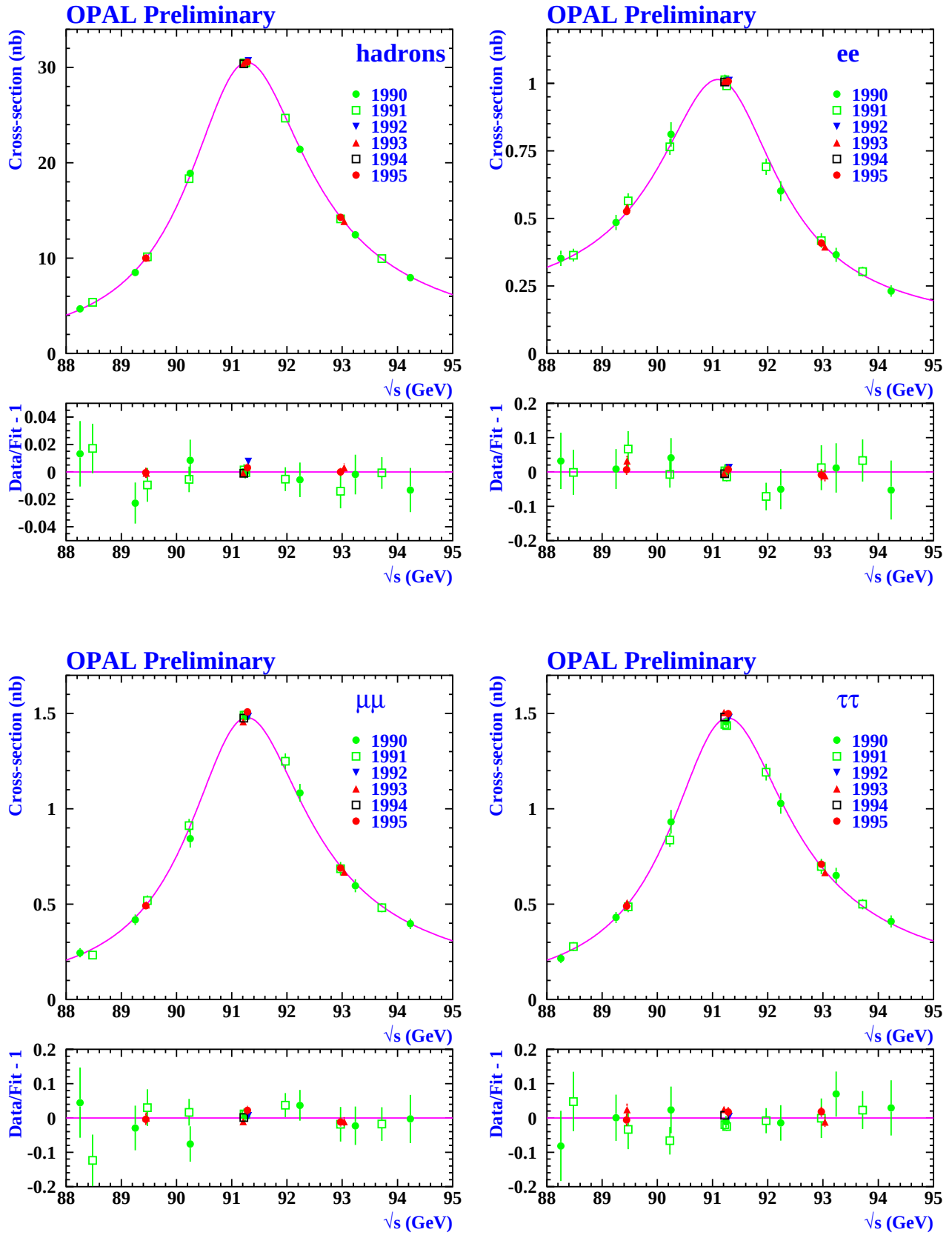


Figure 5.11: Cross sections at the Z pole measured by OPAL for hadrons, electrons, muons and taus [161]. The fiducial volume for the e^+e^- final state is given by the requirement $|\cos\theta_{e^-}| < 0.7$. The result of a fit to the measurements is shown as the line.

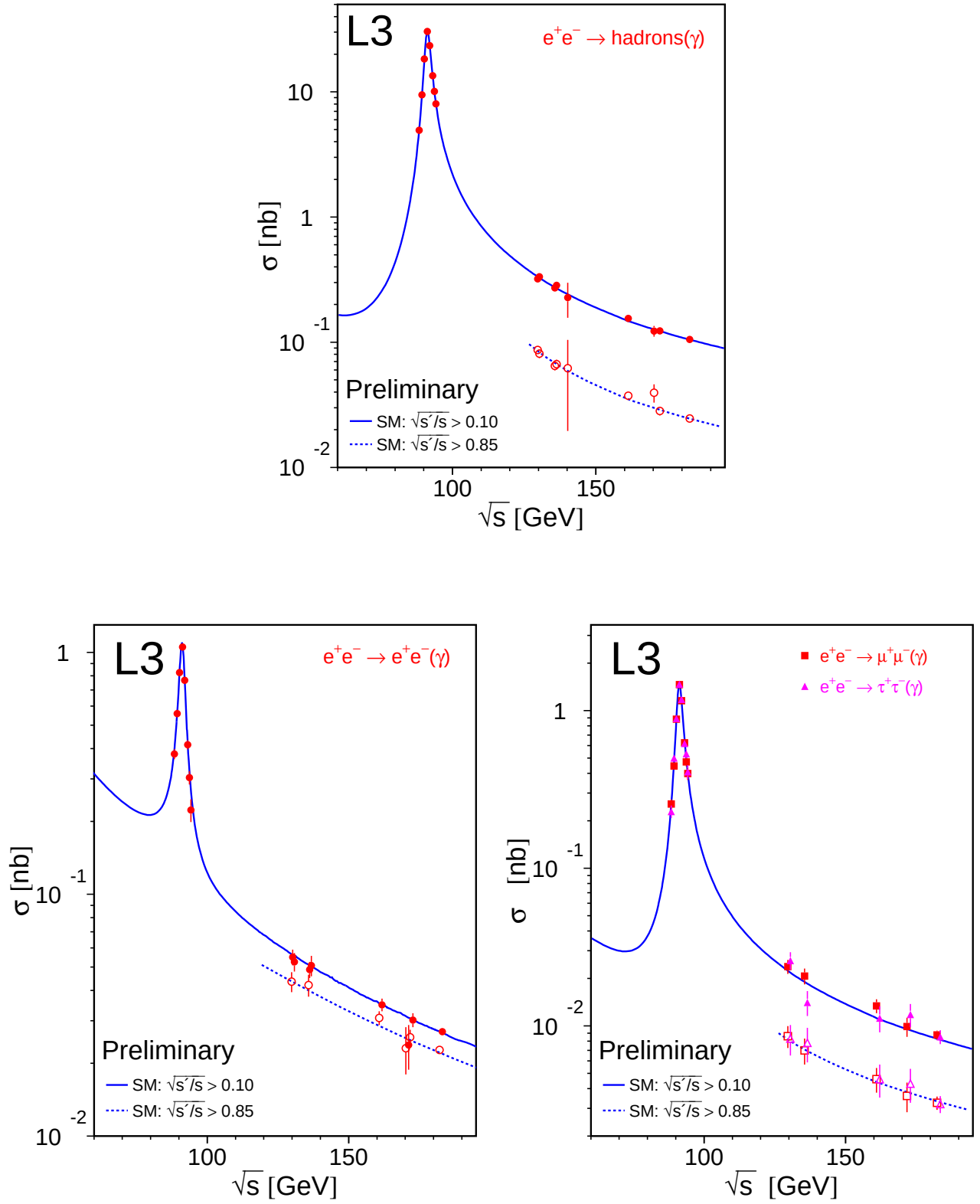


Figure 5.12: Cross sections at and above the Z pole measured for hadrons, electrons, muons and taus by L3 [160]. The fiducial volume for the e^+e^- final state is given by the requirement $44^\circ < \theta < 136^\circ$ for the polar angle of both electron and positron.

5.1.2 Forward-Backward Asymmetries

In order to measure the forward-backward flavour asymmetry in $e^+e^- \rightarrow f\bar{f}(\gamma)$ production, polar angles must be measured, and fermions and anti-fermions must be separated. In case of $\ell^+\ell^-$ production, the fermion is identified by its negative electric charge, while the anti-fermion is identified by its positive charge.

Particle charges are derived from the curvature of tracks measured in the tracking systems of the experiments. Incorrect charge measurement and thus incorrect flavour assignment causes a forward event, $\cos\theta_f > 0$, to be misidentified as a backward event, $\cos\theta_f < 0$, and vice versa. Such charge confusion dilutes the observed asymmetry with respect to the underlying true asymmetry:

$$A_{\text{fb}}^{\text{obs}} = (1 - 2C)A_{\text{fb}}, \quad (5.5)$$

where C denotes the probability for charge confusion of an event. The results quoted by the experiments are corrected for this effect.

For $\ell^+\ell^-$ events, the charge confusion is small, a few % at most. In contrast, for $q\bar{q}$ events, where the original quark charge is obscured due to the hadronisation process, the charge confusion is much larger. In the case of the inclusive hadronic final state, a forward-backward charge asymmetry, discussed in Section 5.7, is measured instead of a flavour-based forward-backward asymmetry.

There are two strategies to measure the forward-backward asymmetry, the counting method and the fitting method. The counting method derives the forward-backward asymmetry from counting the events in the forward and backward hemispheres. As a function of $\cos\theta$, the number of observed events is corrected for background, efficiency and charge confusion. Denoting with N_f and N_b the corrected number of events in the forward and in the backward hemisphere, respectively, the forward-backward asymmetry inside the symmetric fiducial volume defined by the requirement $|\cos\theta| < c$ is simply given by:

$$A_{\text{fb}}(|\cos\theta| < c) = \frac{N_f(c) - N_b(c)}{N_f(c) + N_b(c)}. \quad (5.6)$$

As indicated, the observed asymmetry depends on the phase-space cuts applied in $\cos\theta$. The counting method does not make any assumption on the differential signal cross section $d\sigma/d\cos\theta$.

The functional form of the differential cross section given by the lowest-order Born term formula is a very good approximation to the result expected including all radiative corrections if events with very large acollinearity angle are removed. In case of s -channel processes:

$$\frac{d\sigma(s)}{d\cos\theta} = \sigma(s) \left[\frac{3}{8}(1 + \cos^2\theta) + A_{\text{fb}}(s)\cos\theta \right]. \quad (5.7)$$

Based on this expression for the differential cross section, the forward-backward asymmetry measured within the fiducial volume defined by $|\cos\theta| < c$ is extrapolated to the full solid angle by the relation:

$$A_{\text{fb}}(s) = A_{\text{fb}}(s, |\cos\theta| < c) \cdot \frac{3}{4} \frac{c + \frac{c^3}{3}}{c^2}. \quad (5.8)$$

The fitting method to determine the forward-backward asymmetry relies on a functional form to describe the differential cross section by fitting it to the distribution of observed events. The forward-backward asymmetry is determined by an unbinned maximum likelihood fit maximising the likelihood:

$$L = \prod_i \left[\frac{3}{8}(1 + \cos^2\theta_i) + (1 - 2C_i)A_{\text{fb}}(s)\cos\theta_i \right]. \quad (5.9)$$

where the product runs over all selected events i . The quantity C_i determines the probability for a wrong charge measurement of event i . The factor $(1 - 2C_i)$ in front of the fit parameter A_{fb} takes

into account the effect of charge confusion on an event-by-event basis. The fitted asymmetry, $A_{\text{fb}}(s)$ is directly the asymmetry extrapolated to the full range of polar angles, where the extrapolation is based on the functional form of $d\sigma/d\cos\theta$ used in the likelihood. The unbinned fitting method usually yields smaller statistical errors, since it imposes a functional form describing the differential cross section and makes use of the $\cos\theta$ measurement of each event instead of grouping all events in just two bins of polar angle, forward and backward.

In both methods, all radiative corrections are absorbed in the parameter $A_{\text{fb}}(s)$. The effect of the cut on the acollinearity of the final-state fermion pair, restricting the phase-space for QED radiation, is not corrected for as it ensures the validity of the simple polynomial expression for $d\sigma/d\cos\theta$.

Most experimental systematic effects cancel in the measurements of forward-backward asymmetries, as they are ratios of cross sections. For example, uncertainties on the luminosity do not propagate. In case of the fitting method, fiducial volume effects or $\cos\theta$ dependent efficiencies cancel since they are usually either forward-backward or charge symmetric.

However, background contamination and selection effects may bias the forward-backward asymmetry measurement. For the $e^+e^-(\gamma)$ and $\mu^+\mu^-(\gamma)$ selections, the background from other leptons, mainly $\tau^+\tau^-$ pairs decaying to electrons or muons, respectively, is small. The $\tau^+\tau^-$ sample is selected with the largest background from hadronic and other lepton-pair events. Since the forward-backward asymmetries of $e^+e^-(\gamma)$, $\mu^+\mu^-(\gamma)$ or $\tau^+\tau^-(\gamma)$, and hadronic events are different, the observed asymmetries must be corrected for the asymmetry of the selected background contaminations.

The forward-backward asymmetry, measured using the sample of selected events, may also be affected by a bias due to the event selection procedure. For example, the event selection efficiency may depend on the amount of QED radiation or on the $\ell^+\ell^-$ acollinearity. In case of $\tau^+\tau^-(\gamma)$ production, requirements on the energy of the visible τ decay products also introduce a dependence of the event selection efficiency on the polarisation of the decaying τ leptons. Since such effects do not factorise in the multidimensional differential cross section, a bias is introduced in the forward-backward asymmetry of the selected event sample. This bias is determined based on Monte-Carlo simulations of $\ell^+\ell^-(\gamma)$ production, by comparing the forward-backward asymmetry of selected signal Monte Carlo events with that of all signal Monte Carlo events generated within the idealised phase-space cuts for which $A_{\text{fb}}(s)$ is to be quoted. The selections are designed so that these effects are small or even negligible.

For $\mu^+\mu^-$ and $\tau^+\tau^-$ events, the forward-backward asymmetry is usually determined by the fitting method, as the differential cross section is indeed well described by the simple second-order polynomial in $\cos\theta$ as shown in Figure 5.13. Systematic errors range from 0.0005 to 0.0015 for $\mu^+\mu^-$ events and from 0.0007 to 0.0030 for $\tau^+\tau^-$ events.

For the e^+e^- final state, the forward-backward asymmetry is usually determined by the counting method in the restricted range of large polar angles also used for the e^+e^- cross section measurement. The counting method is used here because the differential cross section exhibits a much more complicated behaviour due to the large contributions arising from t -channel and s/t -interference, as shown in Figure 5.13. Extrapolation to the full solid angle is not useful since for $|\cos\theta| \rightarrow 1$ the t -channel photon exchange dominates leading to divergent cross sections and a forward-backward asymmetry of unity. Systematic errors between 0.001 and 0.003 are obtained.

The leptonic forward-backward asymmetries measured as a function of the centre-of-mass energy are shown in Figure 5.14 for LEP-I energies close to M_Z , and in Figure 5.15 for all centre-of-mass energies.

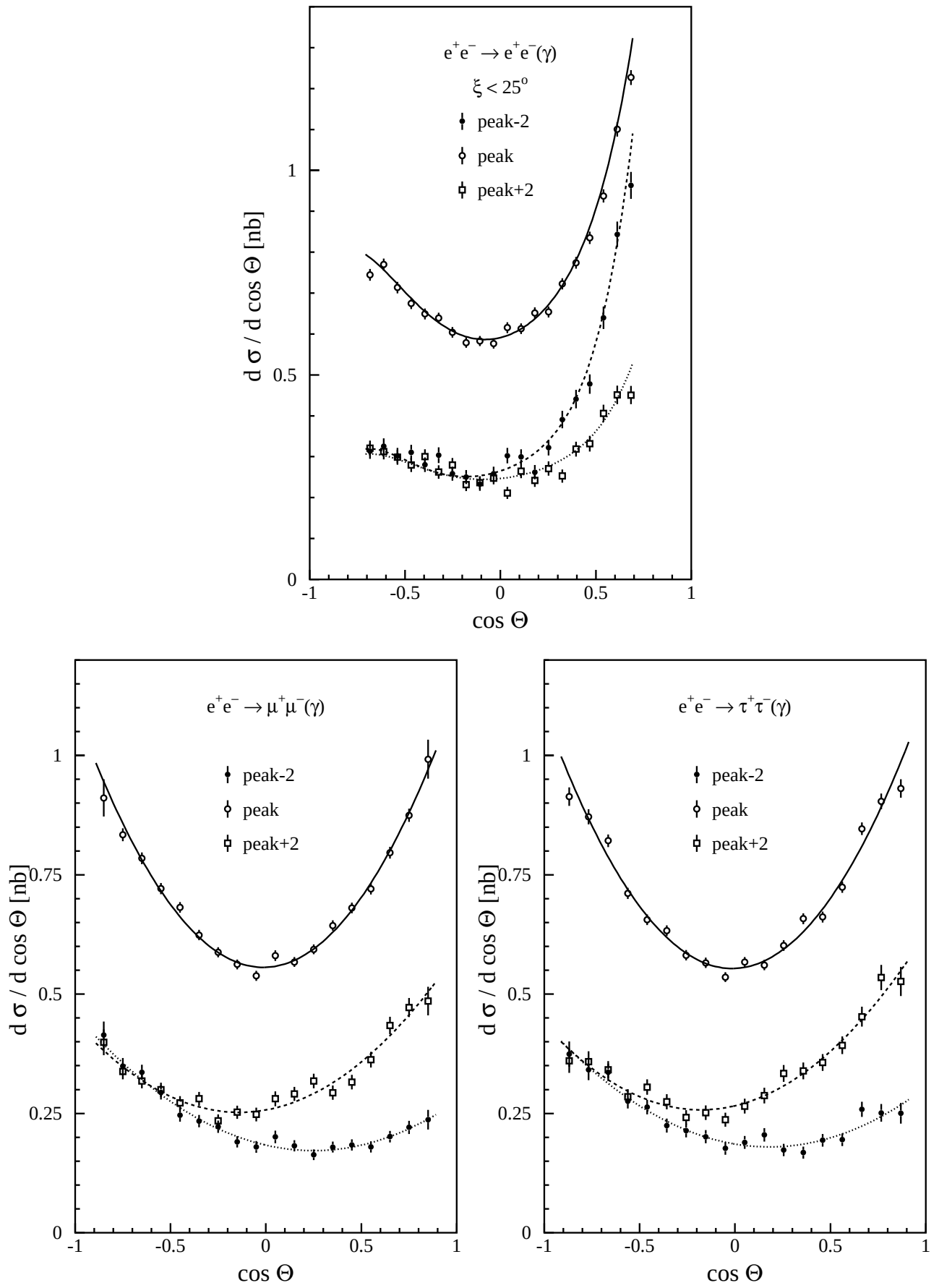


Figure 5.13: Differential cross sections at the Z pole for electrons, muons and taus measured by L3 [139].

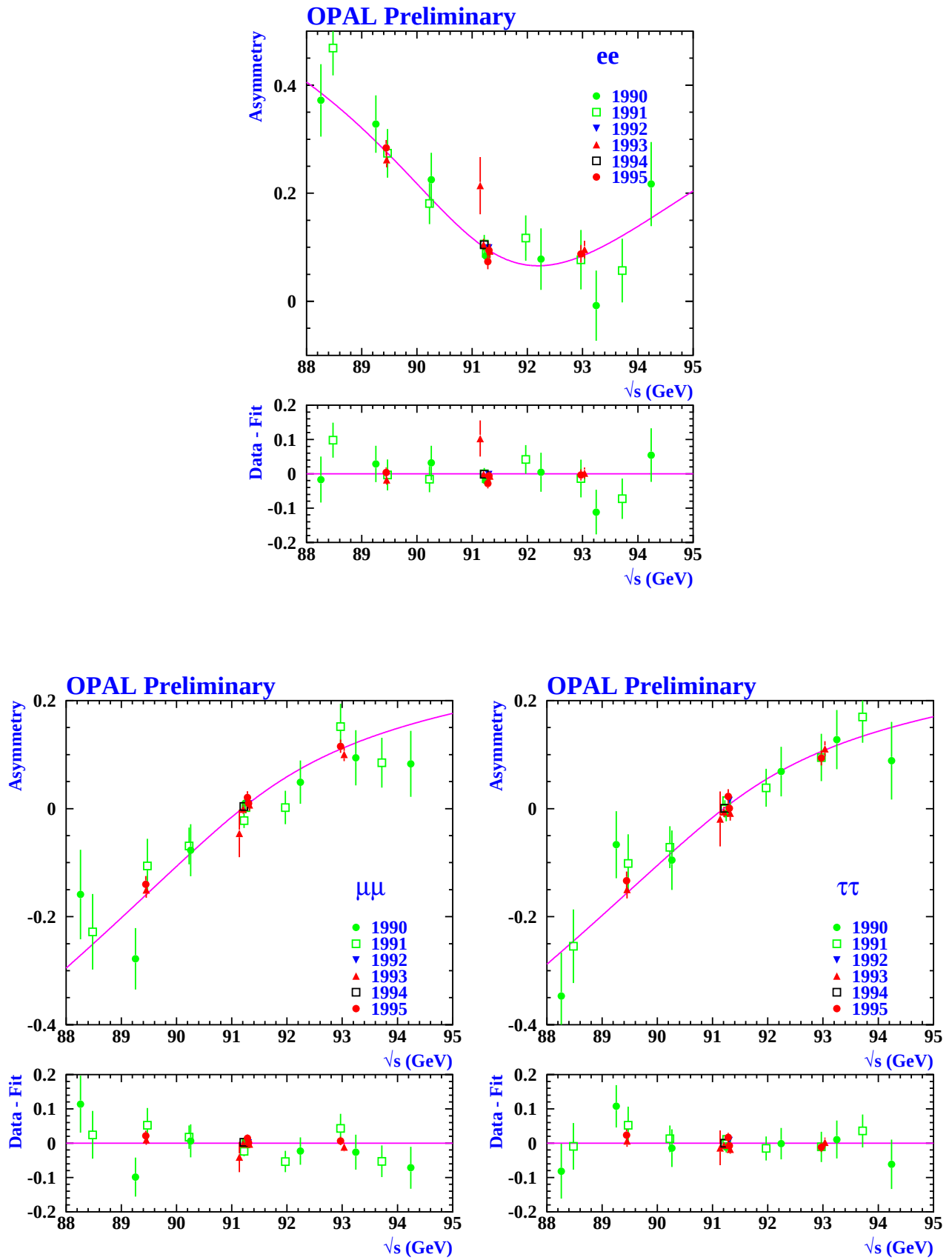


Figure 5.14: Forward-backward asymmetries at the Z pole measured for electrons, muons and taus by OPAL [161]. The fiducial volume for the e^+e^- final state is given by the requirement $\cos\theta_{e^-} < 0.7$. The result of a fit to the measurements is shown as the line.

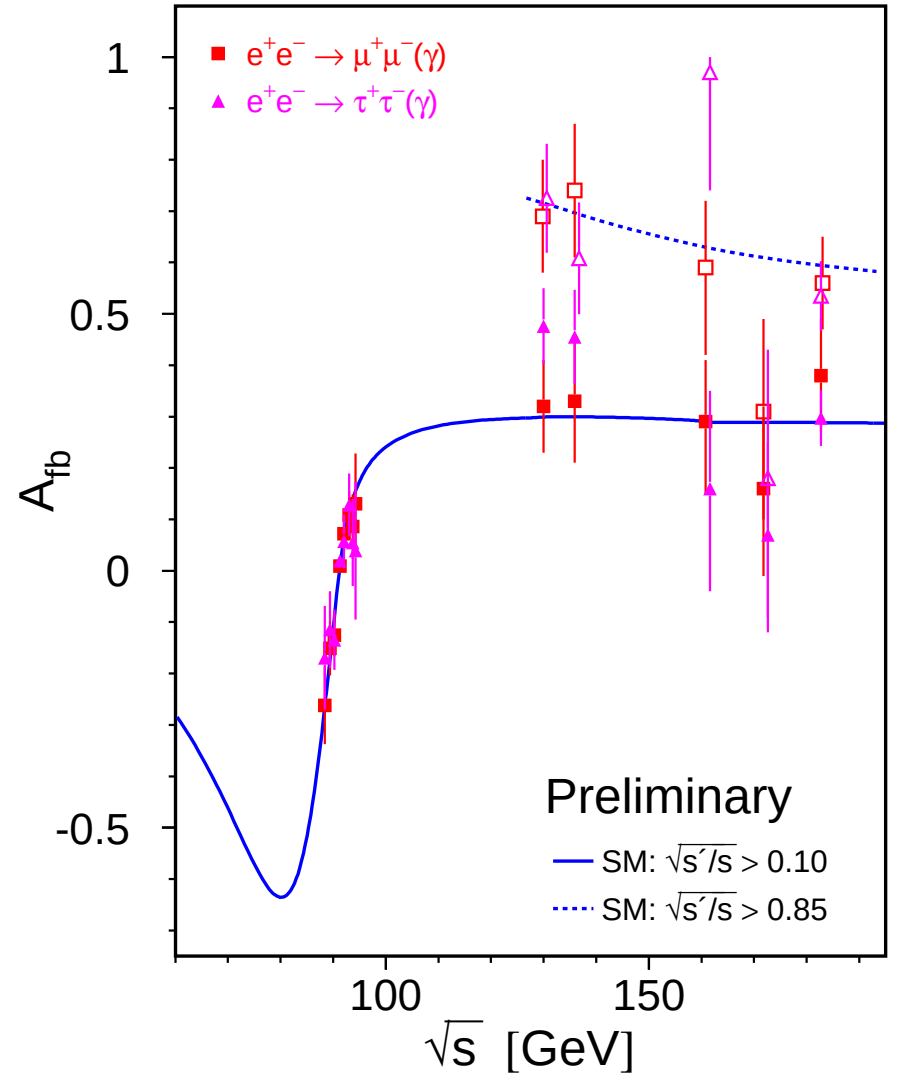
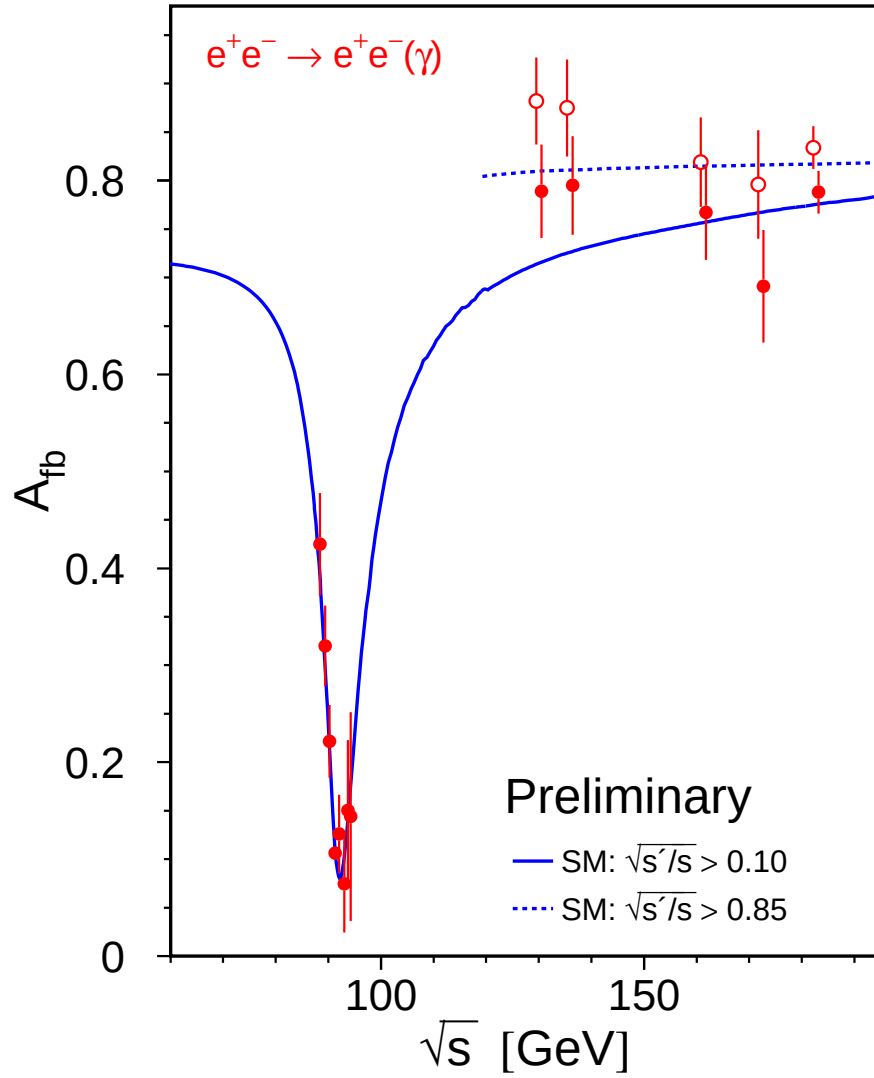


Figure 5.15: Forward-backward asymmetries at and above the Z pole for electrons, muons and taus measured by L3 [160]. The fiducial volume for the e^+e^- final state is given by the requirement $44^\circ < \theta < 136^\circ$ for the polar angle of both electron and positron.

5.1.3 The Z-Boson Parameters

Each LEP experiment determines Z-boson parameters in a fit to its cross sections, $\sigma(s)$, and forward-backward asymmetries, $A_{\text{fb}}(s)$, measured for the different $f\bar{f}$ final states. The $\sqrt{s}$ dependence of the measurement is accounted for by a relativistic Breit-Wigner function describing a spin-1 resonance, as discussed in Section 3.1.1.4. Best values, errors and correlations for two sets of parameters, in the following called the Z-pole parameters and the S-matrix parameters, are determined.

5.1.3.1 The Z-Pole Parameters

The Z-pole parameters describe the s -channel Z-exchange contribution to total cross sections and forward-backward asymmetries. The parameters are chosen such that their determination based on the experimental measurements leads to minimal correlations between them:

- The mass, M_Z , and total width, Γ_Z , of the Z boson, defined using a Breit-Wigner denominator with s -dependent width, $s - M_Z^2 + is\Gamma_Z/M_Z$.
- The hadronic pole cross section, $\sigma_{\text{had}}^\circ$, of Z exchange defined as:

$$\sigma_{\text{had}}^\circ = \frac{12\pi}{M_Z^2} \frac{\Gamma_{ee}\Gamma_{\text{had}}}{\Gamma_Z^2}. \quad (5.10)$$

- The ratio of the Z partial decays widths of hadrons and leptons:

$$R_\ell^Z = \frac{\sigma_{\text{had}}^\circ}{\sigma_\ell^\circ} = \frac{\Gamma_{\text{had}}}{\Gamma_{\ell\ell}} \quad \text{for } \ell = e, \mu, \tau. \quad (5.11)$$

As R_ℓ^Z is also the ratio of the hadronic and leptonic pole cross sections, normalisation uncertainties cancel.

- The leptonic forward-backward pole asymmetries, $A_{\text{fb}}^{0,\ell} = \frac{3}{4}A_e A_f$, where the coupling parameter A_f is defined in terms of the real parts of the effective vector and axial-vector coupling constants, g_{Vf} and g_{Af} :

$$A_{\text{fb}}^{0,\ell} = \frac{3}{4}A_e A_\ell \quad \text{for } \ell = e, \mu, \tau \quad (5.12)$$

$$A_f = 2 \frac{g_{Vf}/g_{Af}}{1 + (g_{Vf}/g_{Af})^2}. \quad (5.13)$$

The total and partial decay widths of the Z boson in the above expression are the physical quantities including fermion-mass effects and absorbing all higher-order radiative corrections. For the forward-backward asymmetries, however, the imaginary parts of the effective couplings and of the photonic vacuum polarisation are accounted for explicitly, and their values are taken from the Standard Model.

The nine parameters describe only the s -channel Z exchange contribution to total cross sections and forward-backward asymmetries. The s -channel γ exchange contributions are fixed to their QED expectations, assuming lepton and quark charges of the Standard Model. For the inclusive hadronic final state, also the γ/Z interference contributions are fixed to the Standard-Model values. For the leptonic final states, the s -channel γ/Z interference contributions are calculated based on the effective couplings. The real and imaginary parts of the photonic vacuum polarisation are taken from QED calculations.

If lepton universality of the neutral weak current is assumed, the nine Z-pole parameters are reduced to five. In that case, $\Gamma_{\ell\ell}$ and R_ℓ^Z are defined based on the decay width of the Z boson into a pair of massless charged leptons. The phase-space suppression due to lepton-mass effects is relevant only in $\tau^+\tau^-$ production, where the partial width $\Gamma_{\tau\tau}$ is reduced by 0.23% and thus R_τ^Z increased by the same amount.

5.1.3.2 The S-Matrix Parameters

The S-Matrix parameters describe the s -channel γ/Z -interference contributions in addition to the s -channel Z -exchange contributions to total cross sections and forward-backward asymmetries. No special parameter transformation is applied to reduce the correlations between these parameters when derived from experimental measurements of cross sections and forward-backward asymmetries. The S-Matrix parameters, see Equation 3.9, are:

- The mass, M_Z , and total width, Γ_Z , of the Z boson, defined using a Breit-Wigner with s -independent width. For the purpose of eased comparisons, mass and total width are transformed according to Equations 3.21 and 3.22 to correspond to the usual definition based on the Breit-Wigner with s -dependent width when listing numerical results.
- The contribution of the Z exchange to the total cross sections, r_f^{tot} , for hadrons and charged leptons.
- The contribution of the γ/Z interference to the total cross sections, j_f^{tot} , for hadrons and charged leptons.
- The contribution of the Z exchange to the forward-backward cross sections, r_f^{fb} , for charged leptons.
- The contribution of the γ/Z interference to the forward-backward cross sections, j_f^{fb} , for charged leptons.

By definition, the S-matrix parameters r_f , j_f and g_f are real numbers absorbing all higher-order radiative corrections. Mass effects are explicitly incorporated in the fitting formulae.

The 16 parameters describe both the s -channel Z exchange and the s -channel γ/Z interference contributions. The s -channel γ exchange contributions are fixed to their QED predictions. If lepton universality of the neutral weak current is assumed the 16 S-Matrix parameters are reduced to eight.

5.1.4 Fitting Procedure

Both parameter sets are determined in the same way. A χ^2 fit is performed, where the χ^2 is constructed from the experimental measurements, the theoretical predictions, the measurement errors, the theoretical errors, and the correlations between the errors:

$$\chi^2 = \Delta V^{-1} \Delta. \quad (5.14)$$

The χ^2 as a function of the fit parameters is minimised using the program MINUIT [162]. The vector Δ is the vector of differences between measurement and prediction, and the matrix V is the error matrix associated with the set of results, taking correlations into account. In case some error components depend on the central values, as is the case for statistical or relative errors, the measured experimental errors are replaced by the expected errors which are calculated based on the predicted central values as a function of the fit parameters. In case of relative errors, the correction of the error component is simply:

$$\delta\sigma_{theo} = \delta\sigma_{exp} \cdot \frac{\sigma_{theo}}{\sigma_{exp}} \quad (5.15)$$

$$\delta A_{fb}^{theo} = \delta A_{fb}^{exp} \cdot \frac{A_{fb}^{theo}}{A_{fb}^{exp}}. \quad (5.16)$$

In case of small backgrounds, the statistical errors δ are $\delta\sigma \propto \sqrt{\sigma}$ for cross sections and $\delta A_{\text{fb}} \propto \sqrt{(1 - A_{\text{fb}}^2)/\sigma}$ for forward-backward asymmetries, so that the correction for the statistical errors is:

$$\delta\sigma_{\text{theo}} = \delta\sigma_{\text{exp}} \cdot \sqrt{\frac{\sigma_{\text{theo}}}{\sigma_{\text{exp}}}} \quad (5.17)$$

$$\delta A_{\text{fb}}^{\text{theo}} = \delta A_{\text{fb}}^{\text{exp}} \cdot \sqrt{\frac{\sigma_{\text{exp}}}{\sigma_{\text{theo}}}} \cdot \sqrt{\frac{1 - A_{\text{fb}}^{\text{theo}}}{1 - A_{\text{fb}}^{\text{exp}}}}. \quad (5.18)$$

The theoretical predictions are calculated as a function of the Z-pole or S-Matrix parameters to be fitted. For this purpose, semianalytical program such as MIZA, SMATASY and ZFITTER, discussed in Section 3.3.2, are used. Effects of initial-state QED radiation are taken into account explicitly by convoluting the theoretical cross sections with the appropriate radiator function. Fixing the s -channel γ exchange contributions to the QED predictions as discussed above is consistent with the use of QED-based radiator functions, which is mandatory to account for the large effects induced by initial-state QED radiative corrections, and with the calculation of the low-angle Bhabha cross section for the measurement of the luminosity.

5.1.4.1 Treatment of Bhabha Scattering

Compared to s -channel fermion-pair production, the additional t -channel Feynman diagrams in case of Bhabha scattering make the calculation of radiatively corrected $e^+e^-(\gamma)$ production technically very difficult. The computations are so complicated that semianalytical programs of the required physical and technical precision, such as ALIBABA [111] and TOPAZ0 [21], are much too slow for fitting purposes. This problem is circumvented by an approximate procedure, which treats the t -channel and s/t -interference contributions as constant, *i.e.*, independent of the interesting electroweak parameters to be fitted. This ansatz is justified as these terms are either small or their dependence on the interesting Z parameters is small or both. Thus, these terms are kept constant while only the s -channel part of the prediction is calculated as a function of the varying fit parameters.

However, corrections, errors and correlations must be calculated based on complete calculations including s -channel, t -channel and s/t -interference contributions, as this is what is measured in the data. An example is given by the energy dependence of cross sections and asymmetries on the centre-of-mass energy, which is needed for the treatment of the LEP energy errors. As shown in Figures 5.11 and 5.14, the energy dependences are drastically different for e^+e^- production compared to $\mu^+\mu^-$ or $\tau^+\tau^-$ production. The t -channel and s/t -interference contributions must be added to the theoretical s -channel calculations, rather than subtracting them from the experimental measurements.

For the purpose of visually comparing cross sections and forward-backward asymmetries of e^+e^- production with those of other charged leptons, the experiments also report s -channel cross section and s -channel forward-backward asymmetries in e^+e^- production. These results are obtained by subtracting the theoretical expectation of t -channel and s/t -interference contributions from the measurements and then extrapolating to the full solid angle. The subtraction procedure introduces a statistical correlation between the s -channel cross section and the s -channel forward-backward asymmetry at each energy point.

5.1.5 Averaging Procedure

The measurements of cross sections and forward-backward asymmetries of the four LEP experiments are averaged based on the four sets of fitted Z-pole or S-Matrix parameters, including the corresponding error matrices [163]. The fitted error matrices describe fully the errors and correlations within each experiment. In order to average correctly, the correlated errors between the experiments are needed for the fitted parameters. The sources of inter-experimental correlations are common to both parameter sets and are discussed in the following.

5.1.5.1 The Centre-of-Mass Energy Calibration

To extract the inter-experimental correlations on the fitted parameters due to the common LEP energy calibration [146–148, 150], an approximate procedure based on the fitted parameters and their error matrix $\mathcal{E}$ is used. Additional fits are performed to the data set of an experiment, where all experimental errors not arising from the LEP energy calibration are scaled by a factor f . Denoting with $\mathcal{E}'$ the error matrix of the second fit, the error matrix $\mathcal{C}$ describing the inter-experimental correlations is then given by:

$$\mathcal{C} = \frac{\mathcal{E}' - f^2 \mathcal{E}}{1 - f^2}. \quad (5.19)$$

A factor of $f = 1/2$ simulates the weight of four experiments in the average. For the scaled fit one has to check that the central values do not move. If they do, this bias, averaged over all four experiments, must be applied to the LEP average. Such a bias is observed in M_Z only, which is understood since the LEP energy errors contributing to M_Z are different for the 1993 and the 1995 scan.

There are two possibilities to account for the bias. One is to use correlated errors as obtained by evaluating $\mathcal{C}$ in the limit $f \rightarrow 1$, which are slightly larger [163]. Another, more intuitive method would be to consider separate M_Z masses in the averaging procedure, one mass for each period of energy calibration [164]. After the average over the experiments is performed, the Z masses corresponding to the different energy-calibration periods can be compared which constitutes an internal cross check of the LEP-I energy calibration. Finally, the mass values are averaged over calibration periods in order to obtain the best Z-mass determination.

The approximative procedure used is checked for the hadronic final state, which essentially determines M_Z and Γ_Z . A fit to the measured hadronic cross sections of all four experiments is performed, taking the full 28-by-28 LEP energy error matrix into account. The results are compared to the results of the approximative procedure also using the hadronic cross sections only. Good agreement is observed [163, 164].

5.1.5.2 The Centre-of-Mass Energy Spread

The uncertainties arising from the error on the spread of the LEP centre-of-mass energy as discussed in Section 4.6 are fully correlated between all experiments. However, the associated uncertainties on the fitted parameters are negligible compared to the other uncertainties on these quantities.

5.1.5.3 The Luminosity Theory Uncertainty

This error, currently 0.11% and soon to be reduced to 0.06%, see Section 4.5, affects in a correlated way all total cross section measurements and thus propagates to all parameters measuring cross sections, *i.e.*, the hadronic pole cross section σ_{had}^0 , or the S-Matrix parameters r_f , j_f and g_f , which scale cross section contributions.

5.1.5.4 The Bhabha Theory Uncertainty

The theoretical errors associated with the calculation of the Bhabha process at large polar scattering angles, in particular with the t -channel and s/t -interference contributions, are discussed in detail in [117]. The uncertainties depend on the centre-of-mass energy, the fiducial volume in polar angle and the maximal allowed final-state electron-positron acollinearity. Scaled by a factor of 0.8 to account for the actual experimental acceptance, they are estimated to be in the range of 1.0 – 1.3 pb on the forward cross section and 0.2 – 0.3 pb on the backward cross section. The size of the uncertainties reflects the importance of the t -channel and s/t -interference contribution in the two angular regions. Since the uncertainty dominates the forward region, there will be a positive correlation between the total cross section and the forward-backward asymmetry.

The error on the total Bhabha cross section propagates to the errors on R_e^Z , r_e^{tot} , j_e^{tot} , and g_e^{tot} , the error on the forward-backward asymmetry enters $A_{\text{fb}}^{0,e}$, and the error on the forward-backward cross section affects r_e^{fb} , j_e^{fb} , and g_e^{fb} . Since the theoretical Bhabha errors are small and the most precise measurements are obtained at the pole itself, it is a good approximation to propagate at least the errors quoted for the peak centre-of-mass energy to the fitted parameters. Because $R_e^Z \propto 1/\sigma_e$, the correlation between R_e^Z and $A_{\text{fb}}^{0,e}$ due to this source of uncertainty will be negative.

5.1.5.5 The Residual Standard Model Dependence

Effective couplings are defined as the real parts of the complex s -dependent formfactors evaluated at $\sqrt{s} = M_Z$. The imaginary parts of the formfactors must be taken from the SM. Thus a dependence on SM calculations and SM input parameters is introduced. Largest effects in the Z-pole fits arise from the dependence on the Higgs mass through the effective electroweak mixing angle fixing the hadronic γ/Z interference term for total cross sections. Varying M_H from 90 GeV to 1000 GeV, the extracted Z mass changes by ± 0.15 MeV, which is very small. This source of SM dependence and associated uncertainty is not present in the S-Matrix analysis, as there the hadronic interference term is determined from the measurements.

5.1.5.6 The QED Convolution

The radiator functions for forward-backward cross sections are known to complete second order. For total cross sections, also the leading $\mathcal{O}(\alpha_{em}^3)$ correction is known, which decreases the convoluted cross sections by an amount ranging from 0.04% at 3 GeV below the Z pole to 0.11% at 3 GeV above the Z pole with respect to the second order calculation [65]. Within the YFS exponentiation scheme [165], these corrections are known already for some time [64]. The changes are comparable to the luminosity theory error and thus must be taken into account. If these corrections are included in the theoretical calculations, it is estimated that the remaining missing higher-order corrections introduce an uncertainty of 0.02% [65] or less. In contrast, for asymmetries, the leading third order correction is not yet known. Based on the structure function approach, it is estimated that the uncertainty on forward-backward asymmetries due to missing higher-orders beyond second order is limited to less than 10^{-4} [166], which is negligible compared to the statistical error of the forward-backward asymmetry measurements.

5.1.5.7 The Radiative Pair Production

The Monte Carlo event generators used to calculate selection efficiencies do not simulate the effect of off-shell photon radiation from the initial or final state leading to additional soft fermion-antifermion pairs. Pairs from the initial state usually remain inside the beam pipe, thus do not affect the selection efficiency. Initial-state pair production reduces the cross section, depending on centre-of-mass energy, by up to 0.3%, with an uncertainty of $1.8 \cdot 10^{-4}$ arising from radiation of hadrons [70, 71]. Pairs from the final state are visible in the detector, however. Corrections due to virtual and real final-state pairs cancel each other [72]. Therefore, the selection procedure should not discriminate against additional soft tracks. Otherwise, a correction of about +0.4% needs to be applied to cross sections, as virtual and real final-state pair production is also not included in the semianalytical programs used to fit the measurements.

5.1.5.8 The Interference of Initial-State and Final-State Radiation

The interference of initial- and final-state radiation combined with box diagrams changes slightly the differential cross section of $f\bar{f}$ production depending on $\cos\theta$ and s' and in particular for $|\cos\theta| \rightarrow 1$. Integrated over the full phase space, the interference effect is suppressed and negligible for LEP-I centre-of-mass energies around M_Z , while it becomes sizeable at the higher LEP-II centre-of-mass energies.

The interference of initial- and final-state radiation is automatically included in the diagrammatic $\mathcal{O}(\alpha_{em})$ calculation of radiative corrections to $f\bar{f}$ production, but not necessarily when concentrating on higher-order initial-state radiative corrections which cause larger effects. The Monte Carlo event generators used to calculate selection efficiencies are therefore operated in the mode to simulate higher-order initial state radiative corrections, then neglecting interference, rather than in $\mathcal{O}(\alpha_{em})$ mode including interference. Thus both the determination of the selection efficiency within the fiducial volume and the extrapolation to full solid angle do not take interference effects into account. However, a bias is only introduced if the selection efficiency within the fiducial volume is different for initial-final interference events. If the extrapolation to full solid angle is performed neglecting initial-final interference, then the program used to calculate the theoretical prediction should also neglect the interference in that region of phase-space. Another possibility is to remove the initial-final interference correction from the data by subtracting the effect expected from theory, and compare with theoretical calculations ignoring initial-final interference effects altogether.

5.1.5.9 The Hadronic Cross Section

The uncertainty on the total hadronic cross section due to fragmentation effects or the subtraction of hadronic two-photon collision processes is potentially correlated between the experiments. However, owing to the close to complete acceptance for hadronic events and since each experiment tunes its Monte Carlo simulations differently, the correlation is assumed to be negligible.

5.1.6 Results on the Z-Pole Parameters

For the determination of the Z-pole parameters, only the LEP-I measurements at centre-of-mass energies in the vicinity of the Z pole, $|\sqrt{s} - M_Z| < 3 \text{ GeV}$, are used. The LEP-II measurement at centre-of-mass energies well above the Z resonance do not improve the determination of Z-pole parameters since in this ansatz the hadronic γ/Z interference term is taken from the Standard Model and the leptonic interference terms are already well constrained by the effective couplings.

The preliminary results on the Z-pole parameters obtained by the LEP experiments and the resulting LEP average are shown in Tables 5.1 and 5.3. The correlation matrices of the LEP averages are shown in Tables 5.2 and 5.4. The determination of the five-parameter average is based on the four nine-parameter sets, imposing lepton universality in the averaging procedure.

As indicated by the $\chi^2/\text{d.o.f}$ of each experiment, the measured cross sections and forward-backward asymmetries are well described by the model. Furthermore, as indicated by the $\chi^2/\text{d.o.f}$ of the average, 28/27 corresponding to a probability of 42%, also the results of the experiments in terms of the fitted Z-pole parameters are in good agreement with each other. The Z-pole parameters measured by the four LEP experiments and the LEP averages will be discussed in the following.

Parameter	ALEPH	DELPHI	L3	OPAL	LEP	MSM
M_Z [GeV]	91.1884 ± 0.0031	91.1866 ± 0.0029	91.1886 ± 0.0029	91.1848 ± 0.0030	91.1867 ± 0.0021	—
Γ_Z [GeV]	2.4950 ± 0.0043	2.4872 ± 0.0041	2.4999 ± 0.0043	2.4939 ± 0.0040	2.4939 ± 0.0024	2.4920 ± 0.0042
$\sigma_{\text{had}}^{\circ}$ [nb]	41.519 ± 0.067	41.553 ± 0.079	41.411 ± 0.074	41.474 ± 0.068	41.491 ± 0.058	41.479 ± 0.011
R_e^Z	20.688 ± 0.074	20.87 ± 0.12	20.78 ± 0.11	20.924 ± 0.095	20.783 ± 0.052	20.735 ± 0.016
R_μ^Z	20.815 ± 0.056	20.67 ± 0.08	20.84 ± 0.10	20.819 ± 0.058	20.789 ± 0.034	20.735 ± 0.016
R_τ^Z	20.719 ± 0.063	20.78 ± 0.13	20.75 ± 0.14	20.855 ± 0.086	20.764 ± 0.045	20.782 ± 0.016
$A_{\text{fb}}^{0,e}$	0.0181 ± 0.0033	0.0189 ± 0.0048	0.0148 ± 0.0063	0.0069 ± 0.0051	0.0153 ± 0.0025	0.0151 ± 0.0011
$A_{\text{fb}}^{0,\mu}$	0.0170 ± 0.0025	0.0160 ± 0.0025	0.0176 ± 0.0035	0.0156 ± 0.0025	0.0164 ± 0.0013	0.0151 ± 0.0011
$A_{\text{fb}}^{0,\tau}$	0.0166 ± 0.0028	0.0244 ± 0.0037	0.0233 ± 0.0049	0.0143 ± 0.0030	0.0183 ± 0.0017	0.0151 ± 0.0011
$\chi^2/\text{d.o.f}$	169/176 (63%)	179/168 (27%)	142/159 (83%)	158/202 (99%)	28/27 (42%)	—

Table 5.1: Z-pole parameters of the four LEP experiments and their average not assuming neutral-current lepton universality [163].

Parameter	M_Z	Γ_Z	$\sigma_{\text{had}}^{\circ}$	R_e^Z	R_μ^Z	R_τ^Z	$A_{\text{fb}}^{0,e}$	$A_{\text{fb}}^{0,\mu}$	$A_{\text{fb}}^{0,\tau}$
M_Z	1.000	0.000	−0.040	0.002	−0.010	−0.006	0.016	0.045	0.038
Γ_Z	0.000	1.000	−0.184	−0.007	0.003	0.003	0.009	0.000	0.003
$\sigma_{\text{had}}^{\circ}$	−0.040	−0.184	1.000	0.058	0.094	0.070	0.006	0.002	0.005
R_e^Z	0.002	−0.007	0.058	1.000	0.098	0.073	−0.442	0.007	0.012
R_μ^Z	−0.010	0.003	0.094	0.098	1.000	0.105	0.001	0.010	−0.001
R_τ^Z	−0.006	0.003	0.070	0.073	0.105	1.000	0.002	0.000	0.020
$A_{\text{fb}}^{0,e}$	0.016	0.009	0.006	−0.442	0.001	0.002	1.000	−0.008	−0.006
$A_{\text{fb}}^{0,\mu}$	0.045	0.000	0.002	0.007	0.010	0.000	−0.008	1.000	0.029
$A_{\text{fb}}^{0,\tau}$	0.038	0.003	0.005	0.012	−0.001	0.020	−0.006	0.029	1.000

Table 5.2: Correlation coefficients of nine-parameter average [163].

Parameter	ALEPH	DELPHI	L3	OPAL	LEP	MSM
M_Z [GeV]	91.1883 ± 0.0031	91.1867 ± 0.0029	91.1886 ± 0.0029	91.1843 ± 0.0029	91.1867 ± 0.0021	—
Γ_Z [GeV]	2.4949 ± 0.0043	2.4876 ± 0.0041	2.4996 ± 0.0043	2.4940 ± 0.0040	2.4939 ± 0.0024	2.4920 ± 0.0042
$\sigma_{\text{had}}^\circ$ [nb]	41.519 ± 0.067	41.553 ± 0.079	41.411 ± 0.074	41.474 ± 0.068	41.491 ± 0.058	41.479 ± 0.011
R_ℓ^Z	20.738 ± 0.038	20.728 ± 0.060	20.788 ± 0.066	20.828 ± 0.045	20.765 ± 0.026	20.735 ± 0.016
$A_{\text{fb}}^{0,\ell}$	0.0169 ± 0.0016	0.0187 ± 0.0019	0.0187 ± 0.0026	0.0141 ± 0.0017	0.01683 ± 0.00096	0.01512 ± 0.00108
$\chi^2/\text{d.o.f}$	173/180 (63%)	184/172 (25%)	144/163 (86%)	160/206 (99%)	31/31 (47%)	—

Table 5.3: Z-pole parameters of the four LEP experiments and their average assuming neutral-current lepton universality [163].

Parameter	M_Z	Γ_Z	$\sigma_{\text{had}}^\circ$	R_ℓ^Z	$A_{\text{fb}}^{0,\ell}$
M_Z	1.000	0.000	−0.040	−0.010	0.062
Γ_Z	0.000	1.000	−0.184	0.002	0.004
$\sigma_{\text{had}}^\circ$	−0.040	−0.184	1.000	0.123	0.006
R_ℓ^Z	−0.010	0.002	0.123	1.000	−0.072
$A_{\text{fb}}^{0,\ell}$	0.062	0.004	0.006	−0.072	1.000

Table 5.4: Correlation coefficients of five-parameter average [163].

5.1.6.1 The Mass of the Z Boson (I)

The comparison of the LEP results for the mass of the Z boson is shown in Figure 5.16. Good agreement between the results is observed. The Z mass is determined with a precision an order of magnitude better than originally anticipated [60]:

$$M_Z = 91186.7 \pm 2.1 \text{ MeV}. \quad (5.20)$$

The successful measurement of M_Z at LEP depends very much on a precise calibration of the LEP energy, in particular of the off-peak energy points, which contributes 1.7 MeV to the total error on M_Z . The relative accuracy of the M_Z measurement, $\delta M_Z/M_Z = 2 \cdot 10^{-5}$, approaches that of the Fermi constant, $\delta G_F/G_F = 1 \cdot 10^{-5}$. As far as Standard-Model calculations are concerned, M_Z , like G_F , is now known with such a precision that uncertainties in theoretical predictions due to the error on M_Z are completely negligible.

5.1.6.2 The Total Width of the Z Boson

The comparison of the LEP results for the total width of the Z boson is given in Figure 5.17. Also for the total width the individual results agree well. The Z width is measured to be:

$$\Gamma_Z = 2493.9 \pm 2.4 \text{ MeV}, \quad (5.21)$$

which is a precision of 0.1%. Also for the measurement of the total width, a precise LEP energy calibration, in particular of the off-peak energy points, is essential, contributing 1.3 MeV of the total error on Γ_Z . Within the Standard Model the total width is calculated, where the prediction depends on the values of the Standard Model input parameters, M_Z , M_t , M_H , α_S and α_{em} . The band associated to the Standard Model calculation shown in Figure 5.17 reflects typical uncertainties in these parameters.

5.1.6.3 The Hadronic Pole Cross Section

The results for the hadronic pole cross sections are summarised in Figure 5.17. The LEP average value is:

$$\sigma_{\text{had}}^\circ = 41.491 \pm 0.058 \text{ nb} \quad (5.22)$$

The error on $\sigma_{\text{had}}^\circ$, 0.13%, is largely dominated by the theoretical error on the luminosity determination, 0.11%. As shown in Figure 5.17 the uncertainty is so large that there is nearly no sensitivity to radiative corrections and Standard Model input parameters.

