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RELATIVISTIC PARTON INTERPRETATION OF THE NEVEU-SCHWARTZ MODEL*

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ABSTRACT

The Lorentz generators for the Neveu-Schwartz theory, recently obtained by Iwasaki and Kikkawa from a zweibein formalism, are here derived with an emphasis on the parton interpretation of the results. This derivation sheds new light on possible ways to construct new dual models.

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The geometric action principle for the dynamics of the dual resonance model[1] has enormously advanced our understanding of the hitherto abstract Nambu-Susskind string. Starting from this principle, Chang and Mansouri have demonstrated[2] that one must view the string and its attendant modes of vibration as occurring in the physical Minkowski space. This approach, coupled with the Brower no-ghost theorem,[3] provides a manifestly covariant description of string dynamics. In addition, these authors[2] observe that the choice of a parametrization for the string and its world sheet such that Lorentz invariance can be made manifest amounts to choosing a particular coordinate gauge.

Different choices of gauge are possible, and Goddard et al. (GGRT)[4] have discussed the case when all potentially trouble-making timelike oscillations are eliminated at the outset. This is analogous to choosing Coulomb as opposed to Lorentz gauge in electrodynamics. Unlike electrodynamics, however, their choice simultaneously corresponds to null-plane quantization instead of ordinary equal-time quantization. It is these two features taken together that make their results extraordinarily interesting from the point of view of finding a bridge between the formal geometric action principle, and the rich body of investigations which attempt to establish a connection between the dual model and parton physics.[5,6]

Recently, the Lorentz invariance of the Neveu-Schwartz (NS) model[7] has been discussed by Iwasaki and Kikkawa,[8] who extend the geometric formalism through the use of "zweibein" fields in order to incorporate the spin degrees of freedom of that model. Because the NS model provides a particularly beautiful example of how the string picture realizes the dynamics of the wee parton sea, it is interesting to rederive the Lorentz generators from a parton point of view.

Following Bjorken,[5] let us first examine the conventional orbital dual model from the point of view of a parton enthusiast: in a near forward hadron-hadron collision, scattering is to be mediated by those partons which have wee fractions of their parent hadron's longitudinal momentum. Suppose that the wee sea is formed (in terms of time-ordered perturbation graphs) by a cascade down from the leading parton. Multiperipheral work indicates that the dominant graphs are those for which the partons are ordered sequentially in rapidity. Further, if these graphs are calculated for ϕ^3 theory in the infinite momentum frame (or using the null-plane quantized theory) an interesting qualitative picture emerges — near neighbor partons in rapidity are also close together in transverse configuration space. It is not implausible, therefore, that rescattering corrections to the basic parton picture involve repeated soft interactions between near neighbors in rapidity, with the basic dynamical variables involved being the distances in transverse configuration space between the interacting partons. One may attempt to describe the evolution of the wee sea by means of an effective Hamiltonian which is a function of the partons' relative transverse momenta, labelled by an ordered parameter corresponding to the partons' rapidity. With the simplest possible dynamical assumption, and a judiciously chosen density of partons along the rapidity axis, the harmonic oscillator Hamiltonian can be obtained as an example in the continuum limit,

$$H = \frac{1}{2\pi} \int_0^{\pi} \left[\left(\frac{\partial x}{\partial \tau}\right)^2 + \left(\frac{\partial x}{\partial \theta}\right)^2 \right] + \text{const.}, \qquad (1)$$

where θ labels the parton, and $x_\perp(\theta)$ is the transverse coordinate of the parton at position θ.

Thus, the semi-quantitative Bjorken argument provides us with the natural variables in terms of which wee sea dynamics might conveniently be discussed.

This is the result of the calculation the dualist desires to abstract. However, it is not obvious from Eq. (1) that the theory so constructed will be relativistically invariant. It is at this point that the string formalism makes its contribution. Equation (1) is just the $(\text{mass})^2$ operator of the GGRT parametrization, with the same interpretation of the transverse coordinate, and of the string labelling parameter, as has been made in Bjorken's parton model. The work of GGRT assures us an almost invariant theory can be constructed (up to a tachyon), but it is of independent interest to ask how one might have proceeded to construct a Lorentz invariant theory without making use of the geometrical formalism.

Fortunately, methods for attacking this type of problem have been discussed in detail by a number of authors.[9] Indeed, Ramond[10] has recently discussed the construction of the orbital dual model as an algebraic problem, based on these methods. In spite of the fact this formalism is not widely in use, it is in keeping with the scope of this paper to simply state its main features, along with nonrigorous plausibility arguments for why it works.

