QCD splitting-function dependence on evolution variable

S. Jadach[1], A. Kusina[2], W. Placzek[3] and M. Skrzypek[1]

[1] Institute of Nuclear Physics, Polish Academy of Sciences,
ul. Radzikowskiego 152, 31-342 Kraków, Poland
[2] Laboratoire de Physique Subatomique et de Cosmologie
53 Rue des Martyrs Grenoble, France
[3] Marian Smoluchowski Institute of Physics, Jagiellonian University,
ul. Łojasiewicza 11, 30-348 Kraków, Poland

Abstract: In this note we present the results of the study performed at IFJ PAN in Krakow in the context of the parton-shower-related research which suggests that QCD splitting function depends on the type of physical evolution variable. We sketch a method of including NLO corrections into a cascade (ladder) of a parton shower (PS). The method is based on the Curci-Furmanski-Petronzio classical paper and matching between PS and NLO distributions obtained from the Feynman diagrams. We note that one does not reproduce the classical CFP NLO kernels if the k_T ordering is used for the calculation. This seems to be a limitation of the definition of the CFP procedure and requires further investigation.

Parton-Shower-related activities at IFJ PAN

Let us give a short review of the results of the group at IFJ PAN related to the parton shower (PS) research.

(A) In the first step we developed a series of works on evolving parton distribution functions (PDFs) using Markovian Monte Carlo (MMC) programs. We solved exactly the LO and NLO DGLAP evolution equations using the MMC methods with long term aim of developing new types of PS MCs. a) The first MMCs were evolving quark and gluon PDFs according to the LO and NLO DGLAP iequations n $t = \ln \mu^2$ variable, with the running $\alpha_s(t)$ [1], [2]. b) Next we developed the MMC for CCFM (1-loop) gluonstrahlung with $t = \ln(k_T/(1-z))$ and $\alpha_s(k_T^2)$ [3]. c) Finally, the most sophisticated MMCs, with various choices of t and an argument of α_s, combining the full DGLAP and CCFM, together with the numerical results, were reported in [4]. d) A corresponding MMC program in C++ implementing variety of the above models was published in [5]. The PDFs $q(t,x)$ and $g(t,x)$ from MMCs agreed (within 3-digits) with non-MC calculations of the QCDnum16 and APCheb programs. e) Let us mention also a more recent study of the MCFM modelling exploiting our MMC done in Ref. [6].

(B) MMCs were designed to test numerically series of constrained MC (CMC) programs, implemented with the same evolution, but with a constrained predefined final x variable (an alternative to the backward evolution in the PS MC) aiming at strict (NLO level) control of the distributions of LO PS MC. a) We began with CMC and MMC modules for single ladder/shower, without a hard process, with exclusive LO kernels, optionally inclusive NLO kernels. CMC programs tested with MMCs were published in [7], [8]. b) Next, we combined two CMCs with a hard process matrix element (ME) into PSMC for Drell-Yan (DY) process

[9], [10]. Such an academic PS MC was instrumental in testing new ideas on the NLO corrections in the exclusive evolution included in all kernels in the initial state ladders/showers, and on the NLO corrections to the hard process (a simpler alternative to MC@NLO [11] and POWHEG [12] approaches) thanks to a perfect numerical and algebraical control over LO distributions.

(C) The next project concerns the NLO corrections in an exclusive form to PS MC (one ladder): a) The first solution, albeit limited to the non-singlet evolution, was proposed and tested numerically in [13], [14]. b) The exclusive NLO kernels had to be recalculated in the Curci-Furmanski-Petronzio (CFP) [15] framework from scratch. The non-singlet 2-real kernels were recalculated in [16] and the non-singlet 1-real-1-virtual ones in the PhD thesis [17]. Also the singlet evolution kernels are now almost complete (unpublished). c) Simplified and faster scheme was reported (numerical tests) in [18]. d) An even simpler and faster scheme of NLO-correcting was presented in [19]. e) A major problem was how to include consistently virtual corrections. The first solution has been formulated but not yet published. It exploits recalculated virtual corrections in the CFP scheme to the non-singlet kernels [20], [17].