5.1.6.4 The Leptonic Quantities

The results on the leptonic quantities R_ℓ^Z and $A_{\text{fb}}^{0,\ell}$ are compared with each other and with the Standard Model in Figure 5.18. The correlated uncertainty arising from Bhabha t -channel and s/t -interference contribution causes the correlation of -44% between R_e^Z and $A_{\text{fb}}^{0,e}$. The leptonic quantities show good agreement among themselves, supporting the hypothesis of lepton universality of the neutral weak current. With this assumption, R_ℓ^Z and $A_{\text{fb}}^{0,\ell}$ are determined with increased precision:

$$R_\ell^Z = 20.765 \pm 0.026 \quad (5.23)$$

$$A_{\text{fb}}^{0,\ell} = 0.01683 \pm 0.00096, \quad (5.24)$$

with a correlation of -7.2% . The parameter R_ℓ^Z is insensitive to the LEP energy, as it is a ratio of cross sections measured at the peak of the resonance where the derivatives with respect to the centre-of-mass energy are small. However, the leptonic pole forward-backward asymmetry $A_{\text{fb}}^{0,\ell}$ depends strongly on the knowledge of the pole centre-of-mass energy in order to transport the measurement to $\sqrt{s} = M_Z$ since $dA_{\text{fb}}(s)/d\sqrt{s} = 8.2 \cdot 10^{-5}/\text{MeV}$ at the pole. An uncertainty of 3 MeV in the centre-of-mass energy therefore translates into an uncertainty of 0.00025 in $A_{\text{fb}}^{0,\ell}$.

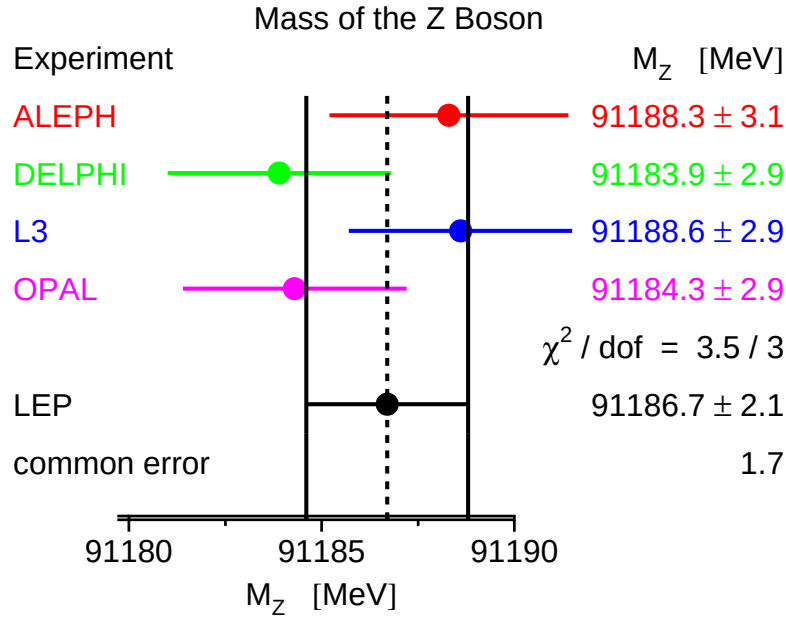


Figure 5.16: Comparison of the LEP results on M_Z . The common error of 1.7 MeV is included in the errors. The χ^2 of the average is calculated based on the uncorrelated errors.

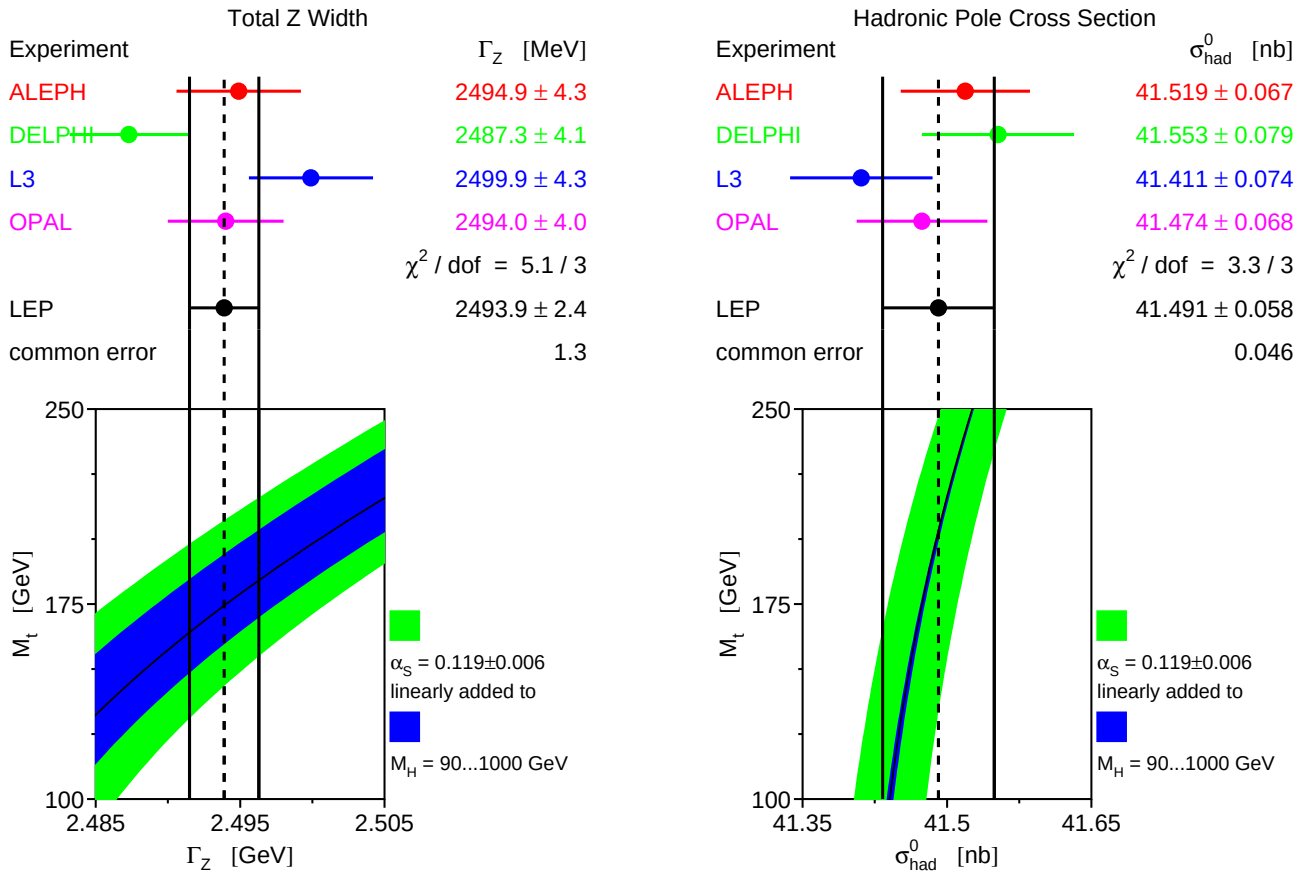


Figure 5.17: Comparison of the LEP results on Γ_Z and σ_{had}^0 . The common error of 1.3 MeV on Γ_Z and 0.046 pb on σ_{had}^0 is included in errors. The χ^2 of the averages are calculated based on the uncorrelated errors.

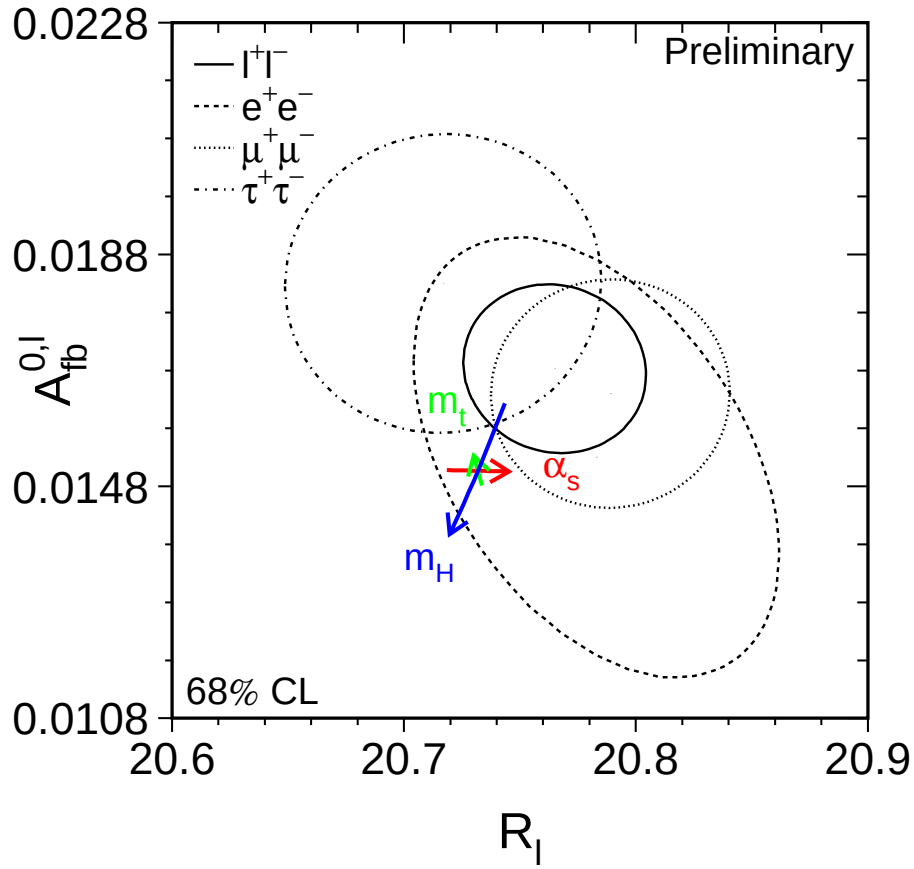


Figure 5.18: Correlation contours of 68% probability in the $(R_\ell^Z, A_{\text{fb}}^{0,\ell})$ plane. In order to be directly comparable to R_e^Z and R_μ^Z , R_τ^Z is reduced by 0.23% to correct for the τ -mass effect. The three lepton species are in good agreement with each other. Imposing neutral-current lepton universality, the small contour is obtained. The Standard-Model prediction is shown for $M_t = 173.8 \pm 5.0$ GeV, $M_H = 300_{-210}^{+700}$ GeV, $\alpha_S(M_Z^2) = 0.119 \pm 0.002$.

5.1.6.5 The Partial Widths of the Z Boson

The partial Z-boson decay widths themselves are calculated from the average set of nine and five Z-pole parameters and their correlations by applying the corresponding parameter transformation. Not assuming lepton universality, the results and correlations are summarised in Table 5.5.

Good agreement between the leptonic partial widths is observed. Assuming lepton universality,

Parameter	Average	Γ_{had}	Γ_{ee}	$\Gamma_{\mu\mu}$	$\Gamma_{\tau\tau}$	MSM
Γ_{had} [MeV]	1743.0 ± 2.9	1.000	-0.146	0.676	0.596	1739.3 ± 3.4
Γ_{ee} [MeV]	83.87 ± 0.14	-0.146	1.000	-0.086	-0.073	83.90 ± 0.12
$\Gamma_{\mu\mu}$ [MeV]	83.84 ± 0.18	0.676	-0.086	1.000	0.443	83.90 ± 0.12
$\Gamma_{\tau\tau}$ [MeV]	83.94 ± 0.22	0.596	-0.073	0.443	1.000	83.71 ± 0.12

Table 5.5: Partial decay widths of the Z boson not assuming lepton universality.

the partial decay widths of the Z boson are determined with increased precision:

$$\Gamma_{\text{had}} = 1742.3 \pm 2.3 \text{ MeV} \quad (5.25)$$

$$\Gamma_{\ell\ell} = 83.90 \pm 0.10 \text{ MeV}, \quad (5.26)$$

with a correlation of 49.6%. Furthermore, the partial decay width of the Z decaying into invisible particles, Γ_{inv} , is calculated. Including correlations, the result is:

$$\begin{aligned} \Gamma_{\text{inv}} &= \Gamma_Z - (\Gamma_{\text{had}} + \Gamma_{\text{ee}} + \Gamma_{\mu\mu} + \Gamma_{\tau\tau}) = \Gamma_Z - (R_\ell^Z + 3 + \delta_\tau) \Gamma_{\ell\ell} \\ &= 500.1 \pm 1.9 \text{ MeV}, \end{aligned} \quad (5.27)$$

where $\delta_\tau = -0.23\%$ takes τ -mass effects into account.

5.1.6.6 The Number of Light Neutrino Species

Within the Standard Model, Z decays into invisible particles are solely given by the decays into neutrino-antineutrino pairs, $Z \rightarrow \nu\bar{\nu}$. Thus the measurement of the invisible width, Γ_{inv} , determines the number of neutrino species, N_ν , which are kinematically accessible in Z decays, $2m_\nu < M_Z$. Instead of using:

$$N_\nu = \frac{\Gamma_{\text{inv}}}{\Gamma_{\nu\nu}^{SM}} \quad (5.28)$$

to determine N_ν , it is more advantageous to calculate:

$$N_\nu = \frac{\Gamma_{\text{inv}}}{\Gamma_{\ell\ell}} \left(\frac{\Gamma_{\ell\ell}}{\Gamma_{\nu\nu}} \right)_{SM}. \quad (5.29)$$

The experimental error on $\Gamma_{\text{inv}}/\Gamma_{\ell\ell}$ is slightly smaller than that of Γ_{inv} . Moreover, radiative corrections in the Standard-Model calculations of the partial widths due to unknown top-quark and Higgs-boson masses cancel to a large extent in the ratio and are not introducing additional sources of error on N_ν . Based on the double ratio, the number of light neutrino species is determined to be:

$$N_\nu = 2.994 \pm 0.011. \quad (5.30)$$

A heavy neutrino with mass m_ν contributes to N_ν by a fractional amount:

$$\Delta N_\nu = \left(1 - \frac{m_\nu^2}{M_Z^2} \right) \sqrt{1 - \frac{4m_\nu^2}{M_Z^2}}. \quad (5.31)$$

Because of the step dependence of ΔN_ν on m_ν for $m_\nu \rightarrow M_Z/2$, the resulting limit on m_ν is just a few tens of MeV below the kinematic limit $M_Z/2$.

The error on the number of hadronic and leptonic events as well as the luminosity theory error contribute to the error δN_ν on the number of neutrinos. Approximately, one finds:

$$\delta N_\nu = 10.5 \frac{\delta N_{\text{had}}}{N_{\text{had}}} \oplus 3.0 \frac{\delta N_{\text{lep}}}{N_{\text{lep}}} \oplus 7.5 \frac{\delta \mathcal{L}}{\mathcal{L}}, \quad (5.32)$$

where $\oplus$ denotes addition in quadrature. The luminosity theory error, $\delta \mathcal{L}/\mathcal{L} = 0.11\%$, is the single largest contribution to the total error on the number of neutrinos, causing an error of 0.008 on N_ν .

5.1.6.7 The Strong Coupling Constant

Measurements involving the hadronic final state are sensitive to the strong coupling constant $\alpha_S(M_Z^2)$, mainly through the final-state QCD correction factor $R_{QCD}^{(\text{had})} = 1 + \Delta_{QCD}^{(\text{had})}(\alpha_S(M_Z^2))$ affecting the hadronic partial width Γ_{had} . Within the Z-pole parameter set, Γ_Z , $\sigma_{\text{had}}^\circ$ and R_ℓ^Z are sensitive to this correction, as is the leptonic pole cross section, $\sigma_\ell^\circ = \sigma_{\text{had}}^\circ / R_\ell^Z$. The dependences are given by:

$$\Gamma_Z \propto 1 + 0.7 \cdot \Delta_{QCD}^{(\text{had})} \quad (5.33)$$

$$\sigma_{\text{had}}^\circ \propto 1 - 0.4 \cdot \Delta_{QCD}^{(\text{had})} \quad (5.34)$$

$$R_\ell^Z \propto 1 + 1.0 \cdot \Delta_{QCD}^{(\text{had})} \quad (5.35)$$

$$\sigma_\ell^\circ \propto 1 - 1.4 \cdot \Delta_{QCD}^{(\text{had})}. \quad (5.36)$$

Within the Z-pole parameter set, R_ℓ^Z has the highest sensitivity to $\Delta_{QCD}^{(\text{had})}(\alpha_S(M_Z^2))$. However, an even larger sensitivity occurs for σ_ℓ° , which is a purely leptonic observable since it is the ratio of the number of lepton-pairs divided by the number of small-angle luminosity Bhabhas. Beyond one-loop order, there are also mixed QCD-electroweak corrections, for example mixed final state radiation of photons and gluons, and gluon exchange in quark-loop insertion of the vector-boson propagators. However, their $\alpha_S(M_Z^2)$ dependence affects observables much less than the direct final-state correction $\Delta_{QCD}^{(\text{had})}(\alpha_S(M_Z^2))$.

Since both R_ℓ^Z and the pole cross sections are ratios of partial widths, other higher-order electroweak radiative corrections depending on top-quark and Higgs-boson masses cancel to a large extent, allowing the determination of $\alpha_S(M_Z^2)$ with reduced uncertainties arising from these electroweak parameters. The systematic error on cross sections due to the luminosity theory uncertainty limits the sensitivity of the pole cross sections to α_S . Considering the leading α_S dependence, $\Delta_{QCD}^{(\text{had})} \simeq \alpha_S/\pi$, the estimated errors on α_S obtained from the four quantities are reported in Table 5.6.

Quantity	Error on $\alpha_S(M_Z^2)$			
	Statistical	Systematic	Luminosity	Total
$\sigma_{\text{had}}^\circ$	$\frac{\pi}{0.4} \sqrt{\frac{1}{N_{\text{had}}}}$ 0.002	$\frac{\pi}{0.4} 5 \cdot 10^{-4}$ 0.004	$\frac{\pi}{0.4} \frac{\delta \mathcal{L}}{\mathcal{L}}$ 0.009	0.010
R_ℓ^Z	$\frac{\pi}{1.0} \sqrt{\frac{1}{N_{\text{had}}} + \frac{1}{N_{\text{lep}}}}$ 0.003	$\frac{\pi}{1.0} 1 \cdot 10^{-3}$ 0.003	— —	0.004
σ_ℓ°	$\frac{\pi}{1.4} \sqrt{\frac{1}{N_{\text{lep}}}}$ 0.002	$\frac{\pi}{1.4} 1 \cdot 10^{-3}$ 0.002	$\frac{\pi}{1.4} \frac{\delta \mathcal{L}}{\mathcal{L}}$ 0.003	0.004

Table 5.6: Estimated errors on α_S derived from different observables showing statistical, experimental systematic and luminosity theory errors as individual error sources. The numerical estimates are based on the combined LEP-I data sample, $N_{\text{had}} = 16 \cdot 10^6$, $N_{\text{lep}} = 17 \cdot 10^5$, and $\delta \mathcal{L}/\mathcal{L} = 0.11\%$.

In the future, when the luminosity theory error is reduced by a factor of two, σ_ℓ° will be the best single quantity to determine $\alpha_S(M_Z^2)$. Currently the quantities R_ℓ^Z and σ_ℓ° determine $\alpha_S(M_Z^2)$ with equal precision. Within the Standard Model framework, taking all $\alpha_S(M_Z^2)$ dependences into account,

one obtains the following results:

$$R_\ell^Z = 20.765 \pm 0.026 \quad (5.37)$$

$$\rightarrow \alpha_S(M_Z^2) = 0.1241 \pm 0.0039 \text{ (exp.)} \pm 0.0017 (M_H), \quad (5.38)$$

and:

$$\sigma_\ell^\circ = 1.9981 \pm 0.0035 \text{ nb} \quad (5.39)$$

$$\rightarrow \alpha_S(M_Z^2) = 0.1221 \pm 0.0039 \text{ (exp.)} \pm 0.0014 (M_H), \quad (5.40)$$

where in each case the constraint $M_t = 173.8 \pm 5.0$ GeV is imposed and the second error arises due to a variation in the mass of the Higgs boson from 90 GeV to 1000 GeV. These results are not independent. A determination of α_S based on the combined data is given in Section 6.2.3.

Theoretical errors on the determination of α_S based on hadronic Z decays are not included in the errors quoted above. They are addressed in [167–169] and are still a matter of debate, with total errors on global averages of α_S ranging from 0.002 to 0.006 [31, 56, 170, 171]. This is discussed again in Section 6.2.3.

5.1.7 Results on the S-Matrix Parameters

For the determination of the S-Matrix parameter, where the γ/Z interference terms are determined from the data, the measurements of fermion-pair production at LEP-II centre-of-mass energies above the Z resonance are included. The reason is that at centre-of-mass energies far away from the Z pole, *e.g.*, $s \gg M_Z^2$, γ exchange and γ/Z interference are no longer dominated by the pure Z exchange since the latter falls off rapidly when moving away from the pole. As shown in Figure 5.19 for hadron production, the LEP-II measurements are particularly sensitive to the γ/Z interference if a cut $s' > M_Z^2$ is applied, rejecting events returning to the Z pole, $s' \approx M_Z^2$, through initial-state photon radiation.

The four sets of S-Matrix parameters obtained by the LEP experiments and the corresponding LEP averages are shown in Tables 5.7 and 5.9. The correlation matrices are shown in Tables 5.8 and 5.10. The determination of the eight-parameter average is based on the four sixteen-parameter sets, where in the averaging procedure lepton universality is imposed. As indicated by the $\chi^2/\text{d.o.f}$ of the averages, the parameters describe the experimental measurements of cross sections and forward-backward asymmetries, and furthermore the results of the four LEP experiments are in agreement. The largest deviation between the individual measurements is seen for the parameter j_ℓ^{fb} , scaling the γZ interference contribution to the leptonic forward-backward cross section and therefore essentially determining the centre-of-mass energy dependence of the leptonic forward-backward asymmetries.

Large correlations appear between the S-Matrix parameters. The correlations between the r_f parameters and the total width Γ_Z are a consequence of the parameter definition. They are not visible in case of the Z-pole parameters, because those are chosen to be as uncorrelated as possible.

Parameter	ALEPH	DELPHI	L3	OPAL	LEP	MSM
M_Z [GeV]	91.1951±0.0056	91.1841±0.0056	91.1870±0.0056	91.1879±0.0055	91.1884±0.0031	—
Γ_Z [GeV]	2.4939±0.0044	2.4897±0.0041	2.5006±0.0043	2.4946±0.0044	2.4945±0.0025	2.4920±0.0042
$r_{\text{had}}^{\text{tot}}$	2.966±0.010	2.957±0.010	2.972±0.010	2.962±0.010	2.9637±0.0063	2.9587±0.0098
r_e^{tot}	0.14361±0.00076	0.1412±0.0010	0.14171±0.00088	0.1418±0.0011	0.14229±0.00049	0.14240±0.00040
r_μ^{tot}	0.14248±0.00062	0.14274±0.00069	0.14257±0.00083	0.14228±0.00066	0.14253±0.00036	0.14240±0.00040
r_τ^{tot}	0.14313±0.00067	0.14140±0.00097	0.1433±0.0011	0.14118±0.00088	0.14247±0.00043	0.14240±0.00040
$j_{\text{had}}^{\text{tot}}$	-0.18±0.27	0.36±0.28	0.30±0.28	0.03±0.27	0.13±0.14	0.2143±0.0084
j_e^{tot}	-0.007±0.041	-0.037±0.045	-0.011±0.045	-0.123±0.060	-0.028±0.023	0.00413±0.00030
j_μ^{tot}	-0.018±0.030	0.052±0.030	0.028±0.036	-0.012±0.037	0.013±0.016	0.00413±0.00030
j_τ^{tot}	-0.012±0.032	0.017±0.037	0.042±0.039	-0.003±0.042	0.010±0.018	0.00413±0.00030
r_e^{fb}	0.00303±0.00072	0.00326±0.00096	0.0025±0.0013	0.0016±0.0010	0.00270±0.00046	0.00262±0.00021
r_μ^{fb}	0.00288±0.00048	0.00267±0.00053	0.00323±0.00067	0.00262±0.00050	0.00279±0.00027	0.00262±0.00021
r_τ^{fb}	0.00288±0.00055	0.00376±0.00075	0.00419±0.00094	0.00318±0.00066	0.00329±0.00034	0.00262±0.00021
j_e^{fb}	0.861±0.058	0.813±0.073	0.644±0.080	0.778±0.068	0.786±0.034	0.7986±0.0010
j_μ^{fb}	0.826±0.036	0.759±0.034	0.838±0.046	0.724±0.036	0.780±0.019	0.7986±0.0010
j_τ^{fb}	0.846±0.041	0.745±0.047	0.788±0.057	0.727±0.042	0.778±0.023	0.7986±0.0010
$\chi^2/\text{d.o.f}$	180/189 (67%)	233/195 (3%)	156/183 (93%)	109/155 (99.8%)	54/48 (26%)	—

Table 5.7: S-Matrix parameters of the four LEP experiments and their average not assuming neutral-current lepton universality [163].

	M_Z	Γ_Z	$r_{\text{had}}^{\text{tot}}$	r_e^{tot}	r_μ^{tot}	r_τ^{tot}	$j_{\text{had}}^{\text{tot}}$	j_e^{tot}	j_μ^{tot}	j_τ^{tot}	r_e^{fb}	r_μ^{fb}	r_τ^{fb}	j_e^{fb}	j_μ^{fb}	j_τ^{fb}
M_Z	1.00	-.10	-.07	-.03	-.06	-.05	-.75	-.25	-.30	-.28	.04	.11	.09	.01	-.02	-.01
Γ_Z	-.10	1.00	.80	.51	.51	.43	.16	.03	.07	.06	.00	.00	.00	.02	.05	.04
$r_{\text{had}}^{\text{tot}}$	-.07	.80	1.00	.59	.66	.55	.13	.01	.05	.05	.01	.01	.01	.03	.06	.05
r_e^{tot}	-.03	.51	.59	1.00	.42	.35	.09	.04	.04	.04	.03	.00	.00	.05	.04	.03
r_μ^{tot}	-.06	.51	.66	.42	1.00	.46	.10	.01	.14	.04	.01	.03	.01	.03	.10	.04
r_τ^{tot}	-.05	.43	.55	.35	.46	1.00	.09	.01	.04	.13	.01	.01	.03	.02	.04	.09
$j_{\text{had}}^{\text{tot}}$	-.75	.16	.13	.09	.10	.09	1.00	.27	.32	.30	-.04	-.11	-.09	.00	.03	.01
j_e^{tot}	-.25	.03	.01	.04	.01	.01	.27	1.00	.11	.10	.01	-.04	-.03	.02	.01	.00
j_μ^{tot}	-.30	.07	.05	.04	.14	.04	.32	.11	1.00	.12	-.01	.00	-.03	.00	.02	.00
j_τ^{tot}	-.28	.06	.05	.04	.04	.13	.30	.10	.12	1.00	-.01	-.04	.00	.00	.01	.01
r_e^{fb}	.04	.00	.01	.03	.01	.01	-.04	.01	-.01	-.01	1.00	.01	.01	.09	.00	.00
r_μ^{fb}	.11	.00	.01	.00	.03	.01	-.11	-.04	.00	-.04	.01	1.00	.03	.00	.17	.00
r_τ^{fb}	.09	.00	.01	.00	.01	.03	-.09	-.03	-.03	.00	.01	.03	1.00	.00	.00	.16
j_e^{fb}	.01	.02	.03	.05	.03	.02	.00	.02	.00	.00	.09	.00	.00	1.00	.00	.00
j_μ^{fb}	-.02	.05	.06	.04	.10	.04	.03	.01	.02	.01	.00	.17	.00	.00	1.00	.00
j_τ^{fb}	-.01	.04	.05	.03	.04	.09	.01	.00	.00	.01	.00	.00	.16	.00	.00	1.00

Table 5.8: Correlation coefficients of sixteen S-Matrix parameters [163].

Parameter	ALEPH	DELPHI	L3	OPAL	LEP	MSM
M_Z [GeV]	91.1951 ± 0.0056	91.1837 ± 0.0056	91.1857 ± 0.0056	91.1866 ± 0.0054	91.1882 ± 0.0031	—
Γ_Z [GeV]	2.4939 ± 0.0044	2.4896 ± 0.0041	2.5002 ± 0.0043	2.4945 ± 0.0044	2.4945 ± 0.0024	2.4920 ± 0.0042
$r_{\text{had}}^{\text{tot}}$	2.966 ± 0.010	2.956 ± 0.010	2.971 ± 0.010	2.962 ± 0.010	2.9637 ± 0.0062	2.9587 ± 0.0098
r_{ℓ}^{tot}	0.14293 ± 0.00055	0.14211 ± 0.00061	0.14264 ± 0.00066	0.14188 ± 0.00060	0.14245 ± 0.00032	0.14240 ± 0.00040
$j_{\text{had}}^{\text{tot}}$	-0.18 ± 0.27	0.38 ± 0.28	0.34 ± 0.28	0.08 ± 0.27	0.14 ± 0.14	0.2143 ± 0.0084
j_{ℓ}^{tot}	-0.012 ± 0.022	0.024 ± 0.023	0.031 ± 0.025	-0.013 ± 0.027	0.004 ± 0.012	0.00413 ± 0.00030
r_{ℓ}^{fb}	0.00292 ± 0.00033	0.00306 ± 0.00040	0.00327 ± 0.00050	0.00264 ± 0.00037	0.00292 ± 0.00019	0.00262 ± 0.00021
j_{ℓ}^{fb}	0.840 ± 0.025	0.761 ± 0.026	0.788 ± 0.033	0.733 ± 0.025	0.780 ± 0.013	0.7986 ± 0.0010
$\chi^2/\text{d.o.f}$	183/197 (75%)	241/203 (3%)	164/191 (92%)	116/163 (99.8%)	59/56 (37%)	—

Table 5.9: S-Matrix parameters of the four LEP experiments and their average assuming neutral-current lepton universality [163].

	M_Z	Γ_Z	$r_{\text{had}}^{\text{tot}}$	r_{ℓ}^{tot}	$j_{\text{had}}^{\text{tot}}$	j_{ℓ}^{tot}	r_{ℓ}^{fb}	j_{ℓ}^{fb}
M_Z	1.00	-0.13	-0.09	-0.08	-0.75	-0.43	0.14	-0.02
Γ_Z	-0.13	1.00	0.80	0.61	0.16	0.09	0.00	0.07
$r_{\text{had}}^{\text{tot}}$	-0.09	0.80	1.00	0.77	0.13	0.06	0.02	0.09
r_{ℓ}^{tot}	-0.08	0.61	0.77	1.00	0.12	0.12	0.03	0.12
$j_{\text{had}}^{\text{tot}}$	-0.75	0.16	0.13	0.12	1.00	0.47	-0.14	0.03
j_{ℓ}^{tot}	-0.43	0.09	0.06	0.12	0.47	1.00	-0.05	0.02
r_{ℓ}^{fb}	0.14	0.00	0.02	0.03	-0.14	-0.05	1.00	0.15
j_{ℓ}^{fb}	-0.02	0.07	0.09	0.12	0.03	0.02	0.15	1.00

Table 5.10: Correlations of eight S-Matrix parameters [163].

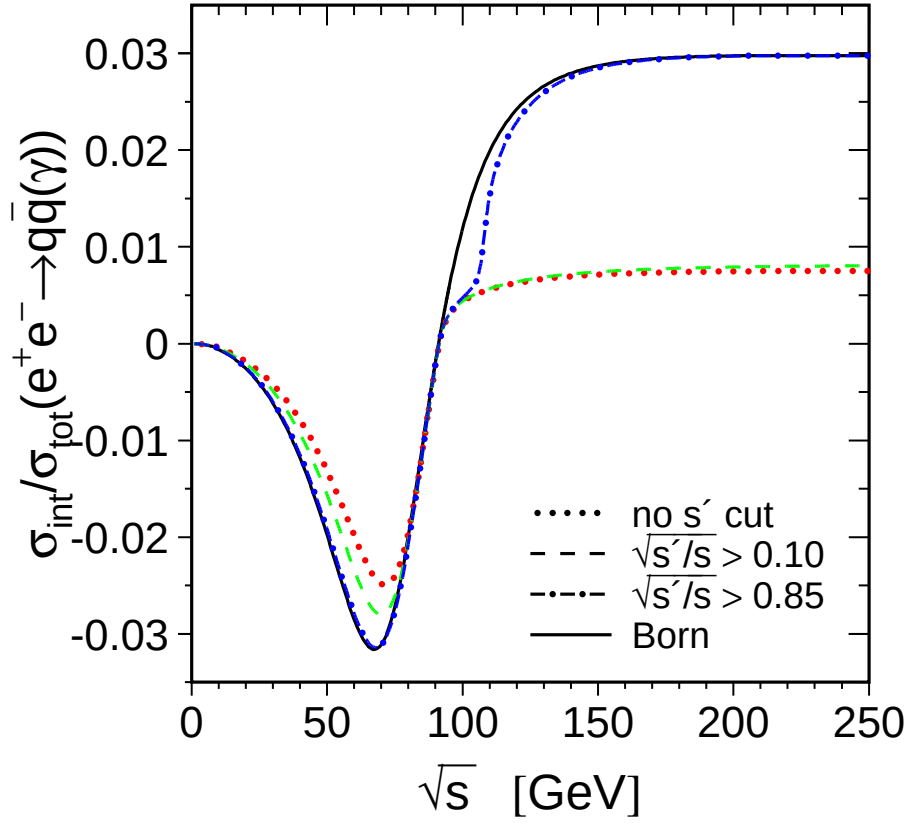


Figure 5.19: Ratio of the interference cross section to the total cross section for $q\bar{q}$ production summed over the five light quark flavours, comparing Born term with radiatively corrected cross sections under various cuts in $\sqrt{s'}$: no $\sqrt{s'}$ cut, $\sqrt{s'}/s > 0.10$, and $\sqrt{s'}/s > 0.85$. Note the step in the latter when the radiative return to the Z is cut out at $\sqrt{s} = M_Z/0.85 = 107$ GeV.

5.1.7.1 The Mass of the Z Boson (II)

A new large correlation, -75% , arises between the mass of the Z boson, M_Z , and the hadronic γ/Z interference term, $j_{\text{had}}^{\text{tot}}$. In the analysis based on the Z-pole parameters, this correlation is not visible as $j_{\text{had}}^{\text{tot}}$ is fixed to its Standard-Model value. The new correlation leads to an increase in the error on M_Z in models where $j_{\text{had}}^{\text{tot}}$ is left free:

$$M_Z = 91188.2 \pm 3.1 \text{ MeV} \quad (5.41)$$

$$j_{\text{had}}^{\text{tot}} = 0.14 \pm 0.14, \quad (5.42)$$

with a correlation between the two results of -75% . The contours of 68% and 95% probability in the $(M_Z, j_{\text{had}}^{\text{tot}})$ plane are shown in Figure 5.20, comparing the results based on LEP-I data only to those including also the LEP-II data. As expected, the error on $j_{\text{had}}^{\text{tot}}$ and consequently the error on M_Z are considerably reduced by using total cross-section measurements at centre-of-mass energies far away from the Z pole [172].

The TOPAZ collaboration at the e^+e^- collider TRISTAN at KEK has performed a measurement of the total hadronic cross section at $\sqrt{s} = 57.77$ GeV, $\sigma_{\text{tot}}^0 = 143.6 \pm 1.5(\text{stat.}) \pm 4.5(\text{syst.})$ pb [173]. Combining this measurement with the LEP results yields:

$$M_Z = 91188.2 \pm 2.9 \text{ MeV} \quad (5.43)$$

$$j_{\text{had}}^{\text{tot}} = 0.14 \pm 0.12, \quad (5.44)$$

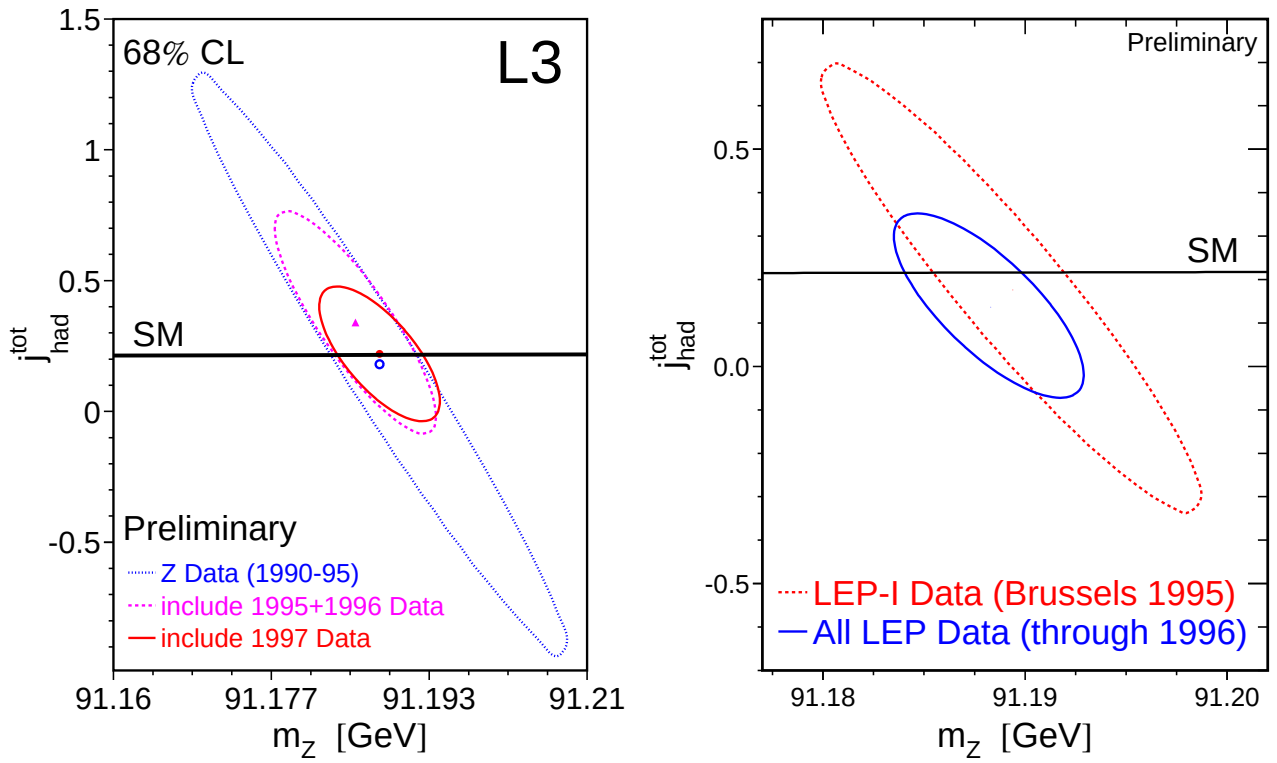


Figure 5.20: Correlation contours of 68% probability in the $(M_Z, j_{\text{had}}^{\text{tot}})$ plane as measured by L3 [160] and the combined LEP result [163].

and the correlation between M_Z and $j_{\text{had}}^{\text{tot}}$ is reduced from -75% to -71% .

Radiative hadron production, $e^+e^- \rightarrow q\bar{q}\gamma$, at the Z pole, $s = M_Z^2$, also explores the energy region below the Z-pole through initial-state radiation lowering the centre-of-mass energy of the colliding e^+e^- pair, and could thus be used to constrain $j_{\text{had}}^{\text{tot}}$. The ALEPH collaboration has performed such an analysis of j_f^{tot} for dimuon production [174]. On the other side of the Z pole in the energy region of LEP-II, $s \gg M_Z^2$, further measurements of the total hadronic cross section excluding the radiative return to the Z will reduce the error on $j_{\text{had}}^{\text{tot}}$ consequently the error on M_Z further, approaching the error on M_Z obtained in the Z-pole parameter set.

5.2 Tau Polarisation

5.2.1 Measurements

The weak charged current decay of the τ lepton violates parity conservation due to its $V - A$ structure. This offers the unique experimental possibility to measure the longitudinal polarisation, *i.e.*, average helicity h , of fermion pairs produced in e^+e^- interactions, here in case of τ pairs.

The LEP experiments analyse up to five different τ decay modes, $\tau \rightarrow e\nu\bar{\nu}$, $\mu\nu\bar{\nu}$, $\pi\nu$, $\rho\nu$ and $a_1\nu$, for polarisation at LEP-I. The τ polarisation, $\mathcal{P}_\tau$, is measured by obtaining the linear combination of the $h_\tau = +1$, $h_\tau = -1$ Monte-Carlo signal and non- τ background distributions of polarisation sensitive variables which best fits the data. Depending on the τ decay mode, different polarisation sensitive variables are analysed. Since fully simulated Monte Carlo events are used, all detector effects influencing the measurement are automatically taken into account and implicitly corrected.

For the e , μ and single- π decay modes, the polarisation sensitive variable is simply the energy, E , of the visible τ decay product. For leptonic τ decays, the energy spectrum of the charged decay product in terms of the scaled energy $x = E/E_\tau$ is given by:

$$\frac{1}{\Gamma} \frac{d\Gamma}{dx} = \frac{1}{3} (5 - 9x^2 + 4x^2) + \mathcal{P}_\tau \cdot \frac{1}{3} (1 - 9x^2 + 8x^3) . \quad (5.45)$$

The measured distribution is shown in Figures 5.21. For hadronic decay modes $\tau \rightarrow \nu_\tau h$, the energy spectrum is linear:

$$\frac{1}{\Gamma} \frac{d\Gamma}{dx} = 1 + \mathcal{P}_\tau \cdot \alpha_h (2x - 1) \quad (5.46)$$

$$\alpha_h = \frac{m_\tau^2 - 2m_h^2}{m_\tau^2 + 2m_h^2} < 1 \quad (5.47)$$

and shown in Figure 5.22. While for pions $\alpha_\pi = 1$, the sensitivity of the energy spectrum to $\mathcal{P}_\tau$ is reduced in case of τ decays to heavy mesons. For the ρ and a_1 decay channels, the polarisation of these spin-1 mesons also carries information on the τ polarisation, which is recovered by also analysing the decay angles of the subsequent ρ and a_1 decays into π mesons. In $\tau^+\tau^-$ events where both τ leptons decay into hadrons, it is possible to reconstruct the τ direction relative to the direction of the decay products, which is needed in order to reach the maximal possible sensitivity to the τ polarisation [175].

The multidimensional analyses in case of τ decays via spin-1 mesons is reduced to one-dimensional analyses by employing the concept of optimal observables [175]. A single optimal variable, ω , is constructed with the same sensitivity to the τ polarisation as the multidifferential cross section. This is possible because in generalisation of the above equations also the fully differential cross section is linear in $\mathcal{P}_\tau$:

$$\frac{d\sigma(\Omega, \mathcal{P}_\tau)}{d\Omega} = C_0(\Omega) + \mathcal{P}_\tau \cdot C_1(\Omega) \quad (5.48)$$

$$\omega(\Omega) = \frac{C_1(\Omega)}{C_0(\Omega)} , \quad (5.49)$$

where Ω is the set of phase-space variables describing the final state. Including the reconstruction of the tau direction, the sensitivity of the ρ and a_1 decays increases to the maximal possible value. The distribution of the optimal observable used in $\tau \rightarrow \rho\nu$ decays is shown in Figure 5.22.

The large number of $\tau^+\tau^-(\gamma)$ events collected by the experiments at LEP-I makes it possible to determine the τ polarisation, $\mathcal{P}_\tau$, as a function of the polar scattering angle, $\cos\theta$. The results of the polarisation measurements are then quoted in terms of the coupling parameters A_e and A_τ as suggested by the lowest order formula describing the dependence of the τ polarisation on the polar scattering angle, evaluated at $\sqrt{s} = M_Z$:

$$\mathcal{P}_\tau(\cos\theta) = -\frac{A_\tau(1 + \cos^2\theta) + 2A_e \cos\theta}{(1 + \cos^2\theta) + 2A_e A_\tau \cos\theta} . \quad (5.50)$$

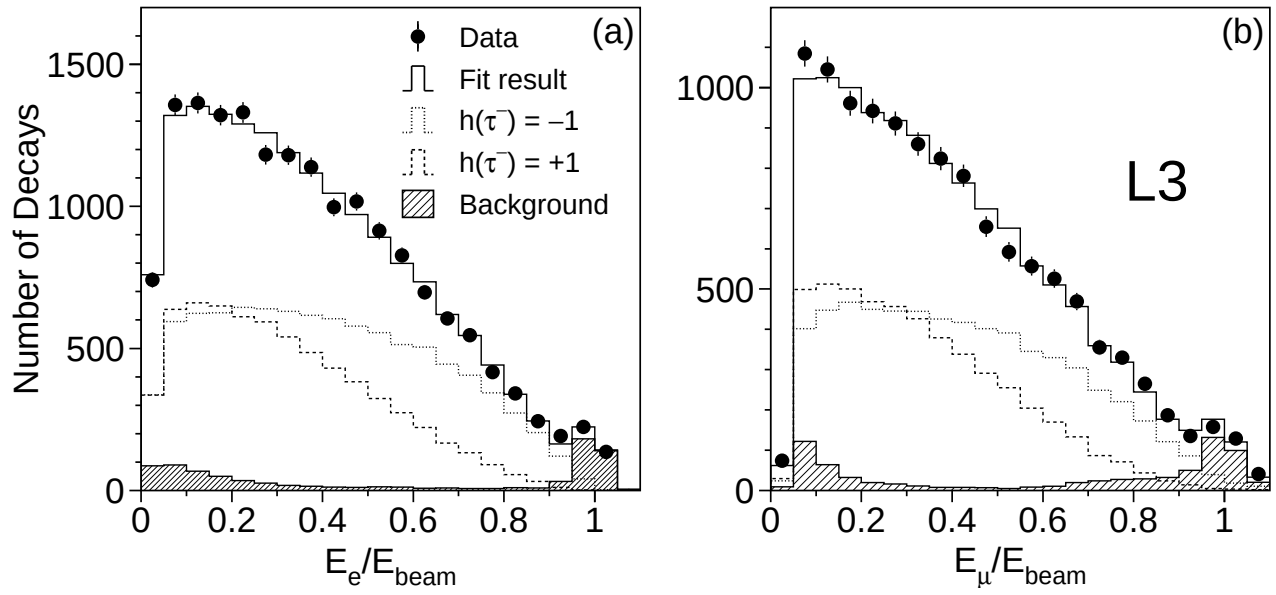


Figure 5.21: Distribution of electron and muon energies in leptonic τ decays as measured by L3 at LEP-I [176].

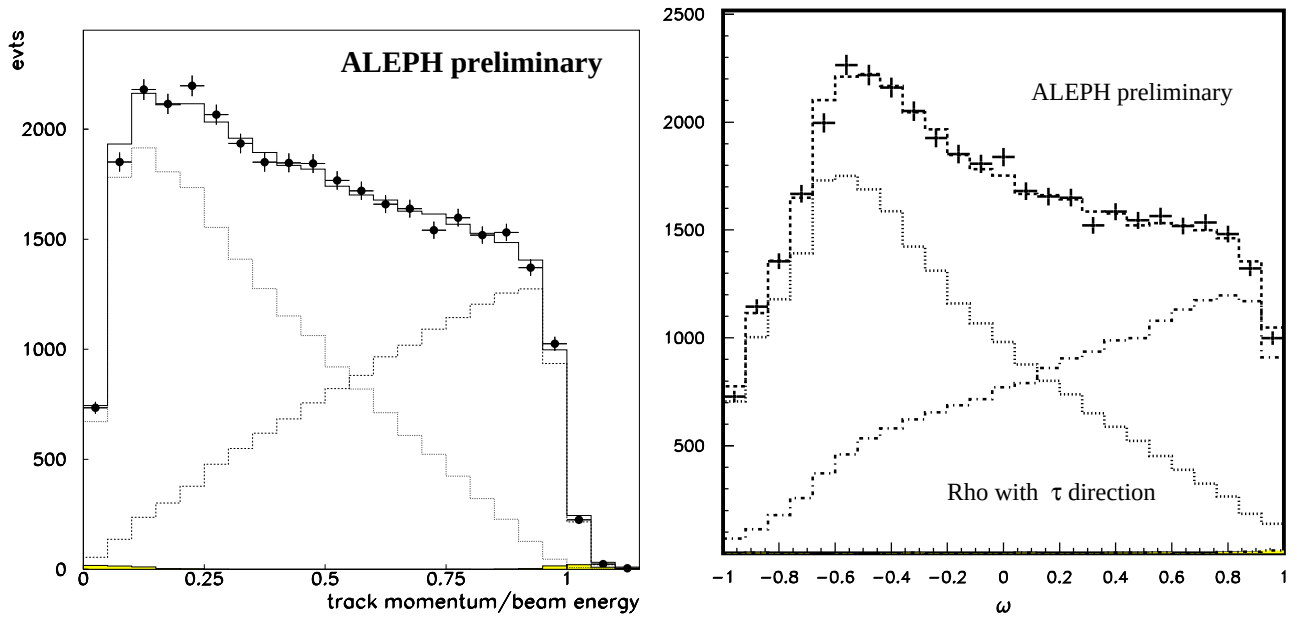


Figure 5.22: Distribution of pion energy in $\tau \rightarrow \pi\nu$ decay and of the optimal observable ω in $\tau \rightarrow \rho\nu$ decays as measured by ALEPH at LEP-I [177].

The above equation for $\mathcal{P}_\tau(\cos \theta)$ neglects corrections for the effects of $\sqrt{s}$ dependence, γ exchange, γ/Z interference, QED initial- and final-state radiation including helicity-flip configurations. They are implicitly taken into account in the experimental analyses through the use of the Monte-Carlo simulations. They amount to corrections of $+0.003$ on both A_e and A_τ with a negligible theoretical uncertainty. The data collected at the off-peak centre-of-mass energies are included. This is possible since the τ polarisation does not depend strongly on $\sqrt{s}$ so that the SM energy dependence may be used to transport the measurements to $\sqrt{s} = M_Z$. The τ polarisation measured as a function of $\cos \theta$ is shown in Figure 5.23.

The experimental analyses implicitly assume the charged current $V - A$ theory for τ decays. In the case this assumption is dropped the uncertainties in the measurements of the Michel parameters [31, 178, 179] describing τ decays increase the error of the extracted $\mathcal{P}_\tau$ polarisation. The additional error on $\mathcal{P}_\tau$ is dominated by the error $\delta\xi$ in the Michel parameter ξ :

$$\delta\mathcal{P}_\tau = |\mathcal{P}_\tau \cdot \delta\xi|, \quad (5.51)$$

as it is mainly the product $\xi\mathcal{P}_\tau$ which is measured. The LEP experiments also analyse the $\tau^+\tau^-$ data in terms of the Michel parameters. In these analyses the τ polarisation is determined together with the Michel parameters. The error is larger but the central values agree, because the Michel parameters are as expected for a pure $V - A$ charged current τ decay. Further details on experimental aspects of the measurement of $\mathcal{P}_\tau$ are reviewed in [180–182]. For the results quoted in the following, the charged current $V - A$ hypothesis is assumed without any associated error.

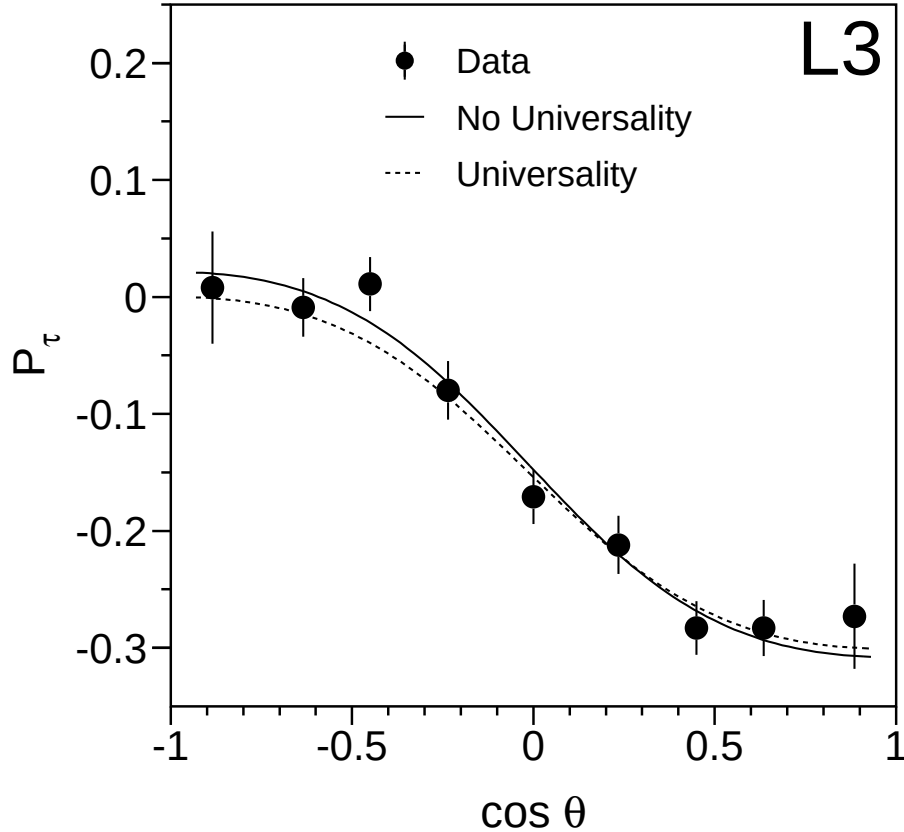


Figure 5.23: Tau polarisation, $\mathcal{P}_\tau$, as a function of the polar scattering angle, $\cos \theta$, as measured by the L3 collaboration based on the complete LEP-I data set [176]. The universality fit imposes the constraint $A_e = A_\tau$.

5.2.2 Averaging Procedure and Results

The averaging procedure consists of several steps. First, within each experiment the τ -polarisation measurements are averaged over all LEP-I centre-of-mass energies and τ -decay channels analysed for polarisation. This average takes correlated experimental systematic errors due to detector effects such as energy calibrations and resolutions properly into account. The results on A_e and A_τ of each LEP experiment, in good agreement with each other, are shown in Table 5.11 and in Figure 5.24. The systematic error on the average τ polarisation, A_τ , is about thrice as large as that of A_e and only slightly smaller than the statistical error. The measurements of A_e are still statistics limited. The errors are treated as uncorrelated when averaging the A_e results and the A_τ results.

In a second step, the results are averaged between the four LEP experiments, resulting in best LEP values for A_e and A_τ :

$$A_e = 0.1479 \pm 0.0051 \quad (5.52)$$

$$A_\tau = 0.1431 \pm 0.0045. \quad (5.53)$$

The two averages have a $\chi^2/\text{d.o.f}$ of 5.2/3 and 1.0/3, corresponding to a probability of 18% and 80%, respectively. The two results agree, as is expected from lepton universality of the neutral weak current. Their average is:

$$A_\ell = 0.1452 \pm 0.0034, \quad (5.54)$$

where the $\chi^2/\text{d.o.f}$ of the average has a value of 0.5/1, corresponding to a probability of 48%. The errors are treated as uncorrelated when combining the averages of A_e and A_τ .

For an improved averaging procedure, errors correlated between the experiments need to be identified and quantified. A direct source of common uncertainties is given by hadronic τ decays, which are not calculated from first principles but modelled using structure functions to describe the hadronic current in the τ decay matrix element. In particular, there are two sources of uncertainties:

1. Radiative corrections in $\tau \rightarrow \pi\nu$ and $\tau \rightarrow \rho\nu$ decays.

These two τ -decay channels carry the largest weight in the τ polarisation analysis. In contrast to leptonic τ decays, which are calculable from first principles since only pointlike particles interacting electroweakly are involved, calculations of QED radiative corrections suffer from uncertainties induced by the presence of a hadronic current.

2. Modelling of the $\tau \rightarrow a_1\nu$ decay.

The $\tau \rightarrow a_1$ decay is least constrained by experimental data on τ decays. Varying the model parameters within their allowed ranges leads to sizeable effects. Because of the relatively small weight of this channel compared to the other decay channels, the uncertainty on the average τ polarisation is much reduced.

For their final τ -polarisation analysis, L3 has estimated the uncertainties due to these effects to be 0.0001 on A_e , 0.0010 on A_τ , and 0.0007 on the average of A_e and A_τ [176].

Another interesting averaging strategy would be to first average the results of the four LEP experiments for each channel separately. This would yield best LEP values for the τ polarisation in each τ decay mode investigated and constitutes an interesting check of possible problems in τ decays or even new physics effects specific for certain final states. For that procedure, the experiments need to provide more details on their analyses in order to treat correlated systematic errors in the average properly.