The BH construction[9] is a way of building a null-plane quantized theory of particle "dynamics in front form."[11] To understand what all this means, we remind the reader of the Bjorken, Kogut, Soper representation[12] of the null-plane quantized Dirac field

$$\psi_+(x) = \sum_{\lambda=\pm 1/2} \int \frac{d\vec{p}_\perp}{(2\pi)^3} \int_0^\infty \frac{d\eta}{2\eta} \Big[b(\vec{p}_\perp, \eta; \lambda)\, \sqrt{2\eta}\, e^{-ipx} \omega(\lambda)$$

$$+\, d^+(\vec{p}_\perp, \eta; \lambda)\, \sqrt{2\eta}\, e^{ipx}\, \omega(-\lambda) \Big] , \tag{2}$$

where $\omega(1/2) = \binom{1}{0}$, and $\omega(-1/2) = \binom{0}{1}$. First quantized versions of the Lorentz generators for the free Dirac theory may be written as follows

$$\mathscr{P}_\perp = -i\partial_\perp ; \tag{3a}$$

$$\mathscr{P}^{+} = \eta \ ; \tag{3b}$$

$$\mathscr{P}^{-} \equiv H = \mathscr{P}_{\perp}^{2} + m^{2}/2\eta \ ; \tag{3c}$$

$$K_3 = \frac{1}{2}\left\{\eta, \partial_\eta\right\} \ ; \tag{3d}$$

$$J_3 = \epsilon_{ab}\ x_a\ p_b + \sigma_3/2 \ ; \tag{3e}$$

$$B_\perp = \eta x_\perp \ ; \tag{3f}$$

$$S_k = \frac{1}{2}\left\{x_k, H\right\} - \frac{1}{2}\left\{\frac{1}{\eta}, K_3\right\}\mathscr{P}_k + \frac{\epsilon_{km}}{2\eta}\left[\frac{\sigma_3}{2}\ P_m - m^2\ \frac{\sigma_m}{2}\right]. \tag{3g}$$

For any number n of free particles, the generators acting on the n-particle wave function are the sum of the generators of the individual particles. For two particles, e.g., one has

$$H_{1+2} = \frac{\vec{p}_1^{\,2} + m^2}{2\eta_1} + \frac{\vec{p}_2^{\,2} + m^2}{2\eta_2} = \frac{\vec{P}^2}{2M} + \frac{\vec{\pi}^2 + m^2}{2\mu} , \tag{4}$$

where we have passed to standard center-of-mass (CM) and relative variables.

There is interaction if H, the generator of displacements in $\tau = x^+$, has an additional nonconstant term on the right-hand side of Eq. (4). Closure of the Poincare algebra imposes requirements on the form such an additional term may take. It also demands the other generators which take the system off a plane τ = const., $S_\perp$, must also be modified to take account of the interaction. This is the nontrivial aspect of the problem.

A mnemonic which paraphrases the results of BH is that if we write the Hamiltonian with interaction in the form

$$H = H_{1+2} + V_{12} \equiv \frac{\vec{\mathscr{P}}^2 + \mathscr{M}^2}{2M} ,$$

$$\mathscr{M}^2 = 2HM - \vec{\mathscr{P}}^2 = 2MV_{12} + \frac{M}{\mu}\left(\vec{\pi}^2 + m^2\right) \tag{5}$$

then the desired generators $S_\perp$ are exactly of the form Eq. (3g), with the following substitutions

$$m^2 \text{ (parameter)} \rightarrow \mathscr{M}^2 \text{ (operator)} ; \tag{6a}$$

$$\sigma^i/2 \text{ (Pauli spin)} \rightarrow j^i \text{ (operator)} ; \tag{6b}$$

$$\eta \rightarrow M; \text{ and the } x_\perp, \ \mathscr{P}_\perp \text{ are CM quantities.} \tag{6c}$$

The idea is that all algebraic relations are to be maintained in making these replacements. Thus, one requires that $\mathscr{M}^2$ be a rotational scalar under the three j^i, which satisfy an SU(2); in addition, $\mathscr{M}^2$ and j^i must commute with all CM variables, and be Galilei invariant.