D) Finally, the last project is the implementation of the NLO corrections to the hard process in a new way, named KrkNLO method. It provides a simpler alternative to MC@NLO and POWHEG methods: a) Methodology for DY process was defined (without numerical tests) in [21]. b) Numerical tests of KrkNLO with the help of Double-CMC PS was done in [10]. c) Further improvements and introduction of PDFs in the MC factorisation scheme can be found in [22], where the the MC implementation is still done with the help of the not-so realistic Double-CMC PS. d) Next, the KrkNLO method was implemented on top of the SHERPA [23] parton shower. Comparisons for the DY process with the fixed-order NLO calculations of MCFM [24] as well as with the NLO-PS matched calculations of MC@NLO and POWHEG were also done in [25] and in the conference note [26]. e) A complete definition of a new scheme for PDFs, called the MC scheme, necessary for the KrkNLO method, both for the DY processes and Higgs-boson production via gluon–gluon fusion was presented in [27] and in the conference note [28]. f) At last, the implementation of the KrkNLO method in the Herwig 7 MC generator [29] and a series of comparisons for Higgs production in gluon–gluon fusion with the programs: MCFM, MC@NLO, POWHEG and HNNLO [30] as well as with experimental data of the ATLAS Collaboration from the LHC Run 1 was published in [31].

In this note we briefly present the findings of Ref. [32]. This is however, a rather technical paper, so let us first explain the background and then summarise on the main results of Ref. [32]. Generally, the context is that of the N+NLO parton shower, with the NLO evolution kernels (PDFs) in the shower (ladder) and the NNLO hard process. As listed earlier, we have already defined and tested some important elements of the complete N+NLO parton shower.

What is NLO Parton Shower?

Let us start with a simpler question: What is LO PS? It is built using the LO-class evolution kernels and the LO PDFs within a certain collinear factorisation scheme. LO PS MC implements the LO DGLAP evolution for semi-inclusive distributions (structure functions, etc.). If the hard process is corrected to the NLO level (N+LO), LO PS MC desirably encapsulates all collinear/soft singularities of the NLO hard process ME in the exclusive form. In the N+LO schemes *one* parton originally generated within LO PS MC gets promoted to the hard process.

Let us now get back to the NLO PS. It is built using the NLO-class evolution kernels and the NLO PDFs, and it implements the NLO DGLAP evolution for certain semi-inclusive distributions (structure functions etc). For the hard process corrected to the NNLO level (N+NLO), it is desirable that NLO PS MC encapsulates all collinear/soft singularities of the NNLO hard process ME in the exclusive form. In the N+NLO scheme up to *two* partons originally generated within the NLO PS MC get promoted to the hard process.

Introducing the complete NLO real and virtual corrections into PS MC in the exclusive form, in accordance with the collinear factorization theorem, is the main problem, theoretically and practically. To this end we proposed a theoretical framework based on the CFP [15] approach. We have formulated and tested three methods of practical implementation of the NLO corrections in the PS MC. One of them turned out to be quite promising.

Consistent matching of NLO PS with CFP is highly nontrivial. The procedure goes like this. One writes PS MC in $d = 4 + 2\epsilon$ space-time dimensions truncated to the 2nd order and applies to it the CFP subtraction procedure. In this step the following problem appears: PS MC has a Sudakov virtual correction with non-pole terms, while CFP has a wave-function renormalisation factor with pure poles. Therefore subtraction has to be done separately for the form factor and for the rest. Once PS MC distributions are processed through the CFP procedure, dummy functions inserted in the PS MC distributions are identified with their counterparts in the CFP distributions representing the complete NLO. There are two requirements of the procedure: 1) Both distributions, that of PS MC and that of the CFP calculations, have to be defined within *the same phase-space* limits. 2) PS MC has to implement exactly all collinear and soft limits of the NLO matrix element.

While constructing NLO PS we had to recalculate the NLO kernels in the exclusive form. We have done this in the CFP scheme, diagrammatically, in the axial gauge, both for real [16] and virtual [17] corrections. In the course of the calculation a technical improvement related to the use of principal value (PV) prescription was proposed [33].