Experiment	Status	A_τ	A_e
ALEPH	(90 - 95), prel.	$0.1452 \pm 0.0052 \pm 0.0032$	$0.1505 \pm 0.0069 \pm 0.0010$
DELPHI	(90 - 95), prel.	$0.1381 \pm 0.0079 \pm 0.0067$	$0.1353 \pm 0.0116 \pm 0.0033$
L3	(90 - 95), final	$0.1476 \pm 0.0088 \pm 0.0062$	$0.1678 \pm 0.0127 \pm 0.0030$
OPAL	(90 - 94), final	$0.134 \pm 0.009 \pm 0.010$	$0.129 \pm 0.014 \pm 0.005$
LEP	Average	$0.1431 \pm 0.0036 \pm 0.0027$	$0.1479 \pm 0.0050 \pm 0.0010$
$\chi^2/\text{d.o.f}$		1.0/3 (80%)	5.2/3 (18%)
MSM		0.1419 ± 0.0051	0.1419 ± 0.0051

Table 5.11: Measurements of A_τ and A_e in τ polarisation and their averages [163]. The first error is statistical and the second systematic.

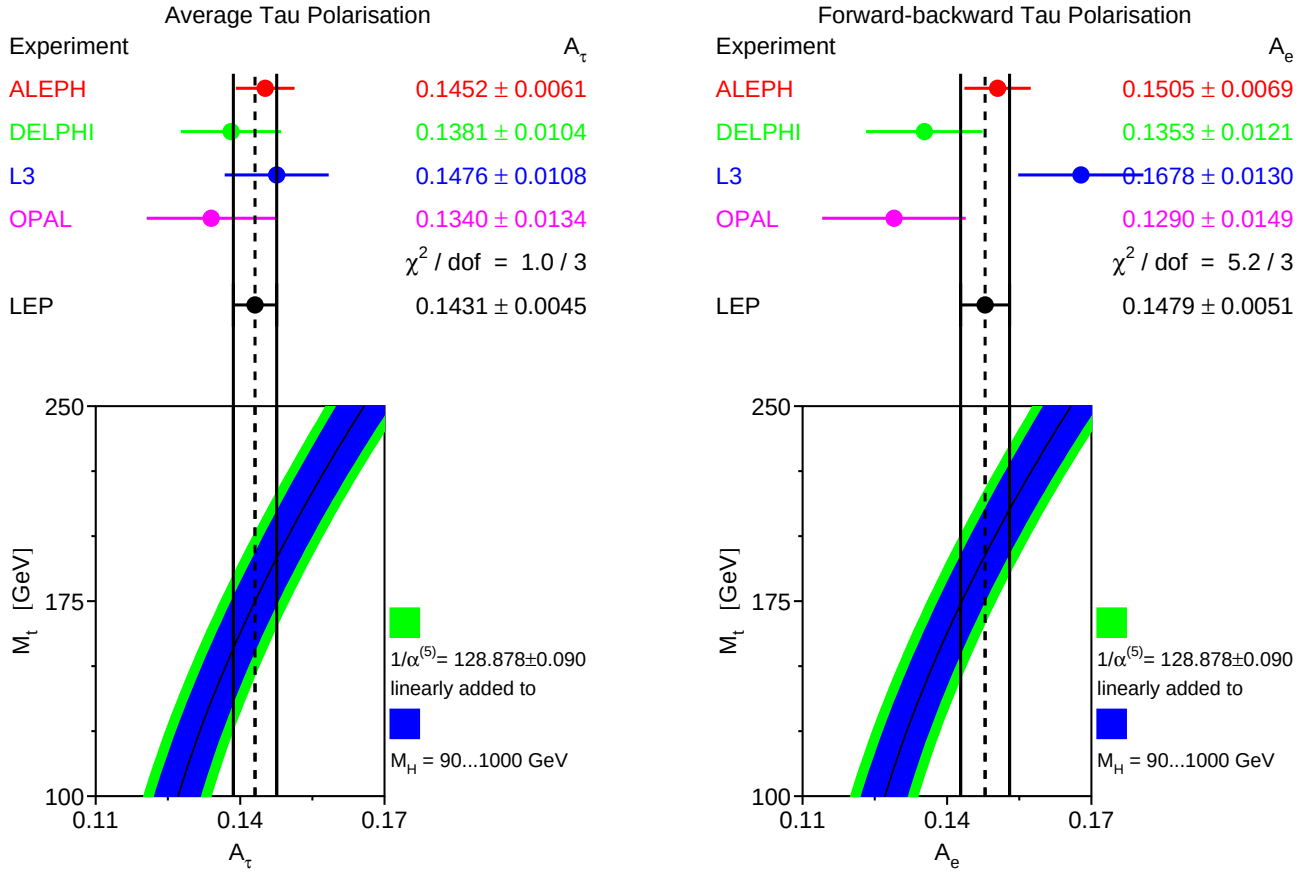


Figure 5.24: Comparison of the LEP results on A_e and A_τ from τ polarisation.

5.3 Left-Right Asymmetries

5.3.1 Measurements

The expressions for the differential and total cross sections of e^+e^- interactions with polarised electron beams are discussed in Section 3.1.1.6. The measurement of the left-right asymmetry is very robust as it requires simply counting Z events for each beam polarisation and forming the corresponding

asymmetry. The analysis is performed for hadronic events, $e^+e^- \rightarrow \text{hadrons}(\gamma)$. For leptonic final states, $e^+e^- \rightarrow \ell^+\ell^-(\gamma)$, it is rather straight forward to measure as well the $\cos\theta$ distribution of the event samples obtained by left-handed and right-handed electron beams. They are shown Figure 5.25 for all three charged lepton flavours. From a fit to the distributions obtained with left-handed and right-handed electron beams, the initial-state coupling parameter A_e and the final-state coupling parameter A_f are determined simultaneously. This way both the left-right asymmetry and the forward-backward left-right asymmetry are determined. In case of e^+e^- production, the additional t -channel and s/t -interference contributions are clearly visible and are accounted for by adding corresponding terms to the s -channel expressions of the fitting formulae.

5.3.2 Results

The average of the A_{lr} measurements from the different SLD data taking periods with different degrees of electron beam polarisation is given by:

$$A_{lr} = 0.1504 \pm 0.0023. \quad (5.55)$$

The average has a $\chi^2/\text{d.o.f}$ of 6.9/5, corresponding to a probability of 23%. The result includes the leptonic polarised forward-backward asymmetries, A_{fblr} , but is dominated by the left-right asymmetry, A_{lr} . The systematic error, about a factor of two smaller than the statistical error, is dominated by the error on the measurement of the beam polarisation P discussed in Section 4.4. The measurement of A_{lr} is by far the most precise determination of the leptonic coupling parameter A_e .

5.4 Heavy Flavours

At SLC and at LEP, the name heavy flavours stands for the heavy c and b quarks, as top quarks are not produced. The large data sample of $q\bar{q}$ events collected by the LEP experiments at the Z pole, $4 \cdot 10^6$ events on average per experiment, corresponds to more than 800,000 $b\bar{b}$ events and nearly 700,000 $c\bar{c}$ events per experiment. The SLD collaboration collected more than 500,000 $q\bar{q}$ events, corresponding to more than 100,000 $b\bar{b}$ events and nearly 100,000 $c\bar{c}$ events.

5.4.1 Measurements

5.4.1.1 Electroweak Parameters

With the help of tagging b and c quark jets as discussed below, samples of $b\bar{b}$ and $c\bar{c}$ events are selected. For $b\bar{b}$ and $c\bar{c}$ final states, the same electroweak parameters as for leptons are determined:

- The ratio of partial Z decay widths for heavy quarks and hadrons:

$$R_q^Z = \frac{\Gamma_{qq}}{\Gamma_{\text{had}}} \quad q = b, c. \quad (5.56)$$

For historical reasons, R_q^Z is defined differently than $R_\ell^Z = \Gamma_{\text{had}}/\Gamma_{\ell\ell}$.

- The pole forward-backward asymmetries in heavy quark production:

$$A_{fb}^{0,q} = \frac{3}{4} A_e A_q \quad q = b, c. \quad (5.57)$$

- The polarised forward-backward asymmetries, measured by SLD at SLC only, written in terms of the coupling parameter A_q :

$$A_q = \frac{4}{3} A_{fblr}^{0,q} \quad q = b, c. \quad (5.58)$$

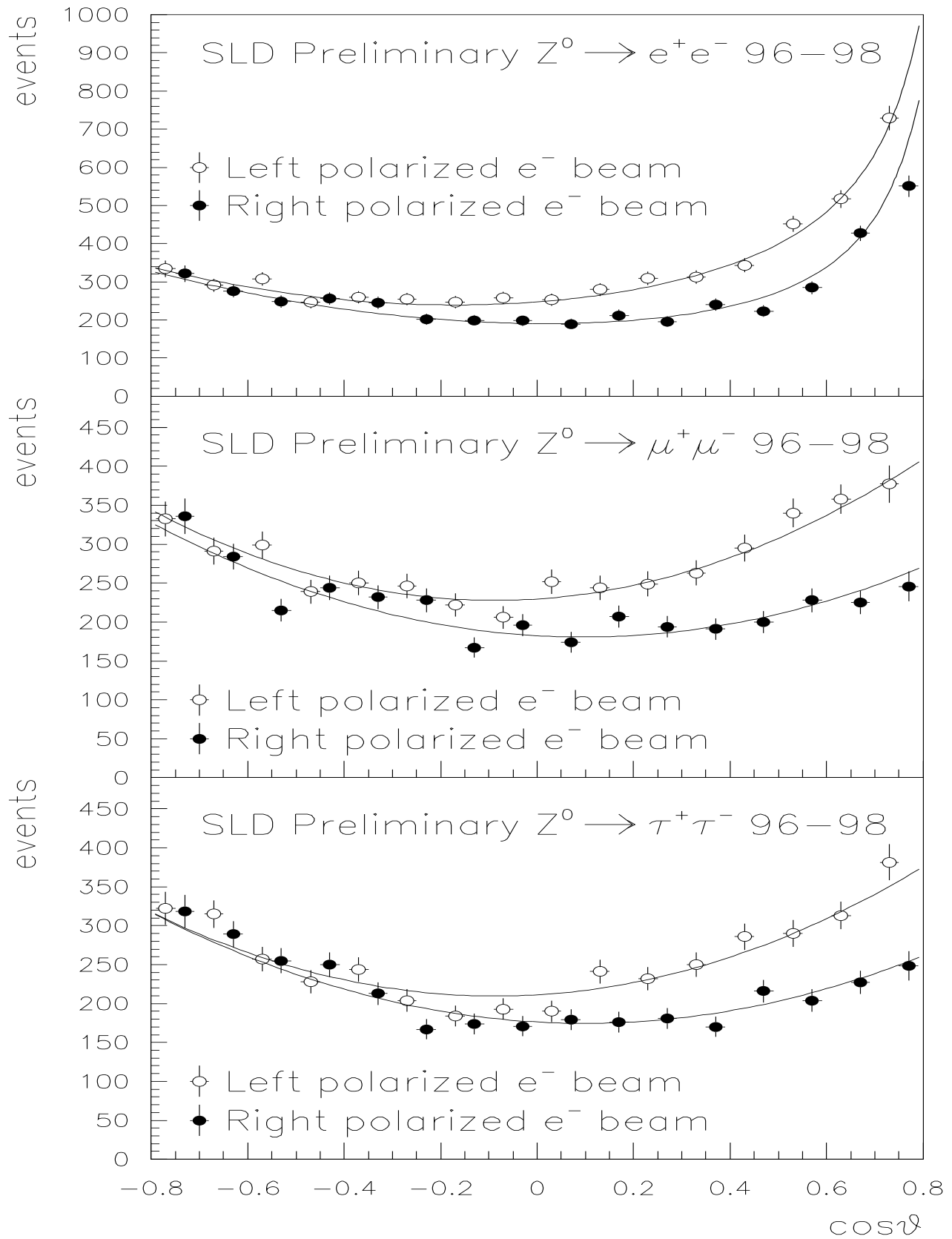


Figure 5.25: Differential cross sections in $\ell^+\ell^-$ production measured by SLD at SLC with polarised electron beams [125].

5.4.1.2 Heavy Flavour Tagging

Samples of $b\bar{b}$ and $c\bar{c}$ events are selected from an inclusive sample of hadronic events based on heavy flavour tags. Several tagging methods have been developed in order to separate $b\bar{b}$ and $c\bar{c}$ events from light quark events. For a review see [183,184]. They are based on specific decay properties of heavy quarks as shown in Figure 5.26, such as semileptonic decays, long lifetimes, high masses, and hard fragmentation of heavy quarks, as well as on exclusive reconstruction of hadrons containing b and c quarks. Distributions of some tagging variables are shown in Figure 5.27. The b tag efficiencies and purities obtained by the experiments are compared in Figure 5.28. The separation of b and c quarks, based on the two variables vertex mass and vertex momentum determined for secondary vertices, is shown in Figure 5.29.

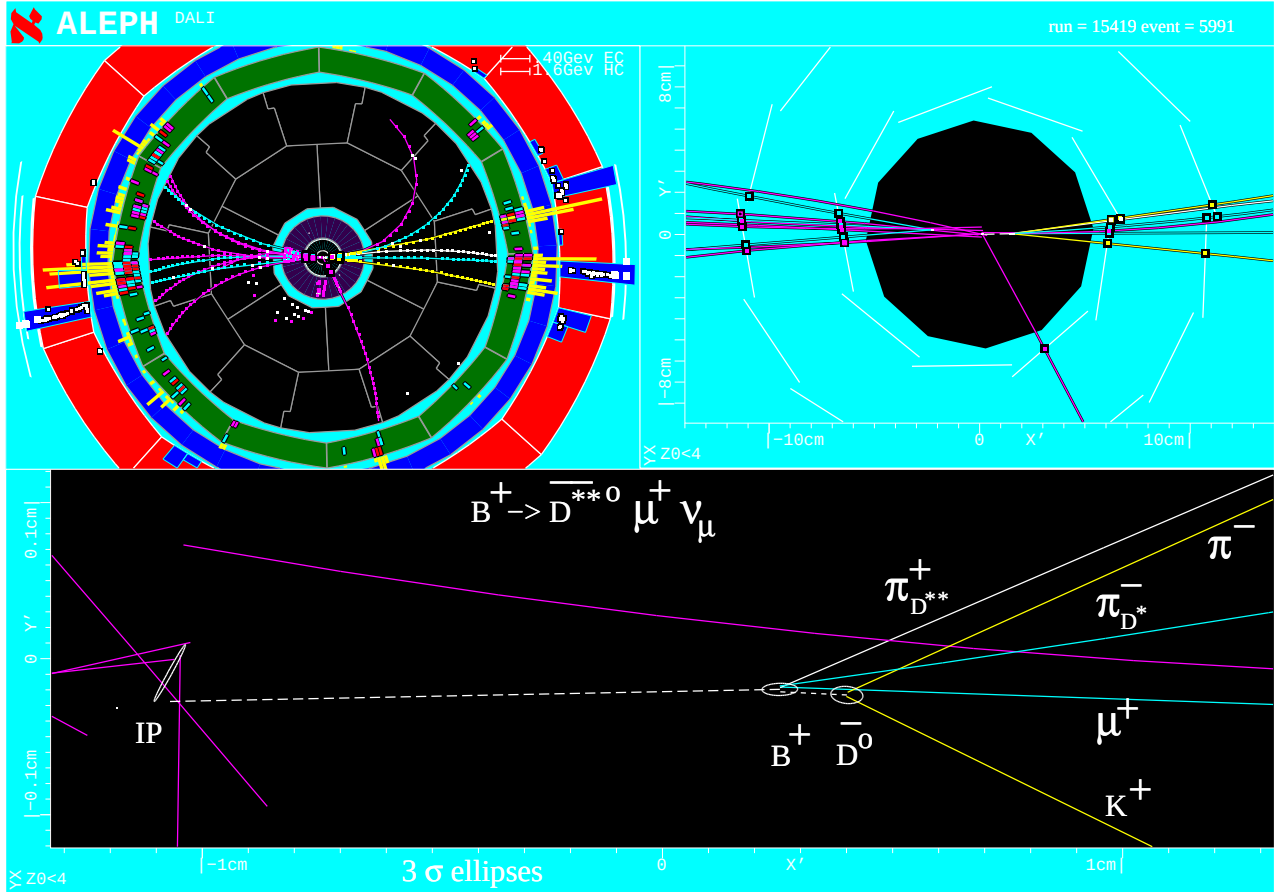


Figure 5.26: Heavy flavour $b\bar{b}$ event recorded at LEP-I by the ALEPH detector. The B meson decay chain is fully reconstructed. Top left: global view showing two hadronic jets with tracks reconstructed in the central tracking chamber and energy depositions in the calorimeters. Top right: Enlarged view showing hits and tracks reconstructed in the silicon micro-strip vertex detector. Bottom: Further expanded view showing the primary interaction vertex and secondary decay vertices.

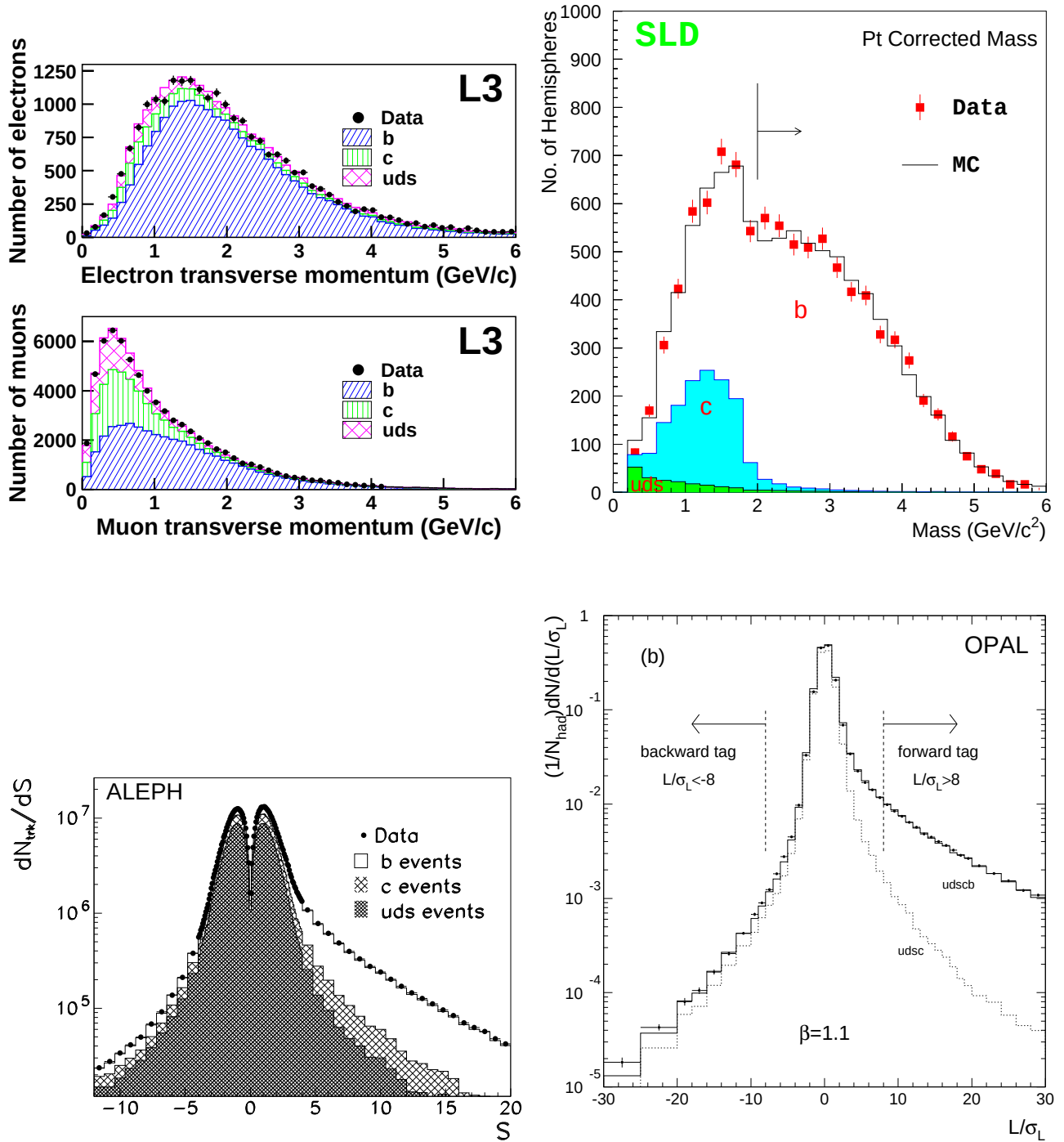


Figure 5.27: Tagging variables in heavy flavour analyses. Top left: transverse momentum of inclusive leptons with respect to the nearest hadronic jet in L3. Samples with high momentum and high-transverse momentum inclusive leptons are enriched in heavy-quark events. Top right: reconstructed invariant mass of system of particles assigned to the secondary vertex (secondary vertex mass) in SLD [185, 186]. The mass distributions are shifted with respect to the expectation from static quark masses due to track losses. Bottom left: impact parameter significance in ALEPH [187]; bottom right: decay length significance in OPAL [188]. The entries on the negative side of the two significance distributions arise from resolution effects, leading to impact parameters and secondary vertices reconstructed in the hemisphere opposite to the one containing the tracks attributed to the decay of the heavy hadron.

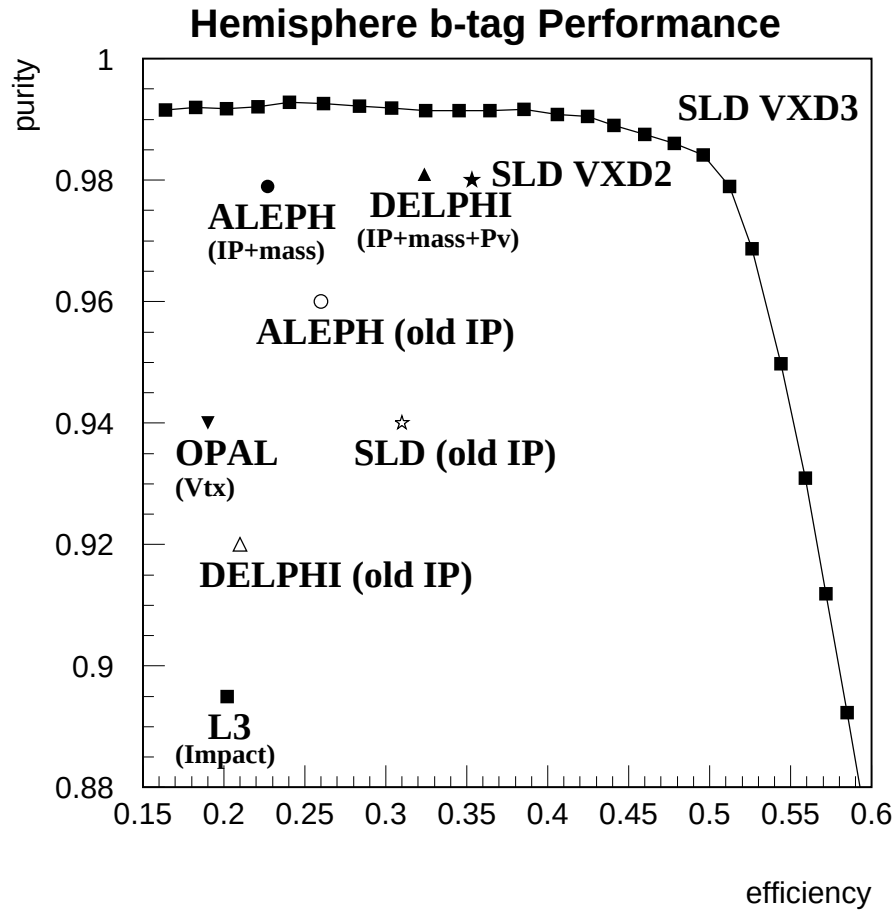


Figure 5.28: Comparison of b-tag efficiency and purity for the four LEP experiments and SLD [186]. Note the suppressed scales.

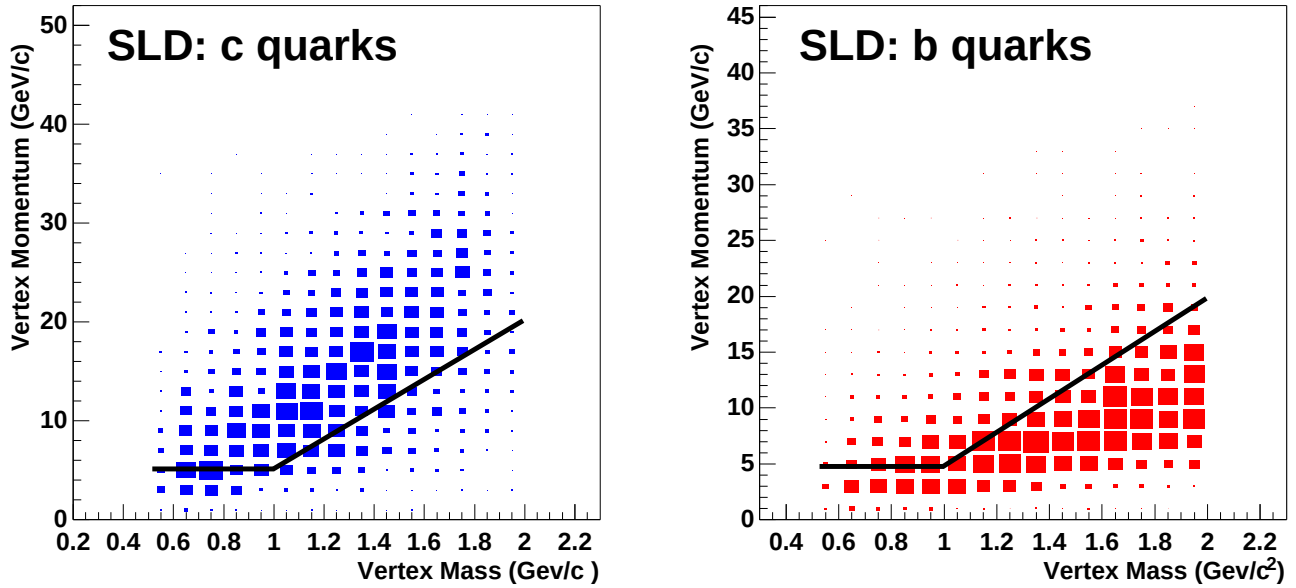


Figure 5.29: Separation of b quarks and c quarks in SLD based on vertex mass and vertex momentum in SLD [185]. The c tag of SLD reaches an efficiency of 14% and a purity of 67%.

5.4.1.3 Partial Widths

Experimentally, the ratio of the number of heavy quark events to all hadronic events is measured, which is the ratio of the corresponding production cross sections at the Z pole:

$$R_q(s, s'_{cut}) = \frac{\sigma(e^+e^- \rightarrow q\bar{q}(\gamma), s' > s'_{cut})}{\sigma(e^+e^- \rightarrow \text{had}, s' > s'_{cut})}, \quad (5.59)$$

where s'_{cut} denotes the s' cut applied in the analysis, $s'_{cut} > 4m_q^2$. In order to derive the ratios of the corresponding Z decay widths, small corrections are applied to account for radiative corrections, γ exchange and γ/Z interference contributions which are $\sqrt{s}$ dependent. Typically, one finds:

$$R_b^Z = \frac{\Gamma_{bb}}{\Gamma_{had}} = R_b(\sqrt{s} = M_Z) + 0.00029 \quad (5.60)$$

$$R_c^Z = \frac{\Gamma_{cc}}{\Gamma_{had}} = R_c(\sqrt{s} = M_Z) - 0.00032, \quad (5.61)$$

where the numerical corrections depend slightly on the invariant-mass or s' -cut applied.

Tagging of heavy quarks is applied to the event as a whole or to the two individual jets or hemispheres of the event. Single-tag methods allow the determination of, for example, R_b , based on the following relation:

$$\frac{N_t}{N_{had}} = \epsilon_b R_b + \epsilon_c R_c + \epsilon_{uds}(1 - R_b - R_c), \quad (5.62)$$

where N_t is the number of tagged events, N_{had} the total number of hadron events analysed, and ϵ_b , ϵ_c and ϵ_{uds} the efficiency of the tag for b, c and light-quark events. Single-tag measurements receive relatively large contributions to the systematic error because the tagging efficiency ϵ_b , multiplying the quantity R_b to be measured, must be taken from Monte Carlo simulations. This drawback is avoided by the double-tag method. In this method, hemispheres or jets rather than the events as a whole are tagged, leading to two relations, one for the number of hemispheres or jets tagged, N_1 , and one for the number of events with both jets or hemispheres are tagged, N_2 , where:

$$\frac{N_1}{2N_{had}} = \epsilon_b R_b + \epsilon_c R_c + \epsilon_{uds}(1 - R_b - R_c) \quad (5.63)$$

$$\frac{N_2}{N_{had}} = \mathcal{C}_b \epsilon_b^2 R_b + \mathcal{C}_c \epsilon_c^2 R_c + \mathcal{C}_{uds} \epsilon_{uds}^2 (1 - R_b - R_c), \quad (5.64)$$

where the correlation coefficients $\mathcal{C}$ take into account possible correlations of the jet tagging efficiencies between the jets or hemispheres. Such correlations arise, for example, through acceptance effects, hard gluon radiation and the position of the primary event vertex.

The double tagging method allows to determine the b tagging efficiency from data, albeit only to the statistical precision set by the size of the data sample, but nevertheless leading to smaller systematic errors on R_b . Furthermore it is possible to combine several different tagging methods, each having its own efficiency determined from the data, in so-called multitag analyses. Such an ansatz improves the statistical sensitivity and reduces systematic errors due to correlations and background.

The individual results on R_b^Z and R_c^Z are summarised in Figure 5.30.

5.4.1.4 Asymmetries

The measurement of the charge of the primary quarks in the reaction $e^+e^- \rightarrow q\bar{q}(\gamma)$ is less straightforward compared to $\ell^+\ell^-$ production, as the quarks manifest themselves in the detector as jets consisting of many neutral and charged hadrons. In the case the heavy quark decays semileptonically, the quark charge is tagged by the charge of the decay lepton associated to the quark jet. For exclusively reconstructed decay vertices of heavy-quark hadrons, the quark charge is inferred from the charge of the decaying meson as given by the sum of the charges of tracks assigned to the decay vertex.

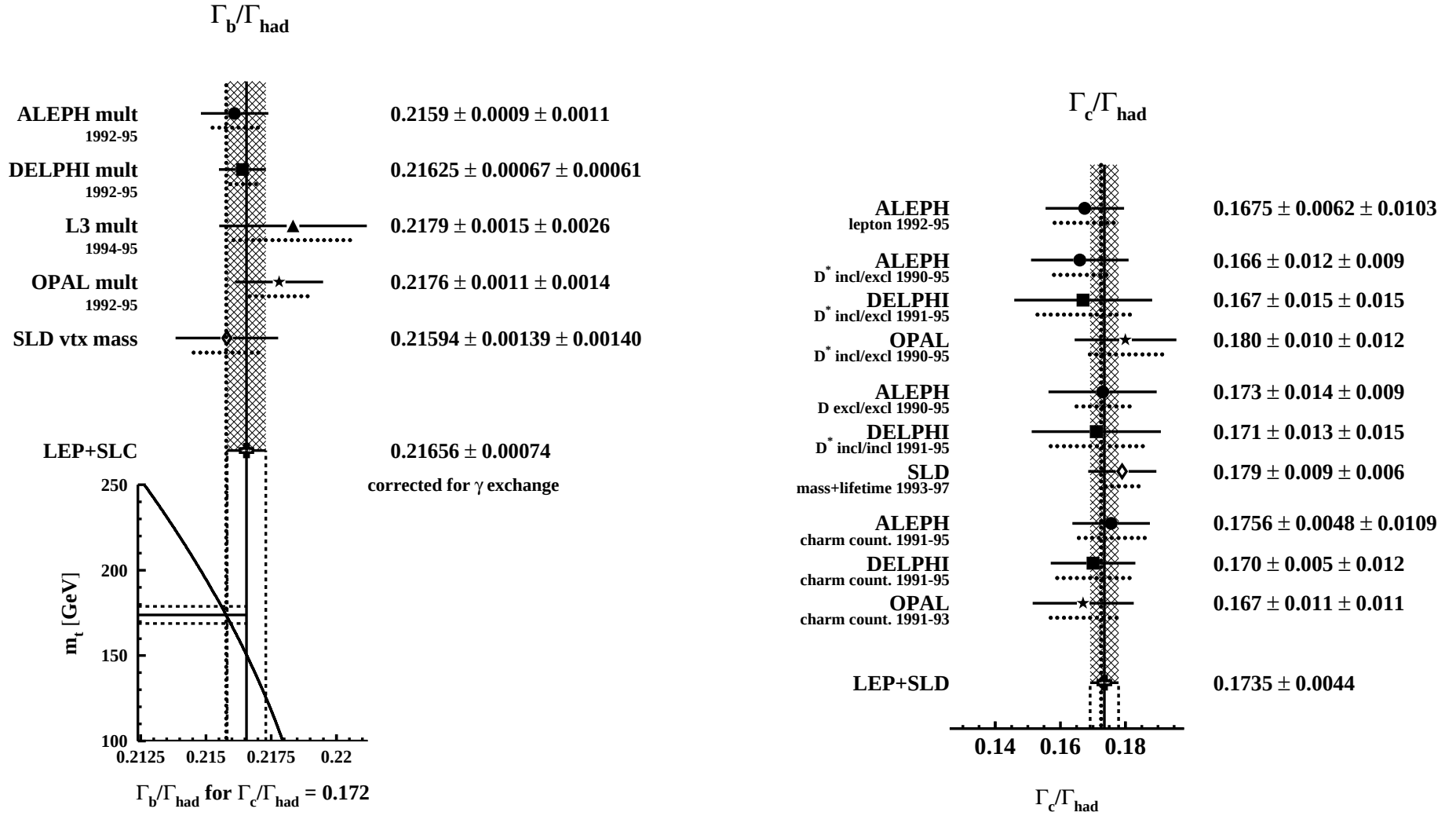


Figure 5.30: Comparison of the SLD and LEP results on R_b^Z and R_c^Z [163]. The dotted line indicates the systematic error of the corresponding measurement.

The complete hadronic jet also carries information about the charge of its parent quark. The jet charge, Q_j , is calculated as the weighted sum of the charges of particles assigned to the jet, where the weight is typically given by the longitudinal momentum of the track with respect to the jet axis:

$$Q_j = \frac{\sum_i q_i |p_i^\parallel|^\kappa}{\sum_i |p_i^\parallel|^\kappa}. \quad (5.65)$$

The sum runs over all particles i with charge q_i assigned to the jet or hemisphere under consideration. The parameter κ is tuned to obtain the best charge separation, resulting in $0.5 < \kappa < 1.5$. The average charge separation, Δ_q , expresses how well a quark jet is separated in measured charge from an antiquark jet:

$$\Delta_q = |\langle Q_q - Q_{\bar{q}} \rangle|. \quad (5.66)$$

For each event, a forward and a backward charge is constructed by using the charge of the jet in the forward and backward hemisphere, respectively:

$$Q_F = Q_{jet}(\cos \theta_{jet} > 0) \quad (5.67)$$

$$Q_B = Q_{jet}(\cos \theta_{jet} < 0). \quad (5.68)$$

The forward-backward charge asymmetry is simply the difference in forward and backward charge, $Q_{FB} = Q_F - Q_B$. In analogy to the counting method to determine asymmetries, the average forward-backward charge asymmetry is determined which is proportional to the usual flavour-based forward-backward asymmetry, A_{fb}^q :

$$\langle Q_{FB}^q \rangle(|\cos \theta| < c) = \text{sign}(q_q) \Delta_q A_{fb}^q(|\cos \theta| < c), \quad (5.69)$$

from which the corresponding pole forward-backward asymmetry, $A_{fb}^{0,q}$, is derived. The use of the average charge asymmetry and charge separation automatically corrects for charge-confusion effects.

Since $A_{fb}^q = \text{sign}(q_q) Q_{FB}^q / \Delta_q$, the charge separation Δ_q needs to be known as precisely as possible. In order to keep the systematic error low, it is determined using data. For this purpose, two experimentally accessible data distributions are analysed, the distribution of the average event charge, $Q = (Q_F + Q_B)/2 = (Q_q + Q_{\bar{q}})/2$, and the distribution of the product of charges, $Q_F Q_B = Q_q Q_{\bar{q}}$. The charge separation Δ is then given by:

$$\Delta = 4 \left(\text{cov}(Q_q, Q_{\bar{q}}) - \langle Q_F Q_B \rangle + \langle Q \rangle^2 \right), \quad (5.70)$$

where only the covariance $\text{cov}(Q_q, Q_{\bar{q}})$ must be taken from Monte-Carlo simulation. The dependence on the simulation is further reduced by also analysing the sum of the Q_F distribution and the Q_B distribution which is nothing else but the sum of the Q_q distribution and the $Q_{\bar{q}}$ distribution.¹ This distribution has the same mean as the distribution of the average event charge, $\langle Q \rangle$. Assuming that the Q_q and $Q_{\bar{q}}$ distributions have the same width σ , the width V of the summed distribution is given by:

$$V^2 = \sigma^2 + \frac{1}{4} \Delta^2, \quad (5.71)$$

so that the charge separation is written as:

$$\Delta^2 = 4 \cdot \frac{\rho(Q_q, Q_{\bar{q}}) V^2 - \langle Q_F Q_B \rangle + \langle Q \rangle^2}{1 + \rho(Q_q, Q_{\bar{q}})}, \quad (5.72)$$

where now only the correlation coefficient $\rho(Q_q, Q_{\bar{q}}) = \text{cov}(Q_q, Q_{\bar{q}})/\sigma^2$ instead of the covariance $\text{cov}(Q_q, Q_{\bar{q}})$ needs to be taken from the simulation.

¹Note that the resulting distribution is not the distribution of the quantity $2Q = Q_F + Q_B = Q_q + Q_{\bar{q}}$.

In order to apply the fitting method to determine the forward backward charge asymmetry, the charge flow is evaluated on an event by event basis. The positive quark is assumed to be in the forward region and the negative quark in the backward region if Q_{FB} is positive, and vice versa if Q_{FB} is negative. Both charge measurements Q_F and Q_B are used to assign the event charge flow. Assuming Gaussian distributions, the charge-confusion probability C of wrong charge assignment is given by:

$$C = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{-\frac{\Delta}{\sigma\sqrt{2(1-\rho)}}} \exp\left(-\frac{1}{2}x^2\right) dx, \quad (5.73)$$

since the width of the $Q_q - Q_{\bar{q}}$ distribution is given by $\sigma\sqrt{2(1-\rho)}$. Thus C depends on the charge separation Δ , the accuracy of the charge determination, σ , and the correlation between the two charges in the event, ρ . A maximum likelihood fit is then performed, where the likelihood is given by:

$$L = \prod_i \left[\frac{3}{8} (1 + \cos^2 \theta_i) + (1 - 2C) A_{fb}(s) \cos \theta_i \right], \quad (5.74)$$

and where the polar angle of the fermion for each event is constructed from the charge flow Q_{FB} and the known quark charge:

$$\cos \theta_i = \text{sign}(q_q Q_{FB}(i)) |\cos \theta_{thrust}(i)|. \quad (5.75)$$

The measured $b\bar{b}$ and $c\bar{c}$ forward-backward asymmetries at centre-of-mass energies around the Z pole are shown in Figure 5.31.

In order to determine pole forward-backward asymmetries, some corrections must be applied to the measurements:

1. Because of the hadronic nature of the final state, the asymmetries are affected by final-state QCD corrections, which modify the differential cross section of the process $e^+e^- \rightarrow q\bar{q}(\gamma)$ as follows:

$$\frac{d\sigma}{d\cos\theta} = \sigma_{\text{tot}} \left[\frac{3}{8} \left(\frac{4}{3 + a_q} \right) (1 + a_q \cos\theta) + A_{fb}^q \cos\theta \right] \quad (5.76)$$

$$A_{fb}^q = A_{fb}^{0,q} \left[1 - c_1 \frac{\alpha_S(M_Z^2)}{\pi} - c_2 \left(\frac{\alpha_S(M_Z^2)}{\pi} \right)^2 \right] \quad (5.77)$$

$$a_q = a_q^0 \left[1 - d_1 \frac{\alpha_S(M_Z^2)}{\pi} - d_2 \left(\frac{\alpha_S(M_Z^2)}{\pi} \right)^2 \right], \quad (5.78)$$

where quantity a_q is called the shape parameter. The coefficients of the series in α_S/π have been calculated to first order for massive quarks and to second order for massless quarks [189–194]. The corrections depend on whether the quark direction or the thrust axis is used to determine the polar angle. For massless quarks to first order and using the quark direction instead of the thrust axis, the results are simply $a_q^0 = 1$, $c_1 = 1$ and $d_1 = 8/3$.

Experimentally, the thrust axis is used to determine the polar angle of the event. Since the corrections are dependent on the experimental details of the analysis, the experiments quote their results with QCD effects removed and extrapolated to full acceptance. Detailed studies on experimental effects are given in [195].

2. Additional corrections are applied to transport the measurements to the pole centre-of-mass energy, $\sqrt{s} = M_Z$, and to remove initial-state QED radiative corrections as well as γ exchange and γ/Z interference contributions. The corrections are reported in Table 5.12. To the number of digits quoted, these corrections have a negligible error.

The individual results on $A_{fb}^{0,b}$ and $A_{fb}^{0,c}$ are summarised in Figure 5.32.

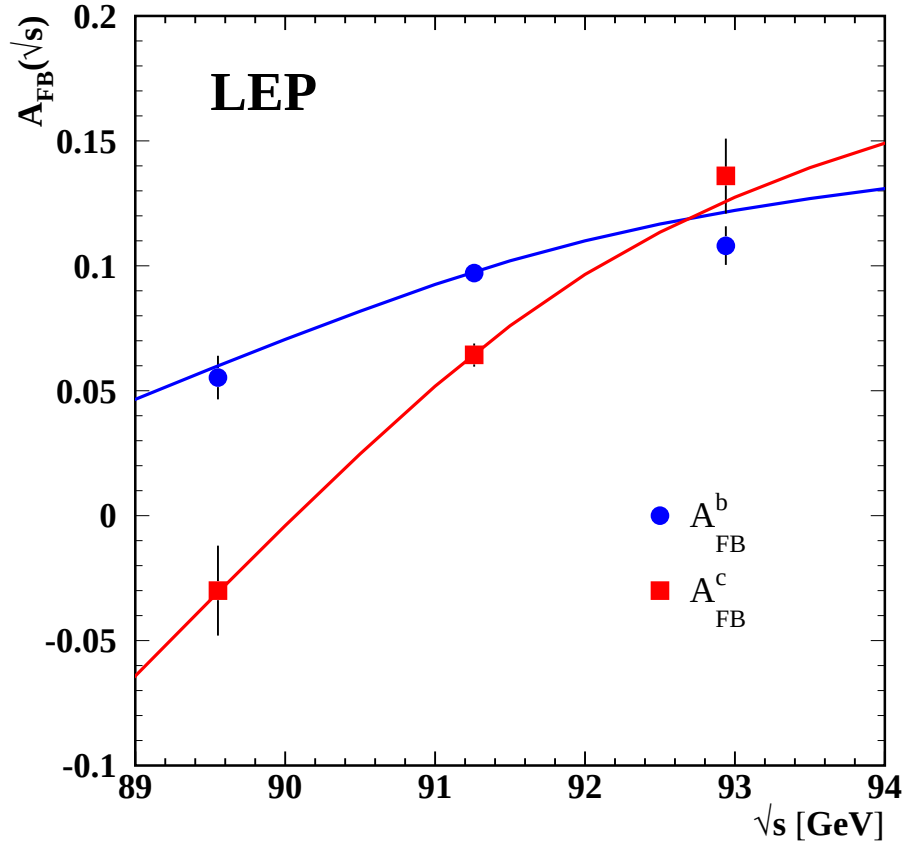


Figure 5.31: Forward-backward asymmetries in $b\bar{b}$ and $c\bar{c}$ production measured as a function of $\sqrt{s}$ around the Z pole [163].

Source	ΔA_{fb}^b	ΔA_{fb}^c
$\sqrt{s} = M_Z$	-0.0013	-0.0034
ISR	+0.0041	+0.0104
$\gamma, \gamma/Z$	-0.0003	-0.0008
Total	+0.0025	+0.0062

Table 5.12: Corrections to be applied to the measured $b\bar{b}$ and $c\bar{c}$ forward-backward asymmetries in order to determine the pole asymmetries [163]. The uncertainty on these corrections is negligible.

5.4.2 Averaging Procedure

The experimental analyses in the heavy flavour sector are much more complicated as compared to lepton and inclusive hadron production. This is reflected in the complicated fitting procedure to obtain the heavy-flavour averages [196]. Uncertainties and correlations arise due to external input, such as fragmentation and decay models and their parameters, as well as from parameters which are measured by the experiments, such as R_c^Z affecting the measurement of R_b^Z due to $c\bar{c}$ background in the $b\bar{b}$ sample.

For the external input parameters, the experiments agree on and constantly revise a common set of central values and uncertainties [197]. All heavy-flavour measurements are corrected in central value and error to correspond to the latest common set of values and uncertainties. Errors arising from the

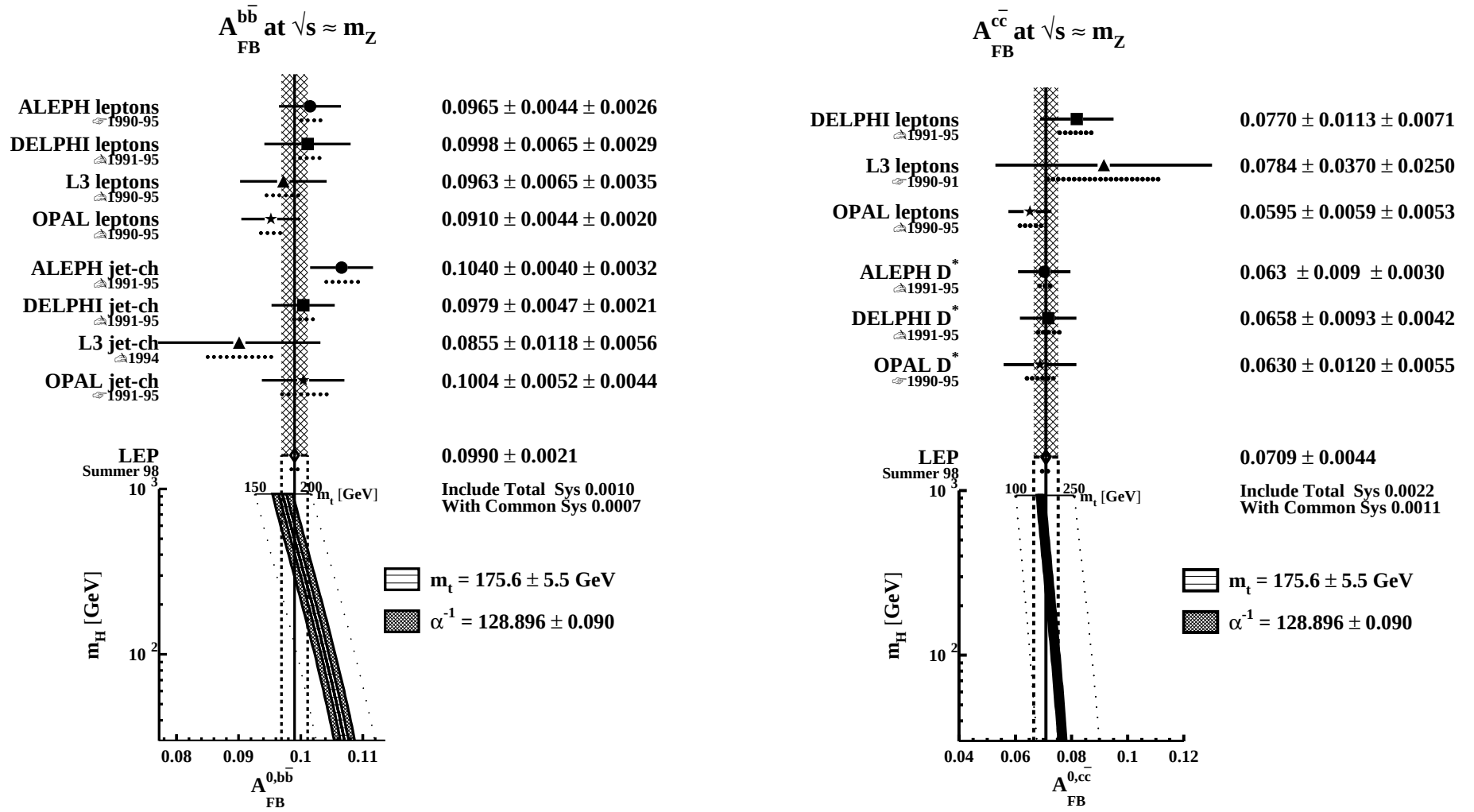


Figure 5.32: Comparison of the LEP results on $A_{fb}^{0,b}$ and $A_{fb}^{0,c}$ [163]. The dotted line indicates the systematic error of the corresponding measurement.

same sources are treated as correlated between the affected measurements.

Other heavy flavour parameters which are directly measured by the experiments are determined in parallel with the heavy flavour electroweak parameters. Besides the four or six electroweak parameters discussed above, the averaging procedure therefore considers and determines the following seven parameters in addition when combining results:

- The semileptonic branching fractions of b quarks, $B(b \rightarrow \ell)$ and $B(b \rightarrow c \rightarrow \bar{\ell})$, which determine the rate of inclusive leptons in $b\bar{b}$ events used by lepton tags;
- The average oscillation or mixing probability of neutral B mesons, $\bar{\chi}$, which leads to a dilution of the $b\bar{b}$ forward-backward asymmetry by a factor $1/(1 - 2\bar{\chi})$;
- The probabilities that a charm quark from Z decays produces one of the weakly decaying charm hadrons, D^+ , D_s , and charmed baryons, $P(c \rightarrow D^+)$, $P(c \rightarrow D_s)$, and $P(c \rightarrow c - \text{Baryon})$, respectively. The probability that a c quark fragments into a D^0 meson is derived from the constraint that all four probabilities add up to unity.
- The probability that a charm quark produces a D^{*+} meson, $P(c \rightarrow D^{*+})$, times the branching fraction for the strong decay $D^{*+} \rightarrow D^0\pi^+$, $P(c \rightarrow D^{*+}) \cdot B(D^{*+} \rightarrow D^0\pi^+)$

The c fractions are important for analyses reconstructing exclusively the various charm hadrons for charm counting.

The dependence of each measurement result on any of the fit parameters is accounted for by parametrising this dependence explicitly. The value of the measured quantity is adjusted to correspond to the set of actual values of the fit for all other parameters. For example, the dependence of the measured value $R_{b\text{measured}}$ on the value of R_c , changed by the fit from the value of $R_{c\text{used}}$ used to determine the measurement result $R_{b\text{measured}}$, is cast as:

$$R_{b\text{measured}} \longrightarrow R_{b\text{measured}} + a_{R_c} \frac{R_c - R_{c\text{used}}}{R_c}. \quad (5.79)$$

This adjusted value of the R_b measurement enters the calculation of the χ^2 to be minimised in the averaging procedure.

5.4.3 Results

The averaging procedure for the LEP heavy flavour results determines eleven parameters, the four electroweak parameters, R_b^Z , R_c^Z , $A_{fb}^{0,b}$, $A_{fb}^{0,c}$, and the seven additional heavy flavour parameters as given above. The results for the four electroweak parameters including their correlations are reported in Table 5.13. The $\chi^2/\text{d.o.f}$ is low, 42/69, resulting in a high probability of 99.5%. In case the SLD heavy flavour results are included, the polarised forward-backward asymmetries are included as additional electroweak parameters, which determine the coupling parameters A_b and A_c . They are treated as independent of the forward-backward asymmetries $A_{fb}^{0,b}$ and $A_{fb}^{0,c}$, *i.e.*, the relation $A_{fb}^{0,q} = \frac{3}{4}A_e A_q$ is not used. The combined results including correlations are reported in Table 5.14. Also in this case, the $\chi^2/\text{d.o.f}$ is rather low, 44/75, thus the probability of the fit very high, 99.8%.

The anticorrelation between R_b^Z and R_c^Z and the correlation between $A_{fb}^{0,b}$ and $A_{fb}^{0,c}$ are mainly caused by the background of $c\bar{c}$ events in the sample of selected $b\bar{b}$ events. Because the results are correlated, in particular (R_b^Z, R_c^Z) , $(A_{fb}^{0,b}, A_{fb}^{0,c})$, and (A_b, A_c) , two-dimensional correlation contours are shown in Figure 5.33. Compared to the experimental errors, the measurements of $A_{fb}^{0,q}$ are the heavy flavour measurements most sensitive to SM parameters, while the SM predictions of A_b and A_c are constant compared to the experimental errors. The largest deviation of the electroweak heavy flavour measurements from the SM is seen for the parameter A_b .

Parameter	Average	R_b^Z	R_c^Z	$A_{fb}^{0,b}$	$A_{fb}^{0,c}$	MSM
R_b^Z	0.21664 ± 0.00076	1.00	-0.17	-0.06	0.02	0.21582 ± 0.00018
R_c^Z	0.1724 ± 0.0048	-0.17	1.00	0.06	-0.05	0.17223 ± 0.00006
$A_{fb}^{0,b}$	0.0991 ± 0.0021	-0.06	0.06	1.00	0.13	0.0994 ± 0.0036
$A_{fb}^{0,c}$	0.0712 ± 0.0045	0.02	-0.05	0.13	1.00	0.0708 ± 0.0028

Table 5.13: Average results on electroweak heavy flavour parameters and their correlations obtained from LEP data. The $\chi^2/\text{d.o.f}$ of the eleven-parameter average is 42/69 (99.5%) [163].

Parameter	Average	R_b^Z	R_c^Z	$A_{fb}^{0,b}$	$A_{fb}^{0,c}$	A_b	A_c	MSM
R_b^Z	0.21656 ± 0.00074	1.00	-0.17	-0.06	0.02	-0.02	0.02	0.21582 ± 0.00018
R_c^Z	0.1735 ± 0.0044	-0.17	1.00	0.05	-0.04	0.01	-0.04	0.17223 ± 0.00006
$A_{fb}^{0,b}$	0.0990 ± 0.0021	-0.06	0.05	1.00	0.13	0.03	0.02	0.0994 ± 0.0036
$A_{fb}^{0,c}$	0.0709 ± 0.0044	0.02	-0.04	0.13	1.00	-0.01	0.07	0.0708 ± 0.0028
A_b	0.867 ± 0.035	-0.02	0.01	0.03	-0.01	1.00	0.04	0.9342 ± 0.0004
A_c	0.647 ± 0.040	0.02	-0.04	0.02	0.07	0.04	1.00	0.6656 ± 0.0022

Table 5.14: Average results on electroweak heavy flavour parameters and their correlations obtained from SLD/LEP data. the $\chi^2/\text{d.o.f}$ of the thirteen-parameter average is 44/75 (99.8%) [163].