The dual model differs radically from this description because there are (in the continuum limit) an infinite number of constituents, not merely two. This is at once a blessing and a curse. The two-dimensional harmonic oscillator has the peculiar property[9] of having its energy levels alternate between integer and half-interger spin with respect to the SU(2) we can form from the degrees of freedom at our disposal. The blessing is that this unphysical result is avoided in the continuum limit. Careful attention to Eq. (3g) reveals that for the $S_\perp$ to have the proper algebra, all that is really required is that a two-vector under j^3 be found, whose components have the same commutator as $(\sqrt{\mathscr{M}^2}\, j^\perp)$,

$$\left[T^1, T^2\right] = i\,\mathscr{M}^2\, T^3 ; \tag{7a}$$

$$\left[T^3, T^i\right] = i\,\epsilon^{ij}\, T^j . \tag{7b}$$

The dual model (almost) realizes this algebra, with T^3 containing only integers in its spectrum. No such operators can be found for the two particle case.

(A second part of this continuum limit blessing is that the operator $\sqrt{\mathscr{M}^2}$ never appears.[13])

The curse accompanying this construction is that by effectively lumping $(\sqrt{\mathscr{M}^2}\, j^i)$ into a single operator, severe singularities have been introduced. These rear their head when proper attention is paid to Schwinger terms when evaluating the commutator Eq. (7a) — a tachyon is required in the spectrum for the dual model.

To see this, we explicitly construct the operators T^i of the orbital dual model. It is evident from Eq. (5) that the $\mathscr{M}^2$-operator is in fact the Hamiltonian of the system in a well-defined frame in which $(P_\perp)_{cm} = 0$. Let us derive this Hamiltonian (for the internal motions) from a canonical formalism, with

$$\mathscr{L}_b = \frac{1}{4\pi}\, \partial_\alpha x^i \partial^\alpha x_i\,; \qquad (i = 1,\ 2 \text{ corresponding to } x_\perp); \tag{8}$$

$$\theta_b^{\mu\nu} = (\partial^\mu x^i)\, \frac{\delta \mathscr{L}}{\delta(\partial_\nu x^i)} - g^{\mu\nu}\mathscr{L}_b\,. \tag{9}$$

(Here $\alpha = 0,\ 1$ refers to τ, conjugate of $\mathscr{M}^2$; and θ.) The Lorentz index "i" labels an external symmetry (like isospin) as far as the two dimensional $(\tau,\ \theta)$ system is concerned. The contraction in this index indicates $\mathscr{L}_b$ is scalar under this symmetry, and the corresponding charge can be computed,

$$T_b^3 = \frac{1}{\pi} : \int d\theta\ \epsilon_{ij}\, x^i\, \partial_\tau x^j : \tag{10}$$

Utilizing the canonical commutation relations

$$[x^i(\theta,\tau),\ \partial_\tau x^j(\theta',\tau)] = i\,\delta(\theta - \theta')\,\delta^{ij} \tag{11}$$

one verifies that "i" labels a two vector under T_b^3.

The remaining task is to construct reasonable scalar operators which can be used together with the two-vectors to form the desired $T_b^{\perp}$. Recall $\mathcal{M}^2$ itself should commute with T_b^{i}. This is a very stringent requirement. If we wish to form local bilinear scalars, we find no solution of Eq. (7) is possible. One must utilize a non-local form (as can be seen for dimensional reasons),

$$T_b^{\perp} = \frac{1}{2} \int\int d\theta \, d\theta' \, S(\theta - \theta') \, \epsilon^{\alpha\beta} \left\{ \partial^{\alpha} \chi^{\perp}(\theta'), \, \theta^{o\beta}(\theta) \right\} \tag{12}$$

where S is the step function. Conservation of this quantity follows at once from $\theta^{\alpha}_{\alpha} = 0$; $\theta^{\mu\nu} = \theta^{\nu\mu}$; $\partial_{\mu} \theta^{\mu\nu} = 0$; and the equations of motion.

The algebra Eq. (7) would follow from Eq. (10), (11) and (12) using naive manipulations. Unfortunately, the anomalous Schwinger term due to[14]

$$< [\theta_b^{oo}(\theta), \, \theta_b^{o1}(\theta')] >_0 = \frac{1}{\pi^2} \sum_{n, m > 0} n \, m \cos(n+m)\, \theta \, \sin(n+m)\, \theta' \tag{13}$$

generates an operator Schwinger term in Eq. (7). The coefficient of this term can be made to vanish only if our transverse oscillators have i = 1 to 24, and if $H = \frac{\mathcal{M}^2}{2M} \to \frac{\mathcal{M}^2 - 1}{2M}$. These are the well-known catastrophes of the string model.