Problem with evolution variable

As sketched earlier, we successfully realised the matching of NLO PS with CFP for bremsstrahlung-type diagrams (C_F^2-type), shown in the right-hand-side plots of Fig. 1. We used various types of the evolution variable: k_T, $k_T/(1-z) \sim \theta$, $k^- = k_T/\sqrt{1-z}$. The situation changed when we included the graphs Vg and Vf, shown in the left-hand-side plots of Fig. 1. We encountered a problem when

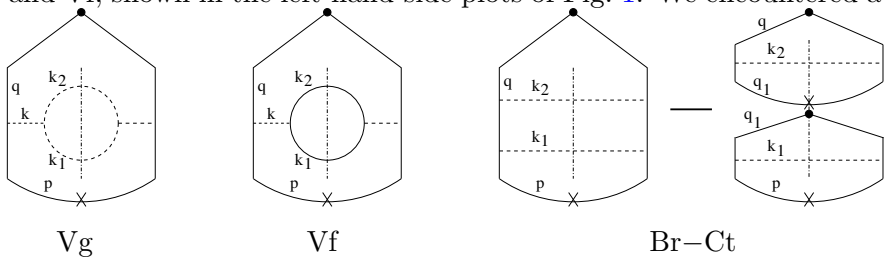

Figure 1: Real graphs with double poles contributing to the NLO non-singlet P_{qq} kernel. The solid lines represent quarks and the dotted lines stand for gluons. Left plots: the FSR-type graphs, right plots: the gluonstrahlung-type graphs.

applying the k_T-ordering to the calculation of the Vg graph of Fig. 1. Namely, we found that in the CFP-based analytical calculation, with phase space as in PS MC and with k_T-ordering, the

contribution of this graph to the NLO kernel is different than in the original CFP paper. This is, of course, rather a problem of CFP, not of PS MC, as according to the theorem proven in the CFP paper the dependence of the MS NLO kernel on the choice of the upper kinematic boundary Q is forbidden. The problem is absent for the gluonstrahlung (non-singlet) kernels shown in the right-hand-side plots of Fig. 1 as well as for all single-pole diagrams. Next, we found that the choice of the type of the "evolution time", i.e. the Q-variable, is essential: for $Q =$ virtuality, $Q = |\vec{k}_{T1} + \vec{k}_{T2}|$ and $Q = |\vec{k}_{T1}| + |\vec{k}_{T2}|$ the problem is absent. It shows up only for the diagrams with the final-state radiation (FSR) gluon splitting into a gluon pair calculated for $Q = \max(k_{T1}, k_{T2})$, combined with the relevant virtual diagram counter-partner. The analytical calculation shows that source of the effect is a mismatch between the upper kinematic limit in the virtual and real diagram contributions. Such a mismatch is usually absent, except for the $Q = \max(k_{T1}, k_{T2})$ case.

Let us discuss a list of possible solutions of this problem: A) The simplest explanation would be that the CFP scheme is not as general as expected, though it is rather unlikely. B) Is it then due to partial introduction of the dimensional regularisation in CFP, only for the collinear singularity, not for the light-cone variable? C) Perhaps it is due to the use of the axial gauge? D) Finally, maybe it cancels when combined with the NNLO hard process?

Summary and outlook

We have reviewed a methodology of including NLO corrections along the parton shower MC ladder. It has been tested numerically. The method is based on matching between PS MC and CFP diagrammatic NLO distributions. We have spotted a problem of not reproducing NLO evolution kernels in the calculations with k_T-ordering kinematics. It is essentially the problem of the CFP scheme itself and requires better understanding.

References

[1] K. Golec-Biernat, S. Jadach, W. Placzek, M. Skrzypek, Markovian Monte Carlo solutions of the NLO QCD evolution equations, Acta Phys. Polon. B37 (2006) 1785–1832. arXiv:hep-ph/0603031.

[2] W. Placzek, K. Golec-Biernat, S. Jadach, M. Skrzypek, Solving the QCD NLO evolution equations with a Markovian Monte Carlo, Acta Phys. Polon. B38 (2007) 2357–2368. arXiv:arXiv:0704.3344.