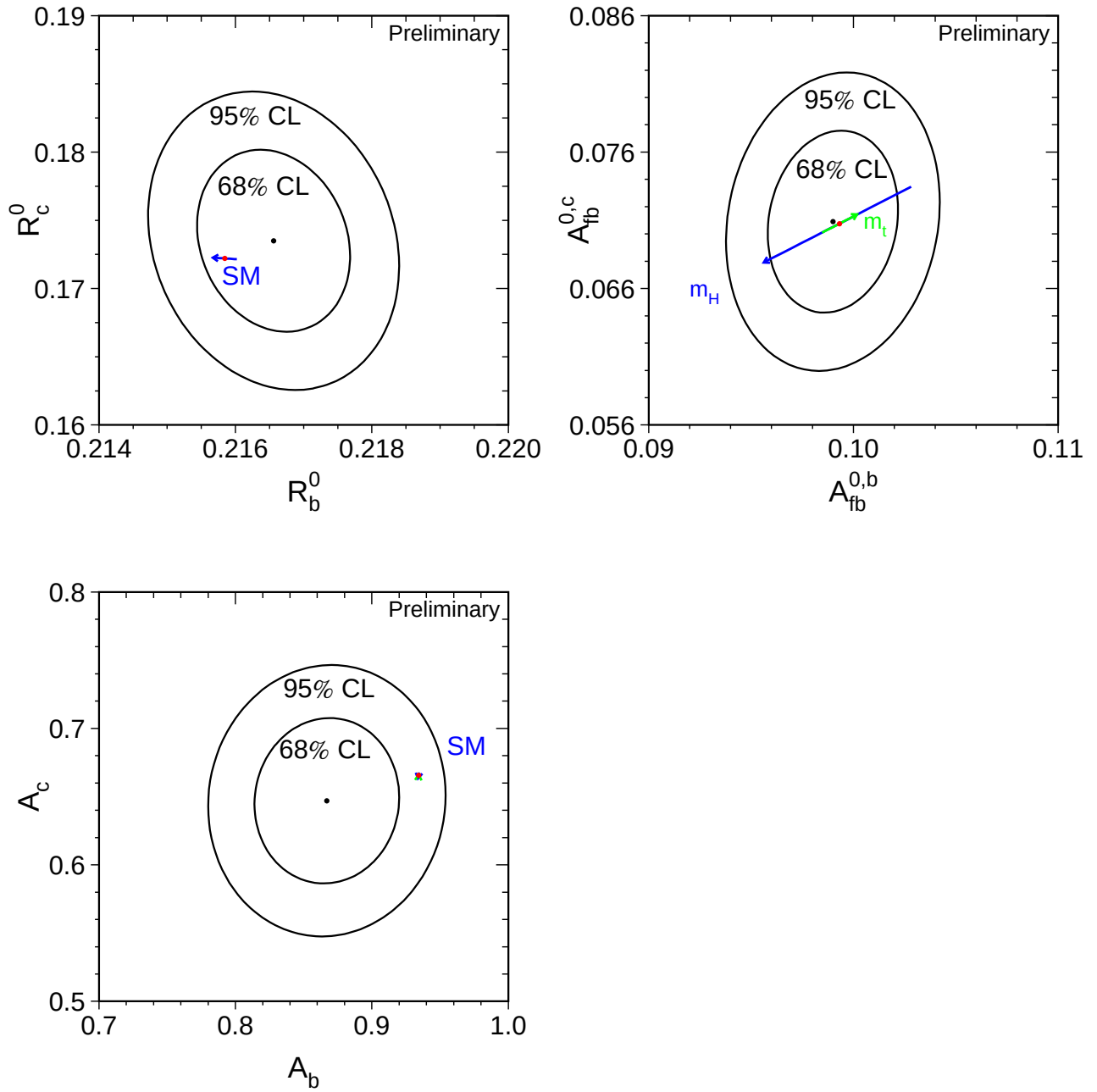


Figure 5.33: Correlation contours of 68% probability for (R_b^Z, R_c^Z) , $(A_{fb}^{0,b}, A_{fb}^{0,c})$, and (A_b, A_c) . The Standard-Model predictions are shown for $M_t = 173.8 \pm 5.0$ GeV and $M_H = 300_{-210}^{+700}$ GeV.

The various sources of errors on the electroweak parameters are reported in Table 5.15. Systematic errors play an important role and sometimes dominate the total error. The small $\chi^2/\text{d.o.f}$ of both fits suggests that the systematic errors, their correlations, or both, are estimated rather conservatively.² The errors on the averages are therefore less than optimal.

²Necessarily, systematic errors are treated like statistical errors in order to be able to apply the averaging procedure based on χ^2 fits with error matrices. However, systematic errors are rarely Gaussian with a well defined spread.

Source	R_b^Z [10^{-3}]	R_c^Z [10^{-3}]	$A_{fb}^{0,b}$ [10^{-3}]	$A_{fb}^{0,c}$ [10^{-3}]	A_b [10^{-2}]	A_c [10^{-2}]
Uncorrelated systematics	0.32	2.4	0.8	1.9	2.4	2.2
QCD effects	0.23	0.2	0.3	0	0.7	0.3
B($D \rightarrow neut.$)	0.17	0.1	0	0	0	0
D decay multiplicity	0.09	0.5	0.1	0.1	0	0
B($D^+ \rightarrow K^- \pi^+ \pi^+$)	0.10	0.4	0	0	0	0.1
B($D_s \rightarrow \phi \pi^+$)	0.02	1.2	0	0	0	0
B($\Lambda_c \rightarrow p K^- \pi^+$)	0.04	0.6	0	0	0	0.1
B($c \rightarrow \ell$)	0.02	1.2	0.3	0.1	0	0
D lifetimes	0.07	0.2	0.1	0.1	0	0
gluon splitting	0.29	0.9	0	0	0.1	0.2
c fragmentation	0.07	0.8	0.1	0	0.1	0.1
light quarks	0.10	0.6	0.5	0	0	0.1
Total systematic	0.57	3.4	1.0	2.2	2.5	2.3
Statistics	0.47	2.7	1.8	3.8	2.5	3.3
Total	0.74	4.4	2.1	4.4	3.5	4.0

Table 5.15: Error sources in heavy-flavour analyses. Except for the statistical error and uncorrelated systematics, all other systematic error sources are treated as correlated between measurements [163].

5.5 The Coupling Parameters

In the previous sections, several measurements are presented which constrain the values of the coupling parameter A_f . These are the forward-backward asymmetries, $A_{fb}^{0,f}$, which determine the product $A_{fb}^{0,f} = \frac{3}{4}A_e A_f$, the τ polarisation results A_e and A_τ , the left-right asymmetry $A_{lr} = A_e$, and the polarised forward-backward asymmetries, $A_{fblr}^{0,f} = \frac{3}{4}A_f$.

5.5.1 Charged Leptons

First, the leptonic coupling parameter A_ℓ will be considered. The results on A_ℓ derived from the various asymmetry measurements are summarised in Table 5.16. As expected from the agreement of the $A_{fb}^{0,\ell}$ values, also good agreement between the different lepton species in terms of A_ℓ is observed. Furthermore, the results on A_e and A_τ agree well with the same quantities determined from the τ -polarisation and A_{lr} measurement. Assuming lepton universality, the leptonic forward-backward asymmetries determine the absolute value of A_ℓ , $|A_\ell| = 0.1498 \pm 0.0043$. Combining this result with the τ -polarisation and A_{lr} measurements, the best value for A_ℓ is obtained:

$$A_\ell = 0.1489 \pm 0.0017. \quad (5.80)$$

The average has a $\chi^2/\text{d.o.f}$ of 2.2/3, corresponding to a probability of 53%.

5.5.2 Heavy Flavours

Second, the quark coupling parameters are considered, based on the measured forward-backward and forward-backward left-right $b\bar{b}$ and $c\bar{c}$ asymmetries. Assuming lepton universality, the coupling

A_ℓ	$A_{\text{fb}}^{0,\ell}$	$\mathcal{P}_\tau$	A_{lr}	Average
A_e	0.143 ± 0.012	0.1479 ± 0.0051	0.1504 ± 0.0023	0.1501 ± 0.0020
A_μ	0.153 ± 0.018			0.145 ± 0.012
A_τ	0.171 ± 0.021	0.1431 ± 0.0045		0.1447 ± 0.0043
A_ℓ	0.1498 ± 0.0043	0.1452 ± 0.0034	0.1504 ± 0.0023	0.1489 ± 0.0017
MSM	0.1419 ± 0.0051	0.1419 ± 0.0051	0.1419 ± 0.0051	0.1419 ± 0.0051

Table 5.16: Results on the leptonic coupling parameters A_ℓ and their combination, with and without the assumption of lepton universality. Note that there is a large anti-correlation between the A_ℓ values derived from the $A_{\text{fb}}^{0,\ell}$ results alone. The correlation matrices are given in Table 5.17. The average of the four A_ℓ determinations based on $A_{\text{fb}}^{0,\ell}$, A_e and A_τ from τ polarisation, and A_{lr} has a $\chi^2/\text{d.o.f}$ of 2.2/3, corresponding to a probability of 53%.

Parameter	A_e	A_μ	A_τ	A_e	A_μ	A_τ
A_e	1.000	-0.720	-0.663	1.000	-0.166	-0.033
A_μ	-0.720	1.000	-0.493	-0.166	1.000	0.014
A_τ	-0.663	-0.493	1.000	-0.033	0.014	1.000

Table 5.17: Correlations between the leptonic coupling parameters derived from $A_{\text{fb}}^{0,\ell}$ only (left) and from all results (right).

parameters for charged leptons, b quarks and c quarks are determined and reported in Table 5.18. The average has a $\chi^2/\text{d.o.f}$ of 2.5/5, corresponding to a probability of 78%.

The mutual consistency between the measurements of A_ℓ , A_q and $A_{\text{fb}}^{0,q} = \frac{3}{4}A_e A_q$ for $q = b, c$ is shown in Figure 5.34. The three bands corresponding to the three measurements overlap in a common region of the (A_ℓ, A_q) plane. The measurement errors on A_b and A_c are very large compared to the effects of electroweak radiative corrections within the SM. This is indicated by the Standard Model expectation as a function of the SM input parameters, which is essentially constant in A_b and nearly so in A_c for any reasonable values of the SM input parameters.

The derived coupling parameter A_b deviates from the Standard Model prediction by about three standard deviations. This is caused by the combined action of several effects. First, the direct SLD measurement of A_b from the left-right forward-backward asymmetry at SLC is low compared to the almost constant Standard Model prediction of this parameter. Second, A_ℓ is high and $A_{\text{fb}}^{0,b}$ is low compared to Standard Model predictions based on the same values for M_t and M_H . Thus A_b

Parameter	Average	A_b	A_c	A_ℓ	MSM
A_b	0.882 ± 0.018	1.000	0.139	-0.412	0.9342 ± 0.0004
A_c	0.640 ± 0.029	0.139	1.000	-0.118	0.6656 ± 0.0022
A_ℓ	0.1491 ± 0.0017	-0.412	-0.118	1.000	0.1419 ± 0.0051

Table 5.18: Results on the heavy-quark coupling parameters A_q and their correlations derived from the SLD/LEP data under the assumption of lepton universality. The average has a $\chi^2/\text{d.o.f}$ of 2.5/5, corresponding to a probability of 78%.

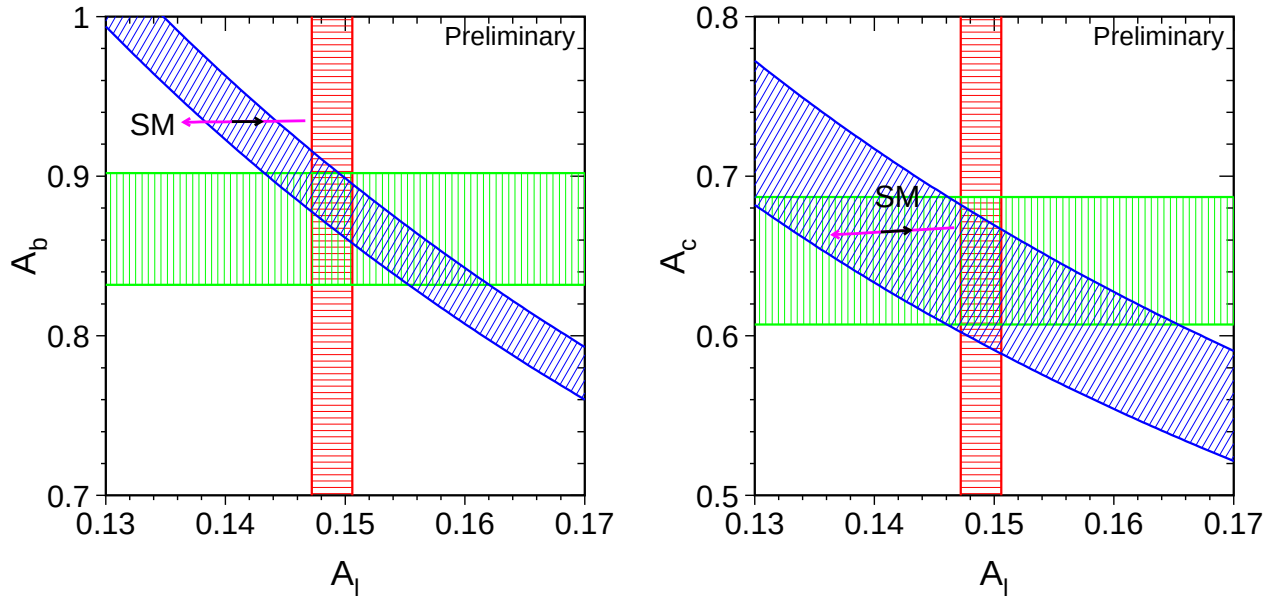


Figure 5.34: Consistency of the measurements of A_ℓ , A_q and $A_{\text{fb}}^{0,q}$ for b quarks (left) and c quarks (right). Bands of ± 1 -sigma width in the (A_ℓ, A_q) plane are shown for the results on A_ℓ (vertical band), A_q (horizontal band) and $A_{\text{fb}}^{0,q} = \frac{3}{4}A_e A_q$ (diagonal band). The SM prediction, shown for $M_t = 173.8 \pm 5.0$ GeV and $M_H = 300_{-210}^{+700}$ GeV, is essentially constant.

determined from $A_b = \frac{4}{3}A_{\text{fb}}^{0,b}/A_\ell$ is also driven towards low values. Combining the two low A_b values, a significantly low A_b average is obtained. It is not possible to lower A_{lr} and to increase A_b through the dependence of their SLD measurement on the SLC electron beam polarisation at the same time, as both observables would change in the same direction. Improved measurements of A_ℓ , $A_{\text{fb}}^{0,b}$ and A_b , combined with a realistic assessment of systematic errors in the heavy-flavour analyses, are clearly desirable.

5.6 The Effective Vector and Axial-Vector Couplings

5.6.1 Charged Leptons

The effective vector and axial-vector coupling constants of the neutral weak current are determined by both asymmetries and partial decay widths of the Z boson. The asymmetries constrain the coupling parameters A_f , which is a function of the ratio of couplings, whereas the partial decay widths determine the sum of the squares of the couplings:

$$A_f = 2 \frac{g_{Vf} g_{Af}}{g_{Vf}^2 + g_{Af}^2} = 2 \frac{g_{Vf}/g_{Af}}{1 + (g_{Vf}/g_{Af})^2} \quad (5.81)$$

$$\Gamma_{ff} = N_C^f \frac{G_F M_Z^3}{6\sqrt{2}\pi} [R_V g_{Vf}^2 + R_A g_{Af}^2], \quad (5.82)$$

where $R_{V,A}$ express QED and QCD corrections of the vector and axial-vector current as discussed in Section 3.1.4.

Both measurements allow to disentangle g_{Vf} and g_{Af} up to the ambiguity of interchanging $g_{Vf} \leftrightarrow g_{Af}$. The analysis of the γ/Z interference terms, *e.g.*, the energy dependence of the forward-backward asymmetry $A_{\text{fb}}(s)$ in the vicinity of $\sqrt{s} = M_Z$, resolves this ambiguity. The measurement of A_f determines only the relative sign between g_{Vf} and g_{Af} . The absolute sign is fixed by the convention that g_{Ae} is chosen to be negative.

The results are reported in Table 5.19. The average has a $\chi^2/\text{d.o.f}$ of 2.1/3, corresponding to a probability of 55%. The graphical comparison between the SLD and LEP measurements is shown in Figure 5.35. The enormous increase in precision on the effective couplings gained through the operation of SLC and LEP is clearly visible when comparing 1998 results with the situation before SLC/LEP in 1987.

Assuming lepton universality, the best values for the leptonic coupling constants are:

$$g_{A\ell} = -0.50102 \pm 0.00030 \quad (5.83)$$

$$g_{V\ell} = -0.03753 \pm 0.00044, \quad (5.84)$$

which have a correlation of -4.1% . The average has a $\chi^2/\text{d.o.f}$ of 5.3/7, corresponding to a probability of 62%. The value of $g_{A\ell}$ is different from the Born-term value of $-1/2$ by more than three standard deviations, showing evidence for higher-order electroweak corrections.

Assuming three generations of neutrinos, $N_\nu = 3$, and defining $g_\nu = g_{V\nu} = g_{A\nu}$, the neutrino coupling constant derived from $\Gamma_{\text{inv}} = 500.1 \pm 1.9$ MeV is given by:

$$|g_\nu| = 0.50123 \pm 0.00095. \quad (5.85)$$

5.6.2 Heavy Flavours

Like for leptons, effective couplings are also determined for the heavy quarks b and c. For quarks, however, additional complications arise in the definition of electroweak effective couplings absorbing all electroweak effects due to the presence of QCD corrections and running-mass effects. For example, M_t dependent effects are contained both in electroweak and in QCD corrections. A clean separation of electroweak and QCD corrections in effective couplings depending on M_t and M_H and QCD correction factors depending on α_S is not possible beyond one-loop order. For the results presented below, the factorisation as implemented in the ZFITTER [22] framework of effective couplings is used as an example.

The results are summarised in Table 5.20 and shown in Figure 5.36. The average has a $\chi^2/\text{d.o.f}$ of 2.5/5, corresponding to a probability of 78%. Of the six effective couplings, the b quark couplings deviate significantly from the Standard Model expectation.

The precise measurement of R_b severely constraints $g_{A_b}^2 + g_{V_b}^2$, squeezing the (g_{V_b}, g_{A_b}) contour curves along a circle in the (g_{V_b}, g_{A_b}) plane, while the measurement of A_b selects a certain region of that circle. With respect to the SM expectation, the circle has the correct radius, while the region selected by A_b is shifted.

This is a consequence of the low value of the coupling parameter A_b discussed before, which forces the ratio g_{V_b}/g_{A_b} to be low as well. In terms of the chiral couplings g_{Lb} and g_{Rb} , the measurement of R_b constrains $g_{Lb}^2 + g_{Rb}^2$ and thus the left handed coupling $|g_{Lb}|$ since $|g_{Lb}| \gg |g_{Rb}|$. The low value of the coupling parameter A_b causes a low absolute value of the right-handed coupling g_{Rb} .

Nevertheless, the experimental results on the neutral current couplings of both c and b quark show that they belong to doublets of weak isospin. This is particularly interesting in the case of b quarks [198] as it proves the existence of the top quark, defined as the weak isospin partner of the b quark.

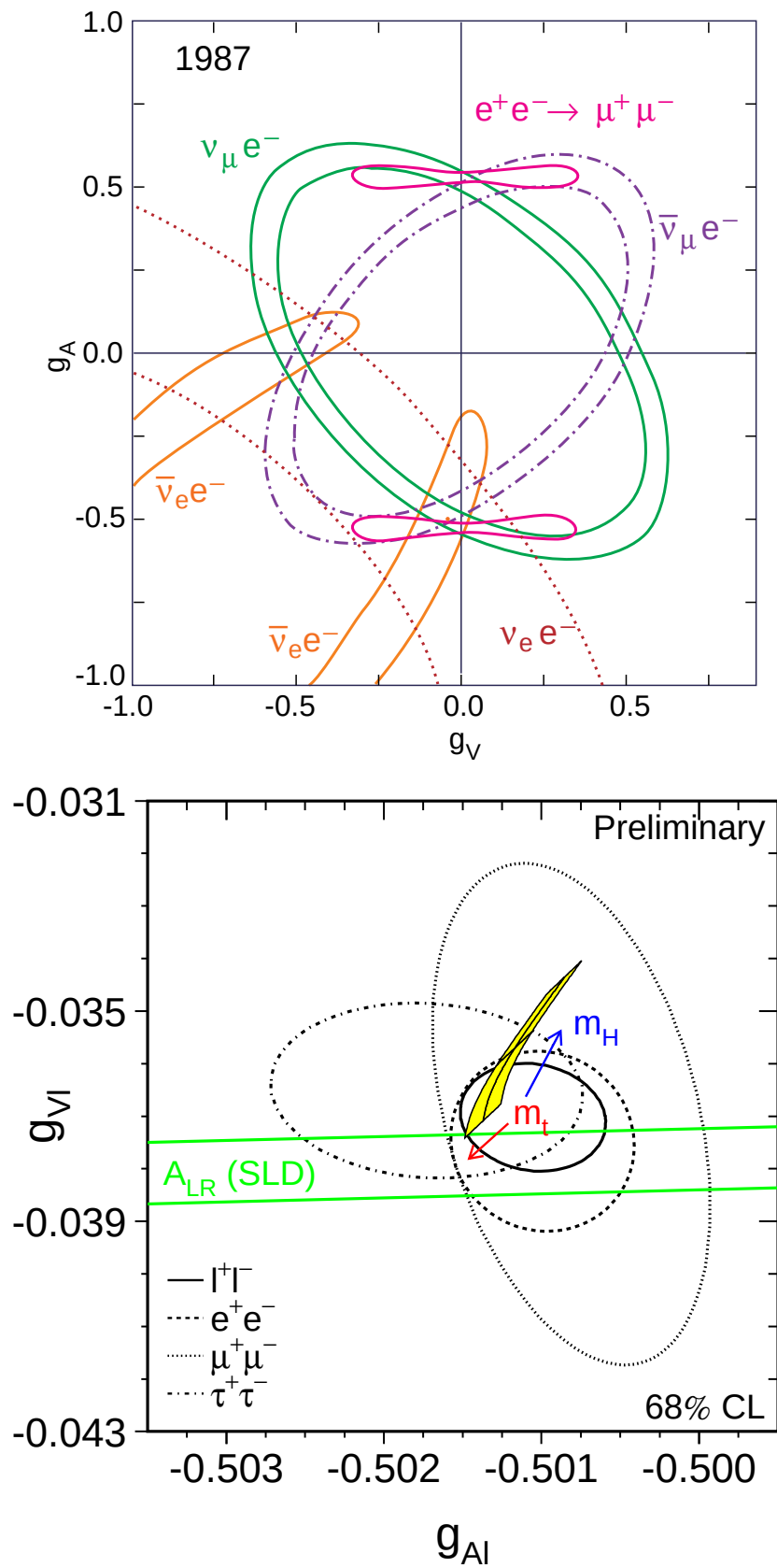


Figure 5.35: Effective vector and axial-vector coupling constants of charged leptons. Top: Status of 1987, before SLC and LEP. Bottom: Status of 1998, based on SLC and LEP-I data.

Parameter	Average	g_{Ae}	$g_{A\mu}$	$g_{A\tau}$	g_{Ve}	$g_{V\mu}$	$g_{V\tau}$	MSM
g_{Ae}	-0.50098 ± 0.00038	1.00	0.00	0.00	0.02	0.00	0.00	-0.50115 ± 0.00027
$g_{A\mu}$	-0.50082 ± 0.00058	0.00	1.00	0.38	0.02	-0.36	0.02	-0.50115 ± 0.00027
$g_{A\tau}$	-0.50171 ± 0.00065	0.00	0.38	1.00	-0.04	0.01	-0.08	-0.50115 ± 0.00027
g_{Ve}	-0.03781 ± 0.00052	0.02	0.02	-0.04	1.00	-0.17	-0.03	-0.03574 ± 0.00131
$g_{V\mu}$	-0.0366 ± 0.0030	0.00	-0.36	0.01	-0.17	1.00	0.01	-0.03574 ± 0.00131
$g_{V\tau}$	-0.0365 ± 0.0011	0.00	0.02	-0.08	-0.03	0.01	1.00	-0.03574 ± 0.00131

Table 5.19: Effective vector and axialvector coupling constants of charged leptons and their correlations derived from SLD/LEP data. The average has a $\chi^2/\text{d.o.f}$ of 2.1/3, corresponding to a probability of 55%.

Parameter	Average	g_{Ab}	g_{Ac}	$g_{A\ell}$	g_{Vb}	g_{Vc}	$g_{V\ell}$	MSM
g_{Ab}	-0.5206 ± 0.0062	1.00	-0.01	-0.01	-0.98	0.15	0.40	-0.49831 ± 0.00029
g_{Ac}	$+0.5067 \pm 0.0075$	-0.01	1.00	-0.02	0.04	-0.29	-0.06	$+0.50130 \pm 0.00027$
$g_{A\ell}$	-0.50102 ± 0.00030	-0.01	-0.02	1.00	0.05	-0.02	-0.04	-0.50115 ± 0.00027
g_{Vb}	-0.3118 ± 0.0101	-0.98	0.04	0.05	1.00	-0.15	-0.41	-0.34314 ± 0.00059
g_{Vc}	$+0.1834 \pm 0.0098$	0.15	-0.29	-0.02	-0.15	1.00	0.12	$+0.19107 \pm 0.00096$
$g_{V\ell}$	-0.03756 ± 0.00042	0.40	-0.06	-0.04	-0.41	0.12	1.00	-0.03574 ± 0.00131

Table 5.20: Effective vector and axial-vector coupling constants of heavy quarks and their correlations derived from SLD and LEP data. The average has a $\chi^2/\text{d.o.f}$ of 2.5/5, corresponding to a probability of 78%.

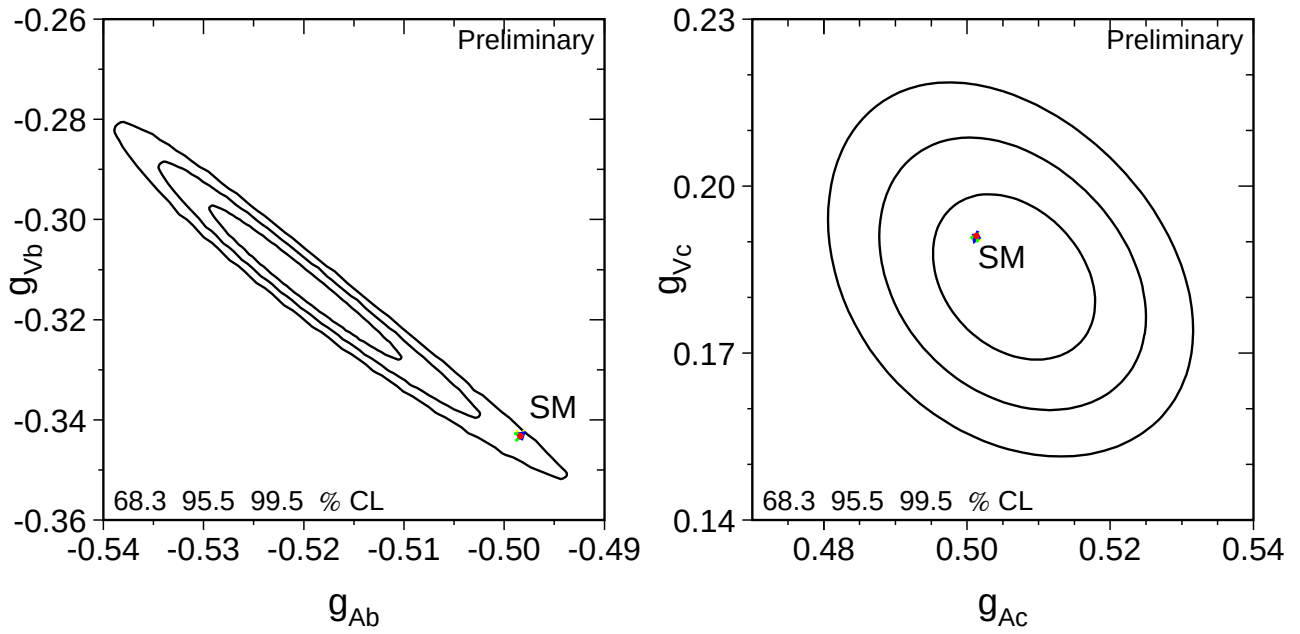


Figure 5.36: Effective vector and axial-vector coupling constants of b quarks (left) and c quarks (right). Shown are the contour curves of 68.3%, 95.5% and 99.5% probability in the (g_{Vq}, g_{Aq}) plane. The SM prediction, shown for $M_t = 173.8 \pm 5.0$ GeV and $M_H = 300^{+700}_{-210}$ GeV, is essentially constant.

5.7 Hadronic Charge Asymmetry

5.7.1 Measurements

The jet-charge technique used to determine the forward-backward asymmetry in $b\bar{b}$ events is also applied to the inclusive hadronic event sample. In this case, however, all the five light quark flavours are mixed, so that the charge asymmetry of the inclusive sample is a weighted sum of the charge asymmetry of the contributing quark flavours:

$$Q_{fb} = \sum_q \frac{\sigma(e^+e^- \rightarrow q\bar{q}(\gamma))}{\sigma(e^+e^- \rightarrow \text{had})} \cdot Q_{fb}^q = \sum_q \frac{\sigma(e^+e^- \rightarrow q\bar{q}(\gamma))}{\sigma(e^+e^- \rightarrow \text{had})} \cdot \text{sign}(q_q) \Delta_q A_{fb}^q. \quad (5.86)$$

Applying small corrections, in analogy to the pole cross section and pole forward-backward asymmetry, the hadronic pole charge asymmetry is obtained:

$$Q_{fb}^0 = \sum_q \frac{\Gamma_{qq}}{\Gamma_{had}} \cdot \text{sign}(q_q) \Delta_q A_{fb}^{0,q} = \frac{3}{2} A_e \frac{\sum_q [\text{sign}(q_q) \delta_q g_{Vq} g_{Aq}]}{\sum_q [g_{Vq}^2 + g_{Aq}^2]}. \quad (5.87)$$

The different quark flavours show different charge separation Δ_q , in particular between up-type quarks and down-type quarks. In addition, fragmentation and hadronisation effects make the charge separation flavour dependent. Experimentally, each term in the above sum must be corrected for selection efficiency and purity of the corresponding quark flavour.

5.7.2 Averaging Procedure and Results

Since the hadronic charge asymmetry is an inclusive quantity, it depends on many electroweak parameters, namely the effective vector- and axial-vector couplings of the five light quarks and the coupling parameter A_e . For a meaningful interpretation of Q_{fb} , the number of eleven quantities needs to be

reduced. Thus quark universality is assumed, meaning that the quark couplings are written in terms of a universal effective electroweak mixing angle:

$$\sin^2 \bar{\theta}_W = \frac{T_3^f}{2q_f} \left(1 - \frac{g_{Vf}}{g_{Af}} \right) \quad (5.88)$$

Since the factor A_e of Q_{fb} has a much stronger $\sin^2 \bar{\theta}_W$ dependence than any of the g_{Vq} , the hadronic charge asymmetry, when interpreted as a measurement of $\sin^2 \bar{\theta}_W$, essentially determines $\sin^2 \bar{\theta}_W$ for electrons. Small flavour-specific electroweak corrections, largest for the b quark, are negligible.

Experiment	Status	$\sin^2 \bar{\theta}_W$
ALEPH	(90 - 94), final	$0.2322 \pm 0.0008 \pm 0.0011$
DELPHI	(91 - 94), prel.	$0.2311 \pm 0.0010 \pm 0.0014$
L3	(91 - 95), prel.	$0.2327 \pm 0.0012 \pm 0.0013$
OPAL	(91 - 94), prel.	$0.2326 \pm 0.0012 \pm 0.0013$
LEP	Average	0.2321 ± 0.0010
MSM		0.23217 ± 0.00064

Table 5.21: Results on the hadronic charge asymmetry obtained at LEP [163].

The experiments quote the results of the hadronic charge asymmetry measurement already in terms of $\sin^2 \bar{\theta}_W$. Their results are summarised in Table 5.21. The systematic error, dominated by fragmentation and decay modelling uncertainties, is typically 10% to 30% larger than the statistical error. In the combination, the systematic error due to fragmentation uncertainties of about 0.005 is assumed to be correlated between the experiments. The LEP average is:

$$\sin^2 \bar{\theta}_W = 0.2321 \pm 0.0010. \quad (5.89)$$

The total systematic part of the error on $\sin^2 \bar{\theta}_W$ for this measurement amounts to 0.0008.

5.8 The Effective Electroweak Mixing Angle

The asymmetry measurements obtained at LEP and SLC determine the ratio of the vector and axial-vector coupling constants, and therefore the effective electroweak mixing angle. A crucial test of the Standard Model is given by the requirement, that it must be possible to interpret all asymmetry measurements in terms of the same value for the effective electroweak mixing angle.

The seven results for $\sin^2 \bar{\theta}_W$ derived from the various asymmetry measurements are shown in Figure 5.37. The sensitivity of $A_{fb}^{0,q} = \frac{3}{4} A_e A_q$ to the electroweak mixing angle arises from the A_e part rather than the A_q part, thus $A_{fb}^{0,q}$ determines $\sin^2 \bar{\theta}_W$ for electrons, and the flavour specific corrections, in particular those of the b quark, do not play a role. Neglecting the small correlation between the measurements of $A_{fb}^{0,b}$ and $A_{fb}^{0,c}$, the weighted average of all $\sin^2 \bar{\theta}_W$ results has a value of:

$$\sin^2 \bar{\theta}_W = 0.23157 \pm 0.00018. \quad (5.90)$$

The average has a $\chi^2/\text{d.o.f}$ of 7.8/6, corresponding to a probability of 25%, showing the consistency of the results. The largest contributions to the χ^2 arise from the two most precise determinations of this quantity, A_{lr} and $A_{fb}^{0,b}$, who lead to low and high values of $\sin^2 \bar{\theta}_W$ with respect to the average. Note that the $\chi^2/\text{d.o.f}$ of the average has decreased from 12.8/6, corresponding to a probability of 5%, by 5.0/6 over the last two years [199, 200].

Preliminary

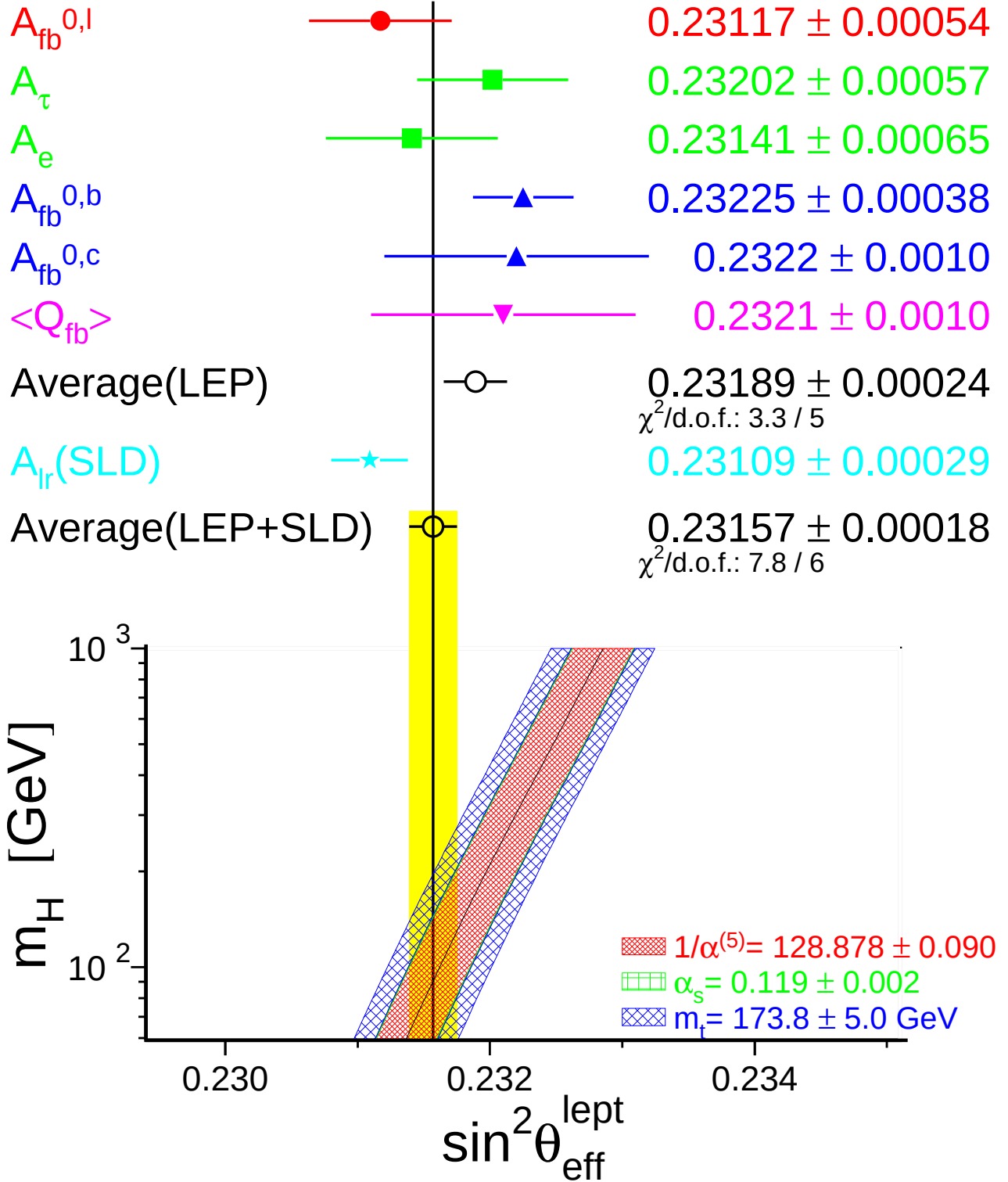


Figure 5.37: Measurements of effective electroweak mixing angle $\sin^2 \bar{\theta}_W$ derived from asymmetry measurements at SLC and LEP-I. For the average, the small correlation between the measurements of $A_{\text{fb}}^{0,b}$ and $A_{\text{fb}}^{0,c}$ is neglected.

5.9 Neutrino-Nucleon Scattering

Neutrino-nucleon scattering allows to measure both charged weak current and neutral weak current interactions. As shown in Figure 5.38, the interactions proceed via the t -channel exchange of a W or Z boson, connecting the incoming neutrino or anti-neutrino to a quark in the nucleons of the target material.



Figure 5.38: Feynman diagrams in neutrino-nucleon scattering on parton level. Left: charged-current reaction. Right: neutral-current reaction.

The differential cross sections for neutral and charged weak interactions are related due to weak-isospin invariance. For isoscalar targets having the same number of up-type quarks and down-type quarks, and considering only valence quarks, this is expressed in the Llewellyn-Smith relations [201]:

$$d\sigma_{NC}(\nu) = g_L^2 d\sigma_{CC}(\nu) + g_R^2 d\sigma_{CC}(\bar{\nu}) \quad (5.91)$$

$$d\sigma_{NC}(\bar{\nu}) = g_L^2 d\sigma_{CC}(\bar{\nu}) + g_R^2 d\sigma_{CC}(\nu), \quad (5.92)$$

where $g_L^2 = g_L^2(u) + g_L^2(d)$ and $g_R^2 = g_R^2(u) + g_R^2(d)$ are the sum of neutral weak current couplings for left- and right-handed quark flavours, while the charged weak current couplings are assumed to be $L = 1, R = 0$. The Llewellyn-Smith relations allow the calculations of cross section ratios, where unknown parton distribution function and experimental effects largely cancel:

$$R_\nu = \frac{\sigma_{NC}(\nu)}{\sigma_{CC}(\nu)} = g_L^2 + g_R^2 \cdot r = \rho^2 \left(\frac{1}{2} - \sin^2 \theta_W + \frac{5}{9} \sin^4 \theta_W [1 + r] \right) \quad (5.93)$$

$$R_{\bar{\nu}} = \frac{\sigma_{NC}(\bar{\nu})}{\sigma_{CC}(\bar{\nu})} = g_L^2 + g_R^2 \cdot \frac{1}{r} = \rho^2 \left(\frac{1}{2} - \sin^2 \theta_W + \frac{5}{9} \sin^4 \theta_W \left[1 + \frac{1}{r} \right] \right), \quad (5.94)$$

with $r = \sigma_{CC}(\bar{\nu})/\sigma_{CC}(\nu) = 1/3$. In case both a neutrino and an antineutrino beam is available, also the Paschos-Wolfenstein ratios $R_\pm$ [202] are measured:

$$R_\pm = \frac{\sigma_{NC}(\nu) \pm \sigma_{NC}(\bar{\nu})}{\sigma_{CC}(\nu) \pm \sigma_{CC}(\bar{\nu})} = \frac{R_\nu \pm r R_{\bar{\nu}}}{1 \pm r} \quad (5.95)$$

$$= g_L^2 \pm g_R^2 = \begin{cases} \rho^2 \left(\frac{1}{2} - \sin^2 \theta_W + \frac{10}{9} \sin^4 \theta_W \right) \\ \rho^2 \left(\frac{1}{2} - \sin^2 \theta_W \right) \end{cases}. \quad (5.96)$$

Scattering of sea quarks and antiquarks introduce systematic errors due to uncertainties in the description of the sea. Since $\sigma(\nu q) = \sigma(\bar{\nu} \bar{q})$ and $\sigma(\bar{\nu} q) = \sigma(\nu \bar{q})$, the effect of scattering from sea quarks and antiquarks, which are symmetric under quark-antiquark exchange, cancels in the difference of the neutrino and antineutrino cross sections. The Paschos-Wolfenstein ratio R_- is thus insensitive to the scattering from the sea.

The ratios $R_\pm$ depend only on the electroweak parameters and $\sin^2 \theta_W$. The ρ parameter appears naturally because the ratio of neutral and charged weak cross sections is taken, which is the ratio of couplings times propagators involving the gauge-boson masses as discussed in Section 2.11, $\rho = (M_W/M_Z)^2 / \cos^2 \theta_W$. This may be reinterpreted as the introduction of effective neutral coupling

constants proportional to $\sqrt{\rho}$ for the $Zq\bar{q}$ vertex and the $Z\nu\bar{\nu}$ vertex, which appear if neutral-current processes are expressed using the charged-current coupling G_F , where the latter cancels in the ratio.

Electroweak radiative corrections modify the above relations [203–205]. The ρ parameter moves away from unity, its leading M_t dependence is the same as the one discussed in Section 2.11.2 before. The parameter κ multiplying the on-shell $\sin^2 \theta_W$ in the above formulae is small because of subtle cancellation effects between radiative corrections caused by bosonic and fermionic loops. The measurement of the various cross section ratios thus remains a determination of the mass ratio M_W/M_Z , which is usually quoted in terms of the on-shell electroweak mixing angle, $\sin^2 \theta_W = 1 - M_W^2/M_Z^2$. Using M_Z as determined at LEP–I, it is also interpreted as a measurement of M_W . However, one has to keep in mind that such a mass determination is not based on an analysis of on-shell W or Z bosons.

5.9.1 Measurements and Results

The most recent neutrino-nucleon scattering experiment, NUTEV, has taken data at Fermilab with the CCFR detector until the fall of 1997. The detector consists of an 18 m long 690 ton neutrino target calorimeter. The calorimeter is made of 168 iron plates, 3 m · 3 m · 5.1 cm in size. The active elements consist of liquid scintillation counters and drift chambers interleaved every two and every four iron plates, respectively. Neutrinos and antineutrinos with energies ranging from 30 GeV to 350 GeV are produced from decays of pions and kaons, which are themselves produced from the interactions of 800 GeV protons in a production target. Mesons of positive or negative charge are selected, yielding either a neutrino or an antineutrino beam, $\pi^+/K^+ \rightarrow \mu^+\nu_\mu$ and $\pi^-/K^- \rightarrow \mu^-\bar{\nu}_\mu$.

For an incident ν_μ or $\bar{\nu}_\mu$ beam, charged-current reactions are tagged by the presence of a μ^- or μ^+ in the final state which traverses the complete detector as a minimum ionising particle. In contrast, neutral-current reactions do not contain a muon in the final state, and the event is much more localised in the detector. This difference in event shape, shown in Figure 5.39, is exploited in order to separate the two types of reactions based on the length of the event in the detector. The distributions of the event length from the NUTEV collaboration for neutrino and antineutrino beams are shown in Figure 5.40 [206]. The contamination of the muon-neutrino beam with electron-neutrinos must be known precisely, as both charged-current and neutral-current electron-neutrino reactions do not lead to muons in the final state.

The neutrino-nucleon scattering experiments CDHS, CHARM and CCFR, used the ratios R_ν and $R_{\bar{\nu}}$ for the determination of $\sin^2 \theta_W$. The NUTEV experiment uses the Paschos-Wolfenstein ratio R_- for reduced systematic errors [206]. The NUTEV result is by far the most precise. An effective R_- is considered:

$$R_- = \frac{R_\nu - rR_{\bar{\nu}}}{1 - r}, \quad (5.97)$$

where the parameter r is adjusted in order to make the measurement insensitive to charm production in charged-current reactions. This is necessary to reduce the corresponding systematic error arising from uncertainties in the charm quark mass and in parton distribution functions. For the NUTEV experimental setup, the optimal value for r turns out to be 0.5136. The preliminary result on $\sin^2 \theta_W$, combined with the CCFR result [207], is given by [206]:

$$\sin^2 \theta_W = 0.2255 \pm 0.0018 \text{ (stat.)} \pm 0.0010 \text{ (syst.)} \quad (5.98)$$

$$-0.00142 \cdot \frac{M_t^2 - (175 \text{ GeV})^2}{(100 \text{ GeV})^2} + 0.00048 \cdot \ln \frac{M_H}{150 \text{ GeV}}, \quad (5.99)$$

where the residual dependence of the result on Standard-Model electroweak radiative corrections depending on M_t and M_H is explicitly parametrised. Within the Standard Model, the value of $\sin^2 \theta_W$ is given by $\sin^2 \theta_W = 0.2245 \pm 0.0017$. Using M_Z from LEP–I, the result of the measurement corresponds to a mass of the W boson of $M_W = 80.25 \pm 0.11 \text{ GeV}$, which is of the same accuracy as the direct measurements of M_W by the experiments at the TEVATRON and at LEP–II.

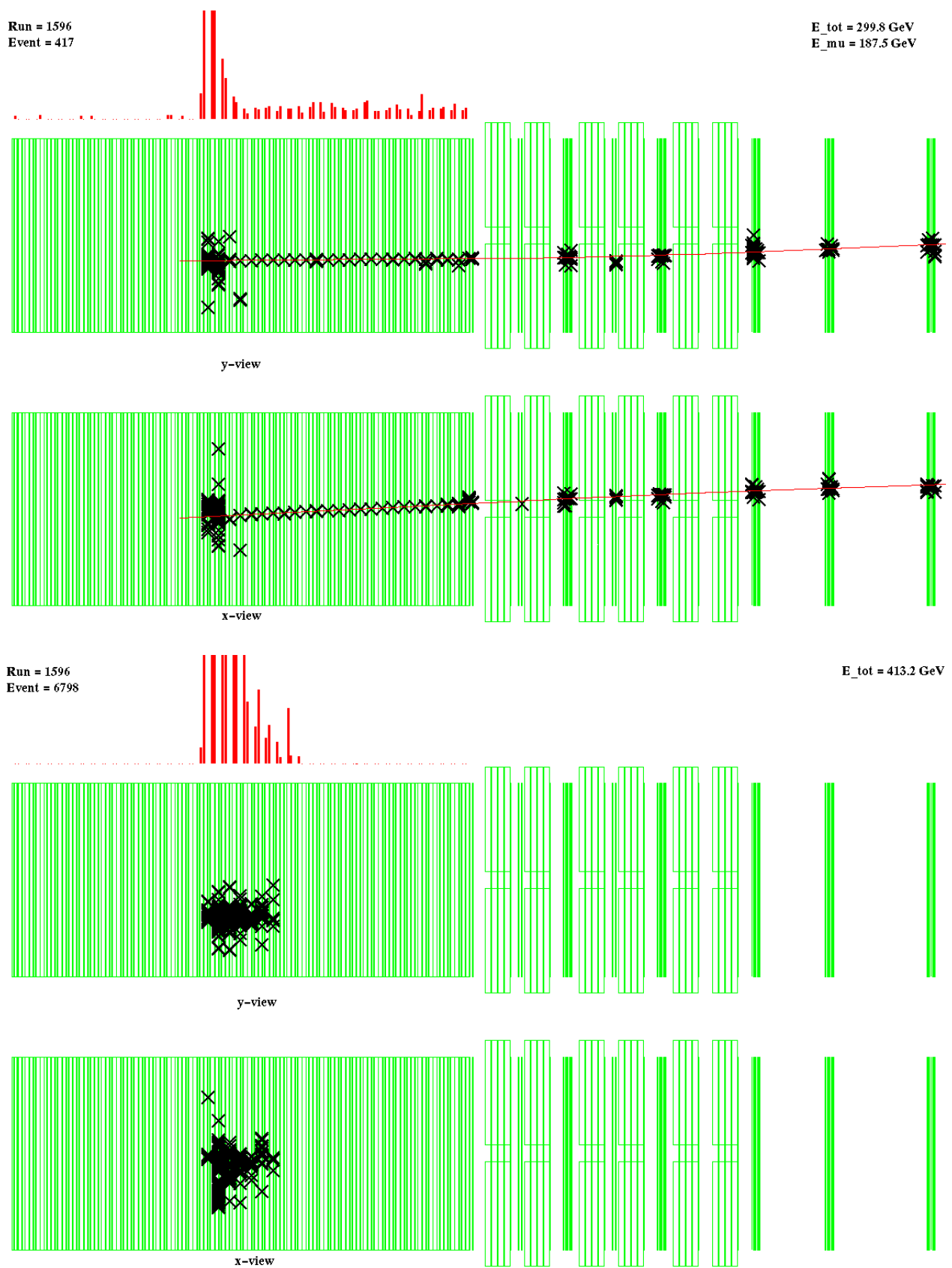


Figure 5.39: Neutrino-nucleon interactions observed in the NUTEV detector. The neutrino or antineutrino beam impinges from the left. The events are shown in both projections. Top: charged current event with a muon in the final state traversing the detector and leading to a long event. Bottom: neutral current event without a muon in the final state leading to a localised shower and a short event.

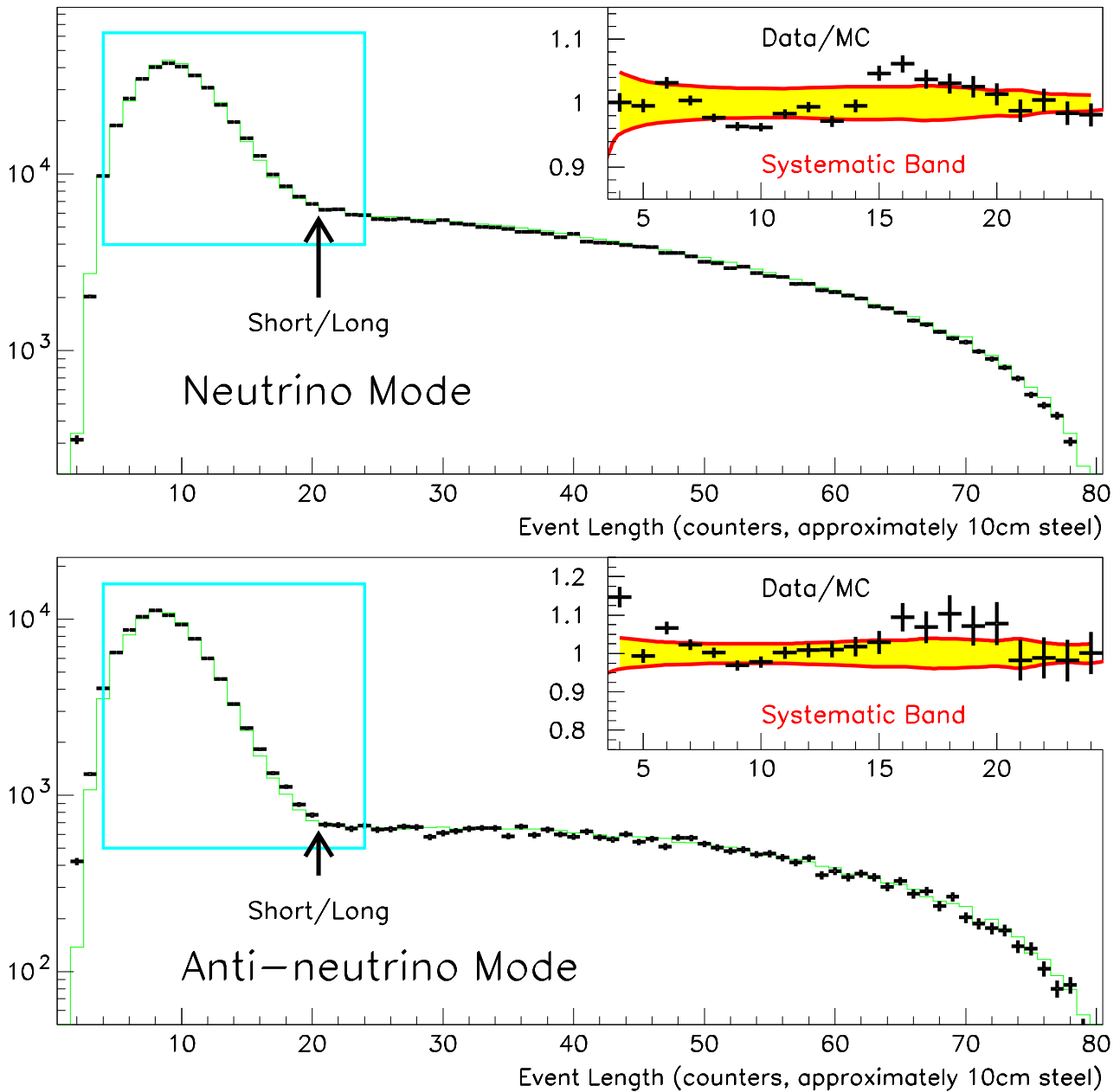


Figure 5.40: Distributions of the event length in neutrino-nucleon interactions observed by the NUTEV collaboration for neutrino and antineutrino beams [206]. The separation between neutral-current and charged-current events is made at a length of 20 counters, approximately 2.1 m of steel, and is indicated by the arrows. The insert shows the ratio between the data and the Monte Carlo expectation together with the band expressing the systematic error.

5.10 Mass of the W Boson

5.10.1 Measurements at Hadron Colliders

In 1983, the W and Z bosons were discovered by the UA1 and UA2 experiments at the CERN SPS $p\bar{p}$ collider. At hadron colliders, W and Z bosons are produced by quark-antiquark fusion:

$$p\bar{p} \rightarrow W + X_W \quad (5.100)$$

$$p\bar{p} \rightarrow Z + X_Z, \quad (5.101)$$

where X_V , $V = W, Z$, denotes the $p\bar{p}$ remnant recoiling against the heavy boson. The W and Z bosons are cleanly identified in the leptonic decay modes, $W \rightarrow \ell\nu_\ell$ and $Z \rightarrow \ell^+\ell^-$, if the leptons are electrons or muons:

$$p\bar{p} \rightarrow X_W \ell\nu_\ell \quad (5.102)$$

$$p\bar{p} \rightarrow X_Z \ell^+\ell^-. \quad (5.103)$$

The Feynman diagrams for W and Z production and decay are shown in Figure 5.41.

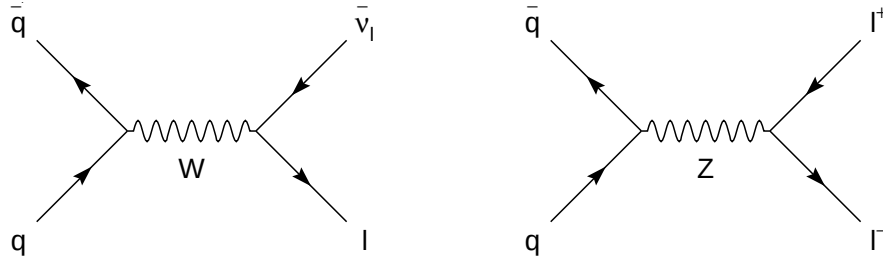


Figure 5.41: Feynman diagrams of W and Z production and decay in $p\bar{p}$ collisions on parton level.

In total a few hundred W and Z events were observed by the SPS experiments UA1 and UA2, which had taken data at a $p\bar{p}$ centre of mass energy between 0.5 TeV and 0.6 TeV. The TEVATRON experiments CDF and DØ, taking data at a centre-of-mass energy of 1.8 TeV, have observed several 10,000 leptonic W and Z decays in run I which ended in 1996. The detectors CDF and DØ are shown in Figure 5.42. They have a similar structure as the LEP and SLD detectors. Viewed from the interaction point, there is a silicon vertex detector, drift chambers, calorimeters and muon chambers. Also in $p\bar{p}$ physics, one aims for a coverage of the full solid as complete as possible. Selected W events are shown in Figures 5.43 and 5.44.

Because of the additional neutrino in leptonic W decay, it is not possible to reconstruct the complete W kinematics, as the W boson is produced with an unknown boost. The longitudinal part of the boost vector cannot be determined as part of the remnants of the proton-antiproton system are lost along the beam pipe. For this reason the W kinematics is only known in the coordinates transverse to the beam axis.