Armed with this experience from the orbital dual model, we are ready to study the NS model. To motivate this model in the parton spirit, recall that in Eqs. (2) and (3), the Dirac field was very conveniently parametrized in terms of two component quantities. There is no approximation involved in this. We now wish to "abstract" from the Bjorken model that, in addition to transverse coordinates and momenta, it is legitimate to include the spin matrices σ^{i} among the possible dynamical variables upon which near-neighbor parton interactions can depend.

Now, examination of spin 1/2 null-plane electrodynamics in the infinite momentum gauge[12] reveals that only $\sigma^{\perp}$ appear in elementary vertices, never σ^3. If this can be stretched to the status of a guide, it may not be unreasonable to take only $\sigma^{\perp}(\theta)$ as our dynamical variables. (To obtain the NS model, this must be done; we postpone further comment on the point to the end.)

In any case, if only $\sigma^{\perp}$ are considered, and if only nearest-neighbor couplings are permitted, a soluble model is obtained.[15] Furthermore, the continuum limit of this theory is, remarkably, the two-dimensional massless Dirac Equation.[16] The energy of the interacting spin system must be appended onto that of the orbital excitations, so our Lagrangian from which $\mathscr{M}^2$ is to be derived is now

$$\mathscr{L} = \mathscr{L}_b + \frac{i}{2} \bar{\chi} \gamma^{\alpha} \overleftrightarrow{\partial}_{\alpha} \chi \quad ; \tag{14}$$

$$\theta^{\mu\nu} = \theta_b^{\mu\nu} + \frac{i}{4} \left(\bar{\chi} \gamma^{\mu} \overleftrightarrow{\partial}^{\nu} \chi + \bar{\chi} \gamma^{\nu} \overleftrightarrow{\partial}^{\mu} \chi \right); \tag{15}$$

where

$$\gamma^o = \sigma^1 \; ; \qquad \gamma^1 = i\sigma^3; \qquad \chi = \begin{pmatrix} \psi \\ \phi \end{pmatrix} \; ; \qquad \bar{\chi} = \chi^{+} \gamma^{o} .$$

When Eq. (15) is used in Eq. (12), expected mixed bose-fermi terms are obtained.[8] A further complication in evaluating the algebra of the $T^{\perp}$ is encountered because of the anomalous terms in the $\theta_f^{\mu\nu} = \theta^{\mu\nu} - \theta_b^{\mu\nu}$ commutators. First, the naive result

$$\iint d\theta \, d\theta' \, f(\theta) \, g(\theta') \quad [\theta_f^{oo}(\theta), \, \theta_f^{o1}(\theta')] = i \int d\theta (f'g - g'f) \, \theta_f^{oo}(\theta) \tag{16}$$

must be modified if f and g are noncommuting operators. One must use

$$\left[\theta_f^{oo}(\theta),\ \theta_f^{o1}(\theta')\right] \overset{\text{naive}}{=} -\frac{i}{4}\left[\left\{\phi^+(\theta)\frac{\partial\psi}{\partial\theta'}\frac{\partial\,\delta(\theta-\theta')}{\partial\theta'} - \phi^+(\theta)\psi(\theta')\frac{\partial^2\delta(\theta-\theta')}{\partial\theta\,\partial\theta'} - \frac{\partial\phi^+}{\partial\theta}\frac{\partial\psi}{\partial\theta'}\delta(\theta-\theta') + \frac{\partial\phi^+}{\partial\theta}\psi(\theta')\frac{\partial\,\delta(\theta-\theta')}{\partial\theta'}\right\} - \left\{(\theta\leftrightarrow\theta')\right\}\right] + \text{H.c.} \tag{17}$$

In addition, one has the Schwinger term due to

$$<[\theta_f^{oo}(\theta),\ \theta_f^{o1}(\theta')]>_0 = \frac{1}{2\pi^2 i}\sum_{n,m>0}(m-n)(m+n-1)\cos(m-n)\,\theta\sin(m+n-1)\theta' \tag{18}$$

(The form of this term depends on the boundary conditions[16] chosen for ψ and ϕ.)