[3] K. Golec-Biernat, S. Jadach, W. Placzek, P. Stephens, M. Skrzypek, Markovian Monte Carlo solutions of the one-loop CCFM equations, Acta Phys. Polon. B38 (2007) 3149–3168. arXiv:hep-ph/0703317.

[4] K. Golec-Biernat, S. Jadach, W. Placzek, M. Skrzypek, Solving QCD evolution equations in rapidity space with Markovian Monte Carlo, Acta Phys. Polon. B39 (2008) 115–146. arXiv:arXiv:0708.1906[hep-ph].

[5] S. Jadach, W. Placzek, M. Skrzypek, P. Stoklosa, Markovian Monte Carlo program EvolFMC v.2 for solving QCD evolution equations, Comput. Phys. Commun. 181 (2010) 393–412. arXiv:0812.3299, doi:10.1016/j.cpc.2009.09.023.

[6] M. Slawinska, S. Jadach, K. Kutak, A new Monte Carlo study of evolution equation with coherence, Phys. Lett. B722 (2013) 151–156. arXiv:1302.0293, doi:10.1016/j.physletb.2013.04.022.

[7] S. Jadach, M. Skrzypek, Solving constrained Markovian evolution in QCD with the help of the non-Markovian Monte Carlo, Comput. Phys. Commun. 175 (2006) 511–527. arXiv:hep-ph/0504263.

[8] S. Jadach, W. Placzek, M. Skrzypek, P. Stephens, Z. Was, Constrained MC for QCD evolution with rapidity ordering and minimum kT*, Comput.Phys.Commun. 180 (2009) 675–698. arXiv:hep-ph/0703281, doi:10.1016/j.cpc.2008.11.014.

[9] S. Jadach, M. Skrzypek, P. Stephens, Z. Was, W. Placzek, Towards new Monte Carlo (QCD + EW) for W / Z production at LHC, Acta Phys. Polon. B38 (2007) 2305–2318.

[10] S. Jadach, M. Jezabek, A. Kusina, W. Placzek, M. Skrzypek, NLO corrections to hard process in QCD shower – proof of concept, Acta Phys.Polon. B43 (2012) 2067. arXiv:1209.4291.

[11] S. Frixione, B. R. Webber, The MC@NLO 3.2 event generatorarXiv:hep-ph/0601192.

[12] S. Frixione, P. Nason, C. Oleari, Matching NLO QCD computations with Parton Shower simulations: the POWHEG method, JHEP 0711 (2007) 070. arXiv:0709.2092, doi:10.1088/1126-6708/2007/11/070.

[13] S. Jadach, M. Skrzypek, QCD evolution in the fully unintegrated form, Acta Phys. Polon. B40 (2009) 2071–2096. arXiv:0905.1399.

[14] S. Jadach, M. Skrzypek, A. Kusina, M. Slawinska, Exclusive Monte Carlo modelling of NLO DGLAP evolution, PoS RADCOR2009 (2010) 069. arXiv:1002.0010.

[15] G. Curci, W. Furmanski, R. Petronzio, Evolution of parton densities beyond leading order: the non-singlet case, Nucl. Phys. B175 (1980) 27.

[16] S. Jadach, A. Kusina, M. Skrzypek, M. Slawinska, Two real parton contributions to non-singlet kernels for exclusive QCD DGLAP evolution, JHEP 08 (2011) 012. arXiv:1102.5083, doi:10.1007/JHEP08(2011)012.

[17] O. Gituliar, Higher-Order Corrections in QCD Evolution Equations and Tools for Their Calculation, Ph.D. thesis, Institute of Nuclear Physics, Cracow (2014), arXiv:1403.6897.

[18] S. Jadach, A. Kusina, M. Skrzypek, M. Slawinska, Monte Carlo modelling of NLO DGLAP QCD evolution in the fully unintegrated form, Nucl. Phys. Proc. Suppl. 205-206 (2010) 295–300. arXiv:1007.2437, doi:10.1016/j.nuclphysbps.2010.09.009.

[19] S. Jadach, A. Kusina, W. Placzek, M. Skrzypek, NLO corrections in the initial-state parton shower Monte Carlo, Acta Phys.Polon. B44 (11) (2013) 2179–2187. arXiv:1310.6090, doi:10.5506/APhysPolB.44.2179.