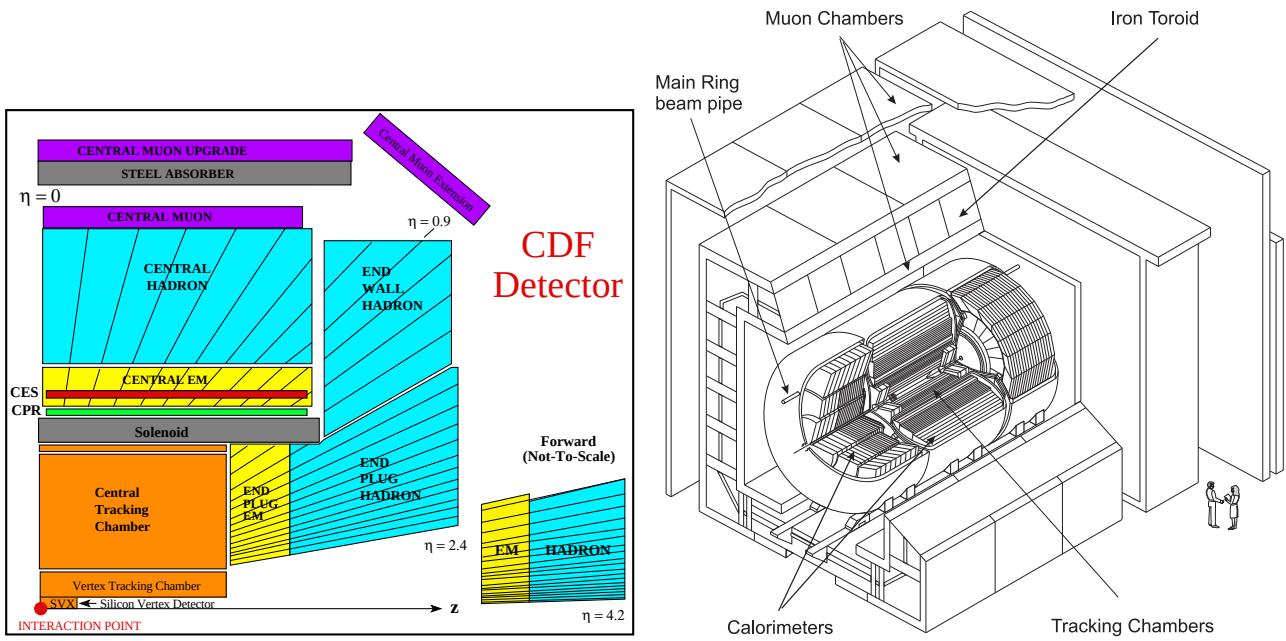


Figure 5.42: The detectors CDF and DØ at the TEVATRON $p\bar{p}$ collider at Fermilab.

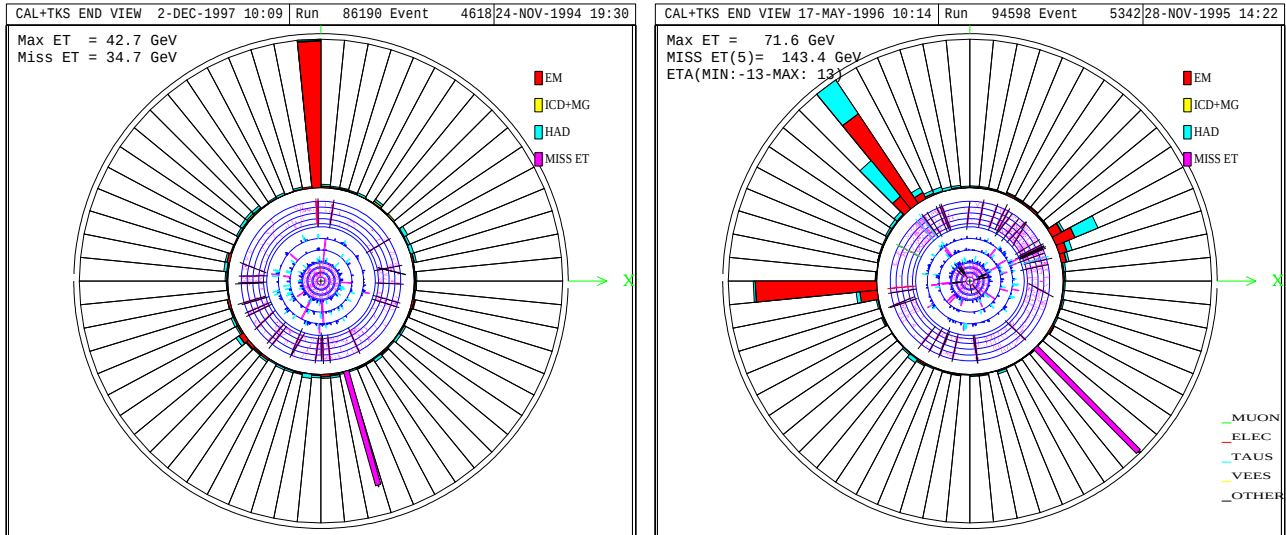


Figure 5.43: W events observed in the DØ detector, where the W decays into an electron and a neutrino. Shown is the end view of the detector with hits and tracks in the central tracking system and energy depositions in the electromagnetic and hadronic calorimeters. Left: W plus zero jets. The electron is causing the electromagnetic jet at $\phi = 90^\circ$. The missing energy attributed to the neutrino is shown as the slim strip at $\phi = 285^\circ$. Right: W plus two jets. The electron is causing the electromagnetic jet at $\phi = 180^\circ$. The missing energy attributed to the neutrino is shown as the slim strip at $\phi = 315^\circ$.

CDF

W + 0,1,2,3 jet(s) Events

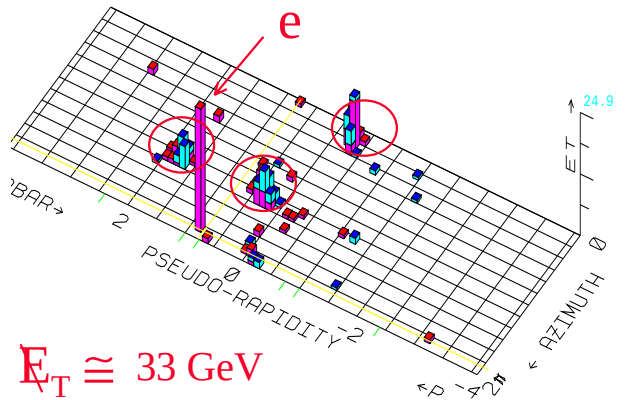
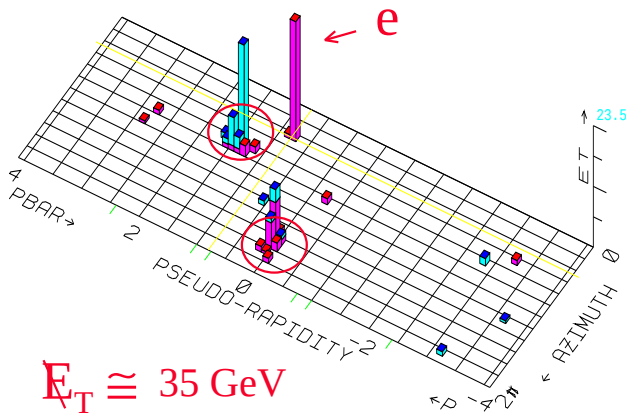
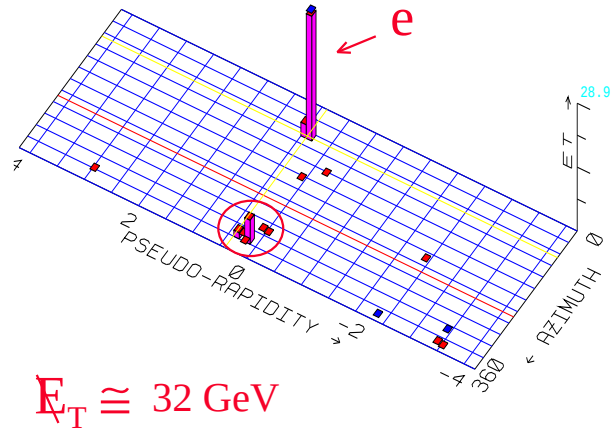
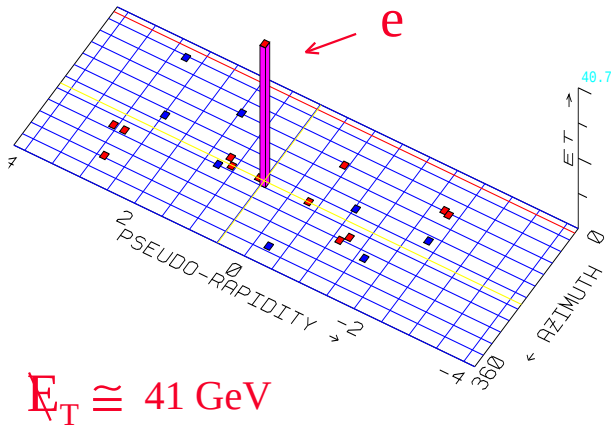


Figure 5.44: W events with 0, 1, 2, 3 additional jets observed in the CDF detector. Shown is the two-dimensional map of energy depositions in the calorimeters.

The so-called transverse invariant mass is calculated based on the transverse energy of the charged lepton, E_T^ℓ , and the transverse energy of the neutrino, E_T^ν , given by the transverse part of the missing energy vector:

$$m_T^2(\ell, \nu) = 2E_T^\ell E_T^\nu (1 - \cos \phi_{\ell\nu}) \leq m_{\text{inv}}^2(\ell, \nu), \quad (5.104)$$

where the angle $\phi_{\ell\nu}$ denotes the azimuthal opening angle between the lepton and the neutrino in the transverse plane. The transverse mass is always smaller than the invariant mass. It approaches the invariant mass if the transverse plane coincides with the decay plane of the W boson. Therefore, information on the W mass is obtained from those events whose transverse mass approaches the upper edge of the transverse-mass spectrum, called Jacobian peak.

The transverse mass m_T is insensitive to transverse boosts and therefore less sensitive to the W boson production model than the transverse energy E_T of lepton or neutrino alone. The energies of the lepton and the neutrino also carry information on M_W . The transverse lepton energy is less sensitive to detector effects than the m_T and E_T^ν spectra which depend strongly on the detector response to the underlying event. This is illustrated in Figure 5.45.

The transverse mass does depend on the longitudinal momentum of the W boson whose distribution depends in turn on the partonic structure functions of the proton. These distributions are constrained by the measured forward-backward asymmetry of W production in hadron collisions.

The CDF experiment uses both $W \rightarrow e\nu$ and $W \rightarrow \mu\nu$ events, while DØ uses $W \rightarrow e\nu$ events only. Distributions of the transverse mass and transverse lepton energy as measured by the DØ and CDF collaborations are shown in Figures 5.46 and 5.47. Detector resolution and the total width of the W boson are responsible for the tail of high transverse masses extending above the nominal value of M_W . Background from $W \rightarrow \tau\nu \rightarrow \ell\nu\nu$, between 0.8% and 1.6%, is concentrated towards lower values of m_T and E_T^ℓ . Background from $Z \rightarrow \ell^+\ell^-$ with one lepton lost is 0.1% to 0.4% for the electron channel and about 3.6% for the muon channel.

A precise energy calibration of the detector is essential. The energy scale and resolution function of leptons is determined by analysing $Z \rightarrow \ell^+\ell^-$ events, as the Z mass is precisely known from LEP-I measurements. Non-linear effects in the energy calibration are studied by using leptonic decays of heavy-quark resonances, $J/\Psi/\Upsilon \rightarrow \ell^+\ell^-$, and $\pi^0 \rightarrow \gamma\gamma$ decays. The understanding of detector effects in the measurement of the hadronic recoil to the W boson is tested by measuring this recoil in Z events, where it is independently determined from the leptonic decay products of the Z. Systematic errors due to this data driven calibration naturally scale down with additional data statistics.

For both TEVATRON experiments, the transverse mass distributions determine M_W with a slightly better statistical and systematic accuracy. Based on fits to their observed transverse-mass spectra, the following results on M_W are obtained by CDF [208–210] and DØ [209–212]:

$$M_W(\text{CDF}) = 80.38 \pm 0.12 \text{ GeV} \quad (5.105)$$

$$M_W(\text{DØ}) = 80.43 \pm 0.11 \text{ GeV}, \quad (5.106)$$

where the errors include systematic uncertainties. Taking a correlated systematic error of 0.05 GeV into account and including the less precise UA2 result, the current preliminary value of M_W obtained at hadron colliders is [209, 210]:

$$M_W = 80.41 \pm 0.09 \text{ GeV}. \quad (5.107)$$

The systematic part of the error on M_W amounts to 0.07 GeV.

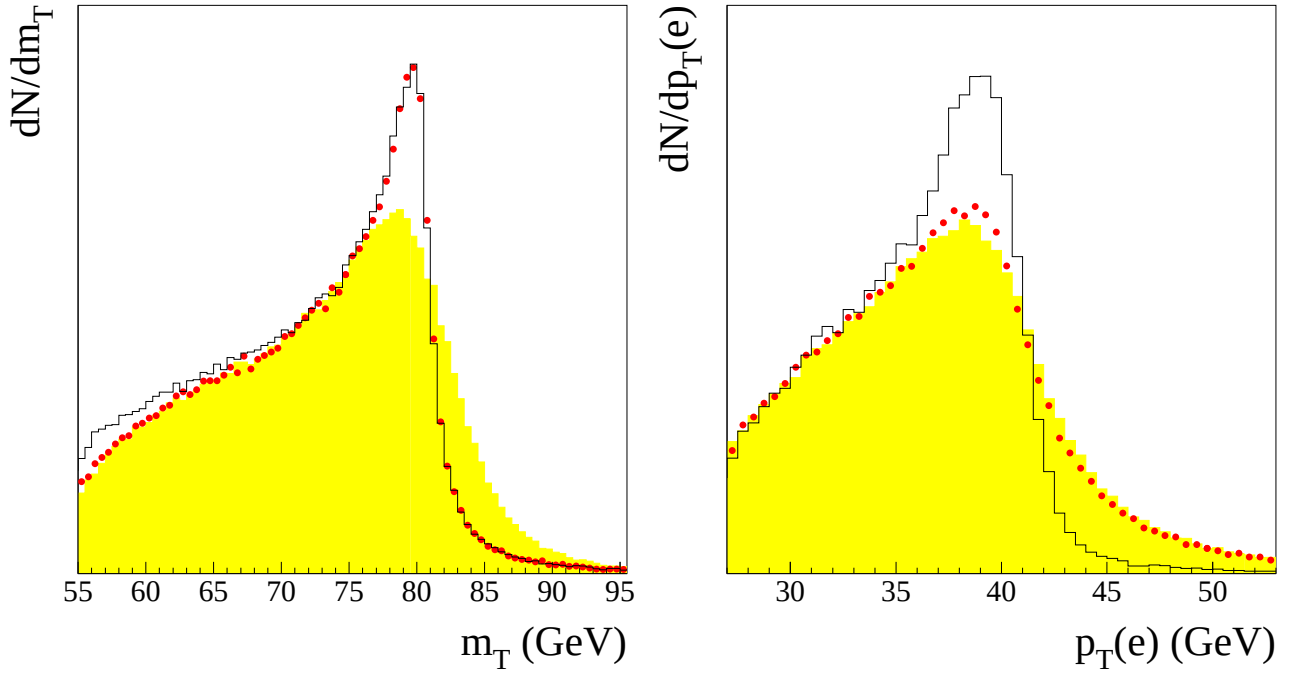


Figure 5.45: Monte Carlo simulation of the transverse lepton-neutrino mass (left) and the transverse lepton energy (right) for DØ [213]. The solid lines show the distributions at generator level with the transverse W momentum set to zero. The dots show the distributions when the transverse W momentum is included according to the W production model, modifying the transverse lepton energy but not the transverse lepton-neutrino mass. The shaded area show the spectra when the resolution of the detector, here DØ, is included, visibly affecting the transverse lepton-neutrino mass but not the transverse lepton energy.

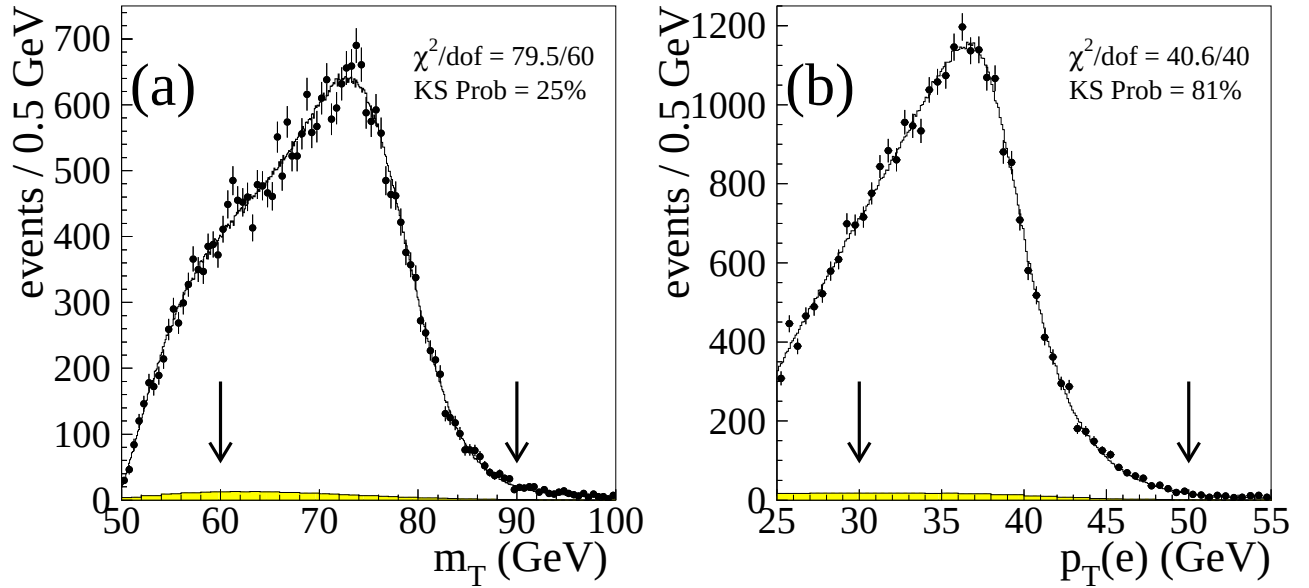


Figure 5.46: Distributions of (a) the transverse electron-neutrino mass and (b) the transverse electron energy as observed in $W \rightarrow e\nu$ events selected by the DØ experiment [213]. The solid line shows the fit result including background. The arrows indicate the fit region. In addition to the $\chi^2/\text{d.o.f}$, the Kolmogorov-Smirnov probability is given. The shaded part denotes the background.

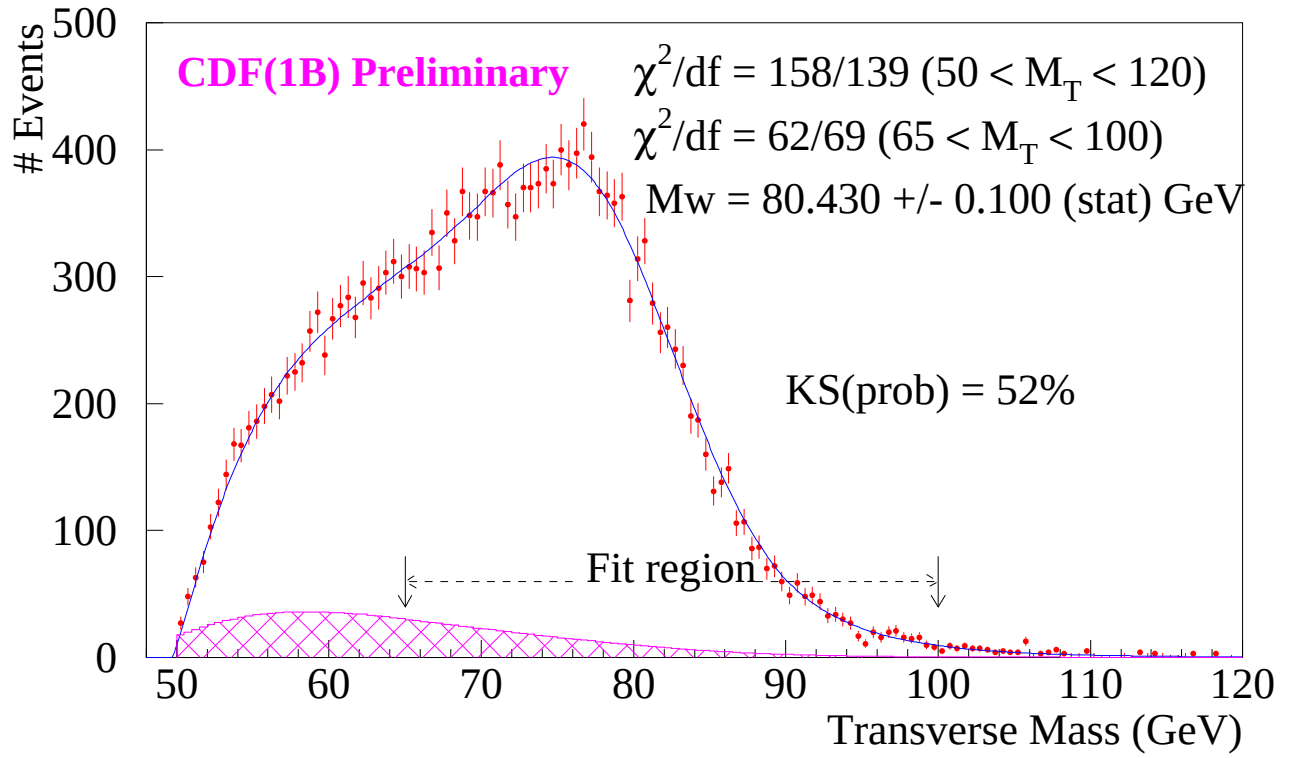


Figure 5.47: Distribution of the transverse muon-neutrino mass as observed in $W \rightarrow \mu\nu$ events selected by the CDF experiment [210]. The solid line shows the fit result including background. The arrows indicate the fit region. In addition to the $\chi^2/\text{d.o.f.}$, the Kolmogorov-Smirnov probability is given. The hatched part denotes the background.

5.10.2.1 Selection

Hadronic events, $W^+W^- \rightarrow q\bar{q}q\bar{q}$, are selected with typical efficiencies of 85% and purities of 80%. Events must contain four well separated hadronic jets and no missing energy. The dominant background arises from QCD multijet production in $e^+e^- \rightarrow q\bar{q}(\gamma)$. At centre-of-mass energies $\sqrt{s} > 2M_Z$, above the ZZ threshold, $e^+e^- \rightarrow ZZ \rightarrow q\bar{q}q\bar{q}(\gamma)$ becomes an important background.

Semileptonic events, $W^+W^- \rightarrow q\bar{q}\ell\nu$, are tagged by the presence of a high-energy charged lepton. In addition, events must contain two hadronic jets and missing energy due to the neutrino. The main background arises through inclusive lepton production in $q\bar{q}(\gamma)$ events, where the missing energy is given by initial-state radiative photons lost in the beam pipe. Semileptonic events are selected with an efficiency between 30% and 90% and a purity between 70% and 95%, where the lower values are obtained for $q\bar{q}\tau\nu$ events and the higher values for $q\bar{q}e\nu$ and $q\bar{q}\mu\nu$ events.

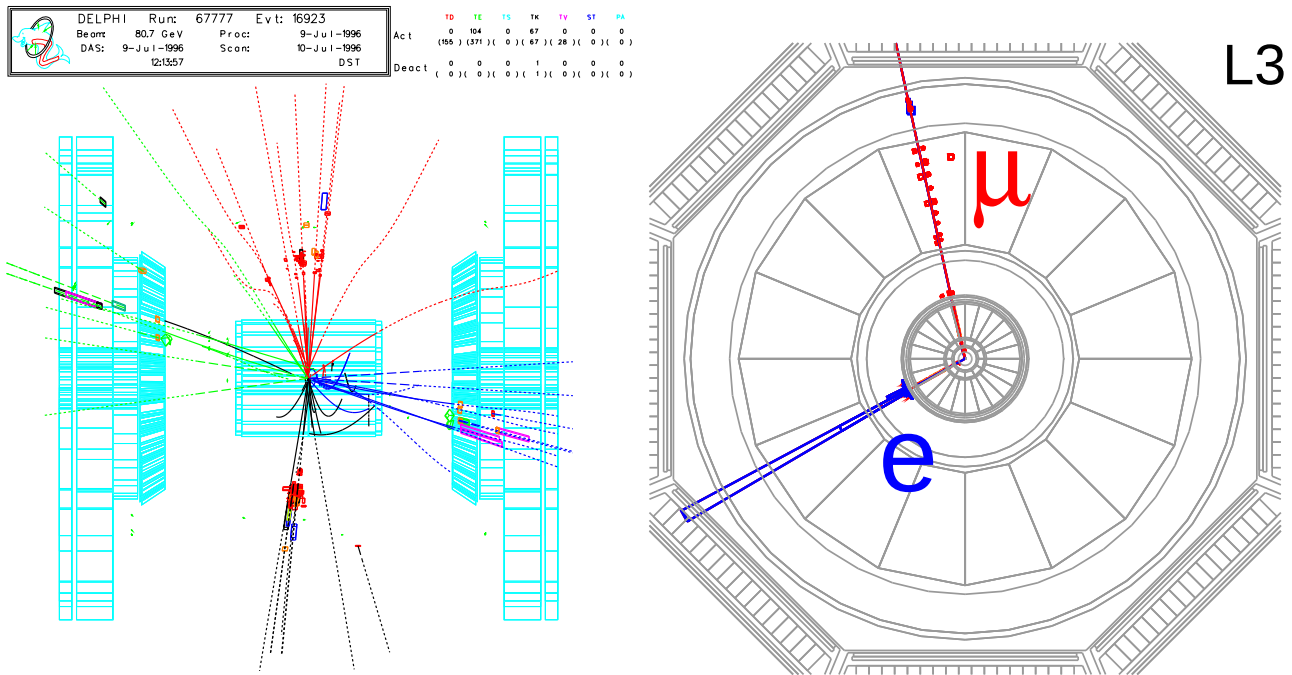


Figure 5.48: W-pair events selected at LEP-II. Left: $W^+W^- \rightarrow q\bar{q}q\bar{q}$ event observed in the DELPHI detector, showing four well separated jets. Right: $W^+W^- \rightarrow e\nu\mu\nu$ event observed in the L3 detector showing apparent lepton flavour violation.

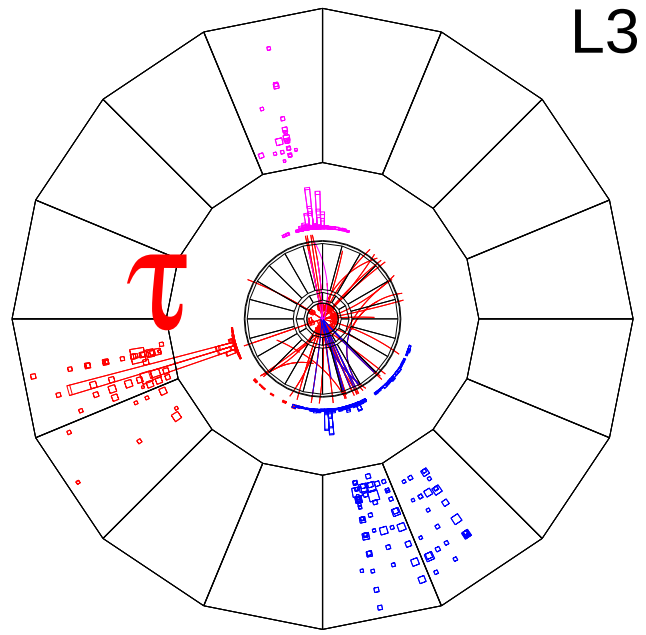
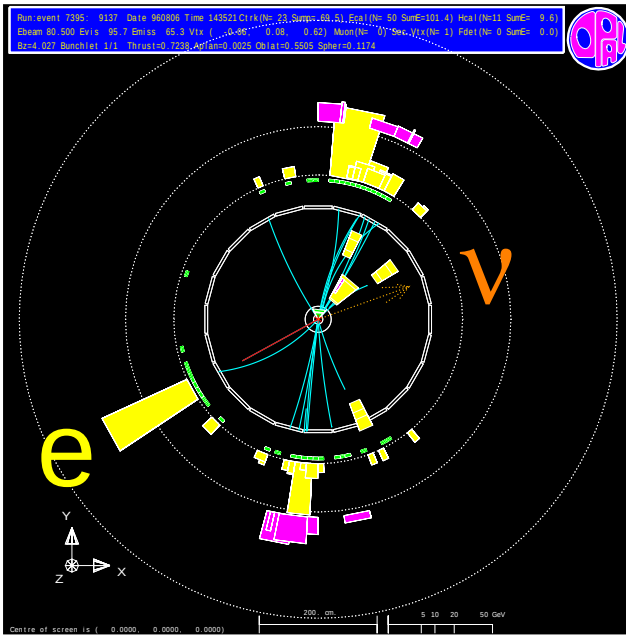
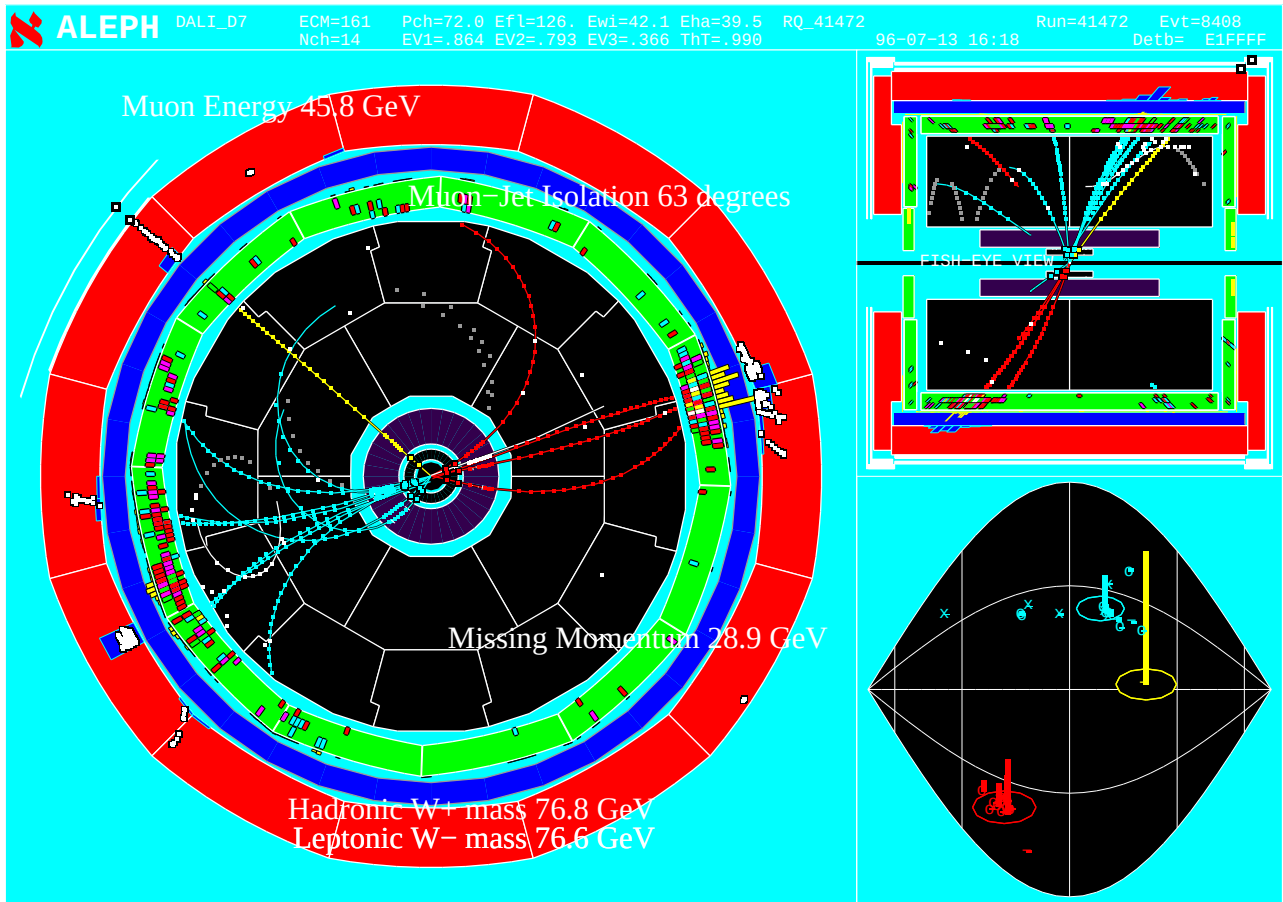


Figure 5.49: W^+W^- events selected at LEP-II. Top: $W^+W^- \rightarrow q\bar{q}\mu\nu$ event observed in the ALEPH detector. Bottom left: $W^+W^- \rightarrow q\bar{q}\nu e$ event observed in the OPAL detector, showing two hadronic jets and an electron. The neutrino is inferred from the missing momentum vector. Bottom right: $W^+W^- \rightarrow q\bar{q}\tau\nu$ event observed in the L3 detector, showing two hadronic jets and a hadronic τ decay.

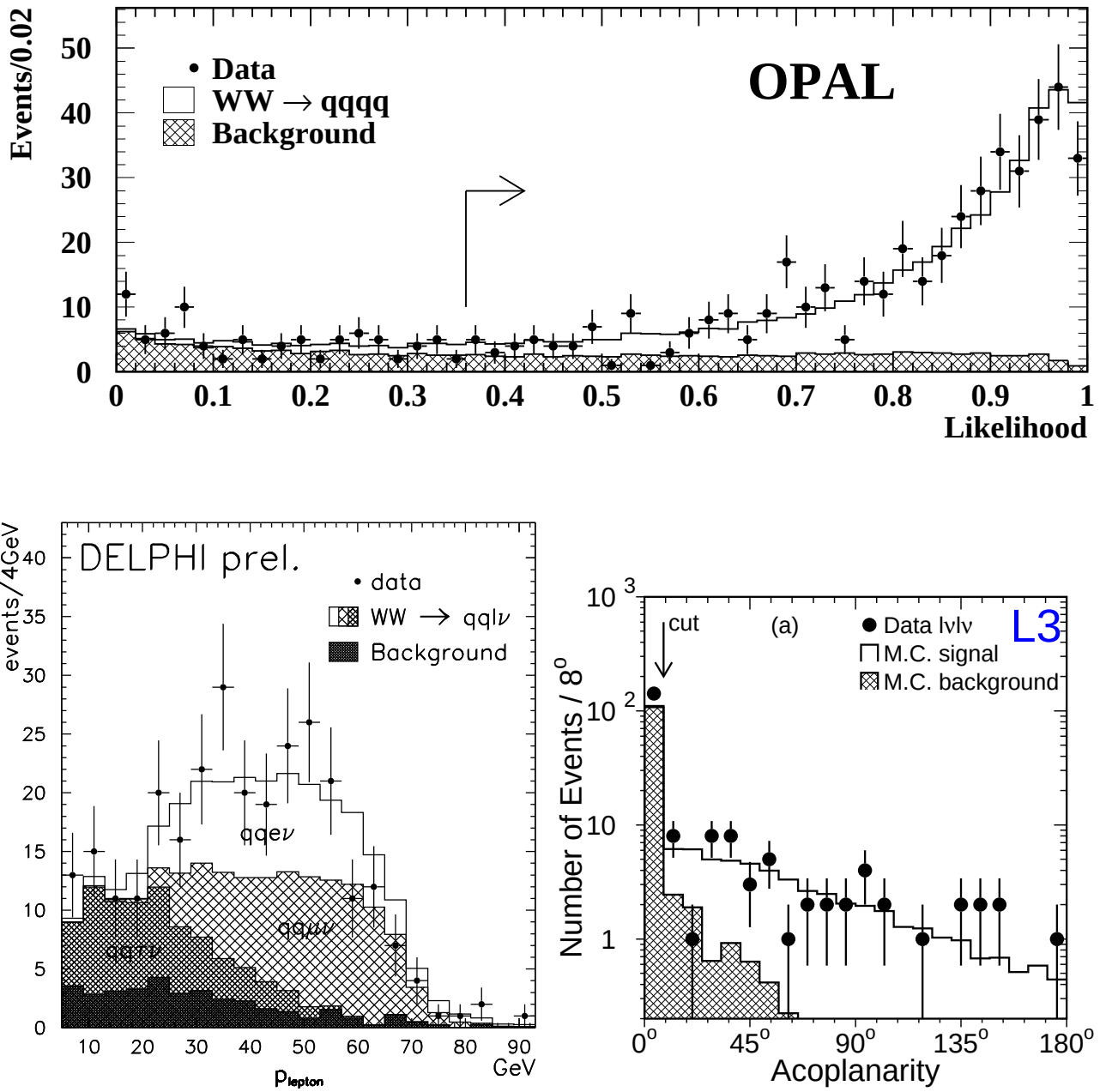


Figure 5.50: Distributions of selection variables in W-pair analyses at $\sqrt{s} = 183$ GeV. Top: Likelihood distribution for $q\bar{q}q\bar{q}$ events selected by OPAL [214]. Bottom left: Momentum of charged lepton in $q\bar{q}l\nu$ events selected by DELPHI [215]. Bottom right: Acoplanarity of charged leptons in $l\nu l\nu$ events selected by L3 [216].

A joint analysis of all W-pair mediated four-fermion channels must be performed in order to determine cross sections and W decay branching fractions, since there is sizeable cross feed between the semileptonic channels and between the purely leptonic channels. Cross sections for channel i are determined by maximising Poisson probabilities $P(N_i, \mu_i)$ to observe N_i events when μ_i are expected:

$$L = \prod_i P(N_i, \mu_i) \quad (5.108)$$

$$\mu_i = \left(\sum_j \epsilon_{ij} \sigma_j + \sigma_i^{bg} \right) \cdot \mathcal{L}, \quad (5.109)$$

where ϵ_{ij} is the efficiency of the selection of channel i to select events of channel j . Within this ansatz, the total W-pair cross section σ_{WW} and the W-decay branching fractions are determined by writing the channel cross sections as follows:

$$\sigma(q\bar{q}q\bar{q}) = \sigma_{WW} \cdot B(W \rightarrow q\bar{q})B(W \rightarrow q\bar{q}) \quad (5.110)$$

$$\sigma(q\bar{q}\ell\nu) = \sigma_{WW} \cdot 2B(W \rightarrow q\bar{q})B(W \rightarrow q\bar{q}) \quad (5.111)$$

$$\sigma(\ell\nu\ell\nu) = \sigma_{WW} \cdot (B(W \rightarrow e\nu) + B(W \rightarrow \mu\nu) + B(W \rightarrow \tau\nu))^2. \quad (5.112)$$

The results on W-decay branching fractions are summarised in Table 5.22 and compared in Figure 5.51. They agree well with the prediction of a universal charged current coupling strength. The measured cross section for W-pair production will be discussed in the following sections.

Experiment	$B(W \rightarrow e\nu)$	$B(W \rightarrow \mu\nu)$	$B(W \rightarrow \tau\nu)$	$B(W \rightarrow q\bar{q})$
ALEPH	$11.2 \pm 0.8 \pm 0.3$	$9.9 \pm 0.8 \pm 0.2$	$9.7 \pm 1.0 \pm 0.3$	$69.0 \pm 1.2 \pm 0.6$
DELPHI	$9.9 \pm 1.1 \pm 0.5$	$11.4 \pm 1.1 \pm 0.5$	$11.2 \pm 1.7 \pm 0.7$	$67.5 \pm 1.5 \pm 0.9$
L3	$10.5 \pm 0.9 \pm 0.2$	$10.2 \pm 0.9 \pm 0.2$	$9.0 \pm 1.2 \pm 0.3$	$70.1 \pm 1.3 \pm 0.4$
OPAL	$11.7 \pm 0.9 \pm 0.3$	$10.1 \pm 0.8 \pm 0.3$	$10.3 \pm 1.0 \pm 0.3$	$67.9 \pm 1.2 \pm 0.6$
Average	10.9 ± 0.5	10.3 ± 0.5	10.0 ± 0.6	68.8 ± 0.8
SM	10.8	10.8	10.8	67.5

Table 5.22: Branching fractions of W decay. The hadronic branching fractions are determined under the assumption of charged-current lepton universality [163].

5.10.2.2 Mass Measurements

There are several methods to measure the mass of the W boson in e^+e^- interactions:

1. The threshold method.

The kinematic threshold of W pair production depends on M_W :

$$\sigma(e^+e^- \rightarrow W^+W^-) = \sigma(M_W, \sqrt{s}). \quad (5.113)$$

2. The method of direct reconstruction.

The W mass is measured by the invariant mass of the $W \rightarrow f\bar{f}$ decay products:

$$m_{\text{inv}}(W) = m_{\text{inv}}(f\bar{f}) \quad (5.114)$$

3. The angle method.

The opening angle between the fermion and the antifermion from W decay, $W \rightarrow f\bar{f}$, has a lower limit depending on M_W :

$$\alpha_{\text{min}} \leq \alpha_{f\bar{f}} \leq 180^\circ \quad (5.115)$$

$$\cos \alpha_{\text{min}} = 1 - \frac{8M_W^2}{s}. \quad (5.116)$$

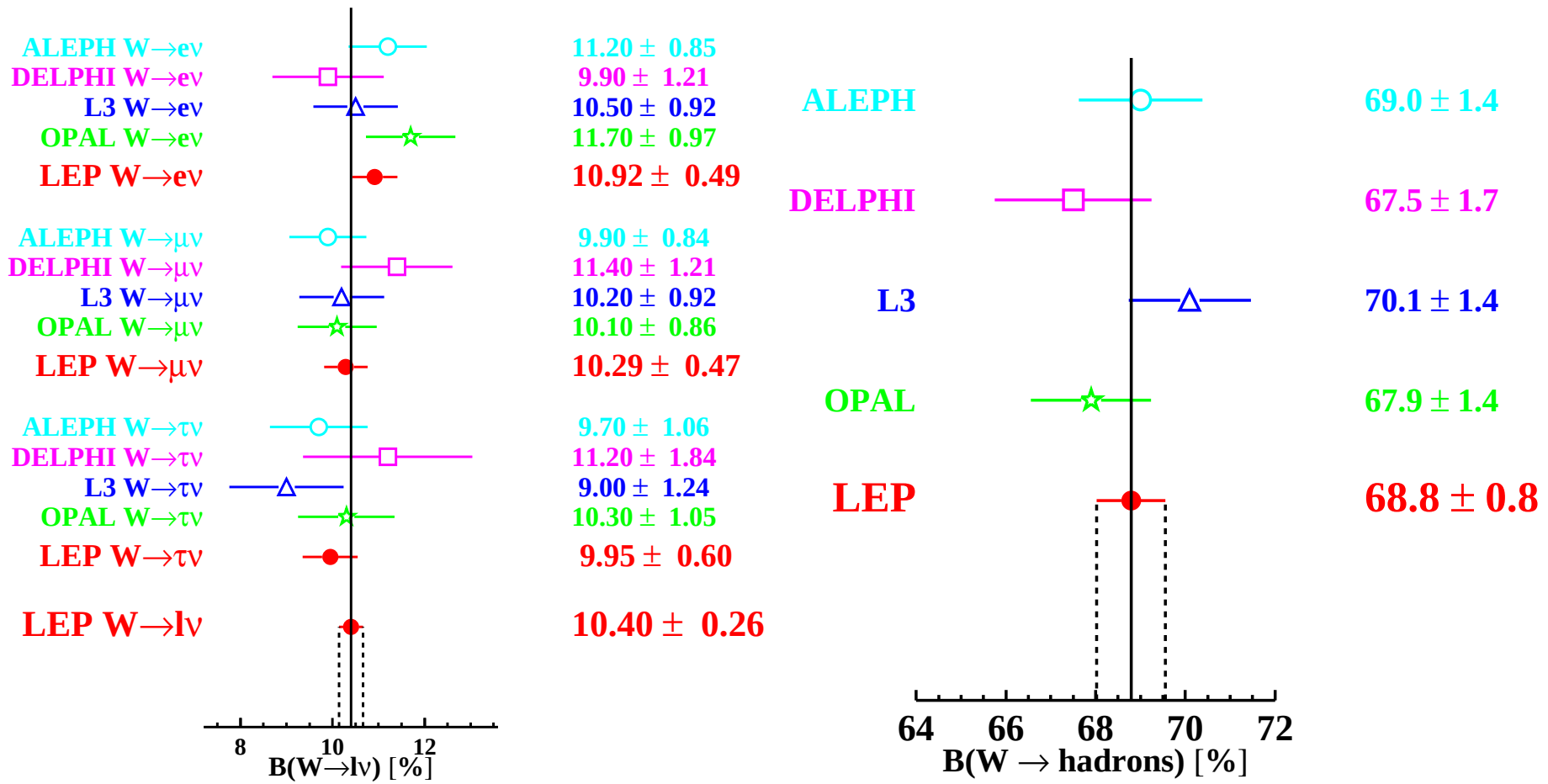


Figure 5.51: Branching fractions of W decay. The hadronic branching fractions are determined under the assumption of charged-current lepton universality [163]. The Standard Model expectations are 10.8% and 67.5%, respectively.

4. The endpoint method.

The energy of each W decay fermion, $W \rightarrow f\bar{f}$, has a lower and an upper limit, both depending on M_W :

$$E_- \leq E_f \leq E_+ \quad (5.117)$$

$$E_{\pm} = \frac{\sqrt{s}}{4} \left(1 \pm \sqrt{1 - \frac{4M_W^2}{s}} \right). \quad (5.118)$$

In each case, the finite width of the W boson has to be taken into account, leading to a softening of otherwise sharp cutoffs. The third method is applicable if both W decay fermions are visible, *i.e.*, for the hadronic decay modes of the W, $W \rightarrow q\bar{q}$. The fourth method is well applicable in case of electrons and muons, whose energies are precisely measured. However, for both methods, only those W decays contribute to the mass measurement which are at the edges of the distributions. This drawback makes the last two method statistically less powerful than the other two, where all events contribute to the mass measurement. Therefore, the experimental effort concentrates on the threshold method for data taken at the kinematic threshold, and on the method of direct reconstruction for data taken at higher centre-of-mass energies.

5.10.2.3 Threshold Method

The threshold method has already been successfully applied by the BES collaboration at the BEPC collider in Beijing in the measurement of the τ lepton mass [217]. The τ -pair cross section is measured at different centre-of-mass energies close to $2m_\tau$, where each $\sqrt{s}$ point is chosen to have the maximal sensitivity of the cross section $\sigma(\sqrt{s})$ on the mass of the τ lepton, given the knowledge on m_τ accumulated so far in the energy scan.

Compared to the τ lepton, the situation for the measurement of the mass of the W boson at LEP-II is somewhat different. The principle of the measurement is shown in Figure 5.52a. The rather large total width of the W boson makes the statistical sensitivity vary only slightly across the range of W masses allowed by earlier measurements. Thus a scan in $\sqrt{s}$ is not necessary and it is sufficient to measure the cross section at a fixed, *a-priori* known centre-of-mass energy.

The sensitivity of the cross section on M_W is given by:

$$\delta M_W = \delta\sigma / \frac{d\sigma_{WW}}{dM_W}. \quad (5.119)$$

Three types of errors, $\delta\sigma$, contribute to the total error of the cross section measurement, the statistical error, $\delta\sigma \propto \sqrt{\sigma_{WW}}$, a systematic scale error, $\delta\sigma \propto \sigma_{WW}$, for example luminosity, and a systematic offset error, $\delta\sigma = \text{const}$, for example background subtraction. The resulting errors on M_W show the following dependence on $\sigma_{WW}(M_W)$:

$$\delta M_W \propto \begin{cases} \frac{dM_W}{d\sigma} \cdot \sqrt{\sigma_{WW}} \\ \frac{dM_W}{d\sigma} \cdot \sigma_{WW} \\ \frac{dM_W}{d\sigma} \end{cases}. \quad (5.120)$$

These error components are shown in Figure 5.52b as a function of M_W . A scale uncertainty of 2% on the theoretical calculation of the W-pair cross section causes an error of 34 MeV on M_W . The error on the background subtraction in the various $W^+W^- \rightarrow f\bar{f}f\bar{f}$ channels is at most 0.1 pb, leading to an error of 50 MeV on M_W . For the amount of luminosity in question, less than 100 pb^{-1} , the statistical error on the cross section measurement is clearly the dominant contribution to the total error on the mass, even in case of 100% efficiency and purity. The curve of the statistical error shows a broad minimum at a centre-of-mass energy about 0.5 GeV above the nominal threshold of $2M_W$. The

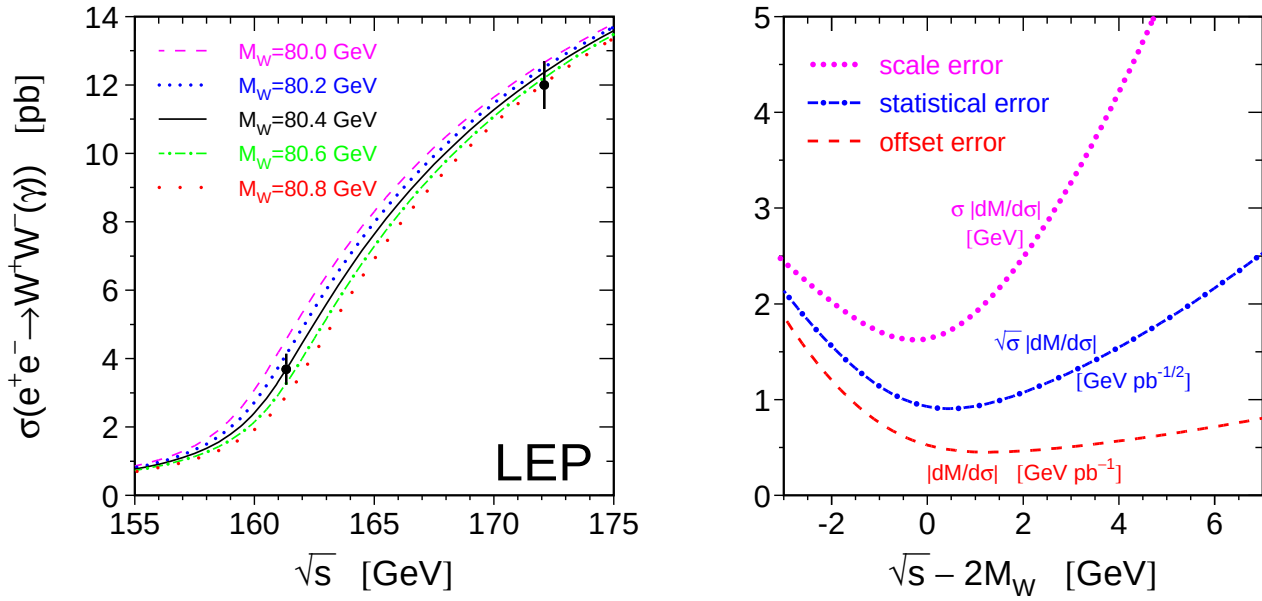


Figure 5.52: W-mass measurement at threshold. Left: Cross section for W-pair production as a function of $\sqrt{s}$ in the vicinity of the kinematic threshold for different values of M_W . Right: Error components of M_W error arising from the statistical error and systematic scale and offset errors on the cross section measurement at threshold. All contributions show a minimum around zero to 2 GeV above the nominal threshold of $\sqrt{s} = 2M_W$.

threshold method is most efficient for the W-mass determination with a cross section measurement at the optimal centre-of-mass energy of $\sqrt{s} = 2M_W + 0.5 \text{ GeV} = 161 \text{ GeV}$.

In the first half of the 1996 data taking period, LEP-II was operated at a centre-of-mass energy of $\sqrt{s} = 161.33 \pm 0.05 \text{ GeV}$. Each LEP experiment collected a luminosity of 10 pb^{-1} and selected about 30 W^+W^- events. The measured total W-pair cross sections of the four LEP experiments are compared in Figure 5.53. The measurements are averaged based on weights given by the expected errors so that the average is unbiased by upward or downward fluctuations of individual measurements. The averaged W-pair cross section at a centre-of-mass energy of $\sqrt{s} = 161.33 \pm 0.05 \text{ GeV}$ is:

$$\sigma_{WW} = 3.69 \pm 0.45 \text{ pb}. \quad (5.121)$$

This value corresponds to a mass of the W boson of:

$$M_W = 80.40^{+0.22}_{-0.21} \pm 0.03 \text{ GeV}, \quad (5.122)$$

also shown in Figure 5.53. The first error is experimental, the second due to the uncertainty in the LEP beam energy calibration as discussed in Section 4.6. For the determination of M_W , this method must assume QED radiative corrections, the Standard-Model calculation of the total width of the W boson in terms of M_W , and the Standard-Model dynamics in the area of gauge couplings of the W boson.

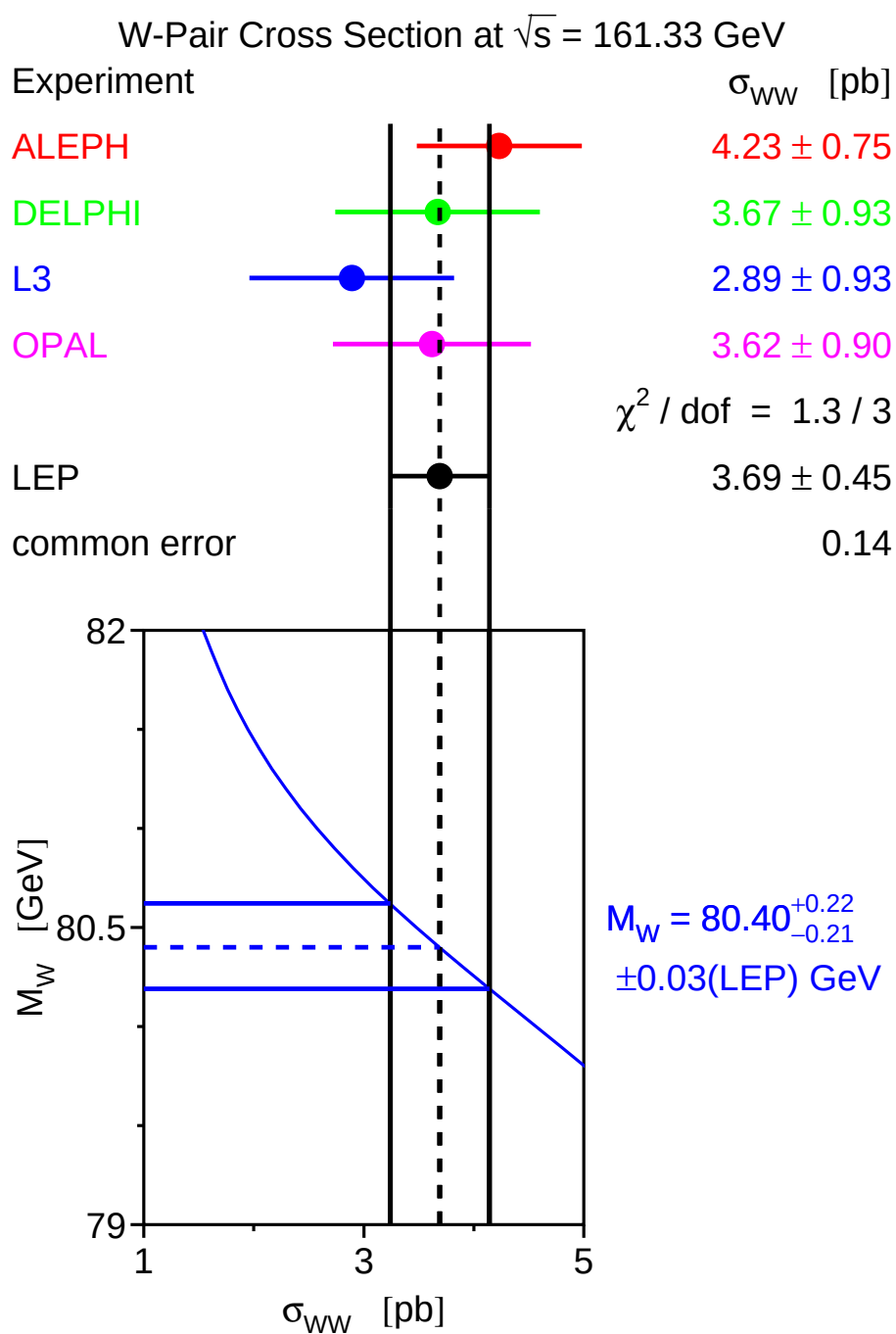


Figure 5.53: Determination of the W-boson mass from the W-pair cross section measured at the kinematic threshold. The χ^2 of the average is calculated based on the uncorrelated errors.

5.10.2.4 Direct Reconstruction Method

At higher centre-of-mass energies, the sensitivity of the total W-pair cross section to the W mass is much reduced. On the other hand, more WW events are available due to the higher cross section, allowing to apply the method of direct reconstruction.