Now, referring back to the interpretation of the dynamics of the NS model in terms of parton spin, it is clear that the conserved charge

$$T_f^3 = \frac{1}{\pi}\int_0^\pi d\theta\ :\ \bar{\chi}\,\gamma^o\chi\ :, \tag{19}$$

is actually the helicity carried by the spin degrees of freedom.[16] Making use of the canonical commutation relations

$$\left\{\chi(\theta,\tau),\quad \chi^+(\theta',\tau)\right\} = \delta(\theta-\theta'), \tag{20}$$

one observes that ψ, ϕ transform like $(\sigma^1 - i\sigma^2)$; and ψ^+, ϕ^+ transform like $(\sigma^1 + i\sigma^2)$ under T_f^3. Since the theory conserves charge, the fermionic energy-momentum tensor Eq. (15) is a scalar under T_f^3, Eq. (19). However, there are new possible structures for $T^\perp$ based upon the "vector" character of χ and χ^+ under this new T_f^3.

Once again, conservation plays a very important role in deciding the form of the possible new candidates for $T^\perp$. Uniquely, a new conserved two-vector not available previously is

$$T_f^{\perp} = \int d\theta \, j^{\alpha}(\theta) \, g^{\alpha\beta} \, \partial_{\beta} X^{\perp}(\theta), \tag{21}$$

where $\qquad j^{\alpha} \equiv : \bar{\chi} \gamma^{\alpha} \chi : .$

Other candidates are either not conserved, vanish identically, or reduce eventually to Eq. (21). It is worth noting that the Schwinger term due to

$$< [j^{0}(\theta, \tau), j^{1}(\theta', \tau)]>_0 = - \frac{2i}{\pi^2} \sum_{n,m>0} \cos(n+m-1)\,\theta \sin(n+m-1)\,\theta' \tag{22}$$

would give rise to problems with the two-vector character of $T_f^{\perp}$, Eq. (21), were it not for the boundary condition $(\partial X^{\perp}/\partial\theta)_{0,\pi} = 0$. The conservation of $T_f^{\perp}$, on the other hand, depends only on the equations of motion of $x^{\perp}$, and on conservation of both vector and axial fermion currents.

The operators to use in the algebra Eq. (7a) may then be chosen to be

$$\Omega^{\perp} = T_b^i + \alpha T_f^i , \tag{23}$$

where now $\theta^{\mu\nu}$ in Eq. (12) defining T_b^i has both Fermi and Bose parts. The requirement that Eq. (7a) be satisfied imposes the condition that $\alpha = (1/\sqrt{2\pi})$ to eliminate mixed Bose-Fermi pieces[17] from the commutator. The usual NS model results that the spectrum include a tachyon at $m_0^2 = -1/2$, and that the dimensions of all transverse operators be extended to 8, then follow. The details of this calculation are tedious, and are adequately presented in Ref. 8. Note that we <u>must</u> have the T_f^i piece.

We conclude with the following comments:

(1) The derivation of the generators based on the BH construction is self-contained. The results are in agreement with the geometrical formalism for both orbital and NS models, as they must be. The geometric formalism, because of its elegance and compactness in terms of an action, makes introduction of interaction straightforward in principle. However, it may prove useful to take advantage of the greater freedom of the BH method to construct new "free" dual models based on parton intuition.

(2) The NS model illustrates the naive Heisenberg spin-spin interaction model can be applied straightforwardly to a hadron system, provided we quantize on a null-plane.[18] In this quantization, the Dirac field is adequately described by two-component spinors, with no approximation involved. Thus it is not necessary to consider four-dimensional $[\gamma^{\mu}(i)\, \gamma_{\mu}(i+1)]$ interactions in order to show the NS model is relativistic.

(3) For the orbital part of the model, one must start from scratch to include dynamics in the longitudinal direction. The reason for this is that the longitudinal fraction cannot at once serve as a label for the transverse degrees of freedom, and as an independent dynamical variable.

However, for the spin degrees of freedom this problem does not arise. Indeed, since $\sigma_z = [\sigma_+, \sigma_-]$, a model with near neighbor σ_z coupling (in addition to the $\sigma_\perp$ coupling) resembles a Thirring model. The interaction between the transverse modes may give rise to new independent (bound-state) longitudinal modes. This is currently under investigation.

(4) As Ramond has stressed,[10] it is possible to treat the construction of new dual models as the purely algebraic problem of finding dynamical variables that realize the algebra Eq. (7). The method employed in this note, making use of

two-dimensional field theory, is, unfortunately, limited to models with $\theta^{\alpha}{}_{\alpha} = 0$. Other methods probably exist.

However, it may be more profitable to examine four-dimensional field theories less trivial than ϕ^3, to discover what the proper combinations of parton variables to use in the description of the hadron wavefunction actually are. Particularly intriguing is the question of how such a wavefunction would remember it was born from gauge invariant dynamics, which nontrivially couple transverse and longitudinal variables. (See, e.g., Ref. 12).

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