[20] O. Gituliar, S. Jadach, A. Kusina, M. Skrzypek, Virtual Corrections to the NLO Splitting Functions for Monte Carlo: the non-singlet case, Acta Phys.Polon. B44 (11) (2013) 2197–2206. arXiv:1310.7537, doi:10.5506/APhysPolB.44.2197.

[21] M. Skrzypek, S. Jadach, A. Kusina, W. Placzek, M. Slawinska, et al., Fully NLO Parton Shower in QCD, Acta Phys.Polon. B42 (2011) 2433–2443. arXiv:1111.5368, doi:10.5506/APhysPolB.42.2433.

[22] S. Jadach, A. Kusina, W. Placzek, M. Skrzypek, M. Slawinska, Inclusion of the QCD next-to-leading order corrections in the quark-gluon Monte Carlo shower, Phys.Rev. D87 (2013) 034029. arXiv:1103.5015, doi:10.1103/PhysRevD.87.034029.

[23] T. Gleisberg, et al., Event generation with SHERPA 1.1, JHEP 02 (2009) 007. arXiv:0811.4622, doi:10.1088/1126-6708/2009/02/007.

[24] J. M. Campbell, R. K. Ellis, MCFM for the Tevatron and the LHC, Nucl. Phys. Proc. Suppl. 205-206 (2010) 10–15. arXiv:1007.3492, doi:10.1016/j.nuclphysbps.2010.08.011.

[25] S. Jadach, W. Placzek, S. Sapeta, A. Sidmok, M. Skrzypek, Matching NLO QCD with parton shower in Monte Carlo scheme the KrkNLO method, JHEP 10 (2015) 052. arXiv:1503.06849, doi:10.1007/JHEP10(2015)052.

[26] S. Jadach, W. Placzek, S. Sapeta, A. Sidmok, M. Skrzypek, NLO Corrections to Hard Process in Parton Shower MC - KrkNLO Method, Acta Phys. Polon. B46 (11) (2015) 2089. doi:10.5506/APhysPolB.46.2089.

[27] S. Jadach, W. Placzek, S. Sapeta, A. Siodmok, M. Skrzypek, Parton distribution functions in Monte Carlo factorisation scheme, Eur. Phys. J. C76 (12) (2016) 649. arXiv:1606.00355, doi:10.1140/epjc/s10052-016-4508-8.

[28] S. Jadach, W. Placzek, S. Sapeta, A. K. Siodmok, M. Skrzypek, New simpler methods of matching NLO corrections with parton shower Monte Carlo, PoS LL2016 (2016) 020. arXiv:1607.00919.

[29] J. Bellm, et al., Herwig 7.0/Herwig++ 3.0 release note, Eur. Phys. J. C76 (4) (2016) 196. arXiv:1512.01178, doi:10.1140/epjc/s10052-016-4018-8.

[30] M. Grazzini, HNNLO: A MC program for Higgs boson production at hadron colliders, in: Proceedings, 43rd Rencontres de Moriond on QCD and high energy interactions: La Thuile, Italy, March 8-15, 2008, 2008. arXiv:0806.3336.
URL http://inspirehep.net/record/788789/files/arXiv:0806.3336.pdf

[31] S. Jadach, G. Nail, W. Placzek, S. Sapeta, A. Siodmok, M. Skrzypek, Monte Carlo simulations of Higgs-boson production at the LHC with the KrkNLO methodarXiv:1607.06799.

[32] S. Jadach, A. Kusina, W. Placzek, M. Skrzypek, On the dependence of QCD splitting functions on the choice of the evolution variable, JHEP 08 (2016) 092. arXiv:1606.01238, doi:10.1007/JHEP08(2016)092.

[33] O. Gituliar, S. Jadach, A. Kusina, M. Skrzypek, On regularizing the infrared singularities in QCD NLO splitting functions with the new Principal Value prescription, Phys.Lett. B732 (2014) 218–222. arXiv:1401.5087, doi:10.1016/j.physletb.2014.03.045.