The fermions in selected W-pair events, $e^+e^- \rightarrow f\bar{f}f\bar{f}(\gamma)$ are reconstructed, yielding the energy, E , and polar and azimuthal angle, θ and ϕ of the visible fermions. Invariant masses of W decay products are calculated for fully hadronic and semileptonic W-pair events, $e^+e^- \rightarrow q\bar{q}q\bar{q}(\gamma)$ and $e^+e^- \rightarrow q\bar{q}\ell\nu(\gamma)$, respectively, while in $e^+e^- \rightarrow \ell\nu\ell\nu(\gamma)$ the presence of two neutrinos does not allow a measurement of the W mass with this method.

The laboratory system coincides with the CM system of the W pair produced in e^+e^- interactions, $e^+e^- \rightarrow W^+W^-$, simplifying the kinematic analysis of the events. A kinematic fit imposing four-momentum conservation and equal mass of the two W bosons allows the determination of the unmeasured neutrino kinematics and improves the resolutions in the kinematics of the measured fermions by a factor of three to four. The spectra of reconstructed invariant masses are shown in Figure 5.54.

A difficulty associated with $q\bar{q}q\bar{q}(\gamma)$ events is the assignment of hadronic jets to W bosons. A priori there are three possibilities to group 4 jets into two pairs, (1,2)(3,4), (1,3)(2,4), and (1,4)(2,3). There are several algorithms to solve this ambiguity. For example, one can choose the combination with the smallest mass difference, or the combination with the largest sum of the two masses. It is also possible to let the kinematic fit evaluate the combinations by imposing an equal mass constraint in addition. In that case a kinematic fit is performed for each of the three possibilities. The kinematic fit with the largest fit probability is most likely indicating the correct combination. In contrast to correctly paired four-jet events, the invariant mass distribution of the incorrectly paired events is much broader, as shown in Figure 5.54. In order to recover events, also the second best pairing is included in the mass determination. This is possible because the W-mass values extracted from correct and incorrect pairings are uncorrelated.

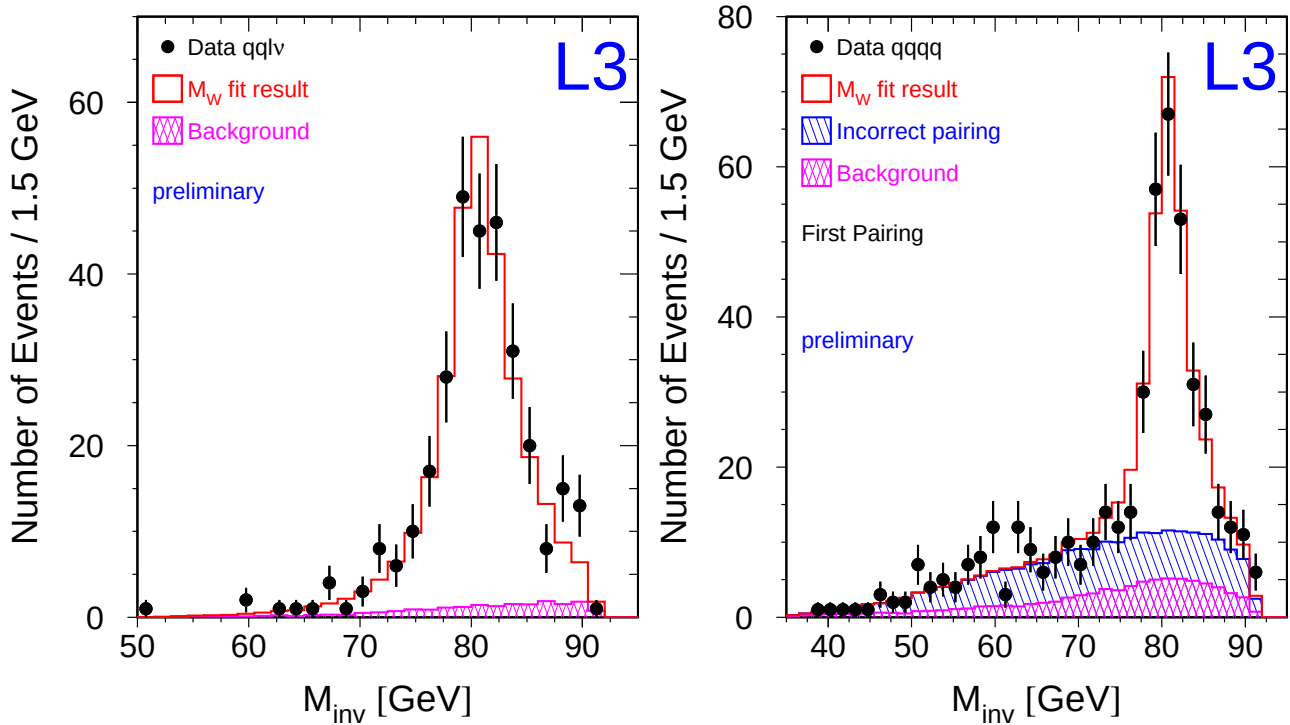


Figure 5.54: Distribution of invariant masses reconstructed in W-pair events selected by L3 at $\sqrt{s} = 183$ GeV [218]. Left: semileptonic events; right: hadronic events.

The mass of the W boson is determined based on the reconstructed mass spectrum. Signal Monte Carlo events are reweighted to construct samples corresponding to different values of M_W . Background, independent of M_W , is added and the resulting Monte Carlo spectrum is compared to the spectrum observed in the data. Technically, an unbinned log-likelihood fit is performed where the M_W -dependent expectation is derived from the reweighted Monte Carlo simulation, thus taking detector and selection effects properly into account.³

The measurements of the four LEP experiments are summarised in Table 5.23 and Figure 5.55, comparing semileptonic and hadronic events. Within the statistical error, the results on M_W agree well between the two channels:

$$M_W(q\bar{q}\ell\nu) = 80.31 \pm 0.11 \text{ GeV} \quad (5.123)$$

$$M_W(q\bar{q}q\bar{q}) = 80.39 \pm 0.14 \text{ GeV}. \quad (5.124)$$

The systematic error on M_W common between the experiments is estimated to be 0.04 GeV for $q\bar{q}\ell\nu$ and 0.10 GeV for $q\bar{q}q\bar{q}$. Systematic errors common to the experiments arise from the calibration of the LEP beam energy, $\delta M_W/M_W = \delta\sqrt{s}/\sqrt{s}$, fragmentation uncertainties and strong final-state interactions (FSI) in the $q\bar{q}q\bar{q}$ channel as discussed in Section 3.2.6. The FSI effects between decay products of different W bosons may lead to four-momentum exchange between the two hadronically decaying W systems so that the invariant masses are not individually conserved, an assumption inherent in the direct reconstruction method. No evidence for FSI between the decay products of different W bosons is seen with the data collected up to 1997, neither in dedicated studies of event shapes and particle multiplicities nor in particle correlations [219, 220]. The error on the mass derived from hadronic events contains a common contribution of 90 MeV to account for possible effects due to strong FSI.

The average of the W mass from the semileptonic and hadronic channels is derived from the combined mass value of the individual experiments, as the treatment of experimental error correlations is better controlled by each experiment:

$$M_W(f\bar{f}f\bar{f}) = 80.36 \pm 0.09 \text{ GeV}. \quad (5.125)$$

The systematic part of the total error on the average amounts to 0.06 GeV. The comparison of the results with the SM is shown in Figure 5.56. Averaging the combined $f\bar{f}f\bar{f}$ results of the four experiments with the threshold measurement improves the error on M_W by less than 10 MeV:

$$M_W = 80.37 \pm 0.09 \text{ GeV}. \quad (5.126)$$

Experiment	Status	$q\bar{q}\ell\nu$	$q\bar{q}q\bar{q}$	$f\bar{f}f\bar{f}$
ALEPH	(96 - 97), prel.	80.34 ± 0.18	80.53 ± 0.18	80.44 ± 0.13
DELPHI	(96 - 97), prel.	80.50 ± 0.24	80.01 ± 0.22	80.24 ± 0.17
L3	(96 - 97), prel.	80.09 ± 0.24	80.59 ± 0.23	80.40 ± 0.18
OPAL	(96 - 97), prel.	80.29 ± 0.19	80.40 ± 0.24	80.34 ± 0.15
LEP	Average	80.31 ± 0.11	80.39 ± 0.14	80.36 ± 0.09
$\chi^2/\text{d.o.f}$		1.6/3 (66%)	5.6/3 (13%)	1.1/3 (78%)

Table 5.23: Measurements of M_W obtained at LEP-II for the $q\bar{q}\ell\nu$ and $q\bar{q}q\bar{q}$ final state, and their combination [163]. The χ^2 of the average is calculated based on the uncorrelated errors. The Standard Model calculation of M_W yields $M_W = 80.30 \pm 0.09 \text{ GeV}$.

³With this method it is also possible to determine the total width, Γ_W , of the W boson, by treating Γ_W as an additional fit parameter independent of M_W instead of imposing the SM relation $\Gamma_W = \Gamma_W(M_W)$.

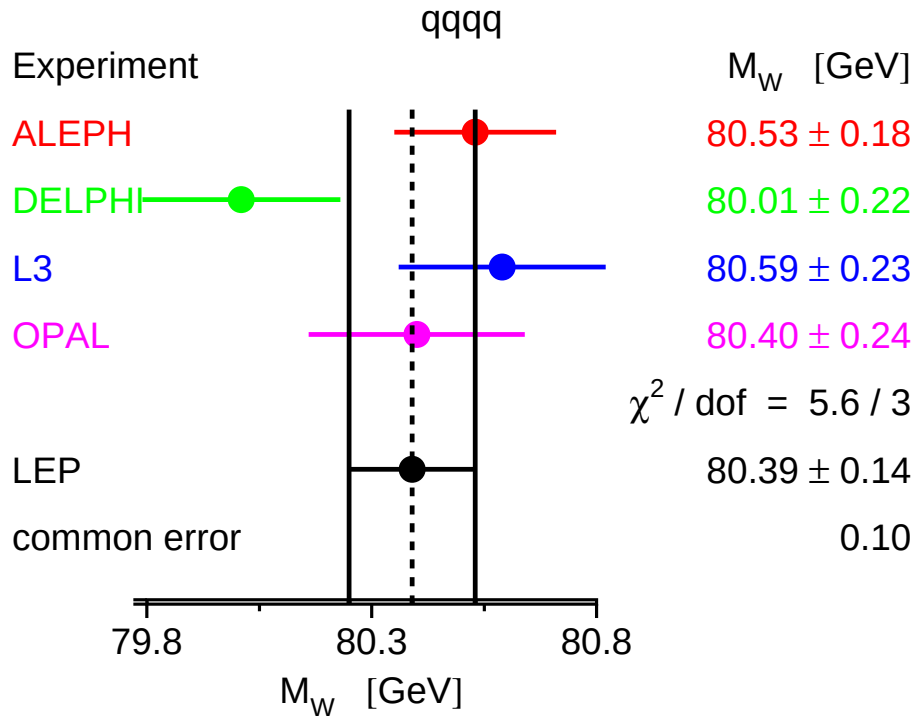
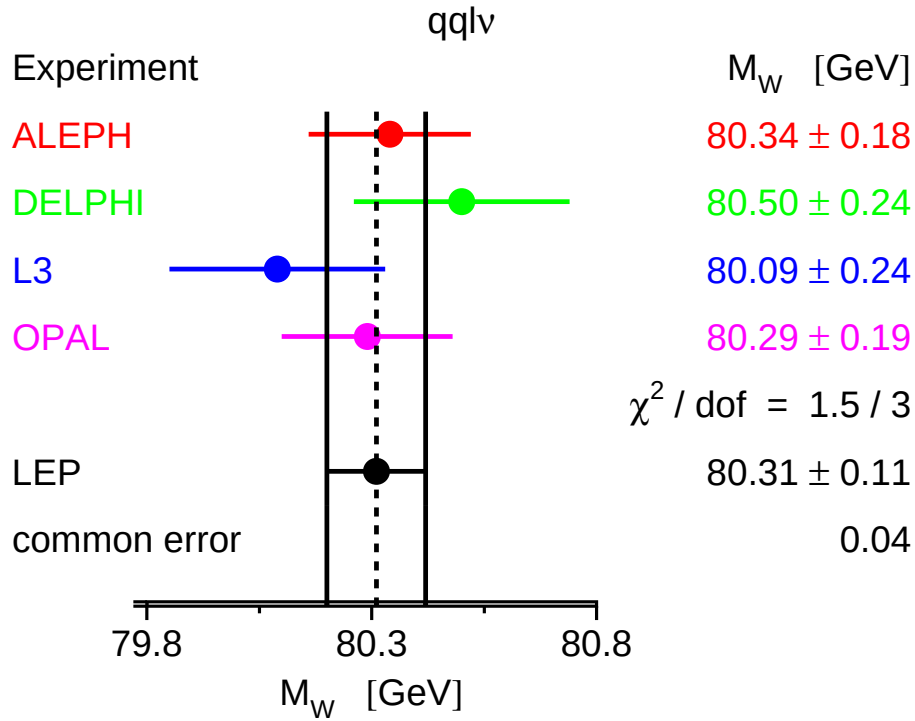


Figure 5.55: W mass as measured by the LEP experiments based on the method of direct reconstruction of invariant masses. Top: semileptonic events; bottom: fully hadronic events. The χ^2 of the averages is calculated based on the uncorrelated errors.

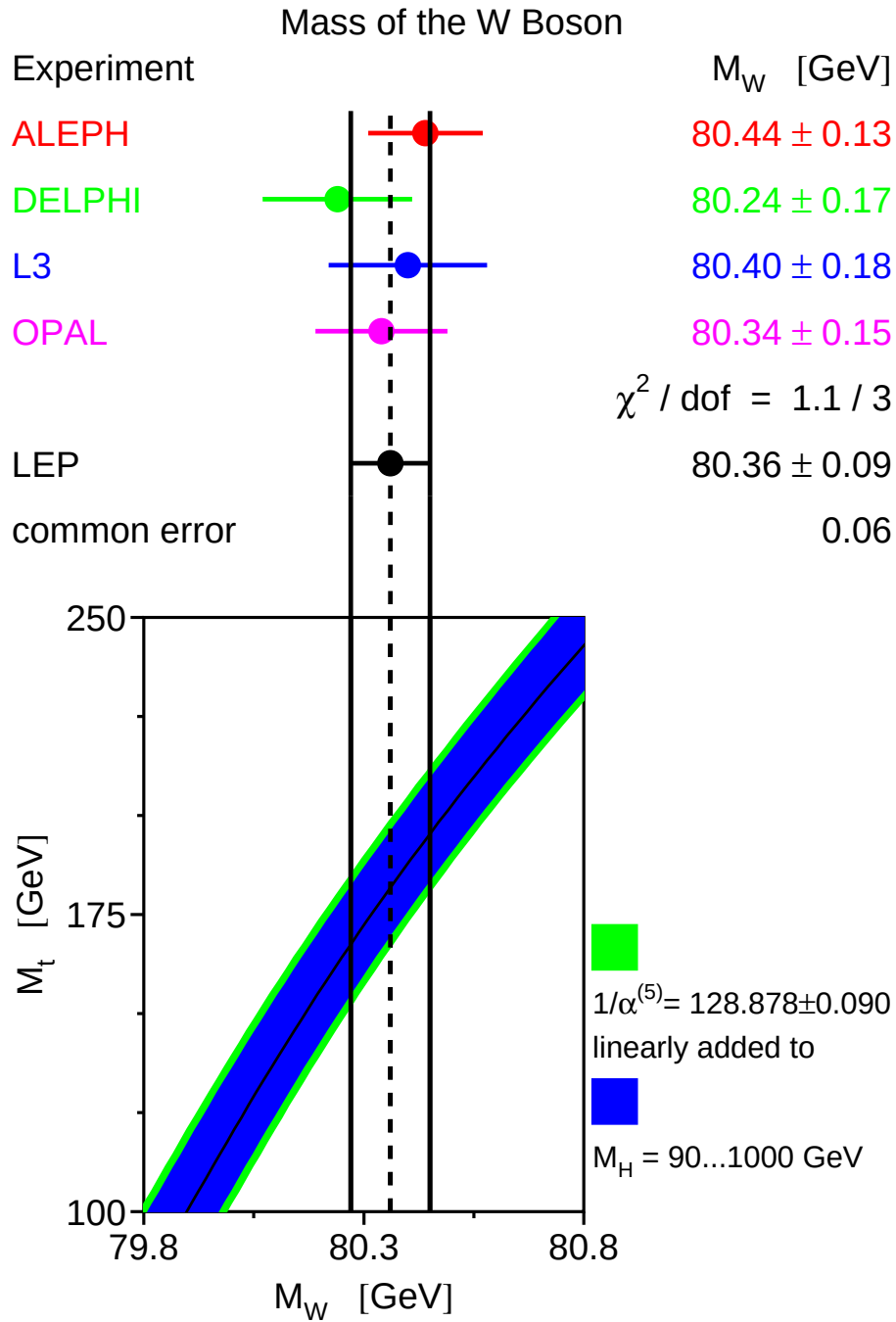


Figure 5.56: W mass as measured by the LEP experiments based on the method of direct reconstruction of invariant masses, combining semileptonic and hadronic events. The χ^2 of the average is calculated based on the uncorrelated errors.

5.10.3 Results

In summary, the direct measurements of M_W at the TEVATRON and at LEP-II are:

$$M_W(\text{TEVATRON}) = 80.41 \pm 0.09 \text{ GeV} \quad (5.127)$$

$$M_W(\text{LEP}) = 80.37 \pm 0.09 \text{ GeV} . \quad (5.128)$$

The combined result is:

$$M_W = 80.39 \pm 0.06 \text{ GeV} , \quad (5.129)$$

which is a measurement of the W boson mass with an accuracy of 0.08%. The Standard Model calculation of M_W yields $M_W = 80.30 \pm 0.09 \text{ GeV}$.

5.11 Gauge Couplings of the W Boson

The measurements of the gauge couplings of the gauge bosons explore and test the SM of electroweak interactions in the fundamental area of gauge-boson selfcouplings and of the Higgs mechanism allowing for massive and thus longitudinal gauge bosons. At current colliders such as the TEVATRON or LEP-II, the error on the measured gauge couplings is large compared to the effect of radiative corrections on these quantities. Thus the measurements test the Standard Model at Born level.

At the TEVATRON, the analysis of triple-gauge-boson couplings relies on the measurement of W boson production accompanied by an additional hard photon or Z boson, assumed to arise from the triple-gauge-boson vertices $\gamma W^+ W^-$ or $Z W^+ W^-$.

At LEP-II, the triple-gauge-boson vertices $\gamma W^+ W^-$ and $Z W^+ W^-$ appear in the s -channel part of W-pair production, $e^+ e^- \rightarrow W^+ W^-$. The electromagnetic triple-gauge-boson vertex $\gamma W^+ W^-$ alone also appears in single-W production, $e^+ e^- \rightarrow W e \nu$, and in single photon production, $e^+ e^- \rightarrow \nu \bar{\nu} \gamma$. The combined analysis of all these processes increases the accuracy in the determination of triple-gauge couplings.

The total cross section of W-pair production alone already contains information on triple-gauge-boson couplings, especially at higher centre-of-mass energies as shown in Figure 5.57. This is due to the fact that the β_W^2 suppression of the gauge-couplings dependent s -channel contribution relative to the gauge-couplings independent t -channel contribution becomes negligible at higher centre-of-mass energies.

Anomalous values of the gauge couplings also modify the angular distribution and the polarisation of the produced $W^+ W^-$ pair as shown in Figure 5.58. The parity-violating charged-current W decays serves as a polarisation analyser. Thus, the multidimensional differential cross section in the five phase-space angles describing the four-fermion final state are exploited for increased accuracy. The set Ω of phase space angles consists of the polar angle of the W^- boson, Θ , and the $W \rightarrow f \bar{f}$ decay angles in the respective W restframes, (θ_+^*, ϕ_+^*) and (θ_-^*, ϕ_-^*) . These angles are graphically shown in Figure 5.59.

The gauge couplings are determined based on the measured total and multidifferential cross sections as shown in Figure 5.60. Similar to the mass analyses, Monte Carlo events are reweighted to correspond to different values of gauge couplings and the reweighted distributions are compared to those observed in the data. Since the matrix element is linear in any anomalous gauge coupling α , both the multidifferential and the total cross section has the form:

$$\frac{d\sigma(\Omega, \alpha)}{d\Omega} = C_0(\Omega) + \alpha \cdot C_1^\alpha(\Omega) + \alpha^2 \cdot C_2^\alpha(\Omega) , \quad (5.130)$$

where $C_0(\Omega)$ is the Standard Model term. In order to avoid fits to multidimensional distributions, the concept of optimal observables as already applied in the $\mathcal{P}_\tau$ -polarisation analysis [175] is also used. For

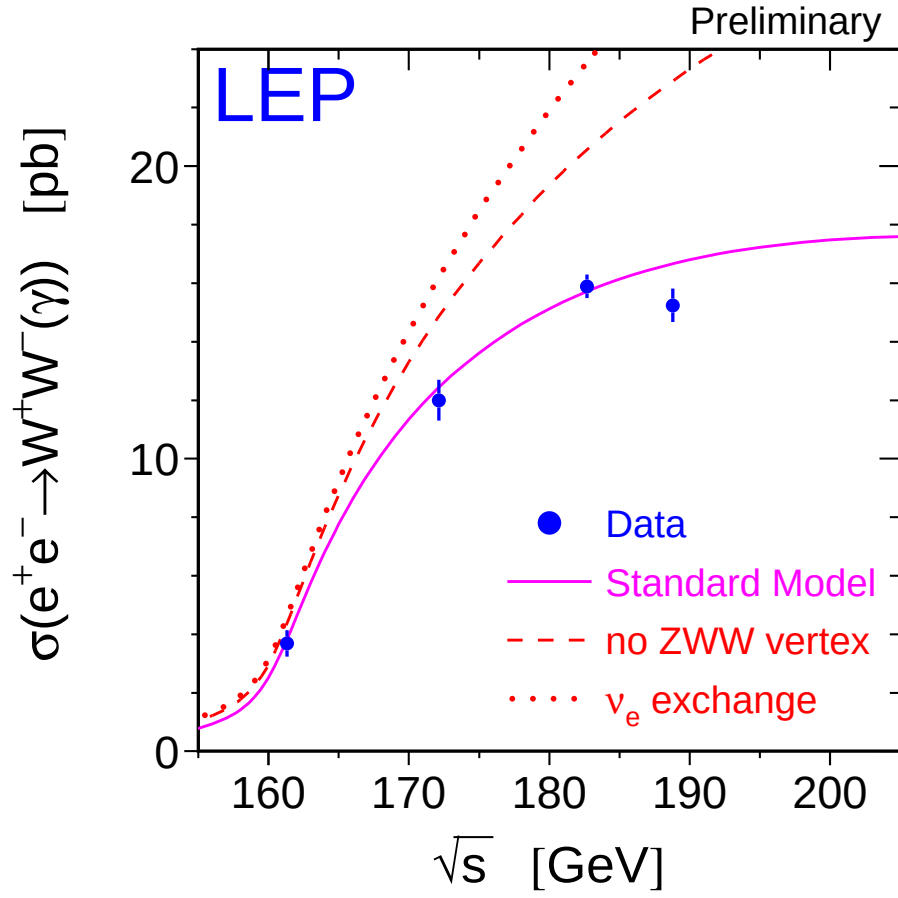


Figure 5.57: Total cross section of W-pair production measured at LEP-II.

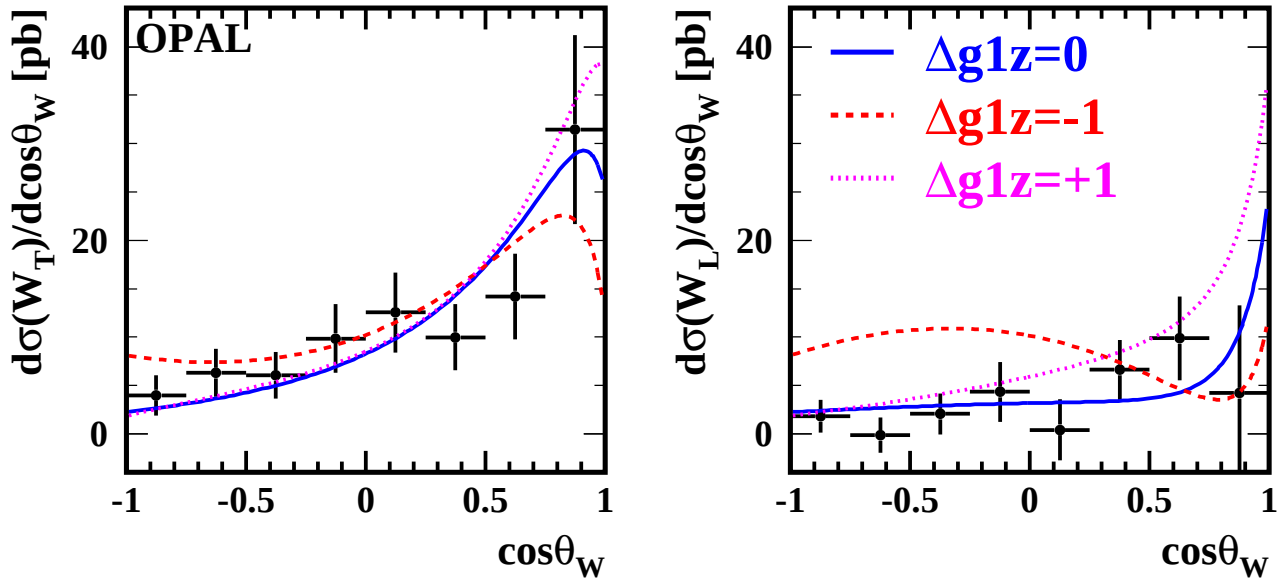


Figure 5.58: Differential W-pair cross section measured by OPAL at $\sqrt{s} = 183$ GeV [214]. Shown are the measured distributions for the production of transverse (right) and longitudinal (left) W bosons as a function of the polar scattering angle, and the expectations for three different values of the anomalous triple-gauge-boson coupling Δg_{1Z}^Z .

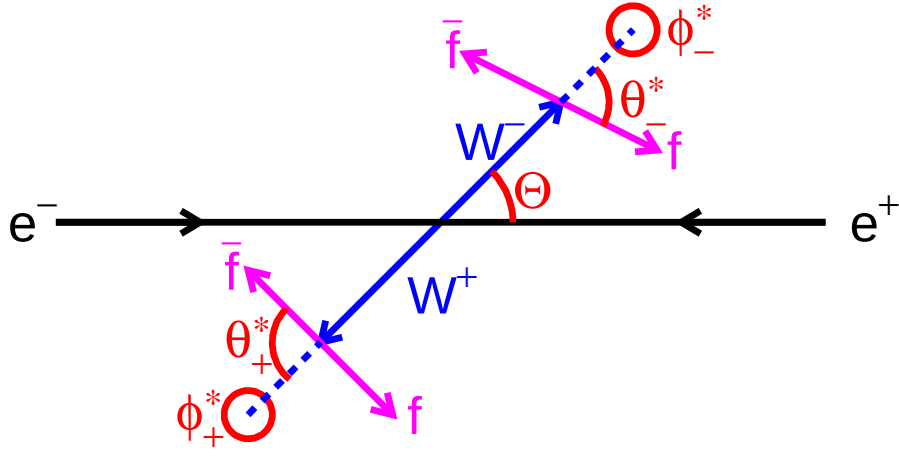


Figure 5.59: Phase-space angles in four-fermion production.

small anomalous couplings values, the quadratic term may be ignored, so that the optimal observable for the anomalous gauge coupling α is simply given by:

$$\omega_\alpha(\Omega) = \frac{C_1^\alpha(\Omega)}{C_0(\Omega)}. \quad (5.131)$$

The distribution of the optimal observable for the gauge coupling g_1^Z in $q\bar{q}\ell\nu$ events is shown in Figure 5.61.

For single-W production, the total cross section is used to derive information on $W^+W^-\gamma$ couplings. in case of single- γ production, the distribution of the energy and the polar angle of the photon are also exploited.

5.11.1 Measurements and Results

The results of the four LEP experiments and of DØ are shown in Tables 5.24 and 5.25 for one- and two-parameter results. The combination of the experimental results is performed by adding the likelihood curves of the individual experiments as indicated in Figure 5.62. Contour curves for two-dimensional results are shown in Figure 5.63. In both cases, systematic errors are included. Systematic errors correlated between the experiments are estimated to be small, and their correlation is neglected in the average.

Currently, the following results on triple-gauge-boson couplings are obtained, combining the four LEP experiments and DØ:

$$g_1^Z = +1.00 \pm 0.08 \quad (5.132)$$

$$\kappa_\gamma = +1.13 \pm 0.16 \quad (5.133)$$

$$\lambda_\gamma = -0.03 \pm 0.07, \quad (5.134)$$

where for the determination of each gauge coupling the other two are set to their Standard-Model value. These results are all in good agreement with the expectations of the SM of electroweak interactions, $g_1^Z = \kappa_\gamma = 1$ and $\lambda_\gamma = 0$. In particular, the results prove the existence of the gauge coupling between the Z boson and a pair of W bosons, *i.e.*, there are selfcouplings among the heavy gauge bosons. The above results completely exclude [221] Kaluza-Klein type theories in which $\kappa_\gamma = -2$ [222].

With these measurements the Standard Model is successfully tested in the area of gauge couplings in the electroweak sector. This justifies the use of the theoretical calculation of $\sigma_{WW}(M_W, \sqrt{s})$ involving SM gauge couplings in order to determine the mass of the W boson from the cross section of W-pair production at the kinematic threshold.

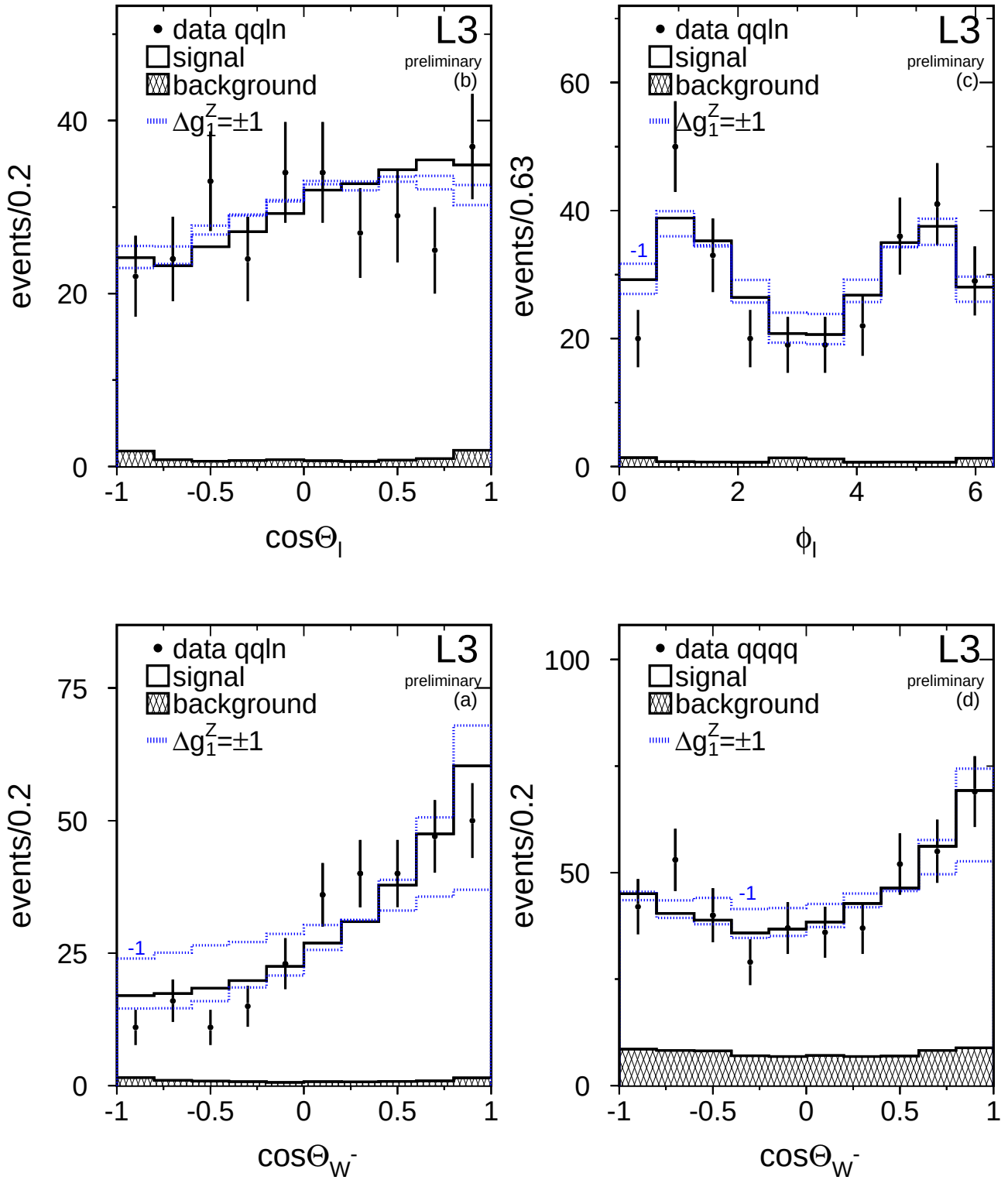


Figure 5.60: Distribution of phase-space angles measured by L3 at $\sqrt{s} = 183$ GeV [223]. The different shapes of the $\cos \Theta$ distributions for $q\bar{q} \ell \nu$ and $q\bar{q} q\bar{q}$ events is caused by wrong assignments of jets to W bosons and increased charge confusion in $q\bar{q} q\bar{q}$ events. The multidifferential distribution is fitted rather than the one-dimensional projections shown here.

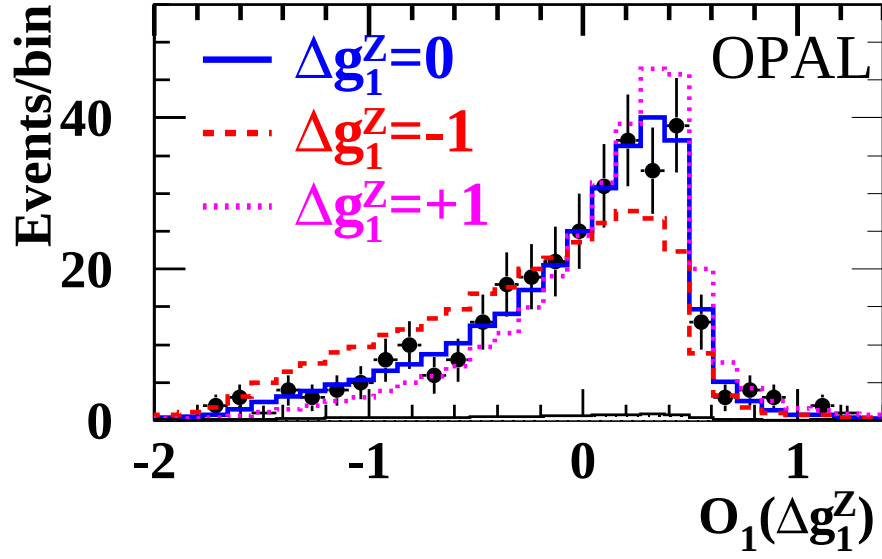


Figure 5.61: Distribution of the optimal observable for the gauge coupling Δg_1^Z in $q\bar{q}\ell\nu$ events as measured by OPAL at $\sqrt{s} = 183$ GeV [214].

Experiment	Δg_1^Z	$\Delta \kappa_\gamma$	λ_γ
ALEPH	—	$-0.02^{+0.28}_{-0.33}$	$+0.05^{+0.50}_{-0.51}$
DELPHI	$+0.04^{+0.14}_{-0.14}$	$+0.34^{+0.26}_{-0.28}$	$-0.07^{+0.19}_{-0.16}$
L3	$-0.03^{+0.18}_{-0.16}$	$+0.16^{+0.40}_{-0.35}$	$+0.01^{+0.19}_{-0.17}$
OPAL	$-0.02^{+0.12}_{-0.11}$	$+0.19^{+0.47}_{-0.37}$	$-0.08^{+0.13}_{-0.12}$
DØ	—	$-0.08^{+0.34}_{-0.34}$	$0.00^{+0.10}_{-0.10}$
Average	$0.00^{+0.08}_{-0.08}$	$+0.13^{+0.14}_{-0.14}$	$-0.03^{+0.07}_{-0.07}$

Table 5.24: One-parameter results on triple gauge boson couplings obtained at LEP and at the TEVATRON [163]. For the determination of each coupling, the other two are set to their Standard Model value of zero.

Experiment	Δg_1^Z	$\Delta \kappa_\gamma$	Δg_1^Z	λ_γ	$\Delta \kappa_\gamma$	λ_γ
ALEPH	—	—	—	—	$+0.05^{+1.2}_{-1.1}$	$-0.05^{+1.6}_{-1.5}$
DELPHI	$+0.06^{+0.21}_{-0.18}$	$+0.31^{+0.50}_{-0.39}$	$+0.17^{+0.16}_{-0.18}$	$-0.23^{+0.22}_{-0.19}$	$+0.27^{+0.39}_{-0.36}$	$-0.10^{+0.36}_{-0.31}$
L3	$-0.06^{+0.20}_{-0.16}$	$+0.33^{+0.62}_{-0.52}$	$-0.06^{+0.23}_{-0.21}$	$+0.04^{+0.24}_{-0.23}$	$+0.35^{+0.64}_{-0.36}$	$-0.04^{+0.21}_{-0.19}$
OPAL	$+0.01^{+0.15}_{-0.17}$	$0.00^{+0.60}_{-0.30}$	$-0.15^{+0.34}_{-0.20}$	$+0.23^{+0.26}_{-0.43}$	$-0.01^{+0.46}_{-0.34}$	$+0.03^{+0.22}_{-0.18}$
Average	$0.00^{+0.12}_{-0.11}$	$+0.28^{+0.33}_{-0.27}$	$+0.05^{+0.13}_{-0.13}$	$-0.07^{+0.16}_{-0.15}$	$+0.09^{+0.17}_{-0.15}$	$+0.02^{+0.13}_{-0.12}$
Correlation	-0.54		-0.79		-0.50	

Table 5.25: Two-parameter results on triple gauge boson couplings obtained at LEP [163]. For the determination of each pair of couplings, the third is set to its Standard Model value of zero.

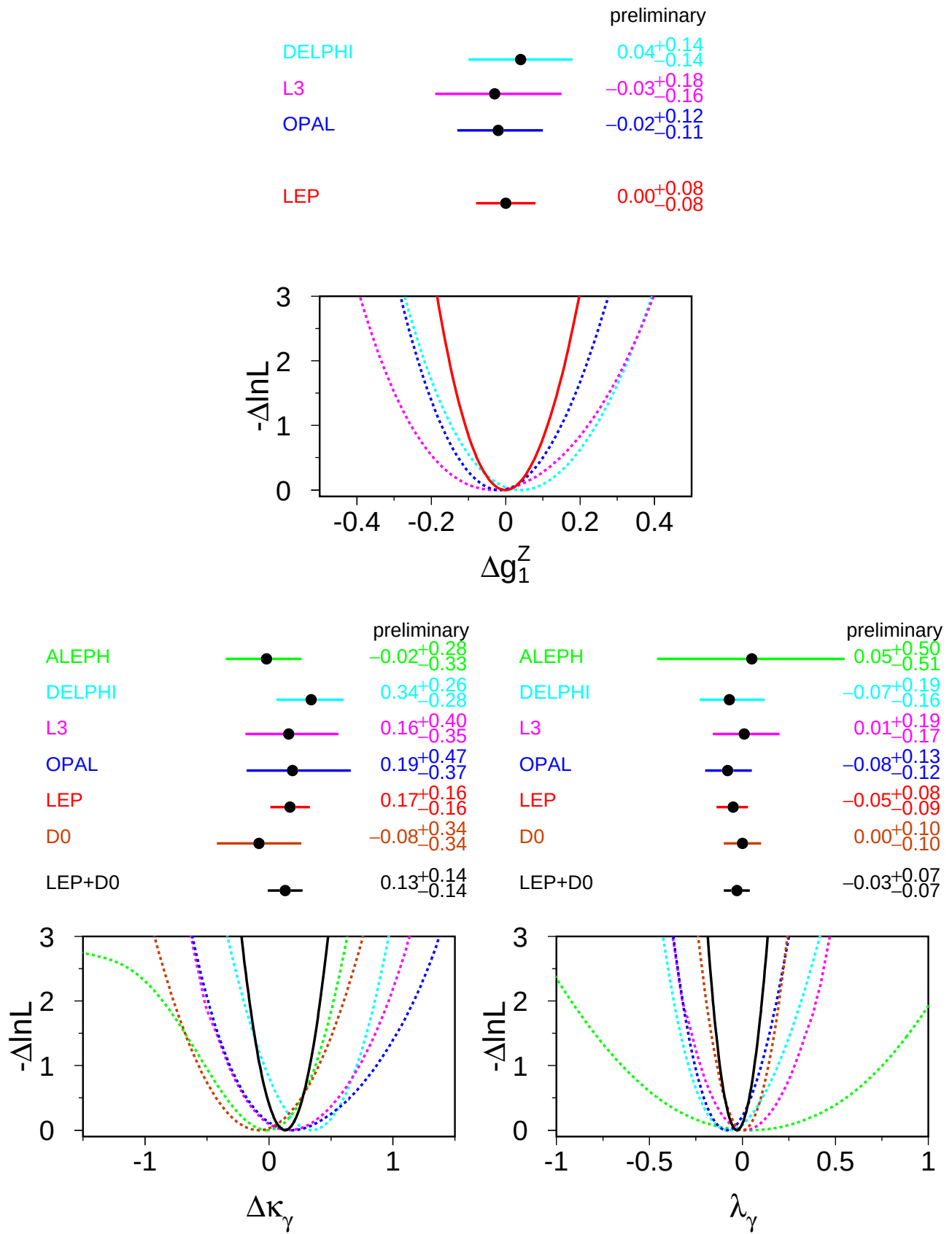


Figure 5.62: Comparison of one-parameter results on triple gauge boson couplings obtained at LEP-II. The results from DØ are also shown and included in the average. For the determination of each coupling, the other two are set to their Standard Model value of zero.

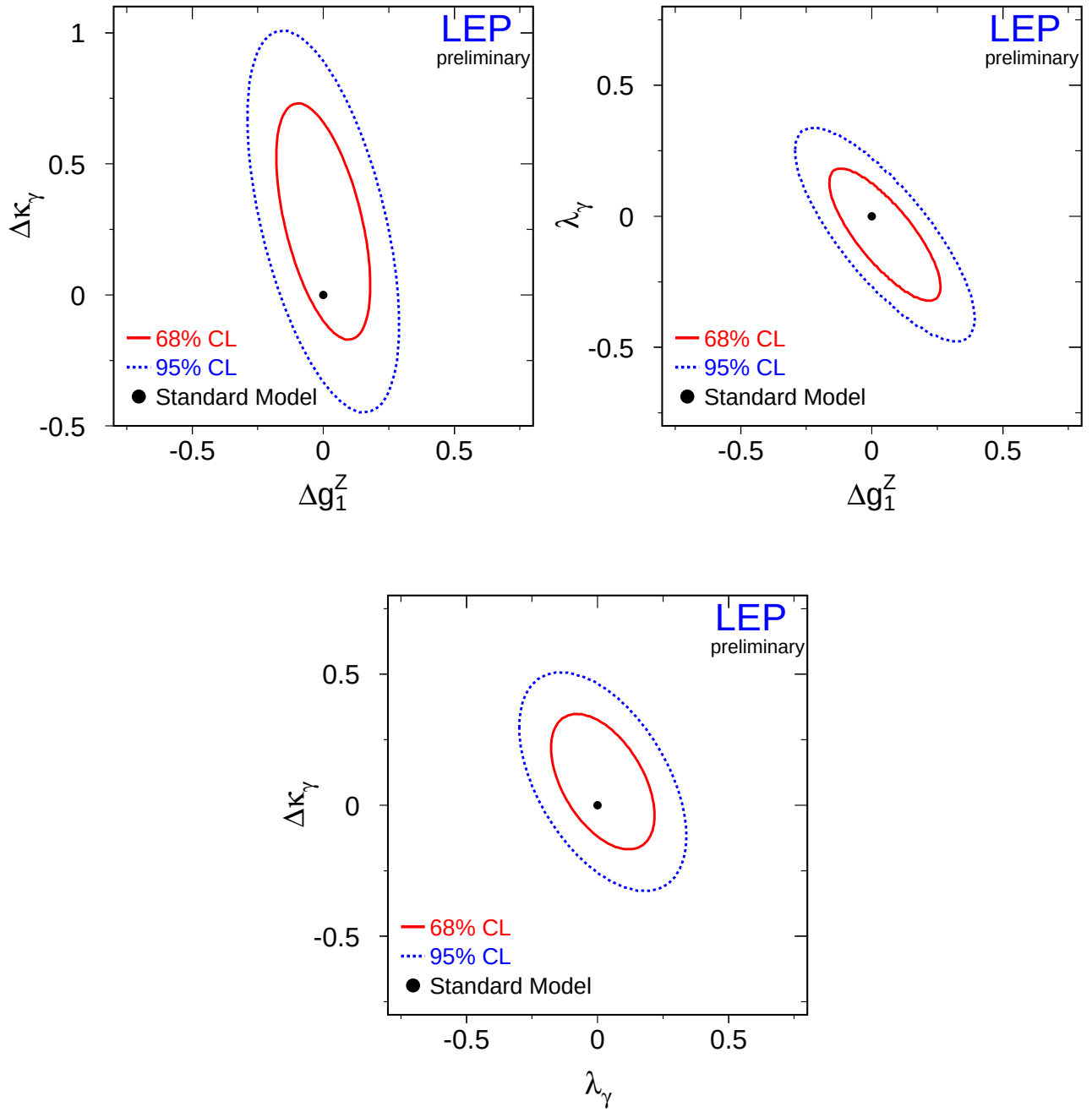


Figure 5.63: Contour curves of 68% and 95% probability for two-parameter results on triple gauge boson couplings obtained at LEP-II. For the determination of each pair of couplings, the third is set to its Standard Model value of zero.

5.12 Mass of the Top Quark

5.12.1 Top Quark Production and Decay

Because of its very high mass, the top quark was discovered at the TEVATRON rather than at the SPS, SLC or at LEP. Proton-antiproton collisions produce $t\bar{t}$ pairs in the reaction:

$$p\bar{p} \rightarrow t\bar{t} + X_t, \quad (5.135)$$

where X_t denotes the $p\bar{p}$ remnant recoiling against the $t\bar{t}$ system. The Feynman diagrams contributing to $t\bar{t}$ production on parton level are shown in Figure 5.64.

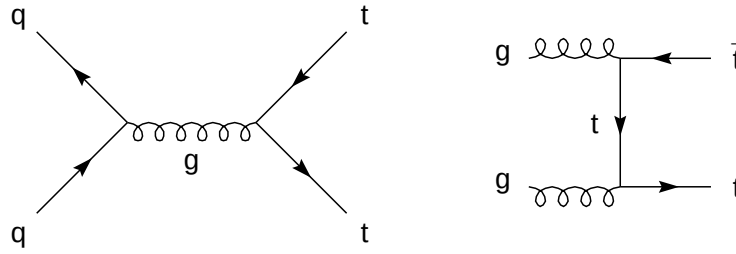


Figure 5.64: Feynman diagrams of $t\bar{t}$ production in $p\bar{p}$ collisions on parton level.

For a given centre-of-mass energy, the total $t\bar{t}$ production cross section decreases rapidly with increasing mass of the top quark, as shown in Figure 5.65. For a top mass of $M_t = 175$ GeV, the theoretical calculations yield cross sections between 4.7 pb and 6.2 pb [224–226], with 90% of the cross section given by the $q\bar{q}$ diagram.

The top quark immediately decays via the charged weak current:

$$t \rightarrow b W \quad (5.136)$$

$$W \rightarrow f\bar{f}', \quad (5.137)$$

since the Cabibbo-Kobayashi-Maskawa matrix element V_{tb} dominates. For sufficiently heavy top quarks, $M_t > M_W + m_b$, the W boson is on-shell. The total decay width of the top, $\Gamma_t \propto G_F M_t^3$, becomes large for high mass, for example, $\Gamma_t = 1.4$ GeV for $M_t = 175$ GeV as discussed in Section 3.2.5.2. Thus the top quark decays before there is enough time to form mesonic or baryonic bound states with any other quarks, and hadrons containing top quarks cannot exist.

The decay channels of $t\bar{t}$ events are those of a W-pair accompanied by a $b\bar{b}$ system. In analogy to W-pair production discussed at LEP-II, there are fully hadronic channels, semileptonic channels and leptonic channels. The branching fractions for these final states are given by the products of W branching fractions.

$$t\bar{t} \rightarrow b\bar{b} W^+ W^- \rightarrow b\bar{b} (f\bar{f}')_1 (f\bar{f}')_2 \quad (5.138)$$

$$\rightarrow \begin{cases} b\bar{b} q\bar{q}' q\bar{q}' & 45.6\% \\ b\bar{b} q\bar{q}' \ell\nu_\ell & 14.6\% \text{ each for } \ell = e, \mu, \tau \\ b\bar{b} \ell^+ \bar{\nu}_\ell \ell'^- \nu_{\ell'} & 10.6\% \text{ for } \ell = e, \mu, \tau \text{ combined} \end{cases} . \quad (5.139)$$

Like in W physics at the TEVATRON, the main leptonic W decay modes analysed are $W \rightarrow e\nu$ and $W \rightarrow \mu\nu$. Since a common experimental signature is the existence of two b quarks in the final state, tagging of hadronic b jets is of crucial importance. It is used to identify $t\bar{t}$ production and to reject the background of QCD multijet events. Examples for each class of $t\bar{t}$ events are shown in Figures 5.66 to 5.70.

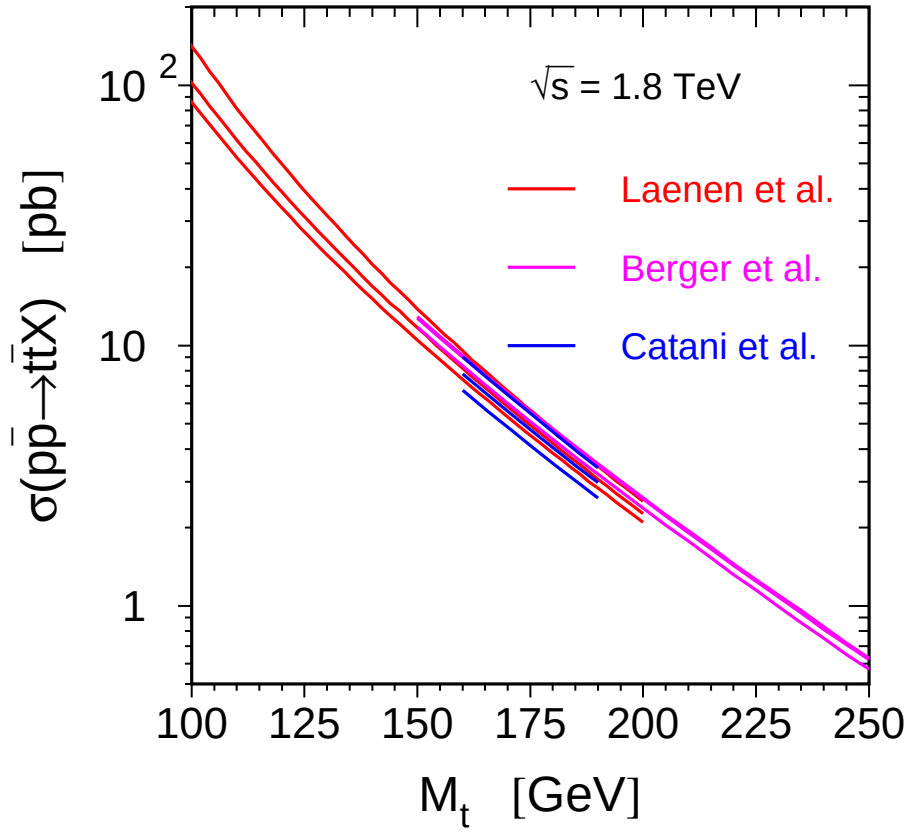


Figure 5.65: Cross section of $t\bar{t}$ production in $p\bar{p}$ collisions at $\sqrt{s} = 1.8$ TeV as a function of the top-quark mass, M_t . For each of the three theoretical calculations [224–226], three lines corresponding to minimal, central and maximal value are shown.

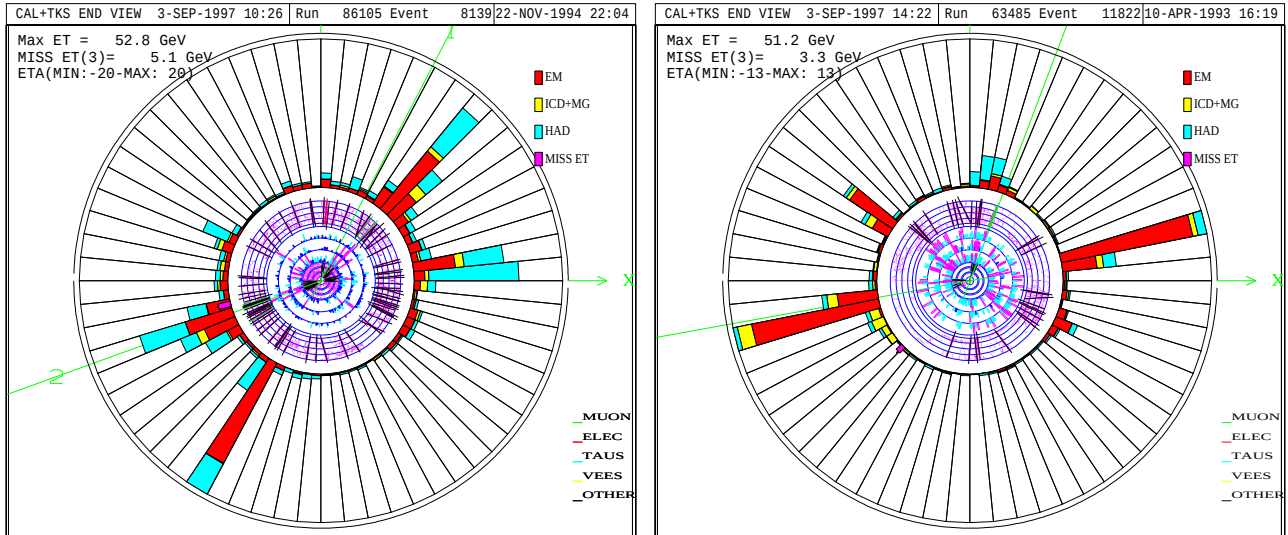


Figure 5.66: Hadronic $t\bar{t}$ events, $t\bar{t} \rightarrow b\bar{b}q\bar{q}'q\bar{q}'$, observed in the DØ detector. Shown is the end view of the detector with hits and tracks in the central tracking system and energy depositions in the electromagnetic and hadronic calorimeters. Both events contain two jets with an inclusive muon, shown as the minimum ionising track traversing the complete detector at $\phi = 60^\circ, 200^\circ$ (left) and at $\phi = 70^\circ, 190^\circ$ (right), which are thus identified as the $b\bar{b}$ jets.

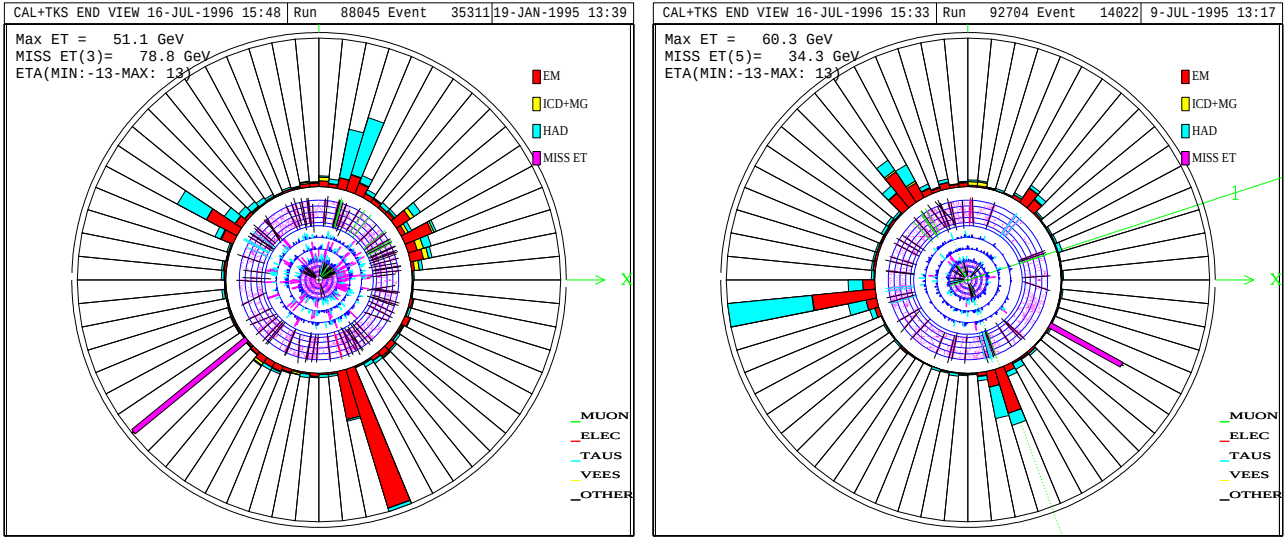


Figure 5.67: Semileptonic $t\bar{t}$ events, $t\bar{t} \rightarrow b\bar{b}q\bar{q}\ell\nu$, observed in the DØ detector. Shown is the end view of the detector with hits and tracks in the central tracking system and energy depositions in the electromagnetic and hadronic calorimeters. Left: The lepton is an electron causing the electromagnetic jet at $\phi = 295^\circ$. The missing energy attributed to the neutrino is shown as the slim strip at $\phi = 220^\circ$. Right: The lepton is a muon shown as the minimum ionising particle at $\phi = 20^\circ$ traversing the complete detector. The missing energy attributed to the neutrino is shown as the slim strip at $\phi = 330^\circ$.

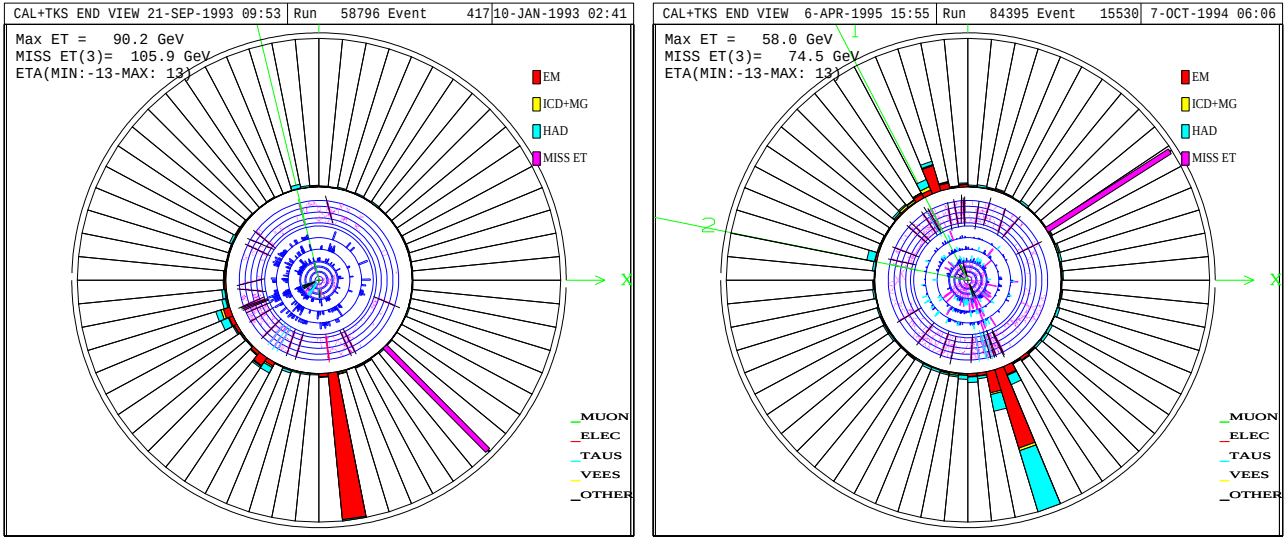


Figure 5.68: Leptonic $t\bar{t}$ events, $t\bar{t} \rightarrow b\bar{b}\ell\nu\ell\nu$, observed in the DØ detector. Shown is the end view of the detector with hits and tracks in the central tracking system and energy depositions in the electromagnetic and hadronic calorimeters. Left: $e\mu$ event. The electron is causing the electromagnetic jet at $\phi = 280^\circ$. The muon is shown as the minimum ionising particle at $\phi = 105^\circ$ traversing the complete detector. The missing energy attributed to the neutrinos is shown as the slim strip at $\phi = 315^\circ$. Right: Both leptons are muons shown as the minimum ionising particles at $\phi = 120^\circ$ and $\phi = 170^\circ$ traversing the complete detector. The missing energy attributed to the neutrinos is shown as the slim strip at $\phi = 35^\circ$.

e + 4 jet event

40758_44414

24-September, 1992

TWO jets tagged by SVX

fit top mass is 170 ± 10 GeV

e^+ , Missing E_T , jet #4 from top

jets 1,2,3 from top (2&3 from W)

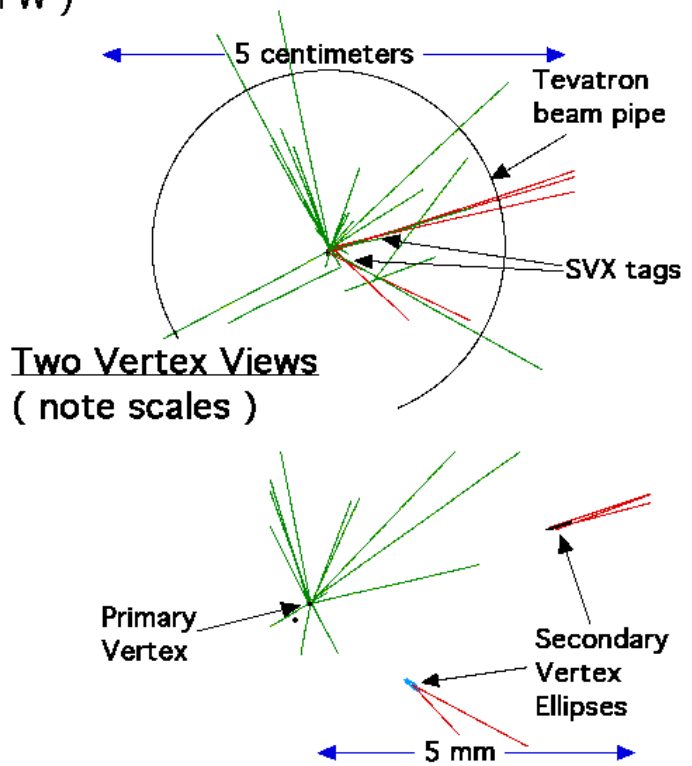
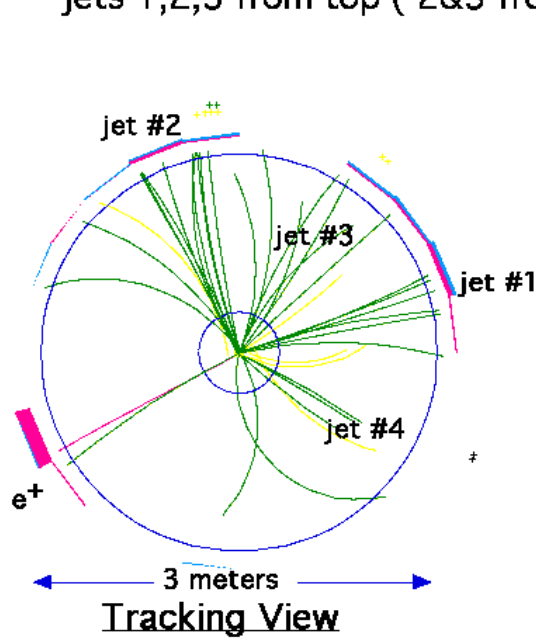
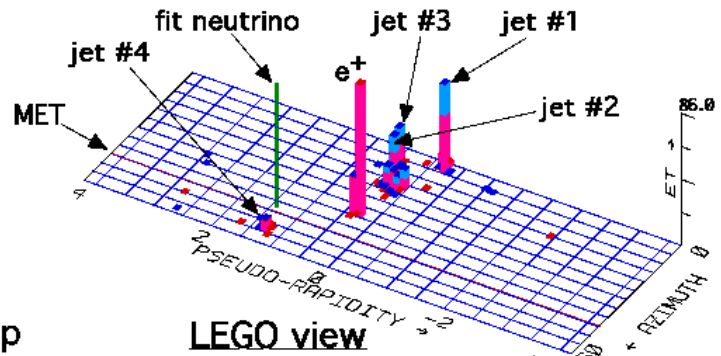
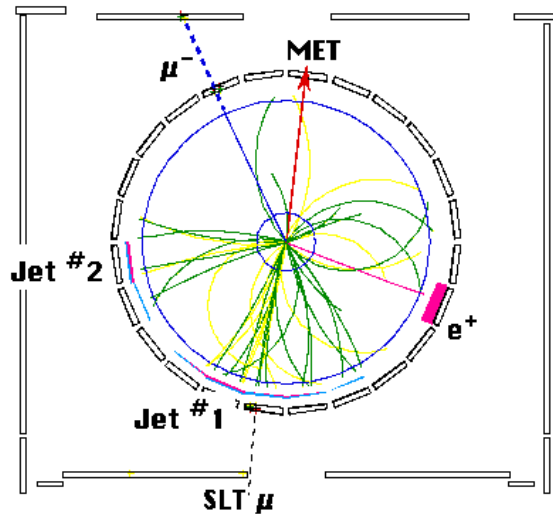


Figure 5.69: Semileptonic $t\bar{t}$ event, $t\bar{t} \rightarrow b\bar{b}q\bar{q}e\nu$, observed in the CDF detector. One top quark is reconstructed from the fourth jet, tagged as a b jet, and the electron and neutrino from W decay. The other top quark is reconstructed from the first jet, also tagged as b jet, and the second and third jet from W decay.

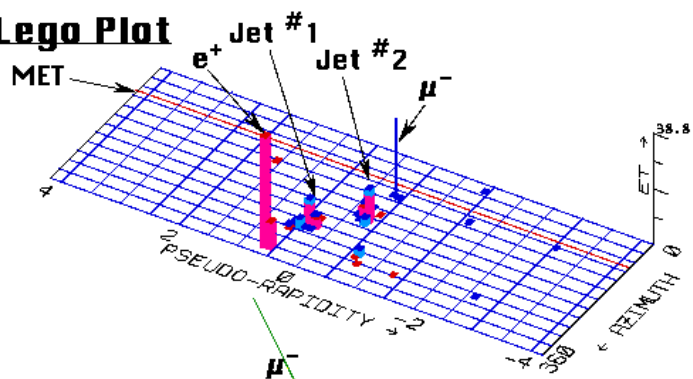
Tracking View



Tagged e μ Event

57621_45230
26-March, 1994
49 GeV -- e^+
25 GeV -- μ^-
26 GeV -- Jet #1
(tagged by SUH and SLT)
25 GeV -- Jet #2
51 GeV -- Missing E_t (MET)

Lego Plot



Vertex Views (note scales)

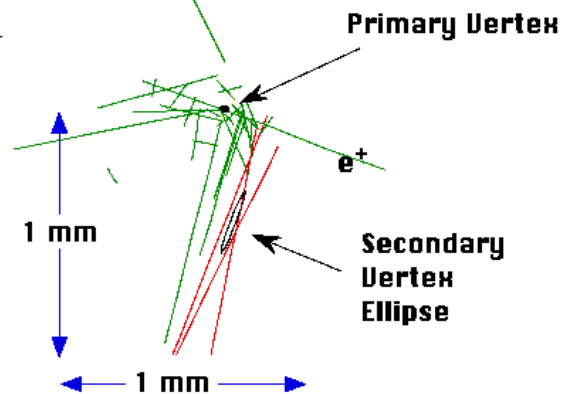
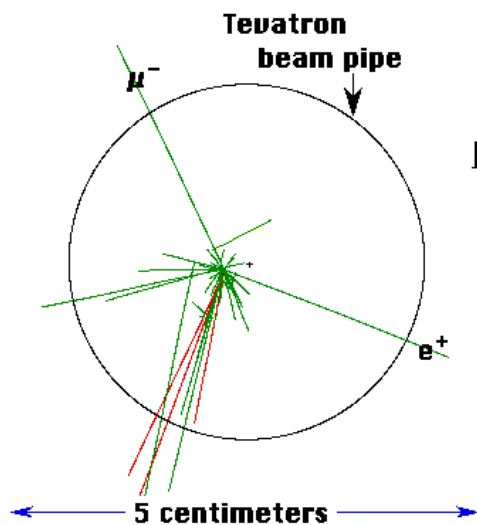


Figure 5.70: Leptonic $t\bar{t}$ event, $t\bar{t} \rightarrow b\bar{b}e\nu\mu\nu$, observed in the CDF detector. The first jet is tagged as a b jet.

5.12.2 Measurements and Results

In $t\bar{t} \rightarrow b\bar{b}W^+W^-$ events, where both W bosons decay leptonically, $t\bar{t} \rightarrow b\bar{b}\ell\nu\ell\nu$, the presence of two neutrinos prevents a complete reconstruction of all invariant masses. However, the mass of the top quark is correlated with the kinematics of the decay products, such as lepton energies, quark-jet energies, and jet-lepton angular separation. The distributions of these variables yield a determination of M_t , albeit with large errors.

In hadronic events, $t\bar{t} \rightarrow b\bar{b}q\bar{q}q\bar{q}$, all decay products are quarks. The six quarks lead to six hadronic jets visible in the detector, two of which are b -quark jets. Knowing quark masses and measuring the three-momenta of the six jets, the two top quarks are fully reconstructed up to combinatorial ambiguities in combining light-quark jets to W bosons, and W bosons and b jets to top quarks. The combination procedure is aided by tagging the b jets and exploiting the mass constraints $m_{\text{inv}}(t) = m_{\text{inv}}(\bar{t})$ and $m_{\text{inv}}(W^+) = m_{\text{inv}}(W^-) = M_W$, yielding a 3C kinematic fit.

Semileptonic events, $t\bar{t} \rightarrow b\bar{b}q\bar{q}'\ell\nu$ also allow a complete reconstruction of the top quark kinematics. It is necessary to consider the complete production and decay chain:

$$\begin{aligned} p\bar{p} &\rightarrow t_1 t_2 + X_t \\ t_1 &\rightarrow b_1 + W_1 \\ t_2 &\rightarrow b_2 + W_2 \\ W_1 &\rightarrow \ell + \nu \\ W_2 &\rightarrow q_1 + q_2. \end{aligned} \tag{5.140}$$

These five four-vector equations contain seven unknown four-vectors, namely those of q_1 , q_2 , ℓ , ν , b_1 , b_2 , X_t , corresponding to $4 \cdot 7 = 28$ unknowns. There are 17 measurements, namely the three-momenta of q_1 , q_2 , ℓ , b_1 , b_2 and the two transverse momentum components of the remnant X_t . Overall four-momentum conservation and the mass conditions:

$$\begin{aligned} m_{\text{inv}}(t_1) &= m_{\text{inv}}(t_2) \\ m_{\text{inv}}(W_1) &= m_{\text{inv}}(W_2) = M_W \\ m_{b_1} &= m_{b_2} = m_b \\ m_\ell &= m_\nu = m_{q_1} = m_{q_2} = 0 \end{aligned} \tag{5.141}$$

establish 13 constraints in total, leading to a 2C kinematic fit for the determination of all four-momenta. Reconstructed mass distributions are shown in Figure 5.71.

The results on M_t obtained by CDF in the hadronic channel [227], the semileptonic channel [228] and the leptonic channel [229], and by DØ in the semileptonic channel [230] and the leptonic channel [231] are summarised in Table 5.26 [232]. By convention, these masses correspond to pole masses. They agree well between channels and experiments. When combining the results of the two experiments, systematic errors and their correlations are taken into account, yielding:

$$M_t = 173.8 \pm 3.2 \text{ (stat.)} \pm 3.9 \text{ (syst.) GeV.} \tag{5.142}$$

The average of the five measurements has a $\chi^2/\text{d.o.f}$ probability of 79%. With a total error of 5.0 GeV, the mass of the top quark is now known with the smallest relative error, 3%, of all quark masses.

The cross section for $t\bar{t}$ production in $p\bar{p}$ collisions is measured in each of the decay channels given above. Combining final states, the cross sections measured by the TEVATRON experiments CDF and DØ are:

$$\sigma_{t\bar{t}}(\text{CDF}) = 7.6_{-1.5}^{+1.8} \text{ pb} \tag{5.143}$$

$$\sigma_{t\bar{t}}(\text{DØ}) = 5.9 \pm 1.6 \text{ pb,} \tag{5.144}$$

which agree within the errors. The measured cross sections depend on the assumed mass of the top quark because of mass-dependent efficiencies, and are evaluated for the masses as measured by each experiment. Measured masses and cross sections are compared in Figure 5.72.

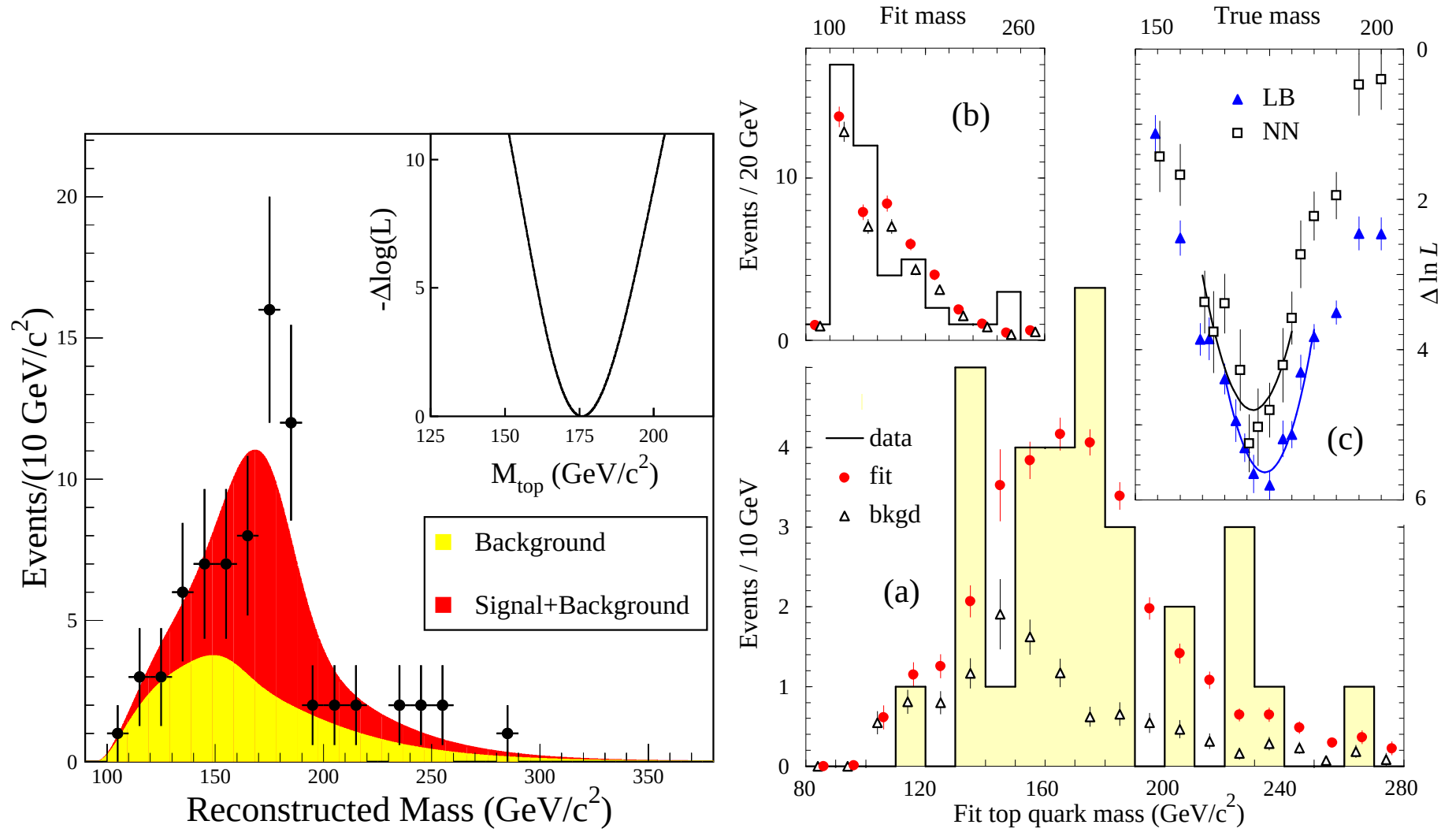


Figure 5.71: Distributions of reconstructed top-quark masses in the semileptonic channel from CDF [233] (left) and DØ [234] (right), including the likelihood curves as a function of the top-quark mass. For DØ, (a) denotes signal enhanced and (b) denotes background enhanced distributions. The likelihood curve in (c) is given for two analyses.

M_t [GeV] Channel	Experiment	
	CDF	DØ
$t\bar{t} \rightarrow b\bar{b}\ell\nu\ell\nu$	$167.4 \pm 10.3 \pm 4.8$	$168.4 \pm 12.3 \pm 3.6$
$t\bar{t} \rightarrow b\bar{b}q\bar{q}\ell\nu$	$175.9 \pm 4.8 \pm 4.9$	$173.3 \pm 5.6 \pm 5.5$
$t\bar{t} \rightarrow b\bar{b}q\bar{q}q\bar{q}$	$186.0 \pm 10.0 \pm 8.2$	—
Average	$175.3 \pm 4.1 \pm 5.0$	$172.1 \pm 5.2 \pm 4.9$
TEVATRON	$173.8 \pm 3.2 \pm 3.9$	

Table 5.26: Measurements of the top-quark mass by the TEVATRON experiments CDF and DØ [232]. The first error is statistical and the second systematic. The overall average has a $\chi^2/\text{d.o.f}$ probability of 79%.

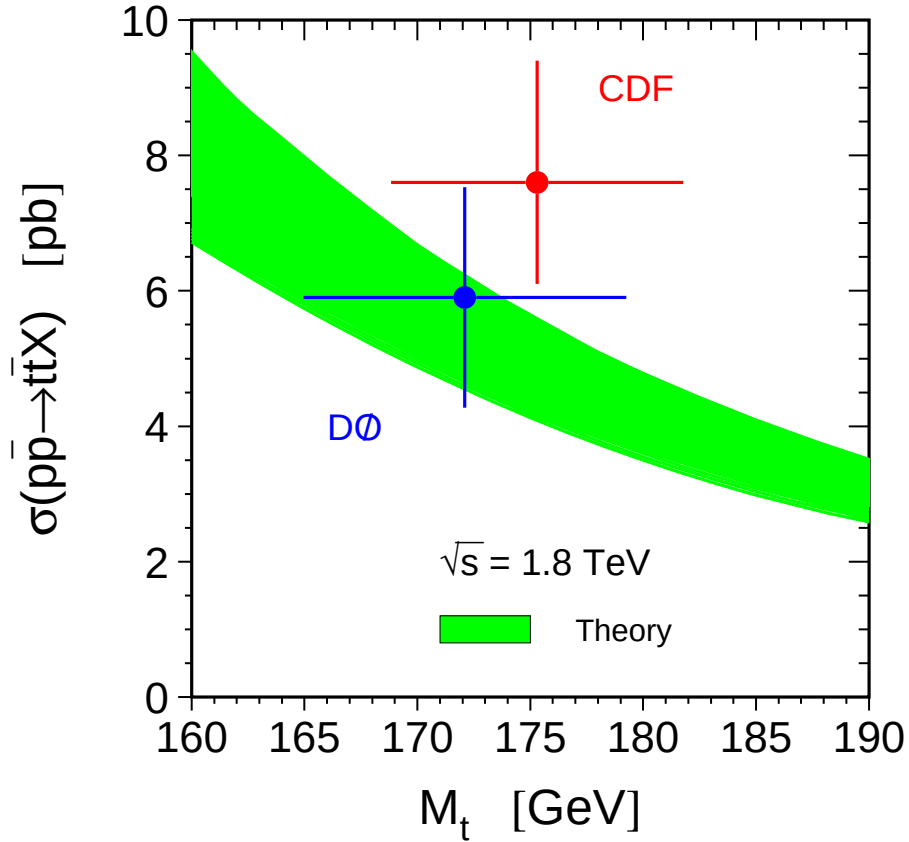


Figure 5.72: Measurements of $t\bar{t}$ production cross section and top-quark mass by the TEVATRON experiments CDF and DØ in $p\bar{p}$ collisions at $\sqrt{s} = 1.8$ TeV.

Chapter 6

Constraints on the Standard Model

The most precise measurements currently available in the area of electroweak interactions have been presented in the previous Chapter. Now these results, mostly still preliminary, are used to test the Standard Model and to constrain its parameters. In particular, electroweak radiative corrections will be analysed in order to constrain the mass of the top quark, the W boson and the Higgs boson. These indirect determinations are then compared to the corresponding direct experimental measurements. The theoretical calculations are performed with the semianalytical programs TOPAZ0 and ZFITTER described in Section 3.3.

6.1 Analysis Procedure

6.1.1 Treatment of Measurements

The results as described in the previous chapter have been obtained by applying several corrections to the raw measurements. As some of these corrections depend on an underlying theory, here the MSM, it must be analysed to which extent these corrections bias any interpretation of the results within this or other theories.

6.1.1.1 Realistic or Convolved Observables

A first set of corrections has already been applied when the experiments quote their results on cross sections and asymmetries. Trivial dependencies of the raw measurements on effects specific to the detector, for example, selection efficiencies or charge confusion, are removed, often with the help of Monte Carlo simulation programs. The aim of the experiments is to present realistic observables which are nevertheless free from effects due to the measurement device.

The validity of the Monte Carlo simulation is tested in the regions of phase space where the detector actually measures, by comparing the output of the simulation to the recorded data. If an extrapolation to unmeasured regions of phase space is required in any variable, a prediction of the differential cross section in that region is needed. Examples are limited solid-angle coverage due to holes in the detector or energy requirements on final-state particles due to the selection procedure.

The removal of detector effects from the measurements is therefore already based on theoretical models used in the Monte Carlo simulation of the various reactions. In order to keep this dependence small, the quoted realistic observables are not necessarily extrapolated to cover the full phase space, but rather extrapolated to correspond to just a few idealised cuts on phase-space variables closely related to the actual selection criteria. For example, cross sections are quoted for a cut on the minimum effective centre-of-mass energy, or asymmetries are quoted for a cut on the maximum $f\bar{f}$ acollinearity.

Main effects are the extrapolation in polar angle, $|\cos\theta| \rightarrow 1$, and energies or invariant mass of the two final state fermions. The extrapolation for these effects depends on two prerequisites. First, the exchange of spin-1 bosons, connecting electron-positron pairs of the initial state to fermion-antifermion

pairs of the final state, determines the $|\cos\theta|$ distribution. Second, the theory of QED determines the energies and angles of radiative photons, thus fixing the energy spectrum of the final state particles through the QED convolution.

Under these quite general assumptions, the extrapolation does not introduce a bias. Furthermore, if the theoretical calculations of convoluted observables are based on the same assumptions which are used to extrapolate the measurements, it does not matter how far the measurements are extrapolated since the extrapolation, being common, effectively cancels out. Nevertheless, even if no bias is introduced, a source of uncertainty arises due to the finite statistical and systematic accuracy of the Monte Carlo simulation used in the correction and extrapolation procedure.

6.1.1.2 Pseudo or Deconvoluted Observables

In a second step, the set of realistic observables, cross sections and asymmetries measured at various centre-of-mass energies, are used to determine so-called pseudo observables, for example properties of the Z boson such as mass and decay widths, or effective coupling constants. Essentially, the effect of the QED convolution to account for photon radiation on cross sections and asymmetries is removed from the measurements, so that these observables are also called deconvoluted observables.

The QED deconvolution requires the knowledge of the QED radiator functions, which can only be calculated assuming the theory of QED. Born term expressions, calculated as a function of the pseudo observables and the centre-of-mass energy, are convoluted to account for QED radiative effects and compared to the measurements. Values and errors for pseudo observables are then determined in a fit to the measured realistic observables. As in principle only the well-known theory of QED is involved, one still regards the determination of pseudo observables as a model-independent analysis of the measurements. At the same time, this is also a test of QED, *i.e.*, whether QED combined with the underlying electroweak model is able to describe the data.

As before, the correction procedure introduces additional sources of errors, as discussed in Section 5.1.5. First, the radiator functions are known only to finite order in perturbation theory. Missing higher-orders impose additional uncertainties on the extracted pseudo observables. Second, the QED deconvolution is independent from the full underlying theory only at one-loop order. At higher order non-factorisable corrections occur, introducing residual dependencies of the fitted pseudo observables on the underlying Standard Model. Another example for Standard Model dependencies are the imaginary parts of the effective couplings which must be taken from the theory, here the Standard Model, in order to calculate realistic observables. Third, model dependencies are introduced by fixing photon-exchange contributions and sometimes also γ/Z interference contributions to their values expected in the Standard Model. For the photon exchange contributions, given by QED only, this approach is consistent with the use of QED-based radiator functions. The γ/Z interference terms, however, are based on the full electroweak theory. The consequences of that particular model dependence are mostly visible in the error on the mass of the Z boson, comparing the results on M_Z obtained in the Z-pole parameter set and the S-Matrix parameter set as discussed in Sections 5.1.6 and 5.1.7. An additional source of model-dependence and associated systematic error arises in the case of Bhabha scattering, $e^+e^- \rightarrow e^+e^-(\gamma)$, due to the contribution of t -channel and s/t -interference diagrams.

Typical uncertainties on the pseudo observables arising from these effects are small but visible compared to the uncertainties arising from experimental errors. The effect of the QED radiator function is estimated by comparing, for example, various third-order radiator functions which differ by subleading terms. The effect of the residual Standard-Model dependence is estimated by repeating the extraction of the pseudo observables varying the Standard Model parameters and the calculational schemes.

6.1.1.3 Standard Model Input Parameters

Within the Standard Model, pseudo observables are calculated as a function of the five Standard-Model input parameters discussed in Section 2.12. From a fit to the pseudo observables, values and errors of the five Standard Model input parameters are determined. The residual Standard-Model dependence of the experimentally measured pseudo observables, neglected in all analyses, should in principle be taken into account explicitly if the effects are non-negligible compared to the experimental errors.

This is possible in two ways. Either, one skips the intermediate step of pseudo observables and calculates predictions for the measured realistic observables directly, based on the Standard-Model input parameters. This is a straight forward solution to the problem, but does not assess the validity of the pseudo observables as such, which are useful quantities in their own right, *e.g.*, the Z boson properties such as mass, total and partial widths, effective couplings, etc. The other possibility is to parameterise the residual dependence of the experimentally determined pseudo observables on the Standard-Model parameters in a way similar to the $\sin^2 \theta_W$ measurement in neutrino-nucleon scattering. These dependences can then be taken into account in the Standard Model analyses. In that case the residual Standard-Model bias is properly accounted for in the determination of both pseudo observables and Standard-Model input parameters.

The effect of missing higher-orders in the QED radiator function on the extraction of Standard-Model input parameters is estimated in a similar way. Fitting directly to realistic observables, the results obtained for the Standard-Model input parameters are compared when varying the QED radiator function. If fits to pseudo observables are performed, the errors on pseudo observables need to include the errors due to the QED radiator function in a correlated manner.

6.2 Tests of the Standard Model

6.2.1 Sensitivity to Radiative Corrections beyond QED

The first question to ask is whether the experimental results show evidence for electroweak radiative corrections, in particular beyond QED and the associated running of the electromagnetic finestructure constant. A particularly straight-forward test is based on those SLD/LEP-I measurements which are free from QCD uncertainties, namely the effective electroweak mixing angle, $\sin^2 \bar{\theta}_W$, measured in leptonic asymmetries, and the ρ parameter determined from the leptonic partial width, $\Gamma_{\ell\ell}$. With the measurement of M_W at TEVATRON and at LEP-II, a further test is possible. In a model based on Born-term expressions including the running of α_{em} only, these parameters are predicted as follows:

$$\rho = 1 \tag{6.1}$$

$$\sin^2 \theta_0 = \frac{1}{2} \left(1 - \sqrt{1 - 4 \frac{\pi \alpha_{em}}{\sqrt{2} G_F M_Z^2}} \right) \tag{6.2}$$

$$\Gamma_{\ell\ell} = \frac{G_F M_Z^3}{24 \sqrt{2} \pi} \rho \left[1 + (1 - 4 \sin^2 \theta_0)^2 \right] \left(1 + \frac{3}{4} \frac{\alpha_{em}}{\pi} \right) \tag{6.3}$$

$$M_W = M_Z \sqrt{1 - \sin^2 \theta_0}. \tag{6.4}$$

Numerical results are shown in Table 6.1 and graphical comparisons are shown in Figure 6.1.

The significant deviation of the Born+QED prediction from the actual measurements shows significant evidence for the need of radiative corrections beyond pure QED. In contrast, the predictions of the Standard Model as a function of top and Higgs mass, also shown in Figure 6.1, are compatible with the experimental results. The comparison shows that the experimental measurements are so precise that it is possible to test the Standard Model to the level of its electroweak radiative corrections.

Parameter	Measurement Result	Born Prediction with				MSM	
		$\alpha_{em} = \alpha_{em}(0)$	pull	$\alpha_{em} = \alpha_{em}(M_Z^2)$	pull	$\alpha_{em}, \alpha_S, M_Z, M_t, M_H$	pull
$\sin^2 \bar{\theta}_W$	0.23157 ± 0.00018	0.21216	108	0.23114 ± 0.00023	1.5	0.23217 ± 0.00064	-0.1
$\Gamma_{\ell\ell}$ [MeV]	83.90 ± 0.10	84.986	-11	83.564 ± 0.011	3.4	83.898 ± 0.118	0.0
M_W [GeV]	80.39 ± 0.06	80.938	-9	79.956 ± 0.012	7.1	80.300 ± 0.087	+0.9
ρ	1.0043 ± 0.0012	1	3.6	1	3.6	1.0046 ± 0.0011	-0.2

Table 6.1: Evidence for radiative corrections, in particular beyond the simple running of α_{em} . Born terms and Born term plus QED predictions are reported. The measurements are also compared to the expectations within the Minimal Standard Model, calculated for $1/\alpha_{em}^{(5)}(M_Z^2) = 128.878 \pm 0.090$, $\alpha_S(M_Z^2) = 0.119 \pm 0.002$, $M_Z = 91186.7 \pm 2.1$ MeV, $M_t = 173.8 \pm 5.0$ GeV and $M_H = 300_{-210}^{+700}$ GeV. The pull is calculated as the difference between the measurement and the model prediction divided by the errors of measurement and prediction added in quadrature. The formfactor ρ is essentially determined by $\Gamma_{\ell\ell}$, with a small dependence on $\sin^2 \bar{\theta}_W$. The resulting correlation between the experimental determinations of ρ and $\sin^2 \bar{\theta}_W$ is 8.8%.

Parameter	Result	$1/\alpha_{em}$	α_S	M_Z	ϵ_1	ϵ_2	ϵ_3	ϵ_b	MSM
$1/\alpha_{em}^{(5)}(M_Z^2)$	128.878 ± 0.090	1.00	0.00	0.00	0.00	-0.07	0.46	0.00	—
$\alpha_S(M_Z^2)$	0.1244 ± 0.0045	0.00	1.00	0.00	-0.45	-0.22	-0.31	-0.62	—
M_Z [GeV]	91.1866 ± 0.0021	0.00	0.00	1.00	-0.06	-0.01	-0.02	0.00	—
ϵ_1	$+0.0042 \pm 0.0012$	0.00	-0.45	-0.06	1.00	0.44	0.80	-0.01	$+0.0046 \pm 0.0011$
ϵ_2	-0.0089 ± 0.0020	-0.07	-0.22	0.00	0.44	1.00	0.26	-0.01	-0.0075 ± 0.0003
ϵ_3	$+0.0042 \pm 0.0012$	0.46	-0.31	-0.02	0.80	0.26	1.00	0.00	$+0.0058 \pm 0.0007$
ϵ_b	-0.0045 ± 0.0019	0.00	-0.62	0.00	-0.01	-0.01	0.00	1.00	-0.0058 ± 0.0005

Table 6.2: Results on the ϵ parameters including their correlations derived from a fit to all measurements, excluding the measurement of M_t at the TEVATRON and of $\sin^2 \theta_W$ from NUTEV, due to its explicit M_t dependence, as M_t is not expressed in terms of the ϵ parameters. The $\chi^2/\text{d.o.f}$ has a value of 11.6/11, corresponding to a probability of 39%.

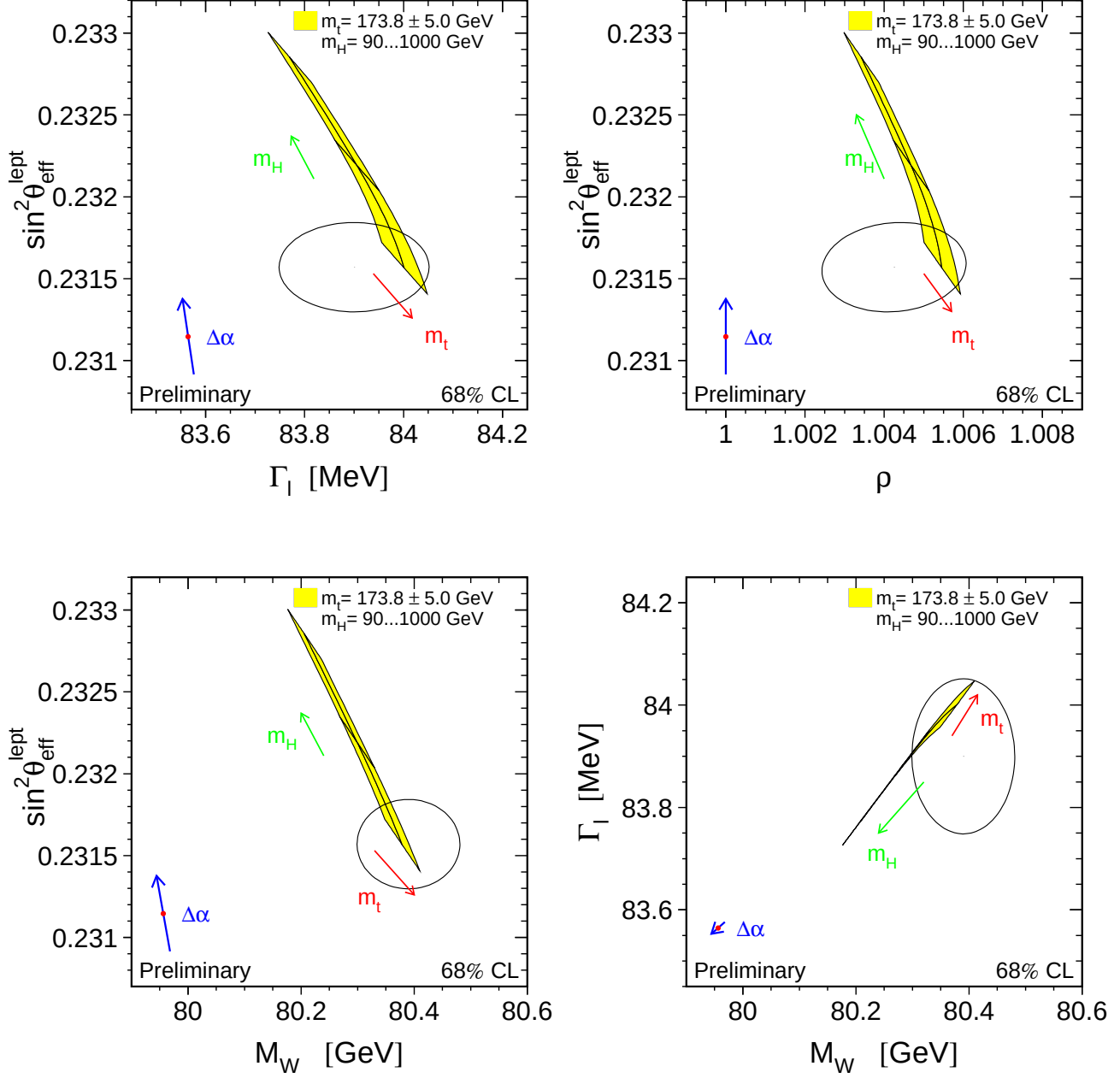


Figure 6.1: Contour curves of 68% probability in the $(\Gamma_{\ell\ell}, \sin^2 \bar{\theta}_W)$, $(M_W, \sin^2 \bar{\theta}_W)$, $(\rho, \sin^2 \bar{\theta}_W)$ and $(M_W, \Gamma_{\ell\ell})$ plane derived from the measurements of $\Gamma_{\ell\ell}$ at LEP-I, $\sin^2 \bar{\theta}_W$ at SLC and LEP-I, and M_W at the TEVATRON and at LEP-II. The prediction of a simplified theory based on Born plus QED with running α_{em} is shown as the point. If $\alpha_{em}(M_Z^2)$ is changed by its error, the arrow associated to the point shows the variation of the prediction. The same uncertainty also affects the complete Standard Model prediction, shown as the hatched region, which is drawn for $M_t = 173.8 \pm 5.0$ GeV and $M_H = 300_{-210}^{+700}$ GeV.

6.2.2 The ϵ Parameters

Within a fit to determine the ϵ parameters, several parameter transformations are needed to arrive at the predictions for the measured observables. Based on G_F , M_Z and $\alpha_{em}(M_Z^2)$, the auxiliary quantity $\sin^2 \theta_0$ is calculated according to Equation 3.100/6.2. The ϵ parameters ϵ_1 , ϵ_2 , and ϵ_3 are used to calculate the quantum correction terms $\Delta\rho$, $\Delta\kappa'$, and Δr_W , according to Equations 3.96, 3.97 and 3.98. This assumes neutral-current universality between neutrinos, charged leptons and light quarks. For b quarks, the specific $Zb\bar{b}$ vertex correction, $\Delta\rho_b$, is expressed with ϵ_b , Equation 3.99. At this stage, the W/Z mass ratio as well as the effective electroweak mixing angle, the effective vector and axial-vector coupling constants, the pole asymmetries and the leptonic partial widths are calculated. The calculations for $q\bar{q}$ final states depend also on the strong coupling constant α_S . When fitting for the four ϵ parameters, the fit must therefore also include $\alpha_{em}(M_Z^2)$, $\alpha_S(M_Z^2)$ and M_Z as parameters in order to take into account their errors in the determination of the ϵ parameters.

The analysis of the measurements in terms of the ϵ parameters yields the numerical results as reported in Table 6.2. All ϵ parameters are significantly different from zero. This shows again significant evidence for radiative corrections beyond simple QCD and QED effects including the running of the electromagnetic finestructure constant.

6.2.3 Standard Model Analyses

The next question is how well the Standard Model is able to describe all experimental results. As the Standard Model predictions depend on the set of values used as Standard Model input parameters, this is tested by fitting to the set of measurements in order to determine the Standard Model input parameters. The probability of the fit, based on the χ^2 value in the minimum and the number of degrees of freedom, expresses the compatibility of the Standard Model with all experimental results for the same values of Standard-Model input parameters. Having determined the Standard-Model input parameters, it is then possible to calculate values for any observable, measured or unmeasured.

The Standard Model input parameters actually being fitted are the inverse of the five-flavour running finestructure constant, $1/\alpha_{em}^{(5)}(M_Z^2)$, the strong coupling constant, $\alpha_S(M_Z^2)$, and the masses of Z boson, top quark and Higgs boson, M_Z , M_t , and $\log_{10}(M_H/\text{GeV})$. In case of the running electromagnetic finestructure constant, the top-quark contribution is removed, because it is M_t dependent. Therefore it is calculated and added inside the programs TOPAZ0 and ZFITTER calculating Standard Model radiative corrections. A value of $1/\alpha_{em}^{(5)}(M_Z^2) = 128.878 \pm 0.090$ [43, 44] is used, where the error is propagated in the fit. For the Higgs mass, the logarithm is used as a fit parameter, because to leading order the radiative corrections depend on $\log M_H$, as discussed in Sections 2.11.2 and 3.1.3. Thus the errors will be more symmetric in $\log_{10}(M_H/\text{GeV})$ than in M_H . Theoretical errors are not included in the results quoted below but discussed separately.

A first fit is performed, where all measurements are included, except the limit on M_H derived from the negative direct search at LEP-II and the direct determinations of M_W and M_t at the TEVATRON and at LEP-II. The predicted values for these quantities are then independent of the direct-search limit and of the measured values so that a comparison tests the predictive power of the Standard Model. The result of this Standard Model fit is shown in Table 6.3.

The χ^2 of the fit has a value of 12.5 for 12 degrees of freedom, corresponding to a probability of 40%, which shows that the set of measurements is well described by the Standard Model, with a unique set of values for the Standard Model input parameters. This is a great success, both experimentally since the measurements are so precise that they are sensitive to the small effects of electroweak radiative corrections, and theoretically since the Standard Model as a theory is able to describe all measurement on this level.

Parameter	Result	$1/\alpha_{em}$	α_S	M_Z	M_t	M_H
$1/\alpha_{em}^{(5)}(M_Z^2)$	128.883 ± 0.087	1.000	-0.146	-0.003	-0.370	0.470
$\alpha_S(M_Z^2)$	0.1202 ± 0.0030	-0.146	1.000	-0.038	-0.214	-0.275
M_Z [GeV]	91.1867 ± 0.0021	-0.003	-0.038	1.000	-0.067	-0.020
M_t [GeV]	$158.2^{+8.7}_{-8.1}$	-0.370	-0.214	-0.067	1.000	0.408
$\log_{10}(M_H/\text{GeV})$	$1.51^{+0.36}_{-0.28}$	0.470	-0.275	-0.020	0.408	1.000
M_H [GeV]	32^{+41}_{-15}					

Table 6.3: Results on the Standard Model parameters derived from a fit to all measurements except limit on M_H derived from the negative direct search at LEP-II and the direct measurements of M_W and M_t at the TEVATRON and LEP-II. The $\chi^2/\text{d.o.f}$ has a value of 12.5/12, corresponding to a probability of 40%.

6.2.3.1 The Mass of the Top Quark

The above constraint on the mass of the top quark must be compared with the result from the direct measurement at the TEVATRON, $M_t = 173.8 \pm 5.0$ GeV. The indirect determination is lower, $M_t = 158.2^{+8.7}_{-8.1}$ GeV, but within the error in agreement with the direct measurement. The accuracy of the constraint on M_t is improved by including the direct measurements of the W boson mass at the TEVATRON and at LEP-II in the Standard Model analysis:

$$M_t = 161.1^{+8.2}_{-7.1} \text{ GeV}, \quad (6.5)$$

increasing the value of M_t towards the current direct measurement and reducing the error by 9%. Nevertheless, the direct measurement of the mass of the top quark is already more precise than the indirect determination of M_t based on the Standard Model.

6.2.3.2 The Mass of the W Boson

Having determined the Standard Model input parameters as reported in Table 6.3, the mass of the W boson is now calculated within the framework of the Standard Model. Compared to the result of the direct measurement of M_W at the TEVATRON and at LEP-II, $M_W = 80.39 \pm 0.09$ GeV, the SM constraint on M_W is given by:

$$M_W = 80.332 \pm 0.037 \text{ GeV}, \quad (6.6)$$

which is in good agreement with the direct measurement. The accuracy of the constraint on M_W is further improved by including the direct measurements of the top quark mass at the TEVATRON in the Standard Model analysis, with the result:

$$M_W = 80.367 \pm 0.029 \text{ GeV}. \quad (6.7)$$

Also here, good agreement with the direct measurement is observed. The SM constraint on M_W is more precise than the current direct measurement. For a stringent test of the SM, the mass of the W boson should thus be measured to an accuracy of at least 30 MeV. The various determinations of M_W are compared in Figure 6.2. All are in good agreement.

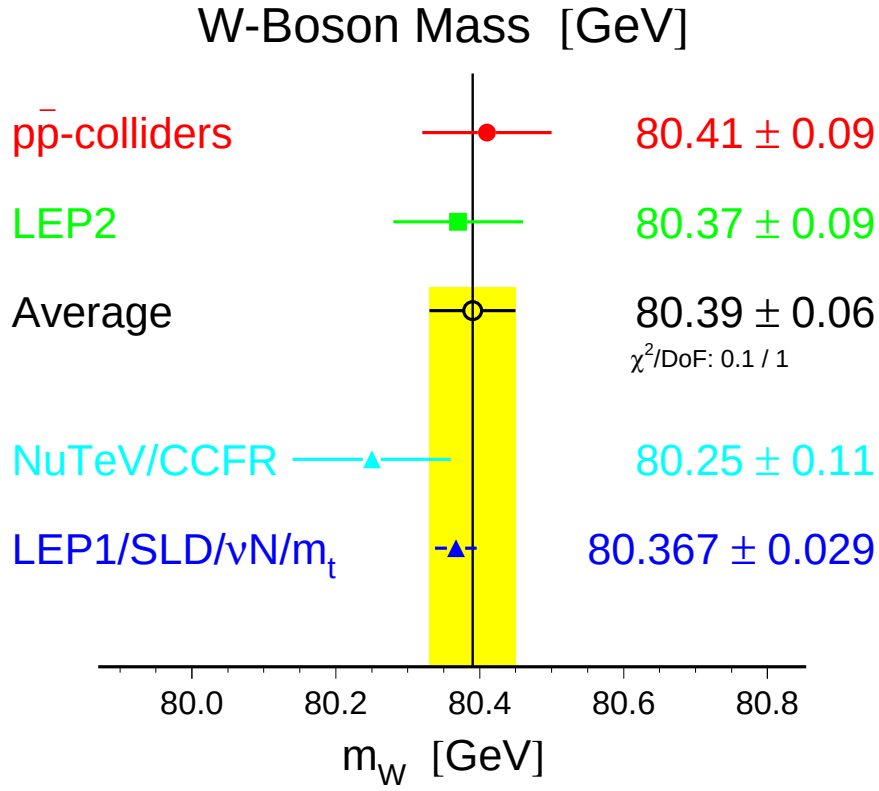


Figure 6.2: Comparison of results on M_W . Shown are the direct measurements of M_W at the TEVATRON and at LEP-II and their average as well as the indirect determinations based on the measurement of $\sin^2 \theta_W$ by the NUTEV experiment and the combined indirect determination based on the measurements of electroweak radiative corrections and the top-quark mass.

6.2.3.3 The Mass of the Higgs Boson

The contour curve of 68% confidence level in the (M_W, M_t) plane corresponding to the results of the SM fit reported in Table 6.3 is shown in Figure 6.3. It is compared to the 68% contour arising from the independent direct measurements of M_W and M_t at the TEVATRON and at LEP-II. The predictions based on the analysis of the measurements in terms of radiative corrections within the framework of the minimal Standard Model agree well with the direct measurements.

The contour curves of Figure 6.3 show that both the direct measurements and the indirect determinations of M_W and M_t prefer low values of M_H . This is seen again when analysing the contour curves in the (M_H, M_t) plane as shown in Figure 6.4. A large correlation, +41%, exists between M_t and M_H . This is a result of the structure of radiative corrections. To leading order, most measurements are sensitive to the same top and Higgs mass dependence. Only the measurement of R_b^Z shows a different leading top-mass dependence due to the specific $Zb\bar{b}$ vertex corrections involving the top quark.

The constraint on M_H is drastically improved if the direct determination of M_t is included in the analysis. The results of a fit to all measurements, except the limit on M_H derived from the negative direct search at LEP-II but including the direct determinations of M_W and M_t at the TEVATRON and at LEP-II, are reported in Table 6.4. The χ^2 of the fit has a value of 14.9 for 15 degrees of freedom, corresponding to a probability of 46%, showing that also the complete set of measurements is well described by the Standard Model.

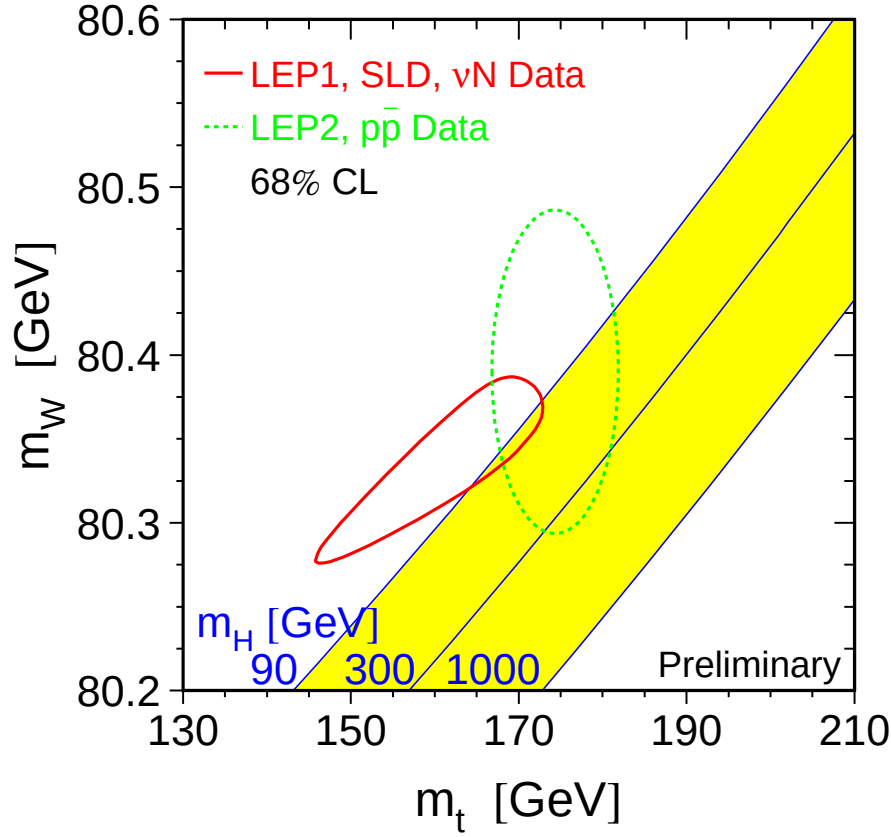


Figure 6.3: Contour curves of 68% probability in the (M_t, M_W) plane, showing the indirect data corresponding to the results reported in Table 6.3, and the direct measurements of these parameters at the TEVATRON and at LEP-II. The band corresponds to the SM relation given by G_F .

Parameter	Result	$1/\alpha_{em}$	α_S	M_Z	M_t	M_H
$1/\alpha_{em}^{(5)}(M_Z^2)$	$128.878 \pm_{0.104}^{0.096}$	1.000	0.039	0.018	0.216	0.769
$\alpha_S(M_Z^2)$	0.1194 ± 0.0029	0.039	1.000	-0.042	0.036	0.125
M_Z	91.1865 ± 0.0021	0.018	-0.042	1.000	-0.009	0.046
M_t	171.1 ± 4.9	0.218	0.036	-0.009	1.000	0.611
$\log_{10}(M_H/\text{GeV})$	$1.88 \pm_{0.41}^{0.33}$	0.769	0.125	0.046	0.611	1.000
M_H [GeV]	$76 \pm_{47}^{85}$					

Table 6.4: Results on the Standard Model parameters derived from a fit to all measurements except the limit on M_H derived from the negative direct search at LEP-II but including the direct determinations of M_W and M_t at the TEVATRON and at LEP-II. The $\chi^2/\text{d.o.f}$ has a value of 14.9/15, corresponding to a probability of 46%.

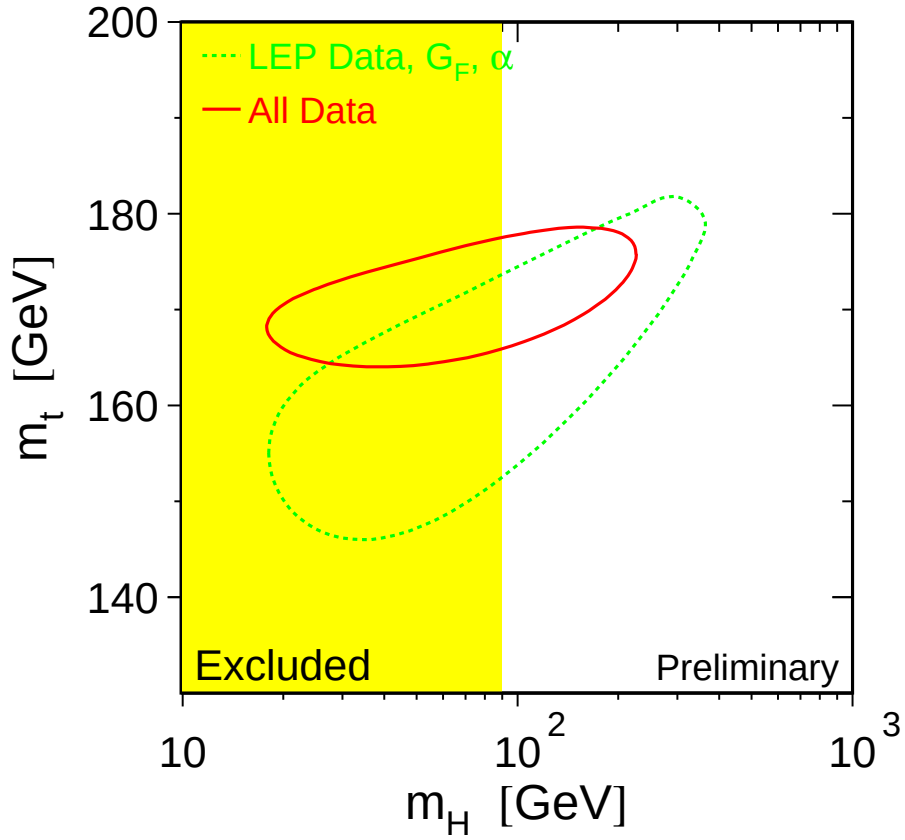


Figure 6.4: Contour curves of 68% probability in the (M_H, M_t) plane, showing the results based on only the LEP-I measurements, and the results based on all measurements, in particular including the direct determination of M_t . Also shown is the region excluded at 95 % CL by the negative direct search for the Higgs boson at LEP-II, 90 GeV [51].

The $\Delta\chi^2_{\min}(M_H)$ curve, defined as:

$$\Delta\chi^2_{\min}(M_H) = \chi^2_{\min}(M_H) - \chi^2_{\min} \quad (6.8)$$

is shown in Figure 6.5. The central line corresponds to the value and error on M_H as reported in Table 6.4:

$$\log_{10}(M_H/\text{GeV}) = 1.88^{+0.33}_{-0.41} \pm 0.05 \quad \text{or} \quad M_H = 76^{+85}_{-47} \pm 9\text{GeV}, \quad (6.9)$$

in agreement with the lower limit on M_H of 90 GeV at 95% CL obtained from the negative direct search at LEP-II [51]. The first error is experimental, while the second accounts for theoretical uncertainties. The theoretical uncertainties are expressed by the shaded band around the central line in Figure 6.5. The band is derived from comparing SM calculations as implemented in the two programs TOPAZ0 and ZFITTER, and varying calculational options within these programs. The programs rely on different renormalisation schemes, the generalised minimal subtraction scheme in case of TOPAZ0 and the on-mass-shell scheme in case of ZFITTER, see [61] for a discussion. The various options of the programs implement different factorisation schemes and momentum-transfer scales. The numerical results differ effectively by as yet uncalculated higher-order terms, thus such a comparison estimates the associated theoretical uncertainties. The width of the band shows, that this uncertainty is small for large Higgs masses and large for small Higgs masses, affecting in particular the lower error on M_H .

The one-sided 95% confidence level upper limit on the Higgs boson is determined including theoretical uncertainties expressed by the width of the shaded band. The determination of the upper limit

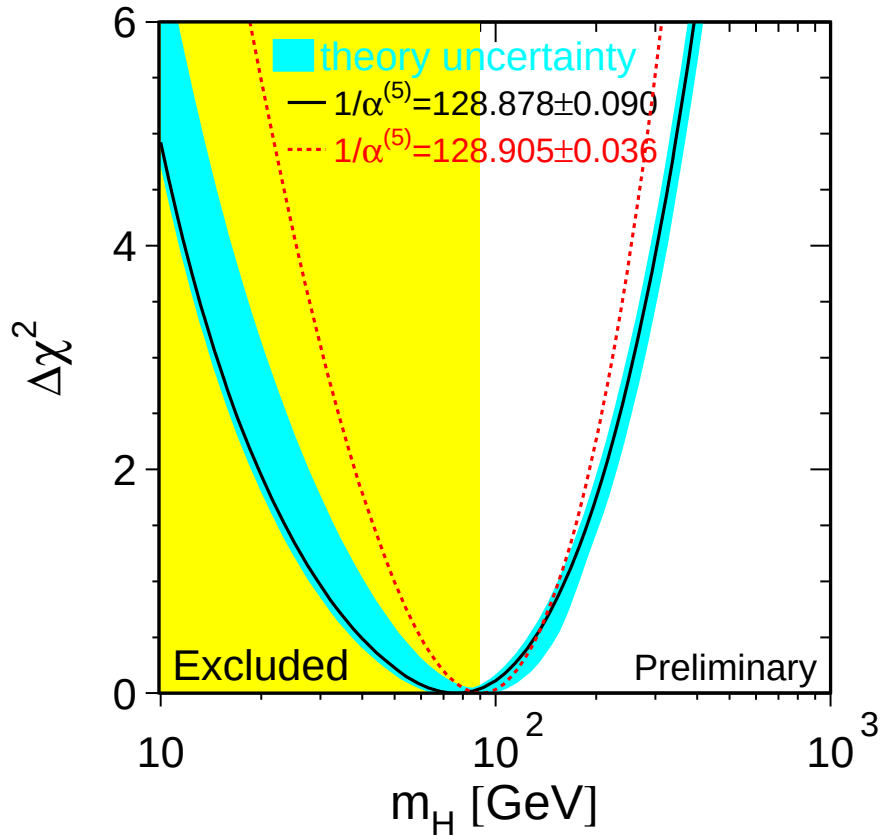


Figure 6.5: Curve showing $\Delta\chi^2_{\min}(M_H) = \chi^2_{\min}(M_H) - \chi^2_{\min}$ as a function of M_H . The width of the shaded band around the curve shows the theoretical uncertainty. The two lines correspond to different evaluations of $\alpha_{em}^{(5)}(M_Z^2)$, namely $1/\alpha_{em}^{(5)}(M_Z^2) = 128.878 \pm 0.090$ [43, 44] and $1/\alpha_{em}^{(5)}(M_Z^2) = 128.905 \pm 36$ [43, 235]. Also shown is the region excluded at 95 % CL by the negative direct search for the Higgs boson at LEP-II, 90 GeV [51].

does not make use of the lower limit on M_H derived from the negative direct search result [51]. Thus the following independent information on the mass of the Higgs boson is available:

$$M_H < 262 \text{ GeV} \quad 95\% \text{ CL SM analysis} \quad (6.10)$$

$$M_H > 90 \text{ GeV} \quad 95\% \text{ CL direct search.} \quad (6.11)$$

A combination of the two is not straight forward as it requires the probability density as a function of M_H describing the negative result from the direct search, rather than just the 95% CL limit. Because of the direct search limit, the impact of the larger theoretical uncertainty at low values of the Higgs mass is of less relevance.

The indirect determination of the Higgs boson mass in a range up to a few hundred GeV shows the selfconsistency of the analysis. Within the **SM**, the Higgs boson must be lighter than about 1 TeV [50]. If the result of the **SM** analysis were a Higgs mass in the TeV region, the **SM** analysis would not be self-consistent and we could draw only one conclusion, namely that the **SM** is not able to describe the data. The indirect determination of M_H is also in agreement with the mass limit on M_H obtained from the negative direct search at LEP-II.

As shown in Table 6.4, the fitted error on $1/\alpha_{em}^{(5)}(M_Z^2)$, ± 0.10 , is larger than that of the direct determination used in the fit, ± 0.09 . The reason for this is that the contribution of all other measurements to the total χ^2 calculated as a function of $1/\alpha_{em}^{(5)}(M_Z^2)$ shows a double minimum structure, creating a negative curvature of $\chi^2_{\min}$ in the region between the two minima. The central value of

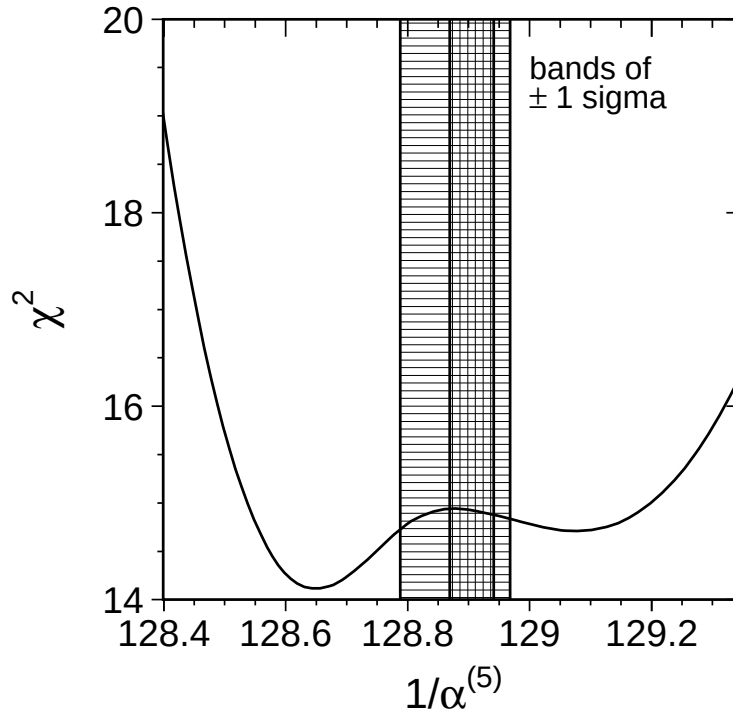


Figure 6.6: Curve showing $\Delta\chi^2_{\min}(\alpha_{em}) = \chi^2_{\min}(\alpha_{em}) - \chi^2_{\min}$ as a function of M_H , excluding the direct constraint on $1/\alpha_{em}^{(5)}(M_Z^2)$. The two direct constraints, $1/\alpha_{em}^{(5)}(M_Z^2) = 128.878 \pm 0.090$ [43, 44] and $1/\alpha_{em}^{(5)}(M_Z^2) = 128.905 \pm 0.036$ [43, 235], are shown as vertical bands with a width of ± 1 sigma. The band corresponding to the first measurement overlaps with the region of negative curvature, while band corresponding to the second measurement, having also a higher central value, lies in the region of about zero curvature.

the direct constraint on $1/\alpha_{em}^{(5)}(M_Z^2)$ lies in that region, as shown in Figure 6.6. The sum of both χ^2 contributions, making up the total χ^2 , has a positive but reduced curvature compared to the direct constraint alone, thus yielding an increased error on $1/\alpha_{em}^{(5)}(M_Z^2)$ in the fit.

The largest correlation, +77%, appears between $\log_{10}(M_H/\text{GeV})$ and $1/\alpha_{em}^{(5)}(M_Z^2)$. In order to improve the accuracy on the prediction of the Higgs boson mass, it is of foremost importance to improve the accuracy on $1/\alpha_{em}^{(5)}(M_Z^2)$. This is shown graphically in Figure 6.5, where the $\Delta\chi^2_{\min}(M_H)$ curve is shown using a more precise constraint on $1/\alpha_{em}^{(5)}(M_Z^2)$, evaluated as discussed in Appendix B. Using $1/\alpha_{em}^{(5)}(M_Z^2) = 128.905 \pm 0.036$ [43, 235], M_H is determined with much improved accuracy:

$$\log_{10}(M_H/\text{GeV}) = 1.96^{+0.23}_{-0.26} \pm 0.05 \quad \text{or} \quad M_H = 91^{+64}_{-41} \pm 11 \text{ GeV}. \quad (6.12)$$

The fitted error on $1/\alpha_{em}^{(5)}(M_Z^2)$ is no longer increased compared to the direct constraint since the central value of the constraint has moved from the region of negative curvature to a region of about zero curvature, as shown in Figure 6.6. Furthermore, the correlation between M_H and $1/\alpha_{em}^{(5)}(M_Z^2)$ is reduced, leaving with 70% the correlation between M_H and M_t as the largest.

The data used in these analyses do not include a direct external constraint on $\alpha_S(M_Z^2)$. This parameter is solely determined by the measurements, mainly through the QCD correction $\Delta_{QCD}^{(\text{had})}$ on the hadronic Z decay width, and its resulting influence on the total Z width and on the hadronic pole cross section. The central value of α_S , 0.120 ± 0.003 , agrees well with the world average of 0.119 ± 0.002 [31]. Since the correlations with other Standard Model parameters are small, including, for example, the world average on $\alpha_S(M_Z^2)$ as a constraint in the fit will neither shift the values of

fitted parameters nor decrease the errors, except the error on α_S , significantly. Note that there is also some controversy on the size of the theoretical error to be assigned in the determination of α_S from the measured QCD correction term Δ_{QCD} in general. Total errors assigned to $\alpha_S(M_Z^2)$ range from 0.002 [31] to 0.004 [56] to 0.006 [170, 171], while the central value of 0.119 is uncontroversial.

The sensitivity of the individual measurements to the Higgs-boson mass is shown in Figures 6.7 and 6.8. Compared to the experimental accuracies, the asymmetries, all determining the effective electroweak mixing angle $\sin^2 \bar{\theta}_W$, are the best Higgs meters. The width of the associated band indicating the SM prediction shows again that $1/\alpha_{em}^{(5)}(M_Z^2)$ needs to be determined with improved precision, as within the SM approximately $\delta(1/\alpha_{em}) \propto \delta \sin^2 \bar{\theta}_W$. The mass of the W boson, still to be improved in accuracy by the measurements at LEP-II, does not suffer as much from $1/\alpha_{em}^{(5)}(M_Z^2)$ induced uncertainties.

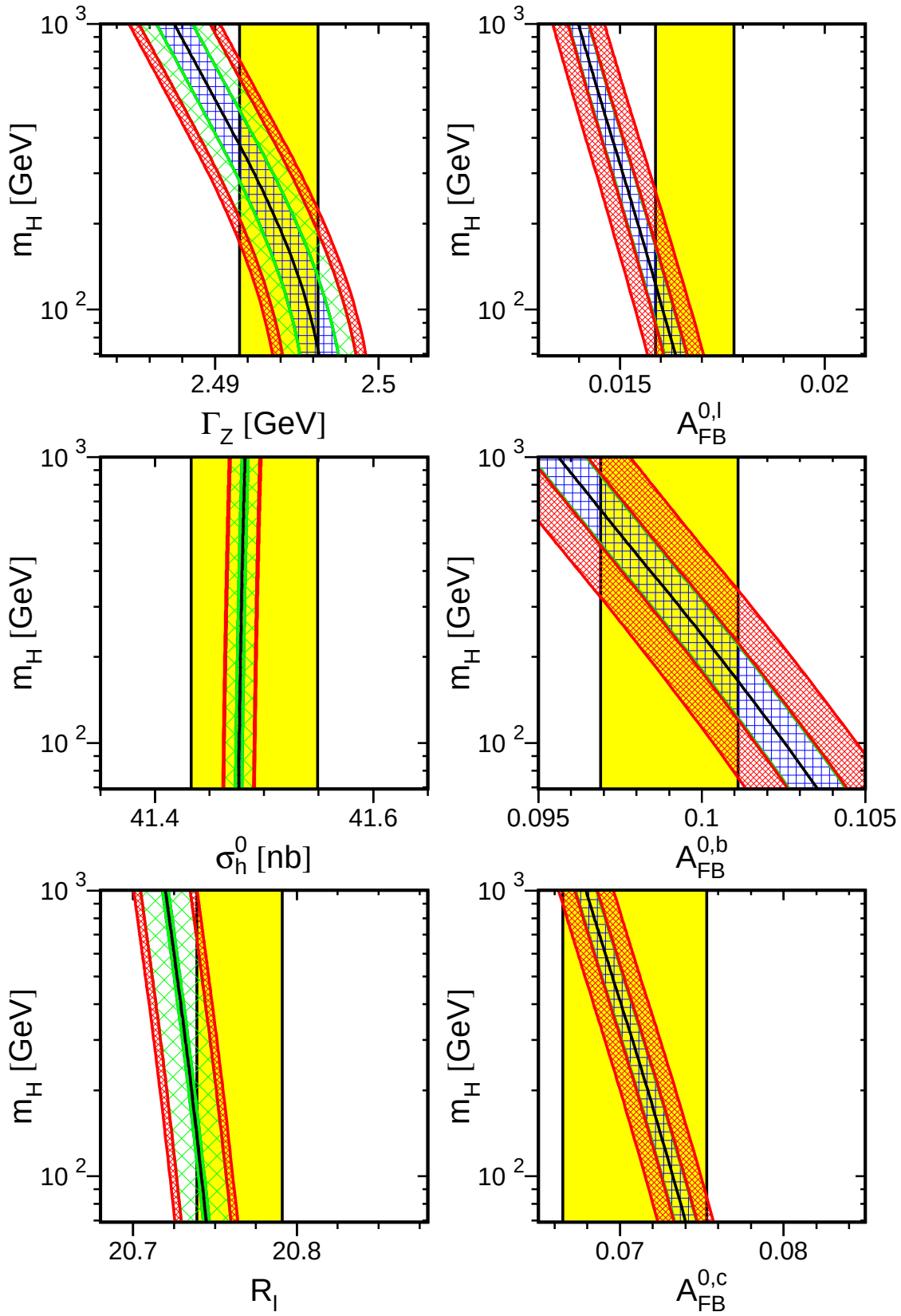


Figure 6.7: Higgs sensitivity (I). The variation of the electroweak observables with the mass of the Higgs boson as calculated within the minimal Standard Model. The vertical bands show the results of the measurements. The Standard Model bands show the uncertainties in the SM calculations due to the uncertainties in M_t , α_S and α_{em} , $M_t = 173.8 \pm 5.0$ GeV, $\alpha_S(M_Z^2) = 0.119 \pm 0.002$, and $1/\alpha_{em}^{(5)}(M_Z^2) = 128.878 \pm 0.090$.

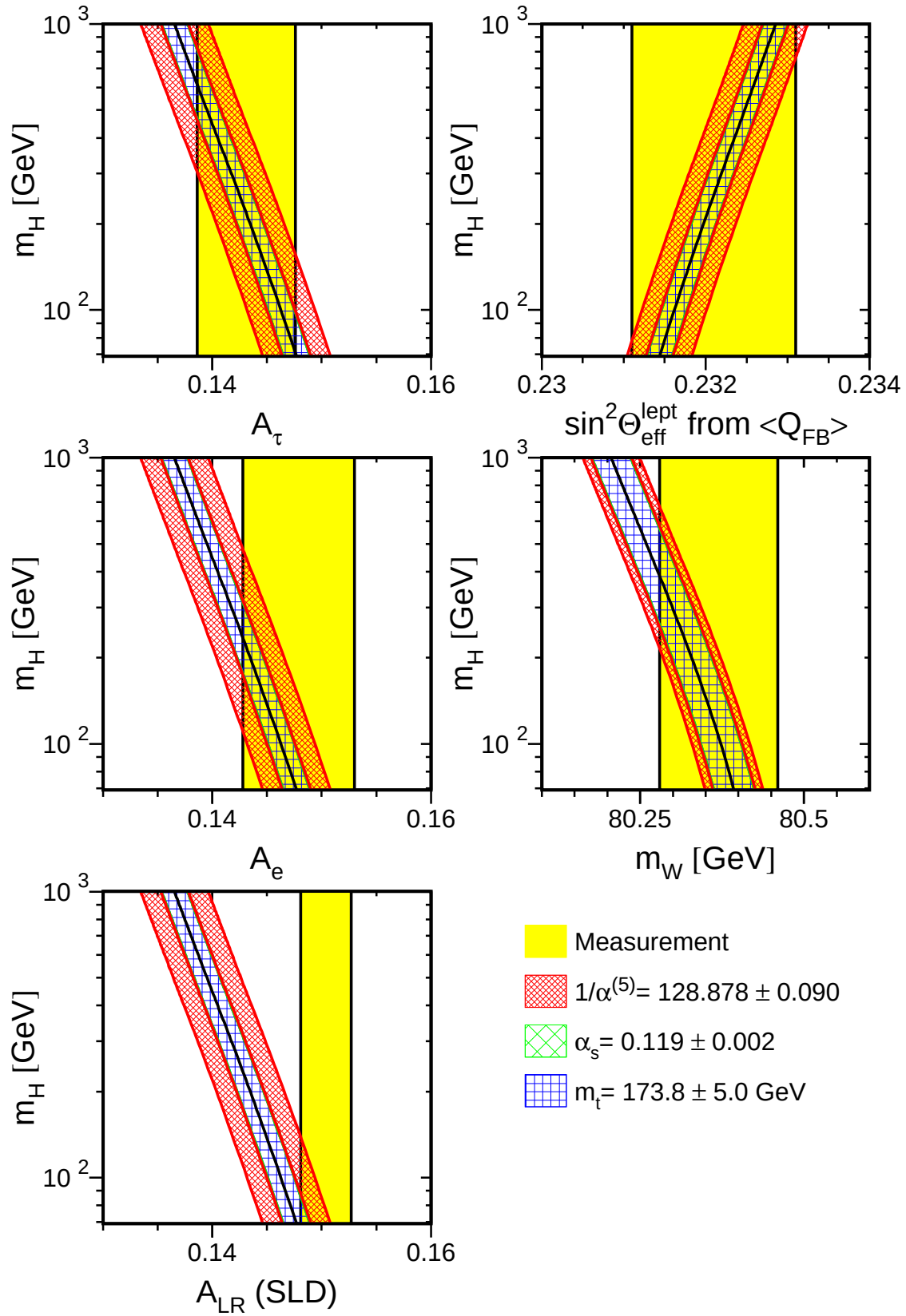


Figure 6.8: Higgs sensitivity (II). The variation of the electroweak observables with the mass of the Higgs boson as calculated within the minimal Standard Model. The vertical bands show the results of the measurements. The Standard Model bands show the uncertainties in the SM calculations due to the uncertainties in M_t , α_s and α_{em} , $M_t = 173.8 \pm 5.0 \text{ GeV}$, $\alpha_s(M_Z^2) = 0.119 \pm 0.002$, and $1/\alpha_{em}^{(5)}(M_Z^2) = 128.878 \pm 0.090$.

6.3 Discussion

The pulls of the individual measurements, defined as the difference between the measurement and the SM prediction, calculated in the minimum of the χ^2 , and divided by the error of the measurement, are shown in Figure 6.9. The largest pulls occur for the measurements related to the b-quark asymmetries, A_b and $A_{fb}^{0,b}$, and for the most precise single measurement of $\sin^2 \bar{\theta}_W$, derived from the left-right asymmetry A_{lr} .

The reasons are already indicated in Section 5.5. Within the SM, A_b is constant. The low value of the measured A_b is thus fixed, does not influence the Standard Model parameters and leads to a constant contribution to the global χ^2 . In contrast, the parameter A_ℓ depends on the values of the Standard Model input parameters. The measurement of $A_{fb}^{0,b} = \frac{3}{4}A_e A_b$ can therefore be accommodated by choosing a combination of SM input parameter values which gives rise to a value of A_ℓ which is low compared to the measurements of A_ℓ from the left-right asymmetry at SLC and from the $\ell^+ \ell^-$ forward-backward and τ polarisation asymmetries at LEP-I. In the minimum of the global χ^2 , the Standard Model parameters are determined such that an equilibrium between the two opposing requirements on A_ℓ is found. Consequently, $A_{fb}^{0,b}$ shows a negative pull. As far as the A_ℓ measurements are concerned, the effect becomes most visible in the pull of the A_{lr} measurement because it is the measurement with both the highest central value and the smallest error. Therefore, A_{lr} shows a positive pull, or equivalently $\sin^2 \bar{\theta}_W$ determined from A_{lr} shows a negative pull, as visible in Figure 6.9.

In interpreting these observations, different conclusions are obtained.

One possibility is that the deviation is attributed to the measurement of A_{lr} . This point of view is supported by two observations: first, the good agreement of all $\sin^2 \bar{\theta}_W$ determinations obtained from asymmetries excluding the A_{lr} determination, as shown in Figure 5.37, and second, the slight incompatibility of the A_{lr} results with the current lower limit on the Higgs boson mass of 90 GeV derived from the negative direct search at LEP-II [51]. The two-sigma deviation of the direct measurement of A_b based on the forward-backward left-right asymmetry is considered as a fluctuation.

The other possibility is that the deviation is attributed to the b-quark sector, in particular to A_b , affecting the direct measurement based on the forward-backward left-right asymmetry and propagating to the forward-backward asymmetry $A_{fb}^{0,b}$ as well. Three years ago, the measurement of R_b^Z deviated by more than three standard deviations from the SM prediction. The measured value of R_b^Z has since moved and is now in agreement with the SM expectation. At the same time, the $b\bar{b}$ forward-backward asymmetry decreased. The largest change in both results occurred during the same time period, between the winter conferences and the summer conferences of the year 1996.

There are a few possible explanations for the observed disagreement:

1. It is simply a statistical fluctuation. This may happen and the measurements should definitely not be excluded from the SM analysis. More data would be welcome, but, unfortunately, additional data taking at Z pole is not foreseen.
2. Some errors are too small. The measurement of A_{lr} is statistics dominated. The low $\chi^2/\text{d.o.f}$ of the heavy flavour average suggests the opposite, *i.e.*, the estimated errors are, if anything, too large. A more refined error analysis may then even cause the discrepancy to become more significant. One may speculate about a bias not identified so far. As discussed before, it is not possible to lower A_{lr} and to increase A_b through the dependence of their SLD measurement on the SLC electron beam polarisation at the same time, as both observables would change in the same direction.
3. There appears new physics in the area of b-quark couplings. The SM analyses discussed in this Section are then intrinsically invalid as they necessarily assume the structure of the Standard Model theory in order to calculate radiative corrections. New physics is of course the most interesting explanation, and thus attracts wide-ranging speculation from many people.

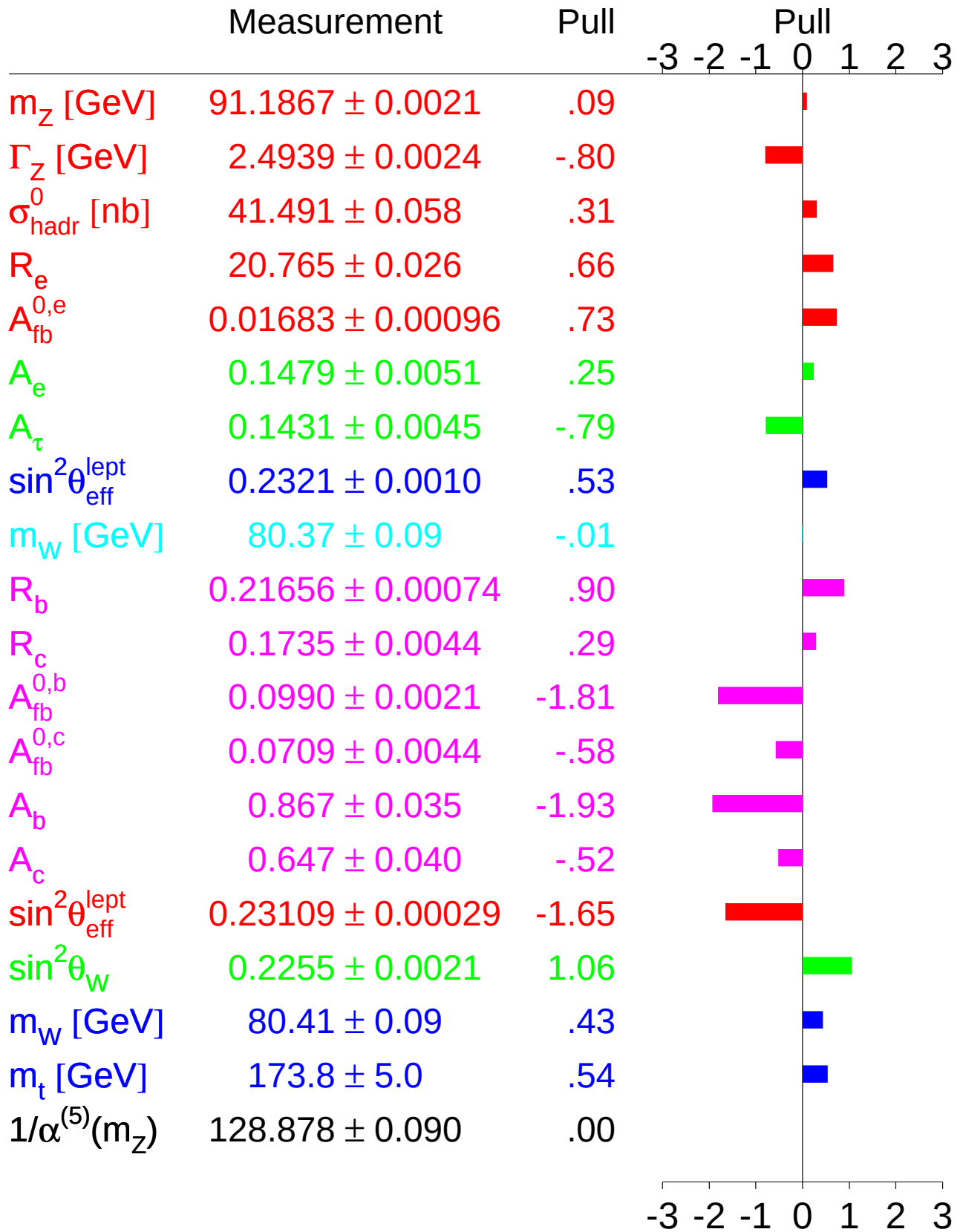


Figure 6.9: Pulls of measurements. The pull is defined as the difference of the measurement and the Standard Model prediction calculated for the central values of the fitted Standard Model input parameters as reported in Table 6.4, divided by the measurement error.

With the complete data at hand, the errors and correlations in the heavy-flavour analyses should be investigated in more detail. The aim should clearly be to understand the problematic low $\chi^2/\text{d.o.f}$ of the average, and to correct possibly overly large errors or correlations assigned. The final results and error assessments of the experimental analyses are needed.

From a statistical point of view, all measurements shows a very acceptable spread around their prediction, which is also quantified by the global $\chi^2/\text{d.o.f}$ of 15/15. With this many measurements, some are expected to deviate by more than one or two standard deviations from their prediction. Thus the observed deviations should be noted and kept in mind but not overinterpreted.

Chapter 7

Future Developments

7.1 Experimental Results

Nearly all of the measurements presented in Chapter 5 are still preliminary. Final results on the data collected at SLC and LEP-I are expected soon, likewise for NUTEV and run-I results from the TEVATRON. Data taking at LEP-II is still in progress.

7.1.1 SLC

According to approved schedule, data taking of SLD at SLC has stopped in June 1998. However, pending funding, SLD and SLAC propose an additional SLC run with the goal of SLD collecting more than 700,000 additional Z decays. The combined result on A_{lr} would permit to determine $\sin^2 \bar{\theta}_W$ with an error of 0.00018, which is as precise as the current preliminary world average. The heavy flavour measurements would become about as precise as the average of the four LEP experiments. In view of the discussion in the previous section, this proposal becomes even more attractive.

7.1.2 LEP

Data taking at LEP is extended for one year and now scheduled to end in September 2000, when civil engineering for the LHC project has to start. All luminosity still to be expected in the years 1999 and 2000, at least 200 pb^{-1} , will be collected at the highest centre-of-mass energy of 200 GeV. Combining the four LEP experiments, a measurement of the mass of the W boson with an accuracy of 30 MeV is within reach. In order to achieve this goal, the LEP-II beam energy must be determined with an accuracy of at most 15 MeV, and final-state interactions in the hadronic W^+W^- channel must be well under control.

7.1.3 HERA

At the ep collider HERA, the experiments H1 and ZEUS measure the cross section of neutral-current and charged-current deep inelastic lepton proton scattering as a function of Q^2 . The Q^2 dependence of the cross section arises from the propagator of the exchanged vector boson. Incorporating radiative corrections by writing the charged-current and neutral-current amplitudes in terms of the Fermi constant G_F , the cross sections are proportional to $G_F^2 M_W^4 / (Q^2 + M_W^2)^2$ and to $G_F^2 M_Z^4 / (Q^2 + M_Z^2)^2$, respectively. The Q^2 dependence increases with increasing Q^2 .

Analysing the Q^2 dependence of the charged-current cross section and keeping G_F fixed, the mass of the W boson is determined. Each experiments achieves an accuracy on M_W of about 5 GeV [236] based on about 40 pb^{-1} of luminosity collected until 1997. As the Q^2 dependence of the propagator is exploited, such a measurement of M_W is comparable to the direct measurements of M_W .

At low Q^2 , the heavy boson masses cancel and the cross sections essentially depend on G_F only, leading to an allowed area of values in the (M_W, M_t) plane which follows closely the lines arising from the G_F constraint, see for example Figure 6.3. With a luminosity of 1000 pb^{-1} , a lepton polarisation of -70% , and a systematic error of 1% , the cross section measurements yield a determination of M_W with an accuracy of 55 MeV when imposing the mass of the top quark [237]. Since electroweak radiative corrections within the SM are exploited, such a measurement is comparable to the indirect determinations of M_W based on the analysis of radiative corrections.

7.1.4 TEVATRON

Run I at the $p\bar{p}$ collider TEVATRON ended in 1996, with each of the two experiments CDF and DØ collecting more than 100 pb^{-1} of luminosity. Run II is scheduled to start in the year 2000 with the goal of accumulating a luminosity of 2 fb^{-1} within 3 years. The mass of the W boson will then be measured with about the same accuracy as at LEP-II, 30 MeV . The mass of the top quark will be measured with an accuracy of 2 GeV , where systematic effects, including final state interactions as discussed in Section 3.2.6, are the limiting factor.

7.1.5 LHC

The principal goal of the large hadron collider, a pp machine, is to find the Higgs boson and manifestations of new physics beyond the minimal Standard Model. Besides the mass of the Higgs boson, also other properties, such as its decay branching fractions, will be measured in order to establish whether the Higgs boson is that of the minimal Standard Model or that of extended theories such as supersymmetry. As far as top quark and W boson are concerned, errors of less than 2 GeV on M_t and less than 30 MeV on M_W are expected.

7.1.6 LC

A future e^+e^- linear collider is the perfect machine to measure the mass of the W boson and the top quark precisely, for example by performing dedicated scans in centre-of-mass energy of the $e^+e^- \rightarrow W^+W^-(\gamma)$ and $e^+e^- \rightarrow t\bar{t}(\gamma)$ kinematic thresholds. Errors of less than 200 MeV on M_t and less than 10 MeV on M_W are possible. Furthermore, the properties of the Higgs boson, if its production is kinematically accessible, will be measured precisely.

7.2 Theoretical Developments

For reduced theoretical errors in the interpretation of radiative corrections, the complete set of two-loop electroweak corrections in $f\bar{f}$ production should be calculated. At LEP-II centre-of-mass energies, one also has to pay attention to QED radiative corrections, for example initial-final interference effects which are no longer suppressed. An ambitious goal is to calculate the complete one-loop electroweak radiative corrections in four-fermion production. The separation of two-fermion production including radiative corrections versus four-fermion production has to be addressed, which continues at the LC with the separation of four-fermion production including radiative corrections versus six-fermion production.

7.3 Future Constraints on the Standard Model

As far as the prediction of the Higgs-boson mass is concerned, the analysis of the correlation matrix shows, that the uncertainty on $\alpha_{em}(M_Z^2)$ should be reduced. For example, reducing the error on $\alpha_{em}(M_Z^2)$ by a factors of two to four as suggested by recent publications on this topic [235, 238–246] reduces the error on $\log M_H$ because of the large correlation between $\log M_H$ and $\alpha_{em}(M_Z^2)$.

Assuming the following uncertainties on Standard-Model parameters to be reached within the next few year:

$$\delta(1/\alpha_{em}(M_Z^2)) = 0.01 \quad (7.1)$$

$$\delta M_t = 2 \text{ GeV} \quad (7.2)$$

$$\delta M_W = 30 \text{ MeV} , \quad (7.3)$$

the relative error on the prediction of the Higgs-boson mass improves by a factor of 2.5, yielding a 35% uncertainty on M_H . The comparison of the current and the expected future $\Delta\chi_{\min}^2(M_H)$ curves is shown in Figure 7.1.

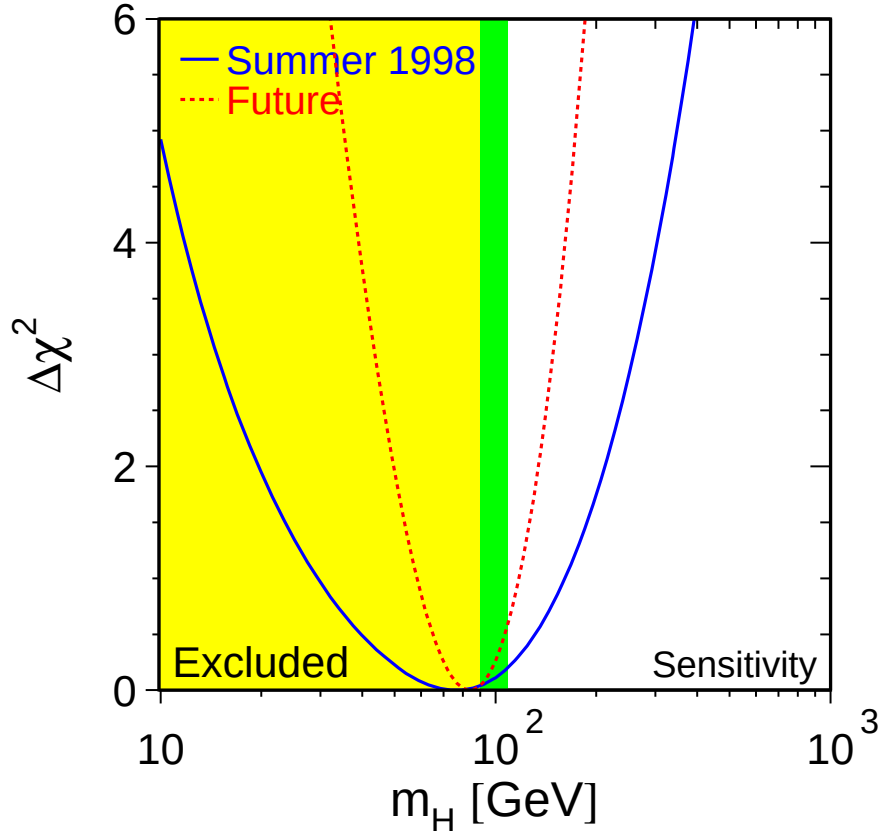


Figure 7.1: Current and expected future curves showing $\Delta\chi_{\min}^2(M_H) = \chi_{\min}^2(M_H) - \chi_{\min}^2$ as a function of M_H . The error on M_H is expected to improve by a factor of 2.5, allowing a prediction of the Higgs mass with an uncertainty of $\delta M_H/M_H = 30\%$. Of course, it is not yet known where the minimum of the χ^2 will be in future. The vertical bands show the region currently excluded at 95 % CL by the negative direct search for the Higgs boson at LEP-II, 90 GeV [51], and the region of the expected sensitivity of the direct search for the final LEP-II centre-of-mass energy of $\sqrt{s} = 200$ GeV, $M_H \rightarrow \sqrt{s} - M_Z = 109$ GeV. The LHC will cover the remaining mass range of a Standard Model Higgs boson.

7.4 What if the Higgs Boson is found?

As soon as the Higgs boson is found and its mass measured, the value of all Standard Model input parameters will be known. The accuracy of SM predictions will improve as the Higgs mass is no longer a free parameter. The Standard Model of electroweak interactions is further tested in the

fundamental sector of mass generation through the comparison of the directly measured Higgs mass with the prediction based on the analysis of radiative corrections.

Since the radiative corrections depend only logarithmically on M_H , the required accuracy on M_H is less compared to the required accuracy on M_t . For example, a 5 GeV error on M_t is equivalent to a 40% error on M_H . Once the Higgs boson is found, be it at LEP-II or LHC, its mass will quickly be known to sufficient precision as far as the calculation of radiative corrections within the Standard Model are concerned. The remaining parametric uncertainties on Standard Model calculations will thus arise due to the error on the running finestructure constant and the error on the mass of the top quark.

Chapter 8

Summary and Conclusion

The last hundred years have seen an exciting development in particle physics, starting with the discovery of radioactivity to the formulation of the theory of electroweak interactions to its successful experimental tests. Recent high-energy physics experiments perform measurements on the electroweak interaction which are so precise that the Standard Model of particle physics is tested at the level of its radiative corrections.

Currently the Standard Model is able to accommodate and explain the mostly still preliminary experimental results in a consistent manner, with the largest deviation observed in the $b\bar{b}$ asymmetries. In particular, the predictions of the mass of the top quark and of the W boson agree well with the direct measurements. From the experimental point of view there is no need to invoke new physics beyond the Standard Model.

The Higgs boson, the fundamental part of the electroweak theory, has not yet been observed directly, presumably because it is too heavy to be produced at a measurable rate at current colliders. Based on the analysis of radiative corrections within the framework of the Standard Model, its mass is predicted and an upper limit of 262 GeV at 95% CL is derived.

With the end of the SLC and LEP-I program there is no significant increase in the accuracy of the measurement of electroweak radiative corrections to be expected. In contrast the measurements of the mass of the W boson and of the top quark will continue to improve. An important quantity in Standard Model analyses is given by the running electromagnetic finestructure constant evaluated at the Z pole, which should be determined with much improved accuracy. This requires the measurement of the hadronic cross section in e^+e^- interactions at centre-of-mass energies between 1 GeV and 7 GeV.

The conceptual problems of the Standard Model as a theory require that it is part of a larger theory, if only to include a quantum mechanical description of gravity. New physics predicted by extended theories is searched for by looking for the production of new particles connected to the new physics. So far, no new particles are found. However, if new particles are too heavy to be produced directly, there is still the possibility to look for virtual effects of such particles changing radiative corrections. So far, the radiative corrections are in agreement with those predicted by the Minimal Standard Model. Extensions of the Standard Model are thus severely constrained.

The direct search for the production of the Higgs boson and new particles takes place at the colliders reaching the highest centre-of-mass energies, at the TEVATRON and LEP-II now, and at the LHC and LC in the future. The mass range which will be explored at the LHC is so large, that either the Higgs boson of the Standard Model or new particles predicted in extensions of the Standard Model must be found. The LC is the machine to perform precision measurements on heavy and new particles in order to determine their exact properties. Thus an exciting period of time in particle physics lies directly ahead.

Appendix A

The Fermi Constant

The Fermi constant G_F is best determined from the precisely measured lifetime of the muon. The decay of the muon into electron and two neutrinos is of purely electroweak nature. It is calculated based on the hypothesis of a universal V – A structure of the charged weak current [247] as follows:

$$\Gamma_e^\mu = \Gamma(\mu \rightarrow e \nu \nu(\gamma)) = \frac{G_F^2 m_\mu^5}{192 \pi^3} F_m(y_e) F_W F_{rad}, \quad (\text{A.1})$$

where the simple four-fermion term is corrected by:

1. a phase space factor $F_m(y_e)$ taking into account the finite mass m_e of the electron¹:

$$y_e = \frac{m_e^2}{m_\mu^2} = 2.339011 \cdot 10^{-5} \quad (\text{A.2})$$

$$F_m(y) = 1 - 8y + 8y^3 - y^4 - 12y^2 \ln y = 0.999813, \quad (\text{A.3})$$

2. a factor F_W for the correction due to W propagator effects:

$$F_W = 1 + \frac{3}{5} \frac{m_\mu^2}{m_W^2} = 1.000001, \quad (\text{A.4})$$

3. a factor F_{rad} expressing QED radiative corrections to first [38, 39] and second [40] order:

$$F_{rad} = 1 + \frac{\alpha(m_\mu)}{\pi} \left(\frac{25}{8} - \frac{\pi^2}{2} \right) + \left(\frac{\alpha(m_\mu)}{\pi} \right)^2 (6.701 \pm 0.002) \approx 0.99580 \quad (\text{A.5})$$

The numerical values above are calculated using world averages [31] for particle masses. The running coupling constant of QED at the muon mass is [40, 248]:

$$\alpha_{em}(m_\mu) = 1/135.90, \quad (\text{A.6})$$

incorporating virtual photon corrections as well as the emission of real photons and electron-positron pairs.

Based on the world average of the muon lifetime [31]:

$$\tau_\mu = \frac{1}{\Gamma_e^\mu} = 2.19703(4) \cdot 10^{-6} \text{ s}, \quad (\text{A.7})$$

the Fermi constant is determined to be [40]:

$$G_F = 1.16637(1) \cdot 10^{-5} \text{ GeV}^{-2}. \quad (\text{A.8})$$

¹In case of a massive muon neutrino and neglecting the electron mass, the correction is given by $F_m(y_{\nu_\mu})$.

Appendix B

The Electromagnetic Finestructure Constant

The finestructure constant at zero momentum transfer, $\alpha_{em}(0)$, is measured based on several methods. The most precise results are obtained from the measurement of the hyperfine splitting in muonic atoms, the quantum hall effect, the anomalous magnetic moment of the electron, and the gyromagnetic ratio of the proton combined with the AC Josephson effect.

The finestructure constant at zero momentum transfer is one of the fundamental physical constants for which the CODATA group provides best values. The last adjustment, performed in 1986 [249], yields for α_{em} :

$$\alpha_{em}(0) = 1/137.0359895(61). \quad (\text{B.1})$$

The next adjustment is planned for the end of 1998 [31].

For Standard Model calculations, the actual number of relevance is not $\alpha_{em} = \alpha_{em}(0)$ but rather the running electromagnetic finestructure constant, $\alpha_{em}(s)$, evaluated at the Z pole, $s = M_Z^2$:

$$\alpha_{em}(s) = \frac{\alpha_{em}(0)}{1 - \Delta\alpha_{em}(s)}. \quad (\text{B.2})$$

The running of α_{em} is caused by fermion loops in the propagator of the photon. The leptonic contributions are calculated to third order with negligible uncertainty [43]. The uncertainty on $\Delta\alpha_{em}(s)$ arises from the contribution of the five light quark flavours, $\Delta\alpha^{(\text{had})}(s)$. It is determined via a dispersion integral:

$$\Delta\alpha_{em}^{(\text{had})}(s) = -\frac{\alpha_{em}}{3\pi} \cdot s \cdot \Re \left[\int_{4m_\pi^2}^{\infty} ds' \frac{R(s')}{s'(s' - s - i\epsilon)} \right] \quad (\text{B.3})$$

$$R(s) = \frac{\sigma^0(e^+e^- \rightarrow \gamma^* \rightarrow \text{had}; s)}{\sigma^0(e^+e^- \rightarrow \gamma^* \rightarrow \mu^+\mu^-; s)}, \quad (\text{B.4})$$

where σ^0 denotes lowest order expressions derived from the experimental measurements by removing effects due to QED radiation and the running of $\alpha_{em}(s)$ [44]. The experimental errors on the hadronic cross sections at centre-of-mass energies between 1 GeV and 7 GeV dominate the final error on $\Delta\alpha_{em}^{(\text{had})}(M_Z^2)$ [44]:

$$\Delta\alpha_{em}^{(\text{had})}(s) = 0.02804 \pm 0.00065. \quad (\text{B.5})$$

Recently, many reevaluations of $\Delta\alpha_{em}^{(\text{had})}(M_Z^2)$ [235, 241–246] have been performed applying perturbative QCD down to low centre-of-mass energies for an improved normalisation or even replacement of the experimental cross section measurements in the continuum region. This approach is grounded in

the successful application of perturbative QCD in hadronic τ decays and indicated in Figure B.1 [235], although some potential systematic effects are not considered [250]. A compilation of the results is shown in Figure B.2 [246]. New and more precise measurements of cross sections in the above mentioned energy range are needed to confirm the theoretical ansatz. Such measurements are currently under way, for example by the BES collaboration at the e^+e^- collider BEPC in Beijing. Preliminary results at centre-of-mass energies of 2.60 GeV and 3.55 GeV show slightly lower results for R than the older measurements [251], in better agreement with the expectation of perturbative QCD.

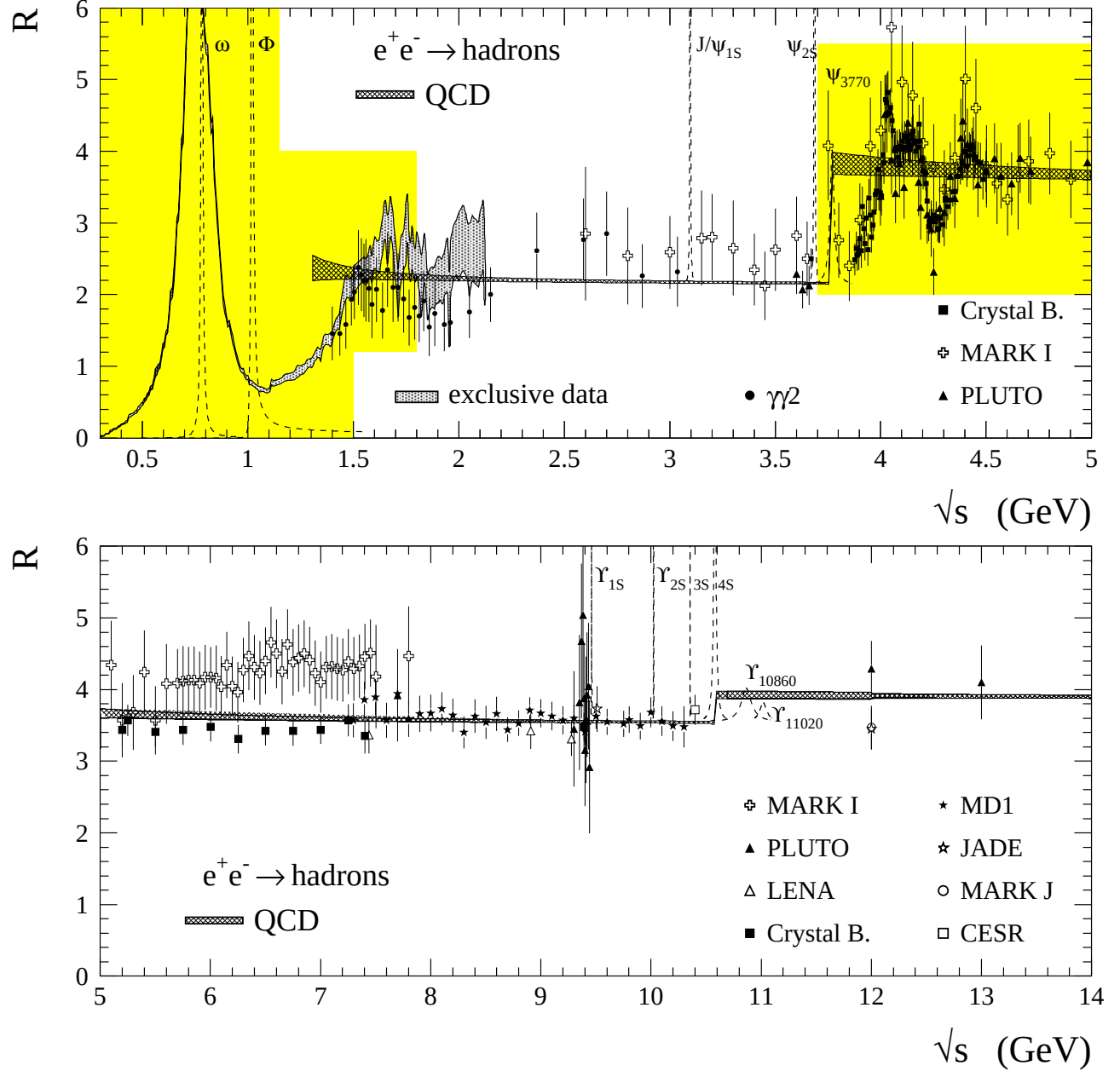


Figure B.1: Measurements of the ratio of the hadronic cross section to the pointlike $e^+e^- \rightarrow \mu^+\mu^-$ cross section at low centre-of-mass energies $\sqrt{s}$ [235]. Also shown is the QCD prediction for the continuum contribution. In addition to the narrow resonances, the experimental data is also used in the shaded regions for the evaluation of the dispersion integral.

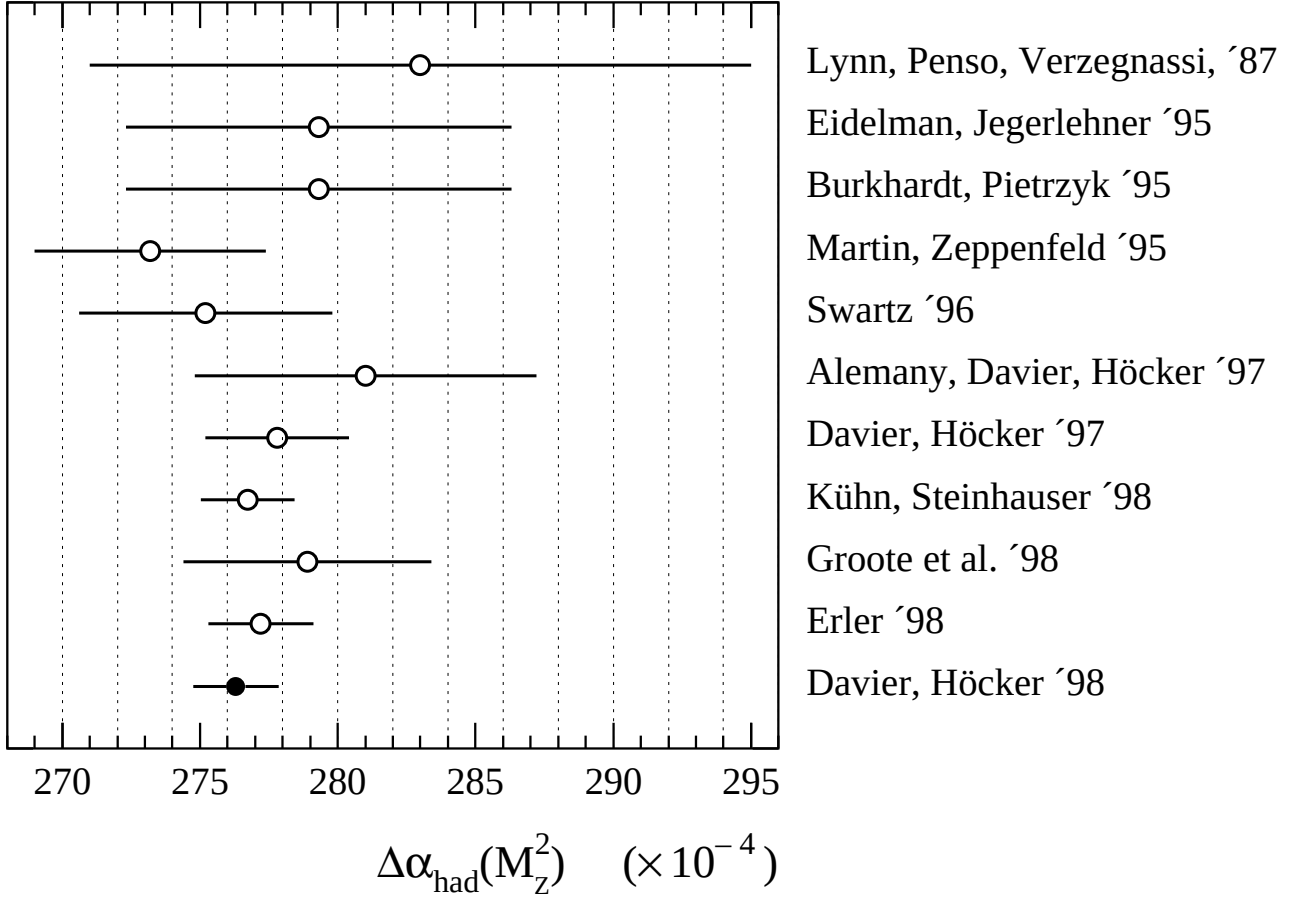


Figure B.2: Determinations of the hadronic contribution to $\Delta\alpha_{em}(M_Z^2)$ arising from the five light quarks and the top quark, $\Delta\alpha_{em}^{(6)}(M_Z^2)$ [246]. The results referred to in the plot are published in [252], [44], [45], [238], [239], [241], [235], [243], [244], [245], and [246].

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